Word Embeddings

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Announcements

- **TBD.**

Overview

Recap on Word Vectors

Vector Semantics

"The meaning of a word is its use in the language." Wittgenstein, 1953.

1. *Pasta is best when cooked just right.*
2. *Pizza should not be too cooked: it burns!*
3. Vector semantics combines two intuitions:
   - **Distributional approach**: define a word by the contexts it occurs into.
   - **Vectorize it**: use vectors to represent word meaning, as a point in space.
4. **Feature engineering** for NLP: word vectors are increasingly used as features for other tasks.
5. (Word) vectors are usually referred as (word) **embeddings** in modern neural network literature.

Co-occurrences

…ound and sonic power of a [new electric	guitar	played through] a guitar amp has play…
…[Some electric	guitar	models feature] piezoelectric pickups…
…[Playing	guitar	with a] pick produces a bright sound …
…ings, he is known for [playing fretless	guitar	in his] performances…
…the neck of [a classical	guitar	is too] wide and the normal position …
…t in the centre of Bristol [playing the	piano	, I was] punched in the head while, a…
…r in Houston, Texanstagram [playing the	piano	in his] flooded home after Hurrican H…
… some supplies, he stopped to [play the	piano	that was] sitting in knee-high water …
…te and one black, who [played classical	piano	together]…
…The [first electric	pianos	from the] late 1920s used metal strin…
…technologies, for example [the electric	car	and the] integration of mobile commun…
…study had each driver of [each electric	car	drive unimpeded], perform a task whil…
…Honda to commence testing of [their new	car	and the] American was no doubt more t…
…mary design considerations for [the new	car	were "safety] innovations, performanc…
…would be possible if almost [all private	cars	requiring drivers], which are not in …
… who donate to groups [providing private	school	scholarships have] written pieces att…
… that students participating [in private	school	choice programs] graduate high school…
…s in the establishment of this [new high	school	, named the] Gavirate Business School…
…Anna heads into her [final high	school	year before] university wanting somet…
… but he can prevent them from [playing at	school]	

Word-Context matrix

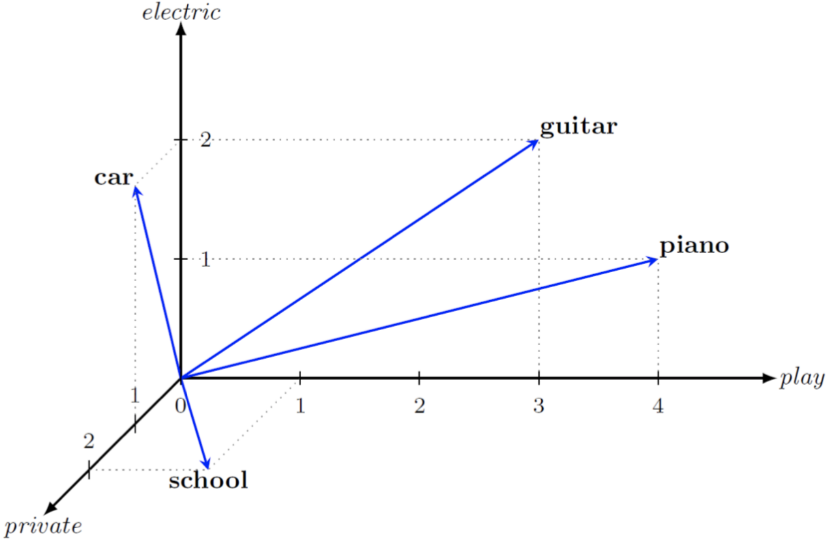
- We have a set of words V and a set of contexts they occur into C, taken from our corpus of documents. X in this case is a $|V| \times |C|$ matrix with word occurrences in contexts.
- The most intuitive context are co-occurrences with other words in V, within a certain **window**. In this case, X would be a $|V| \times |V|$ matrix.

	aardvark	...	computer	data	pinch	result	sugar	...
apricot	0	...	0	0	1	0	1	
pineapple	0	...	0	0	1	0	1	
digital	0	...	2	1	0	1	0	
information	0	...	1	6	0	4	0	

Figure 6.5 Co-occurrence vectors for four words, computed from the Brown corpus, showing only six of the dimensions (hand-picked for pedagogical purposes). The vector for the word *digital* is outlined in red. Note that a real vector would have vastly more dimensions and thus be much sparser.

Credit: J&M, ch. 6.

Vectors



Families of vectors

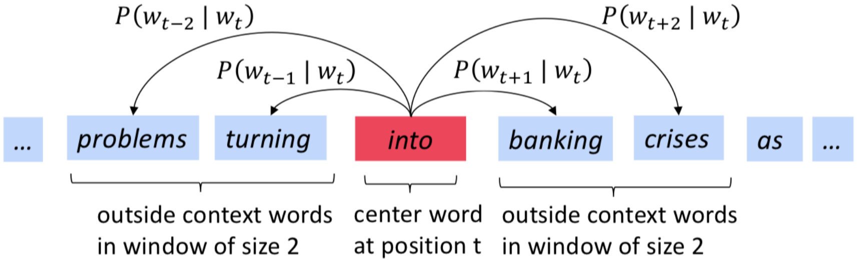
- **Sparse vectors**: many zero values and high-dimensional spaces. E.g., weighted co-occurrence matrices.
- **Dense vectors**: no zero values and comparatively smaller-dimensional spaces.
  - Dimensionality reduction (Singular Value Decomposition, Random indexing, Non-negative matrix factorization).
  - **Neural-network inspired (Word2Vec)**: today.

Word2Vec

Families of vectors

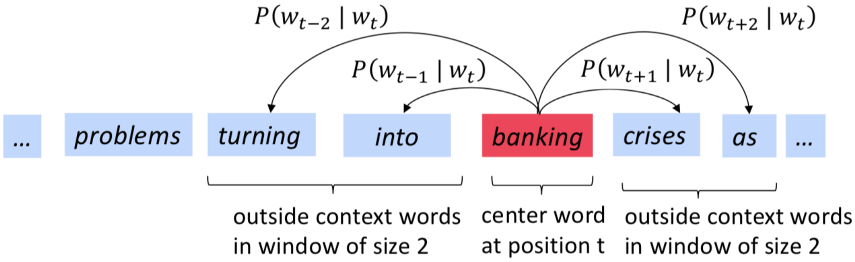
- **Word2Vec**: a framework for learning dense word vectors.
- Idea:
  1. We have a large corpus of text.
  2. We want each word in the vocabulary to be represented by a vector.
  3. We can go through the corpus and establish a *context o* for every *center/focus word c*, using a certain window/span.
  4. **We use the similarity of the word vectors c and o to calculate the probability of context words o given c.**
  5. **We keep adjusting word vectors until our predictions are good.**

Words in context



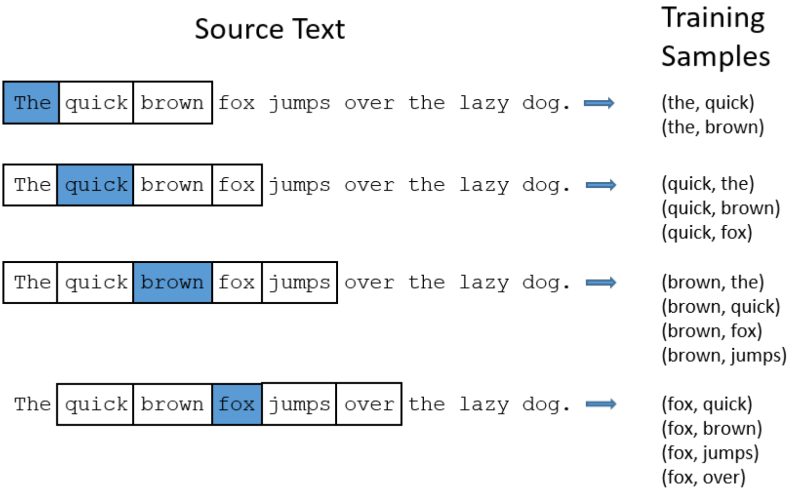
Credit: Stanford CS224N.

Words in context



Credit: Stanford CS224N.

Words in context as data



Credit: `http://mccormickml.com/2016/04/19/word2vec-tutorial-the-skip-gram-model.`

The model

- Our task, for every c (center), o (context) pair, is to estimate high probabilities for:

$$p(w_o|w_c)$$

- *The model parameters are the word embeddings w.*
- For each word position $t = 1 \dots T$, we predict context words within a windows of size m, given the center word w_t (at each position):

$$L(\boldsymbol{w}) = \prod_{t=1}^{T} \prod_{-m \leq j \leq m; j \neq 0} p(\boldsymbol{w}_{t+j}|\boldsymbol{w}_t)$$

The model

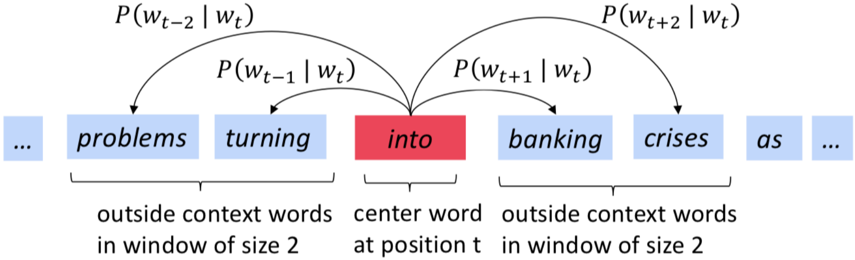
- Loss function (of the negative log likelihood):

$$\mathcal{L}(\boldsymbol{w}) = -\frac{1}{T} logL(\boldsymbol{w}) = -\frac{1}{T} \sum_{t=1}^{T} \sum_{-m \leq j \leq m; j \neq 0} logp(\boldsymbol{w}_{t+j} | \boldsymbol{w}_t)$$

- *Minimizing the loss is equivalent to maximizing the likelihood.*
- How to calculate $p(\boldsymbol{w}_{t+j} | \boldsymbol{w}_t)$? Use two vectors for each word:
  - v_w when w is a center word
  - u_w when w is a context word
- Use the **Softmax** (generalization of the Sigmoid) to predict probabilities of a c (center), o (context) pair:

$$p(o|c) = \frac{exp(u_o^T v_c)}{\sum_{w \in V} exp(u_w^T v_c)}$$

Example



We learn to predict:

- $p(u_{problems} | v_{into})$
- $p(u_{turning} | v_{into})$
- $p(u_{banking} | v_{into})$
- $p(u_{crises} | v_{into})$
- ...

Credit: Stanford CS224N.

Softmax

Exponentiation makes anything positive

Dot product compares similarity of *o* and *c*.
$u^T v = u.v = \sum_{i=1}^{n} u_i v_i$
Larger dot product = larger probability

$$P(o|c) = \frac{\exp(\boxed{u_o^T v_c})}{\boxed{\sum_{w \in V} \exp(u_w^T v_c)}}$$

Normalize over entire vocabulary
to give probability distribution

- The Softmax maps any value to a probability distribution.
- It amplifies large values (*max*) but still gives non-zero probabilities to small values (*soft*).

Credit: Stanford CS224N.

Training via SGD

- Parameters: our word embeddings, **two per word**.
- Usually, these vectors have length d within 50-1000, thus $d \ll |V|$.
- Use gradient descent to optimize and find a minimum of the loss.

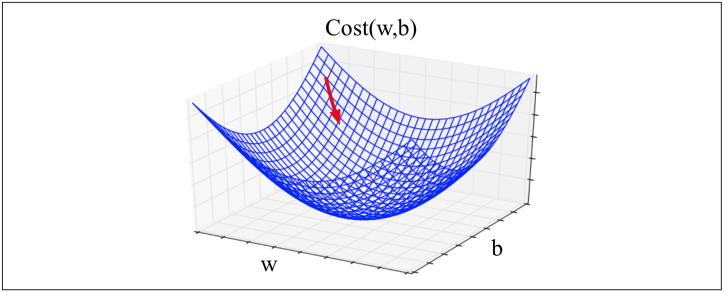


Figure 5.4 Visualization of the gradient vector in two dimensions w and b.

Credit: J&M, ch. 5.

Training via SGD

- Let us ignore for a moment the normalization term $\frac{1}{T}$ and the external summations, which are straightforward.
- Let us take the first (partial) derivative w.r.t. v_c (similarly, you can do this for u_o):

$$\frac{\partial}{\partial v_c} log \frac{exp(u_o^T v_c)}{\sum_{w \in V} exp(u_w^T v_c)} = u_o - \sum_{x \in V} \frac{exp(u_x^T v_c)}{\sum_{w \in V} exp(u_w^T v_c)} \cdot u_x$$
$$= u_o - \sum_{x \in V} p(x|c) \cdot u_x$$

- Thus the derivative w.r.t. the central word vector v_c is the vector for the current context word u_o, minus the weighted average of the model's current representations of other possible contexts!
- *Derivation..*

Conclusion

- After having trained the model, we typically use the vectors v_w or the average of v_w and u_w.
- There are many good implementations of this model: *next lab*.
- This is a very rapidly advancing area, with many, more involved models now-a-days. Still, word embeddings are the main building block of deep learning NLP applications (used as **features**).
- Some more optional references and info below.

References

- Stanford CS224N classes 1 and 2: `http://web.stanford.edu/class/cs224n/index.html`.
- Original Word2Vec paper `https://arxiv.org/pdf/1301.3781.pdf`.
- Negative sampling paper `http://papers.nips.cc/paper/5021-distributed-representations-of-words-and-phrases-and` `pdf`.
- Good tutorial `http://mccormickml.com/2016/04/19/word2vec-tutorial-the-skip-gram-model`.

Note: there is much more. Ask me if you are interested.

Word2Vec Expanded (optional)

Derivation for Softmax

- First, we need some notable derivatives:

$$\frac{\partial log(x)}{\partial x} = \frac{1}{x}$$
$$\frac{\partial exp(x)}{\partial x} = exp(x)$$
$$\frac{\partial f(g(x))}{\partial x} = \frac{\partial f}{\partial g} \cdot \frac{\partial g}{\partial x} \rightarrow \text{chain rule}$$

Derivation for Softmax

- We can divide in two parts:

$$\frac{\partial}{\partial v_c} log \frac{exp(u_o^T v_c)}{\sum_{w \in V} exp(u_w^T v_c)} = \frac{\partial}{\partial v_c} logexp(u_o^T v_c) - \frac{\partial}{\partial v_c} log \sum_{w \in V} exp(u_w^T v_c)$$

- First part:

$$\frac{\partial}{\partial v_c} logexp(u_o^T v_c) = u_o$$

- Second part:

$$\frac{\partial}{\partial v_c} log \sum_{w \in V} exp(u_w^T v_c) = \frac{1}{\sum_{w \in V} exp(u_w^T v_c)} \cdot \frac{\partial}{\partial v_c} \sum_{x \in V} exp(u_x^T v_c)$$

$$= \frac{\sum_{x \in V} exp(u_x^T v_c) u_x}{\sum_{w \in V} exp(u_w^T v_c)}$$

Derivation for Softmax

- Combine:

$$\frac{\partial}{\partial v_c} log \frac{exp(u_o^T v_c)}{\sum_{w \in V} exp(u_w^T v_c)} = u_o - \frac{\sum_{x \in V} exp(u_x^T v_c) u_x}{\sum_{w \in V} exp(u_w^T v_c)}$$
$$= u_o - \sum_{x \in V} p(x|c) u_x$$

Negative sampling

Exponentiation makes anything positive

$$P(o|c) = \frac{\exp(\boxed{u_o^T v_c})}{\boxed{\sum_{w \in V} \exp(u_w^T v_c)}}$$

Dot product compares similarity of o and c.
$u^T v = u.v = \sum_{i=1}^{n} u_i v_i$
Larger dot product = larger probability

Normalize over entire vocabulary
to give probability distribution

- Normalizing over the entire vocabulary is very expensive.
- Idea: let us just create some **negative examples** (collocations absent in the data), and train a **binary logistic regression classifier** to distinguish between positive (real) and negative (fake) pairs.
- For every center-context pair, also sample K negative pairs. The center-context pair is going to be a positive datapoint, the negative pairs are negative datapoints.
- The logistic classifier then uses the same dot product of vectors as features, and a cross-entropy loss (see last class).

Credit: Stanford CS224N.

Variations of Word2Vec

- What we discussed is called the Continuous Bag Of Words model (**CBOW**).
- Alternatively, we can predict the center word using the context words: this is called the **Skip-gram** model.
- **GloVe**: approach that approximates global co-occurrence information instead (which we have seen in previous classes).

More references

- GloVe `https://nlp.stanford.edu/pubs/glove.pdf`.
- Word2Vec implicitly factorizes the (shifted) PPMI matrix we are familiar with: `https://papers.nips.cc/paper/5477-neural-word-embedding-as-implicit-matrix-factorizati pdf`.
- Evaluation of word embeddings: `https://www.aclweb.org/anthology/D15-1036`.