Lab A: Using Python as a Calculator

https://mybinder.org/v2/gh/anniebmcc/pycalclab/master?filepath=mat301a.ipynb

2020 Summer — Calculus 1

Dr Matthew H Sunderland

Jupyter Notebooks

A1. **RUN** the following "code cell" (gray rectangle with `In[ ]` next to it), by CLICKING the code cell and pressing SHIFT+RETURN. Notice that only the last result will display.

```
In [1]: 1 + 2 + 3
        50 - 3
        100*5
```

```
Out[1]: 500
```

A2. **RUN** the following. As always, only the last result displays, but the last result has 2 parts because of the comma.

```
In [2]: 1 + 2 + 3
        50 - 3, 1000*1000
        100*5, 7*7
```

```
Out[2]: (500, 49)
```

A3. The "+" on the toolbar adds a code cell. The "scissors" deletes a cell.

Python arithmetic `+ - * / **`

A4. **RUN** the following.

```
In [3]: 3 + 10*5,  5**2,  27/10
```

```
Out[3]: (53, 25, 2.7)
```

A5. **EXERCISE.**

a) What does each of the 5 arithmetic operations do?

b) Do spaces around the 5 operations matter, or is it just style?

In [4]:
```
# TYPE YOUR ANSWERS BELOW
#
#  a) + is   addition
#     - is   subtraction
#     * is   multiplication
#     / is   division
#    ** is   exponentiation
#
#  b) No, spaces aroung + - * / ** don't matter
```

Python # and =

A6. **RUN** the following. You will notice python ignores everything after #

In [5]:
```
# This is a comment
1 + 1 # This is also a comment
```

Out[5]: 2

A7. **RUN** the following. Notice we assign variables using = Assignment itself does NOT produce output.

In [6]:
```
a = 10
a
```

Out[6]: 10

In [7]:
```
b = 20
```

In [8]:
```
a = 18
b = 21
c = a - b
c
```

Out[8]: -3

A8. **RUN** the following. Notice you can assign multiple variables at once with a comma.

In [9]:
```
x, y = 100, 500
x
```

Out[9]: 100

In [10]:
```
a,b,c = 3,4,5
a + b/c
```

Out[10]: 3.8

A9. **RUN** the following. See that we can compute $\frac{(2-3)*-3}{-1+2}$ all at once (1st cell below), or we can assign variables to help us (2nd cell below).

In [11]:
```
(2 - 3)*-3/(-1 + 2)
```

Out[11]: 3.0

In [12]:
```
top = (2 - 3)*-3
bottom = -1 + 2
top/bottom
```

Out[12]: 3.0

A10. **EXERCISE.** Assign variables to help you compute $3 - \frac{3^2-2\cdot3}{2\cdot3-2}$

In [13]:
```
# Type your answer below and press SHIFT+ENTER

top = 3**2 - 2*3
bottom = 2*3 - 2
3 - top/bottom
```

Out[13]: 2.25

Order of Operations

A11. **RUN** the following. Notice $a - b * c = a - (b * c)$, but they do not equal $(a - b) * c$.

In [14]:
```
a,b,c = 3,4,5

a - b*c,  a - (b*c),  (a - b)*c
```

Out[14]: (-17, -17, -5)

A12. **EXERCISE.** In each row, identify NON-equivalent choice. For example, the answer to (1) is $(a - b) * c$ because $a - b * c = a - (b * c)$

(1)	$a - b * c$	$a - (b * c)$	$(a - b) * c$
(2)	$a * (b - c)$	$(a * b) - c$	$a * b - c$
(3)	$a / b + c$	$a / (b + c)$	$(a / b) + c$
(4)	$(a + b) / c$	$a + (b / c)$	$a + b / c$
(5)	$a ** (b * c)$	$(a ** b) * c$	$a ** b * c$
(6)	$a * (b ** c)$	$a * b ** c$	$(a * b) ** c$
(7)	$a/b ** c$	$(a/b) ** c$	$a/(b ** c)$
(8)	$a ** b/c$	$(a ** b)/c$	$a ** (b/c)$
(9)	$(3 - 3) - 3$	$3 - 3 - 3$	$3 - (3 - 3)$
(10)	$(2 ** 3) ** 2$	$2 ** (3 ** 2)$	$2 ** 3 ** 2$
(11)	$6 / 3 / 2$	$6 / (3 / 2)$	$(6 / 3) / 2$

In [15]:
```
# TYPE YOUR ANSWERS BELOW.
#
#  (1)   (a - b)*c
#  (2)   a*(b - c)
#  (3)   a/(b + c)
#  (4)   (a + b)/c
#  (5)   a ** (b*c)
#  (6)   (a*b) ** c
#  (7)   (a/b) ** c
#  (8)   a ** (b/c)
#  (9)   3 - (3 - 3)
# (10)   (2 ** 3) ** 2
# (11)   6/(3/2)
```

A13. **RUN** the following example, where we add 2 sets of parentheses which show the order of the 2 operations.

In [16]:
```
1 + 3/5
```

Out[16]: 1.6

In [17]:
```
(1 + (3/5))
```

Out[17]: 1.6

A14. **EXERCISE.** Add 4 sets of parentheses, which show the order of the 4 operations.

In [18]:
```
7 - 3 ** 2/9 + 4
```

Out[18]: 10.0

In [19]:
```
# Type your answer below and press SHIFT+ENTER

((7 - ((3 ** 2)/9)) + 4)
```

Out[19]: 10.0

A15. **EXERCISE.** Assign `a,b,c = 4,5,8` and then evaluate $\frac{a^b - c/b}{c-a}, \frac{a^{c-b}}{c-b}, \frac{a^{3/2}}{b}, \frac{a-b(c-a)}{c-a}$

In [20]:
```
# Type your answer below and press SHIFT+ENTER

a,b,c = 4,5,8

(a**b - c/b)/(c-a),  a**(c-b)/(c-b),  a**(3/2)/b,  (a - b*(c-a))/(c-a)
```

Out[20]: (255.6, 21.333333333333332, 1.6, -4.0)

Making python functions

A16. **RUN** the following.

In [21]:
```
def g(x):
    return x**2

g(7)
```

Out[21]: 49

In [22]:
```
def h(n): return n + 100

h(7)
```

Out[22]: 107

A17. **EXERCISE.** Make the function $P(x) = x^2 - 2x + 1$ and find $P(P(7))$.

In [23]:
```
# Type your answer below and press SHIFT+ENTER

def P(x):
    return x**2 - 2*x + 1

P(P(7))
```

Out[23]: 1225

Built-in `%pylab` functions

Python	Math notation	Meaning
`abs(x)`	$\|x\|$	absolute value
`sqrt(x)`	$\sqrt{x}$	square root
`exp(x)`	e^x	exponential function
`log(x)`	$\ln x$	natural logarithm
`sin(x)`	$\sin x$	sine
`arcsin(x)`	$\sin^{-1} x$	inverse sine
`radians(x)`		converts degrees to radians

A18. **RUN** the code cells below. The command `%pylab` only needs to be run once per lab; it loads "built-in functions" (from python packages numpy and matplotlib).

In [24]:
```
%pylab

sqrt(49)
```

```
Using matplotlib backend: MacOSX
Populating the interactive namespace from numpy and matplotlib
```

Out[24]:
```
7.0
```

In [25]:
```
pi, exp(1), sin(pi/2)
```

Out[25]:
```
(3.141592653589793, 2.718281828459045, 1.0)
```

A19. **EXERCISE.** Evaluate

1. $\sin 40^\circ$
2. $\sin^2 65^\circ$
3. $e^{(10-8.5)/3}$
4. $\arcsin(\sin(3\pi/4))$

Note. Python uses radians for all angle measurements, so you need to convert any degrees to radians.

In [26]:
```
# Type your answer below and press SHIFT+ENTER

sin(radians(40)),  sin(radians(65))**2,  exp((10-8.5)/3),  arcsin(sin(3*pi/
4))
```

Out[26]:
```
(0.6427876096865393,
 0.8213938048432696,
 1.6487212707001282,
 0.7853981633974484)
```

Making an array with `r_[ ]`

A20. **RUN** the following. (If you get an error, go back and run A17.) The function `r_[ ]` can make an array of numbers of your choice. We will need arrays for graphing (Lab B).

```
In [27]: x = r_[2,3,4,5,10]
         x**3

Out[27]: array([   8,   27,   64,  125, 1000])
```

A21. **EXERCISE.** Use `r_[ ]` to store the numbers 2,3,5,7,11 in an array named `x`. Find `x*x`.

```
In [28]: # Type your answer below and press SHIFT+ENTER

         x = r_[2,3,5,7,11]
         x*x

Out[28]: array([  4,   9,  25,  49, 121])
```

Making an array with `r_[a:b:stride]`

A22. **RUN** the following. In general, `r_[a:b]` will list integers from a up to but *not* including b. A missing a is the same as 0.

```
In [29]: r_[5:10]

Out[29]: array([5, 6, 7, 8, 9])
```

```
In [30]: r_[:5]

Out[30]: array([0, 1, 2, 3, 4])
```

A23. **EXERCISE.** Use `r_[a:b]` to make the array 1,2,3,4,5,6,7,8,9

```
In [31]: # Type your answer below and press SHIFT+ENTER

         r_[1:10]

Out[31]: array([1, 2, 3, 4, 5, 6, 7, 8, 9])
```

A24. **RUN** the following. In general, `r_[a:b:stride]` spaces out your numbers by the amount `stride`.

```
In [32]: r_[0:100:2]

Out[32]: array([ 0,  2,  4,  6,  8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32,
                34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66,
                68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98])
```

A25. **EXERCISE.** Use `r_[a:b:stride]` to make the array $1, 3, 5, \ldots, 99$

```
In [33]: # Type your answer below and press SHIFT+ENTER

         r_[1:100:2]
```

```
Out[33]: array([ 1,  3,  5,  7,  9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33,
                35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67,
                69, 71, 73, 75, 77, 79, 81, 83, 85, 87, 89, 91, 93, 95, 97, 99])
```

Making an array with `linspace(a,b,n)`

A26. **RUN** the following. Observe that `linspace(a,b,n)` lists n numbers from a to b inclusive. This is useful for generating a lot of evenly-spaced numbers, such as when graphing (Lab B). Observe that `linspace(a,b)` lists 50 numbers from a to b inclusive.

```
In [34]: linspace(0,10,6)
```

```
Out[34]: array([ 0.,  2.,  4.,  6.,  8., 10.])
```

```
In [35]: linspace(0,10)
```

```
Out[35]: array([ 0.        ,  0.20408163,  0.40816327,  0.6122449 ,  0.81632653,
                 1.02040816,  1.2244898 ,  1.42857143,  1.63265306,  1.83673469,
                 2.04081633,  2.24489796,  2.44897959,  2.65306122,  2.85714286,
                 3.06122449,  3.26530612,  3.46938776,  3.67346939,  3.87755102,
                 4.08163265,  4.28571429,  4.48979592,  4.69387755,  4.89795918,
                 5.10204082,  5.30612245,  5.51020408,  5.71428571,  5.91836735,
                 6.12244898,  6.32653061,  6.53061224,  6.73469388,  6.93877551,
                 7.14285714,  7.34693878,  7.55102041,  7.75510204,  7.95918367,
                 8.16326531,  8.36734694,  8.57142857,  8.7755102 ,  8.97959184,
                 9.18367347,  9.3877551 ,  9.59183673,  9.79591837, 10.        ])
```

A27. **EXERCISE.** Use `linspace(a,b,n)` to make the array $1, 1.5, 2, 2.5, 3, 3.5, 4$

```
In [36]: # Type your answer below and press SHIFT+ENTER

         linspace(1,4,7)
```

```
Out[36]: array([1. , 1.5, 2. , 2.5, 3. , 3.5, 4. ])
```

A28. **EXERCISE.**
Convert average body temperature $98.6^\circ F$ to Celsius using $C = 5/9(F - 32)$.

```
In [37]: # Type your answer below and press SHIFT+ENTER

         5/9*(98.6 - 32)
```

```
Out[37]: 37.0
```

A29. **RUN** the following.
Notice that `x` and `y` are arrays,
`c_[x,y]` puts them into a table.

In [38]:
```
x = r_[:10]
y = x**2
c_[x,y]
```

Out[38]:
```
array([[ 0,  0],
       [ 1,  1],
       [ 2,  4],
       [ 3,  9],
       [ 4, 16],
       [ 5, 25],
       [ 6, 36],
       [ 7, 49],
       [ 8, 64],
       [ 9, 81]])
```

A30. **EXERCISE.**
Use `r_` to make an array of Fahrenheit values `x` $= -100, -80, -60, \ldots, 100.$
Make the corresponding array of Celsius values `y`
Use `c_` to put `x` and `y` into a table.

In [39]:
```
# Type your answer below and press SHIFT+ENTER

x = r_[-100:101:20]
y = 5/9*(x - 32)
c_[x,y]
```

Out[39]:
```
array([[-100.        ,  -73.33333333],
       [ -80.        ,  -62.22222222],
       [ -60.        ,  -51.11111111],
       [ -40.        ,  -40.        ],
       [ -20.        ,  -28.88888889],
       [   0.        ,  -17.77777778],
       [  20.        ,   -6.66666667],
       [  40.        ,    4.44444444],
       [  60.        ,   15.55555556],
       [  80.        ,   26.66666667],
       [ 100.        ,   37.77777778]])
```

Lab B: Plotting Graphs in Python

https://mybinder.org/v2/gh/anniebmcc/pycalclab/master?filepath=mat301b.ipynb

2020 Summer — Calculus 1

Dr Matthew H Sunderland

Plotting with `plot`

B1. Example. To graph $f(x) = x^2$ over $[-2, 2]$ by hand, make an xy table: choose some x values,

x	-2	-1	0	1	2
y					

and then use f to compute the corresponding y values.

B2. **RUN** the following. Notice that graphing in python is similar to B1: we make a list of x values and y values.

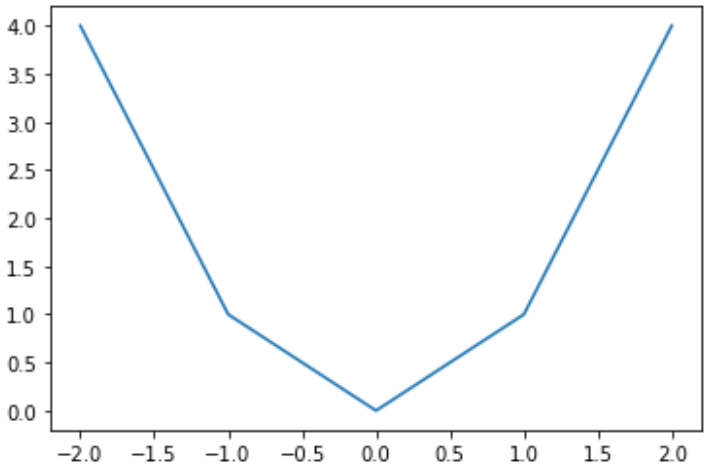
Note to instructor: you may remember that in A18 we wrote `%pylab` and here we write `%pylab inline`; the "inline" tells Jupyter to display images inline instead of as a pop-up.

In [1]:
```
%pylab inline
```

```
Populating the interactive namespace from numpy and matplotlib
```

In [2]:
```
x = r_[-2, -1, 0, 1, 2]
y = r_[4, 1, 0, 1, 4]
plot(x,y)
```

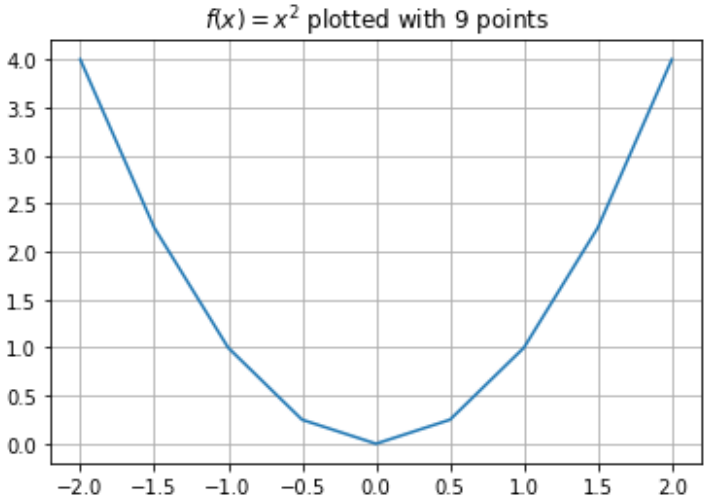
Out[2]:
```
[<matplotlib.lines.Line2D at 0x7f9e427944d0>]
```



B3. **RUN** the following. Notice that we save time by making the x array using `linspace` (see A27) and making the y array by doing arithmetic on x (see A29). For illustrative purposes, we use `c_[x,y]` to make a table out of the arrays x and y (see A29).

In [3]:
```
x = linspace(-2,2,9)
y = x**2

plot(x,y)
title('$f(x) = x^2$ plotted with 9 points')
grid()

c_[x,y]
```

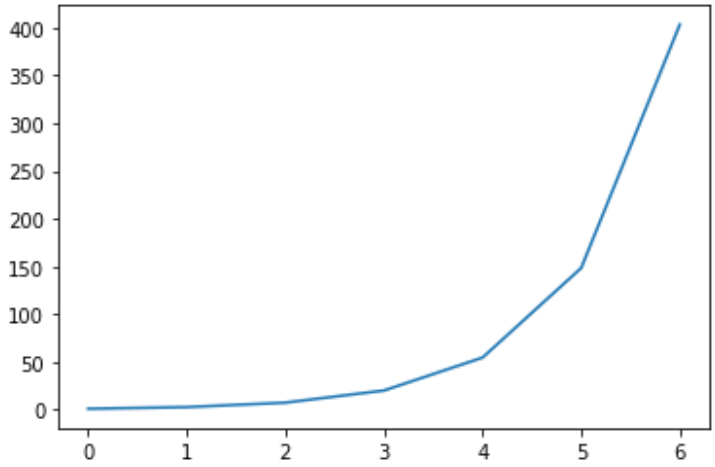
Out[3]:
```
array([[-2.  ,  4.  ],
       [-1.5 ,  2.25],
       [-1.  ,  1.  ],
       [-0.5 ,  0.25],
       [ 0.  ,  0.  ],
       [ 0.5 ,  0.25],
       [ 1.  ,  1.  ],
       [ 1.5 ,  2.25],
       [ 2.  ,  4.  ]])
```



B4. **RUN** the following, which graph $f(x) = e^x$ over the interval $[0, 7]$. Here we make our array x using `r_[a:b:stride]` (see A22). Remember that `exp(x)` is how you write e^x in python (see A18).

In [4]:
```
x = r_[:7]
y = exp(x)
plot(x,y)
```

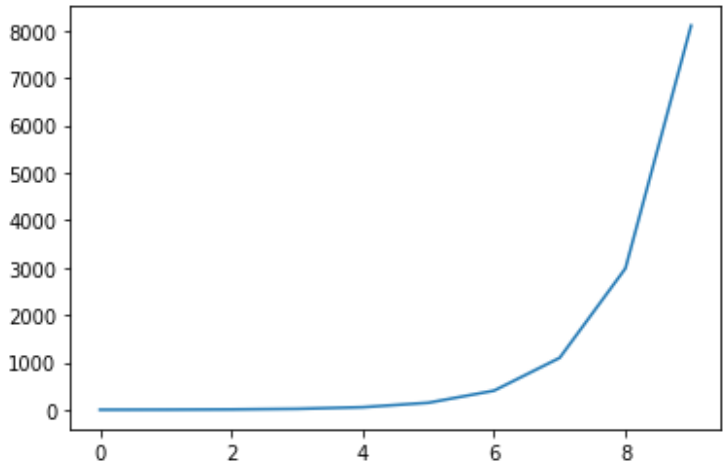
Out[4]:
```
[<matplotlib.lines.Line2D at 0x7f9e42a15a90>]
```



B5. **RUN** the following. When we change the x we must recompute the y; there are two ways to do it (compare B4 to B5).

In [5]:
```
x = r_[:10]
plot(x, exp(x))
```

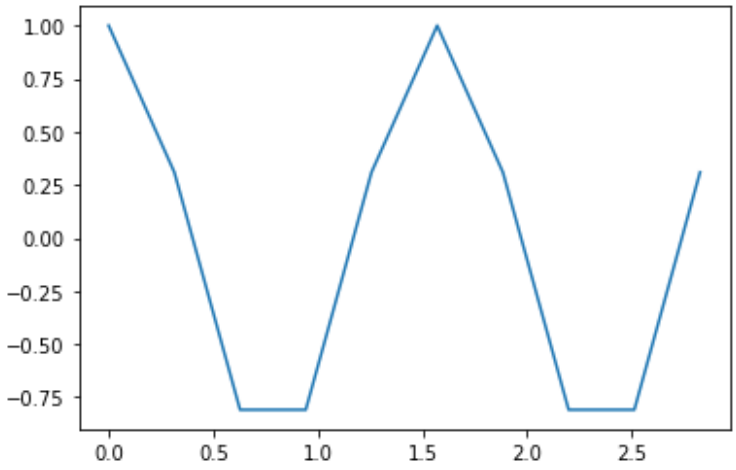
Out[5]:
```
[<matplotlib.lines.Line2D at 0x7f9e42b71850>]
```



B6. **EXERCISE.**
(1) Graph $y = \cos 4x$ over $[0, \pi]$ with a step size of `pi/10`
(2) Redo your plot from iii. using `x = linspace(0,pi)`
(3) Which plot looks more like the plot of a cosine curve?

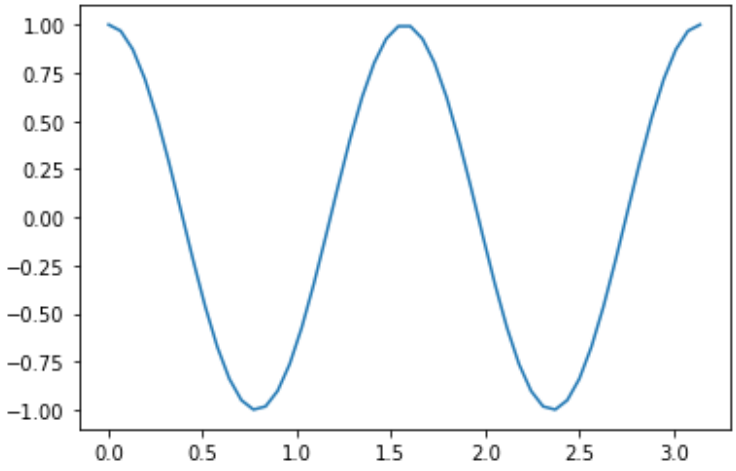
In [6]:
```
# (1) Type your answer below and press SHIFT+ENTER

x = r_[0:pi:pi/10]
y = cos(4*x)
plot(x,y)
```

Out[6]: [<matplotlib.lines.Line2D at 0x7f9e42be5090>]



In [7]:
```
# (2) Type your answer below and press SHIFT+ENTER

x = linspace(0,pi)
y = cos(4*x)
plot(x,y)

# (3) Your answer: the second plot
```

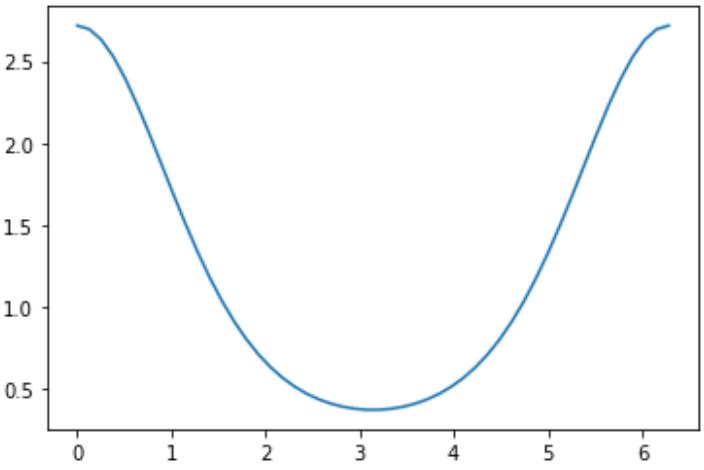
Out[7]: [<matplotlib.lines.Line2D at 0x7f9e42cba150>]



B7. **EXERCISE.** Plot the function $f(x) = e^{\cos x}$ over the interval $[0, 2\pi]$.

In [8]:
```
# Type your answer below and press SHIFT+ENTER

x = linspace(0,2*pi)
y = exp(cos(x))
plot(x,y)
```

Out[8]:
```
[<matplotlib.lines.Line2D at 0x7f9e42f14950>]
```



Doing arthmetic on arrays

B8. **RUN** the following.
We make numpy arrays with `r_` or `linspace`
Numpy arrays "know" how to do "elementwise" arithmetic.
Warning: x^2 is written `x**2` .

In [9]:
```
x = r_[1:5]

x,  10 - x,  x + 10,  10*x,  x**2,  12/x,  x**x,  10**x
```

Out[9]:
```
(array([1, 2, 3, 4]),
 array([9, 8, 7, 6]),
 array([11, 12, 13, 14]),
 array([10, 20, 30, 40]),
 array([ 1,  4,  9, 16]),
 array([12.,  6.,  4.,  3.]),
 array([  1,   4,  27, 256]),
 array([   10,   100,  1000, 10000]))
```

B9. **RUN** the following.

In [10]:
```
# We can add arrays of the same shape (same length)

x = r_[10, 20, 50, 100]
y = r_[3, 0, 7, -1]
x + y
```

Out[10]:
```
array([13, 20, 57, 99])
```

In [11]:
```
# We can add an array (x) and a scalar (y)

x = r_[10, 20, 50, 100]
y = 100
x + y
```

Out[11]:
```
array([110, 120, 150, 200])
```

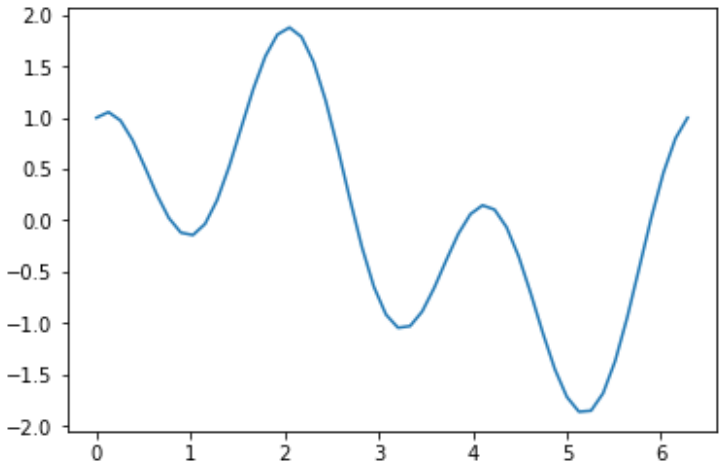
In [12]:
```
# We CANNOT add arrays of DIFFERENT shape

x = r_[10, 20, 50, 100]
y = r_[3, 0, 7]
x + y
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
<ipython-input-12-ab56767c8fea> in <module>
      3 x = r_[10, 20, 50, 100]
      4 y = r_[3, 0, 7]
----> 5 x + y

ValueError: operands could not be broadcast together with shapes (4,) (3,)
```

B10. **RUN** the following.

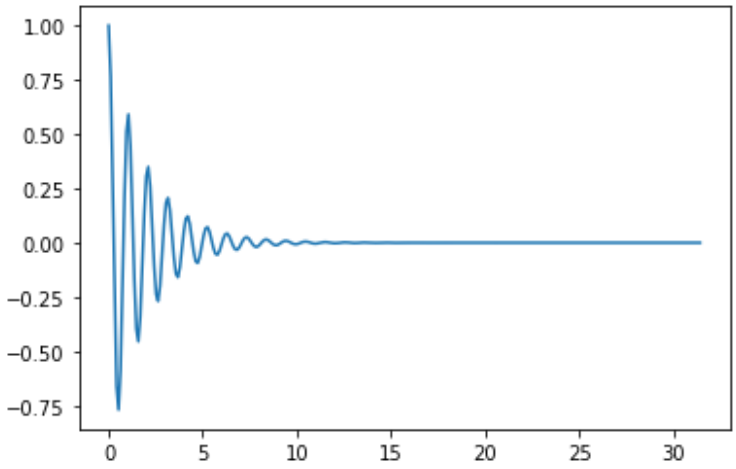
In [13]:
```
# y = sin x + cos 3x over the domain [0,2pi]

x = linspace(0,2*pi)
y = sin(x) + cos(3*x)
plot(x,y)
```

Out[13]:
```
[<matplotlib.lines.Line2D at 0x7f9e43060090>]
```



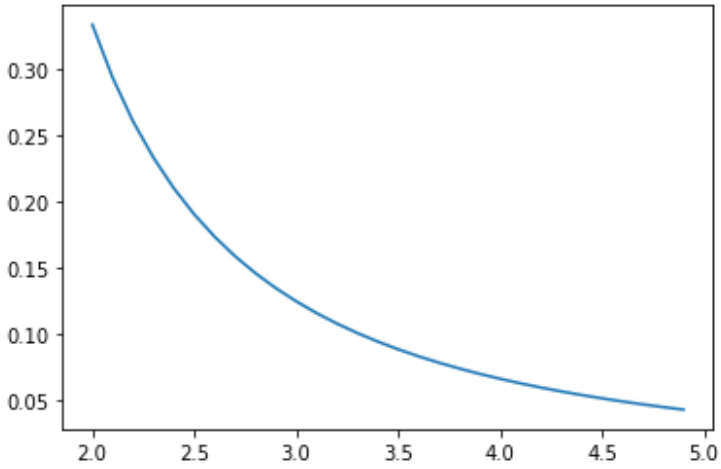
In [14]:
```
# y = e^(-x/2) cos 6x over the domain [0,10pi]

x = linspace(0, 10*pi, 300)
y1 = exp(-x/2) # Here we break up the
y2 = cos(6*x)  # computation into
y = y1*y2      # bite-sized pieces
plot(x,y)
```

Out[14]:
```
[<matplotlib.lines.Line2D at 0x7f9e43144b90>]
```



In [15]:
```
# y = 1/(x^2 - 1) over the domain [2,5]

x = r_[2:5:0.1]
y = 1/(x**2 - 1)
plot(x,y)
```

Out[15]:
```
[<matplotlib.lines.Line2D at 0x7f9e432306d0>]
```



B11. **EXERCISE.** First **RUN** the following.

In [16]:
```
a,b,c = r_[:5], r_[:50:10], r_[:10]
a,b,c
```

Out[16]:
```
(array([0, 1, 2, 3, 4]),
 array([ 0, 10, 20, 30, 40]),
 array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9]))
```

Now, that we've defined a, b, c, which of the following are defined?

$a + b$ $a + c$ $a + 1$ $a * b$ $c ** 2$ $c \hat{} 2$

In [17]:
```
# Type your answer below and press SHIFT+ENTER

a+b, a+1, a*b, c**2
```

Out[17]:
```
(array([ 0, 11, 22, 33, 44]),
 array([1, 2, 3, 4, 5]),
 array([  0,  10,  40,  90, 160]),
 array([ 0,  1,  4,  9, 16, 25, 36, 49, 64, 81]))
```

B12. **RUN** the following example. Let x be the array 1,2,3. Write Python commands to compute x^3.
The output you get should be `array([ 1,  8, 27])` .

In [18]:
```
x = r_[1,2,3]
x**3
```

Out[18]:
```
array([ 1,  8, 27])
```

B13. **EXERCISE.** Using the same array `x = r_[1,2,3]` , find:

$\cos x \sin x$ $\sin^2 x$ $\sin x^2$ $7x^2 \sin \frac{1}{7x^2}$

You should get

```
array([ 0.45464871, -0.37840125, -0.13970775])
array([0.70807342, 0.82682181, 0.01991486])
array([ 0.84147098, -0.7568025 ,  0.41211849])
array([0.99660211, 0.99978743, 0.99995801])
```

In [19]:
```
# Type your answer below and press SHIFT+ENTER

cos(x)*sin(x),  sin(x)**2,  sin(x**2),  7*x**2*sin(1/(7*x**2))
```

Out[19]:
```
(array([ 0.45464871, -0.37840125, -0.13970775]),
 array([0.70807342, 0.82682181, 0.01991486]),
 array([ 0.84147098, -0.7568025 ,  0.41211849]),
 array([0.99660211, 0.99978743, 0.99995801]))
```

B14. **EXERCISE.** Using the same array `x = r_[1,2,3]` , find:

$x - \frac{\cos x - \sin x}{\sin x + \cos x}$ $\qquad$ $\frac{1}{10}(x - \frac{x^{3/2}}{10})^2$

You should get

```
array([1.2179581 , 4.68770694, 1.66751188])
array([0.081     , 0.29486292, 0.61523085])
```

In [20]:
```
# Type your answer below and press SHIFT+ENTER

x - (cos(x)-sin(x))/(sin(x)+cos(x)),   1/10*(x - x**(3/2)/10)**2
```

Out[20]:
```
(array([1.2179581 , 4.68770694, 1.66751188]),
 array([0.081     , 0.29486292, 0.61523085]))
```

Graphing practice

B15 **EXERCISE.**

(1) Graph the function $f(x) = \sin(\frac{\pi}{2}x) + \sin(\frac{2}{5}\pi x)$ over the interval $[0, 40]$.

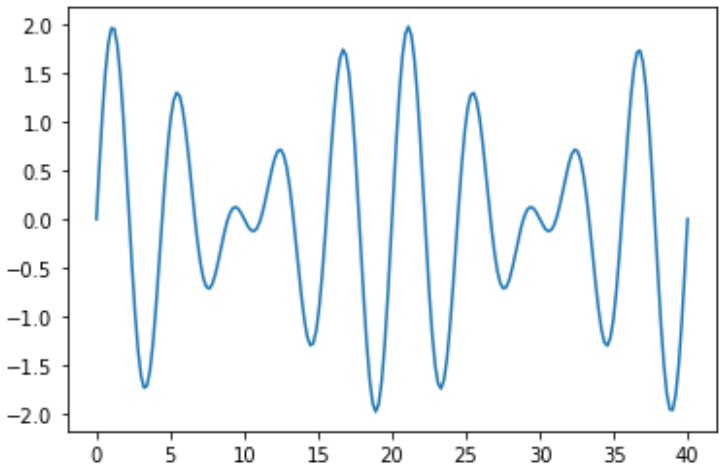
(2) How many peaks (relative maxima) does your graph have?

(3) This function is periodic; how many periods are graphed in $[0, 40]$?

(4) Estimate from your graph the value of $f(10)$ to 1 decimal point.

In [21]:
```
# (1) Type your answer below and press SHIFT+ENTER

x = linspace(0,40,200)
y = sin(pi/2*x) + sin(2/5*pi*x)
plot(x,y)

# (2) Your answer: 10
# (3) Your answer: 2
# (4) Your answer: 0.0
```

Out[21]:
```
[<matplotlib.lines.Line2D at 0x7f9e4296a5d0>]
```



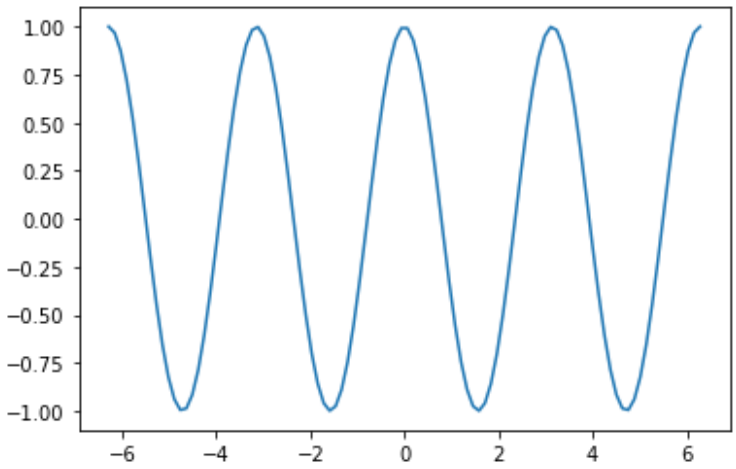
B16. **EXERCISE.**

(1) Graph $f(x) = \cos^2 x - \sin^2 x$ over the interval $[-2\pi, 2\pi]$ using 100 points.

(2) Does the resemble any of the following? $\cos 2x$ $\cos x/2$ $\cos x$

In [22]:
```
# (1) Type your answer below and press SHIFT+ENTER

x = linspace(-2*pi, 2*pi, 100)
y = cos(x)**2 - sin(x)**2
plot(x,y)

# (2) Your answer:  cos(2x)
```

Out[22]: `[<matplotlib.lines.Line2D at 0x7f9e428f5810>]`



B17. **EXERCISE.**

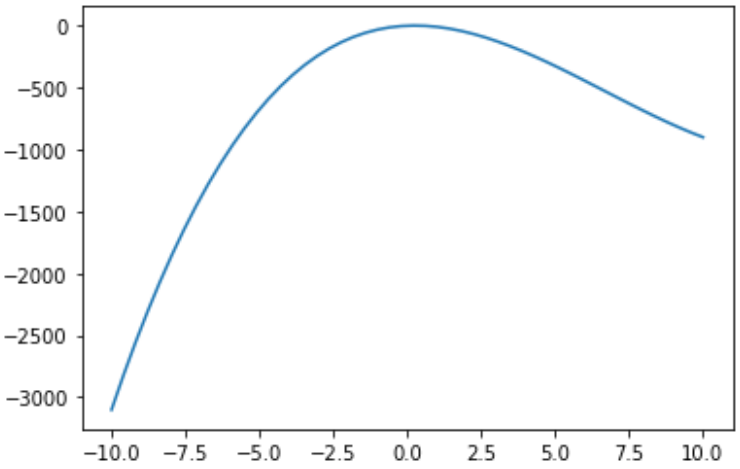
(1) Plot the polynomial function $f(x) = x^3 - 20x^2 + 10x - 1$ over the interval $[-10, 10]$.

(2) Which is the approximate range for the y-axis?

$[-10, 10]$ $(-10, 10)$ $[-3100, 0]$ $[0, 2\pi]$

In [23]:
```
# (1) Type your answer below and press SHIFT+ENTER

x = linspace(-10,10)
y = x**3 - 20*x**2 + 10*x - 1
plot(x,y)

# (2) Your answer:  [-3100,0]
```

Out[23]:
```
[<matplotlib.lines.Line2D at 0x7f9e433ff3d0>]
```



B18. **EXERCISE.** We wish to investigate when (if) the function in B17 is positive. We can't readily tell from our graph in B17 so we will replot over a smaller domain.

(1). Which of these domains seems appropriate for this task?

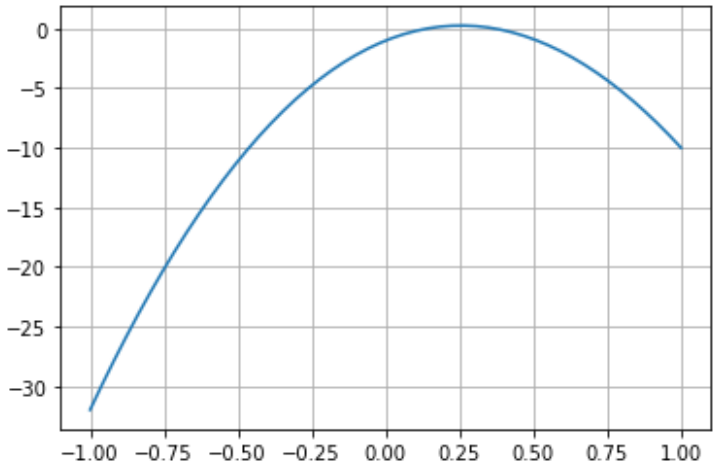
$[0, 500]$ $[0, 10]$ $[-1, 1]$ $[0, 2\pi]$

(2) Replot the graph over the selected domain. Turn on the grid using `grid()`

(3) From your graph, which of these x values have $f(x) > 0$? Indicate all that apply:

0 0.25 0.50 0.75

In [24]:
```
# (1) Your answer:  [-1,1]
# (2) Type your answer below and press SHIFT+ENTER

x = linspace(-1,1)
y = x**3 - 20*x**2 + 10*x - 1
plot(x,y)
grid()

# (3) Your answer:  0.25
```



Lab C: Finding Limits in Python

https://mybinder.org/v2/gh/anniebmcc/pycalclab/master?filepath=mat301c.ipynb

2020 Summer — Calculus 1

Dr Matthew H Sunderland

C1. *RUN the following.*

In [1]:
```
%pylab inline
```

```
Populating the interactive namespace from numpy and matplotlib
```

The magic word `%pylab` loads a bunch of useful functions (including `pi` and `sin`).
The option `inline` tells Jupyter to display any graphs we make on the page instead of in a popup.

C2. Example. Let's say we want to find $\lim_{x \to \pi/2} \frac{\sin x}{x}$. The first thing we try is plugging in.

RUN the following.

In [2]:
```
x = pi/2
sin(x)/x
```

Out[2]:
```
0.6366197723675814
```

C3. **DO EXERCISE.** Based on the result in C2, what is the value of the limit $\lim_{x \to \pi/2} \frac{\sin x}{x}$?

In [3]:
```
# RECORD YOUR ANSWER: the limit is 0.6366197723675814
```

C4. Example. Let's say we want to find $\lim_{x \to 0} \frac{\sin x}{x}$ instead.

RUN the following.

In [4]:
```
x = 0
sin(x)/x
```

```
/Users/sunderland20a/opt/anaconda3/lib/python3.7/site-packages/ipykernel_l
auncher.py:2: RuntimeWarning: invalid value encountered in double_scalars
```

Out[4]:
```
nan
```

Notice that plugging in doesn't work, because sin(0)/0 is undefined.
(Instead of "undefined", Python says "nan," which means "not a number.")

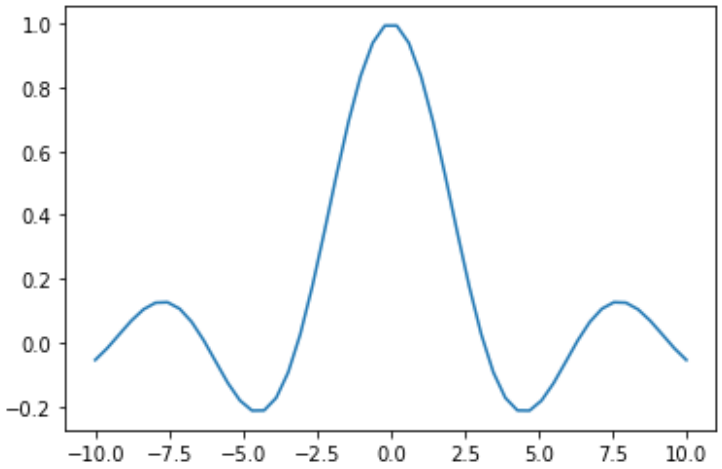
Find the limit with a graph

C5. When plugging in doesn't work, one thing we can try is graphing.

RUN the following.

In [5]:
```
x = linspace(-10,10)
y = sin(x)/x
plot(x,y)
```

Out[5]:
```
[<matplotlib.lines.Line2D at 0x7fed1379aa90>]
```



C6. **DO EXERCISE.** Based on the graph in C5, what is the value of the limit $\lim_{x \to 0} \frac{\sin x}{x}$?

In [6]:
```
# RECORD YOUR ANSWER: the limit is 1
```

Find the limit with a table

C7. The other thing we can try is making a table.

RUN the following.

In [7]:
```
x = r_[1, .1, .01, .001] # Make array of numbers approaching 0 from right
y = sin(x)/x             # Plug the array into our function
c_[x,y]                  # Make a table
```

Out[7]:
```
array([[1.        , 0.84147098],
       [0.1       , 0.99833417],
       [0.01      , 0.99998333],
       [0.001     , 0.99999983]])
```

In [8]:
```
x = r_[-1, -.1, -.01, -.001] # Same but approaching from left
y = sin(x)/x
c_[x,y]
```

Out[8]:
```
array([[-1.        ,  0.84147098],
       [-0.1       ,  0.99833417],
       [-0.01      ,  0.99998333],
       [-0.001     ,  0.99999983]])
```

Note that to display two tables, we use two code cells, because Jupyter only displays one output per cell.

C8. **DO EXERCISE.** Based on the table in C7, what is the value of the limit $\lim_{x \to 0} \frac{\sin x}{x}$?

In [9]:
```
# RECORD YOUR ANSWER: the limit is 1
```

Simplify the code

C9. We can simplify the code for `x` using an array.

RUN the following.

In [10]:
```
x = .1 ** r_[:4]
y = sin(x)/x
c_[x,y]
```

Out[10]:
```
array([[1.        , 0.84147098],
       [0.1       , 0.99833417],
       [0.01      , 0.99998333],
       [0.001     , 0.99999983]])
```

You should get the same table as in C7.
Why does this work? Because `.1 ** r_[:4]` means "0.1 raised to the $0, 1, 2, 3$," which is going to be $1, 0.1, 0.01, 0.001$.

Increase precision

C10. The arrays we have made so far show (up to) 8 decimal places.
As an example, let's change the number of decimal places displayed to 17 decimal places:

RUN the following.

In [11]:
```
set_printoptions(precision=17)

x = .1 ** r_[:4]
y = sin(x)/x
c_[x,y]
```

Out[11]:
```
array([[1.                , 0.8414709848078965],
       [0.1               , 0.9983341664682815],
       [0.01              , 0.9999833334166665],
       [0.001             , 0.9999998333333416]])
```

After you run this cell, from now on, arrays will display (up to) 17 decimal places.

Turn off scientific notation

C11. Arrays with very small numbers (less than 0.001) will automatically change to scientific notation.

RUN the following.

In [12]:
```
x = .1 ** r_[:5]
y = sin(x)/x
c_[x,y]
```

Out[12]:
```
array([[1.0000000000000000e+00, 8.4147098480789650e-01],
       [1.0000000000000001e-01, 9.9833416646828155e-01],
       [1.0000000000000002e-02, 9.9998333341666645e-01],
       [1.0000000000000002e-03, 9.9999983333334164e-01],
       [1.0000000000000002e-04, 9.9999999833333342e-01]])
```

C12. We can turn off scientific notation for arrays using `set_printoptions`

RUN the following.

In [13]:
```
set_printoptions(suppress=True)

x = .1 ** r_[:5]
y = sin(x)/x
c_[x,y]
```

Out[13]:
```
array([[1.                , 0.8414709848078965],
       [0.1               , 0.9983341664682815],
       [0.01              , 0.9999833334166665],
       [0.001             , 0.9999998333333416],
       [0.0001            , 0.9999999983333334]])
```

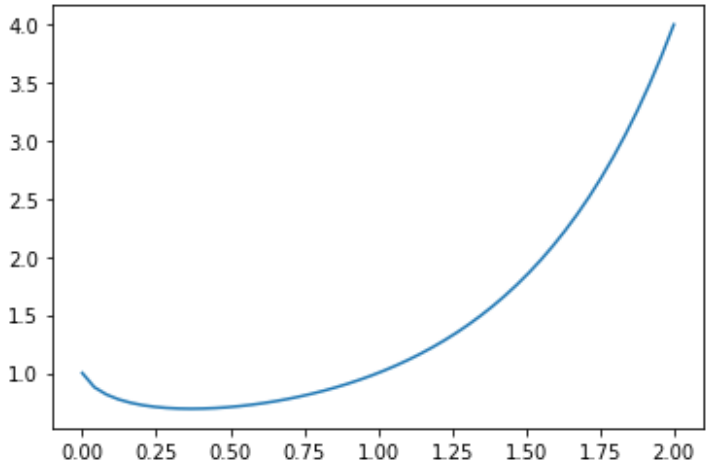
C13. **DO EXERCISE.**

(1) Use the graphical approach to find $\lim_{x \to 0^+} x^x$.

(2) Use the numerical approach (make a table) for the same limit.

In [14]:
```
# (1) TYPE YOUR CODE:

x = linspace(0,2) # Or any interval starting at 0
y = x**x
plot(x,y)

# RECORD YOUR ANSWER: the limit is 1
```

Out[14]:
```
[<matplotlib.lines.Line2D at 0x7fed13905d10>]
```



In [15]:
```
# (2) TYPE YOUR CODE:

x = 0.1 ** r_[:10]
y = x**x
c_[x,y]

# RECORD YOUR ANSWER: the limit is 1
```

Out[15]:
```
array([[1.                , 1.                ],
       [0.1               , 0.7943282347242815],
       [0.01              , 0.954992586021436 ],
       [0.001             , 0.9931160484209338],
       [0.0001            , 0.9990793899844618],
       [0.00001           , 0.9998848773724686],
       [0.000001          , 0.9999861845848758],
       [0.0000001         , 0.9999983881917339],
       [0.00000001        , 0.9999998157932095],
       [0.000000001       , 0.9999999792767343]])
```

Define a function

C14. Example. Let's make a graph and make tables to find $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$

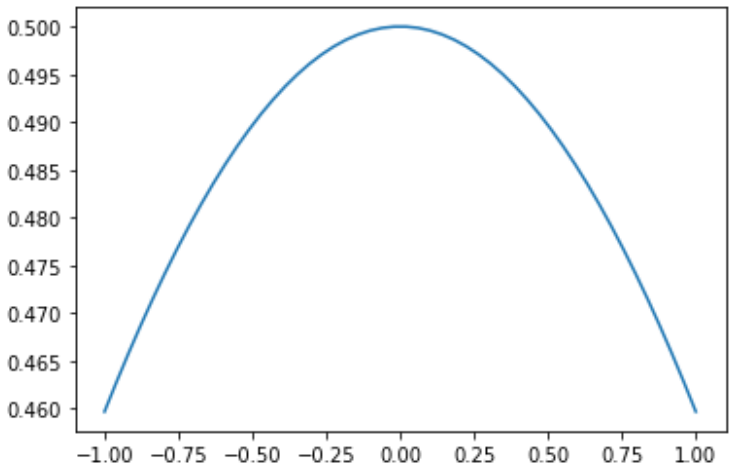
We will use `def` to store the function so we don't have to type the function over and over.

RUN the following.

In [16]:
```
def f(x):
    return (1 - cos(x))/x**2
```

In [17]:
```
x = linspace(-1,1)
y = f(x)
plot(x,y)
```

Out[17]:
```
[<matplotlib.lines.Line2D at 0x7fed139f2e90>]
```



In [18]:
```
x = 0.1 ** r_[:8]
y = f(x)
c_[x,y]
```

Out[18]:
```
array([[1.                , 0.45969769413186023],
       [0.1               , 0.49958347219742893],
       [0.01              , 0.4999958333473662 ],
       [0.001             , 0.4999999583255031 ],
       [0.0001            , 0.49999999696126435],
       [0.00001           , 0.5000000413701853 ],
       [0.000001          , 0.5000444502911701 ],
       [0.0000001         , 0.4996003610813201 ]])
```

In [19]:
```
x = -0.1 ** r_[:8]
y = f(x)
c_[x,y]
```

Out[19]:
```
array([[-1.                ,  0.45969769413186023],
       [-0.1               ,  0.49958347219742893],
       [-0.01              ,  0.4999958333473662 ],
       [-0.001             ,  0.4999999583255031 ],
       [-0.0001            ,  0.49999999696126435],
       [-0.00001           ,  0.5000000413701853 ],
       [-0.000001          ,  0.5000444502911701 ],
       [-0.0000001         ,  0.4996003610813201 ]])
```

So, the graph and the tables tell us that $\lim_{x \to 0} \frac{1 - \cos x}{x^2} = 0.5$

Taking the limit as x approaches a number other than zero

C15. Example. Let's investigate $\lim_{x \to \pi/2} \left(\frac{\pi}{2} - x \right) \tan x$

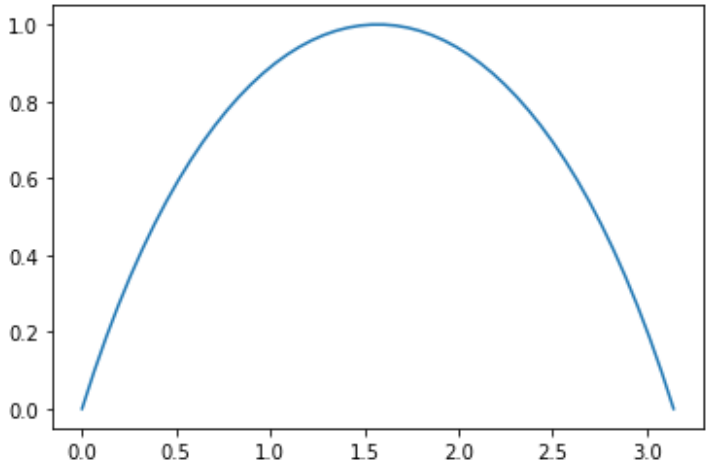
Notice that in this example, x is not approaching 0.

RUN the following.

In [20]:
```
def f(x):
    return (pi/2 - x)*tan(x)
```

In [21]:
```
x = linspace(0,pi)
y = f(x)
plot(x,y)
```

Out[21]:
```
[<matplotlib.lines.Line2D at 0x7fed13adead0>]
```



In [22]:
```
x = pi/2 + 0.1 ** r_[:8]
y = f(x)
c_[x,y]
```

Out[22]:
```
array([[2.5707963267948966, 0.6420926159343308],
       [1.6707963267948966, 0.9966644423259243],
       [1.5807963267948966, 0.9999666664444484],
       [1.5717963267948964, 0.9999996666667056],
       [1.5708963267948965, 0.999999996667279 ],
       [1.5708063267948966, 0.9999999999727899],
       [1.5707973267948965, 1.000000000060899 ],
       [1.5707964267948966, 1.0000000006123202]])
```

In [23]:
```
x = pi/2 - 0.1 ** r_[:8]
y = f(x)
c_[x,y]
```

Out[23]:
```
array([[0.5707963267948966, 0.6420926159343306],
       [1.4707963267948965, 0.9966644423259232],
       [1.5607963267948965, 0.9999666664444362],
       [1.5697963267948967, 0.9999996666665832],
       [1.5706963267948966, 0.9999999966660543],
       [1.5707863267948965, 0.9999999999605436],
       [1.5707953267948966, 0.9999999999384342],
       [1.5707962267948965, 0.9999999993876733]])
```

The graph and the tables tell us that $\lim_{x \to \pi/2} \left(\frac{\pi}{2} - x \right) \tan x = 1$

More Exercises

C16. **DO EXERCISE.**

Let $f(x) = x \ln x$.

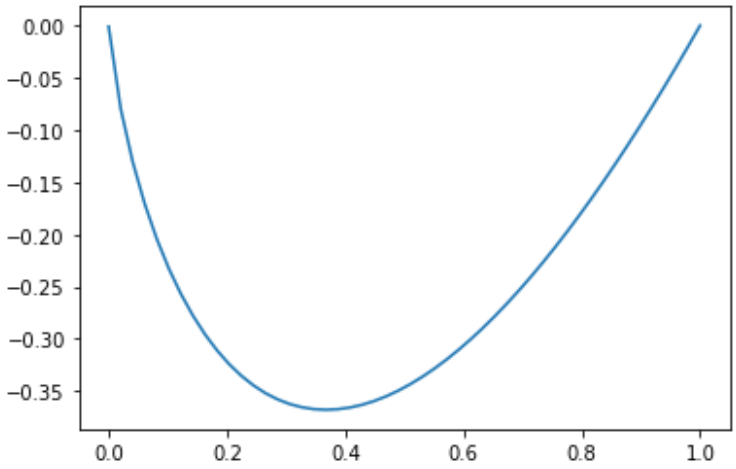
(1) Plot a graph of $f(x)$ over the interval $(0, 1]$ and based on your graph, estimate

$$\lim_{x \to 0^+} f(x).$$

(2) Use the numeric approach to find the same limit.

In [24]:
```
# (1) TYPE YOUR CODE:

x = linspace(0.0001,1)
y = x*log(x)
plot(x,y)

# RECORD YOUR ANSWER: the limit is 0
```

Out[24]:
```
[<matplotlib.lines.Line2D at 0x7fed13bbe5d0>]
```



In [25]:
```
# (2) TYPE YOUR CODE:

x = 0.1 ** r_[:17]
y = x*log(x)
c_[x,y]

# RECORD YOUR ANSWER: the limit is 0
```

Out[25]:
```
array([[ 1.                ,  0.                ],
       [ 0.1               , -0.23025850929940456],
       [ 0.01              , -0.04605170185988092],
       [ 0.001             , -0.00690775527898214],
       [ 0.0001            , -0.00092103403719762],
       [ 0.00001           , -0.0001151292546497 ],
       [ 0.000001          , -0.00001381551055796],
       [ 0.0000001         , -0.0000016118095651 ],
       [ 0.00000001        , -0.00000018420680744],
       [ 0.000000001       , -0.00000002072326584],
       [ 0.0000000001      , -0.00000000230258509],
       [ 0.00000000001     , -0.00000000025328436],
       [ 0.000000000001    , -0.00000000002763102],
       [ 0.0000000000001   , -0.00000000000299336],
       [ 0.00000000000001  , -0.00000000000032236],
       [ 0.000000000000001 , -0.00000000000003454],
       [ 0.0000000000000001, -0.00000000000000368]])
```

C17. **DO EXERCISE.**

(1) Define the python function $f(x) = x^{-1}e^{-1/x}$

(2) Plot the function $f(x)$ over the interval $(0, 1]$ and estimate.

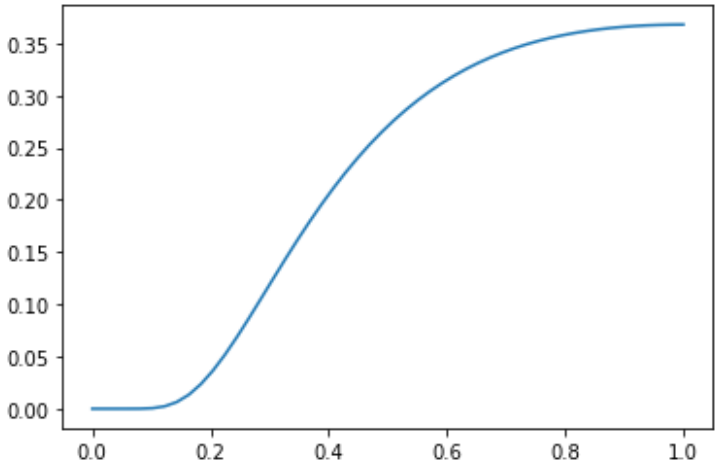
$$\lim_{x \to 0^+} f(x)$$

(3) Use the numeric approach to find the same limit.

In [26]:
```
# (1) TYPE YOUR CODE:

def f(x): return x**-1 * exp(-1/x)
```

In [27]:
```
# (2) TYPE YOUR CODE:

x = linspace(0.0001,1)
plot(x,f(x))

# RECORD YOUR ANSWER: the limit is 0
```

Out[27]:
```
[<matplotlib.lines.Line2D at 0x7fed13c21810>]
```



In [28]:
```
# (3) TYPE YOUR CODE:

x = 0.1 ** r_[:5]
c_[x,f(x)]

# RECORD YOUR ANSWER: the limit is 0
```

Out[28]:
```
array([[1.                , 0.36787944117144233],
       [0.1               , 0.00045399929762485],
       [0.01              , 0.                 ],
       [0.001             , 0.                 ],
       [0.0001            , 0.                 ]])
```

C18. **DO EXERCISE.**
We wish to find the limit of the oscillating function

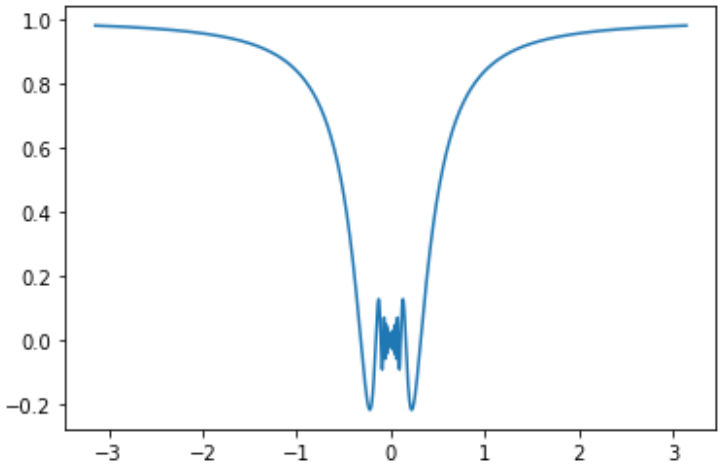
$$f(x) = x \sin \frac{1}{x}$$

as x approaches 0.

(1) Plot the function f over the interval $[-pi, pi]$ using 10000 points for x.
(2) "Zoom in" by replotting the function over $[-0.1, 0.1]$.
(3) Motivated by the squeeze theorem, plot on the same axes the function f over $[-0.1, 0.1]$ as well as $y = |x|$ and $y = -|x|$.
(4) Estimate the limit as $x \to 0$.
(5) How did graphing the absolute value help you find the limit in (3)? (Choose one)
The mean-value theorem *The function is continuous* *The squeeze theorem* *They didnt; I just guessed*

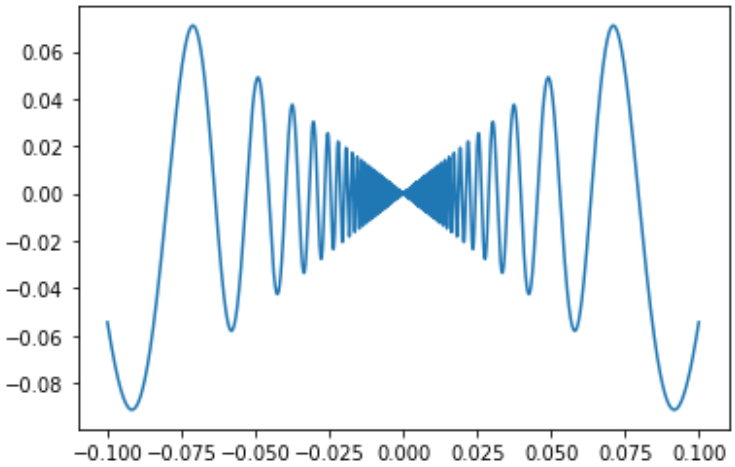
In [29]:
```
# (1) TYPE YOUR CODE:

x = linspace(-pi,pi,10000)
y = x*sin(1/x)
plot(x,y)
```

Out[29]: [<matplotlib.lines.Line2D at 0x7fed13d66b50>]



In [30]:
```
# (2) TYPE YOUR CODE:

x = linspace(-0.1,0.1,10000)
y = x*sin(1/x)
plot(x,y)
```

Out[30]: [<matplotlib.lines.Line2D at 0x7fed13e92b90>]



In [31]:
```
# (3) TYPE YOUR CODE:

x = linspace(-0.1,0.1,10000)
y = x*sin(1/x)
plot(x,y)

y = abs(x)
plot(x,y)

y = -abs(x)
plot(x,y)

# (4) RECORD YOUR ANSWER: the limit is 0
# (5) RECORD YOUR ANSWER: The squeeze theorem
```

Out[31]:
```
[<matplotlib.lines.Line2D at 0x7fed140a3690>]
```