

Week 3 - MATH1012

Practical

This Week: Subspaces & Span

Sets of Vectors: e.g. $\{(0,1),(1,0)\}$, $\{(1,2),(3,-1)\}$, $\{(0.5,0,1),(1,2,3),(2,4,7)\}$

Linear Combination: Some scaling/addition of a set of vectors: $a\vec{v} + b\vec{w}$

Span: The set of all possible Linear Combination's

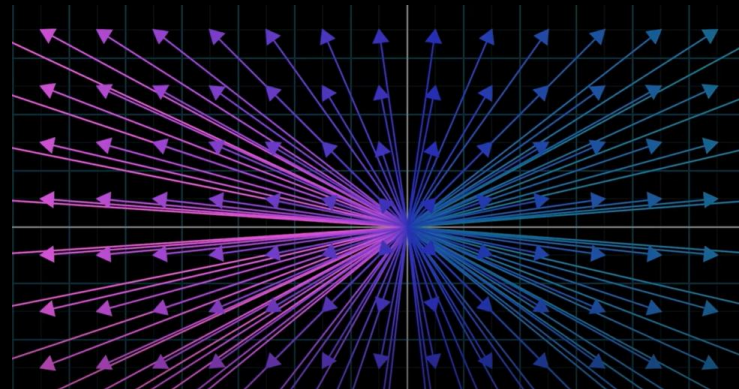
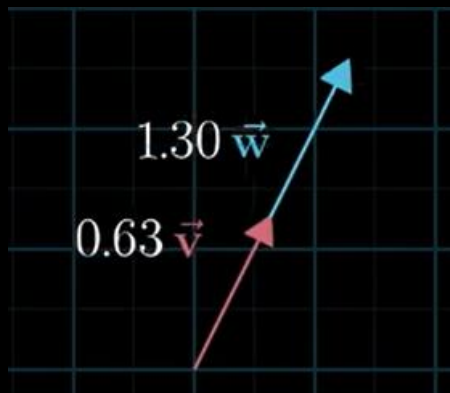
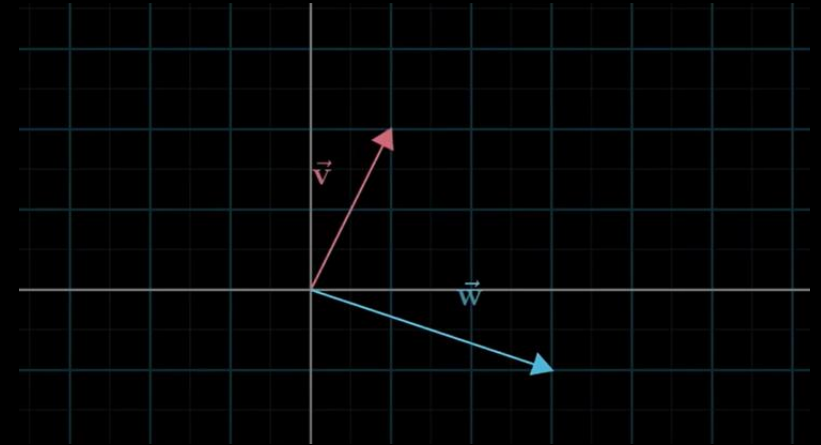
Spanning Set of a (Vector) Space V: Set of vectors which spans the space V.

Basis (of V): Minimal Spanning Set (Vectors which spans V with redundant vectors removed).

Linearly Independent: A set of vector's where each vector cannot be described as a linear combination of other vectors.

Linearly Independent

if: $a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + \dots + a_k\mathbf{v}_k = \mathbf{0}$ When only $v_1 v_2, \dots v_3 = 0$



Determining Linear Independence/Span

$$\text{Is } \underbrace{\begin{pmatrix} -1 \\ 4 \\ 11 \end{pmatrix}}_{\vec{b}} \in \text{Span} \left\{ \underbrace{\begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}}_{\vec{a}_1}, \underbrace{\begin{pmatrix} -3 \\ -5 \\ 13 \end{pmatrix}}_{\vec{a}_2}, \underbrace{\begin{pmatrix} 2 \\ -1 \\ -12 \end{pmatrix}}_{\vec{a}_3} \right\} ?$$

$$x_1 \vec{a}_1 + x_2 \vec{a}_2 + x_3 \vec{a}_3 = \vec{b}$$

$$\Rightarrow \begin{pmatrix} x_1 - 3x_2 + 2x_3 \\ 2x_1 - 5x_2 - x_3 \\ -4x_1 + 13x_2 - 12x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ 11 \end{pmatrix} \Rightarrow \left(\begin{array}{ccc|c} 1 & -3 & 2 & -1 \\ 2 & -5 & -1 & 4 \\ -4 & 13 & -12 & 11 \end{array} \right)$$

$$\text{Are } \underbrace{\begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}}_{\vec{v}_1}, \underbrace{\begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}}_{\vec{v}_2}, \underbrace{\begin{pmatrix} 5 \\ 2 \\ -6 \end{pmatrix}}_{\vec{v}_3} \text{ LD or LI?}$$

$$a_1 \vec{v}_1 + a_2 \vec{v}_2 + a_3 \vec{v}_3 = \vec{0}$$

\Leftrightarrow

$$\begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \end{bmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

\Leftrightarrow

$$\begin{pmatrix} -1 & 3 & 5 \\ 0 & -2 & 2 \\ 2 & 2 & -6 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

This Week: Subspaces & Span.

Subspace – subsets of a Vector Space which can behave as its own vector space.

Three Axioms:

- (1) – Contains the Zero Vector
- (2) - Closed under Scalar Multiplication
- (3) - Closed under Addition

In Euclidean Space, this means that subspaces are going to be points, lines, planes, hyperplanes, et. cetera through the origin.

Summary for the week

Linear Combination: Some scaling/addition of a set of vectors:

Span: The set of all possible Linear Combination's

Spanning Set of a (Vector) Space V: Set of vectors which spans the space V.

Basis (of V): Minimal Spanning Set (Vectors which spans V with redundant vectors removed).

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Linearly Independent

if: $a_1 \mathbf{v}_1 + a_2 \mathbf{v}_2 + \dots + a_k \mathbf{v}_k = \mathbf{0}$ When only $v_1, v_2, \dots, v_3 = 0$

Subspaces:

- (1) - Contains the Zero Vector
- (2) - Closed under Scalar Multiplication
- (3) - Closed under Addition

$$\text{Is } \underbrace{\begin{pmatrix} -1 \\ 4 \\ 11 \end{pmatrix}}_{\vec{b}} \in \text{Span} \left\{ \underbrace{\begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}}_{\vec{a}_1}, \underbrace{\begin{pmatrix} -3 \\ -5 \\ 13 \end{pmatrix}}_{\vec{a}_2}, \underbrace{\begin{pmatrix} 2 \\ -1 \\ -12 \end{pmatrix}}_{\vec{a}_3} \right\} ?$$

$$x_1 \vec{a}_1 + x_2 \vec{a}_2 + x_3 \vec{a}_3 = \vec{b}$$

$$\begin{pmatrix} x_1 - 3x_2 + 2x_3 \\ 2x_1 - 5x_2 - x_3 \\ -4x_1 + 13x_2 - 12x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ 11 \end{pmatrix} \Rightarrow \left(\begin{array}{ccc|c} 1 & -3 & 2 & -1 \\ 2 & -5 & -1 & 4 \\ -4 & 13 & -12 & 11 \end{array} \right)$$

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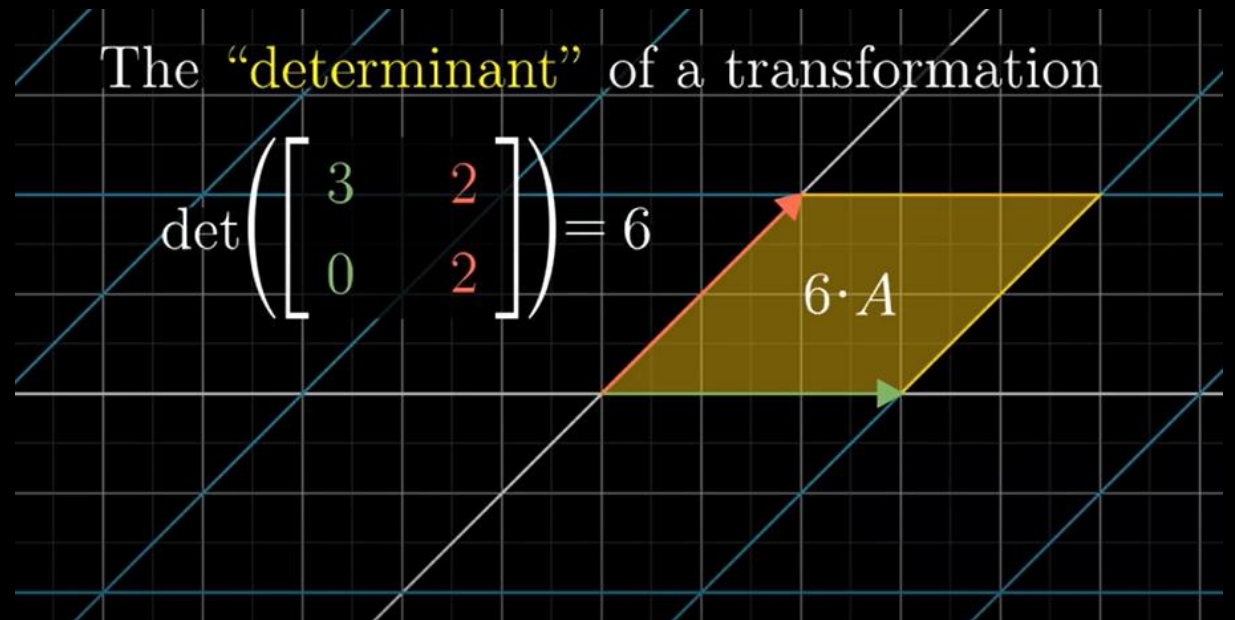
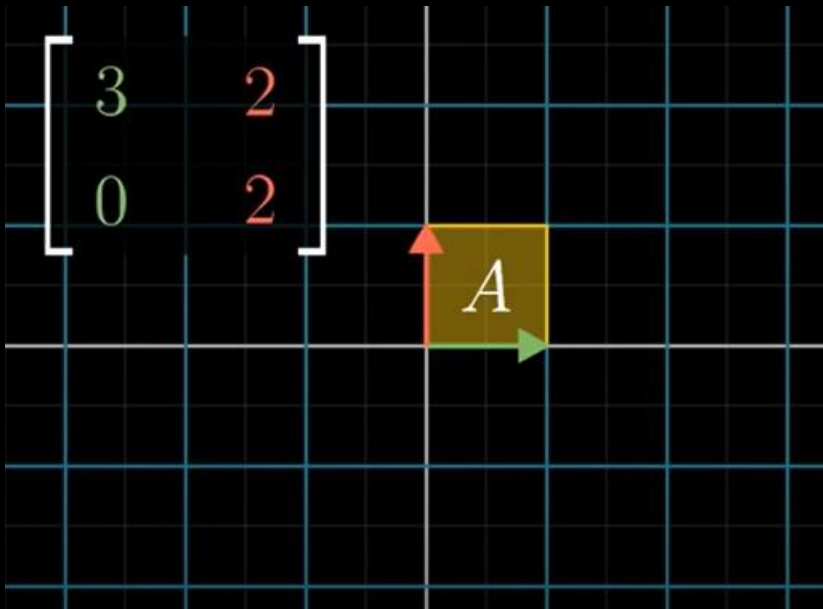
$$a_1 \vec{v}_1 + a_2 \vec{v}_2 + a_3 \vec{v}_3 = \vec{0}$$

$$\begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \end{bmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 3 & 5 \\ 0 & -2 & 2 \\ 2 & 2 & -6 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Determinant: ~Change in Area/Volume under a matrix/transformation.

Linearly Dependence: $\text{Det } A = 0$



Sources

Images:

3Blue1Brown/Grant Sanderson - Slide 2, 6

Trefor Bazett – Slide 3,5