# Week 11 Practical MATH1012

## Forms of Differential Equations

#### Separable:

$$\frac{dy}{dx} = f(x)g(y)$$

Other Examples:

$$y'(x) = x\sin(x)\sqrt{y}$$
$$y'(x) = \frac{x^2}{y^2}$$
$$y'(x) + y^2 = 0$$

e.g.

$$\frac{dy}{dx} = x^2y$$

$$\frac{1}{y}dy = x^2dx$$

$$\int \frac{1}{y}dy = \int x^2dx$$

$$\ln(y) = \frac{x^3}{3} + c \qquad y = e^c e^{\frac{x^3}{3}}$$

$$y = e^{\frac{x^3}{3} + c} \qquad y = Ae^{\frac{x^3}{3}}$$

### Forms of Differential Equations

#### **Linear Differential Equation:**

$$y' + p(x)y = f(x)$$

#### Method:

$$y = \frac{1}{r(x)} \int r(x)f(x)dx$$
$$r(x) = e^{\int p(x)dx}$$

#### Examples:

$$y' + 4y = e^{-x}$$

$$y' + \sin(x) y = 2x$$

$$xy' + (x^3 - 1)y = 2e^{-x}$$

$$y' + \sinh(x) y = e^{-x} + e^x$$

#### Linear – Brief Derivation

Standard Form for 1st Order Linear:

$$y' + p(x)y = f(x)$$

Integrating Factor Method:

$$r(x)y' + r(x)p(x)y = r(x)f(x)$$

$$\stackrel{?}{=}$$

$$\frac{d}{dx}[r(x)y]$$

$$= r(x)y' + r'(x)y$$

$$r'(x) = r(x)p(x)$$

$$\int \frac{r'(x)}{r(x)} dx = \int p(x)dx$$

$$\ln(r(x)) = \int p(x)dx$$

$$r(x) = e^{\int p(x)dx}$$

$$\frac{d}{dx}[r(x)y] = r(x)f(x)$$

$$y = \frac{1}{r(x)} \int r(x) f(x) dx$$

## Forms of Differential Equations

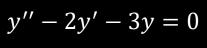
**Constant Coefficients (Homogenous)** 

$$ay'' + by' + cy = 0$$

Characteristic Equation:

$$ar^2 + br + c = 0$$

Real Roots

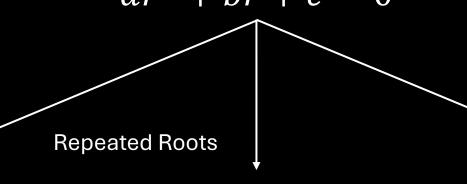


$$r^2 - 2r - 3 = 0$$

$$r = -1,3$$

$$y = c_1 e^{-t} + c_2 e^{3t}$$

 $y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$ 



$$y'' + 4y' + 4y = 0$$

$$r^{2} + 4r + 4 = 0$$

$$r = -2$$

$$y = c_{1}e^{-2t} + c_{2}te^{-2t}$$

$$y = c_1 e^{rt} + c_2 t e^{rt}$$

$$y'' - 2y' - 3y = 0$$

Guess 
$$y = e^{rt}$$

$$r^2e^{rt} - 2re^{rt} - 3e^{rt} = 0$$

$$r^2 - 2r - 3 = 0$$

$$r_1 = 3$$
,  $r_2 = -1$ 

$$y = c_1 e^{3t} + c_2 e^{-t}$$

#### **Complex Roots**

$$y'' + 3y' + 4y = 0$$
$$r^2 + 3r + 4 = 0$$

$$y = c_1 e^{\alpha t} \cos \beta t + c_2 e^{\alpha t} \sin \beta t$$

## Differential Equations (Additional Methods)

#### **Method of Undetermined Coefficients**

$$y'' - 2y' - 3y = 3e^{2t}$$

- 1. Solve homogeneous equation in general  $y_h$
- 2. Find one solution to full equation  $y_p$
- 3. Add solutions together  $y_h + y_p$

Non-homogeneity	Guess
$e^{rt}$	$Ae^{rt}$
$\sin(rt)$ or $\cos(rt)$	Asin(rt) + Bcos(rt)
Degree n polynomial	$A_0 + A_1 t + \dots + A_n t^n$

Guess 
$$y_p = Ae^{2t}$$

$$y'' - 2y' - 3y = 3e^{2t}$$

$$(Ate^{-t})'' - 2(Ate^{-t})' - 3(Ate^{-t}) = 3e^{-t}$$

$$(-2Ae^{-t} + Ate^{t}) - 2(Ae^{-t} - Ate^{-t}) - 3(Ate^{-t}) = 3e^{-t}$$

$$-4Ae^{-t} = 3e^{-t}$$

#### Things I've been asked to mention:

Physics, Mathematics, and Computer Science (PMC) - Teaching Awards are open.

Valuable for Vindication/Progression of your good lecturers and tutors (And un-vindication of bad ones)

Link:

https://forms.office.com/r/jS4c63mQcy

Open until 3<sup>rd</sup> of November

# One more Administrative Thing:

(Website)



## Summary

Separable:

$$\frac{dy}{dx} = f(x)g(y)$$

**Linear Differential Equation:** 

$$y' + p(x)y = f(x)$$

Method:

$$y = \frac{1}{r(x)} \int r(x) f(x) dx$$

$$r(x) = e^{\int p(x) dx}$$

**Constant Coefficients:** 

$$ay'' + by' + cy = 0$$

**Characteristic Equation:** 

$$ar^2 + br + c = 0$$

Real (Unique):

$$y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$$

Real Repeated:

$$y = c_1 e^{rt} + c_2 t e^{rt}$$

Complex:

$$y = c_1 e^{\alpha t} \cos \beta t + c_2 e^{\alpha t} \sin \beta t$$

Method of Undetermined Coefficients:

$$y'' - 2y' - 3y = 3e^{2t}$$

- 1. Solve homogeneous equation in general  $y_h$
- 2. Find one solution to full equation  $y_p$
- 3. Add solutions together  $y_h + y_p$

Non-homogeneity	Guess
$e^{rt}$	Ae <sup>rt</sup>
$\sin(rt)$ or $\cos(rt)$	Asin(rt) + Bcos(rt)
Degree n polynomial	$A_0 + A_1 t + \dots + A_n t^n$