MLT Homework 8

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Question 1

Let $\psi(\lambda) = \frac{\lambda^2}{2}$*. The Legendre-Fenchel transform of* ψ *is given by*

$$\psi^*(\epsilon) = \sup_{\lambda \in \mathbb{R}} \lambda\epsilon - \psi(\lambda).$$

Subquestion 1.1

$\psi^*(\epsilon) = \frac{\epsilon^2}{2}$.

Solution

Let define $\psi_1(\lambda)$:

$$\psi_1(\lambda) = \lambda\epsilon - \frac{\lambda^2}{2}$$

Furthermore:

$$\psi_1'(\lambda) = \epsilon - \lambda$$

So, the maximum is reached at:

$$\lambda = \epsilon \quad \Rightarrow \quad \psi_1(\epsilon) = \epsilon \cdot \epsilon - \frac{\epsilon^2}{2} = \frac{\epsilon^2}{2}$$

We can conclude:

$$\psi^*(\epsilon) = \psi_1(\epsilon) = \frac{\epsilon^2}{2}$$

Subquestion 1.2

$(\psi^*)^{-1}(z) = \pm\sqrt{2z}$.

Solution

From previous point we know:

$$\psi^*(\epsilon) = \frac{\epsilon^2}{2}$$

It follows:

$$\begin{aligned} z &= \frac{\epsilon^2}{2} \\ 2z &= \epsilon^2 \\ \epsilon &= \pm\sqrt{2z} \end{aligned}$$

So:

$$(\psi^*)^{-1}(z) = \pm\sqrt{2z}$$

Question 2

Question 3

We consider an adversarial bandit model with K^2 arms indexed by $i \in [K]$ and $j \in [K]$. For each arm (i, j), the loss at time t is $a_t^i + b_t^i$, where $a_t^i \in [0, 1]$ and $b_t^i \in [0, 1]$ are chosen by the adversary before the start of the interaction. Then each round the learner picks an arm $(I_t, J_t) \in [K]^2$ and observes $a_t^{I_t}$ and $b_t^{J_t}$ separately (and incurs their sum as the loss).

Subquestion 3.1

Consider running a single instance of EXP3 on all K^2 arms (with loss range $[0, 2]$). Show that the expected pseudo-regret compared to the best arm (i^*, j^*) is bounded by

$$\bar{R}_n \leq 2\sqrt{2nK^2 \ln(K^2)}$$

Solution

Subquestion 3.2

Now we will use the a_t^i and b_t^j observations separately. Consider running two K-arm instances of EXP3, one with $i \to a_t^i$ as the loss and one with $j \to b_t^j$ as *the loss. Have the first algorithm control I_t and the second J_t. Show that the overall expected pseudo-regret is bounded by*

$$\bar{R}_n \leq 2\sqrt{2nK \ln K}.$$

Solution