第六章 参数估计

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参 数 估 计

§ 6-1 参数的点估计

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正态总体均值的置信区间

- 单个正态总体均值 μ 的置信区间
- 两个正态总体均值差$\boldsymbol{\mu_1 - \mu_2}$的置信区间

设总体 $X \sim N(\mu_1, \sigma_1^2), Y \sim N(\mu_2, \sigma_2^2), X_1, X_2, \cdots, X_{n_1}$ 为来自总体X的样本，$Y_1, Y_2, \cdots, Y_{n_2}$ 为来自总体 Y 的样本。

样本均值：$\overline{X} = \dfrac{1}{n_1}\sum_{i=1}^{n_1} X_i, \overline{Y} = \dfrac{1}{n_2}\sum_{i=1}^{n_2} Y_i$

样本方差： S_1^2，S_2^2

置信水平：$1-\alpha$

1、$\sigma_1^{\ 2}$, $\sigma_2^{\ 2}$ 已知

① 选统计量

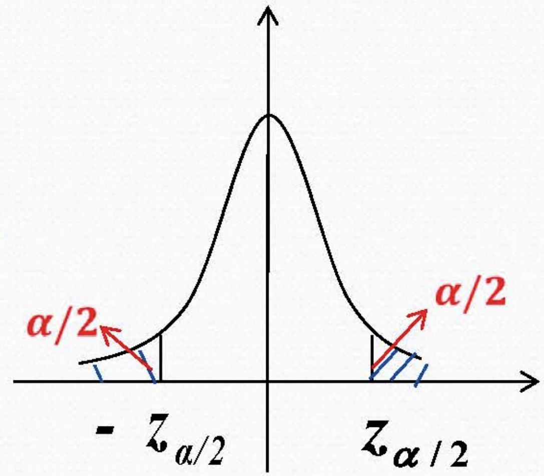
$$\because \overline{X} \sim N(\mu_1, \sigma_1^{\ 2}),\ \overline{Y} \sim N(\mu_2, \sigma_2^{\ 2})$$

$$\therefore \overline{X} - \overline{Y} \sim N(\mu_1 - \mu_2, \frac{\sigma_1^{\ 2}}{n_1} + \frac{\sigma_2^{\ 2}}{n_2})$$

$$\therefore \quad \frac{(\overline{X} - \overline{Y}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^{\ 2}}{n_1} + \frac{\sigma_2^{\ 2}}{n_2}}} \sim N(0,1)$$

1、σ_1^2, σ_2^2 已知

① 选统计量 $\dfrac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1}+\dfrac{\sigma_2^2}{n_2}}} \sim N(0,1)$



② $P\left\{-Z_{\alpha/2} < \dfrac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1}+\dfrac{\sigma_2^2}{n_2}}} < Z_{\alpha/2}\right\} = 1-\alpha$

③ $P\left\{(\bar{X}-\bar{Y})-Z_{\frac{\alpha}{2}}\sqrt{\dfrac{\sigma_1^2}{n_1}+\dfrac{\sigma_2^2}{n_2}} < (\mu_1-\mu_2) < (\bar{X}-\bar{Y})+Z_{\frac{\alpha}{2}}\sqrt{\dfrac{\sigma_1^2}{n_1}+\dfrac{\sigma_2^2}{n_2}}\right\} = 1-\alpha$

$\mu_1-\mu_2$的置信水平为$1-\alpha$的置信区间：

$$\left(\overline{X}-\overline{Y}-Z_{\alpha/2}\cdot\sqrt{\frac{\sigma_1^{\,2}}{n_1}+\frac{\sigma_2^{\,2}}{n_2}},\overline{X}-\overline{Y}\pm Z_{\alpha/2}\cdot\sqrt{\frac{\sigma_1^{\,2}}{n_1}+\frac{\sigma_2^{\,2}}{n_2}}\right)$$

$$=\left(\overline{X}-\overline{Y}+Z_{\alpha/2}\cdot\sqrt{\frac{\sigma_1^{\,2}}{n_1}+\frac{\sigma_2^{\,2}}{n_2}}\right)$$

2、$\sigma_1^{\ 2}$, $\sigma_2^{\ 2}$ 未知

① 选统计量

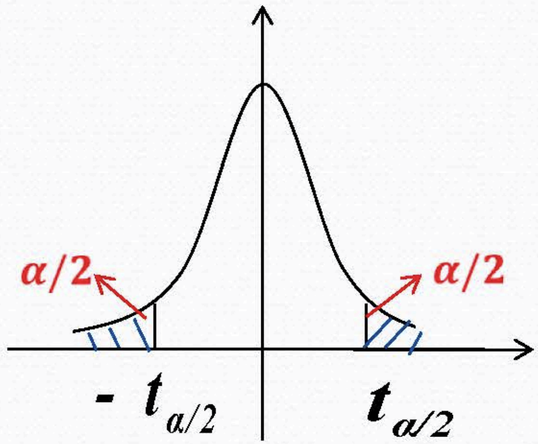
$$\frac{(\overline{X}-\overline{Y})-(\mu_1-\mu_2)}{\sqrt{\dfrac{\sigma_1^{\ 2}}{n_1}+\dfrac{\sigma_2^{\ 2}}{n_2}}} \sim N(0,1)$$

$s_1^{\ 2}$ $s_2^{\ 2}$

$$\boxed{\frac{(\overline{X}-\overline{Y})-(\mu_1-\mu_2)}{\sqrt{\dfrac{S_1^{\ 2}}{n_1}+\dfrac{S_2^{\ 2}}{n_2}}} \sim T(v)}$$

2、σ_1^2, σ_2^2 未知

① 选统计量 $\dfrac{(\overline{X}-\overline{Y})-(\mu_1-\mu_2)}{\sqrt{\dfrac{S_1^2}{n_1}+\dfrac{S_2^2}{n_2}}} \sim T(v)$



其中，$v=\dfrac{\left(\dfrac{S_1^2}{n_1}+\dfrac{S_2^2}{n_2}\right)^2}{\dfrac{1}{n_1-1}\left(\dfrac{S_1^2}{n_1}\right)^2+\dfrac{1}{n_2-1}\left(\dfrac{S_2^2}{n_2}\right)^2}$

② $P\left\{-t_{\alpha/2}<\dfrac{(\overline{X}-\overline{Y})-(\mu_1-\mu_2)}{\sqrt{\dfrac{S_1^2}{n_1}+\dfrac{S_2^2}{n_2}}}<t_{\alpha/2}\right\}=1-\alpha$

③ $$P\left\{(\bar{X}-\bar{Y})-t_{\frac{\alpha}{2}}\sqrt{\frac{S_1^{\ 2}}{n_1}+\frac{S_2^{\ 2}}{n_2}}<(\mu_1-\mu_2)<(\bar{X}-\bar{Y})+t_{\frac{\alpha}{2}}\sqrt{\frac{S_1^{\ 2}}{n_1}+\frac{S_2^{\ 2}}{n_2}}\right\}=1-\alpha$$

$\mu_1-\mu_2$的置信水平为$1-\alpha$的置信区间：

$$\left(\bar{X}-\bar{Y}-t_{\alpha/2}\cdot\sqrt{\frac{S_1^{\ 2}}{n_1}+\frac{S_2^{\ 2}}{n_2}},\bar{X}-\bar{Y}\pm t_{\alpha/2}\cdot\sqrt{\frac{S_1^{\ 2}}{n_1}+\frac{S_2^{\ 2}}{n_2}}\right)$$

$$=\left(\bar{X}-\bar{Y}+t_{\alpha/2}\cdot\sqrt{\frac{S_1^{\ 2}}{n_1}+\frac{S_2^{\ 2}}{n_2}}\right)$$

例1： 随机地取10只A牌灯泡，8只B牌灯泡，测得它们的寿命(以小时计)得到下列数据：A牌寿命的样本均值$\bar{x}=4600$，样本方差 $s_1^2=200^2$；B牌寿命样本均值 $\bar{x}=4300$，样本方差$s_2^2=250^2$.设两样本依次来自正态分布$N(\mu_1,\sigma_1^2),N(\mu_2,\sigma_2^2)$，参数均未知，两样本独立。试求$\mu_1-\mu_2$的置信水平为0.95的近似置信区间.

解析： 这是σ_1^2,σ_2^2未知，求$\mu_1-\mu_2$的置信区间

$$\left(\overline{X}-\overline{Y}-t_{\alpha/2}\cdot\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}},\overline{X}-\overline{Y}\pm t_{\alpha/2}\cdot\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}\right)$$

$$=\left(\overline{X}-\overline{Y}+t_{\alpha/2}\cdot\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}\right)$$

解: 在此 $n_1 = 10$，$n_2 = 8$

$$v = \frac{\left(\frac{200^2}{10} + \frac{250^2}{8}\right)^2}{\frac{1}{9}\left(\frac{40000}{10}\right)^2 + \frac{1}{7}\left(\frac{62500}{8}\right)^2} = 13.29$$

$\therefore [v] = 13$

由$1 - \alpha = 0.95$，知$\alpha = 0.05$

$\therefore t_{0.025}(13) = 2.160$

$\mu_1 - \mu_2$的置信水平为0.95的置信区间为:

$$\boxed{\left(\bar{x}_1 - \bar{x}_2 \pm t_{0.025}(13) \cdot \sqrt{\frac{S_1{}^2}{n_1} + \frac{S_2{}^2}{n_2}}\right)}$$

$$=\left((4600-4300) \pm 2.160\sqrt{\frac{200^2}{10}+\frac{250^2}{8}}\right)$$

$$=(65.20, 534.80)$$

该区间的下限大于零, 因此可认为$\mu_1 > \mu_2$