

Lab A: Using Python as a Calculator

<https://mybinder.org/v2/gh/anniebmcc/pycalclab/master?filepath=mat301a.ipynb>

2020 Summer — Calculus 1

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Jupyter Notebooks

A1. **RUN** the following "code cell" (gray rectangle with `In[]` next to it), by CLICKING the code cell and pressing SHIFT+RETURN. Notice that only the last result will display.

```
In [1]: 1 + 2 + 3  
        50 - 3  
        100*5
```

Out[1]: 500

A2. **RUN** the following. As always, only the last result displays, but the last result has 2 parts because of the comma.

```
In [2]: 1 + 2 + 3  
        50 - 3, 1000*1000  
        100*5, 7*7
```

Out[2]: (500, 49)

A3. The "+" on the toolbar adds a code cell. The "scissors" deletes a cell.

Python arithmetic + - * / **

A4. **RUN** the following.

```
In [3]: 3 + 10*5, 5**2, 27/10
```

Out[3]: (53, 25, 2.7)

A5. **EXERCISE.**

- What does each of the 5 arithmetic operations do?
- Do spaces around the 5 operations matter, or is it just style?

```
In [4]: # TYPE YOUR ANSWERS BELOW
#
# a) + is addition
# - is subtraction
# * is multiplication
# / is division
# ** is exponentiation
#
# b) No, spaces around + - * / ** don't matter
```

Python # and =

A6. **RUN** the following. You will notice python ignores everything after #

```
In [5]: # This is a comment
1 + 1 # This is also a comment
```

Out[5]: 2

A7. **RUN** the following. Notice we assign variables using = Assignment itself does NOT produce output.

```
In [6]: a = 10
a
```

Out[6]: 10

```
In [7]: b = 20
```

```
In [8]: a = 18
b = 21
c = a - b
c
```

Out[8]: -3

A8. **RUN** the following. Notice you can assign multiple variables at once with a comma.

```
In [9]: x, y = 100, 500
x
```

Out[9]: 100

```
In [10]: a,b,c = 3,4,5
a + b/c
```

Out[10]: 3.8

A9. **RUN** the following. See that we can compute $\frac{(2-3)*-3}{-1+2}$ all at once (1st cell below), or we can assign variables to help us (2nd cell below).

```
In [11]: (2 - 3)*-3/(-1 + 2)
```

```
Out[11]: 3.0
```

```
In [12]: top = (2 - 3)*-3
          bottom = -1 + 2
          top/bottom
```

```
Out[12]: 3.0
```

A10. **EXERCISE.** Assign variables to help you compute $3 - \frac{3^2-2\cdot 3}{2\cdot 3-2}$

```
In [13]: # Type your answer below and press SHIFT+ENTER

          top = 3**2 - 2*3
          bottom = 2*3 - 2
          3 - top/bottom
```

```
Out[13]: 2.25
```

Order of Operations

A11. **RUN** the following. Notice $a - b * c = a - (b * c)$, but they do not equal $(a - b) * c$.

```
In [14]: a,b,c = 3,4,5

          a - b*c,  a - (b*c),  (a - b)*c
```

```
Out[14]: (-17, -17, -5)
```

A12. **EXERCISE.** In each row, identify NON-equivalent choice. For example, the answer to (1) is $(a - b) * c$ because $a - b * c = a - (b * c)$

- | | | | |
|------|-----------------|-----------------|----------------|
| (1) | $a - b * c$ | $a - (b * c)$ | $(a - b) * c$ |
| (2) | $a * (b - c)$ | $(a * b) - c$ | $a * b - c$ |
| (3) | $a / b + c$ | $a / (b + c)$ | $(a / b) + c$ |
| (4) | $(a + b) / c$ | $a + (b / c)$ | $a + b / c$ |
| (5) | $a ** (b * c)$ | $(a ** b) * c$ | $a ** b * c$ |
| (6) | $a * (b ** c)$ | $a * b ** c$ | $(a * b) ** c$ |
| (7) | $a / b ** c$ | $(a / b) ** c$ | $a / (b ** c)$ |
| (8) | $a ** b / c$ | $(a ** b) / c$ | $a ** (b / c)$ |
| (9) | $(3 - 3) - 3$ | $3 - 3 - 3$ | $3 - (3 - 3)$ |
| (10) | $(2 ** 3) ** 2$ | $2 ** (3 ** 2)$ | $2 ** 3 ** 2$ |
| (11) | $6 / 3 / 2$ | $6 / (3 / 2)$ | $(6 / 3) / 2$ |

```
In [15]: # TYPE YOUR ANSWERS BELOW.
#
# (1)  (a - b)*c
# (2)  a*(b - c)
# (3)  a/(b + c)
# (4)  (a + b)/c
# (5)  a ** (b*c)
# (6)  (a*b) ** c
# (7)  (a/b) ** c
# (8)  a ** (b/c)
# (9)  3 - (3 - 3)
# (10) (2 ** 3) ** 2
# (11) 6/(3/2)
```

A13. **RUN** the following example, where we add 2 sets of parentheses which show the order of the 2 operations.

```
In [16]: 1 + 3/5
```

```
Out[16]: 1.6
```

```
In [17]: (1 + (3/5))
```

```
Out[17]: 1.6
```

A14. **EXERCISE.** Add 4 sets of parentheses, which show the order of the 4 operations.

```
In [18]: 7 - 3 ** 2/9 + 4
```

```
Out[18]: 10.0
```

```
In [19]: # Type your answer below and press SHIFT+ENTER
```

```
((7 - ((3 ** 2)/9)) + 4)
```

```
Out[19]: 10.0
```

A15. **EXERCISE.** Assign $a, b, c = 4, 5, 8$ and then evaluate $\frac{a^b - c/b}{c - a}, \frac{a^{c-b}}{c - b}, \frac{a^{3/2}}{b}, \frac{a - b(c - a)}{c - a}$

```
In [20]: # Type your answer below and press SHIFT+ENTER
```

```
a,b,c = 4,5,8
```

```
(a**b - c/b)/(c-a), a**(c-b)/(c-b), a**(3/2)/b, (a - b*(c-a))/(c-a)
```

```
Out[20]: (255.6, 21.333333333333332, 1.6, -4.0)
```

Making python functions

A16. **RUN** the following.

```
In [21]: def g(x):
```

```
    return x**2
```

```
g(7)
```

```
Out[21]: 49
```

```
In [22]: def h(n): return n + 100
```

```
h(7)
```

```
Out[22]: 107
```

A17. **EXERCISE.** Make the function $P(x) = x^2 - 2x + 1$ and find $P(P(7))$.

```
In [23]: # Type your answer below and press SHIFT+ENTER
```

```
def P(x):
```

```
    return x**2 - 2*x + 1
```

```
P(P(7))
```

```
Out[23]: 1225
```

Built-in %pylab functions

Python	Math notation	Meaning
<code>abs(x)</code>	$ x $	absolute value
<code>sqrt(x)</code>	\sqrt{x}	square root
<code>exp(x)</code>	e^x	exponential function
<code>log(x)</code>	$\ln x$	natural logarithm
<code>sin(x)</code>	$\sin x$	sine
<code>arcsin(x)</code>	$\sin^{-1} x$	inverse sine
<code>radians(x)</code>		converts degrees to radians

A18. **RUN** the code cells below. The command `%pylab` only needs to be run once per lab; it loads "built-in functions" (from python packages numpy and matplotlib).

```
In [24]: %pylab
```

```
sqrt(49)
```

Using matplotlib backend: MacOSX

Populating the interactive namespace from numpy and matplotlib

```
Out[24]: 7.0
```

```
In [25]: pi, exp(1), sin(pi/2)
```

```
Out[25]: (3.141592653589793, 2.718281828459045, 1.0)
```

A19. **EXERCISE.** Evaluate

1. $\sin 40^\circ$
2. $\sin^2 65^\circ$
3. $e^{(10-8.5)/3}$
4. $\arcsin(\sin(3\pi/4))$

Note. Python uses radians for all angle measurements, so you need to convert any degrees to radians.

```
In [26]: # Type your answer below and press SHIFT+ENTER
```

```
sin(radians(40)), sin(radians(65))**2, exp((10-8.5)/3), arcsin(sin(3*pi/4))
```

```
Out[26]: (0.6427876096865393,  
          0.8213938048432696,  
          1.6487212707001282,  
          0.7853981633974484)
```

Making an array with `r_[]`

A20. **RUN** the following. (If you get an error, go back and run [A17](#).) The function `r_[]` can make an array of numbers of your choice. We will need arrays for graphing (Lab B).

```
In [27]: x = r_[2,3,4,5,10]
         x**3
```

```
Out[27]: array([  8,   27,   64,  125, 1000])
```

A21. **EXERCISE.** Use `r_[]` to store the numbers 2,3,5,7,11 in an array named `x`. Find `x*x`.

```
In [28]: # Type your answer below and press SHIFT+ENTER

         x = r_[2,3,5,7,11]
         x*x
```

```
Out[28]: array([  4,   9,  25,  49, 121])
```

Making an array with `r_[a:b:stride]`

A22. **RUN** the following. In general, `r_[a:b]` will list integers from *a* up to but *not* including *b*. A missing *a* is the same as 0.

```
In [29]: r_[5:10]
```

```
Out[29]: array([5, 6, 7, 8, 9])
```

```
In [30]: r_[ :5]
```

```
Out[30]: array([0, 1, 2, 3, 4])
```

A23. **EXERCISE.** Use `r_[a:b]` to make the array 1,2,3,4,5,6,7,8,9

```
In [31]: # Type your answer below and press SHIFT+ENTER

         r_[1:10]
```

```
Out[31]: array([1, 2, 3, 4, 5, 6, 7, 8, 9])
```

A24. **RUN** the following. In general, `r_[a:b:stride]` spaces out your numbers by the amount `stride`.

```
In [32]: r_[0:100:2]
```

```
Out[32]: array([ 0,  2,  4,  6,  8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32,
                34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66,
                68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98])
```

A25. **EXERCISE.** Use `r_[a:b:stride]` to make the array 1,3,5,...,99

```
In [33]: # Type your answer below and press SHIFT+ENTER
r_[1:100:2]
```

```
Out[33]: array([ 1,  3,  5,  7,  9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33,
                35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67,
                69, 71, 73, 75, 77, 79, 81, 83, 85, 87, 89, 91, 93, 95, 97, 99])
```

Making an array with `linspace(a,b,n)`

A26. **RUN** the following. Observe that `linspace(a,b,n)` lists `n` numbers from `a` to `b` inclusive. This is useful for generating a lot of evenly-spaced numbers, such as when graphing (Lab B). Observe that `linspace(a,b)` lists 50 numbers from `a` to `b` inclusive.

```
In [34]: linspace(0,10,6)
```

```
Out[34]: array([ 0.,  2.,  4.,  6.,  8., 10.])
```

```
In [35]: linspace(0,10)
```

```
Out[35]: array([ 0.          ,  0.20408163,  0.40816327,  0.6122449 ,  0.81632653,
                1.02040816,  1.2244898 ,  1.42857143,  1.63265306,  1.83673469,
                2.04081633,  2.24489796,  2.44897959,  2.65306122,  2.85714286,
                3.06122449,  3.26530612,  3.46938776,  3.67346939,  3.87755102,
                4.08163265,  4.28571429,  4.48979592,  4.69387755,  4.89795918,
                5.10204082,  5.30612245,  5.51020408,  5.71428571,  5.91836735,
                6.12244898,  6.32653061,  6.53061224,  6.73469388,  6.93877551,
                7.14285714,  7.34693878,  7.55102041,  7.75510204,  7.95918367,
                8.16326531,  8.36734694,  8.57142857,  8.7755102 ,  8.97959184,
                9.18367347,  9.3877551 ,  9.59183673,  9.79591837, 10.          ])
```

A27. **EXERCISE.** Use `linspace(a,b,n)` to make the array 1,1.5,2,2.5,3,3.5,4

```
In [36]: # Type your answer below and press SHIFT+ENTER
linspace(1,4,7)
```

```
Out[36]: array([1. , 1.5, 2. , 2.5, 3. , 3.5, 4. ])
```

A28. **EXERCISE.**

Convert average body temperature $98.6^{\circ}F$ to Celsius using $C = 5/9(F - 32)$.

```
In [37]: # Type your answer below and press SHIFT+ENTER
5/9*(98.6 - 32)
```

```
Out[37]: 37.0
```


A29. **RUN** the following.

Notice that `x` and `y` are arrays,
`c_[x,y]` puts them into a table.

```
In [38]: x = r_[:10]
         y = x**2
         c_[x,y]
```

```
Out[38]: array([[ 0,  0],
               [ 1,  1],
               [ 2,  4],
               [ 3,  9],
               [ 4, 16],
               [ 5, 25],
               [ 6, 36],
               [ 7, 49],
               [ 8, 64],
               [ 9, 81]])
```

A30. **EXERCISE.**

Use `r_` to make an array of Fahrenheit values `x = -100, -80, -60, ..., 100`.

Make the corresponding array of Celsius values `y`

Use `c_` to put `x` and `y` into a table.

```
In [39]: # Type your answer below and press SHIFT+ENTER

         x = r_[-100:101:20]
         y = 5/9*(x - 32)
         c_[x,y]
```

```
Out[39]: array([[ -100.      , -73.33333333],
               [  -80.      , -62.22222222],
               [  -60.      , -51.11111111],
               [  -40.      , -40.        ],
               [  -20.      , -28.88888889],
               [    0.      , -17.77777778],
               [   20.      ,  -6.66666667],
               [   40.      ,   4.44444444],
               [   60.      ,  15.55555556],
               [   80.      ,  26.66666667],
               [  100.      ,  37.77777778]])
```

Lab B: Plotting Graphs in Python

<https://mybinder.org/v2/gh/anniebmcc/pycalclab/master?filepath=mat301b.ipynb>

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Plotting with `plot`

B1. Example. To graph $f(x) = x^2$ over $[-2, 2]$ by hand, make an xy table: choose some x values,

x	-2	-1	0	1	2
y					

and then use f to compute the corresponding y values.

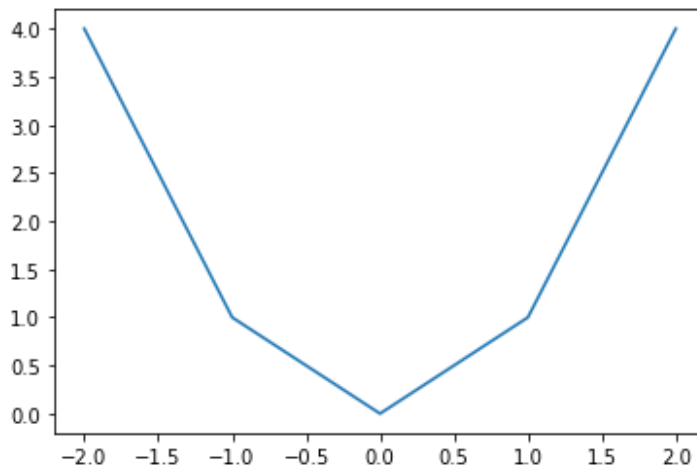
B2. **RUN** the following. Notice that graphing in python is similar to B1: we make a list of x values and y values.

```
In [1]: %pylab inline
```

Populating the interactive namespace from numpy and matplotlib

```
In [2]: x = r_[-2, -1, 0, 1, 2]
y = r_[4, 1, 0, 1, 4]
plot(x,y)
```

```
Out[2]: [<matplotlib.lines.Line2D at 0x7f9e427944d0>]
```



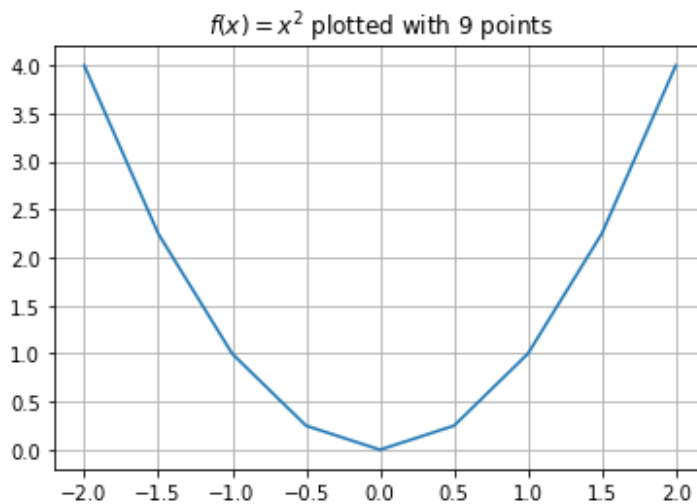
B3. **RUN** the following. Notice that we save time by making the x array using `linspace` (see A27) and making the y array by doing arithmetic on x (see A29). For illustrative purposes, we use `c_[x y]` to make a table out of the arrays x and y (see A29).

```
In [3]: x = linspace(-2,2,9)
        y = x**2

        plot(x,y)
        title('$f(x) = x^2$ plotted with 9 points')
        grid()

        c_[x,y]
```

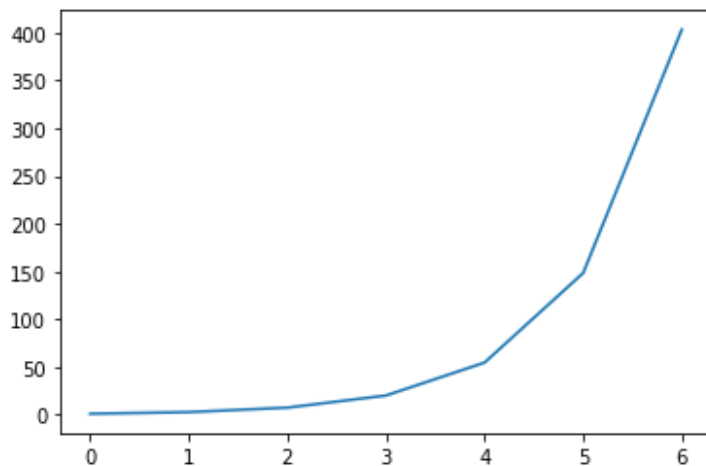
```
Out[3]: array([[ -2.   ,  4.   ],
               [ -1.5   ,  2.25 ],
               [ -1.   ,  1.   ],
               [ -0.5   ,  0.25 ],
               [  0.   ,  0.   ],
               [  0.5   ,  0.25 ],
               [  1.   ,  1.   ],
               [  1.5   ,  2.25 ],
               [  2.   ,  4.   ]])
```



B4. **RUN** the following, which graph $f(x) = e^x$ over the interval $[0, 7]$. Here we make our array x using `r_[a:b:stride]` (see A22). Remember that `exp(x)` is how you write e^x in python (see A18).

```
In [4]: x = r_[:7]
        y = exp(x)
        plot(x,y)
```

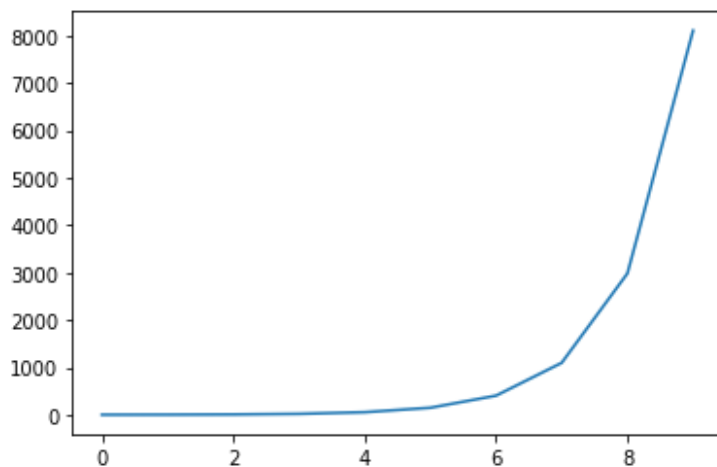
```
Out[4]: [<matplotlib.lines.Line2D at 0x7f9e42a15a90>]
```



B5. **RUN** the following. When we change the x we must recompute the y ; there are two ways to do it (compare B4 to B5).

```
In [5]: x = r_[:10]
        plot(x, exp(x))
```

```
Out[5]: [<matplotlib.lines.Line2D at 0x7f9e42b71850>]
```



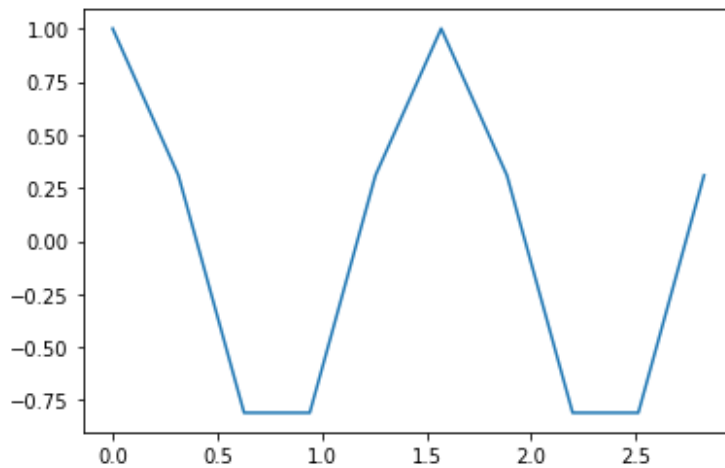
B6. EXERCISE.

- (1) Graph $y = \cos 4x$ over $[0, \pi]$ with a step size of $\pi/10$
- (2) Redo your plot from iii. using `x = linspace(0,pi)`
- (3) Which plot looks more like the plot of a cosine curve?

In [6]: # (1) Type your answer below and press SHIFT+ENTER

```
x = r_[0:pi:pi/10]
y = cos(4*x)
plot(x,y)
```

Out[6]: [<matplotlib.lines.Line2D at 0x7f9e42be5090>]

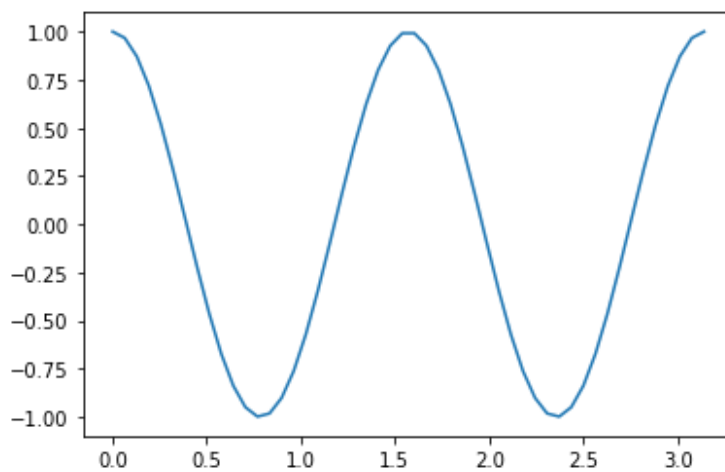


In [7]: # (2) Type your answer below and press SHIFT+ENTER

```
x = linspace(0,pi)
y = cos(4*x)
plot(x,y)

# (3) Your answer: the second plot
```

Out[7]: [<matplotlib.lines.Line2D at 0x7f9e42cba150>]

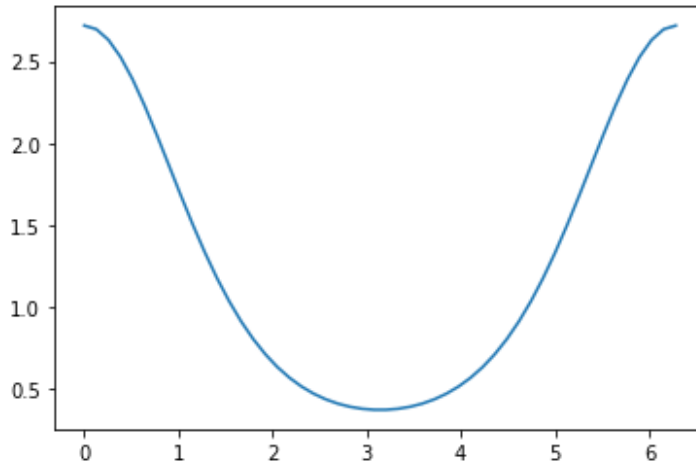


B7. **EXERCISE.** Plot the function $f(x) = e^{\cos x}$ over the interval $[0, 2\pi]$.

```
In [8]: # Type your answer below and press SHIFT+ENTER
```

```
x = linspace(0,2*pi)
y = exp(cos(x))
plot(x,y)
```

```
Out[8]: [<matplotlib.lines.Line2D at 0x7f9e42f14950>]
```



Doing arithmetic on arrays

B8. **RUN** the following.

We make numpy arrays with `r_` or `linspace`

Numpy arrays "know" how to do "elementwise" arithmetic.

Warning: x^2 is written `x**2`.

```
In [9]: x = r_[1:5]
```

```
x, 10 - x, x + 10, 10*x, x**2, 12/x, x**x, 10**x
```

```
Out[9]: (array([1, 2, 3, 4]),
         array([9, 8, 7, 6]),
         array([11, 12, 13, 14]),
         array([10, 20, 30, 40]),
         array([ 1,  4,  9, 16]),
         array([12.,  6.,  4.,  3.]),
         array([ 1,  4, 27, 256]),
         array([ 10, 100, 1000, 10000]))
```

B9. **RUN** the following.

```
In [10]: # We can add arrays of the same shape (same length)
```

```
x = r_[10, 20, 50, 100]
y = r_[3, 0, 7, -1]
x + y
```

```
Out[10]: array([13, 20, 57, 99])
```

```
In [11]: # We can add an array (x) and a scalar (y)
```

```
x = r_[10, 20, 50, 100]
y = 100
x + y
```

```
Out[11]: array([110, 120, 150, 200])
```

```
In [12]: # We CANNOT add arrays of DIFFERENT shape
```

```
x = r_[10, 20, 50, 100]
y = r_[3, 0, 7]
x + y
```

```
-----
ValueError                                Traceback (most recent call last)
<ipython-input-12-ab56767c8fea> in <module>
      3 x = r_[10, 20, 50, 100]
      4 y = r_[3, 0, 7]
----> 5 x + y

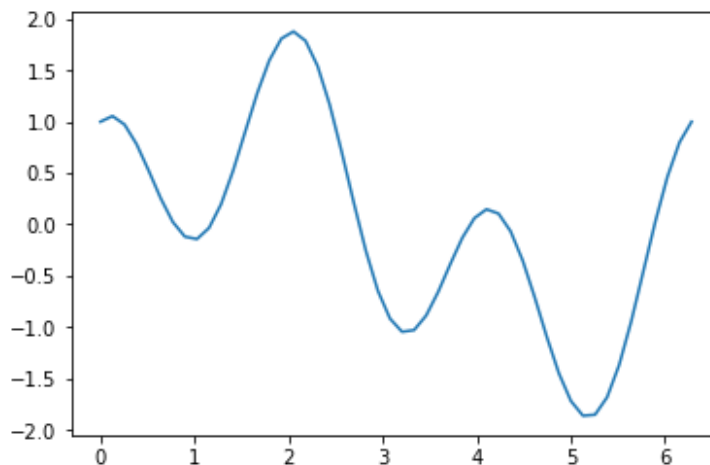
ValueError: operands could not be broadcast together with shapes (4,) (3,)
```

B10. **RUN** the following.

```
In [13]: #  $y = \sin x + \cos 3x$  over the domain  $[0, 2\pi]$ 
```

```
x = linspace(0, 2*pi)
y = sin(x) + cos(3*x)
plot(x, y)
```

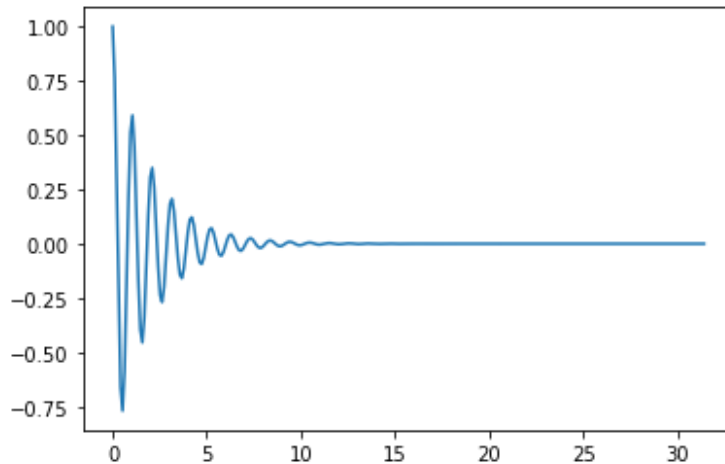
```
Out[13]: [matplotlib.lines.Line2D at 0x7f9e43060090]
```



```
In [14]: #  $y = e^{-x/2} \cos 6x$  over the domain  $[0, 10\pi]$ 

x = linspace(0, 10*pi, 300)
y1 = exp(-x/2) # Here we break up the
y2 = cos(6*x)  # computation into
y = y1*y2      # bite-sized pieces
plot(x,y)
```

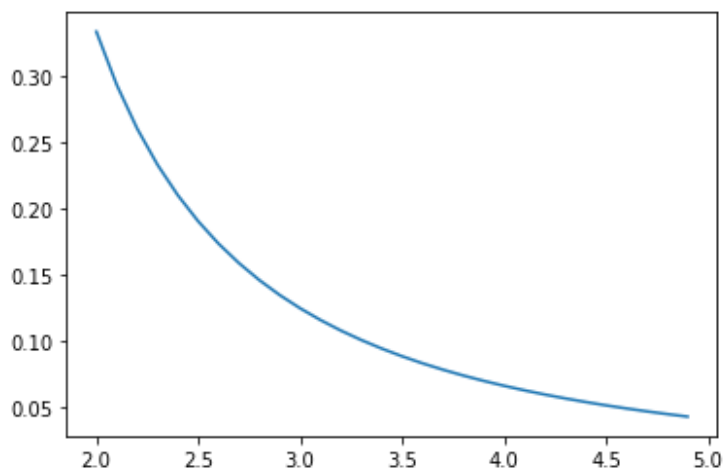
Out[14]: [



```
In [15]: #  $y = 1/(x^2 - 1)$  over the domain  $[2, 5]$ 

x = r_[2:5:0.1]
y = 1/(x**2 - 1)
plot(x,y)
```

Out[15]: [



B11. **EXERCISE.** First **RUN** the following.


```
In [16]: a,b,c = r_[:5], r_[:50:10], r_[:10]
a,b,c
```

```
Out[16]: (array([0, 1, 2, 3, 4]),
array([ 0, 10, 20, 30, 40]),
array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9]))
```

Now, that we've defined a, b, c , which of the following are defined?

$a + b$ $a + c$ $a + 1$ $a * b$ $c ** 2$ $c ^ 2$

```
In [17]: # Type your answer below and press SHIFT+ENTER
a+b, a+1, a*b, c**2
```

```
Out[17]: (array([ 0, 11, 22, 33, 44]),
array([1, 2, 3, 4, 5]),
array([ 0, 10, 40, 90, 160]),
array([ 0, 1, 4, 9, 16, 25, 36, 49, 64, 81]))
```

B12. **RUN** the following example. Let x be the array 1,2,3. Write Python commands to compute x^3 .
The output you get should be `array([1, 8, 27])`.

```
In [18]: x = r_[1,2,3]
x**3
```

```
Out[18]: array([ 1, 8, 27])
```

B13. **EXERCISE.** Using the same array $x = r_[1,2,3]$, find:

$\cos x \sin x$ $\sin^2 x$ $\sin x^2$ $7x^2 \sin \frac{1}{7x^2}$

You should get

```
array([ 0.45464871, -0.37840125, -0.13970775])
array([0.70807342, 0.82682181, 0.01991486])
array([ 0.84147098, -0.7568025 , 0.41211849])
array([0.99660211, 0.99978743, 0.99995801])
```

```
In [19]: # Type your answer below and press SHIFT+ENTER
cos(x)*sin(x), sin(x)**2, sin(x**2), 7*x**2*sin(1/(7*x**2))
```

```
Out[19]: (array([ 0.45464871, -0.37840125, -0.13970775]),
array([0.70807342, 0.82682181, 0.01991486]),
array([ 0.84147098, -0.7568025 , 0.41211849]),
array([0.99660211, 0.99978743, 0.99995801]))
```

B14. **EXERCISE.** Using the same array $x = r_{[1,2,3]}$, find:

$$x - \frac{\cos x - \sin x}{\sin x + \cos x} \quad \frac{1}{10} \left(x - \frac{x^{3/2}}{10} \right)^2$$

You should get

```
array([1.2179581 , 4.68770694, 1.66751188])
```

```
array([0.081      , 0.29486292, 0.61523085])
```

```
In [20]: # Type your answer below and press SHIFT+ENTER

x - (cos(x)-sin(x))/(sin(x)+cos(x)), 1/10*(x - x**(3/2)/10)**2
```

```
Out[20]: (array([1.2179581 , 4.68770694, 1.66751188]),
          array([0.081      , 0.29486292, 0.61523085]))
```

Graphing practice

B15 **EXERCISE.**

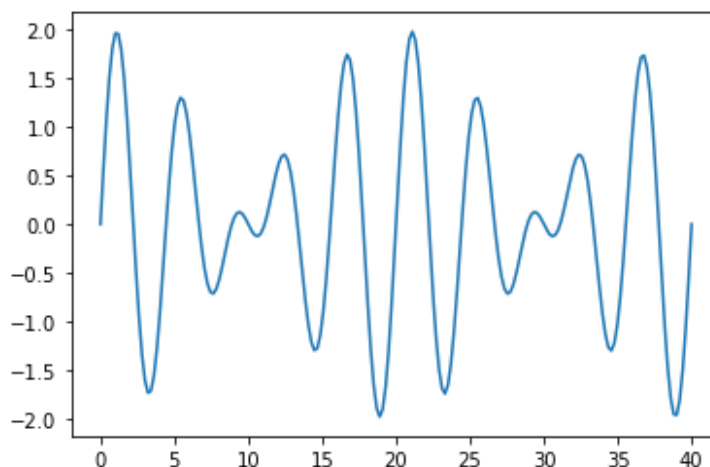
- (1) Graph the function $f(x) = \sin(\frac{\pi}{2}x) + \sin(\frac{2}{5}\pi x)$ over the interval $[0, 40]$.
- (2) How many peaks (relative maxima) does your graph have?
- (3) This function is periodic; how many periods are graphed in $[0, 40]$?
- (4) Estimate from your graph the value of $f(10)$ to 1 decimal point.

```
In [21]: # (1) Type your answer below and press SHIFT+ENTER

x = linspace(0,40,200)
y = sin(pi/2*x) + sin(2/5*pi*x)
plot(x,y)

# (2) Your answer: 10
# (3) Your answer: 2
# (4) Your answer: 0.0
```

```
Out[21]: [<matplotlib.lines.Line2D at 0x7f9e4296a5d0>]
```



B16. EXERCISE.

(1) Graph $f(x) = \cos^2 x - \sin^2 x$ over the interval $[-2\pi, 2\pi]$ using 100 points.

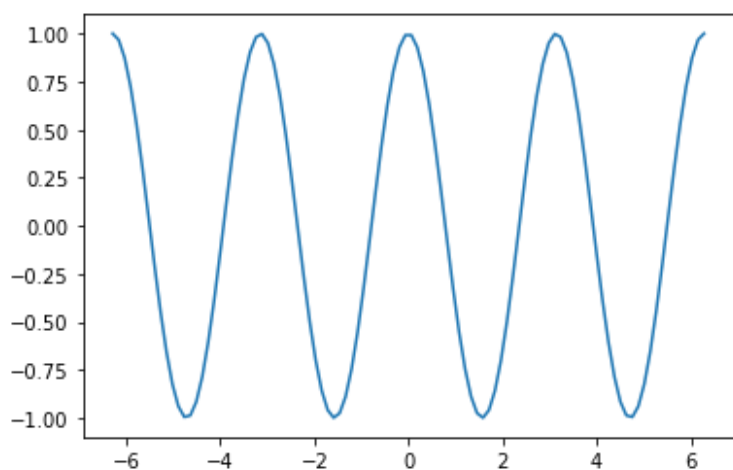
(2) Does the resemble any of the following? $\cos 2x$ $\cos x/2$ $\cos x$

```
In [22]: # (1) Type your answer below and press SHIFT+ENTER
```

```
x = linspace(-2*pi, 2*pi, 100)
y = cos(x)**2 - sin(x)**2
plot(x,y)
```

```
# (2) Your answer: cos(2x)
```

```
Out[22]: [<matplotlib.lines.Line2D at 0x7f9e428f5810>]
```

**B17. EXERCISE.**

(1) Plot the polynomial function $f(x) = x^3 - 20x^2 + 10x - 1$ over the interval $[-10, 10]$.

(2) Which is the approximate range for the y -axis?

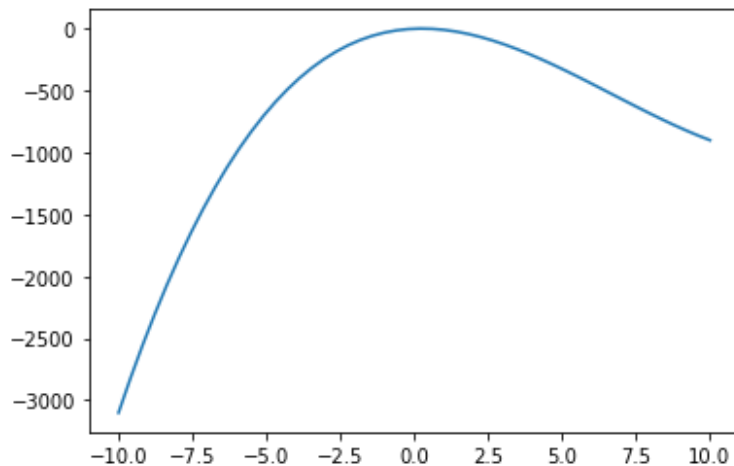
$[-10, 10]$ $(-10, 10)$ $[-3100, 0]$ $[0, 2\pi]$

In [23]: # (1) Type your answer below and press SHIFT+ENTER

```
x = linspace(-10,10)
y = x**3 - 20*x**2 + 10*x - 1
plot(x,y)
```

(2) Your answer: [-3100,0]

Out[23]: [<matplotlib.lines.Line2D at 0x7f9e433ff3d0>]



B18. **EXERCISE.** We wish to investigate when (if) the function in B17 is positive. We can't readily tell from our graph in B17 so we will replot over a smaller domain.

(1). Which of these domains seems appropriate for this task?

[0, 500] [0, 10] [-1, 1] [0, 2 π]

(2) Replot the graph over the selected domain. Turn on the grid using `grid()`

(3) From your graph, which of these x values have $f(x) > 0$? Indicate all that apply:

0 0.25 0.50 0.75

```
In [24]: # (1) Your answer: [-1,1]
# (2) Type your answer below and press SHIFT+ENTER

x = linspace(-1,1)
y = x**3 - 20*x**2 + 10*x - 1
plot(x,y)
grid()

# (3) Your answer: 0.25
```

