Lab A: Using Python as a Calculator

https://mybinder.org/v2/gh/anniebmcc/pycalclab/master?filepath=mat301a.ipynb 2020 Summer — Calculus 1 Dr Matthew H Sunderland

Jupyter Notebooks

A1. **RUN** the following "code cell" (gray rectangle with In[] next to it), by CLICKING the code cell and pressing SHIFT+RETURN. Notice that only the last result will display.

```
In [1]: 1 + 2 + 3
50 - 3
100*5
Out[1]: 500
```

A2. **RUN** the following. As always, only the last result displays, but the last result has 2 parts because of the comma.

```
In [2]: 1 + 2 + 3
50 - 3, 1000*1000
100*5, 7*7
Out[2]: (500, 49)
```

A3. The "+" on the toolbar adds a code cell. The "scissors" deletes a cell.

Python arithmetic + - * / **

A4. **RUN** the following.

```
In [3]: 3 + 10*5, 5**2, 27/10
Out[3]: (53, 25, 2.7)
```

A5. EXERCISE.

- a) What does each of the 5 arithmetic operations do?
- b) Do spaces around the 5 operations matter, or is it just style?

```
In [4]: # TYPE YOUR ANSWERS BELOW
#
# a) + is addition
# - is subtraction
# * is multiplication
# / is division
# ** is exponentiation
# # b) No, spaces aroung + - * / ** don't matter
```

Python # and =

A6. RUN the following. You will notice python ignores everything after #

```
In [5]: # This is a comment
1 + 1 # This is also a comment
Out[5]: 2
```

A7. **RUN** the following. Notice we assign variables using = Assignment itself does NOT produce output.

```
In [6]: a = 10
a
Out[6]: 10
In [7]: b = 20
In [8]: a = 18
b = 21
c = a - b
c
Out[8]: -3
```

A8. **RUN** the following. Notice you can assign multiple variables at once with a comma.

```
In [9]: x, y = 100, 500
x
Out[9]: 100
In [10]: a,b,c = 3,4,5
a + b/c
Out[10]: 3.8
```

A9. **RUN** the following. See that we can compute $\frac{(2-3)*-3}{-1+2}$ all at once (1st cell below), or we can assign variables to help us (2nd cell below).

```
In [11]: (2 - 3)*-3/(-1 + 2)
Out[11]: 3.0
In [12]: top = (2 - 3)*-3
bottom = -1 + 2
top/bottom
Out[12]: 3.0
```

A10. **EXERCISE.** Assign variables to help you compute $3 - \frac{3^2 - 2 \cdot 3}{2 \cdot 3 - 2}$

```
In [13]: # Type your answer below and press SHIFT+ENTER
top = 3**2 - 2*3
bottom = 2*3 - 2
3 - top/bottom
Out[13]: 2.25
```

Order of Operations

A11. **RUN** the following. Notice a - b * c = a - (b * c), but they do not equal (a - b) * c.

```
In [14]: a,b,c = 3,4,5

a - b*c, a - (b*c), (a - b)*c

Out[14]: (-17, -17, -5)
```

A12. **EXERCISE.** In each row, identify NON-equivalent choice. For example, the answer to (1) is (a - b) * c because a - b * c = a - (b * c)

```
(1)
        a - b * c a - (b * c)
                                    (a-b)*c
(2)
       a*(b-c) \qquad (a*b)-c
                                    a*b-c
(3)
       a/b+c
                      a/(b+c)
                                     (a/b) + c
                                     a + b/c
(4)
       (a+b)/c
                      a + (b/c)
                                     a ** b * c
(5)
        a ** (b * c)
                      (a ** b) * c
(6)
       a * (b ** c)
                      a * b ** c
                                     (a * b) ** c
(7)
       a/b ** c
                      (a/b) ** c
                                     a/(b ** c)
(8)
       a ** b/c
                     (a ** b)/c
                                     a ** (b/c)
(9)
       (3-3)-3
                      3 - 3 - 3
                                     3 - (3 - 3)
(10)
       (2 ** 3) ** 2
                     2 ** (3 ** 2)
                                    2 ** 3 ** 2
(11)
        6/3/2
                      6/(3/2)
                                     (6/3)/2
```

```
In [15]: # TYPE YOUR ANSWERS BELOW.
         # (1)
                  (a - b)*c
         # (2)
                  a*(b - c)
         # (3)
                  a/(b + c)
         # (4)
                  (a + b)/c
         # (5)
                  a ** (b*c)
            (6)
                  (a*b) ** c
         # (7)
                  (a/b) ** c
         # (8)
                  a ** (b/c)
         # (9)
                  3 - (3 - 3)
                  (2 ** 3) ** 2
         # (10)
                  6/(3/2)
         # (11)
```

A13. **RUN** the following example, where we add 2 sets of parentheses which show the order of the 2 operations.

```
In [16]: 1 + 3/5
Out[16]: 1.6
In [17]: (1 + (3/5))
Out[17]: 1.6
```

A14. **EXERCISE.** Add 4 sets of parentheses, which show the order of the 4 operations.

```
In [18]: 7 - 3 ** 2/9 + 4

Out[18]: 10.0
```

A15. **EXERCISE.** Assign a,b,c = 4,5,8 and then evaluate $\frac{a^b - c/b}{c-a}$, $\frac{a^{c-b}}{c-b}$, $\frac{a^{3/2}}{b}$, $\frac{a-b(c-a)}{c-a}$

Making python functions

A16. **RUN** the following.

```
In [21]: def g(x):
    return x**2
g(7)
Out[21]: 49
In [22]: def h(n): return n + 100
    h(7)
Out[22]: 107
```

A17. **EXERCISE.** Make the function $P(x) = x^2 - 2x + 1$ and find P(P(7)).

```
In [23]: # Type your answer below and press SHIFT+ENTER

def P(x):
    return x**2 - 2*x + 1

P(P(7))
```

Out[23]: 1225

Built-in %pylab functions

Meaning	Math notation	Python
absolute value	x	abs(x)
square root	$\sqrt{\overline{X}}$	sqrt(x)
exponential function	e^x	exp(x)
natural logarithm	ln x	log(x)
sine	$\sin x$	sin(x)
inverse sine	$\sin^{-1} x$	arcsin(x)
converts degrees to radians		radians(x)

A18. **RUN** the code cells below. The command <code>%pylab</code> only needs to be run once per lab; it loads "built-in functions" (from python packages numpy and matplotlib).

A19. EXERCISE. Evaluate

```
1. sin 40°
```

- 2. $\sin^2 65^\circ$
- 3. $e^{(10-8.5)/3}$
- 4. $\arcsin(\sin(3\pi/4))$

Note. Python uses radians for all angle measurements, so you need to convert any degrees to radians.

Making an array with r_{\parallel}

A20. **RUN** the following. (If you get an error, go back and run A17.) The function $r_{[]}$ can make an array of numbers of your choice. We will need arrays for graphing (Lab B).

A21. **EXERCISE.** Use \mathbf{r} 1 to store the numbers 2,3,5,7,11 in an array named \mathbf{x} . Find $\mathbf{x} \star \mathbf{x}$.

Making an array with r [a:b:stride]

A22. **RUN** the following. In general, $r_{a:b}$ will list integers from a up to but *not* including b. A missing a is the same as 0.

```
In [29]: r_[5:10]
Out[29]: array([5, 6, 7, 8, 9])
In [30]: r_[:5]
Out[30]: array([0, 1, 2, 3, 4])
```

A23. **EXERCISE.** Use $r_{a:b}$ to make the array 1,2,3,4,5,6,7,8,9

```
In [31]: # Type your answer below and press SHIFT+ENTER
    r_[1:10]
Out[31]: array([1, 2, 3, 4, 5, 6, 7, 8, 9])
```

A24. **RUN** the following. In general, r [a:b:stride] spaces out your numbers by the amount stride.

Making an array with linspace(a,b,n)

A26. **RUN** the following. Observe that linspace(a,b,n) lists n numbers from a to b inclusive. This is useful for generating a lot of evenly-spaced numbers, such as when graphing (Lab B). Observe that linspace(a,b) lists 50 numbers from a to b inclusive.

```
In [34]: linspace(0,10,6)
Out[34]: array([ 0., 2., 4., 6., 8., 10.])
In [35]: linspace(0,10)
Out[35]: array([ 0.
                             0.20408163, 0.40816327,
                                                      0.6122449 ,
                                                                  0.81632653,
                1.02040816, 1.2244898,
                                                      1.63265306,
                                        1.42857143,
                                                                  1.83673469,
                2.04081633, 2.24489796, 2.44897959,
                                                     2.65306122,
                                                                  2.85714286,
                             3.26530612, 3.46938776,
                3.06122449,
                                                      3.67346939,
                                                                  3.87755102,
                4.08163265, 4.28571429, 4.48979592,
                                                     4.69387755,
                                                                  4.89795918,
                5.10204082, 5.30612245, 5.51020408,
                                                      5.71428571,
                                                                  5.91836735,
                6.12244898, 6.32653061, 6.53061224,
                                                     6.73469388,
                                                                  6.93877551,
                7.14285714, 7.34693878, 7.55102041,
                                                     7.75510204,
                                                                  7.95918367,
                8.16326531, 8.36734694, 8.57142857, 8.7755102, 8.97959184,
                9.18367347, 9.3877551, 9.59183673, 9.79591837, 10.
                                                                            1)
```

A27. **EXERCISE.** Use linspace(a,b,n) to make the array 1, 1.5, 2, 2.5, 3, 3.5, 4

A28. EXERCISE.

Convert average body temperature $98.6^{\circ}F$ to Celsius using C = 5/9(F - 32).

A29. **RUN** the following.

Notice that x and y are arrays, c[x,y] puts them into a table.

```
In [38]: x = r_{[:10]}
          y = x**2
          c_[x,y]
Out[38]: array([[ 0,
                       0],
                       1],
                 [ 1,
                 [ 2,
                       4],
                 [3, 9],
                 [ 4, 16],
                 [ 5, 25],
                 [ 6, 36],
                 [7, 49],
                 [ 8, 64],
                 [ 9, 81]])
```

A30. EXERCISE.

Use r_ to make an array of Fahrenheit values $x = -100, -80, -60, \dots, 100$. Make the corresponding array of Celsius values y Use c_ to put x and y into a table.

```
In [39]: # Type your answer below and press SHIFT+ENTER
         x = r [-100:101:20]
         y = 5/9*(x - 32)
         c_[x,y]
Out[39]: array([[-100.
                             , -73.33333333],
                             , -62.2222222],
               [ -80.
               [ -60.
                             , -51.111111111,
                            , -40.
               [-40.
                           , -28.88888889],
               [-20.
                           , -17.7777778],
                   0.
                 20.
                            , -6.66666667],
                 40.
                                4.4444444],
                            , 15.5555556],
               [ 60.
                 80.
                               26.66666667],
               [ 100.
                                37.7777778]])
```

Lab B: Plotting Graphs in Python

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Plotting with plot

B1. Example. To graph $f(x) = x^2$ over [-2, 2] by hand, make an xy table: choose some x values,

and then use f to compute the corresponding y values.

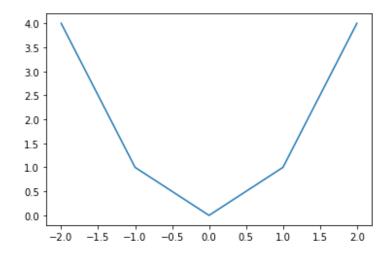
B2. **RUN** the following **twice**. Notice that graphing in python is similar to B1: we make a list of x values and y values.

```
In [1]: %pylab inline

x = r_[-2, -1, 0, 1, 2]
y = r_[4, 1, 0, 1, 4]
plot(x,y)
```

Populating the interactive namespace from numpy and matplotlib

Out[1]: [<matplotlib.lines.Line2D at 0x7fc8a9e8a490>]



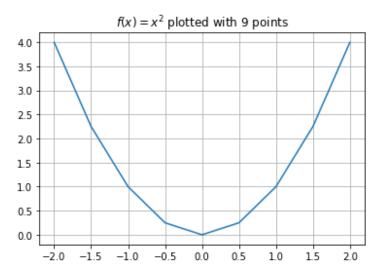
B3. **RUN** the following. Notice that we save time by making the x array using linspace (see A27) and making the y array by doing arithmetic on x (see A29). For illustrative purposes, we use $c_[x y]$ to make a table out of the arrays x and y (see A29).

```
In [2]: x = linspace(-2,2,9)
y = x**2

plot(x,y)
title('$f(x) = x^2$ plotted with 9 points')
grid()

c_[x,y]
```

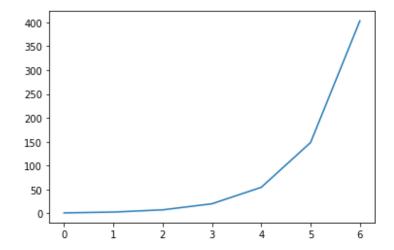
```
Out[2]: array([[-2.
                       4. ],
              [-1.5]
                       2.25],
              [-1.,
                       1. ],
              [-0.5]
                       0.25],
              [ 0.
                       0.],
              [ 0.5 ,
                       0.25],
              [ 1. ,
                       1. ],
              [ 1.5 ,
                      2.25],
              [ 2. ,
                       4. ]])
```



B4. **RUN** the following, which graph $f(x) = e^x$ over the interval [0, 7]. Here we make our array x using $r_{a:b:stride}$ (see A22). Remember that exp(x) is how you write e^x in python (see A18).

```
In [3]: x = r_[:7]
y = exp(x)
plot(x,y)
```

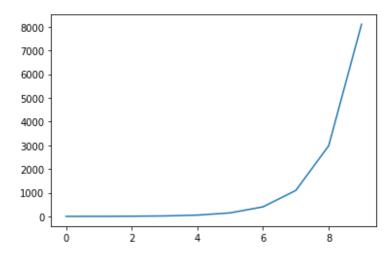
Out[3]: [<matplotlib.lines.Line2D at 0x7fc8aa32d5d0>]



B5. **RUN** the following. When we change the x we must recompute the y; there are two ways to do it (compare B4 to B5).

```
In [4]: x = r_[:10]
plot(x, exp(x))
```

Out[4]: [<matplotlib.lines.Line2D at 0x7fc8aa477210>]



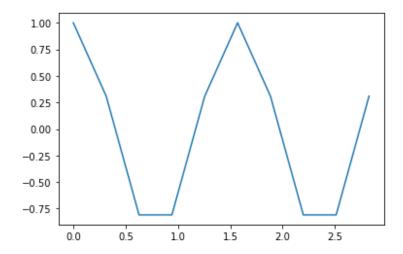
B6. EXERCISE.

- (1) Graph $y = \cos 4x$ over $[0, \pi]$ with a step size of pi/10
- (2) Redo your plot from iii. using x = linspace(0,pi)
- (3) Which plot looks more like the plot of a cosine curve?

```
In [5]: # (1) Type your answer below and press SHIFT+ENTER

x = r_[0:pi:pi/10]
y = cos(4*x)
plot(x,y)
```

Out[5]: [<matplotlib.lines.Line2D at 0x7fc8aa4df510>]

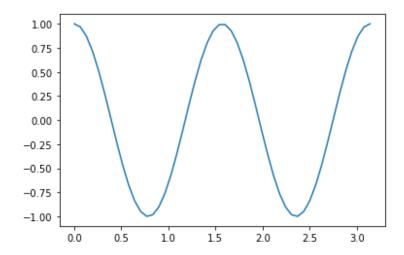


```
In [6]: # (2) Type your answer below and press SHIFT+ENTER

x = linspace(0,pi)
y = cos(4*x)
plot(x,y)

# (3) Your answer: the second plot
```

Out[6]: [<matplotlib.lines.Line2D at 0x7fc8aa621a90>]

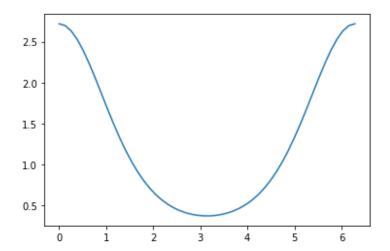


B7. **EXERCISE.** Plot the function $f(x) = e^{\cos x}$ over the interval $[0, 2\pi]$.

```
In [7]: # Type your answer below and press SHIFT+ENTER

x = linspace(0,2*pi)
y = exp(cos(x))
plot(x,y)
```

Out[7]: [<matplotlib.lines.Line2D at 0x7fc8aa6ff910>]



Doing arthmetic on arrays

B8. **RUN** the following.

We make numpy arrays with r_{-} or linspace

Numpy arrays "know" how to do "elementwise" arithmetic.

Warning: x^2 is written x**2.

B9. **RUN** the following.

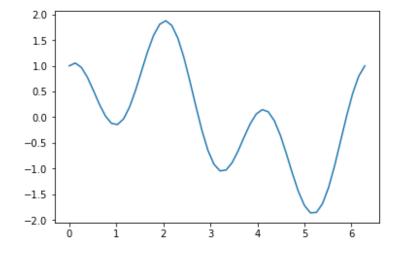
```
In [9]: # We can add arrays of the same shape (same length)
    x = r_[10, 20, 50, 100]
    y = r_[3, 0, 7, -1]
    x + y
Out[9]: array([13, 20, 57, 99])
```

```
In [10]: # We can add an array (x) and a scalar (y)
         x = r_{10}, 20, 50, 100
         y = 100
         x + y
Out[10]: array([110, 120, 150, 200])
In [11]: # We CANNOT add arrays of DIFFERENT shape
         x = r [10, 20, 50, 100]
         y = r_[3, 0, 7]
         x + y
         ValueError
                                                 Traceback (most recent call last)
         <ipython-input-11-ab56767c8fea> in <module>
               3 x = r_{10}, 20, 50, 100
               4 y = r_[3, 0, 7]
         ---> 5 x + y
         ValueError: operands could not be broadcast together with shapes (4,) (3,)
```

B10. **RUN** the following.

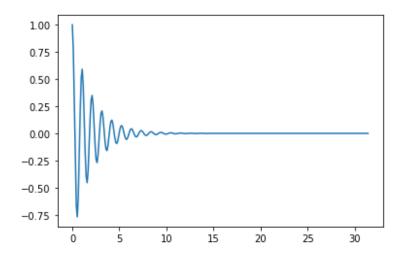
```
In [12]: # y = sin x + cos 3x over the domain [0,2pi]
x = linspace(0,2*pi)
y = sin(x) + cos(3*x)
plot(x,y)
```

Out[12]: [<matplotlib.lines.Line2D at 0x7fc8aa84b710>]



```
In [13]: \# y = e^{(-x/2)} \cos 6x \text{ over the domain } [0,10pi]
x = \text{linspace}(0, 10*pi, 300)
y1 = \exp(-x/2) \# \text{ Here we break up the}
y2 = \cos(6*x) \# \text{ computation into}
y = y1*y2 \# \text{ bite-sized pieces}
\text{plot}(x,y)
```

Out[13]: [<matplotlib.lines.Line2D at 0x7fc8aa94b310>]



```
In [14]: \# y = 1/(x^2 - 1) over the domain [2,5]

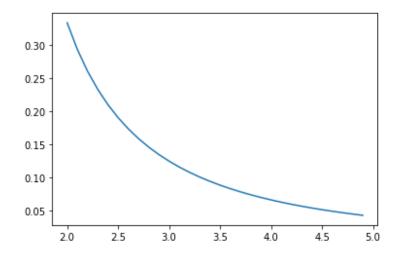
x = r_{2:5:0.1}

y = 1/(x^2 - 1)

y = 1/(x^2 - 1)

y = 1/(x^2 - 1)
```

Out[14]: [<matplotlib.lines.Line2D at 0x7fc8aa166490>]



B11. **EXERCISE.** Define $a,b,c = r_{[1:21:2]}, r_{[1:11]}, r_{[1:12:2]}$

Which of the following are defined?

$$b+c$$
 $a+b$ $a./b$ $a*b$ $a \wedge 2$

```
In [15]: a,b,c = r_{1:21:2}, r_{1:11}, r_{1:12:2}
            # Type your answer below and press SHIFT+ENTER
            a+b, a*b
 Out[15]: (array([ 2, 5, 8, 11, 14, 17, 20, 23, 26, 29]),
             array([ 1, 6, 15, 28, 45, 66, 91, 120, 153, 190]))
B12. RUN the following example. Let x be the array 1,2,3. Write Python commands to compute x^3.
The output you get should be array([ 1, 8, 27]).
 In [16]: x = r_{1,2,3}
 Out[16]: array([ 1, 8, 27])
B13. EXERCISE. Using the same array x = r_{1,2,3}, find:
              \sin^2 x \qquad \sin x^2 \qquad 7x^2 \sin \frac{1}{7x^2}
\cos x \sin x
You should get
array([ 0.45464871, -0.37840125, -0.13970775])
array([0.70807342, 0.82682181, 0.01991486])
array([ 0.84147098, -0.7568025 , 0.41211849])
array([0.99660211, 0.99978743, 0.99995801])
 In [17]: # Type your answer below and press SHIFT+ENTER
           \cos(x) \cdot \sin(x), \sin(x) \cdot x^2, \sin(x \cdot x^2), 7 \cdot x \cdot x^2 \cdot \sin(1/(7 \cdot x \cdot x^2))
 Out[17]: (array([ 0.45464871, -0.37840125, -0.13970775]),
             array([0.70807342, 0.82682181, 0.01991486]),
             array([ 0.84147098, -0.7568025 , 0.41211849]),
             array([0.99660211, 0.99978743, 0.99995801]))
B14. EXERCISE. Using the same array x = r_{1,2,3}, find:
x - \frac{\cos x - \sin x}{\sin x + \cos x}
             \frac{1}{10}(x-\frac{x^{3/2}}{10})^2
You should get
array([1.2179581 , 4.68770694, 1.66751188])
array([0.081 , 0.29486292, 0.61523085])
 In [18]: # Type your answer below and press SHIFT+ENTER
            x - (\cos(x) - \sin(x))/(\sin(x) + \cos(x)), \quad 1/10*(x - x**(3/2)/10)**2
 Out[18]: (array([1.2179581 , 4.68770694, 1.66751188]),
             array([0.081 , 0.29486292, 0.61523085]))
```

Graphing practice

B15 EXERCISE.

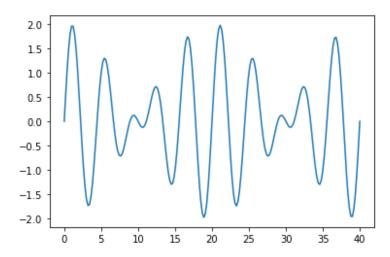
- (1) Graph the function $f(x) = \sin(\frac{\pi}{2}x) + \sin(\frac{2}{5}\pi x)$ over the interval [0, 40].
- (2) How many peaks (relative maxima) does your graph B21 have?
- (3) The function in B21 is periodic; how many periods are graphed in [0, 40]?
- (4) Estimate from your graph B21 the value of f(10) to 1 decimal point.

```
In [19]: # (1) Type your answer below and press SHIFT+ENTER

x = linspace(0,40,200)
y = sin(pi/2*x) + sin(2/5*pi*x)
plot(x,y)

# (2) Your answer: 10
# (3) Your answer: 2
# (4) Your answer: 0.0
```

Out[19]: [<matplotlib.lines.Line2D at 0x7fc8aa0eb090>]



B16. EXERCISE.

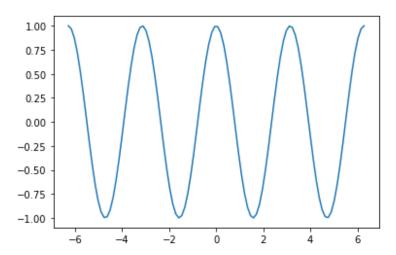
- (1) Graph $f(x) = \cos^2 x \sin^2 x$ over the interval $[-2\pi, 2\pi]$ using 100 points.
- (2) Does the resemble any of the following? $\cos 2x \qquad \cos x/2 \qquad \cos x$

```
In [20]: # (1) Type your answer below and press SHIFT+ENTER

x = linspace(-2*pi, 2*pi, 100)
y = cos(x)**2 - sin(x)**2
plot(x,y)

# (2) Your answer: cos(2x)
```

Out[20]: [<matplotlib.lines.Line2D at 0x7fc8aab1d790>]



B17. EXERCISE.

- (1) Plot the polynomial function $f(x) = x^3 20x^2 + 10x 1$ over the interval [-10, 10].
- (2) Which is the approximate range for the *y*-axis?

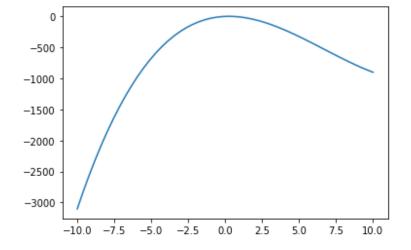
[-10, 10] (-10, 10) [-3100, 0] $[0, 2\pi]$

```
In [21]: # (1) Type your answer below and press SHIFT+ENTER

x = linspace(-10,10)
y = x**3 - 20*x**2 + 10*x - 1
plot(x,y)

# (2) Your answer: [-3100,0]
```

Out[21]: [<matplotlib.lines.Line2D at 0x7fc8aab84d50>]



B18. **EXERCISE.** We wish to investigate when (if) the function in B17 is positive. We can't readily tell from our graph in B17 so we will replot over a smaller domain.

 $[0, 2\pi]$

(1). Which of these domains seems appropriate for this task?

[0, 500]

- [0, 10]
- [-1, 1]
- (2) Replot the graph over the selected domain. Turn on the grid using grid()
- (3) From your graph, which of these x values have f(x) > 0? Indicate all that apply:

0

- 0.25
- 0.50
- 0.75

```
In [22]: # (1) Your answer: [-1,1]
# (2) Type your answer below and press SHIFT+ENTER

x = linspace(-1,1)
y = x**3 - 20*x**2 + 10*x - 1
plot(x,y)
grid()

# (3) Your answer: 0.25
```

