Zhengzhou University

Final Mark:

Student to complete:

Family name	
Other names	
Student number	
Table number	

ECTE344 Control Theory

Examination Paper 2023 - Solution

Exam duration	3 hours
Items permitted by examiner	Calculators, rulers
Directions to students	Attempt all questions. Write your final answers in the boxes provided on this paper. Marks for questions are as indicated – allow appropriate time. Make sure your answers are CLEAR and READABLE. Candidates should note that questions are to be answered as written – no consultation (individual or group) on questions will be given. Any assumptions made should be recorded with your answer.

This exam paper must not be removed from the exam venue.

USE THE SPACE PROVIDED IN BOX FOR YOUR ANSWERS - [100 Marks in Total]

QUESTION 1 - (10 marks)

Consider the system shown in Figure 1, where R(s) is the input and C(s) is the output. Determine the transfer function $\frac{C(s)}{R(s)}$.

Transfer function:

$$\frac{C(s)}{R(s)} = \frac{G_1G_2(G_3 - G_5)G_4}{1 - G_2G_6 + G_1G_2(G_3 - G_5)G_4G_7G_8}$$

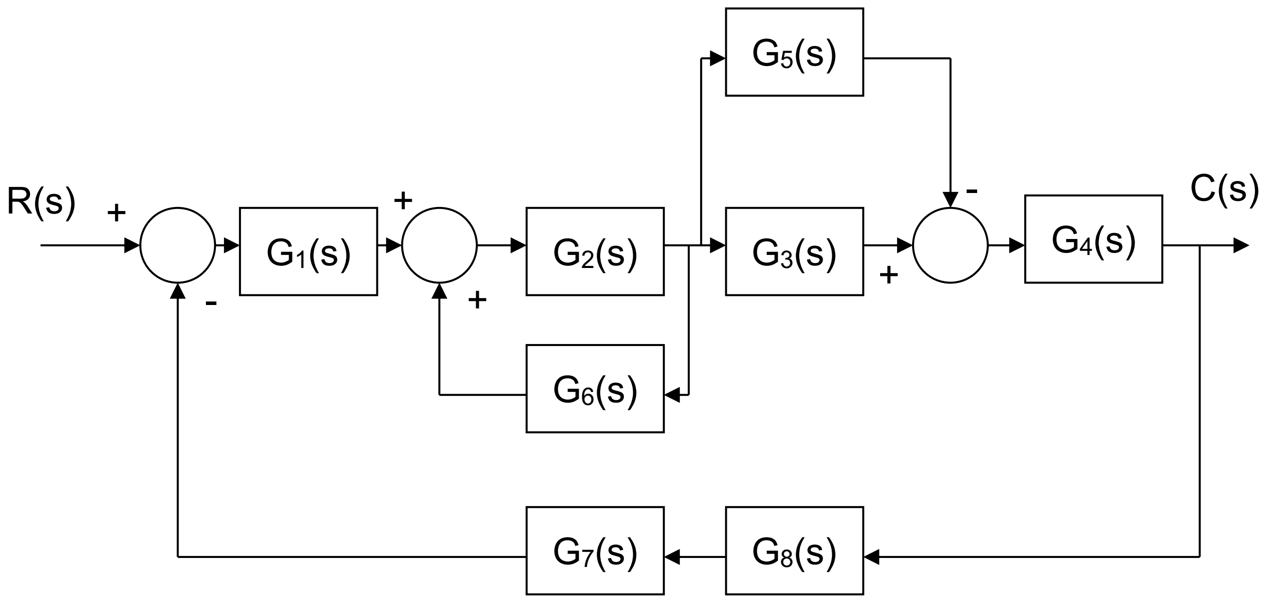


Figure 1

QUESTION 2 - (10 marks)

Consider the system shown in Figure 2.

$$G(s) = \frac{10s+2}{s^2(5s+12)} \tag{1}$$

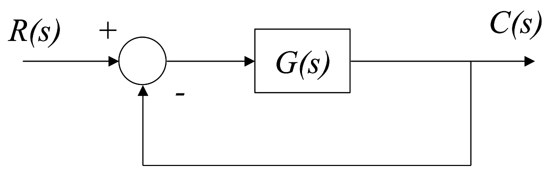


Figure 2

Determine the following characteristics of the system:

(a) the error transfer function;

Formula for error transfer function:

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)}$$

Obtained error transfer function:

```
   5 s^3 + 12 s^2
----------------------------------
5 s^3 + 12 s^2 + 10 s + 2
```

QUESTION 2 (cont.)

(b) the position error constant, velocity error constant and acceleration error constant;

Formula for position error constant: $K_p = \lim_{s \to 0} G(s) = G(0)$

Obtained position error constant: ∞

Formula for velocity error constant: $K_v = \lim_{s \to 0} sG(s)$

Obtained velocity error constant: ∞

Formula for acceleration error constant: $K_a = \lim_{s \to 0} s^2 G(s)$

Obtained acceleration error constant: 2/12=0.1667

(c) the steady state error when the input $r(t)= 10+20t+30t^2$;

Steady state error: 60*12/2=360

QUESTION 3 – (10 marks)

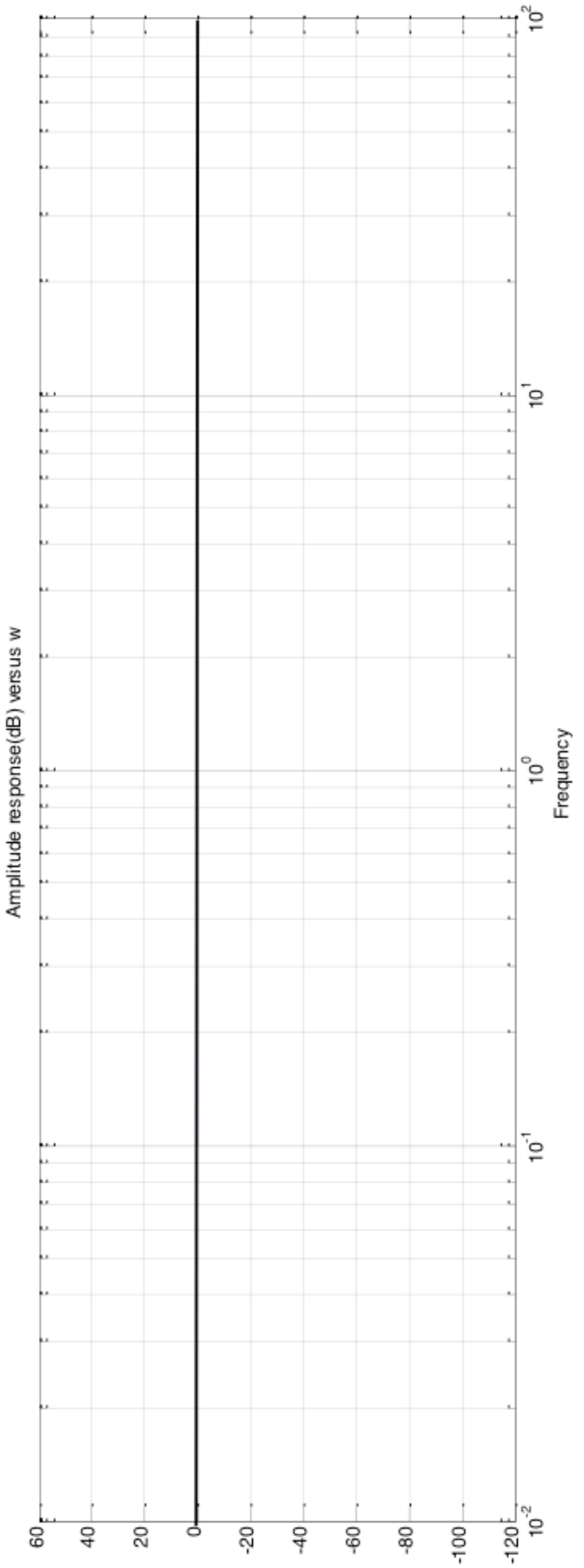
Consider the closed loop system with the following open-loop transfer function:

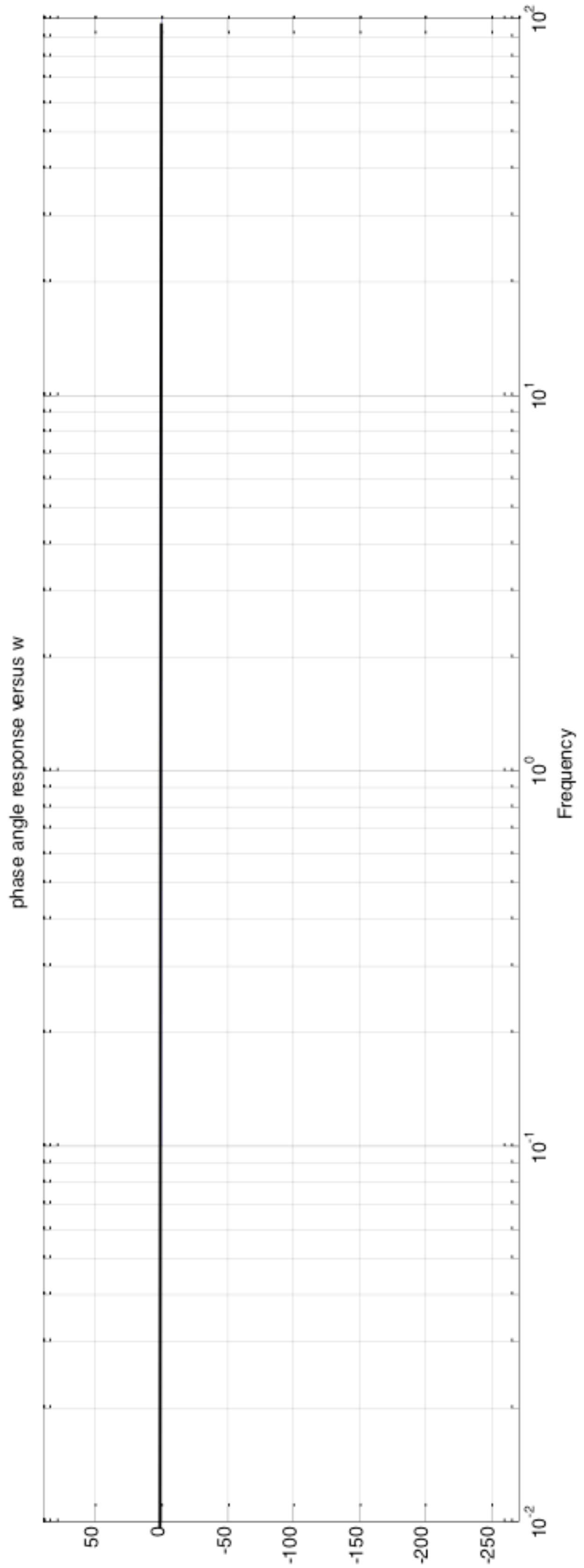
$$G(s)H(s) = \frac{2(s+4)}{s(s^2+2s+4)(s+2)}$$

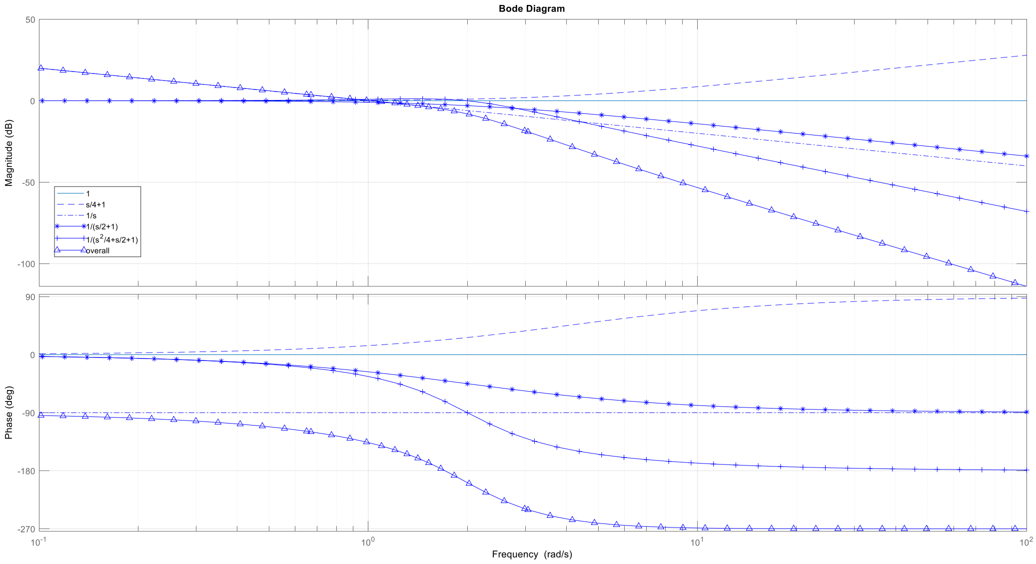
(a) Determine the basic factors of the above transfer function.

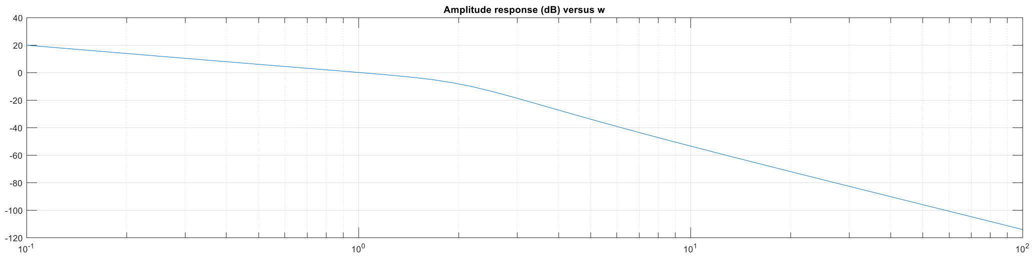
Basic Factors: $\frac{1}{4}s+1$, $\frac{1}{s}$, $\frac{1}{\frac{1}{4}s^2+\frac{1}{2}s+1}$, $\frac{1}{\frac{1}{2}s+1}$

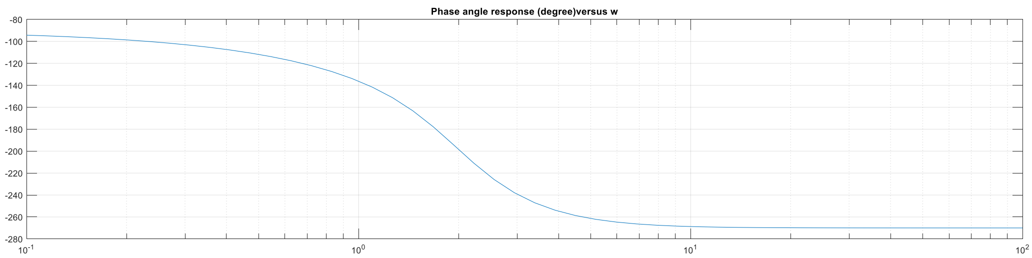
(b) Draw the approximate bode plot of the above system **manually** on the semilog paper provided on the following page. Plot each basic factor separately and clearly mark them on the graph. Then combine them to produce the approximate plot for gain and phase of the system.


Amplitude response(dB) versus w
Amplitude (dB)
Frequency
60
40
20
0
-20
-40
-60
-80
-100
-120
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}


phase angle response versus w
Phase (degrees)
Frequency
50
0
-50
-100
-150
-200
-250
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
1
s/4+1
1/s
1/(s/2+1)
1/(s^2/4+s/2+1)
overall

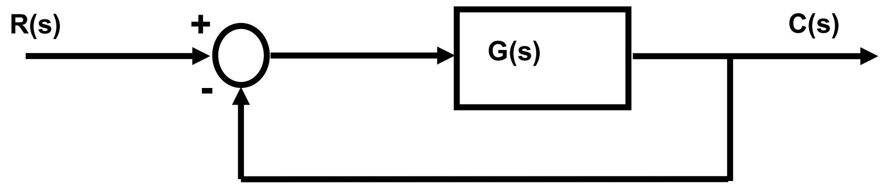

Amplitude response (dB) versus w


Phase angle response (degree)versus w

QUESTION 4 - (6 marks)

Consider the unity feedback system shown below:

$$G(s) = \frac{60}{(s+2)(s+5)}$$



This system has a Nyquist diagram shown in Figure 3:

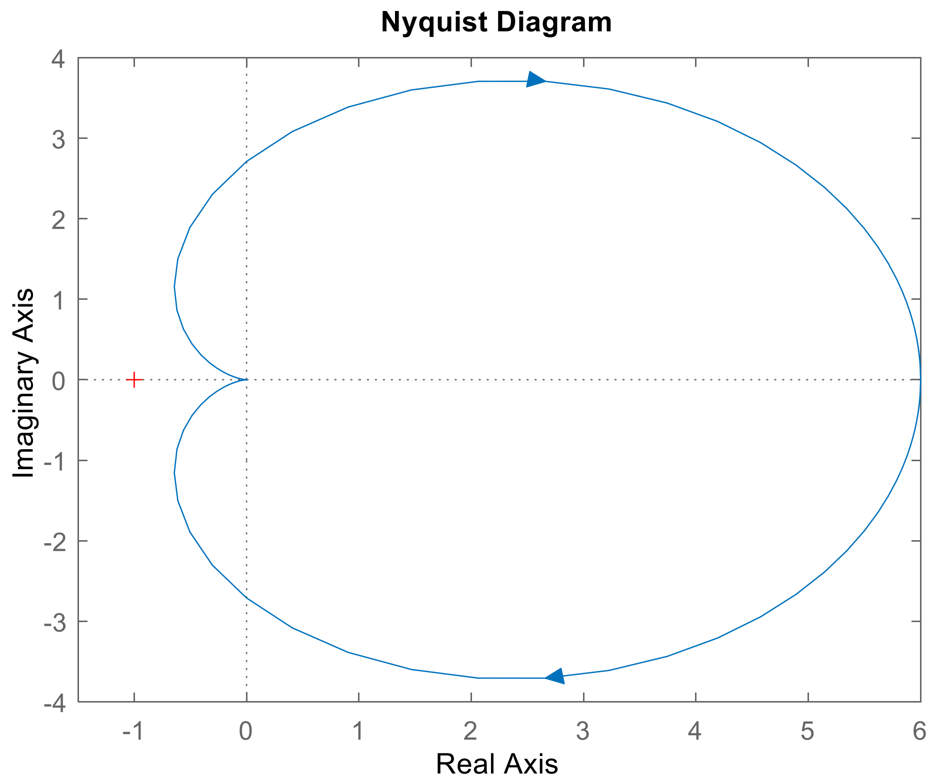


Figure 3 – Nyquist diagram of the system

(a) How many encirclements of -1+0j are made by this diagram?

Number of encirclements of -1+0j: 0

QUESTION 4 (cont.)

(b) How many of the open loop poles are on the right hand side of s plane?

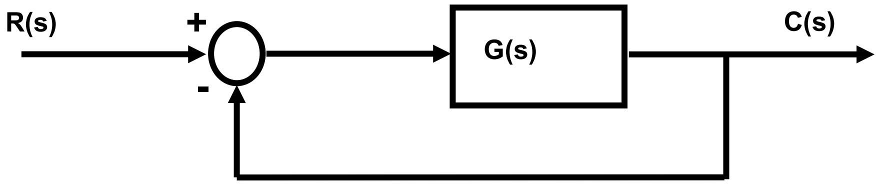
Number of open loop poles on the right hand side of s plane: 0

(c) Based on the Nyquist stability criterion, is this system stable? Why?

Is this system stable? Why? Stable. Because it has no open loop poles on the right hand side of s plane and it does not encircle -1+0j.

QUESTION 5 - (14 marks)

The following system is given: $G(s) = \frac{K}{s^3+4s^2+3s}$ (3)



Sketch the root locus of this system **manually** on the s plane provided at the end of this question. Initially, answer the following questions. Explain the methods used and show all your calculations.

(a) Calculate the poles and zeros of the system, show them and plot them on the s plane provided at the end of this question (draw x for pole and o for zero in the plot).

Poles: 0, -3, -1 Zeros: ∞, ∞, ∞

(b) Indicate the real axis segment of the root locus on the s plane provided at the end of this question.

(c) The number of zeros at infinity.

No. of zeros at infinity: 3

(d) The number of asymptotes and their angles.

No of asymptotes: 3 Write formula for angles of asymptotes: $\theta = \frac{r180}{n-m}, r = \pm 1, \pm 3, \pm 5, \ldots$ Angles of asymptotes: $\theta = \pm 60^{\circ}, 180^{\circ}$

QUESTION 5 (cont.)

(e) The centroid or point of intersection of asymptotes.

Write formula for centroid: $\sigma_a = \dfrac{\sum \text{poles of } G(s)H(s) - \sum \text{zeros of } G(s)H(s)}{n-m}$

Centroid: $\frac{-3-1-0}{3-0} = -1.33$

(f) Which of the following ways is used to calculate the breakaway and re-entry points?

(i) Using the closed loop poles
(ii) Using the characteristic equation
(iii) Using Routh array
(iv) Using $\frac{dK}{ds} = 0, \text{where, } K = -\frac{1}{GH(s)}$

Write the correct way for calculating the breakaway and re-entry points:
(iv) Using $\frac{dK}{ds} = 0, \text{where, } K = -\frac{1}{GH(s)}$

(g) Calculate the breakaway points.

Breakaway point: -0.4514, -2.2153 (remove this due to not in the locus)

(h) Which of the following ways is used to calculate the intersection of the root locus with the imaginary axis?

(i) Using the closed loop poles
(ii) Using the characteristic equation
(iii) Using Routh array
(iv) Using $\frac{dK}{ds} = 0, \text{where, } K = -\frac{1}{GH(s)}$

Write the correct way for calculating the intersection of the root locus with the imaginary axis:
(iii) Using Routh array

(i) Calculate K value at which the root locus intersects with the imaginary axis.

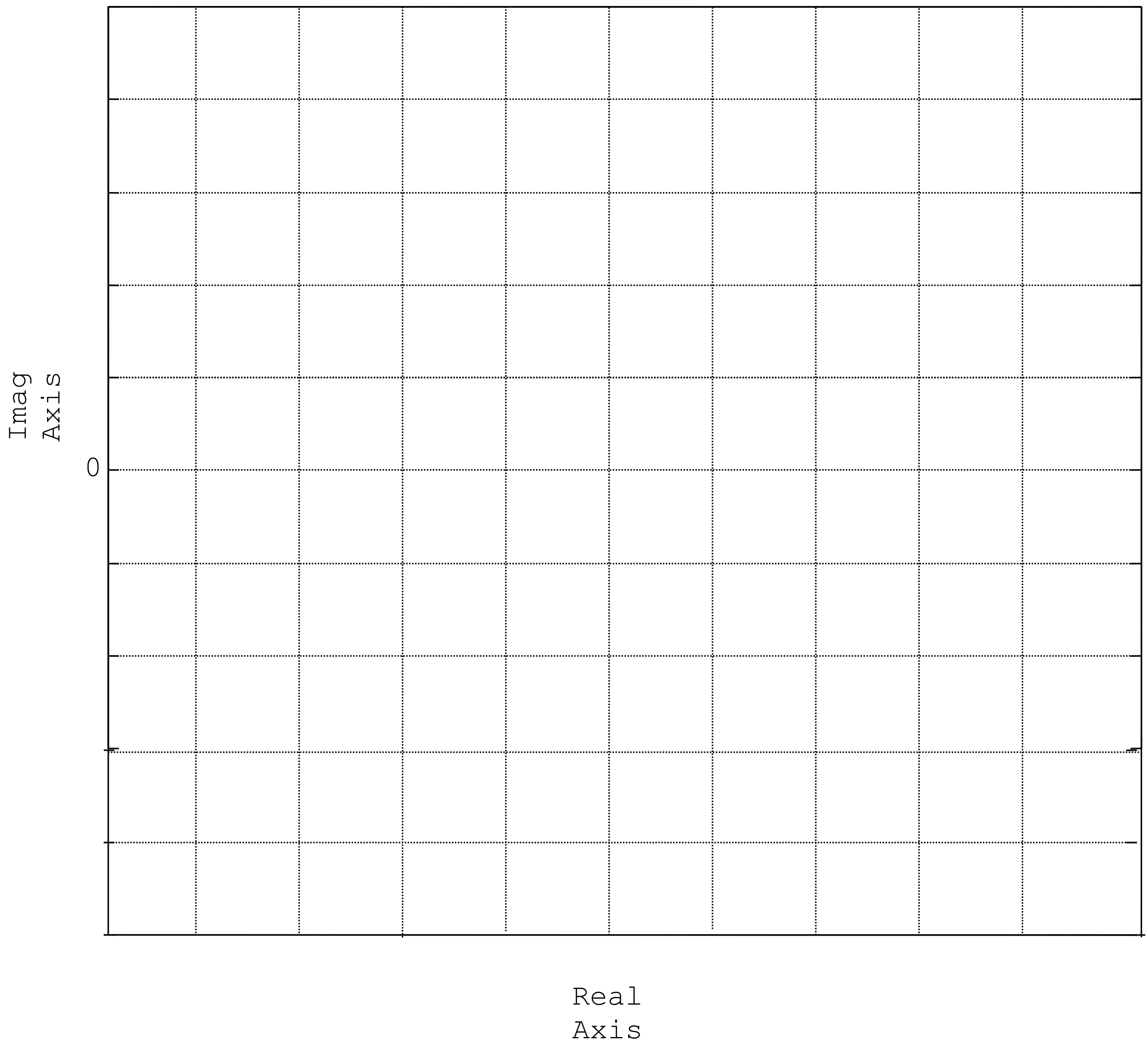
K: 12

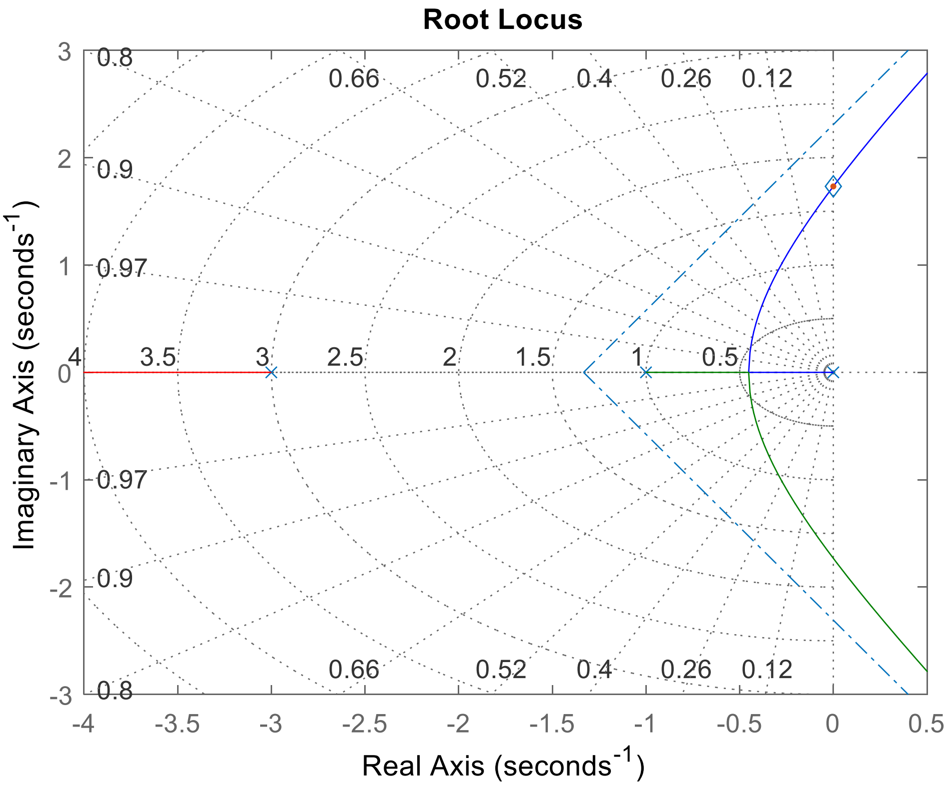
QUESTION 5 (cont.)

(j) For what values of K is the system stable? Why?

Values of K: $K<12$. When $K>12$, poles will be in right hand side of s plane.

(k) Sketch the root locus on the s plane provided.

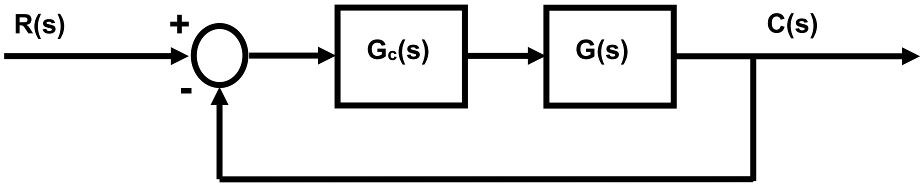



Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
0.8
0.66
0.52
0.4
0.26
0.12
0.9
0.97
4
3.5
3
2.5
2
1.5
1
0.5
3
2
1
0
-1
-2
-3
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5

QUESTION 6 – (10 marks)

Consider the following system:

$$G(s) = \frac{5}{s(s+1)(s+5)}$$



It is desired that the system has a velocity error constant 2.5 1/sec, and a phase margin of 50º.

In order to achieve the desired performance measures for the system, a phase lead compensator in the form

$$G_c(s) = K\frac{Ts+1}{\alpha Ts+1} \qquad 0 < \alpha < 1$$

is required to be designed for it.

(a) Determine the gain K to satisfy the requirement given for the velocity error constant.

K: 2.5

QUESTION 6 (cont.)

(b) If the necessary phase lead angle ϕ_m =48 degrees to be added to the system, determine the attenuation factor α by using one correct relationship between the maximum phase ϕ_m and the attenuation factor α among the following four expressions $sin\phi_m = \frac{1+\alpha}{1-\alpha}$, $sin\phi_m = \frac{1-\alpha}{1+\alpha}$, $sin\phi_m = \frac{1}{1+\alpha}$, $sin\phi_m = \frac{\alpha}{1-\alpha}$, and calculating it.

Correct relationship between ϕ_m and α: $sin\phi_m = \frac{1-\alpha}{1+\alpha}$
Calculated α: 0.1474

(c) If the new gain crossover frequency of the system is $\omega_{gc}=\omega_m$=11.4 rad/sec, determine the zero $\frac{1}{T}$ of the compensator by choosing one correct expression among the following four expressions $\alpha\omega_m$, $\frac{\omega_m}{\alpha}$, $\frac{\omega_m}{\sqrt{\alpha}}$, $\omega_m\sqrt{\alpha}$, and calculating it.

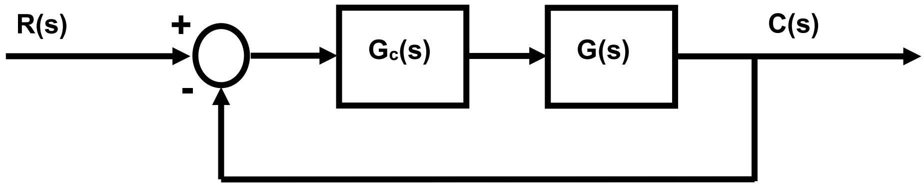
Correct expression for $\frac{1}{T}$: $\omega_m\sqrt{\alpha}$
Calculated zero: 4.3760

(d) If the new gain crossover frequency of the system is $\omega_{gc}=\omega_m$=11.4 rad/sec, determine the pole $\frac{1}{\alpha T}$ of the compensator by using one correct expression among the following four expressions $\alpha\omega_m$, $\frac{\omega_m}{\alpha}$, $\frac{\omega_m}{\sqrt{\alpha}}$, $\omega_m\sqrt{\alpha}$, and calculating it.

Correct expression for $\frac{1}{\alpha T}$: $\frac{\omega_m}{\sqrt{\alpha}}$,
Calculated pole: 29.6980

QUESTION 7 – (10 marks)

A feedback control system is shown as below



where

$$G(s) = \frac{1}{s^2}$$
$$G_c(s) = \frac{K(s+a)}{(s+b)}$$

Determine K, a and b so that the transient response of the overall system has an overshoot of 35% for a step input and settling time with a 2% criterion of 4 s. Assume that phase contribution from zero of the compensator is twice as much as its pole. Follow the steps given below:

(a) Determine the desired damping factor from the overshoot requirement by using one correct expression for the relationship between maximum overshoot and damping factor among the following four expressions $M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$, $M_p = e^{\frac{-\zeta}{\sqrt{1-\zeta^2}}}$, $M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta}}}$, $M_p = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}}$, and calculating it.

> Correct expression for M_p and ζ : $M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$
>
> Calculated damping factor ζ: 0.3169

(b) Determine the real part $\sigma = \zeta\,\omega_n$ of the desired closed loop poles of the compensated system by using one correct expression for the relationship between settling time and damping factor among the following four expressions $T_s = \frac{4}{\zeta\omega_n^2}$, $T_s = \frac{4}{\zeta^2\omega_n^2}$, $T_s = \frac{4}{\zeta^2\omega_n}$, $T_s = \frac{4}{\zeta\omega_n}$, and calculating it.

> Correct expression for T_s and ζ : $T_s = \frac{4}{\zeta\omega_n}$
>
> Calculated real part $\sigma = \zeta\,\omega_n$: 1

(c) Determine the desired closed loop poles of the compensated system.

QUESTION 7 (cont.)

Desired closed loop poles: -1.0000 ± 2.9925j

(d) Determine a and b based on the condition that phase contribution from zero of the compensator is twice as much as its pole.

a: 1.8631

b: 4.9775

(e) Determine K by using one correct expression among the following four expressions $K = \frac{1}{\sqrt{|GH(s)|}}$, $K = |GH(s)|$, $K = \frac{1}{|GH(s)|}$, $K = \sqrt{|GH(s)|}$, and calculating it.

Correct expression for K: $K = \frac{1}{|GH(s)|}$

Calculated K: 15.9101

QUESTION 8 – (30 marks)

Consider the system,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0.6 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad (2)$$

Determine the following:

(a) Determine the transfer function of system (2) by using one correct expression among the following four expressions $G(s) = C(sI - A)^T B + D, G(s) = C(sI - A)^{-1}B + D,\ G(s) = B^T(sI - A)^{-1}C + D, G(s) = C(sI - A)B + D$ and calculating it.

Correct expression for determining transfer function from state space model:

$$\boldsymbol{G(s) = C(sI - A)^{-1}B + D}$$

Determined transfer function:

```
 9 s + 30.2
 -------------------
 s^2 + 4 s + 2.4
```

QUESTION 8 (cont.)

(b) the controllable canonical form of system (2)

```
Controllable canonical form:

Ac =[0         1.0000
    -2.4000   -4.0000]

Bc = [  0
        1 ]

Cc =[30.2000   9.0000]

Dc=0
```

QUESTION 8 (cont.)

(c) Transform the above controllable canonical form in (b) to the observable canonical form by answering the following requirements:

Write the formula for observability matrix W_o: $W_o = \begin{bmatrix} C \\ CA \\ \dots \\ CA^{n-1} \end{bmatrix}$

Write the obtained observability matrix W_o:

Wo =

[30.2000 9.0000
-21.6000 -5.8000]

Is the system observable? Why? Yes. Rank=2 or determinant is not equal to zero

Write the matrix M:

M =

4.0000 1.0000
1.0000 0

Write the formula for the similarity transformation matrix T:

$$T = (MW_o)^{-1}$$

Write the obtained similarity transformation matrix T:

T =
-0.4678 1.5696
1.5696 -5.1559

Write the formula for the transformed matrices A_o, B_o, C_o if the matrices in the controllable form are A_c, B_c, C_c:

$$A_o = T^{-1}A_cT, B_o = T^{-1}B_c, C_o = C_cT$$

Write the observable canonical form:

Ao =
-0.0000 -2.4000
1.0000 -4.0000

Bo =
30.2000
9.0000

Co =
0.0000 1.0000

QUESTION 8 (cont.)

(d) the general zero input solution of system (2)

$x(t)$: $\boldsymbol{x(t) = e^{At}x(0), where\ A = \begin{bmatrix} -1 & 1 \\ 0.6 & -3 \end{bmatrix}}$

(e) Determine the state feedback controller that places the closed-loop poles at *–2* and *–2* for system (2) by answering the following requirements.

Write the formula for controllability matrix W_c: Wc=[B AB ... A^{n-1}B]

Write the obtained controllability matrix W_c:
Wc =

2.0000 -1.0000
1.0000 -1.8000

Is the system controllable? Why? Yes. Wc rank=2

Write the closed-loop system characteristic polynomial when assuming $\boldsymbol{K} = [\boldsymbol{k_1} \quad \boldsymbol{k_2}]$:

s^2 + 2*k1*s + k2*s + 4*s + 7*k1 + (11*k2)/5 + 12/5

Write the desired characteristic polynomial:

s^2 + 4s + 4

Write the obtained feedback gain matrix *K:*
K = [0.6154 -1.2308]

QUESTION 8 (cont.)

(g) Determine the state observer that has the eigenvalues *–10+2j* and *–10-2j* for system (2) by answering the following requirements.

Write the observer state space model assuming the gain matrix $\boldsymbol{K_e}$:

$$\dot{\boldsymbol{x}}_{\boldsymbol{e}} = \boldsymbol{A}\boldsymbol{x}_{\boldsymbol{e}} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{K}_{\boldsymbol{e}}(\boldsymbol{y} - \boldsymbol{C}\boldsymbol{x}_{\boldsymbol{e}})$$

Write the characteristic polynomial assuming $\boldsymbol{K_e} = \begin{bmatrix} \boldsymbol{k_1} \\ \boldsymbol{k_2} \end{bmatrix}$:

s^2 + 4*s + 4*k1*s + k2*s + (63*k1)/5 + 5*k2 + 12/5

Write the desired characteristic polynomial:

s^2 + 20*s + 104

Write the state observer gain matrix $\boldsymbol{K_e}$:

```
Ke =

  [-2.9189;   27.6757]
```

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