Advantages of Digital System:-

→ Ease of programmability

→ Reduction in cost of hardware

→ High Speed

→ High Reliability

→ Design is easy

→ Result can be reproduced easily.

* Binary Number / Binary Number System

Binary:- Binary describes a numbering scheme in which there are only two possible values for each digit: 0 and 1. The term also refers to any digital encoding / decoding system in which there are exactly two possible states. In digital data memory, storage, processing and communications, the 0 and 1 values are sometimes called "low" and "high", respectively.

Number System:- The number system is used for representing the information. A numeral system is a writing system for expressing numbers, that is, a mathematical notation for representing numbers of a given set, using digits or other symbols in a consistent manner.

Ex:- The counting numbers (1, 2, 3 ..) together with the operations of addition, subtraction, multiplication, and division

(or)

A number system is defined as the representation of numbers by using digits or other symbols in a consistent manner. The value of any digit in a number can be determined by a digit, its position in the number, and the base of the number system. The numbers are represented in a unique manner and allow us to operate arithmetic operations like addition, subtraction, and division.

General format of a number:-

$$N_r = a_{n-1} r^{n-1} + a_{n-2} r^{n-2} + \dots + a_1 r^1 + a_0 r^0 + a_{-1} r^{-1} + \dots a_{-m} r^{-n}$$

a_{n-1}	a_{n-2}	$\dots a_1$	a_0	.	a_{-1}	a_{-2}	a_{-3}	$\dots a_{-m}$
↑ MSD	Integer part			↑ Radix point	Fraction part			↑ LSD

$N_r \rightarrow$ Number with base r.

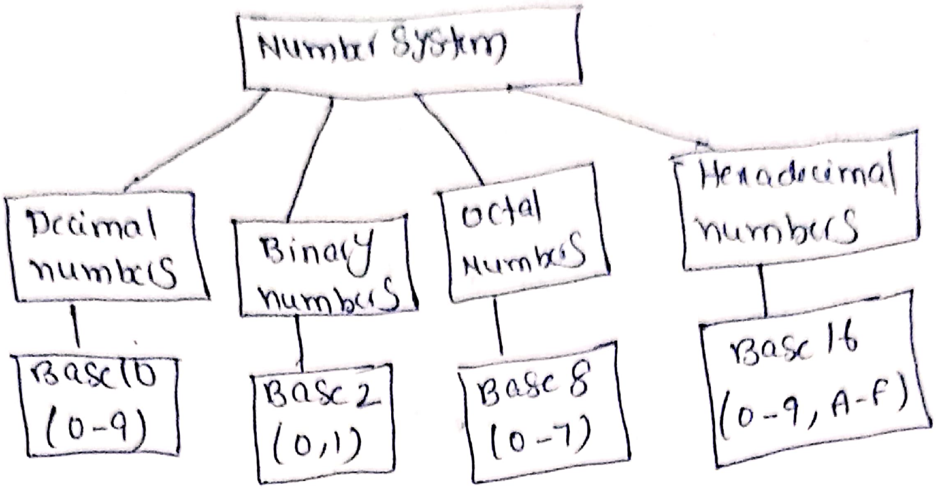
$r \rightarrow$ radix or base

$a \rightarrow$ integer in the range $0 \leq a_i \leq (r-1)$

Types of Number Systems:

There are different types of number systems in which the four main types are;

- → Binary number system (Base-2)
- → Octal number system (Base-8)
- → Decimal number system (Base-10)
- → Hexadecimal number system (Base-16)



1) Binary number System:-

The binary number system is a numbering system that represents numeric values using two unique digits (0 and 1). Most of the computing devices use binary numbering to represent electronic circuit voltage state (i.e; on/off switch) which considers 0 voltage input as off and 1 input as on.

→ computers cannot operate on decimal number, they do their operation on binary.

→ So, it is important to study the binary number system.

→ For example, we enter decimal number for booking the bus ticket; but system will convert to binary.

· The coefficients of binary number system have only two values "0 & 1".

bit → (0 or 1) → smallest unit of memory or instruction that can be given or stored on a computer

4 bits → (Nibble) → A Gr

8 bits → (Byte) → A group of 8 bits.

16 bits → (word)

32 bits → (Double word)

2^{10} ← 1 kilo byte → 1024 bytes

2^{20} ← 1 mega byte → 1024 kilobytes

1 gigabyte = 1024 mega byte (2^{30})

1 tera byte = 1024 gigabyte (2^{40})

We have to consider Base (or) Radix

Base or Radix:- The value representing particular number system. (or)

It defines the no. of digits or symbols or letters used in a number system.

→ How Base or Radix is defined? [How many numbers we have to use?]

To find how many digits or numbers used in a number system we have the formula;

→ If 'r' is the base then 0 to r-1 are the digits present in that number system.

→ So, For Binary the base value is '2'.

i.e; The base or radix for Binary number system is 2.

→ Then the number of digits in binary number system are;

For binary, r = 2

0 to r-1 digits

0 to (2-1)

0 to 1 (∵ i.e; 0 and 1)

→ So, Binary digits (0 and 1) are called as "Bits".

The binary number system uses positional notation. But in this case, each digit is multiplied by the appropriate power of two based on its position.

1) Example: $(101101)_2$ in decimal is

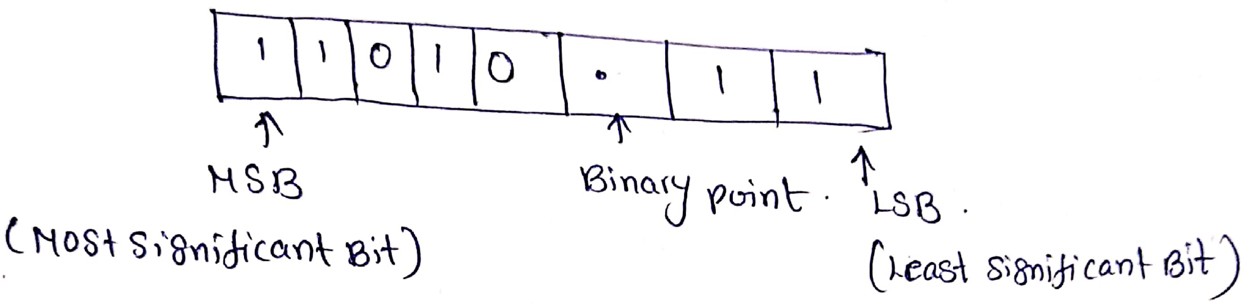
$= 1\times2^5 + 0\times2^4 + 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$

$= 1\times32 + 0\times16 + 1\times8 + 1\times4 + 0\times2 + 1\times1$

$= 32 + 8 + 4 + 1$

$= (45)_{10}$.

Ex: - (2) The decimal equivalent of the binary number $(11010.11)_2$ is 26.75



Multiplication of the coefficient by powers of 2

$1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0 + 1\times2^{-1} + 1\times2^{-2} = 26.75$

$16+8+0+2+0+0.5+0.25 = 26.75$

To distinguish between numbers of different bases we enclose the coefficients in parenthesis and write a subscript equal to the base used.

Ex: - Base-5 number $(4021.2)_5$

$= 4\times5^3 + 0\times5^2 + 2\times5^1 + 1\times5^0 + 2\times5^{-1} = (511.4)_{10}$

The coefficient of values for base 5 can be only 0, 1, 2, 3, 4.

Few Examples of binary numbers are as follows:

→ 10

→ 111

→ 10101

→ 11110

(2) Octal number System:

A number System with its base as eight and uses digits from 0 to 7 is called Octal number System. The word octal is used to represent the numbers that have eight as the base. The octal numbers have many applications and importance such as it is used in computers and digital numbering systems.

The octal number system is a base-8 system that has eight digits i.e; 0, 1, 2, 3, 4, 5, 6, 7

Ex:- (1) $(347)_8 = 3\times8^2 + 4\times8^1 + 7\times8^0$

(2) $(127.4)_8 = 1\times8^2 + 2\times8^1 + 7\times8^0 + 4\times8^{-1} = (87.5)_{10}$

Note:- The digits 8 & 9 cannot appear in an octal number.

By adding each digit of an octal number in a power of 8. we can find the decimal equivalent of octal numbers.

The position of the digits in an octal number System is given as:

---	8^3	8^2	8^1	8^0	.	8^{-1}	8^{-2}	8^{-3}	- - -
	MSB				Octal point			LSB	

For octal ;. $r=8$

0 to $r-1$ digits

0 to $(8-1)$

= 0 to 7

i.e; 0, 1, 2, 3, 4, 5, 6, 7 digits are present in octal

(3) Decimal number System:-

Decimal number system has 10 symbols. The symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. So the radix or base of this number system is '10'. The position of each digit in a decimal number indicates with the magnitude of the quantity represented and can be assigned a weight.
Hence it is called positional weight number system.

For decimal number system; the base value is 10;

then $r = 10$.

$r = 10$

0 to $(r-1)$

0 to $(10-1)$

0 to 9.

i.e; 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 digits are present in decimal.

...	10^2	10^1	10^0	.	10^{-1}	10^{-2}	....

↑ Decimal point

In the decimal number system we can express any decimal number in units, tens,

Ex:- (1) 7392, it can be written as

$7\times10^3 + 3\times10^2 + 9\times10^1 + 2\times10^0$.

$= 7000 + 300 + 90 + 2$

(2) (548), it can be written as

$5\times10^2 + 4\times10^1 + 1\times10^0$

(3) $(212\cdot367)_{10} = 2\times10^2 + 1\times10^1 + 2\times10^0 + 3\times10^{-1} + 6\times10^{-2} +$
7×10^{-3}

(4) Hexa decimal number System:-

The "Hexadecimal" or simply "Hex" numbering system uses the Base of 16 system and are a popular choice for representing long binary values because their format is quite compact and much easier to understand compared to the long binary strings of 1's and 0's.

It is a base 16 number system. The first ten digits are borrowed from the decimal system i.e; 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.

→ For Hexadecimal number System; the Base value is 16;

then r=16.

0 to (r-1)

0 to (16-1)

0 to 15

i.e; 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 digits are present in Hexadecimal.

→ In Hexadecimal we are having decoding from 10 to 15.

i.e; 10-A, 11-B, 12-C, 13-D, 14-E, 15-F

→ Why decoding is Required?

If we want to send Hexadecimal data to some one. and if the data is 2 and 15; we will send it as 215.

→ So, the person on other side will understand it as 215 i.e; two hundred and fifteen; for that reason if we decode 15 with F

→ Then we send the data as 2F.

Ex:- (1) $(B65F)_{16}$

$= 11\times16^3 + 6\times16^2 + 5\times16^1 + 15\times16^0 = (46,687)_{10}$

(2) $(1795.AB)_{16}$

$= 16^3\times1 + 16^2\times7 + 16^1\times9 + 16^0\times5 + 16^{-1}\times10 + 16^{-2}\times11$

Number system	Base	First digit	last digit	All digits/characters
Binary	2	0	1	0, 1
octal	8	0	7	0,1,2,3,4,5,6,7
Decimal	10	0	9	0,1,2,3,4,5,6,7,8,9
Hexadecimal	16	0	F	0,1,2,3,4,5,6,7,8,9, A,B,C,D,E,F.

Decimal	Binary	octal	Hexadecimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

* Number Base Conversions:-

The human beings use decimal number system while computer uses binary number system. Therefore it is necessary to convert decimal number into its equivalent binary number while feeding to computer and again convert to decimal while displaying result.

→ However, dealing with a large quantity of binary number of many bits is inconvenient for humans.

→ Therefore, octal and hexadecimal numbers are used as a short hand means of expressing large binary numbers.

→ Digital circuits strictly work in Binary.

Why conversions?

– We have decimal values, but system understand binary.

So, we cannot give binary every time

Number Base Conversions

Binary to other number system

→ Binary to Decimal
→ Binary to octal
→ Binary to Hexadecimal

Decimal to other Number System

→ Decimal to Binary
→ Decimal to octal
→ Decimal to Hexa decimal

Octal to other number system

→ Octal to Binary
→ Octal to Decimal
→ Octal to Hexa decimal

Hexadecimal to other number system

→ Hexa to Binary
→ Hexa to Decimal
→ Hexa to ~~Decimal~~ octal.

I) Binary to other Number Systems:-

There are 3 conversions possible for binary number, i.e; binary to decimal, binary to octal, and binary to hexadecimal.

(a) Binary to decimal conversion:- The process starts from multiplying the bits of binary number with its corresponding positional weights.

Ex:- (1) convert $(1100)_2 \rightarrow X_{10}$

$$\overset{2^3}{1}\overset{2^2}{1}\overset{2^1}{0}\overset{2^0}{0}$$

$$= 1\times2^3 + 1\times2^2 + 0\times2^1 + 0\times2^0$$
$$= 8 + 4 + 0 + 0 = 12$$

The decimal equivalent value for $(1100)_2 = (12)_{10}$.

(2) convert $(1100\cdot01)_2 \rightarrow X_{10}$

$$(\overset{3}{1}\overset{2}{1}\overset{1}{0}\overset{0}{0}\cdot\overset{-1}{0}\overset{-2}{1})$$

$$\Rightarrow 1\times2^3 + 1\times2^2 + 0\times2^1 + 0\times2^0 + 0\times2^{-1} + 1\times2^{-2}$$
$$\Rightarrow 8 + 4 + 0 + 0 + 0 + 0\cdot25$$
$$\Rightarrow (12\cdot25)_{10}$$

The decimal equivalent value for $(1100\cdot01)_2 = (12\cdot25)_{10}$

(3) convert $(10110\cdot0101)_2 \rightarrow X_{10}$

$$\overset{4}{1}\overset{3}{0}\overset{2}{1}\overset{1}{1}\overset{0}{0}\cdot\overset{-1}{0}\overset{-2}{1}\overset{-3}{0}\overset{-4}{1}$$

$$\Rightarrow 1\times2^4 + 0\times2^3 + 1\times2^2 + 1\times2^1 + 0\times2^0 + 0\times2^{-1} + 1\times2^{-2} + 0\times2^{-3} + 1\times2^{-4}$$
$$\Rightarrow 16 + 4 + 2 + (1/4) + (1/16)$$
$$\Rightarrow 22\cdot3125$$

The decimal equivalent value for $(10110\cdot0101)_2 = (22\cdot312$

(4) Ex:- $(10110.001)_2$

We multiplied each bit of $(10110.001)_2$ with its respective positional weight, and last we add the products of all the bits with its weight.

$$(10110.001)_2 = (1\times2^4)+(0\times2^3)+(1\times2^2)+(1\times2^1)+(0\times2^0)+ (0\times2^{-1})+(0\times2^{-2})+(1\times2^{-3})$$

$$= (1\times16)+(0\times8)+(1\times4)+(1\times2)+(0\times1)+ (0\times\tfrac{1}{2})+(0\times\tfrac{1}{4})+(1\times\tfrac{1}{8})$$

$$= 16+0+4+2+0+0+0+0.125$$

$$=(22.125)_{10}$$

The decimal equivalent value for $(10110.001)_2 = (22.125)_{10}$

(b) Binary to octal conversions :- The base numbers of binary and octal are 2 and 8, respectively. In a binary number, the pair of three bits is equal to one octal digit

→ The base for octal number is the third power of the base for binary numbers.

→ Therefore; by grouping 3 digits of binary numbers and then converting each group digit to its octal equivalent we can convert binary number to its octal equivalent.

1) In the first step, we have to make the pairs of three bits on both sides of the binary point. If there will be one or two bits left in a pair of three bits pair, we add the required no. zeros on extreme sides.

2. In the second step, we write the octal digits corresponding to each pair.

Ex:- (1) $(11111010 1011 \cdot 0011)_2$

(i) Firstly, we make pairs of three bits on both sides of the binary point.

111 110 101 011 · 001 1

(ii) On the right side of the binary point, the last pair has only one bit. To make it a complete pair of three bits, we added two zeros on the extreme side.

111 110 101 011 · 001 100

(iii) Then, we wrote the octal digits, which correspond to each pair.

$(111110101011 \cdot 0011)_2 = (7653 \cdot 14)_8$

Ex:-(2) Convert $(10101101 \cdot 0111)_2$ to octal number

Sol:- Step 1:- Make group of 3 bits starting from LSB for integer part and MSB for fractional part, by adding 0's at the end; if required.

Step 2:- Write equivalent octal number for each group of 3 bits.

Step 1	0	1	0	1	0	1	1	0	1	·	0	1	1	1	0	0
Step 2		2			5			5		·		3			4	

Adding 0 to make a group of 3 bits

Adding 0's to make a group of 3-bits.

$(10101101 \cdot 0111)_2 = (255 \cdot 34)_8$

Ex:- (3) convert $(1010.11)_2 \rightarrow X_8$

Sol:- Need 3 binary bits; so group 3 bits left grouping for decimal before and right grouping for after decimal.

1010.11

001010.110 ← padded zero

2 padded zero. → 1, 2, 6 → $(12.6)_8$

$(1010.11)_2 = (12.6)_8$

(c) Binary to Hexadecimal conversion:- The base numbers of binary and hexadecimal are 2 and 16, respectively. In a binary number, the pair of four bits is equal to one hexadecimal digit.

Step 1:- In the first step, we have to make the pairs of four bits on both sides of the binary point. If there will be one, two or three bits left in a pair of four bits pair, we add the required number of zeros on extreme sides.

Step 2:- In the second step, we write the hexadecimal digits corresponding to each pair.

Example 1:- $(10110101011.0011)_2$

(i) Firstly, we make pairs of four bits on both sides of the binary point

0101,1010,1011.0011

(ii) On the left side of the binary point, the first pair has three bits. To make it a complete pair of four bits, add one zero on the extreme side

$$\underbrace{0101}_{5}\ \underbrace{1010}_{A}\ \underbrace{1011}_{B}\ .\ \underbrace{0011}_{3}$$

(iii) Then, we write the hexadecimal digits, which correspond to each pair

$$(10110101011.0011)_2 = (5AB.3)_{16}$$

Example 2:- $(11011011110.1001101)_2$

0	0	1	1	0	1	1	0	1	1	1	0	.	1	0	0	1	1	0	1	0
		3				6				E		.			9				A	

Adding 0's to make group of 4 bits

Adding 0 to make a group of 4 bits

$$(11011011110.1001101)_2 = 36E.9A$$

Ex:- (3) convert $(1101101100)_2 \longrightarrow X_{16}$

padding ← $\underbrace{0011}_{3} \,|\, \underbrace{0110}_{6} \,|\, \underbrace{1100}_{C}$

$$(1101101100)_2 \rightarrow (36C)_{16}$$

4) Convert $(101011.01101)_2 \rightarrow X_{16}$

padding zeros ← $\underbrace{0010}_{2} \,|\, \underbrace{1011}_{B} \,.\, \underbrace{0110}_{6} \,|\, \underbrace{1000}_{8}$ → padding zeros

$$(101011.01101)_2 \rightarrow (2B.68)_{16}$$

II Decimal to Other Number System:-

The decimal number can be an integer or floating point integer. When the decimal number is a floating-point integer, then we convert both part (integer and fractional) of the decimal number in the isolated form (individually). There are the following steps that are used to convert the decimal number in to a similar number of any base 'r'.

→ **Step 1:-** we perform the division operation on integer and successive part with base 'r'. we will list down all the remainders till the quotient is zero. then we find out the remainders in reverse order for getting the integer part of the equivalent number of base 'r'. In this, the least and most significant digits are denoted by the first and the last remainders.

→ **Step 2:-** In the next step, the multiplication operation is done with base 'r' of the fractional and successive fraction. The carries are noted until the result is zero or when the required number of the equivalent digit is obtained. For getting the fractional part of the equivalent number of base 'r', the normal sequence of carrying is considered.

(a) Decimal to Binary conversion:-

1. In the first step, we perform the division operation on the integer and the successive quotient with the base binary (2).

2. Next, we perform the multiplication on the integer and successive quotient with the base of binary (2).

Example (1): $(152.25)_{10}$

Step 1: Divide the number 152 and its successive quotients with base 2.

Operation	Quotient	Remainder
152/2	76	0
76/2	38	0
38/2	19	0
19/2	9	1
9/2	4	1
4/2	2	0
2/2	1	0
1/2	0	1

$(152)_{10} = (10011000)_2$

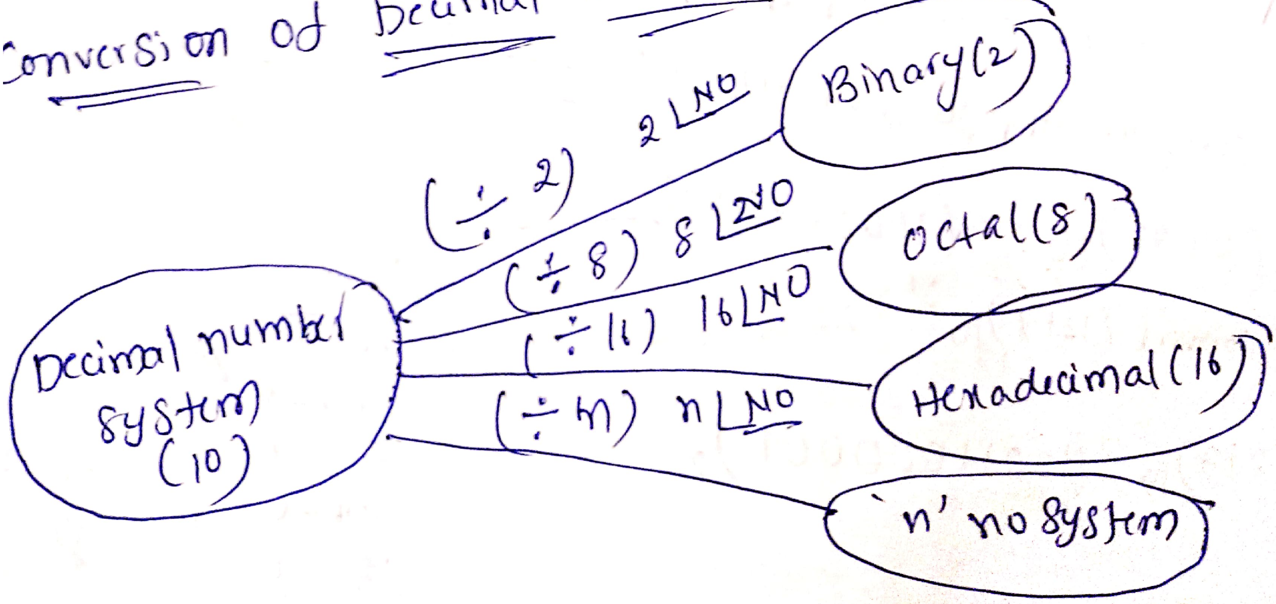
Step 2: Now, perform the multiplication of 0.25 and successive fraction with base 2.

Operation	Result	carry
0.25×2	0.50	0
0.50×2	1.0	1

$(0.25)_{10} = (.01)_2$

$(152.25)_{10} = (10011000.01)_2$

Conversion of Decimal number system to Any Base (or) Radix



Ex (2): Convert $(24)_{10} \to X_2$ (Binary)

For Binary Base is 2

Divide 24 by 2

2	24	Remainder
2	12	0
2	6	0
2	3	0
	1	1

Bottom to top.

Consider the remainder values from bottom to top i.e; 11000 is the binary equivalent for $(24)_{10}$.

$\therefore (24)_{10} = (11000)_2$

Ex (3): Convert $(24.25)_{10} \to X_2$ (Binary)

Same procedure as above for decimal before values.

→ For after decimal digits we have to multiply by '2' i.e; base value.

For 24 we know binary equivalent is 11000

(24.25)

↓ ↳ Multiply by base value

11000

$0.25 \times 2 = 0.5 = 0$

$0.5 \times 2 = 1.0 = 1$

Top to Bottom

Consider top to bottom $(0.25) = 01$

$(24.25)_{10} = (11000.01)_2$

1) Convert $(1217)_{10} = X_2$

2	1217	
2	608	1
2	304	0
2	152	0
2	76	0
2	38	0
2	19	0
2	9	1
2	4	1
2	2	0
2	1	0

Bottom to top

$(1217)_{10} = (10011000001)_2$

(5) convert $(24.625)_{10} = X_2$

$$\begin{array}{r|l} 2 & 24 \\ 2 & 12 - 0 \\ 2 & 6 - 0 \\ 2 & 3 - 0 \\ & 1 - 1 \end{array}$$

Bottom to top

$0.625 \times 2 = 1.25 = 1$
$0.25 \times 2 = 0.5 = 0$
$0.5 \times 2 = 1.0 = 1$

Top to bottom.

$(24.625)_{10} = (11000.101)_2$

(6) convert $(41.6875)_{10} = X_2$

$$\begin{array}{r|l} 2 & 41 \\ 2 & 20 - 1 \\ 2 & 10 - 0 \\ 2 & 5 - 0 \\ 2 & 2 - 1 \\ & 1 - 0 \end{array}$$

Bottom to Top.

$0.6875 \times 2 = 1.375 = 1$
$0.375 \times 2 = 0.75 = 0$
$0.75 \times 2 = 1.50 = 1$
$0.50 \times 2 = 1.0 = 1$

Top to Bottom

$(41.6875)_{10} = (101001.1011)_2$

(b) Conversion of Decimal to octal:-

1) In the step1, we perform the division operation on the integer and the successive quotient with the base of octal (8).

2) Next, we perform the multiplication on the integer and the successive quotient with the base of octal (8).

Example (1):- $(152.25)_{10}$

Step 1:- Divide the number 152 and its successive quotients with base 8.

Operation	Quotient	Remainder
152/8	19	0
19/8	2	3
2/8	0	2

$(152)_{10} = (230)_8$

Step 2: Now perform the multiplication of 0.25 and successive fraction with base 8

Operation	Result	Carry
0.25×8	$2.0 = 0$	2

$(0.25)_{10} = (2)_8$

The octal number of the decimal number 153.25 is 231.2

Ex (2): $(24)_{10} \rightarrow \times 8$

Divide the given decimal number with base value (8).

$$\begin{array}{r|l} 8 & 24 \\ \hline & 3 - 0 \end{array}$$

Bottom to top i.e; 30

So, the octal value is $(30)_8$

$(24)_{10} = (30)_8$

Ex (3): Convert $(153)_{10} \rightarrow \times 8$

$$\begin{array}{r|l} 8 & 153 \\ \hline 8 & 19 - 1 \\ \hline & 2 - 3 \end{array}$$

Bottom to top i.e; $(231)_8$

The octal equivalent value for $(153)_{10} = (231)_8$

Ex (4): Convert $(0.513)_{10} \rightarrow \times 8$

Top to bottom

$0.513 \times 8 = 4.104 = 4$

$0.104 \times 8 = 0.832 = 0$

$0.832 \times 8 = 6.656 = 6$

$0.656 \times 8 = 5.248 = 5$

$0.248 \times 8 = 1.984 = 1$

$0.984 \times 8 = 7.872 = 7$

⋮

The octal equivalent value for $(0.513)_{10} = (.406517\ldots)_8$

Ex (5):- Convert $(204.25)_{10} \rightarrow X_8$

$$\begin{array}{r|l} 8 & 204 \\ \hline 8 & 25 - 4 \\ \hline & 3 - 1 \end{array}$$

i.e; 314

$0.25 \times 8 = 2.00$

The octal equivalent for $(204.25)_{10} = (314.2)_8$

(C) Conversion of Decimal to Hexadecimal :-

Divide the given decimal number with the base value 16.

Ex (1):- Convert $(24)_{10} \rightarrow X_{16}$

$$\begin{array}{r|l} 16 & 24 \\ \hline & 1 - 8 \end{array}$$

Bottom to top.

$(24)_{10} \rightarrow (18)_{16}$ (or) H

Ex (2):- Convert $(4769)_{10} \rightarrow X_{16}$

$$\begin{array}{r|l} 16 & 4769 \\ \hline 16 & 298 - 1 \\ \hline 16 & 18 - 10(A) \\ \hline & 1 - 2 \end{array}$$

Bottom to top.

The Hexadecimal equivalent value for $(4769)_{10} = (12A1)_{16}$

Ex (3):- Convert $(422.675)_{10} \rightarrow X_{16}$

$$\begin{array}{r|l} 16 & 422 \\ \hline 16 & 26 - 6 \\ \hline & 1 - 10(A) \end{array}$$

Bottom to top

$0.675 \times 16 = 10.8$ A

$0.8 \times 16 = 12.8$ C

$0.8 \times 16 = 12.8$ C

top to bottom

The Hexadecimal equivalent value for $(422.675)_{10} = (1A6.ACC)_{16}$

Ex (4):. Convert decimal $(24)_{10} \rightarrow x_4$ (any base)

Sol:- Base = 4

$$\begin{array}{r|l} 4 & 24 \\ \hline 4 & 6 - 0 \\ \hline & 1 - 2 \end{array}$$

$\therefore (24)_{10} \rightarrow (120)_4$

Ex (5):. Convert $(164.25)_{10} \rightarrow x_6$

$$\begin{array}{r|l} 6 & 164 \\ \hline 6 & 27 - 2 \\ \hline & 4 - 3 \end{array}$$

$164 \rightarrow 432$

$0.25 \times 6 = 1.5 = 1$

$0.5 \times 6 = 3.0 = 3$

$(.25) = 13$

The base 6 equivalent value for $(164.25)_{10} = (432.13)_8$

(iv) Octal to Other Number System

The Process of converting octal to decimal differs from the remaining one

(a) Octal to Binary Conversion:-

The process of converting octal to binary is the reverse process of binary to octal. We write the three bits binary code of each octal number digit

Ex 1:- $(152.25)_8 \rightarrow x_2$

We write the three-bit binary digit for 1, 5, 2, and 5.

$(152.25)_8 = (001\ 101\ 010.\ 010\ 101)_2$

So, the binary number of the octal number 152.25 is $(001101010.010101)_2$

The relation b/w octal and Binary number system is

Octal 8, Binary 2.

Binary $(2^3) = 8$

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

To convert octal to Binary, each digit of the octal number is individually converted to its binary equivalent to get octal to binary conversion of the number.

Ex (2):- Convert $(64)_8 \rightarrow X_2$

Sol:- Binary value for 6 is 110

Binary value for 4 is 100

$\therefore (64)_8 \rightarrow (110100)_2$

Ex (3):- Convert $(64.25)_8 \rightarrow X_2$

$(110100.010101)_2$ (for two value is 010
5 value is 101)

$\therefore (64.25)_8 \rightarrow (110100.010101)_2$

[illegible] from multiplying the digits of octal numbers [illegible] corresponding positional weights.

Sol:- $(152.25)_8 \rightarrow X_{10}$

Step 1:- We multiply each digit of (152.25) with its respective positional weight, and last we add the products of all the bits with its weight.

$$(152.25)_8 = 1\times8^2+(5\times8^1)+(2\times8^0)+(2\times8^{-1})+(5\times8^{-2})$$
$$= 64+40+2+(2\times 1/8)+(5\times 1/64)$$
$$= 64+40+2+0.25+0.078125$$
$$= 106.328125$$

$(152.25)_8 \rightarrow (106.328125)_{10}$

radix = 8 → radix 10

Σ digit $\times 8^{pv}$

Sum (digits) multiplied with 8 power place value

Ex (2):- Convert $(12)_8 \rightarrow X_{10}$

$(\overset{1}{1}\overset{0}{2})_8 \Rightarrow 2\times8^0+1\times8^1$
$\Rightarrow 2+8=10$

$(12)_8 \rightarrow (10)_{10}$

Ex (3):- convert $(12.10)_8 \rightarrow X_{10}$

$(12.10) \Rightarrow 1\times8^1+2\times8^0+1\times8^{-1}+0\times8^{-2}$
$\Rightarrow 8+2+0.125+0$
$\Rightarrow 10.125$

$\therefore (12.10)_8 \rightarrow (10.125)_{10}$

Ex (4): Convert $(126.5)_8 \to X_{10}$

$(126.5)_8 \Rightarrow 1\times8^2 + 2\times8^1 + 6\times8^0 + 5\times8^{-1}$

$\Rightarrow 64+16+6+(5/8)$

$\Rightarrow 86.625$

$\therefore (126.5)_8 \Rightarrow (86.625)_{10}$

(C) Octal to Hexadecimal Conversion :-

→ Convert octal number to its binary equivalent

→ Convert binary number to its hexadecimal equivalent

Ex(1):- Convert $(72.1)_8 \to X_{16}$ (or) H

Mediator is binary: $8 \to 2^3 \to$ 3 bits in binary data.

72.1

111 010 001

$(111010.001)_2$

→ Binary to Hexadecimal $(2 = 2^4) \to$ four bits.

padding two zeros: $\underbrace{0011}_{3}\,\underbrace{1010}_{A}.\underbrace{0010}_{2}$ padding zero $\Rightarrow (3A.2)_{16}$ (or) H

$\therefore (72.1)_8 \to (3A.2)_{16}$

Ex(2):- Convert $(147)_8 \to X_{16}$

1 4 7

001 100 111

$\underbrace{0000}_{0}\,\underbrace{0110}_{6}\,\underbrace{0111}_{7} \to (67)_{16}$

$\therefore (147)_8 \to (67)_{16}$

Ex(3):. $(152.25)_8$

We write the three - bit binary digit for 1, 5, 2 and 5

$(152.25)_8 = (001\ 101010.010101)_2$

→ padding 3 zeros   → padding two zeros

$\underline{0000}\ \underline{01\ 10}\ \underline{1010}\ .\ \underline{0101}\ \underline{0100}$

0 6 A . 5 4

$(6A54)_{16}$

∴ $(152.25)_8 \rightarrow (6A.54)_{16}$

IV Hexa decimal to Other Number System

The process of converting hexadecimal to decimal differs from the remaining one

(a) Hexadecimal to Binary conversion:-

The process of converting hexadecimal to binary is the reverse process of binary to hexadecimal. We write the four bits binary code of each hexadecimal number digit.

→ conversion depends on relation of Hexa radix and Binary radix (16, 2)

Ex(1):- Convert $(64.25)_{16} \rightarrow X_2$

6 4 . 2 5 → 0101

0110 0100 0010

$(01100100.00100101)_2$

∴ $(64.25)_{16} \rightarrow (01100100.00100101)_2$

Hexadecimal = 16 = 2^4	Binary = 2 2^3 2^2 2^1 2^0
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
10	1 0 1 0
11	1 0 1 1
12	1 1 0 0
13	1 1 0 1
14	1 1 1 0
15	1 1 1 1

Ex(2):- convert $(BABA)_{16} \rightarrow X_2$

B A B A

1011 1010 1011 1010

$(BABA)_{16} \rightarrow (1011\ 1010\ 1011)_2$

Ex(3):- convert $(16.5)_{16} \rightarrow X_2$

16 . 5

0001 0110 0101

$\therefore (16.5)_{16} \rightarrow (00010110 \cdot 0101)_2$

(b) Hexa decimal to octal conversion:-

Step 1:- We will find the binary equivalent of the hexadecimal number.

Step 2:- We have to make the pairs of three bits on both sides of the binary point. If there will be one or two bits left in a pair of three bits pair, we add the required number of zeros on extreme sides and write the octal digits corresponding to each pair.

Ex(1):- Convert $(BC66.AF)_{16} \rightarrow \times 8$

Sol:- Step 1:- Write equivalent 4-bit binary number for each hexadecimal digit

Step 2:- Make group of 3 bits starting from LSB for integer part and MSB for fractional part by adding 0's at the end; if required.

Step 3:- Write equivalent octal number for each group of 3 bits

B C 6 6 . A F → 1111

→ 1010

1011 1100 0110 0110

1011110001100110.10101111 → Binary Mediator

1011110001100110.101011110 ← padding zeros

two zeros padding: 001 011 110 001 100 110 . 101 011 110

1 3 6 1 4 6 . 5 3 6

$(BC66.AF)_{16} \rightarrow (136146.536)_8$

Ex(2):. convert $(AF.2B)_{16} \rightarrow X_8$

Sol:. convert first hexadecimal to Binary then Binary to octal.

AF . 2B → 1011

1010 1111 0010

Binary Mediator → 10101111.00101011

So, binary to octal, we need to group 3bits

010 101 111 . 001 010 110

2 5 7 1 2 6

$(AF\ 2B)_{16} \rightarrow (257.126)_8$

Ex(3):. Convert $(16.5)_{16} \rightarrow X_8$

16 . 5 → 0101

0001 0110

0001 0110.0101

We can add 0's left and right

padding zero

000 010 110 . 010 100 → padding zero's

0 2 6 2 4

$(26.24)_8$

$\therefore (16.5)_{16} \rightarrow (26.24)_8$

(Hexa - 10) PV = Place value

digit $\times 16^{PV}$

The process of converting hexadecimal to decimal is the same as binary to decimal. The process starts from multiplying the digits of hexadecimal numbers with its corresponding positional weights.

Ex: (1) Convert $(129)_{16} \rightarrow \times_{10}$

$\Rightarrow 16^2 \times 1 + 16^1 \times 2 + 16^0 \times 9$

$\Rightarrow 256 + 32 + 9$

$\Rightarrow 297$

$\therefore$ The decimal equivalent value for $(129)_{16}$ is $(297)_{10}$

(2) Convert $(1F)_{16} \rightarrow \times_{10}$?

$\Rightarrow 15 \times 16^0 + 1 \times 16^1$

$\Rightarrow 15 + 16 = 31$

$\therefore$ The decimal equivalent value for $(1F)_{16}$ is $(31)_{10}$

Ex (3):- convert $(AB.24)_{16} \rightarrow \times_{10}$?

$\Rightarrow 16^1 \times 10 + 16^0 \times 11 + 2 \times 16^{-1} + 4 \times 16^{-2}$

$\Rightarrow 160 + 11 + 0.125 + 0.015625$

$\Rightarrow 171.140625$

$\therefore$ The decimal equivalent value for $(AB.24)_{16}$ is $(171.140625)_{10}$

Ex (4):- Convert $(BABA)_{16} \rightarrow \times_{10}$

$\Rightarrow 11 \times 16^3 + 10 \times 16^2 + 11 \times 16^1 + 10 \times 16^0$

$\Rightarrow 45056 + 2560 + 176 + 10$

$\Rightarrow 47802$

$\therefore$ The decimal equivalent value for $(BABA)_{16}$ is $(47802)_{10}$

Problem 1:- Given that $(64)_{10} = (100)_b$ · determine the value of b.

Sol:-
$$(64)_{10} = (100)_b$$
$$6\times10^1 + 4\times10^0 = (1\times b^2) + (0\times b^1) + (0\times b^0)$$
$$60+4 = b^2$$
$$64 = b^2$$
$$(8)^2 = b^2$$
$$\Rightarrow \therefore b = 8$$

∴ The base is 8.

Problem (2):- Solve for x

(i) $(367)_8 = (x)_2$ (ii) $(378\cdot93)_{10} = (x)_8$

(iii) $(B9F\cdot AE)_{16} = (x)_8$ (iv) $(16)_{10} = (100)_x$

(i) $(367)_8 = (x)_2$

3 6 7
↓ ↓ ↓
011 110 111

$\therefore (367)_8 = (011\ 110111)_2$

(ii) $(378\cdot93)_{10} = (x)_8$

8 | 378
8 | 47 − 2
5 − 7

572

$0\cdot93\times8 = 7\cdot44 \to 7$
$0\cdot44\times8 = 3\cdot52 \to 3$
$0\cdot52\times8 = 4\cdot16 \to 4$
$0\cdot16\times8 = 1\cdot28 \to 1$

$\therefore (378\cdot93)_{10} = (572\cdot7341)_8$

(iii) $(B9F\cdot AE)_{16} = (x)_8$

10|1 10|01 1|111|·|10|10 11|100

5 6 3 7 · 5 3 4

$\therefore (B9F\cdot AE)_{16} = (5637\cdot534)_8$

(iv) $(16)_{10} = (100)_x$

$16 = 1\times x^2 + 0\times x^1 + 0\times x^0$

$16 = x^2$

$\Rightarrow x = 4$

$(16)_{10} = (100)_4$

Problem:- Determine the value of 'b' for the following

(i) $(292)_{10} = (1204)_b$

$= 1 \times b^3 + 2 \times b^2 + 0 \times b^1 + 4 \times b^0$ ~~$b^2(b+2)+4$~~

$= b^3 + 2b^2 + 4 \Rightarrow$

$\therefore b = 6$

* Signed Binary Numbers

The binary numbers which can be identified by their MSB (Most Significant Bit), whether they are positive or negative are called "signed binary numbers". This is the simplest way of representing the both positive and negative numbers in binary system

positive numbers including zero can be represented as unsigned numbers & Negative numbers can be represented as signed Numbers.

→ In ordinary arithmetic, a negative number is indicated by minus sign and positive number is by plus sign

→ computers should represent everything with binary digits. The sign should be represented in the left most position of the number.

→ The leftmost bit [sign bit] in the number represents sign of the number.

→ sign bit is '0' for positive numbers.

→ sign bit is '1' for Negative numbers.

These numbers are represented by the sign magnitude format

MSB →

B_7	B_6	B_5	B_4	B_3	B_2	B_1	B_0
sign	magnitude						

If MSB = 0 then +ve'
If MSB = 1 then -ve'.

Ex:- $+9 = \underset{\text{sign}}{0}\ \underline{1001} \rightarrow$ magnitude

$-9 = \underset{\text{sign}}{1}\ \underline{1001} \rightarrow$ magnitude

If the string of bits $\underline{01001}$ can be represented as 9 [unsigned binary]
because left most bit is '0'

→ String of bits $\underline{11001}$ can be represented as binary equivalent of 25' when considered as unsigned number.

→ Another system mostly used is "signed complement system" for representing -ve numbers. In this system negative number is indicated by its complement.

→ Where as signed magnitude system negates a number by changing its sign

→ The signed complement system negates a numbers by taking its complement

→ In this system we can use both 1's or 2's complement

• positive signed binary numbers:-

The binary numbers having their MSB 0 are called ... signed binary numbers".

Positive Number	Binary equivalent
0	000
1	001
2	010
3	011

• Negative signed binary numbers:

The binary numbers having their MSB 1 are called "negative signed binary numbers".

Negative Number	Binary equivalent
1	1001
2	1010
3	1011
4	1100

unsigned numbers can have a wide range of representation. But whereas, in case of signed numbers, we can represent their range only from $-(2^{(n-1)}-1)$ to $+(2^{(n-1)}-1)$

where 'n' is the number of bits (including sign bit)

Ex:- for a 5 bit signed number (including 4 magnitude bits & 1 sign bit), the range will be

$-(2^{(5-1)}-1)$ to $+(2^{(5-1)}-1)$

$-(2^{(4)}-1)$ to $+(2^{(4)}-1)$

-15 to $+15$

Ex:-
⇒ (1) −9 = 11001 → sign magnitude
⇒ 10110 → signed 1's complement
⇒ 10111 → signed 2's complement

Ex:- (2) −6 is represent in 3 formats

consider the number 6 represented in binary with 8 bits

signed magnitude representation : 10000110

signed - 1's complement : 11111001

signed - 2's complement : 11111010

→ In signed - magnitude −6 is obtained from +6 by changing the sign bit in the left most (MSB) position from 0 to 1

→ In signed 1's complement, −6 is obtained by complementing all the bits of +6 ; including sign bit

→ The signed 2's complement representation of −6 is obtained by taking 2's complement of positive number ; including the sign bit.

Decimal	signed - 2's Complement	signed - 1's Complement	signed Magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
−0	—	1111	1000
−1	1111	1110	1001
−2	1110	1101	1010
−3	1101	1100	1011
−4	1100	1011	1100
−5	1011	1010	1101
−6	1010	1001	1110
−7	1001	1000	1111