

2. Consider an infinitely long, hollow cylindrical wire. The wire has outer diameter b and the cylindrical hole at its center has diameter a .

- a) Find the magnetic field everywhere if a known current i flows from left to right and the current is uniformly spread over the region between a and b .

Ampere's Law

$$\oint \vec{B} dl = \mu_0 I_{\text{Enclosed}}$$

When $x < a$, the magnetic field is the same at each point.

$$I_{\text{Enclosed}} = 0$$

$$\vec{B} \oint dl = \mu_0 I_{\text{Enclosed}}$$

$$\vec{B} \cdot 2\pi x = \mu_0 \cdot 0$$

$$\vec{B} = 0 \text{ T}$$

$$\text{When } a < x < b, A = \pi \left(\frac{b}{2}\right)^2 - \left(\frac{a}{2}\right)^2 = \pi \left(\frac{b^2}{4} - \frac{a^2}{4}\right)$$

Total current passing through A is I

$$\text{Current per unit area } \frac{I}{A} = \frac{I}{\pi \left(\frac{b^2}{4} - \frac{a^2}{4}\right)}$$

$$I_{\text{Enclosed}} = \frac{I}{\pi \left(\frac{b^2}{4} - \frac{a^2}{4}\right)} \cdot \pi \left(x^2 - \frac{a^2}{4}\right) = \frac{I(x^2 - \frac{a^2}{4})}{\frac{b^2}{4} - \frac{a^2}{4}}$$

$$\vec{B} \oint dl = \mu_0 I_{\text{Enclosed}}$$

$$\vec{B} \cdot 2\pi x = \mu_0 \frac{I(x^2 - \frac{a^2}{4})}{\frac{b^2}{4} - \frac{a^2}{4}}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi x} \frac{x^2 - \frac{a^2}{4}}{\frac{b^2}{4} - \frac{a^2}{4}} = \frac{\mu_0 I}{2\pi x} \frac{4x^2 - a^2}{b^2 - a^2} \text{ T}$$

When $x > b$,

$$I_{\text{Enclosed}} = I$$

$$\vec{B} \oint dl = \mu_0 I_{\text{Enclosed}}$$

$$\vec{B} \cdot 2\pi x = \mu_0 \cdot I$$

$$\vec{B} = \frac{\mu_0 I}{2\pi x} \text{ T}$$

- b) Find the force that this wire would exert on a thin, straight length of wire W located a distance $2b$ from the axis of the cylinder if that thin wire had a current $3I$ flowing from left to right.

$F = IL \times \vec{B}$ where I is current, L is Length, and \vec{B} is Magnetic Field

For $x = 2b$,

$$\begin{aligned}\vec{B} &= \frac{\mu_0 I}{2\pi x} \\ &= \frac{\mu_0 I}{4\pi b}\end{aligned}$$

For $3I$ and W ,

$$\begin{aligned}F &= IL \times \vec{B} \\ &= 3IW \frac{\mu_0 I}{4\pi b} \\ &= \frac{\mu_0}{4\pi} \frac{3I^2 W}{b}\end{aligned}$$