Zhengzhou University

Final Mark:

Student to complete:

Family name	
Other names	
Student number	
Table number	

ECTE344 Control Theory

Examination Paper 2022 - Solution

Exam duration	3 hours
Items permitted by examiner	Calculators, rulers
Directions to students	Attempt all questions. Write your final answers in the boxes provided on this paper. Marks for questions are as indicated – allow appropriate time. Make sure your answers are CLEAR and READABLE. Candidates should note that questions are to be answered as written – no consultation (individual or group) on questions will be given. Any assumptions made should be recorded with your answer.

This exam paper must not be removed from the exam venue.

USE THE SPACE PROVIDED IN BOX FOR YOUR ANSWERS - [100 Marks in Total]

QUESTION 1 - (10 marks)

Consider the system shown in Figure 1, where R(s) is the input and C(s) is the output. Determine the transfer function $\dfrac{C(s)}{R(s)}$.

Transfer function:

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1 H_1 G_3 G_4 + G_1 G_2 G_3 G_4}{1 + H_2 G_3 + G_1 H_1 G_3 G_4 + G_1 G_2 G_3 G_4}}$$

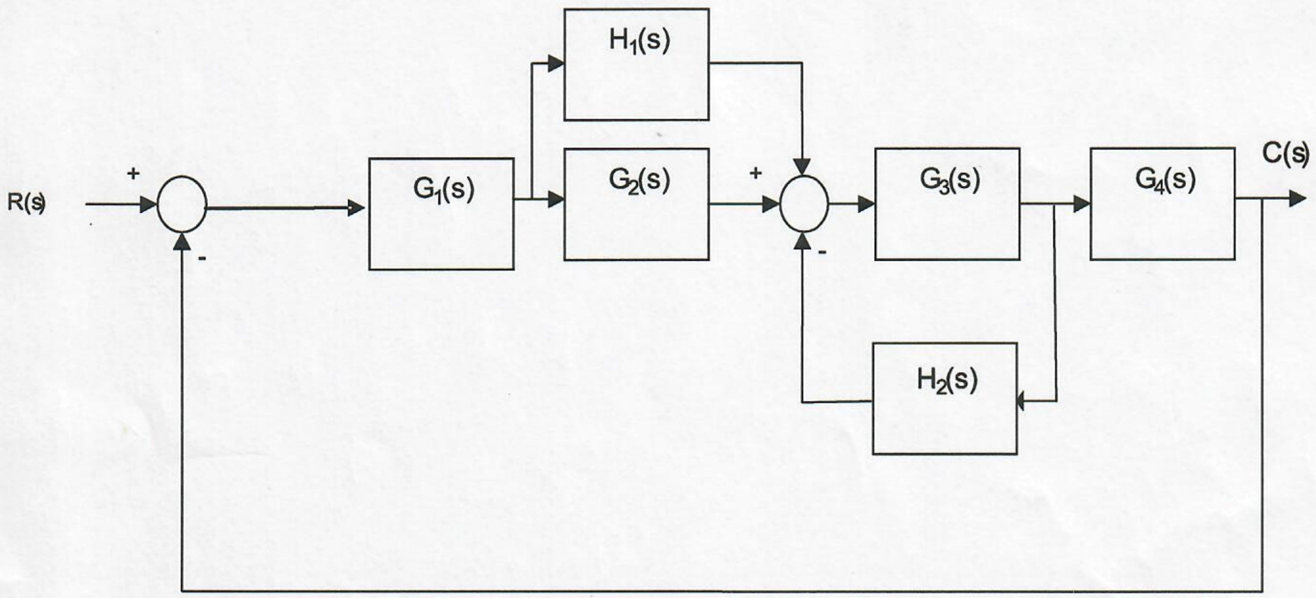

Figure 1

QUESTION 2 - (10 marks)

Consider the system shown in Figure 2.

$$G(s) = \frac{5}{s(s+2)(s+6)} \tag{1}$$

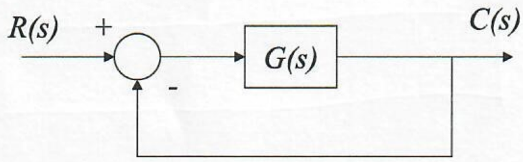

Figure 2

Determine the following characteristics of the system:

(a) the error transfer function;

Formula for error transfer function:

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)}$$

Obtained error transfer function:

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  s^3 + 8 s^2 + 12 s
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s^3 + 8 s^2 + 12 s + 5
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QUESTION 2 (cont.)

(b) the position error constant, velocity error constant and acceleration error constant;

Formula for position error constant: $K_p = \lim_{s\to 0} G(s) = G(0)$

Obtained position error constant: ∞

Formula for velocity error constant: $K_v = \lim_{s\to 0} sG(s)$

Obtained velocity error constant: 0.4167 或 $\frac{5}{12}$

Formula for acceleration error constant: $K_a = \lim_{s\to 0} s^2 G(s)$

Obtained acceleration error constant: 0

(c) the steady state error when the input r(t)=100+0.1t;

Steady state error: 0.1/0.4167=0.24

QUESTION 8 – (30 marks)

Consider the system,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-1 & 1\\ 0.6 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}2\\ 1\end{bmatrix}u$$

$$y = \begin{bmatrix}4 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \qquad (2)$$

Determine the following:

(a) Determine the transfer function of system (2) by using one correct expression among the following four expressions $G(s) = C(sI - A)^T B + D, G(s) = C(sI - A)^{-1}B + D,\ G(s) = B^T(sI - A)^{-1}C + D,\ G(s) = C(sI - A)B + D$ and calculating it.

Correct expression for determining transfer function from state space model: $$\mathbf{G(s) = C(sI - A)^{-1}B + D}$$ Determined transfer function: 9 s + 30.2 ------------------ s^2 + 4 s + 2.4

QUESTION 8 (cont.)

(b) the controllable canonical form of system (2)

Controllable canonical form:

Ac =[0 1.0000
-2.4000 -4.0000]

Bc = [0
1]

Cc =[30.2000 9.0000]

Dc=0

QUESTION 8 (cont.)

(c) Transform the above controllable canonical form in (b) to the observable canonical form by answering the following requirements:

Write the formula for observability matrix W_o: $W_o = \begin{bmatrix} C \\ CA \\ \dots \\ CA^{n-1} \end{bmatrix}$

Write the obtained observability matrix W_o:

Wo =
[30.2000 9.0000
-21.6000 -5.8000]

Is the system observable? Why? Yes. Rank=2 or determinant is not equal to zero

Write the matrix M:

M =

4.0000 1.0000
1.0000 0

Write the formula for the similarity transformation matrix T:

$$T = (MW_o)^{-1}$$

Write the obtained similarity transformation matrix T:

T =

-0.4678 1.5696
1.5696 -5.1559

求 $\frac{1}{-19.24}\begin{bmatrix} 9 & -30.2 \\ -30.2 & 99.2 \end{bmatrix}$

Write the formula for the transformed matrices A_o, B_o, C_o if the matrices in the controllable form are A_c, B_c, C_c:

$$A_o = T^{-1}A_cT, B_o = T^{-1}B_c, C_o = C_cT$$

Write the observable canonical form:

Ao =
-0.0000 -2.4000
1.0000 -4.0000

Bo =
30.2000
9.0000

Co =
0.0000 1.0000

QUESTION 8 (cont.)

(d) the general zero input solution of system (2)

$x(t)$: $x(t) = e^{At}x(0)$, *where* $A = \begin{bmatrix} -1 & 1 \\ 0.6 & -3 \end{bmatrix}$

(e) Determine the state feedback controller that places the closed-loop poles at *–2* and *–2* for system (2) by answering the following requirements.

Write the formula for controllability matrix W_c: Wc=[B AB ... A^{n-1}B]

Write the obtained controllability matrix W_c:

Wc=

2.0000 -1.0000
1.0000 -1.8000

Is the system controllable? Why? Yes. Wc rank=2

Write the closed-loop system characteristic polynomial when assuming $\boldsymbol{K} = [\boldsymbol{k_1} \quad \boldsymbol{k_2}]$:

s^2 + 2*k1*s + k2*s + 4*s + 7*k1 + (11*k2)/5 + 12/5

Write the desired characteristic polynomial:

s^2 + 4s + 4

$$\begin{cases} 2k_1 + k_2 + 4 = 4 \\ 7k_1 + 2.2k_2 + 2.4 = 4 \end{cases}$$

Write the obtained feedback gain matrix *K:*

K = [0.6154 -1.2308] 或 $[\frac{8}{13}, -\frac{16}{13}]$

QUESTION 8 (cont.)

(g) Determine the state observer that has the eigenvalues *−10+2j* and *−10-2j* for system (2) by answering the following requirements.

Write the observer state space model assuming the gain matrix K_e:

$$\dot{x_e} = Ax_e + Bu + K_e(y - Cx_e)$$

Write the characteristic polynomial assuming $K_e = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$:

s^2 + 4*s + 4*k1*s + k2*s + (63*k1)/5 + 5*k2 + 12/5

Write the desired characteristic polynomial:

s^2 + 20*s + 104

$$\begin{cases} 4k_1 + k_2 + 4 = 20 \\ 12.6k_1 + 5k_2 + 2.4 = 104 \end{cases}$$

Write the state observer gain matrix K_e:

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Ke =

  [-2.9189;   27.6757]
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$[-\frac{108}{37}, \frac{1024}{37}]$

Paper Ends - Intentionally Blank from this line