

DECOMPOSITION, ABSTRACTION, FUNCTIONS

HOW DO WE WRITE CODE?

- so far...
 - covered language mechanisms
 - know how to write different files for each computation
 - each file is some piece of code
 - each code is a sequence of instructions
- problems with this approach
 - easy for small-scale problems
 - messy for larger problems
 - hard to keep track of details
 - how do you know the right info is supplied to the right part of code

GOOD PROGRAMMING

- more code not necessarily a good thing
- measure good programmers by the amount of functionality
- introduce **functions**
- mechanism to achieve **decomposition** and **abstraction**

EXAMPLE -- PROJECTOR

- a projector is a black box
- don't know how it works
- know the interface: input/output
- connect any electronics to it that can communicate with that input
- black box somehow converts image from input source to a wall, magnifying it
- **ABSTRACTION IDEA**: do not need to know how projector works to use it



<http://www.myprojectorlamps.com/blog/wp-content/uploads/Dell-1610HD-Projector.jpg>

EXAMPLE -- PROJECTOR

- projecting large image for Olympics decomposed into separate tasks for separate projectors
- each projector takes input and produces separate output
- all projectors work together to produce larger image
- **DECOMPOSITION IDEA:** different devices work together to achieve an end goal



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APPLY THESE IDEAS TO PROGRAMMING

- **DECOMPOSITION**

- Break problem into different, self-contained, pieces

- **ABSTRACTION**

- Suppress details of method to compute something from use of that computation

CREATE STRUCTURE with DECOMPOSITION

- in example, separate devices
- in programming, divide code into **modules**
 - are **self-contained**
 - used to **break up** code
 - intended to be **reusable**
 - keep code **organized**
 - **keep code coherent**
- this lecture, achieve decomposition with **functions**
- in a few weeks, achieve decomposition with **classes**

SUPPRESS DETAILS with ABSTRACTION

- in example, no need to know how to build a projector
- in programming, think of a piece of code as a **black box**
 - cannot see details
 - do not need to see details
 - do not want to see details
 - hide tedious coding details
- achieve abstraction with **function specifications** or **docstrings**

DECOMPOSITION & ABSTRACTION

- powerful together
- code can be used many times but only has to be debugged once!

FUNCTIONS

- write reusable piece/chunks of code, called **functions**
- functions are not run in a program until they are “**called**” or “**invoked**” in a program
- function characteristics:
 - has a **name**
 - has **parameters** (0 or more)
 - has a **docstring** (optional but recommended)
 - has a **body**

HOW TO WRITE and CALL/INVOKE A FUNCTION

keyword name parameters
or arguments

```
def is_even( i ):
```

```
    """
```

```
    Input: i, a positive int
```

```
    Returns True if i is even, otherwise False
```

```
    """
```

Specification,
docstring

body

```
    print("hi")
```

```
    return i%2 == 0
```

```
is_even(3)
```

later in the code, you call the
function using its name and
values for parameters

IN THE FUNCTION BODY

```
def is_even( i ):  
    """  
    Input: i, a positive int  
    Returns True if i is even, otherwise False  
    """
```

```
    print("hi")
```

```
    return i%2 == 0
```

evaluate some
expressions

keyword

expression to
evaluate and return

VARIABLE SCOPE

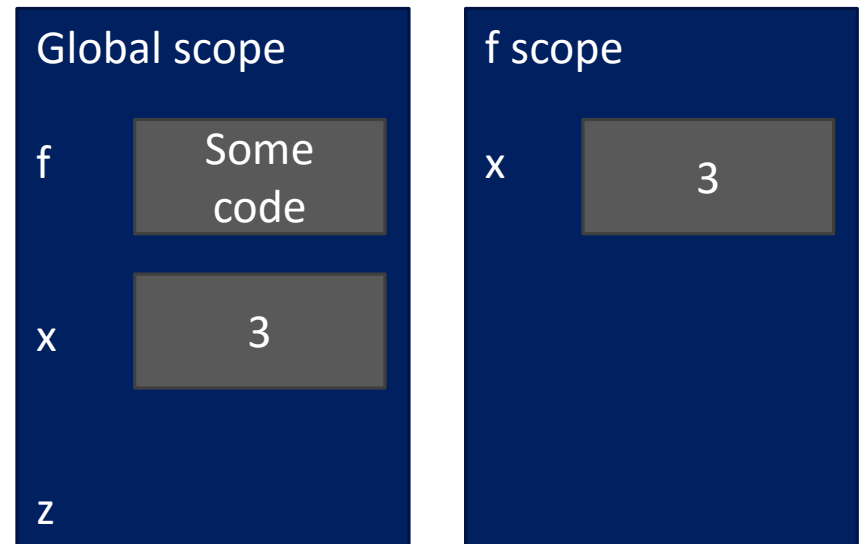
- **formal parameter** gets bound to the value of **actual parameter** when function is called
- new **scope/frame/environment** created when enter a function
- **scope** is mapping of names to objects

```
def f(x):  
    x = x + 1  
    print('in f(x): x =', x)  
    return x  
  
x = 3  
z = f(x)
```

formal parameter

actual parameter

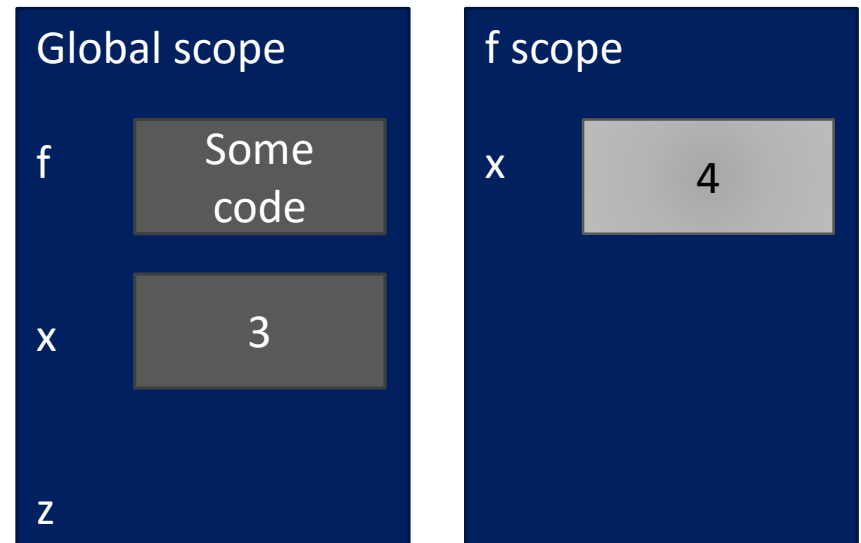

call to f, before body evaluated



VARIABLE SCOPE

- **formal parameter** gets bound to the value of **actual parameter** when function is called
- new **scope/frame/environment** created when enter a function
- **scope** is mapping of names to objects

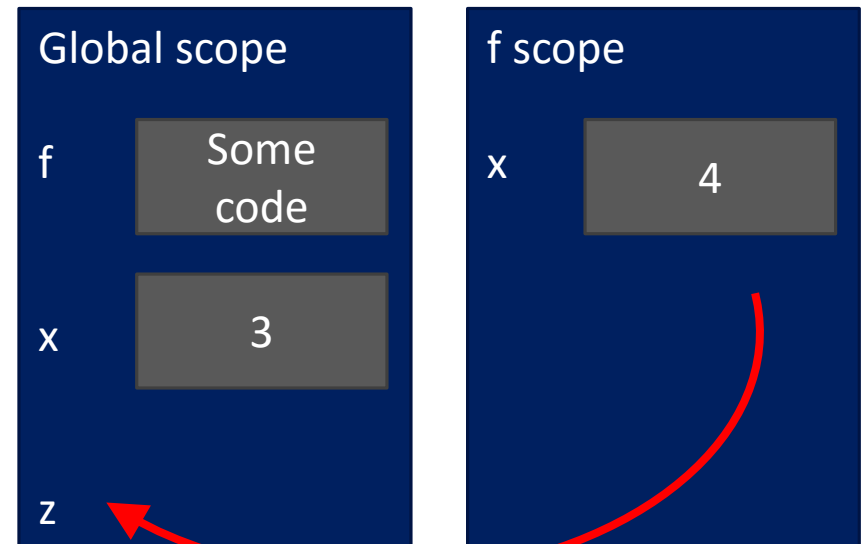
```
def f( x ) :  
    x = x + 1  
    print('in f(x): x =', x)  
    return x  
  
x = 3  
z = f( x )
```



VARIABLE SCOPE

- **formal parameter** gets bound to the value of **actual parameter** when function is called
- new **scope/frame/environment** created when enter a function
- **scope** is mapping of names to objects

```
def f( x ) :  
    x = x + 1  
    print('in f(x): x =', x)  
    return x  
  
x = 3  
z = f( x )
```



VARIABLE SCOPE

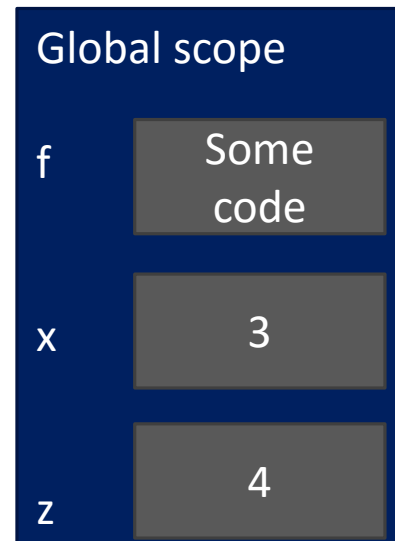
- **formal parameter** gets bound to the value of **actual parameter** when function is called
- new **scope/frame/environment** created when enter a function
- **scope** is mapping of names to objects

```
def f( x ) :  
    x = x + 1  
  
    print('in f(x): x =', x)  
  
    return x
```

```
x = 3
```

```
z = f( x )
```

binding of returned value to
variable z



ONE WARNING IF NO return STATEMENT

```
def is_even( i ):  
    """  
    Input: i, a positive int  
    Does not return anything  
    """
```

```
i%2 == 0
```

*without a return
statement*

- Python returns the value **None, if no return given**
- represents the absence of a value

return vs. print

- return only has meaning **inside** a function
 - only **one** return executed inside a function
 - code inside function but after return statement not executed
 - has a value associated with it, **given to function caller**
- print can be used **outside** functions
 - can execute **many** print statements inside a function
 - code inside function can be executed after a print statement
 - has a value associated with it, **outputted** to the console

FUNCTIONS AS ARGUMENTS

- arguments can take on any type, even functions

```
def func_a():  
    print('inside func_a')  
def func_b(y):  
    print('inside func_b')  
    return y  
def func_c(z):  
    print('inside func_c')  
    return z()  
  
print(func_a())  
print(5 + func_b(2))  
print(func_c(func_a))
```

call func_a, takes no parameters
call func_b, takes one parameter
call func_c, takes one parameter, another function

SCOPE EXAMPLE

- inside a function, **can access** a variable defined outside
- inside a function, **cannot modify** a variable defined outside

```
def f(y):  
    x = 1  
    x += 1  
    print(x)
```

*x is re-defined
in scope of f*

```
x = 5  
f(x)  
print(x)
```

*different x
objects*

```
def g(y):  
    print(x)  
    print(x + 1)
```

*x from
outside g*

```
x = 5  
g(x)  
print(x)
```

*x inside g is picked up
from scope that called
function g*

```
def h(y):  
    x = x + 1
```

```
x = 5  
h(x)  
print(x)
```

*UnboundLocalError: local variable
'x' referenced before assignment*

SCOPE EXAMPLE

- inside a function, **can access** a variable defined outside
- inside a function, **cannot modify** a variable defined outside

```
def f(y):  
    x = 1  
    x += 1  
    print x
```

```
x = 5  
f(2)  
print x
```

```
def g(y):  
    print x
```

```
x = 5  
g(2)  
print x
```

```
def h(y):  
    x = x + 1
```

```
x = 5  
h(2)  
print x
```

x from
global/main
program scope

HARDER SCOPE EXAMPLE



IMPORTANT
and
TRICKY!

***Python Tutor is your best friend to
help sort this out!***

<http://www.pythontutor.com/>

SCOPE DETAILS

```
def g(x):  
    def h():  
        x = 'abc'  
    x = x + 1  
    print('in g(x): x =', x)  
    h()  
    return x
```

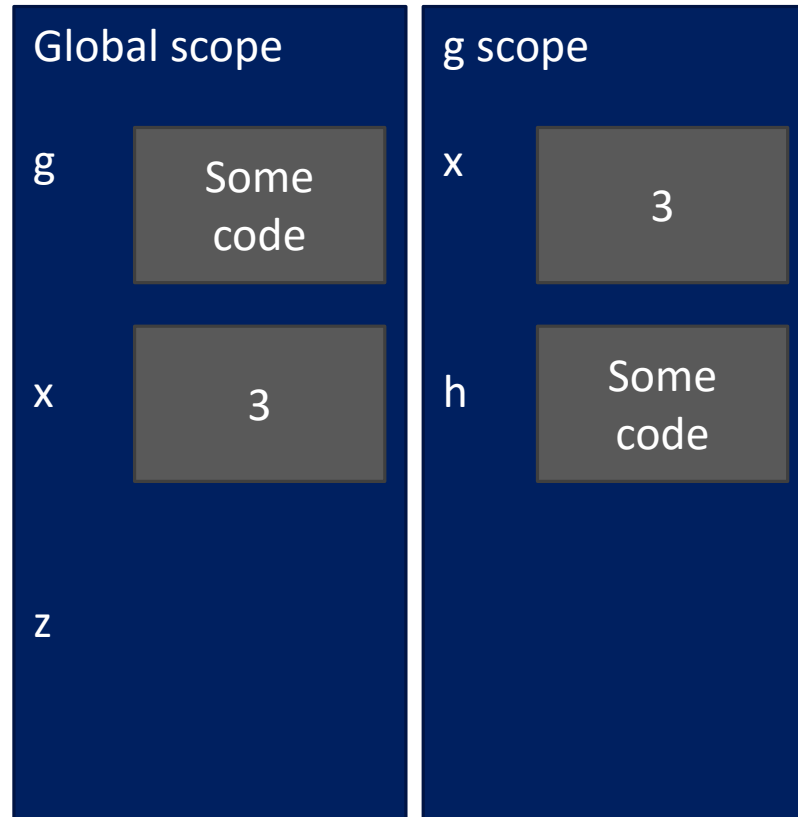
Some code

```
x = 3  
z = g(x)
```



SCOPE DETAILS

```
def g(x):  
    def h():  
        x = 'abc'  
    x = x + 1  
    print('in g(x): x =', x)  
    h()  
    return x
```



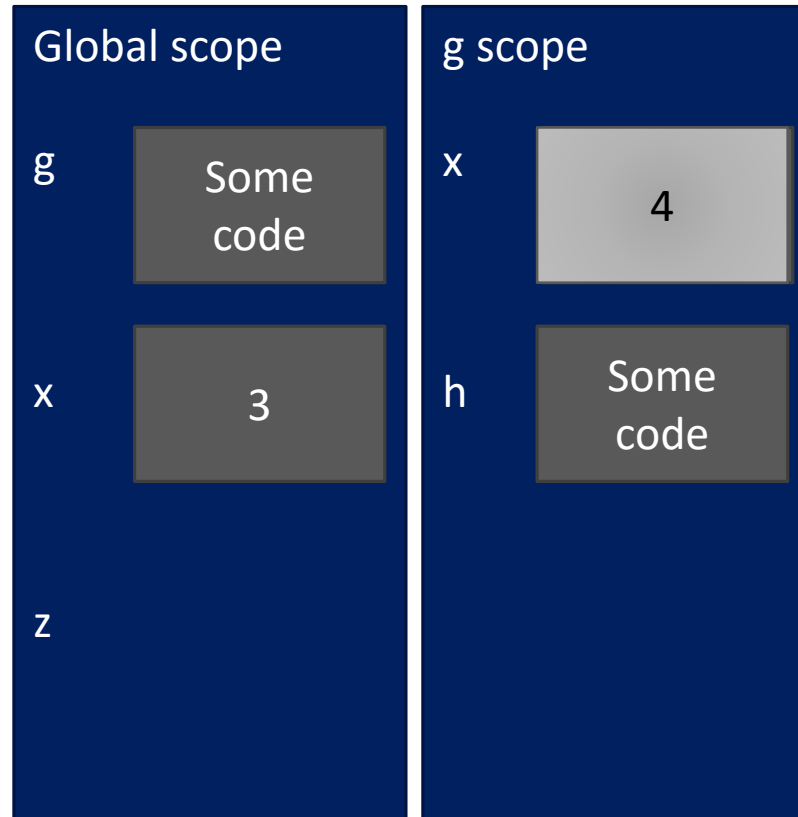
```
x = 3  
z = g(x)
```

SCOPE DETAILS



```
def g(x):  
    def h():  
        x = 'abc'  
    x = x + 1  
    print('in g(x): x =', x)  
    h()  
    return x
```



```
x = 3  
z = g(x)
```

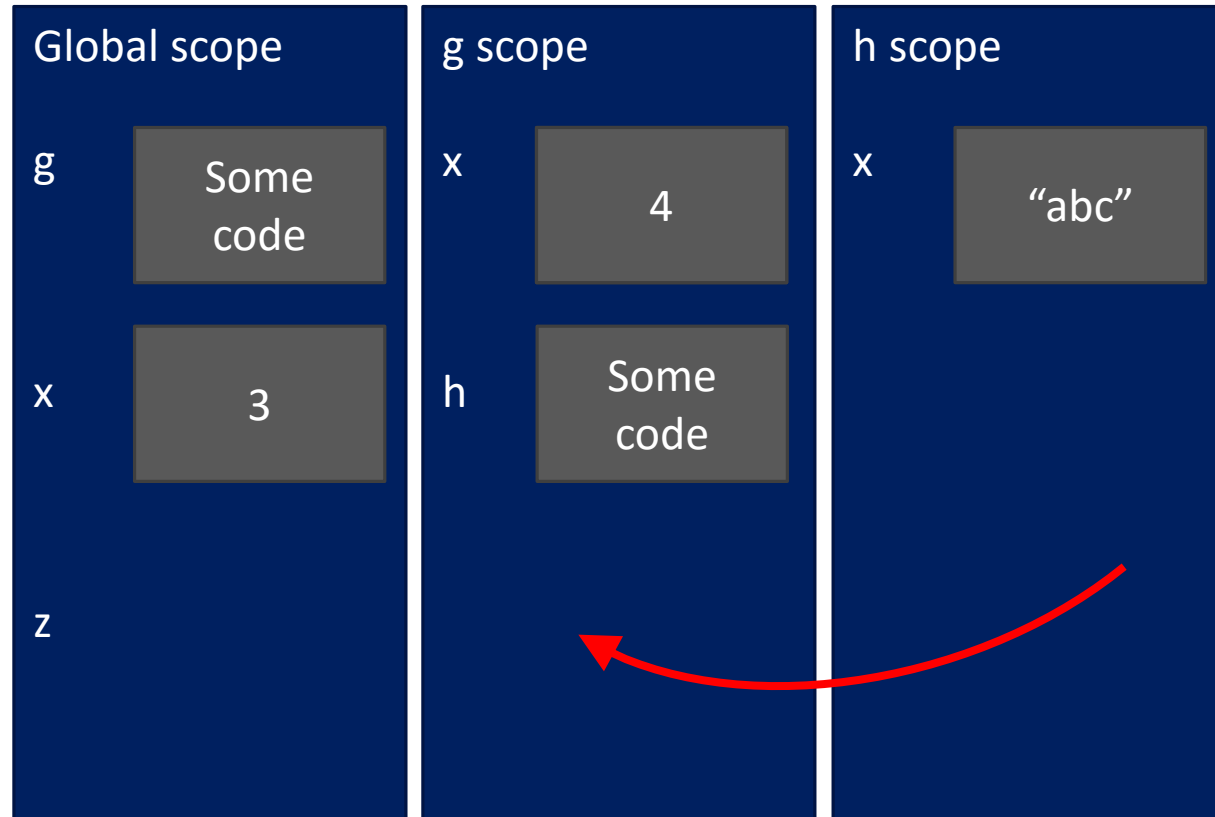


SCOPE DETAILS

```
def g(x):  
    def h():  
        x = 'abc'   
    x = x + 1  
    print('in g(x): x =', x)  
    h()   
    return x
```

x = 3

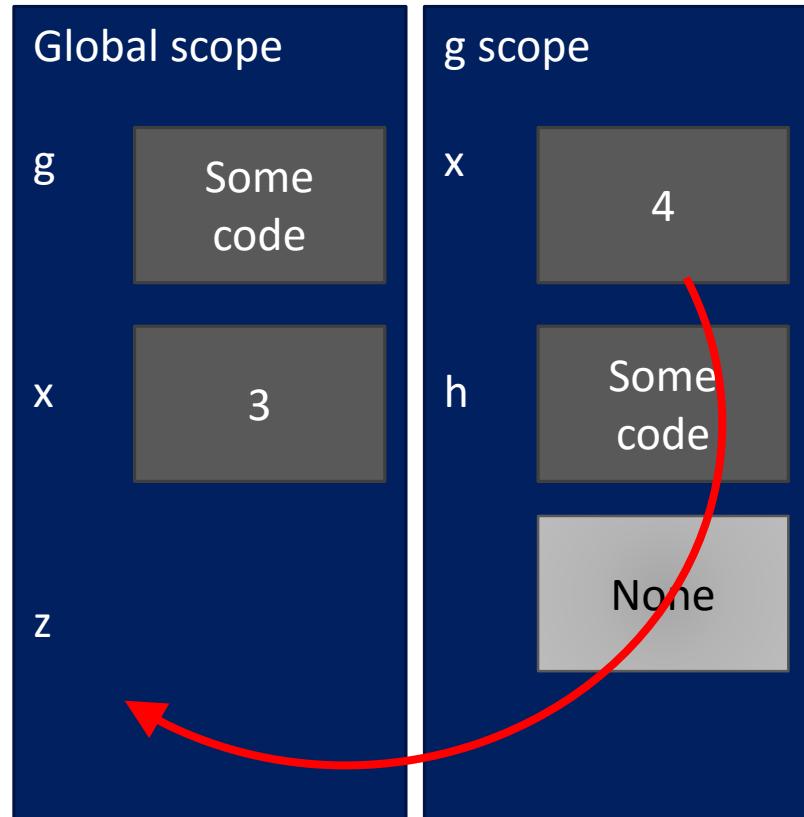
z = g(x)



SCOPE DETAILS

```
def g(x):  
    def h():  
        x = 'abc'  
    x = x + 1  
    print('in g(x): x =', x)  
    h()  
    return x
```

```
x = 3  
z = g(x)
```

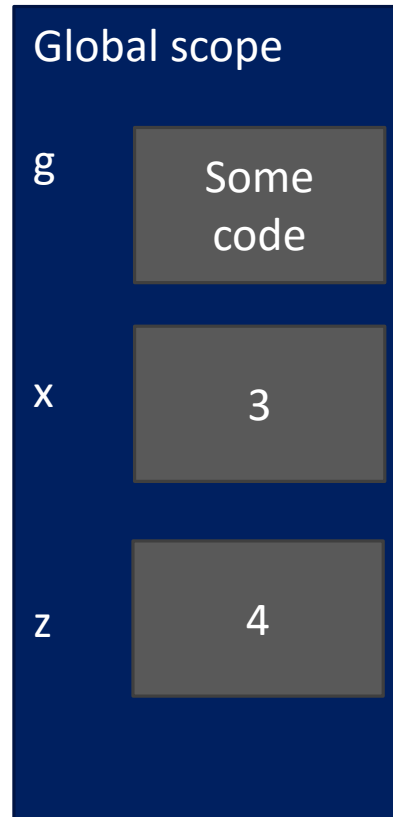


SCOPE DETAILS

```
def g(x):  
    def h():  
        x = 'abc'  
    x = x + 1  
    print('in g(x): x =', x)  
    h()  
    return x
```

```
x = 3
```

```
z = g(x)
```



KEYWORD ARGUMENTS AND DEFAULT VALUES

- Simple function definition, if last argument is TRUE, then print lastName, firstName; else firstName, lastName

```
def printName(firstName, lastName, reverse):  
    if reverse:  
        print(lastName + ', ' + firstName)  
    else:  
        print(firstName, lastName)
```


KEYWORD ARGUMENTS AND DEFAULT VALUES

- Each of these invocations is equivalent

```
printName('Eric', 'Grimson', False)
```

```
printName('Eric', 'Grimson', reverse = False)
```

```
printName('Eric', lastName = 'Grimson', reverse = False)
```

```
printName(lastName = 'Grimson', firstName = 'Eric',  
          reverse = False)
```

KEYWORD ARGUMENTS AND DEFAULT VALUES

- Can specify that some arguments have default values, so if no value supplied, just use that value

```
def printName(firstName, lastName, reverse = False):  
    if reverse:  
        print(lastName + ', ' + firstName)  
    else:  
        print(firstName, lastName)
```

```
printName('Eric', 'Grimson')
```

```
printName('Eric', 'Grimson', True)
```


SPECIFICATIONS

- a **contract** between the implementer of a function and the clients who will use it
 - **Assumptions:** conditions that must be met by clients of the function; typically constraints on values of parameters
 - **Guarantees:** conditions that must be met by function, providing it has been called in manner consistent with assumptions

```
def is_even( i ):
```

```
    """
```

```
    Input: i, a positive int
```

```
    Returns True if i is even, otherwise False
```

```
    """
```

```
    print "hi"
```

```
    return i%2 == 0
```

```
is_even(3)
```


WHAT IS RECURSION

- a way to design solutions to problems by **divide-and-conquer or decrease-and-conquer**
- a programming technique where a **function calls itself**
- in programming, goal is to NOT have infinite recursion
 - must have **1 or more base cases** that are easy to solve
 - must solve the same problem on **some other input** with the goal of simplifying the larger problem input

ITERATIVE ALGORITHMS SO FAR

- looping constructs (while and for loops) lead to **iterative** algorithms
- can capture computation in a set of **state variables** that update on each iteration through loop

MULTIPLICATION – ITERATIVE SOLUTION

- “multiply $a * b$ ” is equivalent to “add a to itself b times”
- capture **state** by
 - an **iteration** number (i) starts at b
 $i \leftarrow i-1$ and stop when 0
 - a current **value of computation** ($result$)
 $result \leftarrow result + a$

```
def mult_iter(a, b):  
    result = 0  
    while b > 0:  
        result += a  
        b -= 1  
    return result
```

iteration
current value of computation,
a running sum
current value of iteration variable

MULTIPLICATION – RECURSIVE SOLUTION

■ recursive step

- think how to reduce problem to a **simpler/smaller version** of same problem

$$\begin{aligned} a * b &= \underbrace{a + a + a + a + \dots + a}_{b \text{ times}} \\ &= a + \underbrace{a + a + a + \dots + a}_{b-1 \text{ times}} \\ &= a + a * (b-1) \end{aligned}$$

■ base case

- keep reducing problem until reach a simple case that can be **solved directly**
- when $b = 1$, $a * b = a$

```
def mult(a, b):
```

```
    if b == 1:
        return a
```

```
    else:
        return a + mult(a, b-1)
```

FACTORIAL

$$n! = n * (n-1) * (n-2) * (n-3) * \dots * 1$$

- what n do we know the factorial of?

```
n = 1      →      if n == 1:
                        return 1
```

base case

- how to reduce problem? Rewrite in terms of something simpler to reach base case

```
n*(n-1)!      →      else:
                        return n*factorial(n-1)
```

recursive step

RECURSIVE FUNCTION SCOPE EXAMPLE

```
def fact(n):  
    if n == 1:  
        return 1  
    else:  
        return n*fact(n-1)  
  
print(fact(4))
```

Global scope

fact

Some
code

fact scope
(call w/ n=4)

n

4

fact scope
(call w/ n=3)

n

3

fact scope
(call w/ n=2)

n

2

fact scope
(call w/ n=1)

n

1

print fact(4)

print 24

return 4*fact(3)

return 4*6

return 3*fact(2)

return 3*2

return 2*fact(1)

return 2*1

return 1
base case

SOME OBSERVATIONS

- each recursive call to a function creates its **own scope/environment**
- **bindings of variables** in a scope is not changed by recursive call
- flow of control passes back to **previous scope** once function call returns value

using the same variable names but they are different objects in separate scopes

ITERATION vs. RECURSION

```
def factorial_iter(n):  
    prod = 1  
    for i in range(1, n+1):  
        prod *= i  
    return prod  
  
def factorial(n):  
    if n == 1:  
        return 1  
    else:  
        return n*factorial(n-1)
```

- recursion may be simpler, more intuitive
- recursion may be efficient from programmer POV
- recursion may not be efficient from computer POV

INDUCTIVE REASONING

- How do we know that our recursive code will work?
- `mult_iter` terminates because `b` is initially positive, and decreases by 1 each time around loop; thus must eventually become less than 1
- `mult` called with `b = 1` has no recursive call and stops
- `mult` called with `b > 1` makes a recursive call with a smaller version of `b`; must eventually reach call with `b = 1`

```
def mult_iter(a, b):  
    result = 0  
    while b > 0:  
        result += a  
        b -= 1  
    return result
```

```
def mult(a, b):  
    if b == 1:  
        return a  
    else:  
        return a + mult(a, b-1)
```


MATHEMATICAL INDUCTION

- To prove a statement indexed on integers is true for all values of n :
 - Prove it is true when n is smallest value (e.g. $n = 0$ or $n = 1$)
 - Then prove that if it is true for an arbitrary value of n , one can show that it must be true for $n+1$

EXAMPLE OF INDUCTION

- $0 + 1 + 2 + 3 + \dots + n = (n(n+1))/2$
- Proof
 - If $n = 0$, then LHS is 0 and RHS is $0 \cdot 1/2 = 0$, so true
 - Assume true for some k , then need to show that
 - $0 + 1 + 2 + \dots + k + (k+1) = ((k+1)(k+2))/2$
 - LHS is $k(k+1)/2 + (k+1)$ by assumption that property holds for problem of size k
 - This becomes, by algebra, $((k+1)(k+2))/2$
 - Hence expression holds for all $n \geq 0$

RELEVANCE TO CODE?

- Same logic applies

```
def mult(a, b):
```

```
    if b == 1:
```

```
        return a
```

```
    else:
```

```
        return a + mult(a, b-1)
```

- Base case, we can show that `mult` must return correct answer
- For recursive case, we can assume that `mult` correctly returns an answer for problems of size smaller than `b`, then by the addition step, it must also return a correct answer for problem of size `b`
- Thus by induction, code correctly returns answer

TOWERS OF HANOI

- The story:
 - 3 tall spikes
 - Stack of 64 different sized discs – start on one spike
 - Need to move stack to second spike (at which point universe ends)
 - Can only move one disc at a time, and a larger disc can never cover up a small disc



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TOWERS OF HANOI

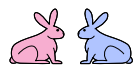
- Having seen a set of examples of different sized stacks, how would you write a program to print out the right set of moves?
- **Think recursively!**
 - Solve a smaller problem
 - Solve a basic problem
 - Solve a smaller problem

```
def printMove(fr, to):  
    print('move from ' + str(fr) + ' to ' + str(to))  
  
def Towers(n, fr, to, spare):  
    if n == 1:  
        printMove(fr, to)  
    else:  
        Towers(n-1, fr, spare, to)  
        Towers(1, fr, to, spare)  
        Towers(n-1, spare, to, fr)
```

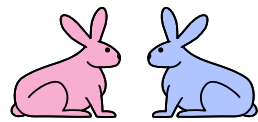

RECURSION WITH MULTIPLE BASE CASES

■ Fibonacci numbers

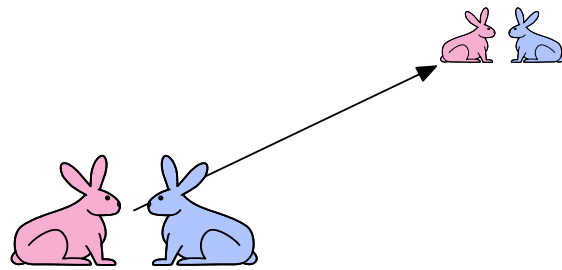
- Leonardo of Pisa (aka Fibonacci) modeled the following challenge
 - Newborn pair of rabbits (one female, one male) are put in a pen
 - Rabbits mate at age of one month
 - Rabbits have a one month gestation period
 - Assume rabbits never die, that female always produces one new pair (one male, one female) every month from its second month on.
 - How many female rabbits are there at the end of one year?



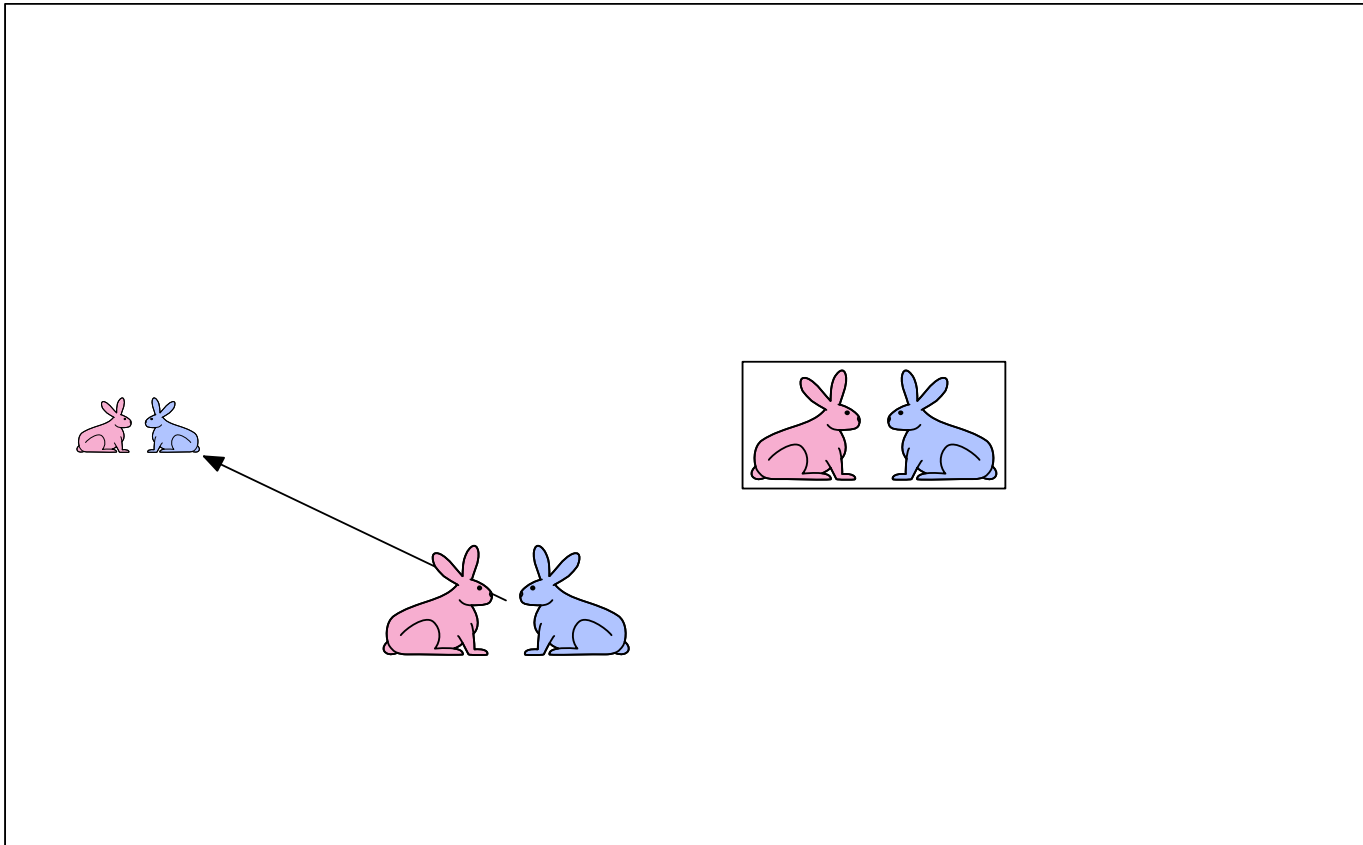
Demo courtesy of Prof. Denny Freeman and Adam Hartz



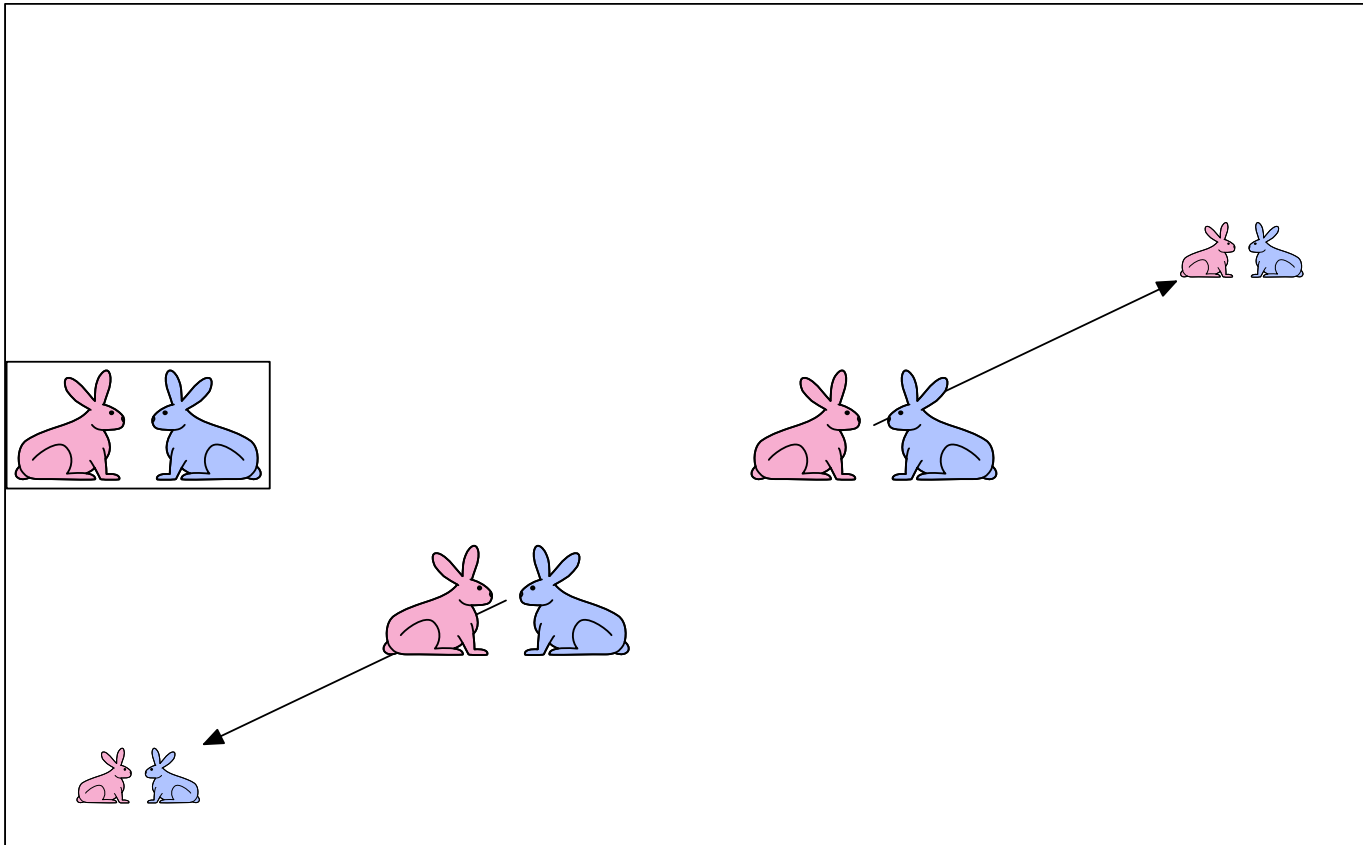
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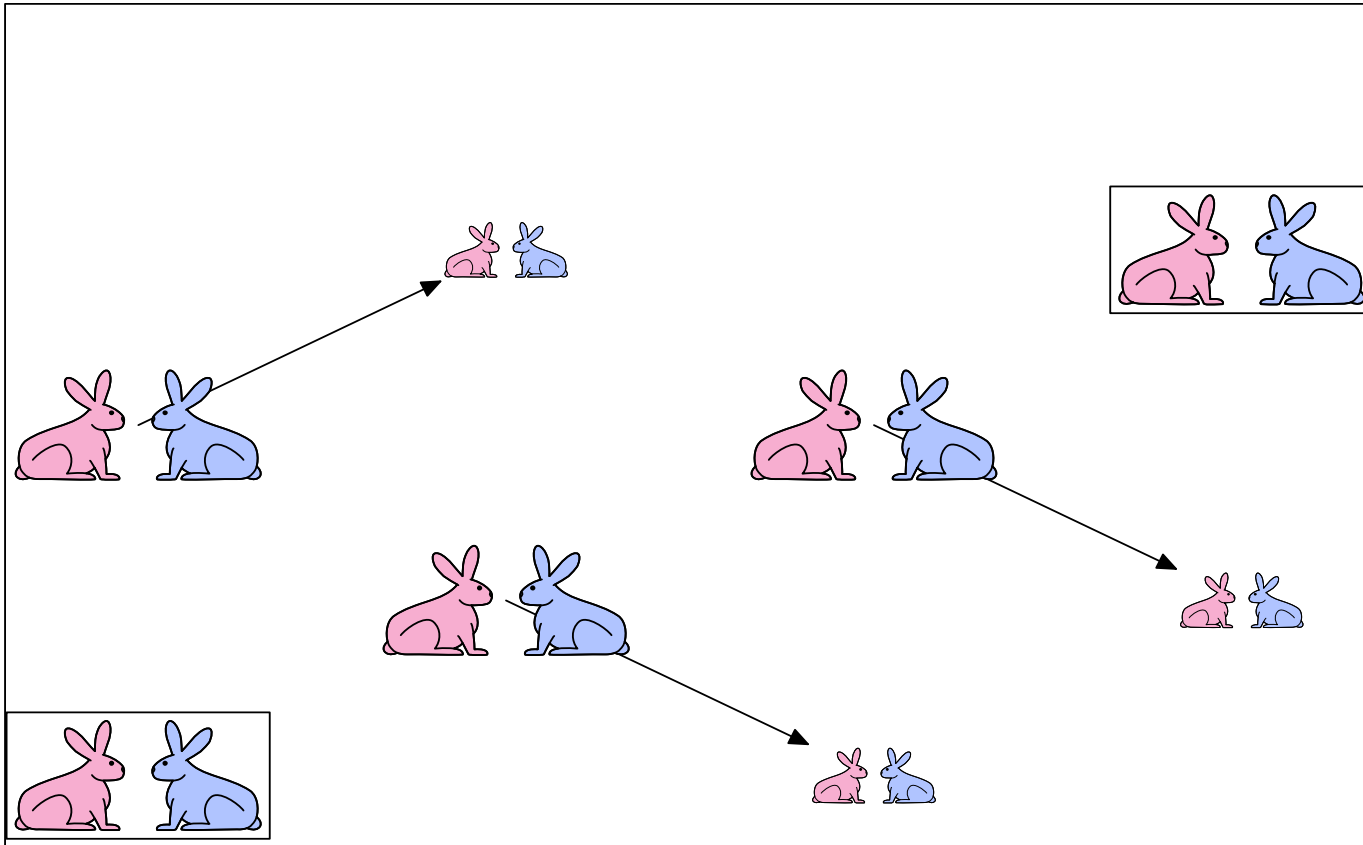
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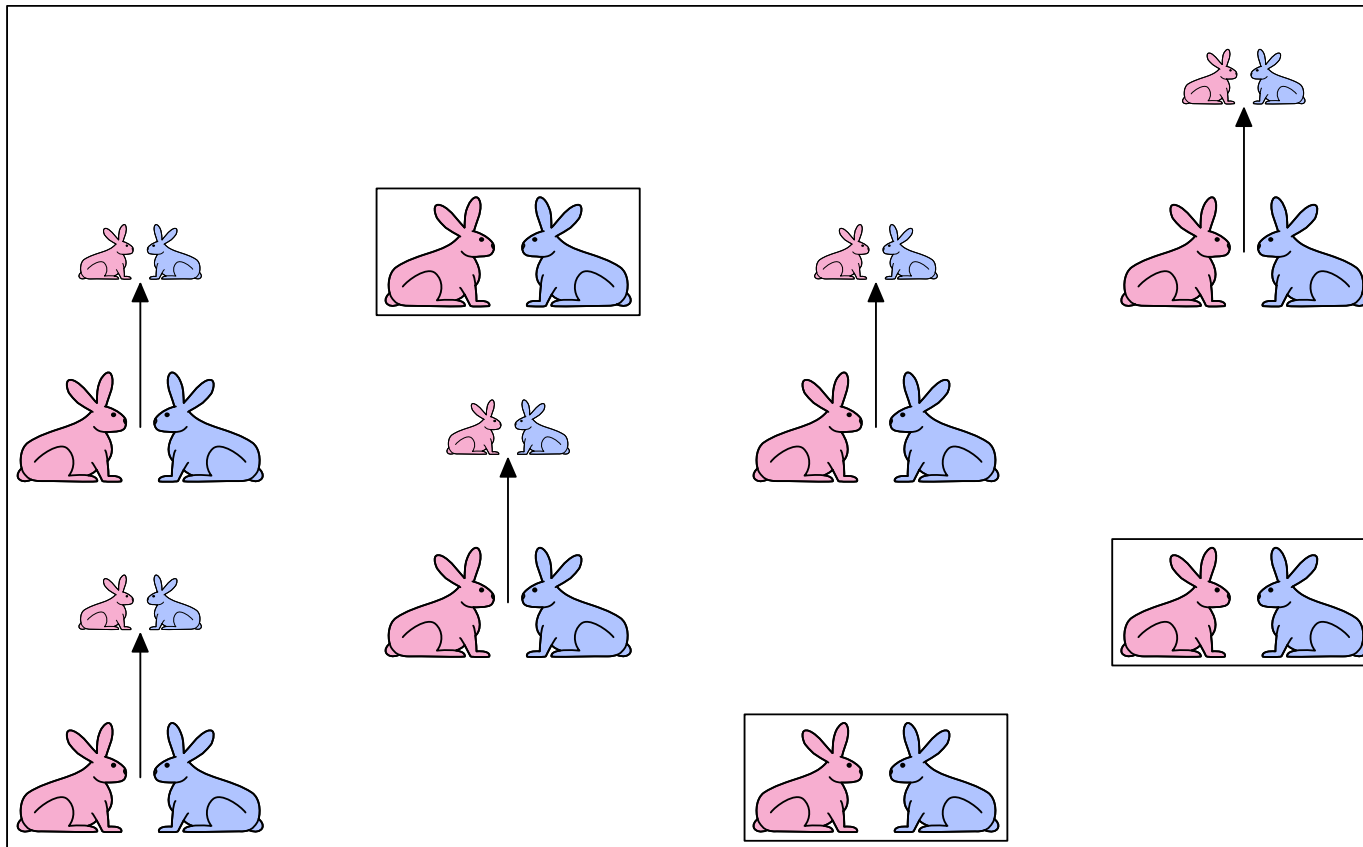
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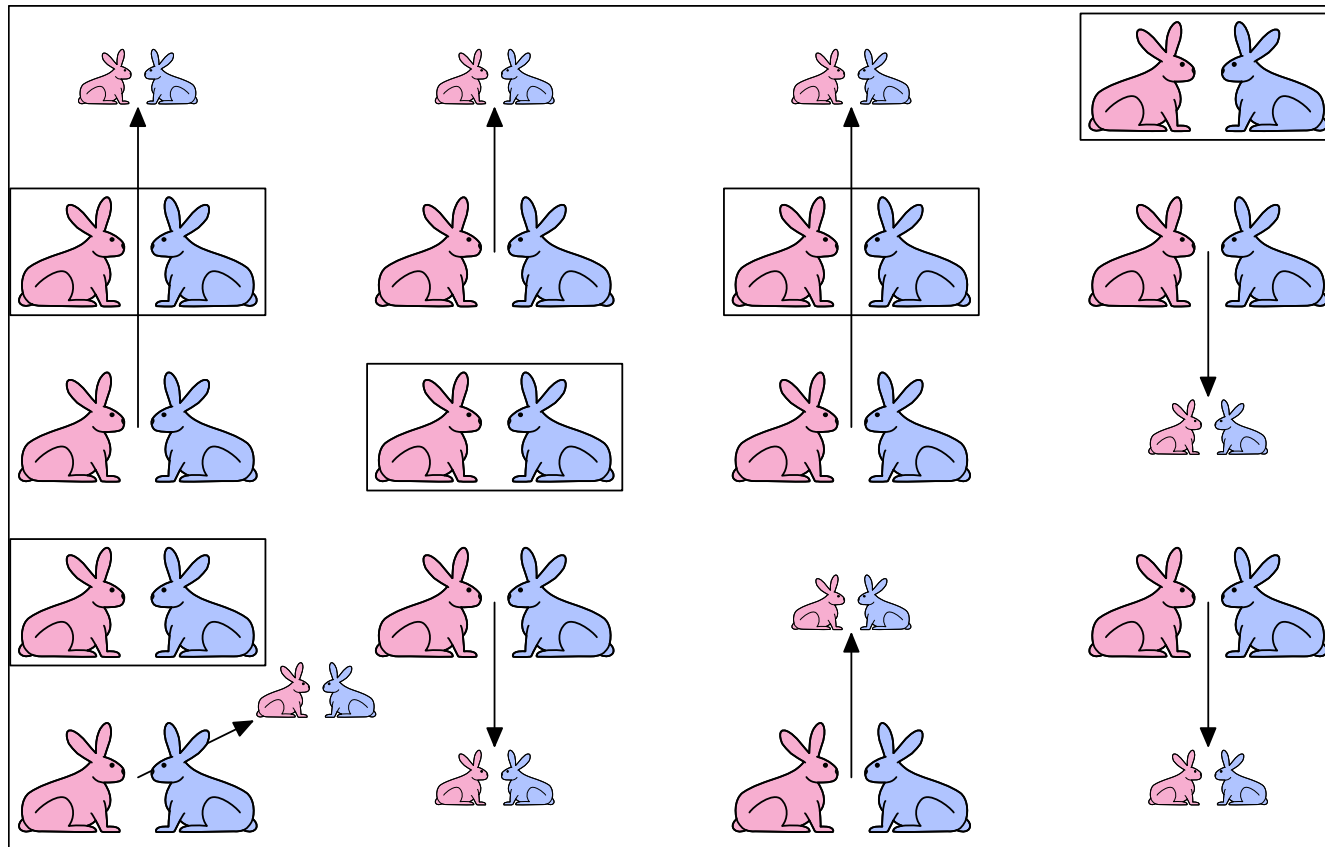
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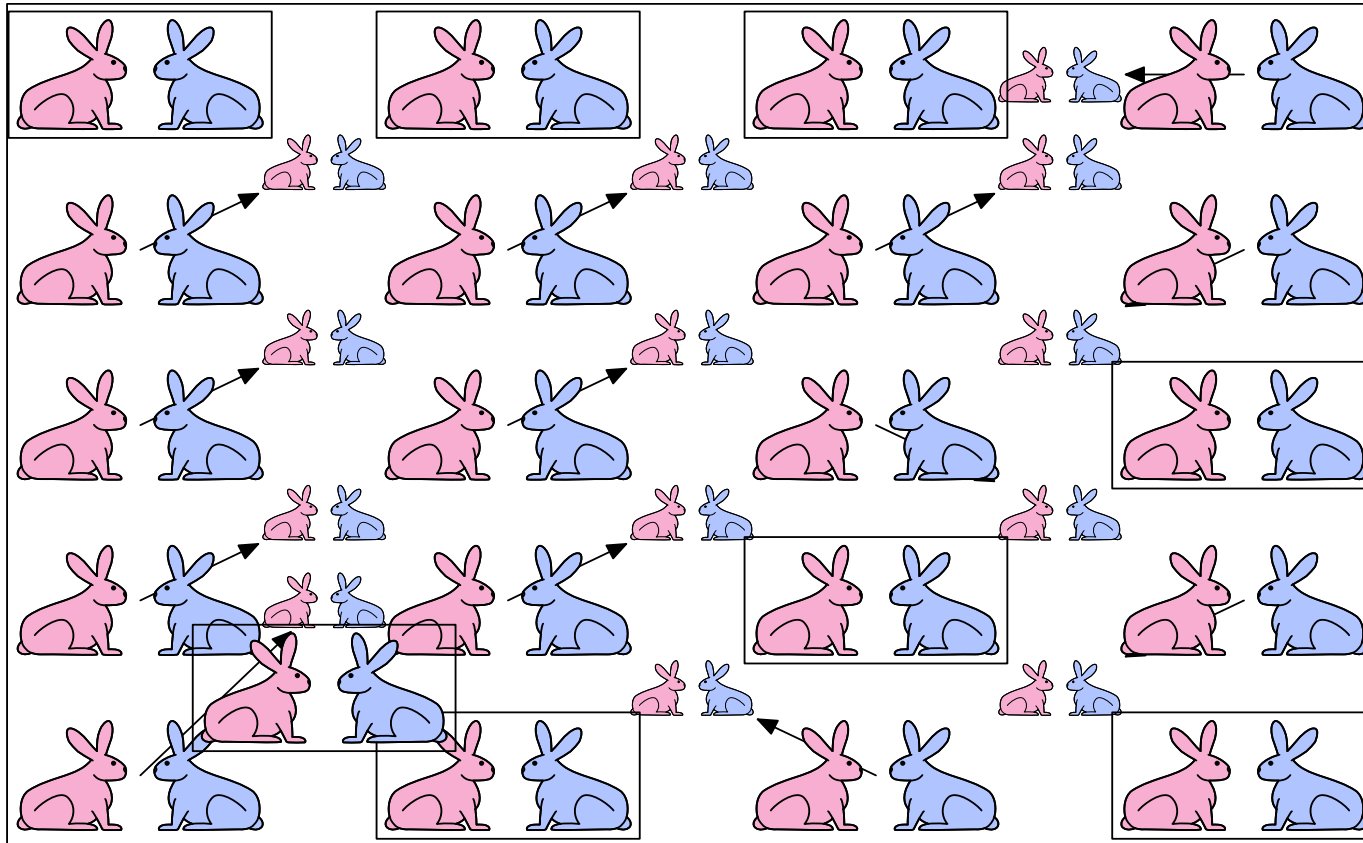


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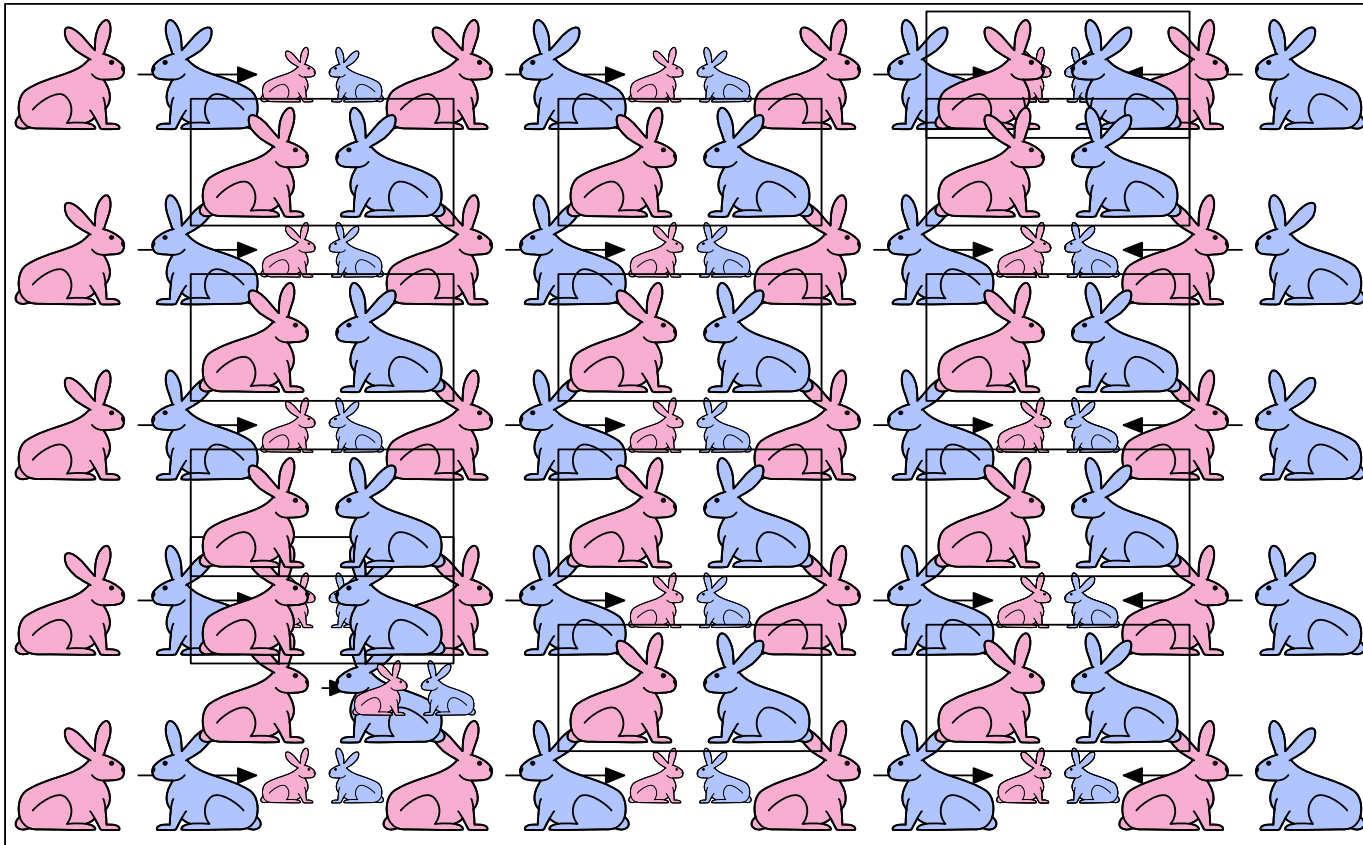


Demo courtesy of Prof. Denny Freeman and Adam Hartz





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FIBONACCI

After one month (call it 0) – 1 female

After second month – still 1 female (now pregnant)

After third month – two females, one pregnant, one not

In general, $\text{females}(n) = \text{females}(n-1) + \text{females}(n-2)$

- Every female alive at month $n-2$ will produce one female in month n ;
- These can be added those alive in month $n-1$ to get total alive in month n

Month	Females
0	1
1	1
2	2
3	3
4	5
5	8
6	13

FIBONACCI

- Base cases:
 - $\text{Females}(0) = 1$
 - $\text{Females}(1) = 1$
- Recursive case
 - $\text{Females}(n) = \text{Females}(n-1) + \text{Females}(n-2)$

```
def fib(x):  
    """assumes x an int >= 0  
        returns Fibonacci of x"""  
    if x == 0 or x == 1:  
        return 1  
    else:  
        return fib(x-1) + fib(x-2)
```


RECURSION ON NON-NUMERICS

- how to check if a string of characters is a palindrome, i.e., reads the same forwards and backwards
 - “Able was I, ere I saw Elba” – attributed to Napoleon
 - “Are we not drawn onward, we few, drawn onward to new era?” – attributed to Anne Michaels



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SOLVING RECURSIVELY?

- First, convert the string to just characters, by stripping out punctuation, and converting upper case to lower case
- Then
 - Base case: a string of length 0 or 1 is a palindrome
 - Recursive case:
 - If first character matches last character, then is a palindrome if middle section is a palindrome

EXAMPLE

- 'Able was I, ere I saw Elba' → 'ablewasiereisawleba'
- `isPalindrome('ablewasiereisawleba')`
is same as
 - `'a' == 'a'` and
`isPalindrome('blewasiereisawleb')`

```

def isPalindrome(s):

    def toChars(s):
        s = s.lower()
        ans = ''
        for c in s:
            if c in 'abcdefghijklmnopqrstuvwxyz':
                ans = ans + c
        return ans

    def isPal(s):
        if len(s) <= 1:
            return True
        else:
            return s[0] == s[-1] and isPal(s[1:-1])

    return isPal(toChars(s))

```

DIVIDE AND CONQUER

- an example of a “divide and conquer” algorithm
- solve a hard problem by breaking it into a set of sub-problems such that:
 - sub-problems are easier to solve than the original
 - solutions of the sub-problems can be combined to solve the original

MODULES AND FILES

- have assumed that all our code is stored in one file
- cumbersome for large collections of code, or for code that should be used by many different other pieces of programming
- a **module** is a `.py` file containing a collection Python definitions and statements

EXAMPLE MODULE

- the file `circle.py` contains

```
pi = 3.14159
```

```
def area(radius):
```

```
    return pi*(radius**2)
```

```
def circumference(radius):
```

```
    return 2*pi*radius
```

■

EXAMPLE MODULE

- then we can import and use this module:

```
import circle
pi = 3
print(pi)
print(circle.pi)
print(circle.area(3))
print(circle.circumference(3))
```

- results in the following being printed:

```
3
3.14159
28.27431
18.849539999999998
```


OTHER IMPORTING

- if we don't want to refer to functions and variables by their module, and the names don't collide with other bindings, then we can use:

```
from circle import *  
  
print(pi)  
  
print(area(3))
```

- this has the effect of creating bindings within the current scope for all objects defined within `circle`
- statements within a module are executed only the first time a module is imported

FILES

- need a way to save our work for later use
- every operating system has its own way of handling files; Python provides an operating-system independent means to access files, using a **file handle**

```
nameHandle = open('kids', 'w')
```

- creates a file named `kids` and returns file handle which we can name and thus reference. The `w` indicates that the file is to be opened for writing into.

FILES: example

```
nameHandle = open('kids', 'w')
for i in range(2):
    name = input('Enter name: ')
    nameHandle.write(name + '\n')
nameHandle.close()
```

FILES: example

```
nameHandle = open('kids', 'r')  
for line in nameHandle:  
    print(line)  
nameHandle.close()
```