We will construct a sequence in Σ^+ to be able to apply Fatou's Lemma. Since we know $|f_n| \leq g_n$ for all $n \in \mathbb{N}$ and $f_n \to f$ and $g_n \to g$, we have

$$|f_n - f| \leq g_n + g$$

Therefore we can conclude $g_n + g - |f_n - f| \geq 0$ and in Σ^+. Applying fatou's lemma now gives us

$$\liminf_{n\to\infty} \mu(g_n + g - |f_n - f|) \geq \mu(\liminf_{n\to\infty}(g_n + g - |f_n - f|)).$$

The RHS equals $2\mu(g)$, since both f and g converge pointwise. The LHS is equal to $2\mu(g) - \limsup_{n\to\infty} \mu(|f_n - f|)$. By assumption we have $\lim_{n\to\infty} \mu(g_n) = \mu(g)$. Putting it together we obtain

$$0 \geq \limsup_{n\to\infty} \mu(|f_n - f|)$$

MLT Homework 3

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