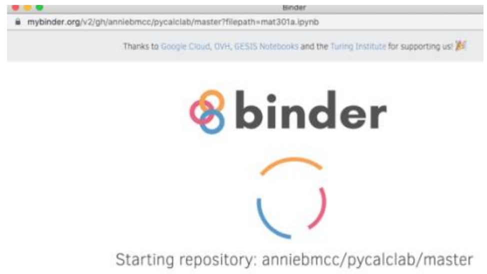
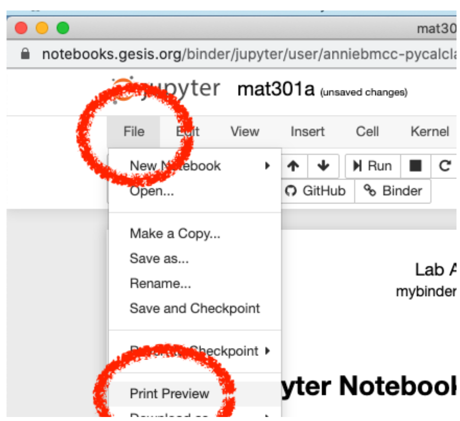
MAT301 Lab Directions

1. Click link in table of contents
   Wait for page to load

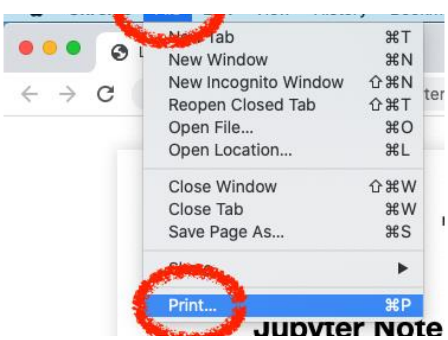


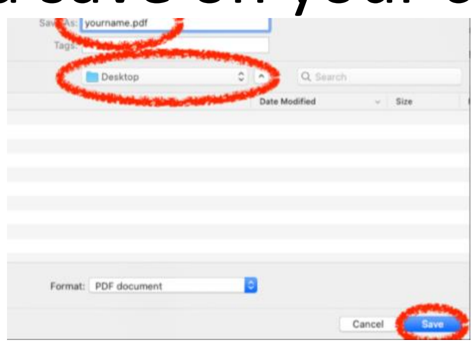
2. Run each cell that says RUN
   Type your answer to each EXERCISE

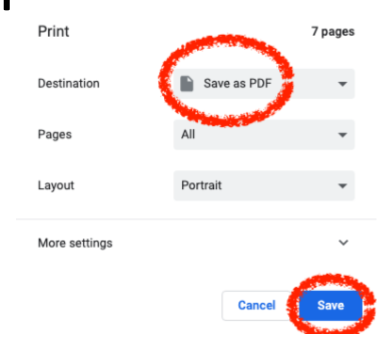
3. Open print preview



4. Print to PDF and save on your computer







5. Upload your PDF to blackboard

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Lab A: Using Python as a Calculator
https://mybinder.org/v2/gh/anniebmcc/pycalclab/master?filepath=mat301a.ipynb
2020 Summer — Calculus 1
Dr Matthew H Sunderland

Jupyter Notebooks

A1. **RUN** the following "code cell" (gray rectangle with `In[ ]` next to it), by CLICKING the code cell and pressing SHIFT+RETURN. Notice that only the last result will display.

In [ ]:
```
1 + 2 + 3
50 - 3
100*5
```

A2. **RUN** the following. As always, only the last result displays, but the last result has 2 parts because of the comma.

In [ ]:
```
1 + 2 + 3
50 - 3, 1000*1000
100*5, 7*7
```

A3. The "+" on the toolbar adds a code cell. The "scissors" deletes a cell.

Python arithmetic + - * / **

A4. **RUN** the following.

In [ ]:
```
3 + 10*5,  5**2,  27/10
```

A5. **EXERCISE.**
a) What does each of the 5 arithmetic operations do?
b) Do spaces around the 5 operations matter, or is it just style?

In [ ]:
```
# TYPE YOUR ANSWERS BELOW
#
#  a) + is
#     - is
#     * is
#     / is
#    ** is
#
#  b)
```

Python # and =

A6. **RUN** the following. You will notice python ignores everything after #

In [ ]:
```
# This is a comment
1 + 1 # This is also a comment
```

A7. **RUN** the following. Notice we assign variables using = Assignment itself does NOT produce output.

In [ ]:
```
a = 10
a
```

In [ ]:
```
b = 20
```

In [ ]:
```
a = 18
b = 21
c = a - b
c
```

A8. **RUN** the following. Notice you can assign multiple variables at once with a comma.

In [ ]:
```
x, y = 100, 500
x
```

In [ ]:
```
a,b,c = 3,4,5
a + b/c
```

A9. **RUN** the following. See that we can compute $\frac{(2-3)*-3}{-1+2}$ all at once (1st cell below), or we can assign variables to help us (2nd cell below).

In [ ]:
```
(2 - 3)*-3/(-1 + 2)
```

In [ ]:
```
top = (2 - 3)*-3
bottom = -1 + 2
top/bottom
```

A10. **EXERCISE.** Assign variables to help you compute $3 - \frac{3^2-2\cdot 3}{2\cdot 3-2}$

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

Order of Operations

A11. **RUN** the following. Notice $a - b * c = a - (b * c)$, but they do not equal $(a - b) * c$.

In [ ]:
```
a,b,c = 3,4,5

a - b*c,   a - (b*c),   (a - b)*c
```

A12. **EXERCISE.** In each row, identify NON-equivalent choice. For example, the answer to (1) is $(a - b) * c$ because $a - b * c = a - (b * c)$

(1)	$a - b * c$	$a - (b * c)$	$(a - b) * c$
(2)	$a * (b - c)$	$(a * b) - c$	$a * b - c$
(3)	$a / b + c$	$a / (b + c)$	$(a / b) + c$
(4)	$(a + b) / c$	$a + (b / c)$	$a + b / c$
(5)	$a ** (b * c)$	$(a ** b) * c$	$a ** b * c$
(6)	$a * (b ** c)$	$a * b ** c$	$(a * b) ** c$
(7)	$a/b ** c$	$(a/b) ** c$	$a/(b ** c)$
(8)	$a ** b/c$	$(a ** b)/c$	$a ** (b/c)$
(9)	$(3 - 3) - 3$	$3 - 3 - 3$	$3 - (3 - 3)$
(10)	$(2 ** 3) ** 2$	$2 ** (3 ** 2)$	$2 ** 3 ** 2$
(11)	$6 / 3 / 2$	$6 / (3 / 2)$	$(6 / 3) / 2$

In [ ]:
```
# TYPE YOUR ANSWERS BELOW.
#
#  (1)   (a - b)*c
#  (2)
#  (3)
#  (4)
#  (5)
#  (6)
#  (7)
#  (8)
#  (9)
# (10)
# (11)
```

A13. **RUN** the following example, where we add 2 sets of parentheses which show the order of the 2 operations.

In [ ]:
```
1 + 3/5
```

In [ ]:
```
(1 + (3/5))
```

A14. **EXERCISE.** Add 4 sets of parentheses, which show the order of the 4 operations.

In [ ]:
```
7 - 3 ** 2/9 + 4
```

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

A15. **EXERCISE.** Assign `a,b,c = 4,5,8` and then evaluate $\dfrac{a^b - c/b}{c-a}, \dfrac{a^{c-b}}{c-b}, \dfrac{a^{3/2}}{b}, \dfrac{a-b(c-a)}{c-a}$

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

Making python functions

A16. **RUN** the following.

In [ ]:
```
def g(x):
    return x**2

g(7)
```

In [ ]:
```
def h(n): return n + 100

h(7)
```

A17. **EXERCISE.** Make the function $P(x) = x^2 - 2x + 1$ and find $P(P(7))$.

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

Built-in `%pylab` functions

Python	Math notation	Meaning
`abs(x)`	$\|x\|$	absolute value
`sqrt(x)`	$\sqrt{x}$	square root
`exp(x)`	e^x	exponential function
`log(x)`	$\ln x$	natural logarithm
`sin(x)`	$\sin x$	sine
`arcsin(x)`	$\sin^{-1} x$	inverse sine
`radians(x)`		converts degrees to radians

A18. **RUN** the code cells below. The command `%pylab` only needs to be run once per lab; it loads "built-in functions" (from python packages numpy and matplotlib).

In [ ]:
```
%pylab

sqrt(49)
```

In [ ]:
```
pi, exp(1), sin(pi/2)
```

A19. **EXERCISE.** Evaluate

1. $\sin 40^\circ$
2. $\sin^2 65^\circ$
3. $e^{(10-8.5)/3}$
4. $\arcsin(\sin(3\pi/4))$

Note. Python uses radians for all angle measurements, so you need to convert any degrees to radians.

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

Making an array with `r_[ ]`

A20. **RUN** the following. (If you get an error, go back and run A17.) The function `r_[ ]` can make an array of numbers of your choice. We will need arrays for graphing (Lab B).

In [ ]:
```
x = r_[2,3,4,5,10]
x**3
```

A21. **EXERCISE.** Use `r_[ ]` to store the numbers 2,3,5,7,11 in an array named `x`. Find `x*x`.

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

Making an array with `r_[a:b:stride]`

A22. **RUN** the following. In general, `r_[a:b]` will list integers from a up to but *not* including b. A missing a is the same as 0.

In [ ]:
```
r_[5:10]
```

In [ ]:
```
r_[:5]
```

A23. **EXERCISE.** Use `r_[a:b]` to make the array 1,2,3,4,5,6,7,8,9

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

A24. **RUN** the following. In general, `r_[a:b:stride]` spaces out your numbers by the amount `stride` .

In [ ]:
```
r_[0:100:2]
```

A25. **EXERCISE.** Use `r_[a:b:stride]` to make the array $1, 3, 5, \ldots, 99$

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

Making an array with `linspace(a,b,n)`

A26. **RUN** the following. Observe that `linspace(a,b,n)` lists n numbers from a to b inclusive. This is useful for generating a lot of evenly-spaced numbers, such as when graphing (Lab B). Observe that `linspace(a,b)` lists 50 numbers from a to b inclusive.

In [ ]:
```
linspace(0,10,6)
```

In [ ]:
```
linspace(0,10)
```

A27. **EXERCISE.** Use `linspace(a,b,n)` to make the array $1, 1.5, 2, 2.5, 3, 3.5, 4$

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

A28. **EXERCISE.**
Convert average body temperature $98.6^{\circ}\ F$ to Celsius using $C = 5/9(F - 32)$.

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

A29. **RUN** the following.
Notice that `x` and `y` are arrays,
`c_[x,y]` puts them into a table.

In [ ]:
```
x = r_[:10]
y = x**2
c_[x,y]
```

A30. **EXERCISE.**
Use `r_` to make an array of Fahrenheit values `x` $= -100, -80, -60, \ldots, 100.$
Make the corresponding array of Celsius values `y`
Use `c_` to put `x` and `y` into a table.

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

Lab B: Plotting Graphs in Python
https://mybinder.org/v2/gh/anniebmcc/pycalclab/master?filepath=mat301b.ipynb
2020 Summer — Calculus 1
Dr Matthew H Sunderland

Plotting with `plot`

B1. Example. To graph $f(x) = x^2$ over $[-2, 2]$ by hand, make an xy table: choose some x values,

x	-2	-1	0	1	2
y					

and then use f to compute the corresponding y values.

B2. **RUN** the following. Notice that graphing in python is similar to B1: we make a list of x values and y values.

In [ ]:
```
%pylab inline
```

In [ ]:
```
x = r_[-2, -1, 0, 1, 2]
y = r_[4, 1, 0, 1, 4]
plot(x,y)
```

B3. **RUN** the following. Notice that we save time by making the x array using `linspace` (see A27) and making the y array by doing arithmetic on x (see A29). For illustrative purposes, we use `c_[x,y]` to make a table out of the arrays x and y (see A29).

In [ ]:
```
x = linspace(-2,2,9)
y = x**2

plot(x,y)
title('$f(x) = x^2$ plotted with 9 points')
grid()

c_[x,y]
```

B4. **RUN** the following, which graph $f(x) = e^x$ over the interval $[0, 7]$. Here we make our array x using `r_[a:b:stride]` (see A22). Remember that `exp(x)` is how you write e^x in python (see A18).

In [ ]:
```
x = r_[:7]
y = exp(x)
plot(x,y)
```

B5. **RUN** the following. When we change the x we must recompute the y; there are two ways to do it (compare B4 to B5).

In [ ]:
```
x = r_[:10]
plot(x, exp(x))
```

B6. **EXERCISE.**
(1) Graph $y = \cos 4x$ over $[0, \pi]$ with a step size of `pi/10`
(2) Redo your plot from iii. using `x = linspace(0,pi)`
(3) Which plot looks more like the plot of a cosine curve?

In [ ]:
```
# (1) Type your answer below and press SHIFT+ENTER
```

In [ ]:
```
# (2) Type your answer below and press SHIFT+ENTER

# (3) Your answer:
```

B7. **EXERCISE.** Plot the function $f(x) = e^{\cos x}$ over the interval $[0, 2\pi]$.

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

Doing arthmetic on arrays

B8. **RUN** the following.
We make numpy arrays with `r_` or `linspace`
Numpy arrays "know" how to do "elementwise" arithmetic.
Warning: x^2 is written `x**2` .

In [ ]:
```
x = r_[1:5]

x,  10 - x,  x + 10,  10*x,  x**2,  12/x,  x**x,  10**x
```

B9. **RUN** the following.

In [ ]:
```
# We can add arrays of the same shape (same length)

x = r_[10, 20, 50, 100]
y = r_[3, 0, 7, -1]
x + y
```

In [ ]:
```
# We can add an array (x) and a scalar (y)

x = r_[10, 20, 50, 100]
y = 100
x + y
```

In [ ]:
```
# We CANNOT add arrays of DIFFERENT shape

x = r_[10, 20, 50, 100]
y = r_[3, 0, 7]
x + y
```

B10. **RUN** the following.

In [ ]:
```
# y = sin x + cos 3x over the domain [0,2pi]

x = linspace(0,2*pi)
y = sin(x) + cos(3*x)
plot(x,y)
```

In [ ]:
```
# y = e^(-x/2) cos 6x over the domain [0,10pi]

x = linspace(0, 10*pi, 300)
y1 = exp(-x/2) # Here we break up the
y2 = cos(6*x)  # computation into
y = y1*y2      # bite-sized pieces
plot(x,y)
```

In [ ]:
```
# y = 1/(x^2 - 1) over the domain [2,5]

x = r_[2:5:0.1]
y = 1/(x**2 - 1)
plot(x,y)
```

B11. **EXERCISE.** First **RUN** the following.

In [ ]:
```
a,b,c = r_[:5], r_[:50:10], r_[:10]
a,b,c
```

Now, that we've defined a, b, c, which of the following are defined?
$a + b \qquad a + c \qquad a + 1 \qquad a * b \qquad c ** 2 \qquad c\ \hat{}\ 2$

In [ ]:
```
# Type your answer below and press SHIFT+ENTER
```

B12. **RUN** the following example. Let x be the array 1,2,3. Write Python commands to compute x^3.
The output you get should be `array([ 1,  8, 27])` .

In [ ]:
```
x = r_[1,2,3]
x**3
```

B13. **EXERCISE.** Using the same array `x = r_[1,2,3]` , find:

$\cos x \sin x \qquad \sin^2 x \qquad \sin x^2 \qquad 7x^2 \sin \frac{1}{7x^2}$

You should get

```
array([ 0.45464871, -0.37840125, -0.13970775])
array([0.70807342, 0.82682181, 0.01991486])
array([ 0.84147098, -0.7568025 ,  0.41211849])
array([0.99660211, 0.99978743, 0.99995801])
```

```
In [ ]: # Type your answer below and press SHIFT+ENTER
```

B14. **EXERCISE.** Using the same array `x = r_[1,2,3]` , find:

$x - \frac{\cos x - \sin x}{\sin x + \cos x} \qquad \frac{1}{10}(x - \frac{x^{3/2}}{10})^2$

You should get

```
array([1.2179581 , 4.68770694, 1.66751188])
array([0.081     , 0.29486292, 0.61523085])
```

```
In [ ]: # Type your answer below and press SHIFT+ENTER
```

Graphing practice

B15 **EXERCISE.**

(1) Graph the function $f(x) = \sin(\frac{\pi}{2}x) + \sin(\frac{2}{5}\pi x)$ over the interval $[0, 40]$.

(2) How many peaks (relative maxima) does your graph have?

(3) This function is periodic; how many periods are graphed in $[0, 40]$?

(4) Estimate from your graph the value of $f(10)$ to 1 decimal point.

```
In [ ]: # (1) Type your answer below and press SHIFT+ENTER

        # (2) Your answer:
        # (3) Your answer:
        # (4) Your answer:
```

B16. **EXERCISE.**

(1) Graph $f(x) = \cos^2 x - \sin^2 x$ over the interval $[-2\pi, 2\pi]$ using 100 points.

(2) Does the resemble any of the following? $\qquad \cos 2x \qquad \cos x/2 \qquad \cos x$

```
In [ ]: # (1) Type your answer below and press SHIFT+ENTER

        # (2) Your answer:
```

B17. **EXERCISE.**
(1) Plot the polynomial function $f(x) = x^3 - 20x^2 + 10x - 1$ over the interval $[-10, 10]$.
(2) Which is the approximate range for the y-axis?

$[-10, 10]$ $\quad$ $(-10, 10)$ $\quad$ $[-3100, 0]$ $\quad$ $[0, 2\pi]$

In [ ]:
```
# (1) Type your answer below and press SHIFT+ENTER

# (2) Your answer:
```

B18. **EXERCISE.** We wish to investigate when (if) the function in B17 is positive. We can't readily tell from our graph in B17 so we will replot over a smaller domain.

(1). Which of these domains seems appropriate for this task?

$[0, 500]$ $\quad$ $[0, 10]$ $\quad$ $[-1, 1]$ $\quad$ $[0, 2\pi]$

(2) Replot the graph over the selected domain. Turn on the grid using `grid()`

(3) From your graph, which of these x values have $f(x) > 0$? Indicate all that apply:

0 $\quad$ 0.25 $\quad$ 0.50 $\quad$ 0.75

In [ ]:
```
# (1) Your answer:
# (2) Type your answer below and press SHIFT+ENTER

# (3) Your answer:
```

Lab C: Finding Limits in Python
https://mybinder.org/v2/gh/anniebmcc/pycalclab/master?filepath=mat301c.ipynb
2020 Summer — Calculus 1
Dr Matthew H Sunderland

C1. *RUN the following.*

In [ ]:
```
%pylab inline
```

The magic word `%pylab` loads a bunch of useful functions (including `pi` and `sin`).
The option `inline` tells Jupyter to display any graphs we make on the page instead of in a popup.

C2. Example. Let's say we want to find $\lim_{x \to \pi/2} \frac{\sin x}{x}$. The first thing we try is plugging in.

RUN the following.

In [ ]:
```
x = pi/2
sin(x)/x
```

C3. **DO EXERCISE.** Based on the result in C2, what is the value of the limit $\lim_{x \to \pi/2} \frac{\sin x}{x}$?

In [ ]:
```
# RECORD YOUR ANSWER: the limit is ________
```

C4. Example. Let's say we want to find $\lim_{x \to 0} \frac{\sin x}{x}$ instead.

RUN the following.

In [ ]:
```
x = 0
sin(x)/x
```

Notice that plugging in doesn't work, because sin(0)/0 is undefined.
(Instead of "undefined", Python says "nan," which means "not a number.")

Find the limit with a graph

C5. When plugging in doesn't work, one thing we can try is graphing.

RUN the following.

In [ ]:
```
x = linspace(-10,10)
y = sin(x)/x
plot(x,y)
```

C6. **DO EXERCISE.** Based on the graph in C5, what is the value of the limit $\lim_{x \to 0} \frac{\sin x}{x}$?

In [ ]:
```
# RECORD YOUR ANSWER: the limit is _________
```

Find the limit with a table

C7. The other thing we can try is making a table.

RUN the following.

In [ ]:
```
x = r_[1, .1, .01, .001] # Make array of numbers approaching 0 from right
y = sin(x)/x             # Plug the array into our function
c_[x,y]                  # Make a table
```

In [ ]:
```
x = r_[-1, -.1, -.01, -.001] # Same but approaching from left
y = sin(x)/x
c_[x,y]
```

Note that to display two tables, we use two code cells, because Jupyter only displays one output per cell.

C8. **DO EXERCISE.** Based on the table in C7, what is the value of the limit $\lim_{x \to 0} \frac{\sin x}{x}$?

In [ ]:
```
# RECORD YOUR ANSWER: the limit is _________
```

Simplify the code

C9. We can simplify the code for `x` using an array.

RUN the following.

```
In [ ]: x = .1 ** r_[:4]
        y = sin(x)/x
        c_[x,y]
```

You should get the same table as in C7.
Why does this work? Because `.1 ** r_[:4]` means "0.1 raised to the $0, 1, 2, 3$," which is going to be $1, 0.1, 0.01, 0.001$.

Increase precision

C10. The arrays we have made so far show (up to) 8 decimal places.
As an example, let's change the number of decimal places displayed to 17 decimal places:

RUN the following.

```
In [ ]: set_printoptions(precision=17)

        x = .1 ** r_[:4]
        y = sin(x)/x
        c_[x,y]
```

After you run this cell, from now on, arrays will display (up to) 17 decimal places.

Turn off scientific notation

C11. Arrays with very small numbers (less than 0.001) will automatically change to scientific notation.

RUN the following.

```
In [ ]: x = .1 ** r_[:5]
        y = sin(x)/x
        c_[x,y]
```

C12. We can turn off scientific notation for arrays using `set_printoptions`

RUN the following.

```
In [ ]: set_printoptions(suppress=True)

        x = .1 ** r_[:5]
        y = sin(x)/x
        c_[x,y]
```

C13. **DO EXERCISE.**

(1) Use the graphical approach to find $\lim_{x \to 0^+} x^x$.

(2) Use the numerical approach (make a table) for the same limit.

```
In [ ]: # (1) TYPE YOUR CODE:

        # RECORD YOUR ANSWER: the limit is ________
```

```
In [ ]: # (2) TYPE YOUR CODE:

        # RECORD YOUR ANSWER: the limit is ________
```

Define a function

C14. Example. Let's make a graph and make tables to find $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$

We will use `def` to store the function so we don't have to type the function over and over.

RUN the following.

```
In [ ]: def f(x):
            return (1 - cos(x))/x**2
```

```
In [ ]: x = linspace(-1,1)
        y = f(x)
        plot(x,y)
```

```
In [ ]: x = 0.1 ** r_[:8]
        y = f(x)
        c_[x,y]
```

```
In [ ]: x = -0.1 ** r_[:8]
        y = f(x)
        c_[x,y]
```

So, the graph and the tables tell us that $\lim_{x \to 0} \frac{1 - \cos x}{x^2} = 0.5$

Taking the limit as x approaches a number other than zero

C15. Example. Let's investigate $\lim_{x \to \pi/2} \left(\frac{\pi}{2} - x \right) \tan x$

Notice that in this example, x is not approaching 0.

RUN the following.

In [ ]:
```
def f(x):
    return (pi/2 - x)*tan(x)
```

In [ ]:
```
x = linspace(0,pi)
y = f(x)
plot(x,y)
```

In [ ]:
```
x = pi/2 + 0.1 ** r_[:8]
y = f(x)
c_[x,y]
```

In [ ]:
```
x = pi/2 - 0.1 ** r_[:8]
y = f(x)
c_[x,y]
```

The graph and the tables tell us that $\lim_{x \to \pi/2} \left(\frac{\pi}{2} - x \right) \tan x = 1$

More Exercises

C16. **DO EXERCISE.**
Let $f(x) = x \ln x$.
(1) Plot a graph of $f(x)$ over the interval $(0, 1]$ and based on your graph, estimate

$$\lim_{x \to 0^+} f(x).$$

(2) Use the numeric approach to find the same limit.

In [ ]:
```
# (1) TYPE YOUR CODE:

# RECORD YOUR ANSWER: the limit is _________
```

In [ ]:
```
# (2) TYPE YOUR CODE:

# RECORD YOUR ANSWER: the limit is _________
```

C17. **DO EXERCISE.**

(1) Define the python function $f(x) = x^{-1}e^{-1/x}$

(2) Plot the function $f(x)$ over the interval $(0, 1]$ and estimate.

$$\lim_{x \to 0^+} f(x)$$

(3) Use the numeric approach to find the same limit.

In [ ]:
```
# (1) TYPE YOUR CODE:
```

In [ ]:
```
# (2) TYPE YOUR CODE:

# RECORD YOUR ANSWER: the limit is _________
```

In [ ]:
```
# (3) TYPE YOUR CODE:

# RECORD YOUR ANSWER: the limit is _________
```

C18. **DO EXERCISE.**

We wish to find the limit of the oscillating function

$$f(x) = x \sin \frac{1}{x}$$

as x approaches 0.

(1) Plot the function f over the interval $[-pi, pi]$ using 10000 points for x.

(2) "Zoom in" by replotting the function over $[-0.1, 0.1]$.

(3) Motivated by the squeeze theorem, plot on the same axes the function f over $[-0.1, 0.1]$ as well as $y = |x|$ and $y = -|x|$.

(4) Estimate the limit as $x \to 0$.

(5) How did graphing the absolute value help you find the limit in (3)? (Choose one)

The mean-value theorem *The function is continuous* *The squeeze theorem* *They didnt; I just guessed*

In [ ]:
```
# (1) TYPE YOUR CODE:
```

In [ ]:
```
# (2) TYPE YOUR CODE:
```

In [ ]:
```
# (3) TYPE YOUR CODE:

# (4) RECORD YOUR ANSWER: the limit is _________
# (5) RECORD YOUR ANSWER:
```

Lab D: Finding First Derivatives in Python
https://mybinder.org/v2/gh/anniebmcc/pycalclab/master?filepath=mat301d.ipynb
2020 Summer — Calculus 1
Dr Matthew H Sunderland

D1. *RUN the following.*

In [ ]:
```
%pylab inline
set_printoptions(suppress=True)
```

D2. **Warmup (1)–(4).**

Let $f(x) = \dfrac{x^2 - x}{\sqrt{(x-1)^2}} + 1$

(1) Find $\lim_{x\to 1^+} f(x)$

In [ ]:
```
# (1) TYPE YOUR CODE:

# RECORD YOUR ANSWER: the limit is _____
```

(2) Find $\lim_{x\to 1^-} f(x)$

In [ ]:
```
# (2) TYPE YOUR CODE:

# RECORD YOUR ANSWER: the limit is _____
```

(3) Based on your answer to (1) and (2), find $\lim_{x\to 1} f(x)$

In [ ]:
```
# (3) RECORD YOUR ANSWER: the limit _____
```

(4) Graph the function f over $[-2, 1) \cup (1, 4]$.

In [ ]:
```
# (4) TYPE YOUR CODE:
```

Computing the derivative at a point using the limit definition of derivative

In lecture we learned that limit definition of derivative is $f'(x) := \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

D3. EXAMPLE. Let us approximate $\frac{d}{dx}x^x\Big|_{0.6}$ to 4 decimal places.

RUN the following.

In [ ]:
```
def f(x): return x**x

x = 0.6
h = 0.1 ** r_[:9]    # = array of numbers going to 0
m = (f(x+h)-f(x))/h
c_[h,m]              # make a table
```

Thus the limit to 4 decimal places is 0.3600

D4. ASSIDE. Later on in calculus you will learn how to compute $\frac{d}{dx}x^x\Big|_{0.6}$ algebraically. It turns out that

$$(x^x)' = (e^{(\ln x)(x)})' = e^{(\ln x)(x)}(x/x + \ln x) = x^x(1 + \ln x)$$

and so $(x^x)'(0.6) = 0.6^{0.6}(1 + \ln 0.6)$.

RUN the following

In [ ]:
```
0.6**0.6*(1 + log(0.6))
```

So we see that $\frac{d}{dx}x^x\Big|_{0.6} = 0.6^{0.6}(1 + \ln 0.6) = 0.3600430649889697$, which matches our approximation in D3.

D5. **EXERCISE.** Approximate $\frac{d}{dx}\left(\sin x\right)\Bigg|_{x\,=\,\pi/4}$ to 4 decimal places.

In [ ]:
```
# TYPE YOUR CODE:

# RECORD YOUR ANSWER:
# the answer to 4 decimal places is _____
```

Secant and tangent lines

The fraction inside the limit in the limit definition of derivative is called the "difference quotient"

difference quotient = $\dfrac{f(x+h)-f(x)}{h}$

The difference quotient is the slope of the secant line though the graph at x and $x+h$.

D6. EXAMPLE.
Consider the same function as in D3, $f(x) = x^x$.
We'll plot f over $[0,2]$ and the secants through $x_0 = 1$ with $h = 1, 0.1, 0.01$

RUN the following.

In [ ]:
```
def f(x): return x**x

x = linspace(0,2,10000)
plot(x,f(x))

x0 = 0.6

for h in [1, 0.1, 0.01]:
    m = (f(x0+h) - f(x0))/h
    y = f(x0) + m*(x-x0)
    plot(x,y)

legend(['Function f', 'Secant (h=1)', 'Secant (h=0.1)', 'Secant (h=0.01)'])
grid()
```

We actually have already computed the slope of these 3 secant lines in D3.
Their slopes are:
1.385228648279758,
0.430339898526158, and
0.367055009340134.

As h approaches 0, the secant lines approach the tangent line,
and the difference quotient (the slope of the secant line)
approaches the derivative (the slope of the tangent line).

D7. **EXERCISE (1)–(2).** Consider the same function as in D5, $f(x) = \sin x$.
(1) Plot f over $[0,\pi]$ and the secants through $x_0 = \pi/4$ with $h = \pi/2, 1, 0.1$.

PRO TIP. Change the x-axis labels using
`xticks([0,pi/4,pi/2,3*pi/4,pi],['0','pi/4','pi/2','3pi/4','pi'])`

(2) What is the slope of the three secant lines? Hint: look at the table you made in D5.

In [ ]:
```
# (1) TYPE YOUR CODE:

# (2) RECORD YOUR ANSWER:
#     the slope of the 3 secant lines are
#     __________,
#     __________,
#     __________
```

Graphing the difference quotient

You can approximate the derivative using the difference quotient with very small h.

D8. EXAMPLE. Consider the same function as in D3, $f(x) = x^x$.
Let's plot f over $[0, 3]$ and its difference quotient with $h = 0.001$.

RUN the following.

In [ ]:
```
def f(x): return x**x

x = linspace(0,3,10000)
plot(x,f(x))
grid()

h = 0.001
m = (f(x+h) - f(x))/h
plot(x,m,':r',linewidth=5) # Dotted, red, thick line

legend(['Function f','Difference quotient (h=0.01)']);
```

The difference quotient for small h is approximately the derivative.

D9. **EXERCISE (1)–(3).**
Consider the same function as in D5, $f(x) = \sin x$.

(1) Plot f and its difference quotient using $h = 0.001$ over the domain $[0, 2\pi]$.

PRO TIP. Change the x-axis labels using
`xticks([0,pi/2,pi,3*pi/2,2*pi], ['0','pi/2','pi','3pi/2','2pi'])`

In [ ]:
```
# TYPE YOUR CODE:
```

(2) What function do the difference quotients appear to approach?

In [ ]:
```
# (2) RECORD YOUR ANSWER: __________
```

(3) Fill-in-the-blank: At the roots of the difference quotients, the original function f has _____.

In [ ]:
```
# (3) RECORD YOUR ANSWER:
#     At the roots of the difference quotients,
#     the original function f has __________
```

D10. **EXERCISE (1)–(5).**
(1) Let $f(x) = e^x$.
Graph over $[-2, 2]$ the function f and its difference quotient for $h = 0.01$.

In [ ]:
```
# (1) TYPE YOUR CODE:
```

(2) TRUE/FALSE. Since f is always increasing, $f' > 0$
(3) TRUE/FALSE. Tangent lines for f are never flat, so f has no critical points.
(4) TRUE/FALSE. Since f increases faster and faster, f' should be increasing.
(5) Take a guess as to what the derivative of $f(x) = e^x$ is.

In [ ]:
```
# (2) RECORD YOUR ANSWER: _____
# (3) RECORD YOUR ANSWER: _____
# (4) RECORD YOUR ANSWER: _____
# (5) RECORD YOUR ANSWER: _____
```

Differentiability

In lecture we learned that corners and cusps mean "continuous but not differentiable."

D11. EXAMPLE.
Consider the function $f(x) = (x^2)^{1/3}$ which has a cusp at 0.

Let's plot the function f over $[-2, 2]$, and
its difference quotient for $h = 0.0001$ over $[-2, 0) \cup (0, 2]$.

RUN the following

In [ ]:
```
def f(x): return (x**2)**(1/3)

x = linspace(-2,2,10000)
plot(x,f(x)) # Plot original function f
ylim([-2,2]) # Zoom y-axis to [-2,2]
grid()       # Add gridlines

h = 0.0001

# Here we plot the difference quotient
# over the two pieces of [-2,0)u(0,2]:

x = linspace(-2,-0.0001,10000)
m = (f(x+h) - f(x))/h
plot(x,m,':r',linewidth=5) # Dotted, red, thick line.

x = linspace(0.0001,2,10000)
m = (f(x+h) - f(x))/h
plot(x,m,':r',linewidth=5); # Dotted, red, thick line.
```

The graph of the difference quotient approaches the graph of the derivative of $x^{2/3}$, which is $(2/3)x^{-1/3}$. Since f has a cusp at 0, the derivative has an asymptote at 0.

D12. **EXERCISE (1)–(2).**
(1) Let $f(x) = |x|$.
Plot the function f over $[-2, 2]$.
Plot the difference quotient for $h = 0.0001$ over $[-2, 0) \cup (0, 2]$.

(2) Fill in the blank: Since f has a corner at 0, thus f' has a _____ at 0.

In [ ]:
```
# (1) TYPE YOUR CODE:

# (2) RECORD YOUR ANSWER:
#     Since f has a corner at 0,
#     thus f' has a ____________ at 0.
```