Homework 2

1) (a) $(a+b)^*(a+b+bb+ba+ab)$

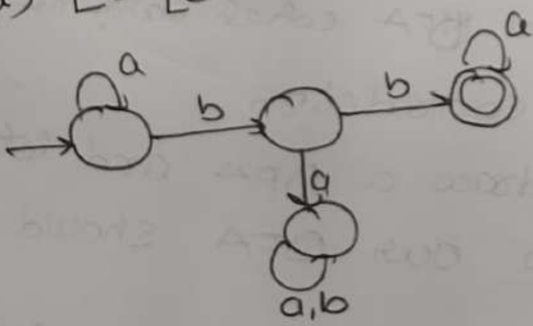
(b) $a^* + (a^*b)(a+\varepsilon+ba^*b)^*(ba^*)$

	0	1	2
r_{11}	$a+\varepsilon$	a^*	
r_{12}	b	a^*b	
r_{21}	b	ba^*	
r_{22}	$a+\varepsilon$	$a+\varepsilon+ba^*b$	

(c) a^*b^*

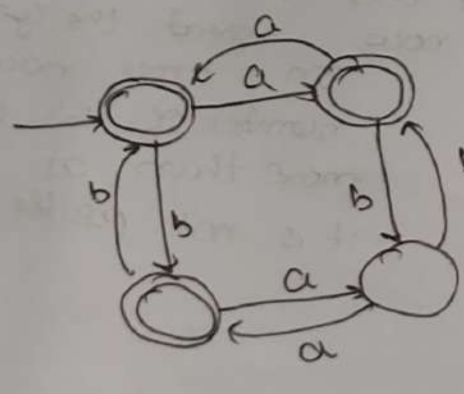
(d)

2) (a) $L = \{a^*bba^*\}$



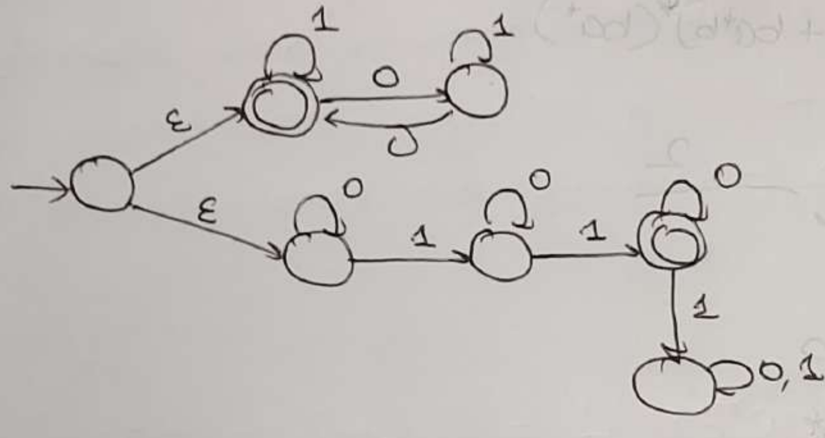
We need to distinguish 0 b's with 1 b and 2 b's hence we need 3 states for it and we need one more trap state.

(b)

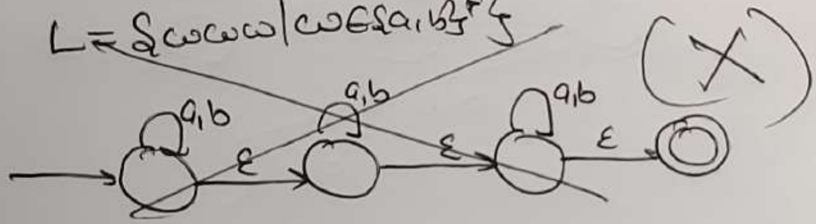


We need to 4 state to store parity of a's, b's.

4 (a) L = { w ∈ {0,1}* | w contains an even number of 0s, or exactly 2 1s }



(c) ~~L = { www | w ∈ {a,b}* }~~



(d) L = { w ∈ {a,b}* | w has more a's than b's }

It's not possible to construct DFA which satisfy this

Proof: Let us prove it by contradiction.

Assume it's possible to draw a DFA and let it has n-states. now our DFA should accept $a^{3n} \cdot b^{2n}$

$\{\underbrace{a \ldots a}_{3n}\}\{\underbrace{b \ldots b}_{2n\ \text{tim}}\}$ → now this must contain a cycle now repeat the cycle 5n times now this number of b's will be more than a's hence it is not possible.

(c) Assume it is regular, then there is a DFA with n states satisfying this condition.
now consider word xx where $|x| = 2n$, now x must contain a cycle let it be huv, now $(huuv)(x)$ must also be accepted but $huuv \neq x$ hence it is also accepting other strings hence it is not possible to have a DFA.

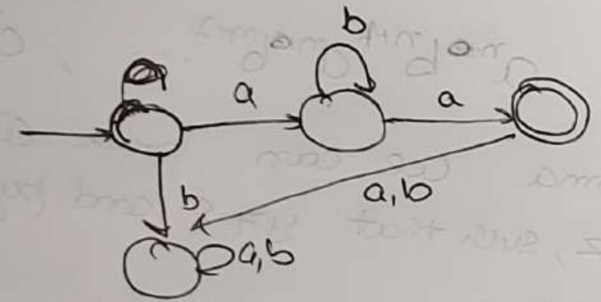
(e) Let us assume there is a DFA with n states satisfying it.
→ our DFA satisfies $a^{n}b^{n+1}, a^{n}b^{n+2} \dots, a^{n+1}b^{2n}$
now by pumping lemma we can break $a^{n+1}b^{n+2}$ in to three strings xyz, such that $y \neq \varepsilon$ and $|xy| \leq n$
and xy^kz is also in L.
now ~~repeating~~ k increasing k increases no of a's
hence xy^2z contains no's of a's between $[n, 2n]$ and hence it should also be accepted but it's a contradiction.

(f) $L = \{a^i b^j a^k \mid k \neq i+j\}$
→ Consider a DFA exists.
→ let $N = n$, where we can apply pumping lemma from
Consider $i = n$, $j = n$, then $k \neq n + jn \Rightarrow k \neq 2n$
So every word $a^n b^n a^k$ where $k \neq 2n$ must be accepted.
→ now by pumping lemma $xy^k b^n a^k$ must be accepted where $xy = a^n$ and $y \neq \varepsilon$. So by this we are accepting (m, n, r) where $m > n$, but we know $m + n = r$ is possible, hence ~~we cannot accept~~ : contradiction

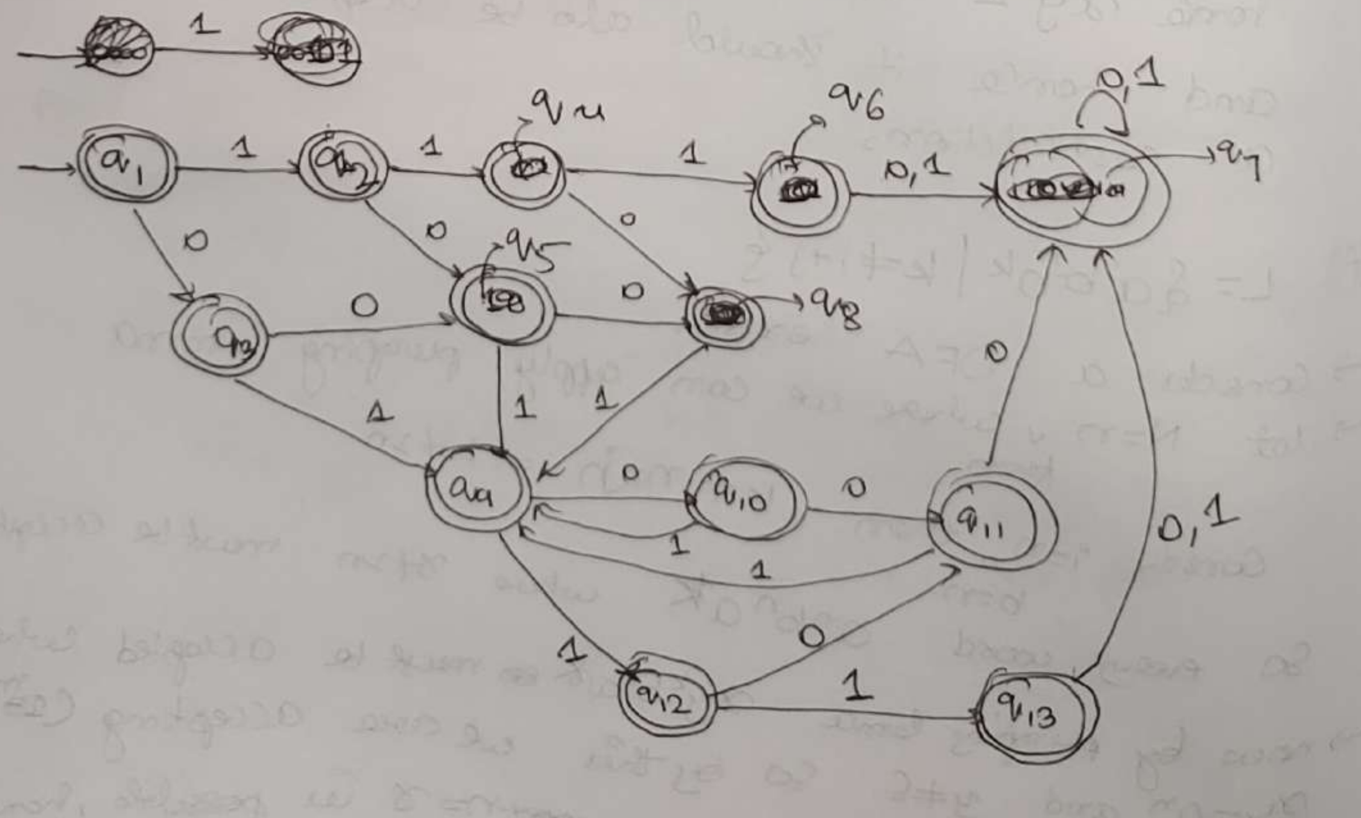
(a) $L = \{a^n \mid n \text{ is not a perfect square}\}$

→ Assume for sake of contradiction that L is regular. By the pumping lemma there exists a positive constant n such that any string in L of atleast n can be pumped. Consider the string $w = a$

(b) $L(ab^*a^*) \cap L(a^*b^*a) = L(ab^*a)$



1(c)



3.(a) Let L be a regular language ~~with~~ for some DFA $A = (Q, \Sigma, \delta, q_0, F)$. Then $\bar{L} = L(B)$, where B is the DFA $(Q, \Sigma, \delta, q_0, Q-F)$. That is B is exactly like A, but the accepting states of A have become non accepting states of B and vice versa.

Then w is in $L(B)$ if and only if $\hat{\delta}(q_0, w)$ is in $Q-F$, which occurs if and only if w is not in $L(A)$.