Week 9 - MATH1012 Tutorial

Overview

Finishing off Convergence of Series (Ratio Test)

Taylor Series

- Know how to find Coefficients
- Know how to find Upper Bound for Error

Power Series

- Know how to find Coefficients
- Radius of Convergence

(Non-Assessable but may be useful in the future: A Power/Taylor Series with an Infinite Radius of Convergence is called Analytic)

Summary

Ratio Test:

1) $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1 \Rightarrow \sum_{n=1}^{\infty} a_n$ *is absolutely convergent*

2) $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L > 1$ *or diverges* $\Rightarrow \sum_{n=1}^{\infty} a_n$ *is divergent*

3) $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1 \Rightarrow$ *Inconclusive*

Taylor Series:

If $f(x)$ has a power series representation

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n \qquad |x-a| < R$$

Then

$$c_n = \frac{f^{(n)}(a)}{n!}$$

Power Series:

$$\sum_{n=0}^{\infty} c_n (x-a)^n \qquad c_n \text{ is a sequence}$$

Taylor Series Error:

$$R_n(x) = \frac{\max\left(f^{(n+1)}(a)\right)}{(n+1)!}(x-a)^{n+1}$$

Quick Review – Taylor Approximation Error

$$\begin{aligned} P_{n+1}(x) &= f(a) + \frac{f'(a)}{1!}(x-a) + \cdots + \frac{f^n(a)}{n!}(x-a)^n + \frac{f^{(n+1)}(a)}{(n+1)!}(x-a)^{n+1} \\ &= P_n(x) + \frac{f^{(n+1)}(a)}{(n+1)!}(x-a)^{n+1}. \end{aligned}$$

Since the difference between $P_n(x)$ and $P_{n+1}(x)$ is just that last term, the error of $P_n(x)$ can be no larger than that term. In other words, the error R_n is

$$R_n(x) = \max\left(\frac{f^{(n+1)}(a)}{(n+1)!}(x-a)^{n+1}\right).$$

Since a and n are constant in this formula, terms depending only on those constants and x are unaffected by the $\max$ operator and can be pulled outside:

$$R_n(x) = \frac{\max\left(f^{(n+1)}(a)\right)}{(n+1)!}(x-a)^{n+1}.$$

Source: Brilliant.org wiki

Sources

Images on Page 3 – Trefor Bazett

Error Formula on Page 3, Slide 4 – Brilliant Wiki