Homework 3

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Written Portion

Part 1

1. T (cool, slow, cool) = 1 R (cool, slow, cool) = +2

   T (cool, fast, cool) = 1/3 R (cool, fast, cool) = +2

   T (cool, fast, warm) = 2/3 R (cool, fast, warm) = +4

   T (warm, slow, cool) = 1 R (warm, slow, cool) = +2

   T (warm, fast, off) = 1 R (warm, fast, off) = 0

   T (off, wait, off) = 1 R (off, wait, off) = 0

   $V^*(cool) = max_a[1 * (2 + 0.5 * V^*(cool)), 1/3 * (2 + 0.5 * V^*(cool)) + 2/3 * (4 + 0.5 * V^*(warm))]$

   $V^*(warm) = max_a[1 * (2 + 0.5 * V * (cool)), 1 * (0 + 0.5 * V * (off))]$

   $V^*(off) = max_a[1 * (0 + 0.5 * V^*(off))]$

   Solve the equations above, we can get that:

   $V^*(cool) = 6$ $V^*(warm) = 5$ $V^*(off) = 0$

2.

Transition	Q(cool, slow)	Q(cool, fast)	Q(warm, slow)	Q(warm, fast)	Q(off, wait)
(cool,slow,cool,+2)	1	0	0	0	0
(cool,fast,cool,+2)	1	1.5	0	0	0
(cool,fast,warm,+4)	1	2.75	0	0	0
(warm,slow,cool,+2)	1	2.75	2.375	0	0
(cool,fast,warm,+4)	1	4.5625	2.375	0	0
(warm,fast,off,0)	1	4.5625	2.375	0	0
(off,wait,off,0)	1	4.5625	2.375	0	0

3. When cool, go fast; when warm, go slow; when off, wait.

Part 2

1.

B	C	P(B,C)
+b	+c	0.0125
+b	-c	0.0375
-b	+c	0.2375
-b	-c	0.7125

2. $P(B = +b) = 0.05$ $P(B = -b) = 0.95$ $P(C = +c) = 0.25$ $P(C = -c) = 0.75$

   $P(+b)P(+c) = 0.05 * 0.25 = 0.0125 = P(+b, +c)$

   We can also get that P(+b)P(-c) = P(+b, -c), P(-b)P(+c) = P(-b, +c), P(-b)P(-c) = P(-b, -c)

   So we can get that $P(B)P(C) = P(B, C)$, which means that B are independent of C.

3. $P(A = +a) = P(+a, +b, +c) + P(+a, +b, -c) + P(+a, -b, +c) + P(+a, -b, -c) = 0.8025$

4. $P(+b|A=+a) = P(+a,+b)/P(+a) = 0.0025/0.8025 = 0.00312$
   $P(-b|A=+a) = 1 - P(+b|A=+a) = 0.99688$
5. $P(+b|+a,-c) = \frac{P(+a,+b,-c)}{P(+a,-c)} = \frac{0.0025}{0.0025+0.6125} = 0.00407$

   It is now higher. Because we now know that the weather is nice and you arrive on time, since the weather can't stop you arrive on time, we now have more tolerance towards the MTA, which means that we allow it to be more naughty, so the probability of delay increases. For analogy, we need a project done, and there are two workers A and B, if A is sick (bad weather), B have to do more work (lower probability of MTA to delay), but if A is healthy and hardworking (Great weather), B can do less work (higher probability of MTA to delay).

Part 3

1.

X_1	X_2	$P(X_1, X_2)$
cool	cool	0.125
cool	warm	0.375
warm	cool	0.1
warm	warm	0.3
warm	off	0.1

2.

X_2	$P(X_2)$
cool	0.225
warm	0.675
off	0.1

3. $X_3 \coprod X_1|X_2$, $X_1 \coprod X_3|X_2$
   $P(X_3|X_1,X_2) = P(X_3|X_2)$

X_2	X_3	$P(X_3|X_1, X_2)$
cool	cool	0.25
cool	warm	0.75
warm	cool	0.2
warm	warm	0.6
warm	off	0.2
off	warm	0.2
off	off	0.8

   $P(X_1|X_3,X_2) = P(X_1|X_2) = \frac{P(X_1,X_2)}{P(X_2)}$

X_1	X_2	$P(X_1 \vert X_3, X_2)$
cool	cool	5/9
cool	warm	5/9
warm	cool	4/9
warm	warm	4/9
warm	off	1

4. When n is large enough, we have $P(X_n) = P(X_{n-1})$

   $P_\infty(cool) = P(cool|cool)P_\infty(cool) + P(cool|warm)P_\infty(warm)$

   $P_\infty(warm) = P(warm|cool)P_\infty(cool) + P(warm|warm)P_\infty(warm) + P(warm|off)P_\infty(off)$

   $P_\infty(off) = P(off|warm)P_\infty(warm) + P(off|off)P_\infty(off)$

   $P_\infty(cool) + P_\infty(warm) + P_\infty(off) = 1$

   Solve the equations above, we can get:

   $P_\infty(cool) = \frac{2}{17}$

   $P_\infty(warm) = \frac{15}{34}$

   $P_\infty(warm) = \frac{15}{34}$

Part 4

1.

X_{t+1}	$P(X_{t+1} \vert X_t)$
(1,0)	1/2
(1,1)	1

E_t	$P(E_t \vert X_t)$
S	1/2
C	1/2

2.

X_1	$P(X_1 \vert plant)$
(1,2)	1/3
(2,2)	1/3
(3,1)	1/3

3. $P(X_2|e_1) = \sum_{x1} P(X_2|X_1)P(X_1|e_1)$

   $B(X_2) = P(X_2|e_1, e_2) \propto P(e_2|X_2)P(X_2|e_1)$

   And we know that only when $X_2 \in (1,2), (2,2), (3,1)$, $P(e_2|X_2)! = 0$, So we only need to calculate these three beliefs.

   $P(X_2 = (1,2), (2,2), (3,1)|e_1, e_2) \propto (1 * 2/9, 1 * 2/9, 1 * 1/9) = (2/9, 2/9, 1/9)$

   So we can get that:

   $P(X_2 = (1,2), (2,2), (3,1)|e_1, e_2) = (2/5, 2/5, 1/5)$

4. The state for X_3 is $(1,1)$

5. From 4 we know $P(X_3 = (1,1)|plant, plant, sofa) = 1$

   $P(X_4 = (1,0), (1,1)|plant, plant, sofa, sofa) \propto P(sofa|X_4) \sum_{x3} P(X_4|X_3)P(X_3|plant, plant, sofa) = (1/2 * 1/4, 1 * 1/4)$

So we can get that:

$P(X_4 = (1,0),(1,1)|plant, plant, sofa, sofa) = (1/3, 2/3)$