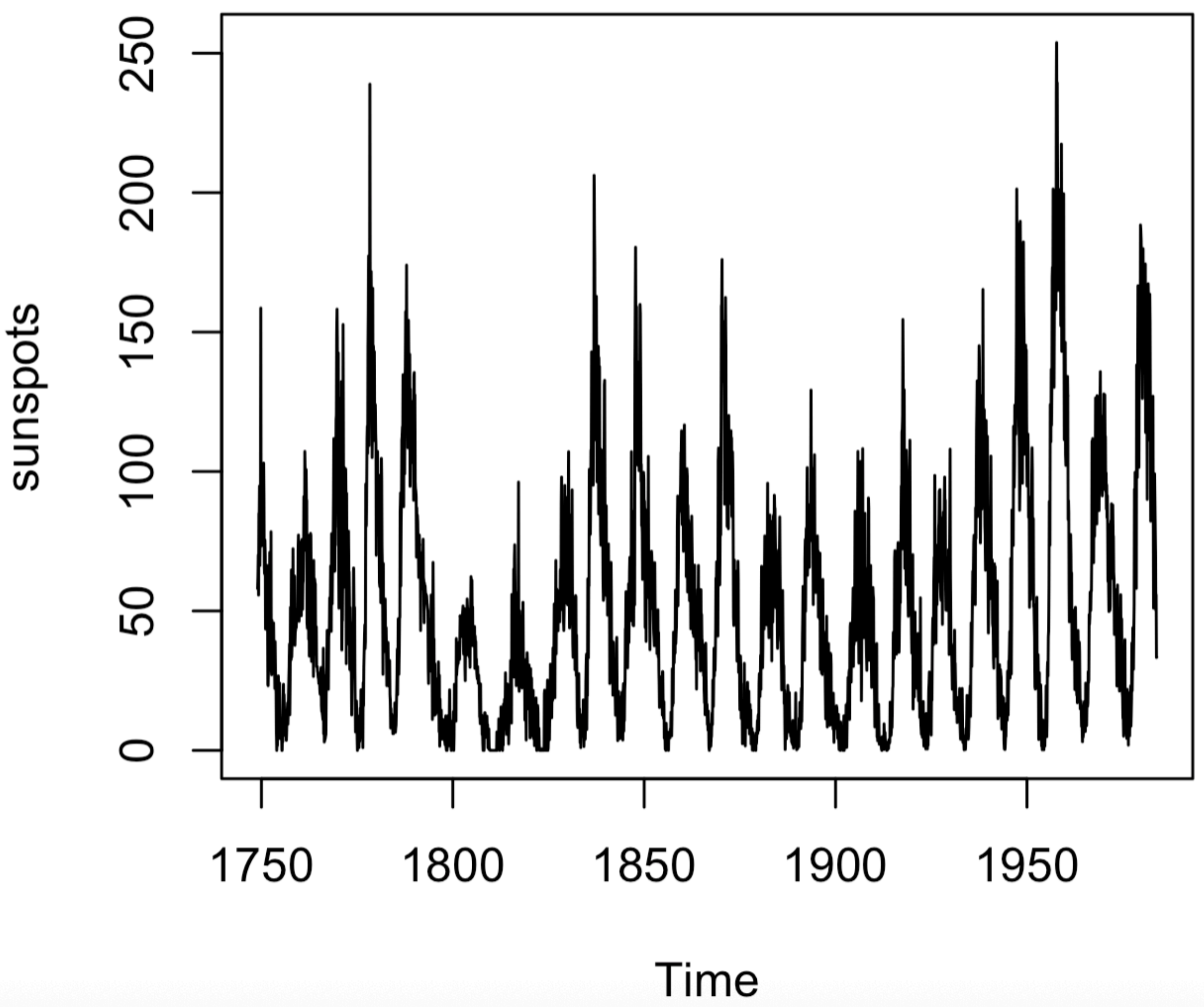
sunspots

We now build a model for the monthly time series of sunspot numbers.
The model we aim to construct consists of an integrated random walk, a monthly seasonality with stochastic dummies, a stochastic cycle, and white noise.

```
data("sunspots")
plot(sunspots)
```



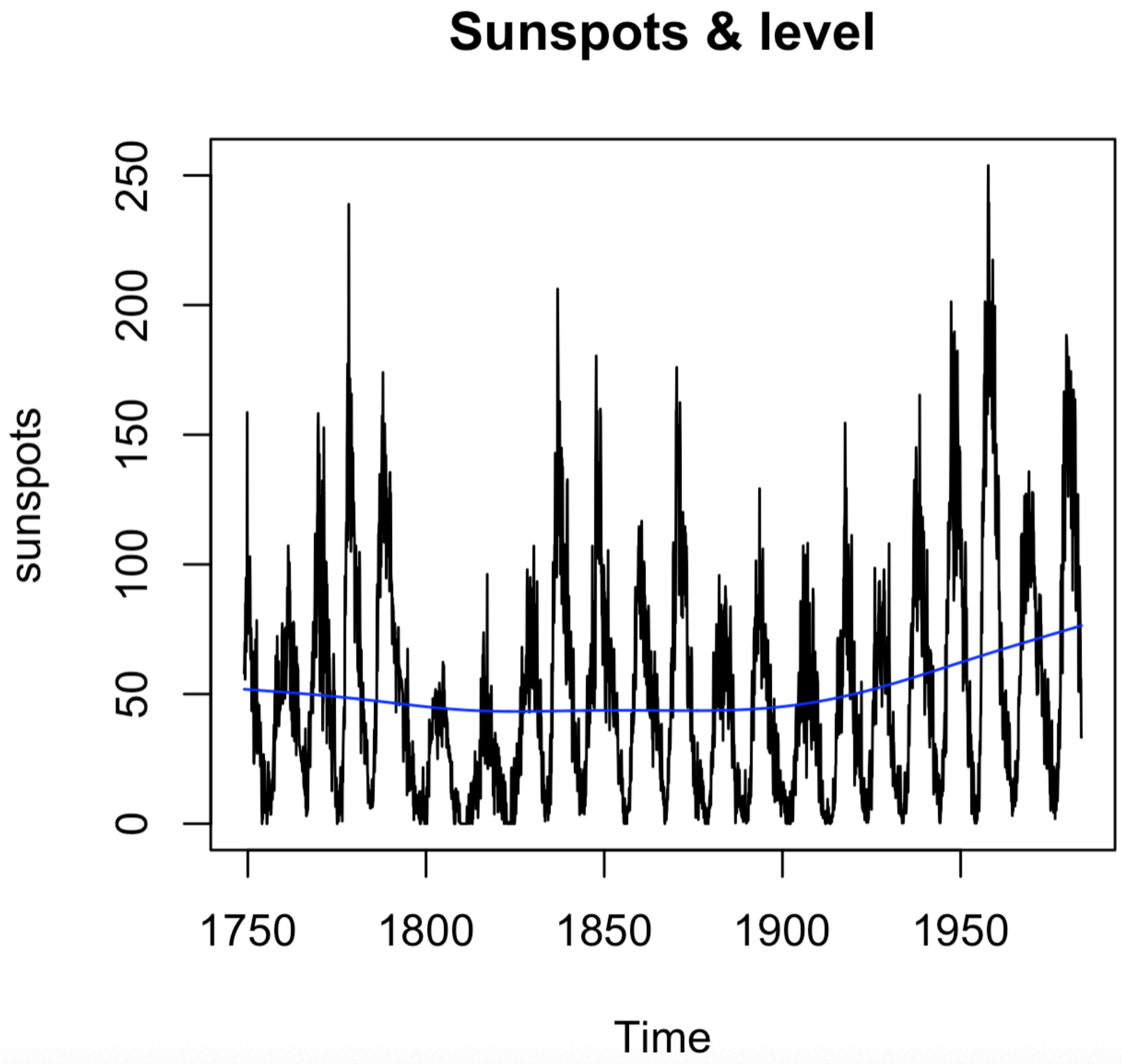

```
modello3 <- SSModel(sunspots~0+SSMtrend(2, list(0, NA))+
SSMcycle(11, NA)+
SSMseasonal(12, NA, "dummy"),
H = NA)

updtfn <- function(pars, model) {
model$Q[2, 2, 1] <- exp(pars[1])
model$Q[3, 3, 1] <- exp(pars[2])
model$Q[4, 4, 1] <- model$Q[5, 5, 1] <- exp(pars[3])
model$H[1, 1, 1] <- exp(pars[4])
rho <- 1/(1+exp(-pars[5]))
```

```
cperiod <- 24 + 156/(1+exp(-pars[6]))
lambda <- 2*pi/cperiod
model$T[14, 14, 1] <- model$T[15, 15, 1] <- rho*cos(lambda)
model$T[15, 14, 1] <- -rho*sin(lambda)
model$T[14, 15, 1] <- rho*sin(lambda)
model
}

fit3 <- fitSSM(modello3, c(-1, 2, 5, 5, 2, 0.81), updtfn)


cat("period =", 24 + 156/(1+exp(-fit3$optim.out$par[6])))

## period = 144.0089


cat("rho =", 1/(1+exp(-fit3$optim.out$par[5])))
## rho = 0.9999996
```

We note that logit(ρ) is a period transformation that ensures the parameter lies between 2 and 15 years.

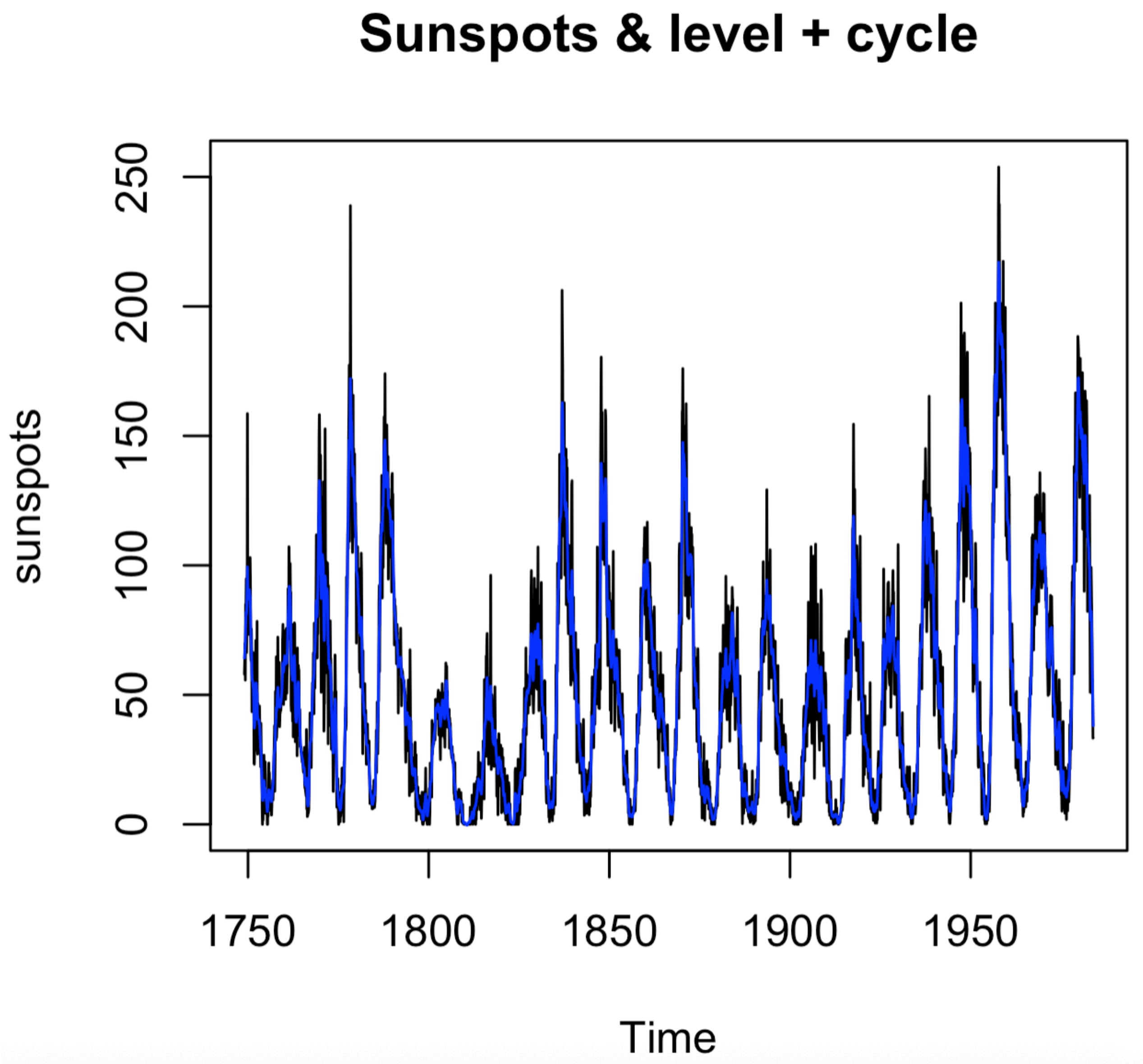
The cycle is highly persistent and has an average periodicity of about 144/12 = 12 years. Let us examine the components.

```
smo3 <- KFS(fit3$model, smoothing = "state")
plot(sunspots, main = "Sunspots & level")
lines(smo3$alphahat[, 1], col = "blue")
```



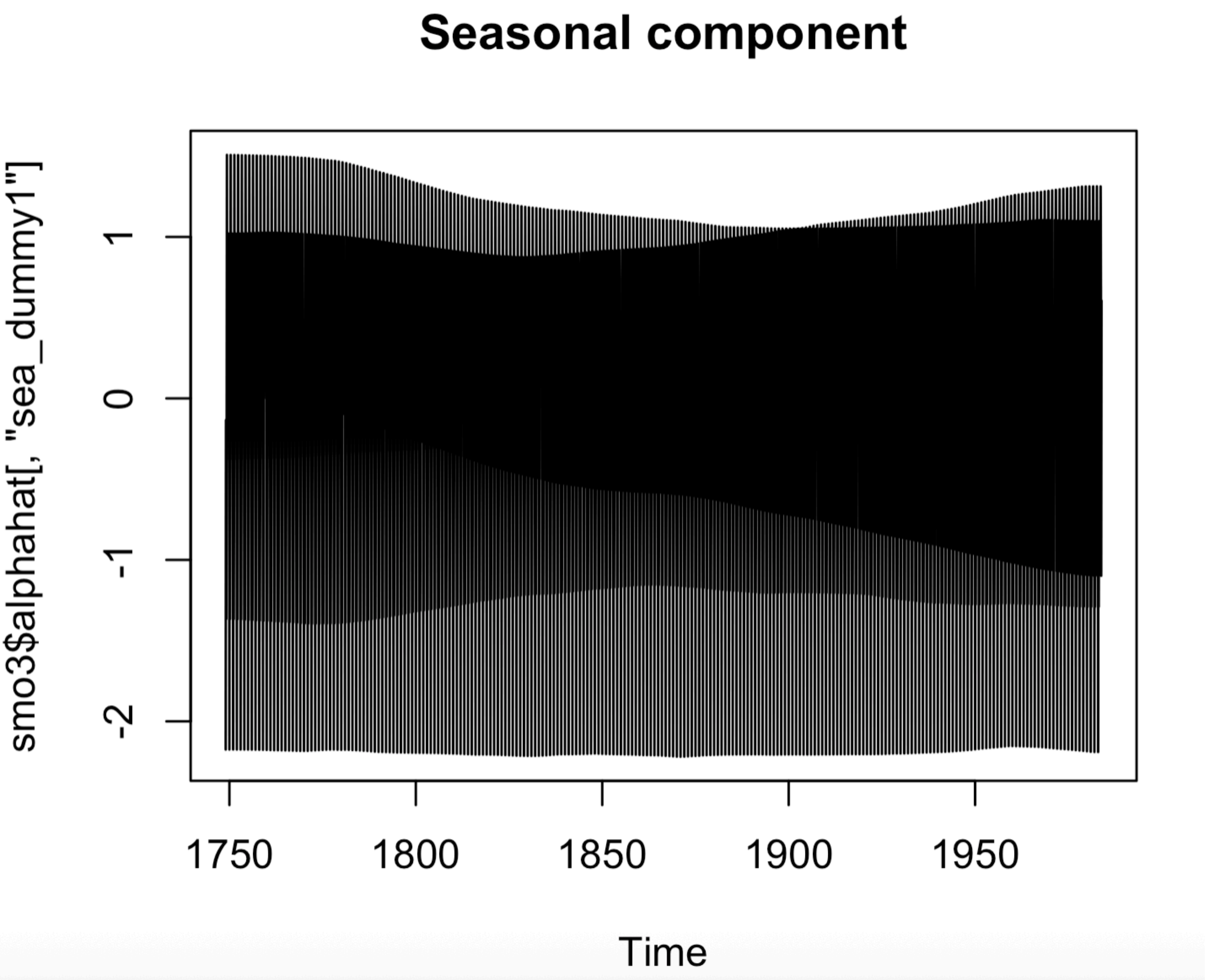
Adding the cycle as well:

```
plot(sunspots, main = "Sunspots & level + cycle")
lines(smo3$alphahat[, "level"] + smo3$alphahat[, "cycle"], col = "blue")
```



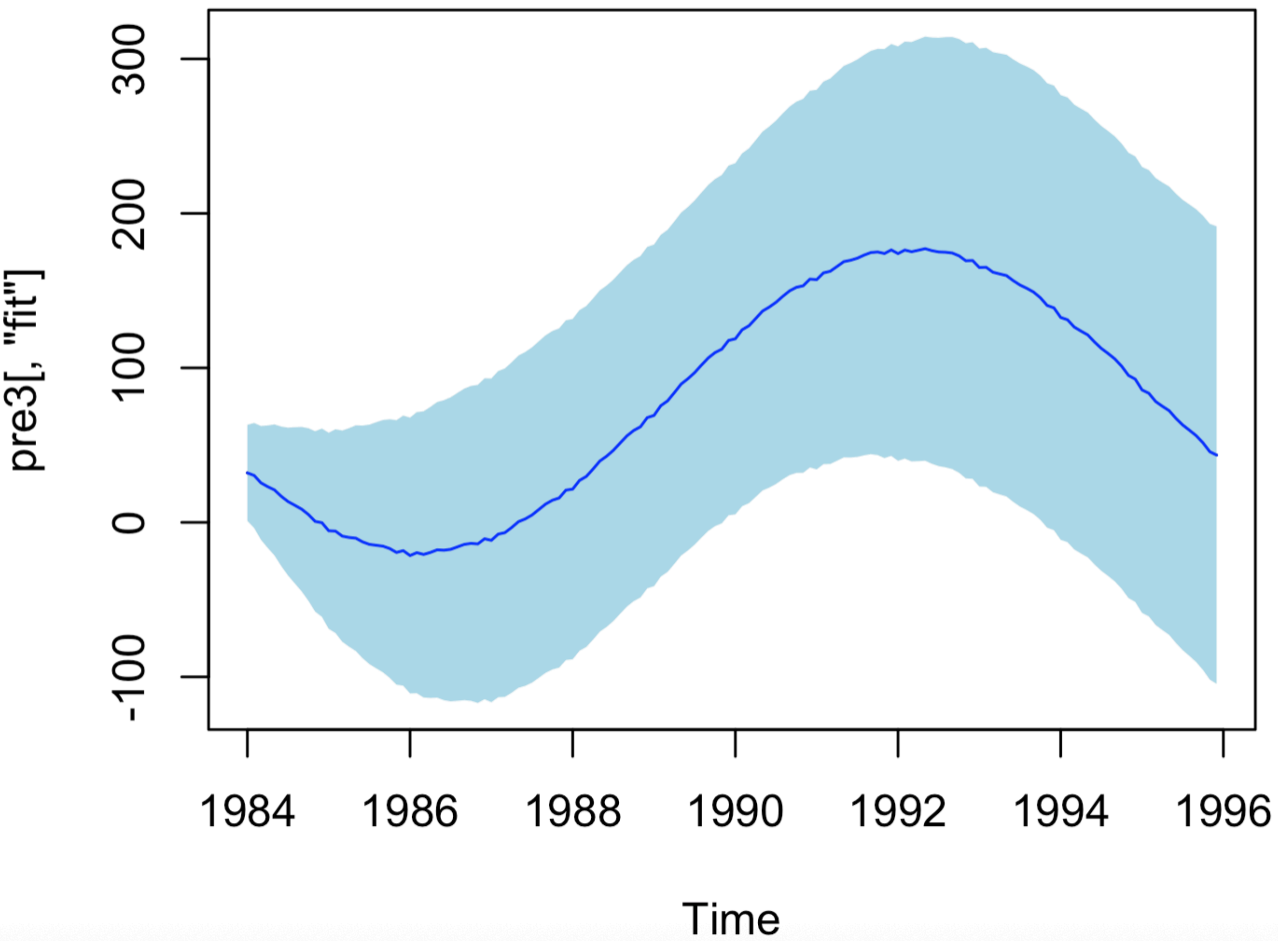
while the seasonal component:

```
plot(smo3$alphahat[, "sea_dummy1"], main = "Seasonal component")
```



To make forecasts, we can use the `predict()` function.

```
pre3 <- predict(fit3$model, n.ahead = 144, interval = "prediction")
plot(pre3[, "fit"], ylim = c(min(pre3), max(pre3)))
polygon(c(time(pre3), rev(time(pre3))),
        c(pre3[, "lwr"], rev(pre3[, "upr"])),
col = "lightblue", border = FALSE)
lines(pre3[, "fit"], col = "blue")
```



The result is an interval of values within which, with a given confidence level, we expect the future series to lie, taking into account the uncertainty of both the latent components and the observation noise.

Thanks to the state space formulation and the Kalman filter, the forecast is strictly optimal in the mean squared error sense, and dynamically reflects the complex structure of the model (trend, seasonality, and cycle). This allows anticipating cyclical and seasonal changes, providing valuable tools for long-term analysis and planning.