2. Consider an infinitely long, hollow cylindrical wire. The wire has outer diameter b and the cylindrical hole at its center has diameter a.

   a) Find the magnetic field everywhere if a known current i flows from left to right and the current is uniformly spread over the region between a and b.

Ampere's Law

$$\oint \vec{B} dl = \mu_0 I_{\text{Enclosed}}$$

When $x < a$, the magnetic field is the same at each point.
$I_{\text{Enclosed}} = 0$

$$\begin{aligned} \vec{B} \oint dl &= \mu_0 I_{\text{Enclosed}} \\ \vec{B} \cdot 2\pi x &= \mu_0 \cdot 0 \\ \vec{B} &= 0 \text{ T} \end{aligned}$$

When $a < x < b$, $A = \pi b^2 - \pi a^2 = \pi(b^2 - a^2)$
Total current passing through A is I
Current per unit area $\frac{I}{A} = \frac{I}{\pi(b^2-a^2)}$
$I_{\text{Enclosed}} = \frac{I}{\pi(b^2-a^2)} \pi(x^2 - a^2) = \frac{I(x^2-a^2)}{b^2-a^2}$

$$\begin{aligned} \vec{B} \oint dl &= \mu_0 I_{\text{Enclosed}} \\ \vec{B} \cdot 2\pi x &= \mu_0 \frac{I(x^2 - a^2)}{b^2 - a^2} \\ \vec{B} &= \frac{\mu_0 I}{2\pi x} \frac{x^2 - a^2}{b^2 - a^2} \text{ T} \end{aligned}$$

When $x > b$,
$I_{\text{Enclosed}} = I$

$$\begin{aligned} \vec{B} \oint dl &= \mu_0 I_{\text{Enclosed}} \\ \vec{B} \cdot 2\pi x &= \mu_0 \cdot I \\ \vec{B} &= \frac{\mu_0 I}{2\pi x} \text{ T} \end{aligned}$$

b) Find the force that this wire would exert on a thin, straight length of wire W located a distance $2b$ from the axis of the cylinder if that thin wire had a current $3I$ flowing from left to right.

$F = IL \times \vec{B}$ where I is current, L is Length, and $\vec{B}$ is Magnetic Field

For $x = 2b$,

$$\begin{aligned}\vec{B} &= \frac{\mu_0 I}{2\pi x} \\ &= \frac{\mu_0 I}{4\pi b}\end{aligned}$$

For $3I$ and W,

$$\begin{aligned}F &= IL \times \vec{B} \\ &= 3IW \frac{\mu_0 I}{4\pi b} \\ &= \frac{u_0}{4\pi} \frac{3I^2 W}{b}\end{aligned}$$