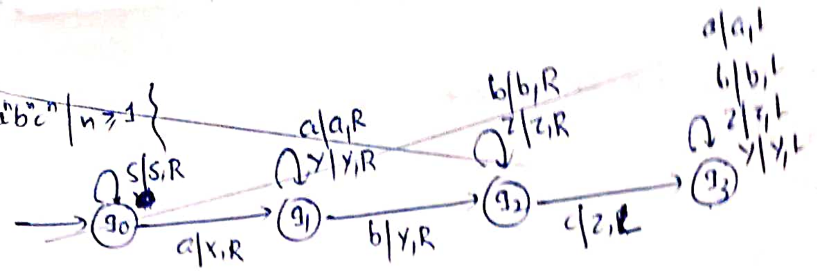
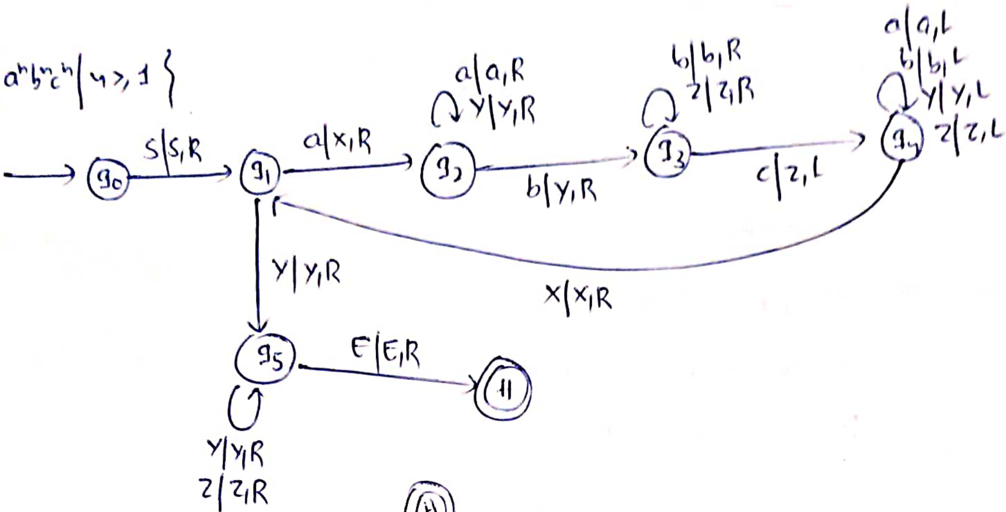
1

a) $\{a^n b^n c^n \mid n \geq 1\}$

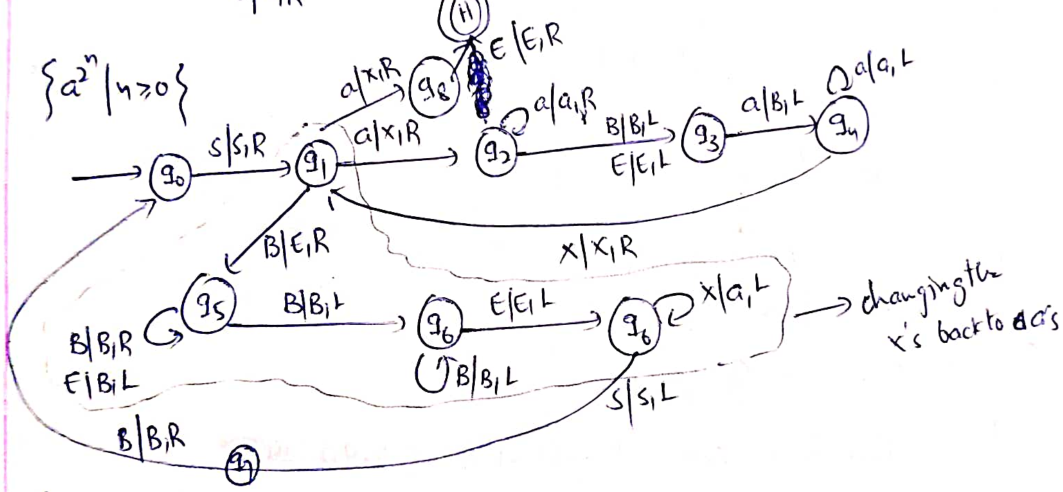


1

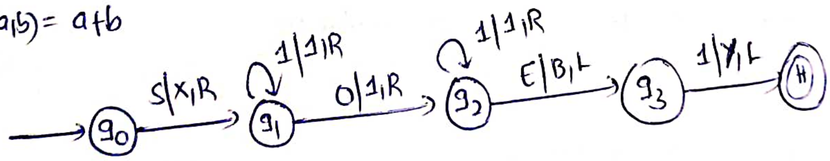
a) $\{a^n b^n c^n \mid n \geq 1\}$



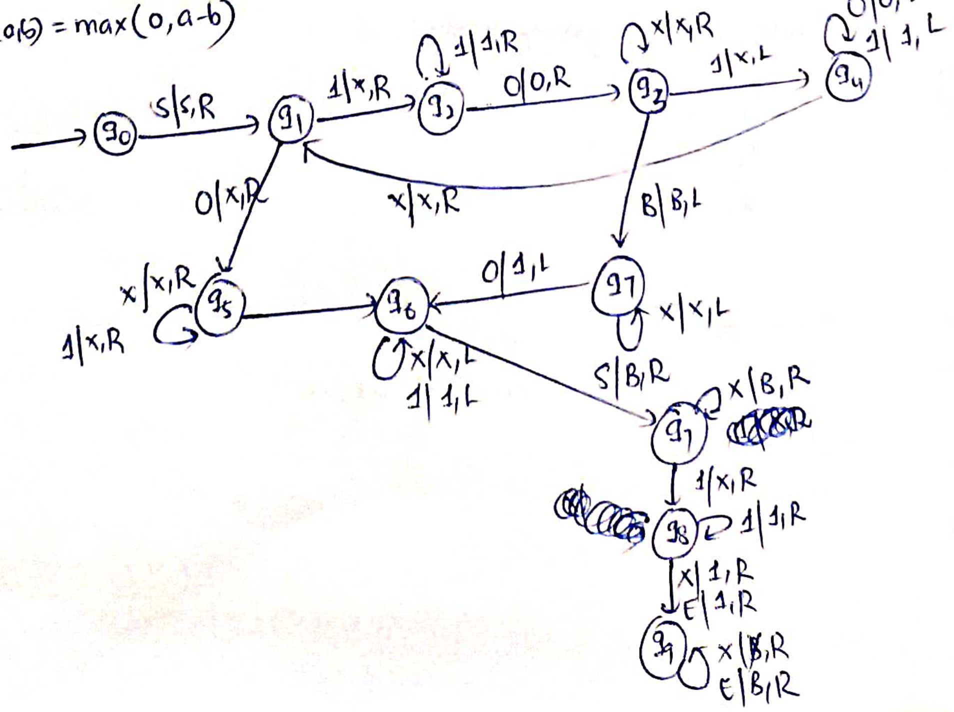
b) $\{a^{2^n} \mid n \geq 0\}$



c) $f(a,b) = a+b$



d) $f(a,b) = \max(0, a-b)$



3

a) All finite subsets of N are countable.

$\{1\}, \{2\}, \{3\}, \ldots$

$\{1,2\}, \{1,3\}, \ldots$

$\{1,2,3\}, \ldots$

let $P_1, P_2, \ldots P_k$ be primes such that $P_1 < P_2 < \ldots P_k$.

let $g(\{a_1, a_2, \ldots a_k\}) = P_1^{a_1} P_2^{a_2} \ldots P_k^{a_k}$

Injective function.

$\Rightarrow$ Countable.

b) All finite $f: N \rightarrow \{0,1\}$

Each function 'f' can be thought of as $\{a_1, a_2, \ldots a_N\}$ where $a_i \in \{0,1\}$

proof by diagonilisation:

f_1 0 1 0 1 0 0 - -

f_2 1 1 0 - -

⋮

Let $f = (1 0 \ldots)$

complement of f_i at i^{th} index.

$\Rightarrow$ Not countable

c) All functions $f: \{0,1\} \rightarrow N$

Each function can be thought of as (a_1, a_2) $a_i \in N$.

We have seen that 2-tuple is countable.

$\Rightarrow$ Countable.

d) All regular languages over $\{0,1\}$

Similar to 'b' proof by diagonlisation.

R_1 0 1 0 0 0 1 - -

R_2 1 0 0 1 - -

R_3

⋮

create $R = (1 1 \ldots)$

complement of R_i at i^{th} index.

$\Rightarrow$ Not countable.