Homework4

Part 1: HMM Grading

1. $P(X_1 = c|e_1) \propto P(e_1|c)P(X_1 = c) = 0.8 \times 0.5 = 0.4$
   $P(X_1 = w|e_1) \propto P(e_1|w)(P(X_1 = w) = 0.5 \times 0.5 = 0.25$
   $P(X_1 = c|e_1) = \frac{0.4}{0.25+0.4} = 0.615$
   $P(X_1 = w|e_1) = \frac{0.25}{0.4+0.25} = 0.385$

   $P(X_2 = c|e_1, e_2) \propto P(e_2|c) \sum_{X_1} P(X_2 = c|X_1)P(X_1|e_1) \propto 0.2 \times (0.5 \times 0.4 + 0.2 \times 0.25) = 0.05$
   $P(X_2 = w|e_1, e_2) \propto P(e_2|w) \sum_{X_1} P(X_2 = w|X_1)P(X_1|e_1) \propto 0.5 \times (0.5 \times 0.4 + 0.8 \times 0.25) = 0.2$
   $P(X_2 = c|e_1, e_2) = \frac{0.05}{0.05+0.2} = 0.2$
   $P(X_2 = w|e_1, e_2) = \frac{0.2}{0.05+0.2} = 0.8$

   $P(X_3 = c|e_1, e_2, e_3) \propto P(e_3|c) \sum_{X_2} P(X_3 = c|X_2)P(X_2|e_2) \propto 0.8 \times (0.5 \times 0.2 + 0.2 \times 0.8) = 0.208$
   $P(X_3 = w|e_1, e_2, e_3) \propto P(e_3|w) \sum_{X_2} P(X_3 = w|X_2)P(X_2|e_2) \propto 0.5 \times (0.5 \times 0.2 + 0.8 \times 0.8) = 0.37$
   $P(X_3 = c|e_1, e_2, e_3) = \frac{0.208}{0.208+0.37} = 0.360$
   $P(X_3 = w|e_1, e_2, e_3) = 0.640$
   The autograder will grade the first part to right, and grade the second part and third part to wrong.
2. $m_0(c) = 0.5$
   $m_0(w) = 0.5$

   $m_1(c) = P(e_1|c)m_0(c) = 0.8 \times 0.5 = 0.4$
   $m_1(w) = P(e_1|w)m_0(w) = 0.5 \times 0.5 = 0.25$

   $m_2(c) = P(e_2|c) \max[P(c|c)m_1(c), P(c|w)m_1(w)] = 0.2 \times \max[0.5 \times 0.4, 0.2 \times 0.25] = 0.2 \times 0.2 = 0.04$
   $m_2(w) = P(e_2|w) \max[P(w|c)m_1(c), P(w|w)m_1(w)] = 0.5 \times \max[0.5 \times 0.4, 0.8 \times 0.25] = 0.5 \times 0.2 = 0.1$
   So the autograder will grade the second part to wrong, but it can't determine whether the first part is right or wrong.

Part 2: Independence Overload

1. $(A, B), (A, D)$
2. $(C, D), (A, D)$
3. None
4. $(A, E), (B, E)$
5. None
6. $P(A, C) = P(A)P(C|A) = \sum_B P(B)P(A)P(C|A, B)$
7. $P(C, D) = \sum_{A,B} P(C, D, A, B) = \sum_{A,B} P(A)P(B)P(C|A, B)P(D|B)$
8. $P(E|C) = \frac{P(E,C)}{P(C)} = \frac{\sum_{A,B,D} P(A)P(B)P(D|B)P(C|A,B)P(E|C,D)}{\sum_{A,B} P(A)P(B)P(C|A,B)}$
9. $P(A, B|C) = \frac{P(A,B,C)}{P(C)} = \frac{P(A)P(B)P(C|A,B)}{\sum_{A,B} P(A)P(B)P(C|A,B)}$
10. $P(A, D|C, E) = \frac{P(A,D,C,E)}{P(C,E)} = \frac{\sum_B P(A,B,C,D,E)}{\sum_{A,B,D} P(A,B,C,D,E)} = \frac{\sum_B P(A)P(B)P(C|A,B)P(D|B)P(E|C,D)}{\sum_{A,B,D} P(A)P(B)P(D|B)P(C|A,B)P(E|C,D)}$

Part 3: Samples for Everyone

1.

C	B	P(B|C)
+c	+b	2/3
+c	-b	1/3
-c	+b	2/5
-c	-b	3/5

2. Discard sample 1, 3, 5, 6, 7.

   The resulting distribution is:
   $\hat{P}(+e| - b, -c) = \frac{1}{3}$
   $\hat{P}(-e| - b, -c = \frac{2}{3}$
3. Sample in the order A, B, C, D, E; fix -b and -c whenever we get to them.

   Weight each sample by probability of -b, -c given the sample:
   - $(-a, -b, -c, -d, +e),\ w = P(-b)P(-c| - a, -b) = 0.75P(-b)$
   - $(+a, -b, -c, -d, -e),\ w = P(-b)P(-c| + a, -b) = 0.5P(-b)$
   - $(-a, -b, -c, -d, -e),\ w = P(-b)P(-c| - a, -b) = 0.75P(-b)$

     Normalize the weights above, we can get:
     $\hat{P}(+e| - b, -c) = \frac{0.75P(-b)}{(0.75+0.5+0.75)P(-b)} = 0.375$
     $\hat{P}(-e| - b, -c) = \frac{(0.5+0.75)P(-b)}{(0.75+0.5+0.75)P(-b)} = 0.625$