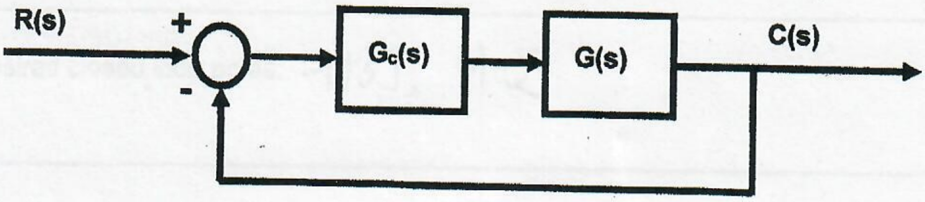
QUESTION 7 – (10 marks)

A feedback control system is shown as below



where

$$G(s) = \frac{1}{s^2}$$
$$G_c(s) = \frac{K(s+a)}{(s+b)}$$

Determine K, a and b so that the transient response of the overall system has an overshoot of 35% for a step input and settling time with a 2% criterion of 4 s. Assume that phase contribution from zero of the compensator is twice as much as its pole. Follow the steps given below:

(a) Determine the desired damping factor from the overshoot requirement by using one correct expression for the relationship between maximum overshoot and damping factor among the following four expressions $M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$, $M_p = e^{\frac{-\zeta}{\sqrt{1-\zeta^2}}}$, $M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta}}}$, $M_p = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}}$, and calculating it.

Correct expression for M_p and ζ:	$M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$
Calculated damping factor ζ:	$e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = 0.35$, $\zeta = 0.317$

(b) Determine the real part $\sigma = \zeta\,\omega_n$ of the desired closed loop poles of the compensated system by using one correct expression for the relationship between settling time and damping factor among the following four expressions $T_s = \frac{4}{\zeta\omega_n^2}$, $T_s = \frac{4}{\zeta^2\omega_n^2}$, $T_s = \frac{4}{\zeta^2\omega_n}$, $T_s = \frac{4}{\zeta\omega_n}$, and calculating it.

Correct expression for T_s and ζ:	$T_s = \frac{4}{\zeta\omega_n}$
Calculated real part $\sigma = \zeta\,\omega_n$:	$\sigma = \zeta\omega_n = \frac{4}{T_s} = 1$

(c) Determine the desired closed loop poles of the compensated system.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{9.955}{s^2+2s+9.955}$$

$s^2+2s+9.955=0$

$\begin{cases} s_1 = -1+3j \\ s_2 = -1-3j \end{cases}$

$-1+3j \quad -1-3j$

$1.86 \quad 4.98$

$\frac{1}{|G(s)|} \quad 16.04$

1. $M_p = \frac{-3.70}{e^{\sqrt{1-\zeta^2}}}$ $\zeta = 0.317$

QUESTION 7 (cont.) 2. $\frac{4}{3\omega_n}$ 1

3. Desired closed loop poles: $-1+3j,\ -1-3j$

(d) Determine a and b based on the condition that phase contribution from zero of the compensator is twice as much as its pole.

We have: $G_c(s)\cdot G(s) = \frac{K(s+a)}{s^2(s+b)}$ and $\boxed{\angle s+a = 2\angle s+b}$

So: $\angle G_c(s)G(s)\Big|_{s=-1+3j} = \cancel{\frac{\angle a-1+3j}{\angle(-1+3j)^2}}\ \angle a-1+3j - \angle(-1+3j)^2 - \angle b-1+3j = 180^\circ$

so: $\arctan\frac{3}{a-1} - \arctan\frac{3}{b-1} = 37^\circ$

And: $\begin{cases} \arctan\frac{3}{a-1} = 2\times 37^\circ = 74^\circ \\ \arctan\frac{3}{b-1} = 37^\circ \end{cases}$ So: $\begin{cases} \frac{3}{a-1} = \tan 74^\circ = 3.487 \\ \frac{3}{b-1} = \tan 37^\circ = 0.754 \end{cases}$

$\begin{cases} a = 1.86 \\ b = 4.98 \end{cases}$

a: 1.86

b: 4.98

(e) Determine K by using one correct expression among the following four expressions $K = \frac{1}{\sqrt{|GH(s)|}}$, $K = |GH(s)|$, $K = \frac{1}{|GH(s)|}$, $K = \sqrt{|GH(s)|}$, and calculating it.

Correct expression for K: $K = \frac{1}{|GH(s)|}$

Calculated K: $\left|\frac{K(s+1.86)}{s^2(s+4.98)}\right|_{s=-1+3j} = 1$, $K = 16.04$