Annexe_Resonance

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Expression de A_{ventre} :

$$A = \sum_p A_p$$

$k = \frac{2\pi}{\lambda}$

déphasage de π à la réflexion

$$A(x) = a_0 \times \left[e^{-ikx} + e^{i\pi} r e^{-ik(L-x)} e^{-ikL} + e^{2i\pi} rr' e^{-ikx} e^{-2ikL} + e^{i\pi} rr'r e^{-ik(L-x)} e^{-3ikL} + \dots \right]$$

$$= a_0 \times \left[\sum_n (rr')^n e^{-ikx} e^{-2inkL} - \sum_m r^{m+1} r'^m e^{ikx} e^{-2imkL} e^{-2ikL} \right]$$

$$= a_0 \times \left[e^{-ikx} \times \frac{1}{1 - rr' e^{-2ikL}} - e^{ikx} \times \frac{r e^{-2ikL}}{1 - rr' e^{-2ikL}} \right]$$

$$= a_0 \times \frac{1}{1 - rr' e^{-2ikL}} \times \left[e^{-ikx} - e^{ikx} r e^{-2ikL} \right]$$

$$|A(x)|^2 = a_0^2 \times \frac{1}{(1 - rr'\cos(2kL))^2 + (rr'\sin(2kL))^2} \times \left\{ 1 + r^2 - 2r \underbrace{\left[\cos(kx)\cos(k(x-2L)) - \sin(kx)\sin(k(x-2L)) \right]}_{\cos(2k(x-L)) \,=\, 1 - 2\sin^2(k(x-L))} \right\}$$

$$= a_0^2 \times \frac{1 + r^2 - 2r + 4r\sin^2(k(x-L))}{1 + (rr')^2 - 2rr' \underbrace{\cos(2kL)}_{1 - 2\sin^2(kL)}}$$

$$= a_0^2 \times \frac{(1-r)^2 + 4r\sin^2(k(x-L))}{(1-rr')^2 + 4rr'\sin^2(kL)}$$

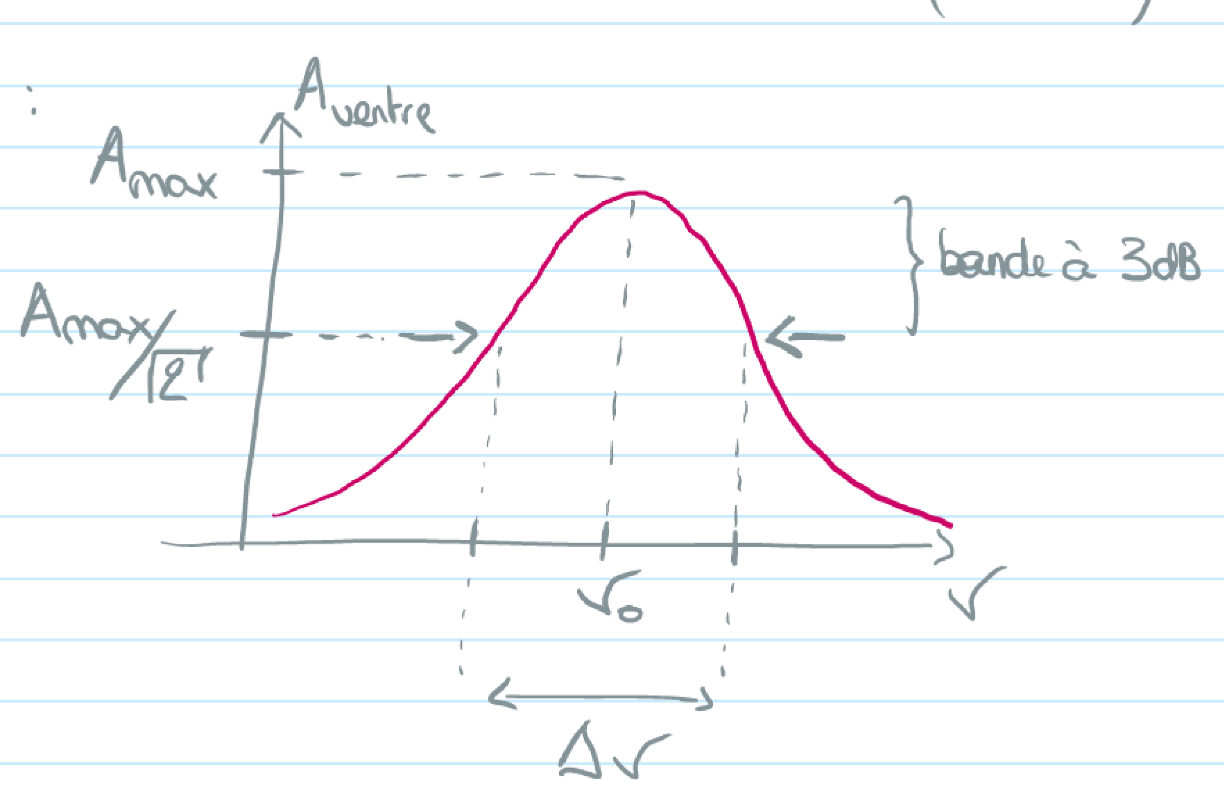
(Pour un ventre, $x_n = (2n+1)\frac{\lambda}{4}$, $n \in \mathbb{N}$) Pour un ventre, $A(x)$ est maximal $\Rightarrow \sin^2(kx - kL) = 1$

Donc $A_{ventre}^2 = a_0^2 \frac{1 - 2r + r^2 + 4r}{\quad} = a_0^2 \frac{(1+r)^2}{(1-rr')^2 + 4rr'\sin^2(kL)}$

A la résonance, les ventres sont maximals :

$\Rightarrow \sin(kL) = 0$ $\qquad A_{res}(\nu) = a_0^2 \frac{(1+r)^2}{(1-rr')^2}$

Si on trace $A(\nu)$: $\lambda = \frac{c}{\nu}$



$\rightarrow Q = \frac{\nu}{\Delta\nu}$:

$$A^2 = \frac{A_{max}^2}{2} \iff \frac{1}{(1-rr')^2 + 4rr'\sin^2(\frac{2\pi\nu L}{c})} = \frac{1}{2} \times \frac{1}{(1-rr')^2}$$

$$\iff \frac{(2-1) \times (1-rr')^2}{4rr'} = \sin^2\left(\frac{2\pi\nu L}{c}\right)$$

$$\iff \frac{1-rr'}{2\sqrt{rr'}} = \sin\left(\frac{2\pi}{c}\nu L\right) \qquad \text{et } \nu_{1/2} = \nu_0 \pm \varepsilon$$

$$\sin(\nu L) = \sin \nu_0 L + \varepsilon L \overbrace{\cos(\nu_0 L)}^{=1 \text{ par déf de } \nu_0}$$

$$\rightarrow \frac{\nu}{\Delta\nu} = n\pi \frac{\sqrt{rr'}}{1-rr'}$$