Planet Migration in Disks

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OUTLINE

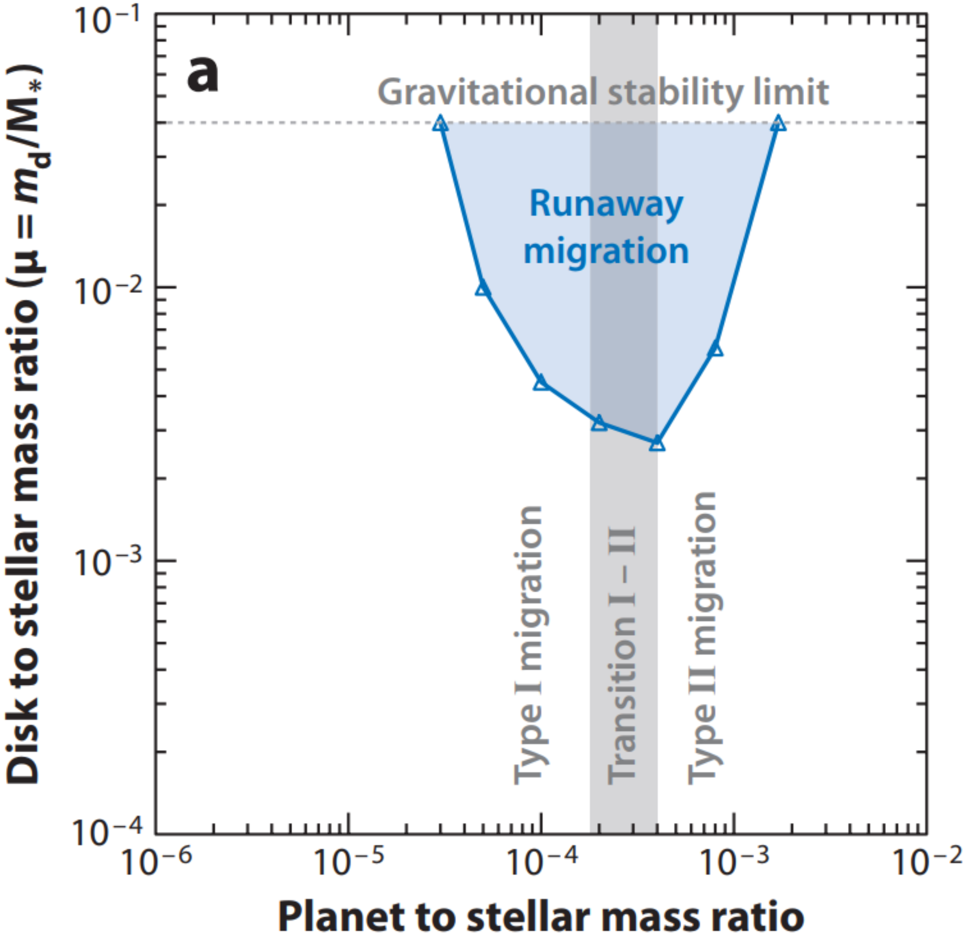
- Type I Migration
  - Lindblad Resonance
  - Torques
  - Numerical Simulation
  - Horseshoe Drag and Saturation
- Type II Migration
- Type III Migration

PLANET MIGRATION: OVERVIEW

- Planet orbits change over time.

Mechanisms

- **Protoplanetary Disk**
- Gravitational Scattering
- Tidal Forces
- Kozai Mechanism
- Planetesimals



LINDBLAD RESONANCE

Non-axisymmetric rotating potential (Ω_p):

$$\Phi = \Phi_0(R) + \Phi_1(R, \varphi), \quad \left|\frac{\Phi_1}{\Phi_0} \ll 1\right|$$

$$\Phi_1(R, \varphi) = \sum_{m=1}^{\infty} \Phi_m(R) \cos m\varphi$$

R and φ: polar coordinates in the corotating frame

Divide R and φ into zeroth- and first-order parts:

$$R(t) = R_0 + R_1(t), \quad \varphi(t) = \varphi_0(t) + \varphi_1(t)$$

LINDBLAD RESONANCE

$$R_1(\varphi_0) = \cdots + C\cos m\varphi_0$$

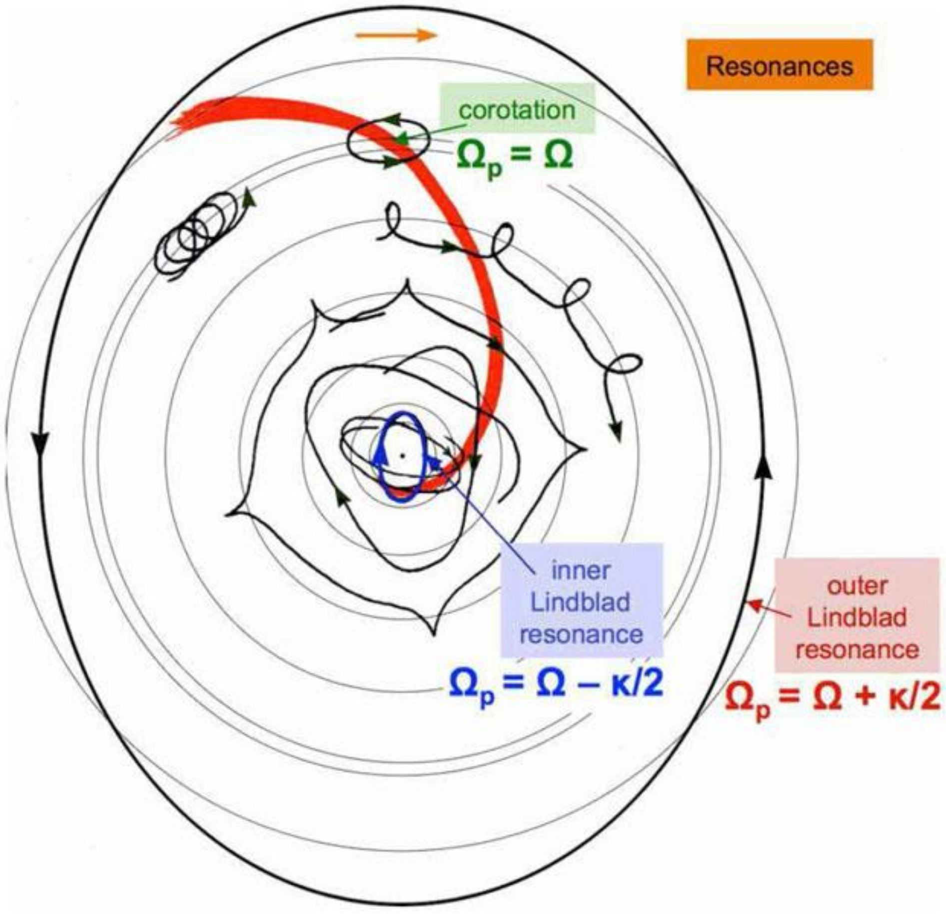
$$C = -\frac{1}{\Delta}\left[\frac{\mathrm{d}\Phi_m}{\mathrm{d}R} + \frac{2\Omega(R)\Phi_m}{R(\Omega(R)-\Omega_p)}\right]_{R=R_0}$$

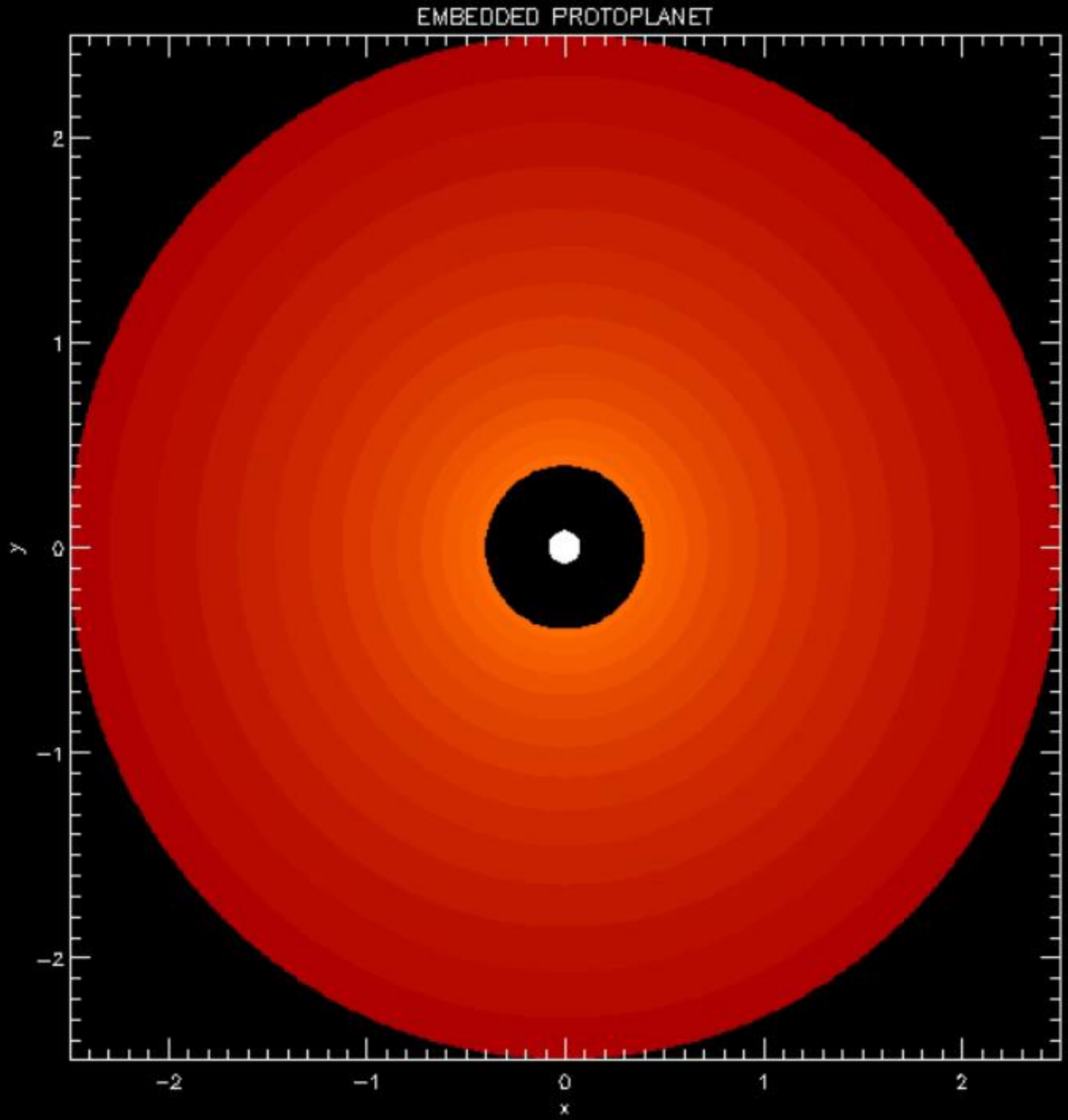
$$\Delta = \kappa_0^2 - m^2(\Omega(R_0)-\Omega_p)$$

κ_0: epicyclic frequency $\Omega(R)$: circular orbit frequency

Resonance:

- Corotation: $\Omega(R_0) = \Omega_p$
- Lindblad: $m(\Omega(R_0) - \Omega_p) = \pm\kappa_0$




EMBEDDED PROTOPLANET
y
x
2
1
0
−1
−2
−2
−1
0
1
2

Torques

- Lindblad Torques:

$$\Gamma_m^{\mathrm{L}} = \mathrm{sign}(\Omega - \Omega_p) \frac{\pi^2 \Sigma}{3\Omega\Omega_p} \left(r \frac{\mathrm{d}\Phi_m}{\mathrm{d}r} + \frac{2m^2(\Omega - \Omega_p)}{\Omega} \Phi_m \right)^2$$

- Corotation Torques:

$$\Gamma_m^{C} = \frac{m\pi^2}{2} \frac{\Phi_m}{r} \left(\frac{\mathrm{d}\Omega}{\mathrm{d}r} \right)^{-1} \frac{\mathrm{d}}{\mathrm{d}r} \left(\frac{4\Sigma\Omega}{\kappa^2} \right)$$

NUMERICAL SIMULATION

$$\Gamma_{\rm tot} = -(1.36 + 0.62\beta_\Sigma + 0.43\beta_T)\Gamma_0$$

$$\Sigma \propto r^{-\beta_\Sigma}, \quad T \propto r^{-\beta_T}$$

Migration Timescale:

$$\tau_{\rm mig} = \frac{a_p}{\dot{a}_p} = \frac{L_p}{2\Gamma_{\rm tot}}$$

a_p: orbit radius L_p: angular momentum

Earth-mass, 1 au, MMSN disk model:

$$\tau_{\rm mig} \sim 10^5 \text{ yr}$$

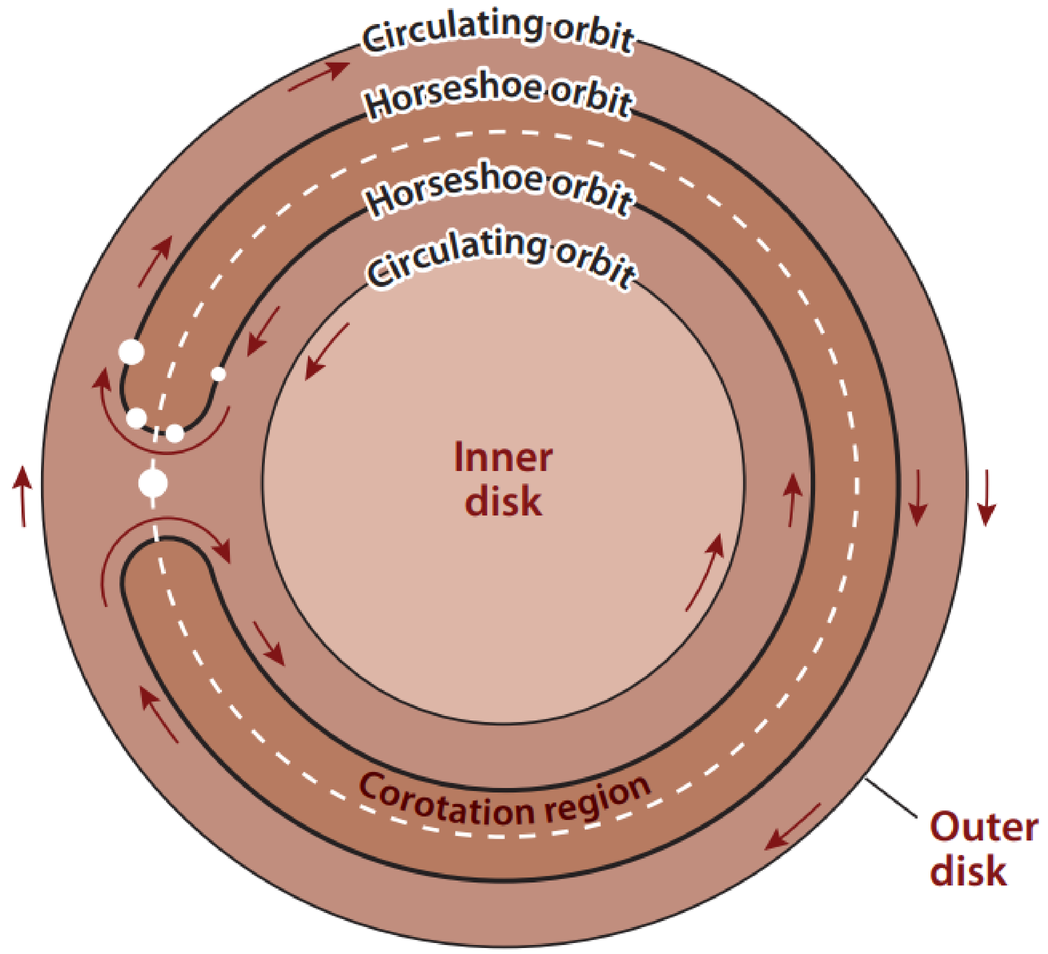
HORSESHOE DRAG (NONLINEAR)

Horseshoe region

Assume: planet in circular Keplerian orbit with radius R_p and angular velocity Ω_p (Counterclockwise)

In corotational frame, disk particles in horseshoe orbit

Angular momentum exchange at each U-turn



HORSESHOE DRAG

U-turn: disk particle goes from inner part to outer part

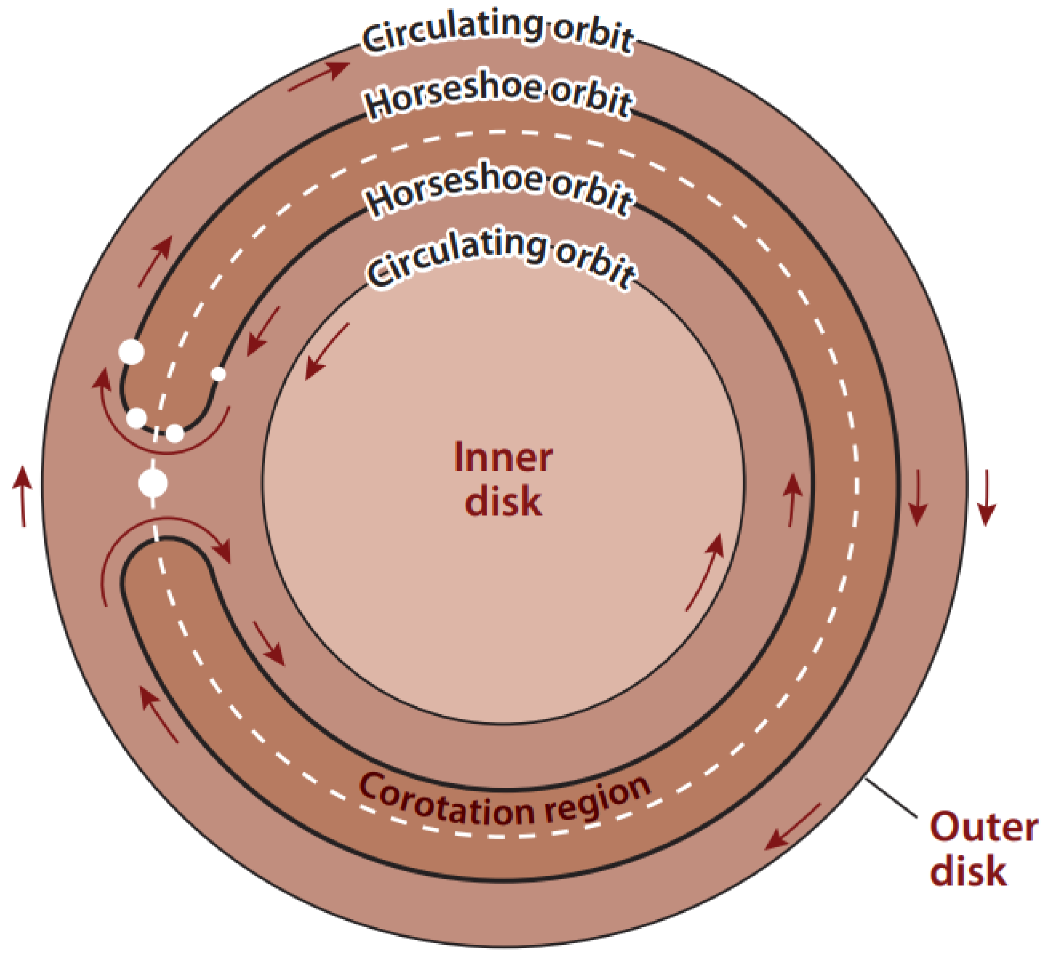
Particle angular momentum gain:

$$\Delta L_i = \Delta m_i(\Omega_o R_o^2 - \Omega_i R_i^2)$$

$\Omega_k,\ R_k\ (k = i, o)$ are angular velocities and radii of inner and outer horseshoe orbit, Δm_i is the amount of mass transfer.

Outer to inner:

$$\Delta L_o = -\Delta m_o(\Omega_o R_o^2 - \Omega_i R_i^2)$$



HORSESHOE DRAG

Net torque on the planet:

$$\Gamma = -\frac{(\Delta L_i + \Delta L_o)}{\Delta t} = \frac{\Delta m_o - \Delta m_i}{\Delta t}(\Omega_o R_o^2 - \Omega_i R_i^2)$$

Consider mass transfer (k= i,o):

$$\frac{dm_k}{dt} = \frac{\sigma_k R_k dR_k d\theta_k}{dt} = \sigma_k R_k \left|\Omega_k - \Omega_p\right| dR_k$$

σ is the surface density and $R\ dR\ d\theta$ is the area element in 2D polar coordinate system.

Density gradient and unequal differential areas contribute to net torque.

HORSESHOE DRAG

Assume density of the disk: $\sigma \propto r^{-s}$

Through some calculations with Jacobian energy conservation,*

for Keplerian orbits:

$$\Gamma = \frac{3}{4}(\frac{3}{2} - s)\sigma w^4 \Omega^2$$

w is the half width of the horseshoe region.

Small s: planet migrates outwards (disk particles flow inwards); and vice versa.

A steady mass flow?

TORQUE SATURATION

Mass flow can’t stay constant: altered density gradient

Net torque decay to 0 while local density exponent $s \rightarrow 3/2$

- Ways to slow down saturation:

    Viscosity: Angular momentum refueled by other parts of disk

    Fast migration: radial drift large enough to go across horseshoe region

TYPE II MIGRATION

Lindblad torques wants to open a gap in disk, $\Gamma_L \propto M_p^2$

Viscosity + pressure want to close the gap → balance

- Results:

  Corotational torque vanishes (no corotating matter)

  Very massive M_p → Lindblad resonance orbits cleared

  → torque weaken → slow down migration

TYPE III MIGRATION

- When migrating, planet carries with it material that is trapped in horseshoe orbits and bound to the planet within its Hill sphere.

$$\dot{a}_p = \frac{2\Gamma^{\mathrm{L}}}{\Omega_p a_p (m_p - \delta m)}$$

Halve the semimajor axis in 50 orbits! "Runaway Migration"

QUESTION

What are the causes of horseshoe drag?

1. Lindblad resonance.
2. Disk viscosity.
3. Density gradient.
4. Temperature gradient.
5. Unequal differential areas.

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Thanks for your listening!

Q&A