Definition. A semiring is an algebraic system $(\mathbb{K},\ \oplus,\ \otimes,\ \bar{0},\ \bar{1})$ such that,

- $(\mathbb{K},\ \oplus,\ \bar{0})$ is a **commutative monoid**[a] with $\bar{0}$ as the identity element for $\oplus$,
- $(\mathbb{K},\ \otimes,\ \bar{1})$ is a **monoid** with $\bar{1}$ as the identity element for $\otimes$,
- $\otimes$ distributes over $\oplus$: for all $a, b, c \in \mathbb{K}$,

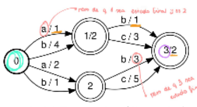
  $(a \oplus b) \otimes c = (a \otimes c) \oplus (b \otimes c)$

  $c \otimes (a \oplus b) = (c \otimes a) \oplus (c \otimes b)$
- $\bar{0}$ is an annihilator for $\otimes$: for all $a \in \mathbb{K}$,

  $a \otimes \bar{0} = \bar{0} \otimes a = \bar{0}$

- **Product** $\otimes$: to compute the weight of a path (product of the weights of constituent transitions)
- **Sum** $\oplus$: to compute the weight of a sequence (sum of the weights of the paths labeled with that sequence)

Semiring	Set	$\oplus$	$\otimes$	$\bar{0}$	$\bar{1}$
Boolean	$\{0, 1\}$	$\vee$	$\wedge$	0	1
Probability	$\mathbb{R}_+ \cup \{+\infty\}$	$+$	$\times$	0	1
Log	$\mathbb{R} \cup \{-\infty, +\infty\}$	$\oplus_{log}$	$+$	$+\infty$	0
Tropical	$\mathbb{R}_+ \cup \{+\infty\}$	min	$+$	$+\infty$	0



Probability semiring $(\mathbb{K}, +, \times, 0, 1)$

$[[T]](ab) = 14 \to \pi_1 = 1 \otimes 1 \otimes 2 = 2$; $\to \pi_2 = 2 \otimes 3 \otimes 2 = 12$ $\}\ 2 \oplus 12 = 14$

Tropical semiring $(\mathbb{K}, \min, +, \infty, 0)$

$[[T]](ab) = 4 \to \pi_1 = 1 \otimes 1 \otimes 2 = 4$; $\to \pi_2 = 2 \otimes 3 \otimes 2 = 7$ $\}\ \min(4,7) = 4$