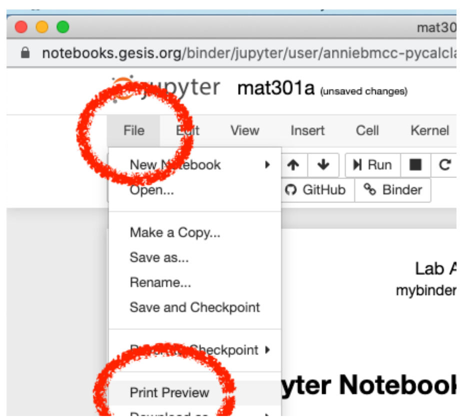
MAT301 Lab Directions

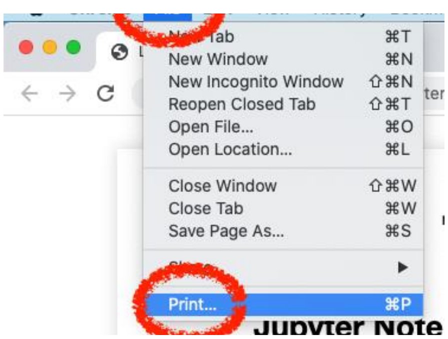
1. Click link in table of contents
   Wait for page to load

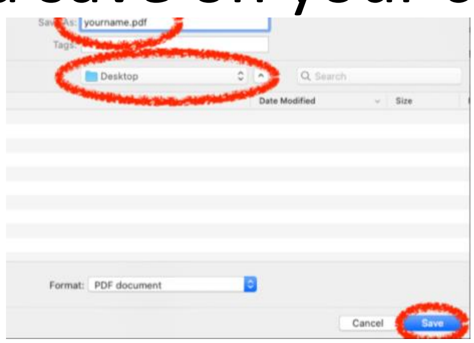
2. Run each cell that says RUN
   Type your answer to each EXERCISE

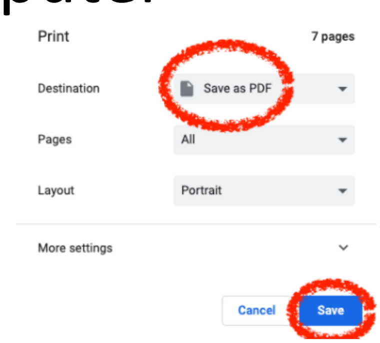
3. Open print preview



4. Print to PDF and save on your computer







5. Upload your PDF to blackboard

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Lab A: Using Python as a Calculator
https://mybinder.org/v2/gh/anniebmcc/pycalclab/master?filepath=mat301a.ipynb
2020 Summer — Calculus 1
Dr Matthew H Sunderland

Jupyter Notebooks

A1. **RUN** the following "code cell" (gray rectangle with `In[ ]` next to it), by CLICKING the code cell and pressing SHIFT+RETURN. Notice that only the last result will display.

In [ ]:
```
1 + 2 + 3
50 - 3
100*5
```

A2. **RUN** the following. As always, only the last result displays, but the last result has 2 parts because of the comma.

In [ ]:
```
1 + 2 + 3
50 - 3, 1000*1000
100*5, 7*7
```

A3. The "+" on the toolbar adds a code cell. The "scissors" deletes a cell.

Python arithmetic + - * / **

A4. **RUN** the following.

In [ ]:
```
3 + 10*5,   5**2,   27/10
```

A5. **EXERCISE.**
a) What does each of the 5 arithmetic operations do?
b) Do spaces around the 5 operations matter, or is it just style?

In [ ]:
```
# TYPE YOUR ANSWERS BELOW
#
#  a) + is
#     - is
#     * is
#     / is
#    ** is
#
#  b)
```

Python # and =

A6. **RUN** the following. You will notice python ignores everything after #

In [ ]:
```
# This is a comment
1 + 1 # This is also a comment
```

A7. **RUN** the following. Notice we assign variables using = Assignment itself does NOT produce output.

In [ ]:
```
a = 10
a
```

In [ ]:
```
b = 20
```

In [ ]:
```
a = 18
b = 21
c = a - b
c
```

A8. **RUN** the following. Notice you can assign multiple variables at once with a comma.

In [ ]:
```
x, y = 100, 500
x
```

In [ ]:
```
a,b,c = 3,4,5
a + b/c
```

A9. **RUN** the following. See that we can compute $\frac{(2-3)*-3}{-1+2}$ all at once (1st cell below), or we can assign variables to help us (2nd cell below).

In [ ]:
```
(2 - 3)*-3/(-1 + 2)
```

In [ ]:
```
top = (2 - 3)*-3
bottom = -1 + 2
top/bottom
```

A10. **EXERCISE.** Assign variables to help you compute $3 - \frac{3^2-2\cdot 3}{2\cdot 3-2}$

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

Order of Operations

A11. **RUN** the following. Notice $a - b * c = a - (b * c)$, but they do not equal $(a - b) * c$.

In [ ]:
```
a,b,c = 3,4,5

a - b*c,   a - (b*c),   (a - b)*c
```

A12. **EXERCISE.** In each row, identify NON-equivalent choice. For example, the answer to (1) is $(a - b) * c$ because $a - b * c = a - (b * c)$

(1)	$a - b * c$	$a - (b * c)$	$(a - b) * c$
(2)	$a * (b - c)$	$(a * b) - c$	$a * b - c$
(3)	$a\,/\,b + c$	$a\,/\,(b + c)$	$(a\,/\,b) + c$
(4)	$(a + b)\,/\,c$	$a + (b\,/\,c)$	$a + b\,/\,c$
(5)	$a ** (b * c)$	$(a ** b) * c$	$a ** b * c$
(6)	$a * (b ** c)$	$a * b ** c$	$(a * b) ** c$
(7)	$a/b ** c$	$(a/b) ** c$	$a/(b ** c)$
(8)	$a ** b/c$	$(a ** b)/c$	$a ** (b/c)$
(9)	$(3 - 3) - 3$	$3 - 3 - 3$	$3 - (3 - 3)$
(10)	$(2 ** 3) ** 2$	$2 ** (3 ** 2)$	$2 ** 3 ** 2$
(11)	$6\,/\,3\,/\,2$	$6\,/\,(3\,/\,2)$	$(6\,/\,3)\,/\,2$

In [ ]:
```
# TYPE YOUR ANSWERS BELOW.
#
#   (1)    (a - b)*c
#   (2)
#   (3)
#   (4)
#   (5)
#   (6)
#   (7)
#   (8)
#   (9)
# (10)
# (11)
```

A13. **RUN** the following example, where we add 2 sets of parentheses which show the order of the 2 operations.

In [ ]:
```
1 + 3/5
```

In [ ]:
```
(1 + (3/5))
```

A14. **EXERCISE.** Add 4 sets of parentheses, which show the order of the 4 operations.

In [ ]:
```
7 - 3 ** 2/9 + 4
```

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

A15. **EXERCISE.** Assign `a,b,c = 4,5,8` and then evaluate $\dfrac{a^b - c/b}{c-a}, \dfrac{a^{c-b}}{c-b}, \dfrac{a^{3/2}}{b}, \dfrac{a-b(c-a)}{c-a}$

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

Making python functions

A16. **RUN** the following.

In [ ]:
```
def g(x):
    return x**2

g(7)
```

In [ ]:
```
def h(n): return n + 100

h(7)
```

A17. **EXERCISE.** Make the function $P(x) = x^2 - 2x + 1$ and find $P(P(7))$.

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

Built-in `%pylab` functions

Python	Math notation	Meaning
`abs(x)`	$\lvert x \rvert$	absolute value
`sqrt(x)`	$\sqrt{x}$	square root
`exp(x)`	e^x	exponential function
`log(x)`	$\ln x$	natural logarithm
`sin(x)`	$\sin x$	sine
`arcsin(x)`	$\sin^{-1} x$	inverse sine
`radians(x)`		converts degrees to radians

A18. **RUN** the code cells below. The command `%pylab` only needs to be run once per lab; it loads "built-in functions" (from python packages numpy and matplotlib).

In [ ]:
```
%pylab

sqrt(49)
```

In [ ]:
```
pi, exp(1), sin(pi/2)
```

A19. **EXERCISE.** Evaluate

1. $\sin 40^\circ$
2. $\sin^2 65^\circ$
3. $e^{(10-8.5)/3}$
4. $\arcsin(\sin(3\pi/4))$

Note. Python uses radians for all angle measurements, so you need to convert any degrees to radians.

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

Making an array with `r_[ ]`

A20. **RUN** the following. (If you get an error, go back and run A17.) The function `r_[ ]` can make an array of numbers of your choice. We will need arrays for graphing (Lab B).

In [ ]:
```
x = r_[2,3,4,5,10]
x**3
```

A21. **EXERCISE.** Use `r_[ ]` to store the numbers 2,3,5,7,11 in an array named `x`. Find `x*x`.

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

Making an array with `r_[a:b:stride]`

A22. **RUN** the following. In general, `r_[a:b]` will list integers from a up to but *not* including b. A missing a is the same as 0.

In [ ]:
```
r_[5:10]
```

In [ ]:
```
r_[:5]
```

A23. **EXERCISE.** Use `r_[a:b]` to make the array 1,2,3,4,5,6,7,8,9

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

A24. **RUN** the following. In general, `r_[a:b:stride]` spaces out your numbers by the amount `stride`.

```
In [ ]: r_[0:100:2]
```

A25. **EXERCISE.** Use `r_[a:b:stride]` to make the array $1, 3, 5, \ldots, 99$

```
In [ ]: # Type your answer below and press SHIFT+ENTER
```

Making an array with `linspace(a,b,n)`

A26. **RUN** the following. Observe that `linspace(a,b,n)` lists n numbers from a to b inclusive. This is useful for generating a lot of evenly-spaced numbers, such as when graphing (Lab B). Observe that `linspace(a,b)` lists 50 numbers from a to b inclusive.

```
In [ ]: linspace(0,10,6)
```

```
In [ ]: linspace(0,10)
```

A27. **EXERCISE.** Use `linspace(a,b,n)` to make the array $1, 1.5, 2, 2.5, 3, 3.5, 4$

```
In [ ]: # Type your answer below and press SHIFT+ENTER
```

A28. **EXERCISE.**
Convert average body temperature $98.6^{\circ} F$ to Celsius using $C = 5/9(F - 32)$.

```
In [ ]: # Type your answer below and press SHIFT+ENTER
```

A29. **RUN** the following.
Notice that `x` and `y` are arrays,
`c_[x,y]` puts them into a table.

```
In [ ]: x = r_[:10]
        y = x**2
        c_[x,y]
```

A30. **EXERCISE.**
Use `r_` to make an array of Fahrenheit values `x` $= -100, -80, -60, \ldots, 100.$
Make the corresponding array of Celsius values `y`
Use `c_` to put `x` and `y` into a table.

```
In [ ]: # Type your answer below and press SHIFT+ENTER
```

Lab B: Plotting Graphs in Python
https://mybinder.org/v2/gh/anniebmcc/pycalclab/master?filepath=mat301b.ipynb
2020 Summer — Calculus 1
Dr Matthew H Sunderland

Plotting with `plot`

B1. Example. To graph $f(x) = x^2$ over $[-2, 2]$ by hand, make an xy table: choose some x values,

x	-2	-1	0	1	2
y					

and then use f to compute the corresponding y values.

B2. **RUN** the following. Notice that graphing in python is similar to B1: we make a list of x values and y values.

In [ ]:
```
%pylab inline
```

In [ ]:
```
x = r_[-2, -1, 0, 1, 2]
y = r_[4, 1, 0, 1, 4]
plot(x,y)
```

B3. **RUN** the following. Notice that we save time by making the x array using `linspace` (see A27) and making the y array by doing arithmetic on x (see A29). For illustrative purposes, we use `c_[x,y]` to make a table out of the arrays x and y (see A29).

In [ ]:
```
x = linspace(-2,2,9)
y = x**2

plot(x,y)
title('$f(x) = x^2$ plotted with 9 points')
grid()

c_[x,y]
```

B4. **RUN** the following, which graph $f(x) = e^x$ over the interval $[0, 7]$. Here we make our array x using `r_[a:b:stride]` (see A22). Remember that `exp(x)` is how you write e^x in python (see A18).

In [ ]:
```
x = r_[:7]
y = exp(x)
plot(x,y)
```

B5. **RUN** the following. When we change the x we must recompute the y; there are two ways to do it (compare B4 to B5).

In [ ]:
```
x = r_[:10]
plot(x, exp(x))
```

B6. **EXERCISE.**
(1) Graph $y = \cos 4x$ over $[0, \pi]$ with a step size of `pi/10`
(2) Redo your plot from iii. using `x = linspace(0,pi)`
(3) Which plot looks more like the plot of a cosine curve?

In [ ]:
```
# (1) Type your answer below and press SHIFT+ENTER
```

In [ ]:
```
# (2) Type your answer below and press SHIFT+ENTER

# (3) Your answer:
```

B7. **EXERCISE.** Plot the function $f(x) = e^{\cos x}$ over the interval $[0, 2\pi]$.

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

Doing arthmetic on arrays

B8. **RUN** the following.
We make numpy arrays with `r_` or `linspace`
Numpy arrays "know" how to do "elementwise" arithmetic.
Warning: x^2 is written `x**2` .

In [ ]:
```
x = r_[1:5]

x,  10 - x,  x + 10,  10*x,  x**2,  12/x,  x**x,  10**x
```

B9. **RUN** the following.

In [ ]:
```
# We can add arrays of the same shape (same length)

x = r_[10, 20, 50, 100]
y = r_[3, 0, 7, -1]
x + y
```

In [ ]:
```
# We can add an array (x) and a scalar (y)

x = r_[10, 20, 50, 100]
y = 100
x + y
```

In [ ]:
```
# We CANNOT add arrays of DIFFERENT shape

x = r_[10, 20, 50, 100]
y = r_[3, 0, 7]
x + y
```

B10. **RUN** the following.

In [ ]:
```
# y = sin x + cos 3x over the domain [0,2pi]

x = linspace(0,2*pi)
y = sin(x) + cos(3*x)
plot(x,y)
```

In [ ]:
```
# y = e^(-x/2) cos 6x over the domain [0,10pi]

x = linspace(0, 10*pi, 300)
y1 = exp(-x/2) # Here we break up the
y2 = cos(6*x)  # computation into
y = y1*y2      # bite-sized pieces
plot(x,y)
```

In [ ]:
```
# y = 1/(x^2 - 1) over the domain [2,5]

x = r_[2:5:0.1]
y = 1/(x**2 - 1)
plot(x,y)
```

B11. **EXERCISE.** First **RUN** the following.

In [ ]:
```
a,b,c = r_[:5], r_[:50:10], r_[:10]
a,b,c
```

Now, that we've defined a, b, c, which of the following are defined?
$a + b \qquad a + c \qquad a + 1 \qquad a * b \qquad c ** 2 \qquad c\ \hat{}\ 2$

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

B12. **RUN** the following example. Let x be the array 1,2,3. Write Python commands to compute x^3.
The output you get should be `array([ 1,  8, 27])`.

In [ ]:
```
x = r_[1,2,3]
x**3
```

B13. **EXERCISE.** Using the same array `x = r_[1,2,3]` , find:

$\cos x \sin x$ $\qquad$ $\sin^2 x$ $\qquad$ $\sin x^2$ $\qquad$ $7x^2 \sin \frac{1}{7x^2}$

You should get

```
array([ 0.45464871, -0.37840125, -0.13970775])
array([0.70807342, 0.82682181, 0.01991486])
array([ 0.84147098, -0.7568025 ,  0.41211849])
array([0.99660211, 0.99978743, 0.99995801])
```

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

B14. **EXERCISE.** Using the same array `x = r_[1,2,3]` , find:

$x - \frac{\cos x - \sin x}{\sin x + \cos x}$ $\qquad$ $\frac{1}{10}(x - \frac{x^{3/2}}{10})^2$

You should get

```
array([1.2179581 , 4.68770694, 1.66751188])
array([0.081     , 0.29486292, 0.61523085])
```

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

Graphing practice

B15 **EXERCISE.**

(1) Graph the function $f(x) = \sin(\frac{\pi}{2}x) + \sin(\frac{2}{5}\pi x)$ over the interval $[0, 40]$.

(2) How many peaks (relative maxima) does your graph have?

(3) This function is periodic; how many periods are graphed in $[0, 40]$?

(4) Estimate from your graph the value of $f(10)$ to 1 decimal point.

In [ ]:
```
# (1) Type your answer below and press SHIFT+ENTER

# (2) Your answer:
# (3) Your answer:
# (4) Your answer:
```

B16. **EXERCISE.**

(1) Graph $f(x) = \cos^2 x - \sin^2 x$ over the interval $[-2\pi, 2\pi]$ using 100 points.

(2) Does the resemble any of the following? $\qquad$ $\cos 2x$ $\qquad$ $\cos x/2$ $\qquad$ $\cos x$

In [ ]:
```
# (1) Type your answer below and press SHIFT+ENTER

# (2) Your answer:
```

B17. **EXERCISE.**

(1) Plot the polynomial function $f(x) = x^3 - 20x^2 + 10x - 1$ over the interval $[-10, 10]$.

(2) Which is the approximate range for the y-axis?

$[-10, 10]$ $\quad$ $(-10, 10)$ $\quad$ $[-3100, 0]$ $\quad$ $[0, 2\pi]$

In [ ]:
```
# (1) Type your answer below and press SHIFT+ENTER

# (2) Your answer:
```

B18. **EXERCISE.** We wish to investigate when (if) the function in B17 is positive. We can't readily tell from our graph in B17 so we will replot over a smaller domain.

(1). Which of these domains seems appropriate for this task?

$[0, 500]$ $\quad$ $[0, 10]$ $\quad$ $[-1, 1]$ $\quad$ $[0, 2\pi]$

(2) Replot the graph over the selected domain. Turn on the grid using `grid()`

(3) From your graph, which of these x values have $f(x) > 0$? Indicate all that apply:

0 $\quad$ 0.25 $\quad$ 0.50 $\quad$ 0.75

In [ ]:
```
# (1) Your answer:
# (2) Type your answer below and press SHIFT+ENTER

# (3) Your answer:
```

Lab C: Finding Limits in Python

https://mybinder.org/v2/gh/anniebmcc/pycalclab/master?filepath=mat301c.ipynb
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C1. **RUN** the following. When we are asked to compute a limit, the first thing we try is plugging in. Sometimes this works, such as for $\lim_{x\to\pi/2}\sin(x)/x$:

In [ ]:
```
%pylab inline
```

In [ ]:
```
x = pi/2
sin(x)/x
```

C2. **RUN** the following. However sometimes plugging in doesn't work, such as for $\lim_{x\to 0}\sin(x)/x$:

In [ ]:
```
x = 0
sin(x)/x
```

C3. **RUN** the following. When plugging in doesn't work, there are two things we can do in Python. First we can look at the graph. The graph tells us that $\lim_{x\to 0}\sin(x)/x = 1$.

In [ ]:
```
x = linspace(-10,10)
y = sin(x)/x
plot(x,y)
```

C4. **RUN** the following. The second thing we can do is make a table of x and $f(x)$ where we plug in numbers closer and closer to the given $x = a$. The table tells us that $\lim_{x\to 0}\sin(x)/x = 1$.

In [ ]:
```
x = r_[1, .1, .01, .001]
y = sin(x)/x
c_[x,y]
```

In [ ]:
```
x = r_[-1, -.1, -.01, -.001]
y = sin(x)/x
c_[x,y]
```

C5. **RUN** the following. Sometimes python will give us scientific notation. We use the command `set_printoptions` to turn off scientific notation and get more digits of precision.

In [ ]:
```
x = r_[1, .1, .01, .001, .0001, .00001]
y = sin(x)/x
c_[x,y]
```

In [ ]:
```
set_printoptions(precision=17, suppress=True)

x = r_[1, .1, .01, .001, .0001, .00001]
y = sin(x)/x
c_[x,y]
```

C6. **RUN** the following. See that we get the same output in C6 and C5, but the code is cleaner in C6 because we make x in a more clever way.

In [ ]:
```
x = .1 ** r_[:6]
y = sin(x)/x
c_[x,y]
```

C7. **EXERCISE.**
(1) Use the graphical approach to find the following right limit of $f(x) = x^x$, $x > 0$,

$$\lim_{x \to 0^+} x^x.$$

(2) What is the value of the limit? (Enter no limit as DNE)

In [ ]:
```
# (1) Type your code below and press SHIFT+ENTER

# (2) YOUR ANSWER:
```

C8. **EXERCISE.**
(1) Use the numeric (table) approach to find the following (two sided) limit

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2}.$$

You will need a table for the right-hand limit and a table for the left-hand limit.

(2) What is the value of the limit?
(3) How does the numerical instability of the problem show up in the output?

In [ ]:
```
# (1) Type your code below and press SHIFT+ENTER
```

In [ ]:
```
```

In [ ]:
```
# (2) YOUR ANSWER:

# (3) YOUR ANSWER:
#
#
```

C9. **EXERCISE.**
(1) Use the numeric approach to find the following limit

$$\lim_{x \to \pi/2} \left(\frac{\pi}{2} - x \right) \tan x.$$

(2) What is the value of the limit?

In [ ]:
```
# (1) Type your code below and press SHIFT+ENTER
```

In [ ]:
```

```

In [ ]:
```
# (2) YOUR ANSWER:
```

C10. **EXERCISE.**
Let $f(x) = x \ln x$ for $x > 0$. We will investigate the right-hand limit $\lim_{x \to 0^+} f(x)$ as follows.
(1) Use the numeric approach to find the limit.
(2) What is the value of the limit?
(3) Plot a graph of $f(x)$ over the interval $(0, 1)$.
(4) Does the graph in (3) confirm your answer in (2)?

In [ ]:
```
# (1) Type your code below and press SHIFT+ENTER
```

In [ ]:
```
# (2) YOUR ANSWER:
```

In [ ]:
```
# (3) Type your code below and press SHIFT+ENTER

# (4) YOUR ANSWER:
```

C11. **EXERCISE.**
We wish to find the limit of the oscillating function

$$f(x) = x \sin \tfrac{1}{x}$$

as x approaches 0.

(1) Plot the function f over the interval $[-pi, pi]$ using 10000 points for x.
(2) "Zoom in" by replotting the function over $[-0.1, 0.1]$.
(3) Motivated by the squeeze theorem, plot on the same axes the function f over $[-0.1, 0.1]$ as well as $y = |x|$ and $y = -|x|$.
(4) Estimate the limit as $x \to 0$.
(5) How did graphing the absolute value help you find the limit in (3)? (Choose one)
The mean-value theorem *The function is continuous* *The squeeze theorem* *They didnt; I just guessed*

In [ ]:
```
# (1) Type your code below and press SHIFT+ENTER
```

In [ ]:
```
# (2) Type your code below and press SHIFT+ENTER
```

In [ ]:
```
# (3) Type your code below and press SHIFT+ENTER

# (4) YOUR ANSWER:
# (5) YOUR ANSWER:
```

Functions

C12. **RUN** the following. We learned how to make python functions in A16. Here we define

$$f(x) = \frac{\sin x}{x}$$

$$g(x) = 7x^2 \sin \frac{1}{7x^2}$$

and then use our functions to find $f(1)$ and $g(1)$.

In [ ]:
```
def f(x):
    return sin(x)/x

def g(x):
    a = 7*x**2
    return a * sin(1/a)

f(1), g(1)
```

C13. **EXERCISE.**

(1) Define the python function $f(x) = x^{-1}e^{-1/x}$
(2) Plot the function $f(x)$ over the interval $(0, 1]$ using your python function `f`
(3) Use your graph in (2) to estimate the right limit as x goes to 0.

In [ ]:
```
# (1) Type your code below and press SHIFT+ENTER

# (2) Type your code below and press SHIFT+ENTER

# (3) YOUR ANSWER:
```

C14. **EXERCISE.**

(1) Create python functions with the following definitions.

$$f(x) = \frac{x-1}{\arccos x}$$

$$g(x) = x^x$$

(2) Make a table to find the limit

$$\lim_{x \to 1^-} g(f(x))$$

(3) What is the value of the limit in (2)?
(4) Does the limit in (1) satisfy the following limit law? If so, what are c, L, M?

If $\lim_{x \to c} f(x) = L$
and $\lim_{x \to L} g(x) = M$
then $\lim_{x \to c} g(f(x)) = M$

In [ ]:
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# (1) Type your code below and press SHIFT+ENTER

# (2) Type your code below and press SHIFT+ENTER

# (3) YOUR ANSWER:
# (4) YOUR ANSWER:
```

C15. **EXERCISE.**
(1) Define $f(x) = x^x$.
(2) Make a table to find

$$f'(x) := \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

for $x = 1$.

(3) What value for $f'(1)$ do you get?
(4) Make the same table as in (2) but with $x = 0$.
(5) What value for $f'(0)$ do you get?
(6) Are you surprised that 43) worked? What part of (4) do we expect to fail, and what did python do instead?

In [ ]:
```
# (1) Type your code below and press SHIFT+ENTER

# (2) Type your code below and press SHIFT+ENTER

# (3) YOUR ANSWER:
```

In [ ]:
```
# (4) Type your code below and press SHIFT+ENTER

# (5) YOUR ANSWER:
```

In [ ]:
```
# (6) YOUR ANSWER:
#
#
```