* Complements:-

* Examples of Binary numbers:-

(i) Addition

augend : 101101

addend : +100111

1111

Sum : 1010100

(ii) Subtraction

minuend : 101101

Subtrahend : 100111

difference : 000110

(iii) Multiplication

Multiplicand : 1011

Multiplier × 101

1011

0000

1011

Product : 110111

* Complements:-

Complements are used in digital computers to simplify the operation of subtraction for logical manipulation.

There are two types of complements for each base-r system:

- → The radix complement (or) r's complement
- → The diminished radix complement (or) (r-1)'s complement
- → For Binary number system - 2's complement and
  - 1's complement
- → For Decimal number system - 10's complement and
  - 9's complement

	r's complement	(r-1)'s complement
r=10	10's complement	9's complement
r=2	2's complement	1's complement

∴ r's complement = $r^n - N$

∴ (r-1)'s complement = $r^n - 1 - N$

(i) r's complement [Radix complement]

The r's complement of an n-digit number N in base r is defined as $r^n - N$ for $N \neq 0$ and 0 for $N = 0$.

→ r's complement is obtained by adding 1 to the (r-1)'s complement.

Examples:- (1) complement of $(7)_{10}$

Sol:- $N = 7$ $\therefore r^n - N = 10^1 - 7 = 3$.
$n = 1$
$r = 10$

9's complement + 1 = 10's complement

(or)

$$\begin{array}{r} 9 \\ 7 \\ \hline 2 \end{array} \rightarrow \text{9's complement}$$

$$\begin{array}{r} +1 \\ \hline 3 \end{array} \rightarrow \text{10's complement}$$

Ex (2):- $(5690)_{10}$

$N = 5690$ $\therefore 10^4 - 5690 = 10000 - 5690$
$r = 10$ $= 4310$.
$n = 4$

9's complement

$$\begin{array}{r} 9999 \\ 5690 \\ \hline 4309 + 1 \end{array} = 4310 \rightarrow \text{10's complement}$$

Ex (3):- Find 2's complement of 1101

$\Rightarrow r^n - N$

$r = 2$ $\therefore 2^4 - 1101 = 16 - 1101$
$N = 1101$ $= (16)_{10} - 1101$
$n = 4$ $= 10000 - 1101$

$(\because 16 - 13 = 3)$

$= 11$

$$\begin{array}{l} 2 \,|\, 16 \\ 2 \,|\, 8 - 0 \\ 2 \,|\, 4 - 0 \\ 2 \,|\, 2 - 0 \\ \quad 1 - 0 \end{array}$$

$$\begin{array}{r} 10000 \\ 1101 \\ \hline 0011 \end{array}$$

→ 2's complement of 1101

1's complement of 1101 → 0010

2's complement
$$\begin{array}{r} 0010 \\ +\ 1 \\ \hline 0011 \end{array}$$
+1 by adding one

0011 → (3)

(ii) (r-1)'s complement :-

If N is a positive number in base 'r' with 'n' integer digits and 'm' fraction digits, the (r-1)'s complement is defined as $[(r^n - 1) - N]$ (or) $[r^n - r^{-m} - N]$

where r = base of the given number system
n = no. of digits [integer]
N = given positive number
m = no. of fraction digits.

Ex:- (1) Find 9's complement for the given decimal numbers.

(a) 7543

n = 4
r = 10

$= (10^4 - 1) - 7543$
$= 9999 - 7543$
$= 2456$

(b) 0.7863

n = 0.
r = 10
m = 4

$= 10^0 - 10^{-4} - 0.7863$
$= 1 - 0.0001 - 0.7863$
$= 0.2136$

(c) 632.456

n = 3
r = 10
m = 3

For integer

n = 3
r = 10

$(10^3 - 1) - 632$
$(1000 - 1) - 632$
$999 - 632$
$= 367$

For fraction

n = 0
r = 10
m = 3

$= 10^0 - 10^{-3} - 0.456$
$= 1 - 0.001 - 0.456$
$= 0.999 - 0.456$
$= 0.543$

∴ (367.543)

1's complement

To find the one's complement of a given number of bits.

$(r^n-1)-N$ The 1's complement of a number is found by changing all 1's to 0's and all 0's to 1's.

$N \rightarrow$ given positive number

$r \rightarrow$ Base (or) radix

$n \rightarrow$ Number of bits

Ex:- (1) Find the 1's complement for 1101

$n = 4$

$N = 1101$

Binary number $\rightarrow$ Base $r=2$

$(r^n-1)-N$

$\Rightarrow (2^4-1)-1101$

$\Rightarrow (16-1)-1101$

$\Rightarrow 15-1101$ Binary value for 15 is 1111

```
    1111
(-) 1101
    ----
    0010
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$\therefore$ 1's complement for 1101 is 0010

(2) 1's complement for 11011001

Sol:- 11011001 $\xrightarrow{\text{1's complement}}$ 00100110

(3) 1111110101 to find 1's complement

Sol:- 1111110101 $\xrightarrow{\text{1's complement}}$ 0000001010

9's complement:-

Formula is $(r^n-1)-N$.

Ex:. (1) Find the 9's complement for 4

$(r^n-1)-N$

$N \rightarrow$ given number

$r=10$, $n=1$

$\Rightarrow (10^1-1)-4$

$\Rightarrow 9-4=5$

(2) $24 \xrightarrow{\text{9's complement}}$

$r=10$, $n=2$

$(r^n-1)-N \Rightarrow (10^2-1)-24 \Rightarrow 99-24 \Rightarrow 75$

(or)

$$\begin{array}{r} 99 \\ -24 \\ \hline 75 \end{array}$$

(3) $789 \xrightarrow{\text{9's complement}} 999-789=210$

(4) $5476000 \xrightarrow{\text{9's complement}} 9999999-5476000$

$= 4523999$

(5) $4716 \xrightarrow{\text{9's complement}} 9999-4716=5283$

2's complement :-

Obtained by adding 1 to the Least significant Bit (LSB) of 1's complement of the number $(r^n - N)$. The 2's complement of binary number is

Ex: (1) Find the 2's complement for $(1010)_2$

$(r^n - N)$

Given no. is binary base $r = 2$

no. of digits $n = 4$

Given number $N = 1010$

$(2^4 - 1010) = (16 - 1010)$

Binary equivalent for 16 is 10000

$$\begin{array}{r} 10000 \\ 1010 \\ \hline 00110 \\ \hline \end{array}$$

(or)

2's complement = 1's complement + 1

Given $(1010)_2$

$1010 \xrightarrow{\text{1's complement}} 0101$

Add '1' to 0101 ;

$$\begin{array}{r} 0101 \\ +\ 1 \\ \hline 0110 \end{array}$$

(2) Find 2's complement for $(11011010)_2$

$(11011010)_2 \xrightarrow{\text{2's complement}} 00100101$

$$\begin{array}{r} 00100101 \\ +\ 1 \\ \hline 00100110 \\ \hline \end{array}$$

2's complement for $(11011010)_2$ is $(00100110)_2$

10's complement:- $(r^n - N)$

Ex: (1) Find the 10's complement for $(24)_{10}$

$(24)_{10}$ $(r^n - N)$

$r=10$, $n=2$, Given number $N=24$

$(10^2 - 24) \Rightarrow 100 - 24 = 76.$

(or)

10's complement = 9's complement + 1

$24 \xrightarrow{\text{10's complement}}$

$$\begin{array}{r} 99 \\ -24 \\ \hline 75 \\ +1 \\ \hline 76 \end{array}$$

→ add '1'

(2) Find 10's complement for $(102398)_{10}$

$(102398)_{10} \xrightarrow{\text{10's complement}}$

$$\begin{array}{r} 999999 \\ -102398 \\ \hline 897601 \\ +1 \\ \hline 897602 \end{array}$$

(3) Find the 10's complement for $(246700)_{10}$

$(246700)_{10} \xrightarrow{\text{10's complement}}$

$$\begin{array}{r} 999999 \\ -246700 \\ \hline 753299 \\ +1 \\ \hline 753300 \end{array}$$

∴ The 10's complement for $(246700)_{10}$ is 753300.