# MLT Homework 5

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## Question 1

### Subquestion 1.1

*Consider a hypothesis class $\mathcal{H} = \cup_{n=1}^{\infty} \mathcal{H}_n$, where for every $n \in \mathbb{N}$, $\mathcal{H}_n$ is finite. Find a weighting function $w : \mathcal{H} \to [0,1]$ such that $\sum_{h \in \mathcal{H}} w(h) \leq 1$ and so that for all $h \in \mathcal{H}$, $w(h)$ is determined by $|\mathcal{H}_{n(h)}|$.*

**Solution**

Since we have a countably infinite union of finite sets, we know that the number of elements is countably infinite. Therefore, we can number them as:

$$h_1, h_2, \ldots$$

If we pick weights as:

$$w(h_i) = \left( \frac{1}{2^{|H_{n(h_i)}|}} \right)^i ; \quad i = 1, 2, \ldots$$

the sum of the weights in the worst case would be, when $|H_{n(h_i)}| = 1$ $(\forall i)$:

$$\sum_{i=1}^{\infty} w(h_i) = \sum_{i=1}^{\infty} \left( \frac{1}{2} \right)^i = 1$$

### Subquestion 1.2

*Define such a function $w$ when for all $n$ $\mathcal{H}_n$ is countable (possibly infinite).*

**Solution**

Countably infinite union of countable sets is again a countable set, so we can choose the same weighted function as before.

# Question 2

*7.5*

**Solution**

# Question 3


**Solution**