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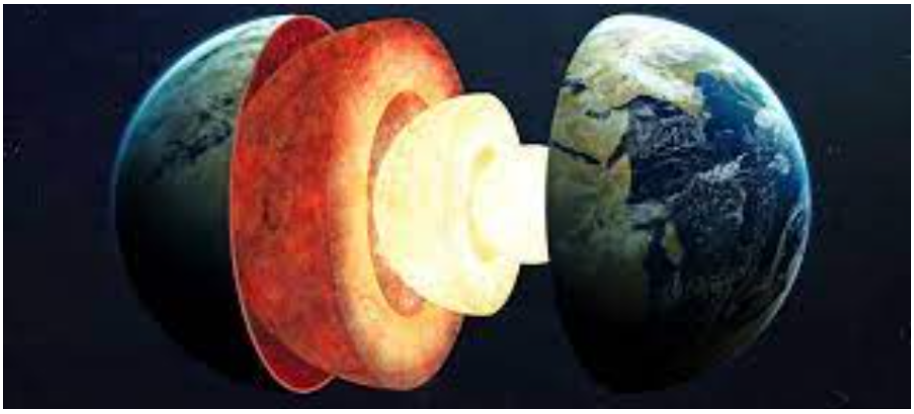
Inverse Problems

Image Deconvolution of HST Saturn Data: Wiener Filtering, Conjugate Gradient & Regularization Effects



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1 Introduction

Observations of Saturn by the Hubble Space Telescope (HST) are affected by instrument-induced blurring, modeled by the Point Spread Function (PSF), and additive noise. Recovering sharp, high-fidelity images requires solving an ill-posed inverse problem. This study compares **frequency-domain Wiener filtering** and **iterative Conjugate Gradient (CG) optimization**, and investigates the effect of strong regularization (masked weights) on image reconstruction.

Keywords

PSF modeling, Fourier analysis, Wiener filter, Conjugate Gradient, regularization, inverse problems, Saturn imaging

2 Methods

2.1 Wiener Filtering

The Wiener filter operates in Fourier space:

$$\hat{X}(k) = \frac{H^*(k)}{|H(k)|^2 + \lambda} Y(k)$$

where $Y(k)$ is the Fourier transform of the observed image, $H(k)$ is the PSF, and λ is a regularization parameter controlling noise amplification. Wiener filtering provides stable reconstructions but smooths fine spatial details.

2.2 Conjugate Gradient (CG) Optimization

The regularized normal equation is solved iteratively:

$$(H^\top W H - \mu D^\top D)x = H^\top W y$$

where W represents data weighting, D is a derivative operator imposing smoothness, and μ controls regularization. CG reconstruction preserves edges but is more sensitive to noise and regularization choices.

2.3 Masked-Weight Reconstruction

Applying very small weights (0.01–0.02) in W suppresses high-frequency contributions. The resulting image emphasizes robust, low-frequency structures and highlights regions where deconvolution is stable versus noise-dominated.

3 Results

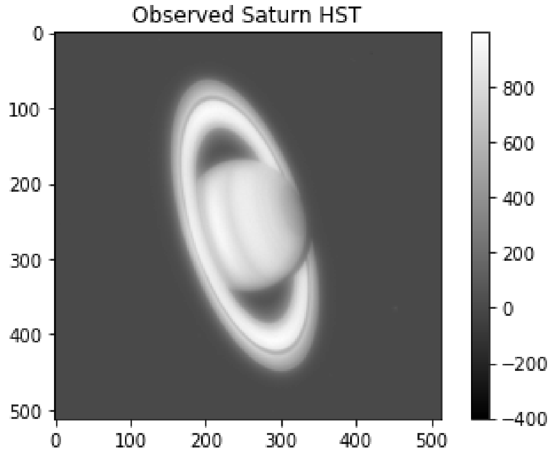

Figure 1: Raw observed HST Saturn image (blurred and noisy).

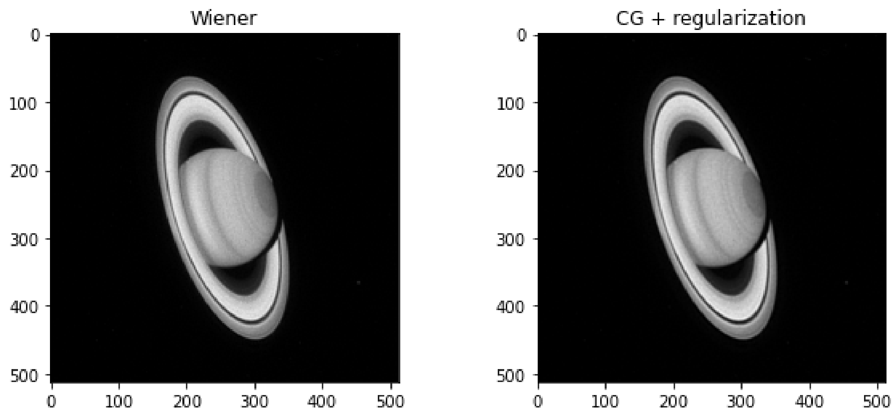

Figure 2: Wiener-filtered reconstruction and Conjugate Gradient reconstruction.

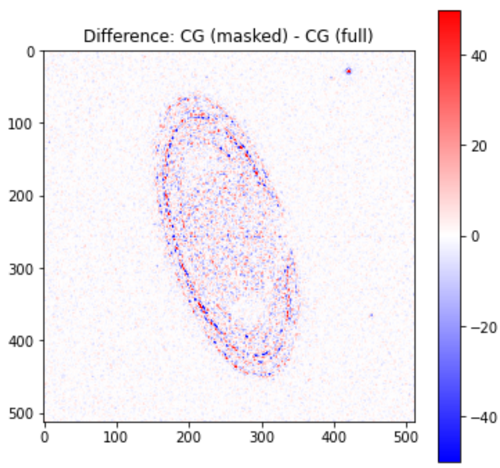

Figure 3: Masked-weight reconstruction emphasizing low-frequency structures.

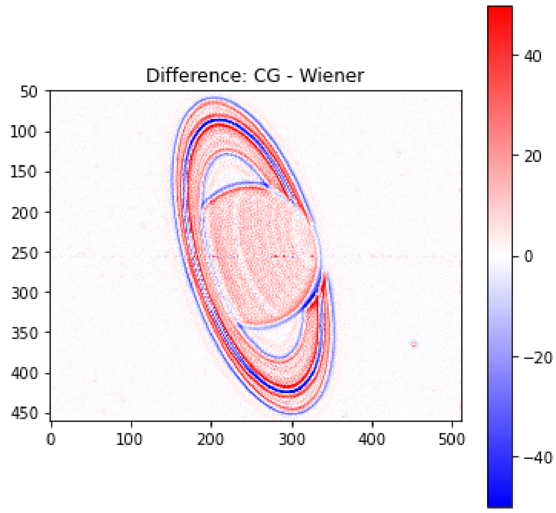

Figure 4: Difference image (CG − Wiener): ring-like residuals indicate sensitivity of sharp edges to reconstruction.

4 Discussion

- Wiener filtering provides stable, noise-limited reconstruction, suitable for general morphology.
- Conjugate Gradient allows edge-preserving recovery, important for fine structures like Saturn's rings.
- Masked-weight reconstruction serves as a diagnostic tool, revealing regions where deconvolution is reliable versus dominated by noise.
- Difference images show that sharp edges are most sensitive to the reconstruction method, while smooth areas (planet center) are consistently recovered.

5 Conclusion

This study demonstrates the trade-offs between frequency-domain filtering and iterative optimization in astronomical imaging. Wiener filtering ensures stability, CG recovers sharp edges, and masked-weight reconstruction provides a diagnostic for assessing reliability. Selecting appropriate regularization is crucial for accurate deconvolution of HST images.