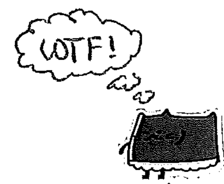
2-22-17 Loss Function partial derivaties in respect to r_{ij}

$$\underset{U,V,B^*,b'}{\operatorname{argmin}} \sum_{i,j \in K} \overbrace{\left(r_{ij} - \mu - b^*_i - b'_j - U_{i-}\cdot V_{-j}\right)^2}^{a} + \overbrace{\lambda_1\left(\|U_{i-}\|_2^2 + \|V_{-j}\|_2^2\right)}^{b} + \overbrace{\lambda_2\left((b^*_i)^2 + (b'_j)^2\right)}^{c}$$



Term a only

$$\frac{\partial}{\partial U_{ij}} \left(r_{ij} - \mu - b^*_i - b'_j - U_{i-}\cdot V_{-j}\right)^2$$

Apply chain Rule $\frac{df(a)}{dx} = \frac{df}{da} \cdot \frac{da}{dx}$

$$\frac{\partial}{\partial a}\left(a^2\right) = 2a$$

$$\frac{\partial}{\partial U_{ij}} = V_{ij}$$

U_{i-} V_{-j}

Mus iterate over entire row/column

$$\frac{\partial}{\partial a} \cdot \frac{\partial}{\partial U_{ij}} = 2a \cdot V_{ij} = \boxed{-2\left(r_{ij} - \mu - b^*_i - b'_j - U_{i-}\cdot V_{-j}\right) V_{ij}}$$

$$\frac{\partial}{\partial V_{ij}} = \boxed{-2\left(r_{ij} - \mu - b^*_i - b'_j - U_{i-}\cdot V_{-j}\right) U_{ij}}$$

(Evans)

Term a only (continued)

$$\frac{\partial}{\partial b_i^*}\left(r_{ij}-\mu-b_i^*-b_j'-u_{i-}\cdot V_{-j}\right)^2$$

$$\frac{\partial}{\partial a}\left(a^2\right) = 2a$$

$$\frac{\partial}{\partial b_i^*}\left(r_{ij}-\mu-b_i^*-b_j'-u_{i-}\cdot V_{-j}\right)^2 = -1$$

$$\frac{\partial}{\partial a}\cdot\frac{\partial}{\partial b_i^*} = -2a = \boxed{-2\left(r_{ij}-\mu-b_i^*-b_j'-u_{i-}\cdot V_{-j}\right)}$$

$$\parallel$$

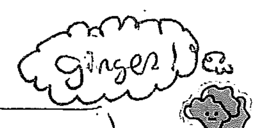
$$\frac{\partial}{\partial b_j'} = \boxed{-2\left(r_{ij}-\mu-b_i^*-b_j'-u_{i-}\cdot V_{-j}\right)}$$

Term B only

$$\frac{\partial}{\partial u_{ij}}\left(\lambda_1\left(\|u_{i-}\|_2^2+\|V_{-j}\|_2^2\right)\right)$$

$$\lambda_1\frac{\partial}{\partial u_{ij}}\left(\|u_{i-}\|_2^2+\cancelto{0}{\|V_{-j}\|_2^2}\right)$$

$$\|u_{i-}\|_2^2 = \sum_i^M u_{ij}^2 = 2u_{ij}$$



$$\therefore \boxed{\frac{\partial}{\partial u_{ij}} = \lambda_1\cdot 2\cdot u_{ij}} \quad \therefore \boxed{\frac{\partial}{\partial V_{ij}} = \lambda_1\cdot 2\cdot V_{ij}}$$

/Evans)

Term C

$$\lambda_2\left(\left(b_i^*\right)^2 + \left(b_j'\right)^2\right)$$

$$\frac{\partial}{\partial b_i^*}\left(\lambda_2\left(\left(b_i^*\right)^2 + \left(b_j'\right)^2\right)\right)$$

$$\lambda_2 \frac{\partial}{\partial b_i^*}\left(\left(b_i^*\right)^2 + \left(b_j'\right)^2\right) = 2b_i^*$$

$$\therefore \quad \cancel{\lambda_2 2 b_i^*}$$

$$\frac{\partial}{\partial b_i^*} = \boxed{\lambda_2 \cdot 2 \cdot b_i^*}$$

$$\frac{\partial}{\partial b_j'} = \boxed{\lambda_2 \cdot 2 \cdot b_j'}$$

All together

Must iterate over u_{i-}

$$\frac{\partial}{\partial u_{ij}} = \boxed{-2(r_{ij} - \mu - b_i^* - b_j' - u_{i-} \cdot V_{-j})\, v_{ij} + \lambda_1 \cdot 2 \cdot u_{ij}}$$

$$\frac{\partial}{\partial v_{ij}} = \boxed{-2(r_{ij} - \mu - b_i^* - b_j' - u_{i-} \cdot V_{-j})\, u_{ij} + \lambda_1 \cdot 2 \cdot v_{ij}}$$

$$\frac{\partial}{\partial b_i^*} = \boxed{-2(r_{ij} - \mu - b_i^* - b_j' - u_{i-} \cdot V_{-j}) + \lambda_2 \cdot 2 \cdot b_i^*}$$

$$\frac{\partial}{\partial b_j^*} = \boxed{-2(r_{ij} - \mu - b_i^* - b_j' - u_{i-} \cdot V_{-j}) + \lambda_2 \cdot 2 \cdot b_j'}$$

$$U_{new} = U_{old} - \alpha \frac{\partial}{\partial u}$$

$$V_{new} = V_{old} - \alpha \frac{\partial}{\partial V}$$

$$b^*_{new} = b^*_{old} - \alpha \frac{\partial}{\partial b^*}$$

$$b'_{old} = b'_{old} - \alpha \frac{\partial}{\partial b'}$$

(evans)