Homework 5

Part 1

1. $P(Y)$

Y	P(Y)
-y	3/7
+y	4/7

   $P(X_1|Y)$

X_1	Y	$P(X_1|Y)$
-1	-y	2/3
0	-y	1/3
1	-y	0
-1	+y	0
0	+y	1/2
1	+y	1/2

   $P(X_2|Y)$

X_2	Y	$P(X_2|Y)$
-1	-y	0
0	-y	2/3
1	-y	1/3
-1	+y	1/2
0	+y	1/4
1	+y	1/4

2. $P(+y|X_1 = +1, X_2 = +1) = P(+y) * p(X_1 = +1| + y) * P(X_2 = +1| + y) = 1/14$
   $P(-y|X_1 = +1, X_2 = +1) = P(-y) * P(X_1 = +1| - y) * P(X_2 = +1| - y) = 0$
   Since $P(+y|X_1 = +1, X_2 = +1) > P(-y|X_1 = +1, X_2 = +1)$, We predict +y.

3. We have the equation $\frac{c(x)+k}{N+k|X|}$

   $P(X_1|Y)$

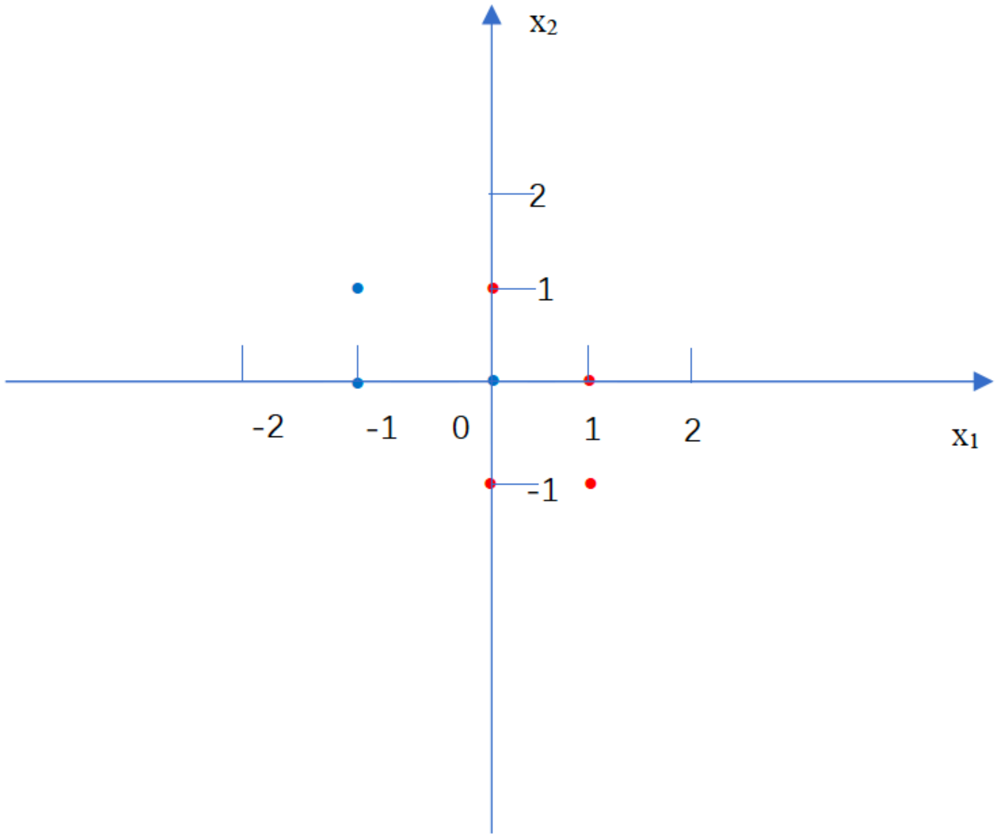
X_1	Y	$P(X_1|Y)$
-1	-y	1/2
0	-y	1/3
1	-y	1/6
-1	+y	1/7
0	+y	3/7
1	+y	3/7

   $P(X_2|Y)$

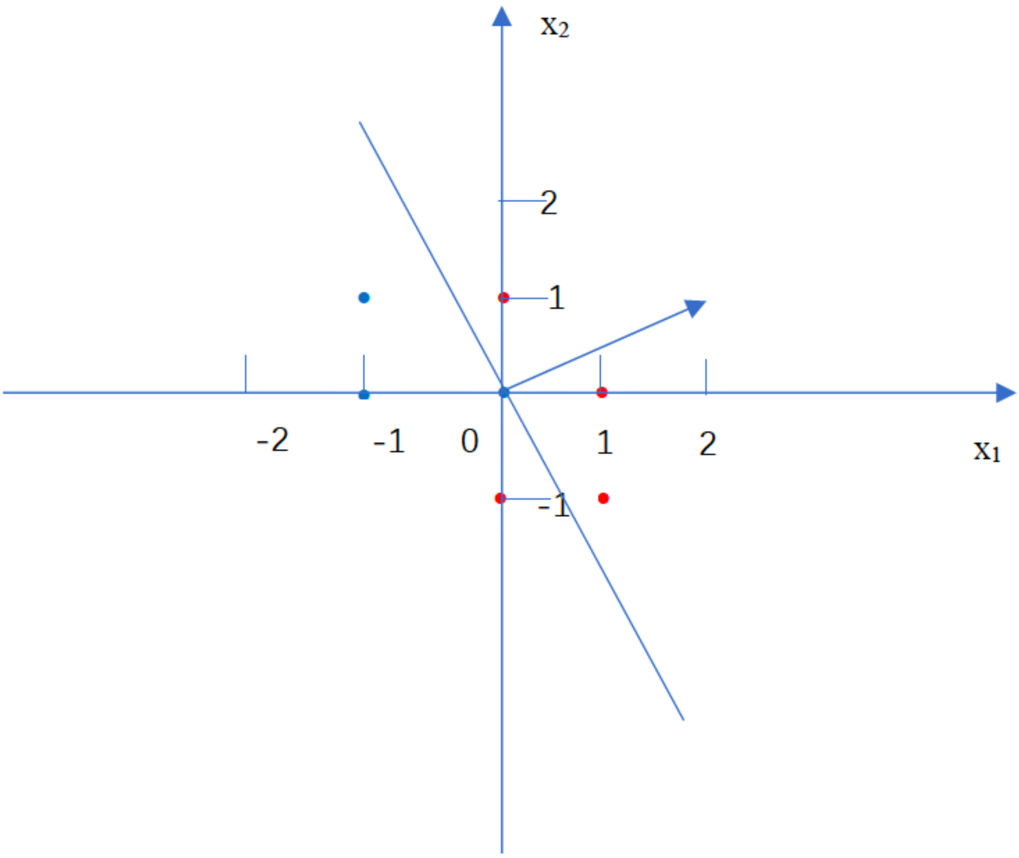
X_2	Y	P
-1	-y	1/6
0	-y	3/6
1	-y	2/6
-1	+y	3/7
0	+y	2/7
1	+y	2/7

4. $X_1 = 0 \ and \ X_2 = -1$ or $X_1 = 1 \ and \ X_2 = -1$, We can get the maximum of $P(X_1, X_2, Y + y)$

Part 2

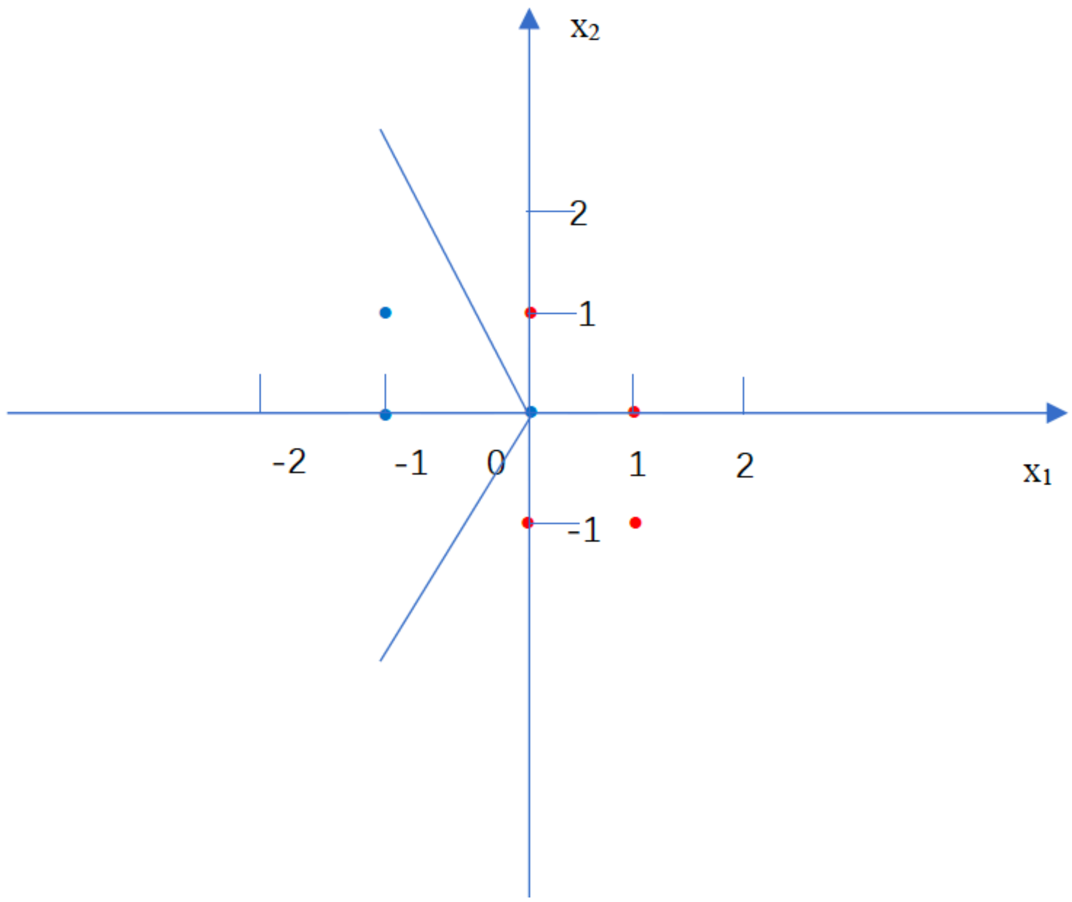


1. As we can see ,the data is not linearly separable.



2. It will predict it as +y.
3.
   - For $X_3 = 1$, doesn't allow because it only add a bias term to the linear classifier but the data is not linearly separable.

- For $X_3 = |X_1|$, doesn't allow because for point (0, 1, 0), (0,-1, 0), w should be (a, 0, b) (a and b are some real number) to classify them to the same category, but in this case the classifier will also judge (0, 0, 0) to the same category, which is wrong.
- For $X_3 = |X_2|$, allow. Let $w = (2, 0, 1)$, the decision boundary is $x_1 = -\frac{1}{2}|x_2|$, which shows as below:



As we can see, this boundary perfectly separate the data.

- For $X_3 = X_1^2$, doesn't allow because for point (0, 1, 0), (0,-1,0), w should be (a, 0, b) (a and b are some real number) to classify them to the same category, but in this case the classifier will also judge (0, 0, 0) to the same category, which is wrong.
- For $X_3 = X_2^2$, allow because the decision boundary could be a parabola going leftwards. Just let $w = (1, 0, 1)$, the decision boundary is $x_1 = -x_2^2$, which is a parabola going leftwards and with original point on it, and it can perfectly separate the data.
- For $X_3 = X_1^2 + X_2^2$, allow because its decision boundary could be a circle. We let $w = (2, 0, 1)$, the decision boundary of the classifier is $2x_1 + x_1^2 + x_2^2 = 0$, which is also $(x_1 + 1)^2 + x_2^2 = 1$. This circle can perfectly separate the data.

4.

$\phi(x)$	$Y(predicated)$	w
(-1, 1, 0)	-y	(0, 0, 0)
(0, 0, 0)	-y	(0, 0, 0)
(-1, 0, 1)	-y	(0, 0, 0)
(0, 1, 1)	-y	(0, 1, 1)
(1, -1, 0)	-y	(1, 0, 1)
(0, -1, 1)	+y	(1, 0, 1)
(1, 0, 1)	+y	(1, 0, 1)

5. $(-1,+1,0) : -1+0+0 = -1 => -y\ , correct$

   $(0,0,0) : 0+0+0 = 0 => -y\ , correct$

   $(-1,0,1) : -1+0+1 = 0 => -y\ , correct$

   $(0,1,1) : 0+0+1 = 1 => +y\ , correct$

   $(1,-1,0) : 1+0+0 = 1 => +y\ , correct$

   $(0,-1,1) : 0+0+1 = 1 => +y\ , correct$

   $(1,0,1) : 1+0+1 = 2 => +y\ , correct$

   So it is able to linearly separate all of them.

Part 3

1. (0, 0) (1, 0) and (0,1)

   The Euclidean distance is $\sqrt{0.5^2 + 0.5^2} = 0.71$

   Since (0, 0) is -y, (1, 0) and (0, 1) are +y, we predict oversell.

2. $score = -1 * (0+1) + (-2) * 1 + 2 * 1.52 + 1 * 1 = 2.5 > 0 => +y$

3. The means:

   $m_1 = (-2/3, 1/3)$

   $m_2 = (0, 1)$

   $m_3 = (2/3, 2/3)$

   The cluster is:

   m1: (-1, 1) (0, 0) (-1, 0)

   m2: (0, 1)

   m3: (1, -1) (0, -1) (1, 0)

   We predict m1 to -y and m2, m3 to +y.

   For every points in the data, we calculate the Euclidean distances between them and the means, then we can find that :

data	nearest mean
(-1, 1)	m1
(0, 0)	m1
(-1, 0)	m1
(0, 1)	m2
(1, -1)	m3
(0, -1)	m3
(1, 0)	m3

So it is stable.