

MLT Homework 13

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Question 1

Finite Θ .

Subquestion 1.1

Verify that \bar{p} is a probability mass function on $\{0, 1\}^m$.

Solution

$$\begin{aligned}\sum_{(z_1, \dots, z_m) \in \{0, 1\}^m} \bar{p}(z_1, \dots, z_m) &= \sum_{(z_1, \dots, z_m) \in \{0, 1\}^m} \sum_{\theta \in \Theta} w(\theta) p_{\theta}(z^m) \\ &= \sum_{(z_1, \dots, z_m) \in \{0, 1\}^m} \sum_{\theta \in \Theta} \frac{1}{N} p_{\theta}(z^m) \\ &= \frac{1}{N} \sum_{\theta \in \Theta} \sum_{(z_1, \dots, z_m) \in \{0, 1\}^m} p_{\theta}(z^m) \\ &= \frac{1}{N} \sum_{\theta \in \Theta} 1 \\ &= \frac{1}{N} N \\ &= 1\end{aligned}$$

Subquestion 1.2

Show that the worst-case case regret of \bar{p} is also bounded by N .

Solution

Subquestion 1.3

Let us fix $N = |\Theta|$ and set $\Theta = \{1/(N+1), \dots, N/(N+1)\}$. For example, if we set $N = 4$ then $\Theta = \{0.2, 0.4, 0.6, 0.8\}$.

Solution

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Subquestion 1.4

Use the law of large numbers to argue that

$$\lim_{m \rightarrow \infty} S_m = \log N.$$

Solution

Subquestion 1.5

Informally explain why, for small sample sizes m , the Shtarkov sum for $\Theta = \{0.47, 0.49, 0.51, 0.53\}$ is significantly smaller than the Shtarkov sum for $\Theta = \{0.2, 0.4, 0.6, 0.8\}$.

Solution

Subquestion 1.6

Suppose that P consists of a finite number of black-box experts, which given each history Z_1, Z_{i-1} provide us a distribution on Z_i , which may depend on the past — we don't know how they come up with their predictions, we just observe their predictions they make on the sample. Explain why we'd rather use the Bayesian than the Shtarkov predictor in such a setting.

Solution

Question 2