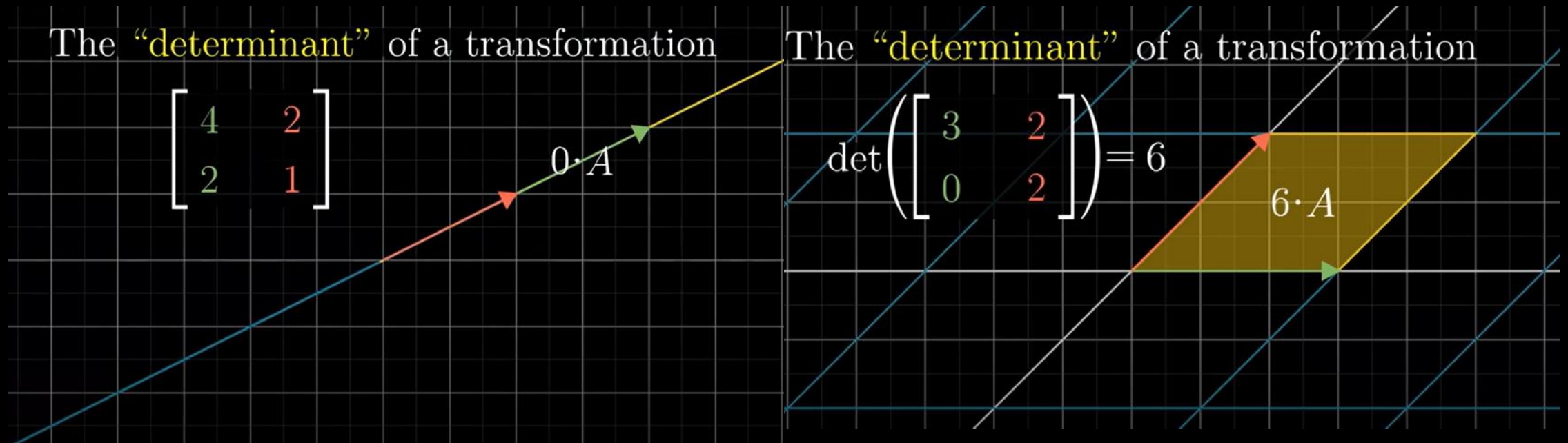


Week 4  
MATH1012 Practical

# Determinants:

Determinant: (Signed) Change in Area/Volume for a matrix under a transformation.

Linearly Dependence:  $\text{Det } A = 0$



# Column Space

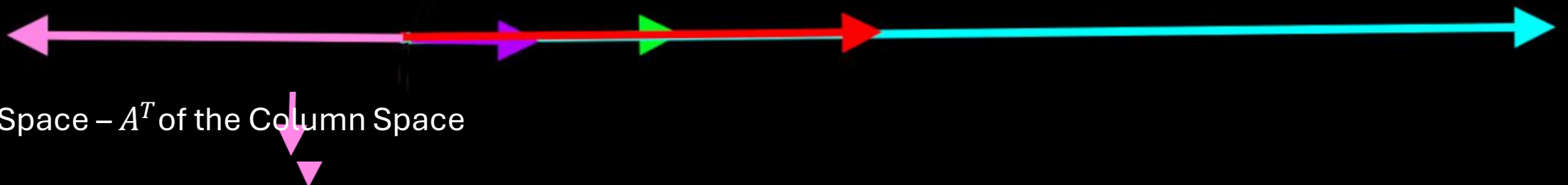
For a given transformation:

Column Space: The Range of a Transformation (Where your vectors end up after a transformation). i.e. The span of a transformation.

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$$s \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

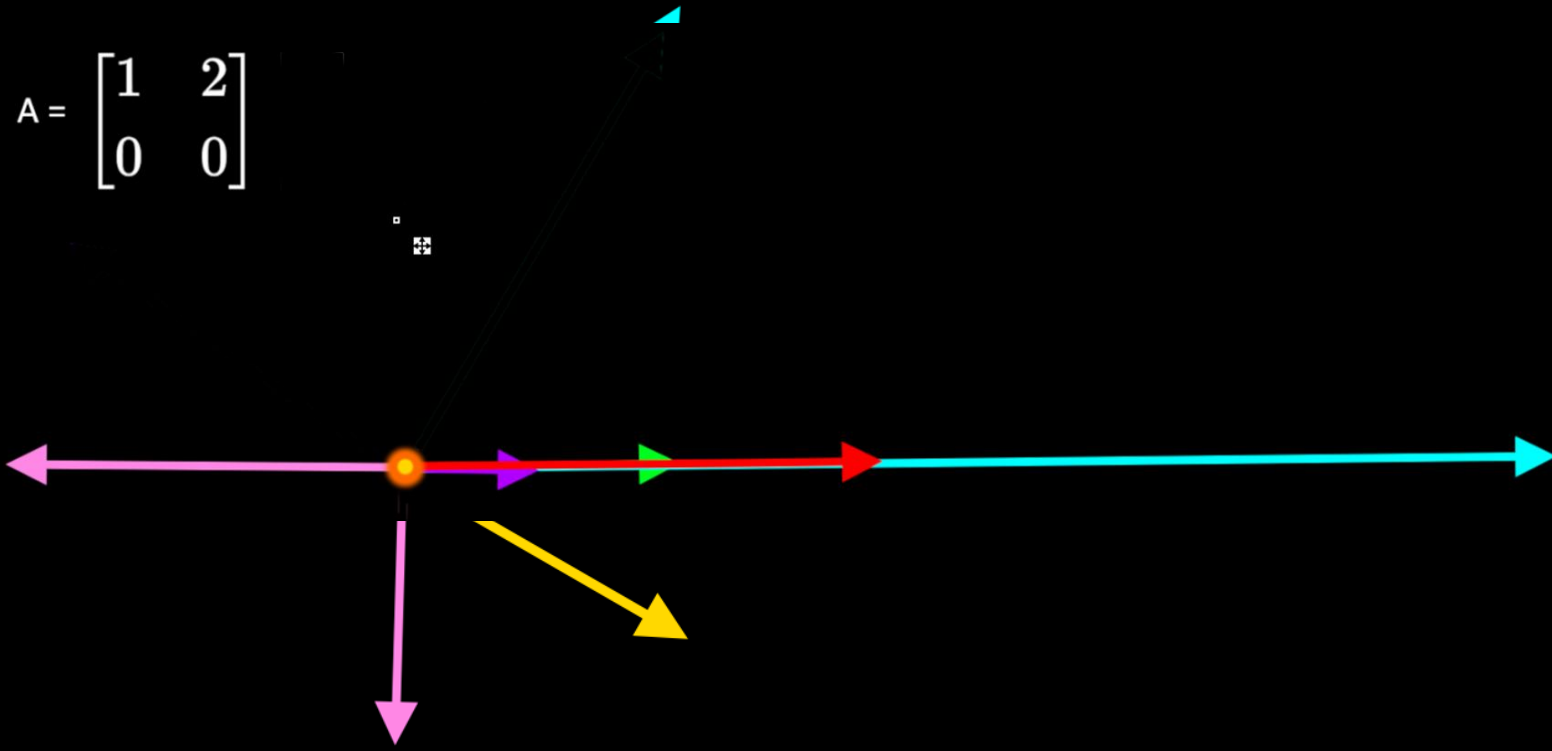
Row Space –  $A^T$  of the Column Space



# Nullspace

All of the vectors in the domain where:  $Ax = 0$

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$



# Nullspace & Column Space (Computation):

Nullspace is: The Parametric/Vector Solution of  $Ax = 0$

Column Space are the original vectors which corresponds to the leading 1's after finding REF.

Ex ✓

$$\left( \begin{array}{cccc|c} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

↑      ↑  
 $x_2 = s$      $x_4 = t$

2<sup>nd</sup> row:  $x_2 = -3t$   
1<sup>st</sup> row:  $x_1 = -2s$

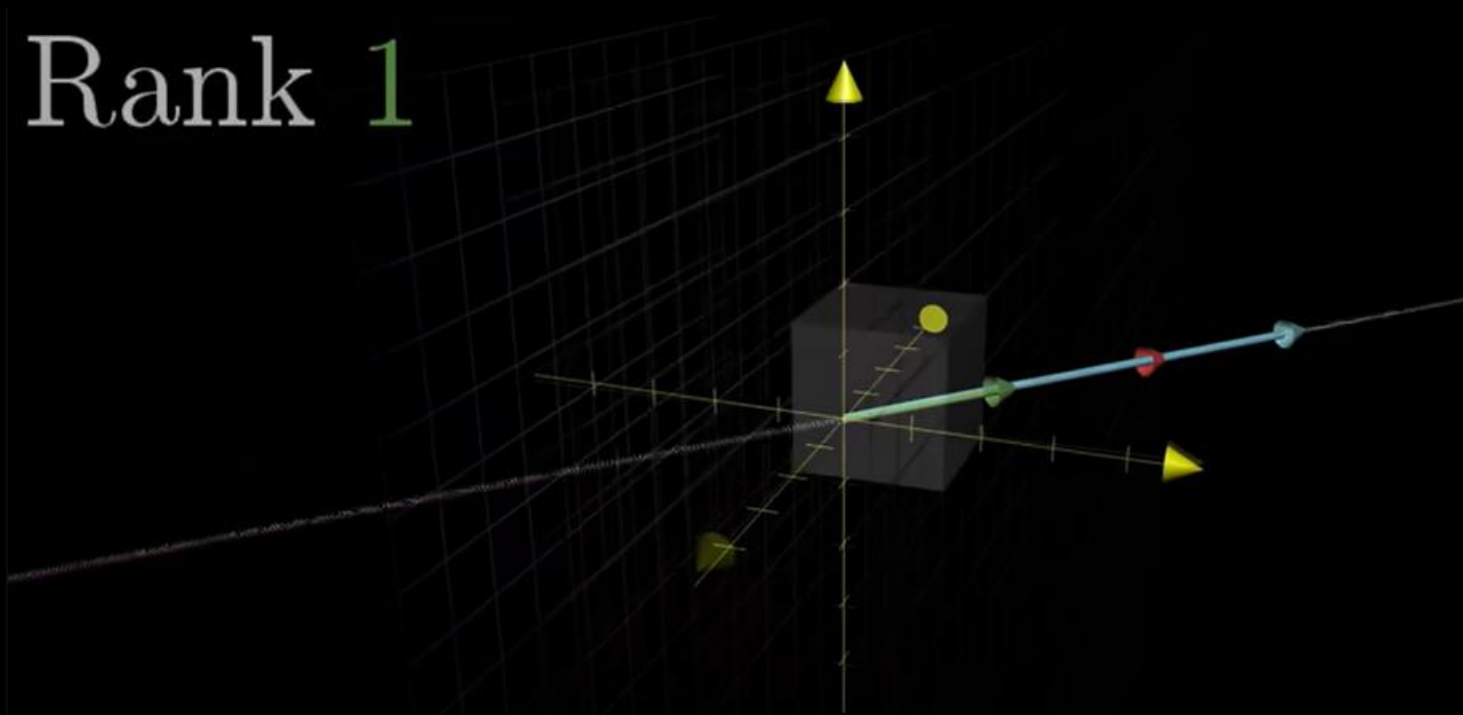
$$\vec{x} = s \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 0 \\ -3 \\ 1 \end{pmatrix}$$

$$A = \left( \begin{array}{cccc} 1 & 2 & 1 & 0 \\ 2 & 4 & 3 & 1 \\ 3 & 6 & 4 & 1 \end{array} \right) \xrightarrow{\text{RREF}} \left( \begin{array}{cccc} 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

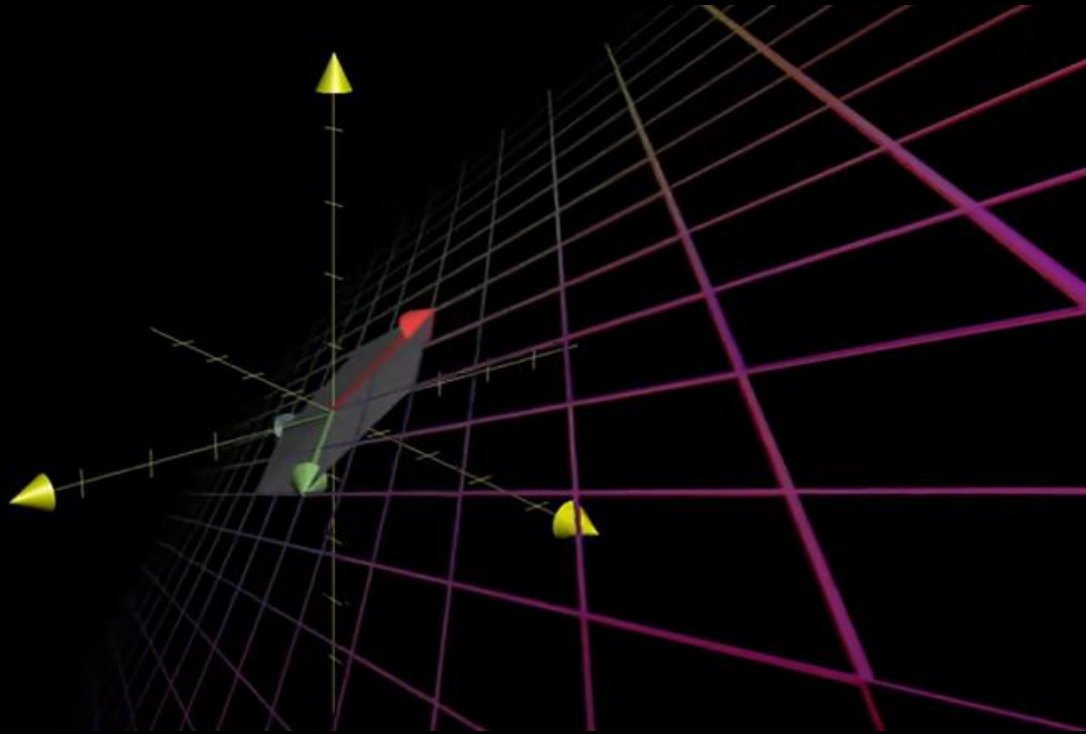
↑      ↑                      ↑      ↑

© Pictures: Trefor Bazett  
(2017)

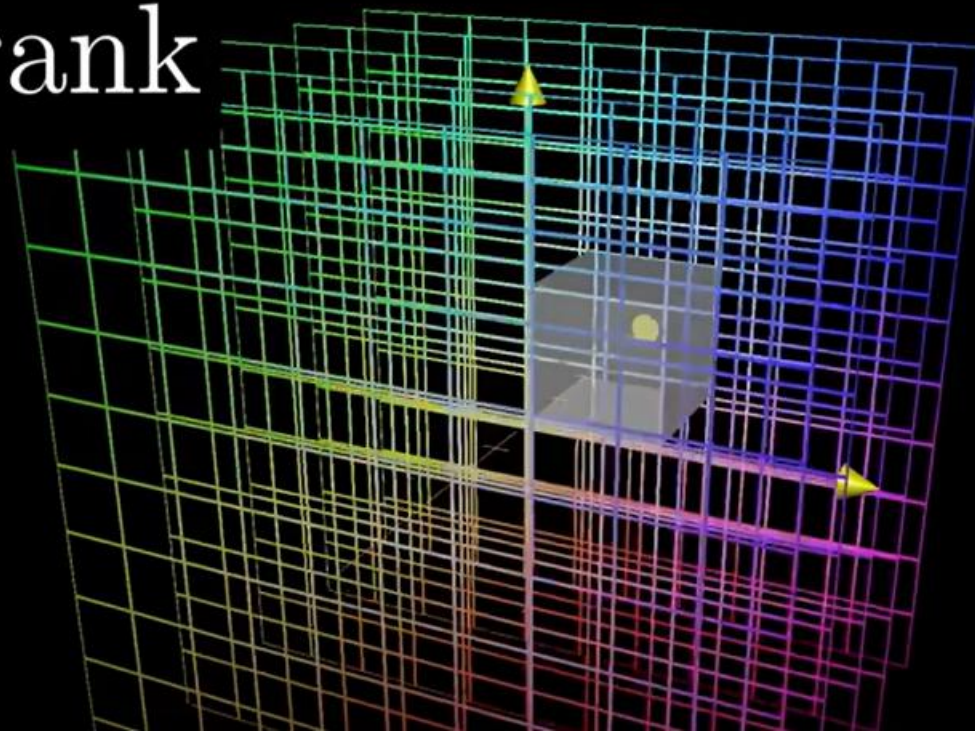
# Rank



Rank 2



Full rank





# Rank – Nullity Theorem

Dimension Theorem:

For  $m \times n$  matrix  $A$ ,

$$\text{Dim}(\text{Col}(A)) + \text{Dim}(\text{Null}(A)) = n$$

$$\text{Rank}(A) + \text{Dim}(\text{Null}(A)) = n$$

Diagram illustrating the Rank-Nullity Theorem with a matrix  $A$  of size  $m \times n$ .

The matrix  $A$  is shown as:

$$\begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The matrix has 4 columns ( $n=4$ ) and 4 rows ( $m=4$ ).

The first two columns are pivot columns (leading 1s), and the last two are free columns.

2 Leading 1s =  $\text{Dim}(\text{Col}(A)) = r$

2 Free columns =  $\text{Dim}(\text{Null}(A)) = n - r$

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# Summary:

Nullspace: All of the vectors in the domain where;  $Ax = 0$

Column Space: The Range of a Transformation (Where your vectors end up after a transformation). i.e. The span of a transformation.

Computation:

Nullspace is: The Parametric/Vector Solution of  $Ax = 0$

Column Space are the original vectors which corresponds to the leading 1's after finding REF.

Rank-Nullity Theorem:

$\text{Rank}(A) + \text{Dim}(\text{Null}(A)) = n$  (with  $n$ . as no. of columns)

# Sources

Images:

3Blue1Brown/Grant Sanderson – Slides 2,6,7,8

Trefor Bazett – Slides 5,9





$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

# Column Space

For a given transformation:

Column Space: The Range of a Transformation (Where your vectors end up after a transformation). i.e. The span of a transformation.

