Week 8 Practical

MATH1012

Convergence of Series

Geometric Series:

$$\sum_{n=1}^{\infty} ar^{n-1} = \begin{cases} \frac{a}{1-r} & |r| < 1 \\ diverges & |r| \geq 1 \end{cases}$$

Absolute and Conditional Convergence

$$\sum_{n=1}^{\infty} |a_n| = \text{convergent} \rightarrow \sum_{n=1}^{\infty} a_n = \text{convergent}$$

P-Series:

A p-series

$$\sum_{n=0}^{\infty} \frac{1}{n^p}$$

converges if and only if $p > 1$.

Convergence of Series

Convergence Tests:

Divergence Test: $\lim_{n\to\infty} a_n \neq 0 \Rightarrow \sum_{n=1}^{\infty} a_n \textit{ diverges}$

Comparison Test: $a_n \leq b_n$

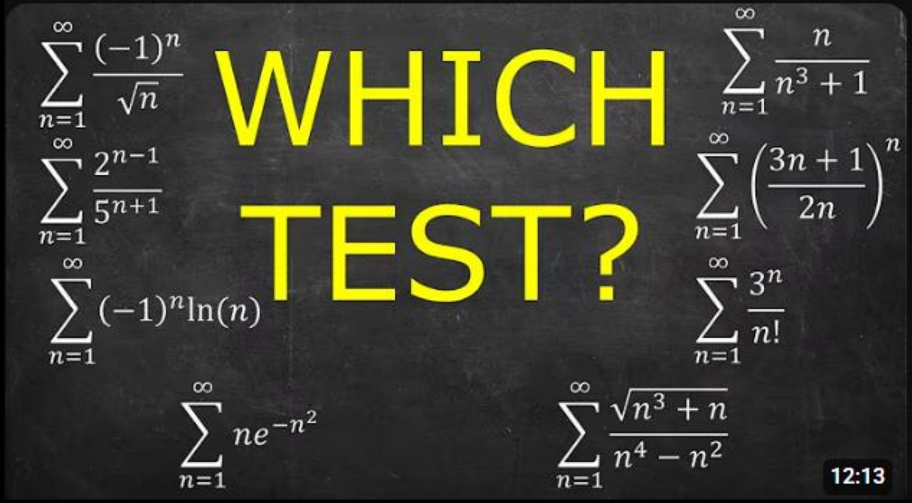
Limit Comparison Test: $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$

Integral Test: $\sum_{n=1}^{\infty} a_n \textit{ converges if } \int_1^{\infty} f(x)dx \textit{ converges}$

Ratio Test: $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$

Alternating Series Test: $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$

Extra Resources



Choosing Which Convergence Test to Apply to 8 Series

580K views • 6 years ago

Dr. Trefor Bazett

Deciding which **convergence** test to apply to a given series is often the hardest part of the unit on series **convergence**. In this video ...

9 chapters Intro | Geometric Series | Integral Test | Alternating Series Test | Divergence Test | Comparison Test |...

Summary:

Geometric Series:

$$\sum_{n=1}^{\infty} a_1(r)^{n-1} \text{ converges if and only if } -1 < r < 1$$

$$\sum_{n=1}^{\infty} a_1(r)^{n-1} = \frac{a_1}{1-r}$$

Convergence Tests:

Divergence Test: $\lim_{n\to\infty} a_n \neq 0 \Rightarrow \sum_{n=1}^{\infty} a_n \ diverges$

Comparison Test: $a_n \leq b_n$

Limit Comparison Test: $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$

Integral Test: $\sum_{n=1}^{\infty} a_n \ converges \ if \int_1^{\infty} f(x)dx \ converges$

Absolute Convergence implies Conditionally Convergent:

$$\sum_{n=1}^{\infty} |a_n| = \text{convergent} \rightarrow \sum_{n=1}^{\infty} a_n = \text{convergent}$$

P Series:

$$\sum_{n=0}^{\infty} \frac{1}{n^p}$$

converges if and only if $p > 1$.

Ratio Test: $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$

Alternating Series Test: $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$

Sources

Images: Slides 2-5 – Trefor Bazett