

Week 9 - MATH1012

Tutorial

Overview

Finishing off Convergence of Series (Ratio Test)

Taylor Series

- Know how to find Coefficients
- Know how to find Upper Bound for Error

Power Series

- Know how to find Coefficients
- Radius of Convergence

(Non-Assessable but may be useful in the future: A Power/Taylor Series with an Infinite Radius of Convergence is called Analytic)

Summary

Ratio Test:

- 1) $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L < 1 \Rightarrow \sum_{n=1}^{\infty} a_n$ is absolutely convergent
- 2) $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L > 1$ or diverges $\Rightarrow \sum_{n=1}^{\infty} a_n$ is divergent
- 3) $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1 \Rightarrow$ Inconclusive

Power Series:

$$\sum_{n=0}^{\infty} c_n (x - a)^n \quad c_n \text{ is a sequence}$$

Taylor Series:

If $f(x)$ has a power series representation

$$f(x) = \sum_{n=0}^{\infty} c_n (x - a)^n \quad |x - a| < R$$

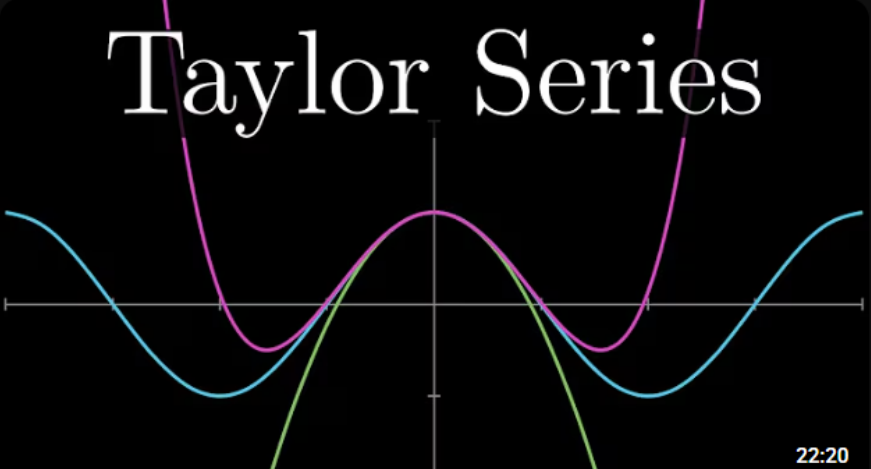
Then

$$c_n = \frac{f^{(n)}(a)}{n!}$$

Taylor Series Error:

$$R_n(x) = \frac{\max \left(f^{(n+1)}(a) \right)}{(n+1)!} (x - a)^{n+1}$$

Extra Resources





A graph illustrating the Taylor series approximation of a function. The function is shown as a blue curve. The Taylor series approximation is shown as a green curve. The graph is titled "Taylor Series" in a large, white, serif font. The x-axis and y-axis are shown with tick marks. The green curve is a polynomial approximation of the blue function, showing the characteristic oscillations of a Taylor series.

22:20



Taylor series | Chapter 11, Essence of calculus

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 3Blue1Brown 

Taylor polynomials are incredibly powerful for approximations and analysis. Help fund future projects: ...

CC

 **5 chapters** Approximating $\cos(x)$ | Generalizing e^x | Geometric meaning of the second term | Convergence issues 

Quick Review – Taylor Approximation Error

$$\begin{aligned} P_{n+1}(x) &= f(a) + \frac{f'(a)}{1!}(x-a) + \cdots + \frac{f^n(a)}{n!}(x-a)^n + \frac{f^{(n+1)}(a)}{(n+1)!}(x-a)^{n+1} \\ &= P_n(x) + \frac{f^{(n+1)}(a)}{(n+1)!}(x-a)^{n+1}. \end{aligned}$$

Since the difference between $P_n(x)$ and $P_{n+1}(x)$ is just that last term, the error of $P_n(x)$ can be no larger than that term. In other words, the error R_n is

$$R_n(x) = \max \left(\frac{f^{(n+1)}(a)}{(n+1)!}(x-a)^{n+1} \right).$$

Since a and n are constant in this formula, terms depending only on those constants and x are unaffected by the max operator and can be pulled outside:

$$R_n(x) = \frac{\max(f^{(n+1)}(a))}{(n+1)!}(x-a)^{n+1}.$$

Sources

Images on Page 3 – Trefor Bazett

Error Formula on Page 3, Slide 4 – Brilliant Wiki