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Ariadne: analysis for machine learning programs

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Ariadne: Analysis for Machine Learning Programs

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Abstract

Machine learning has transformed domains like vision and translation, and is now increasingly used in science, where the correctness of such code is vital. Python is popular for machine learning, in part because of its wealth of machine learning libraries, and is felt to make development faster; however, this dynamic language has less support for error detection at code creation time than tools like Eclipse. This is especially problematic for machine learning: given its statistical nature, code with subtle errors may run and produce results that look plausible but are meaningless. This can vitiate scientific results. We report on *Ariadne*: applying a static framework, WALA, to machine learning code that uses TensorFlow. We have created static analysis for Python, a type system for tracking tensors—Tensorflow’s core data structures—and a data flow analysis to track their usage. We report on how it was built and present some early results.

CCS Concepts • Theory of computation → Program analysis; • Computing methodologies → Machine learning approaches;

Keywords machine learning, program analysis

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1 Introduction

From translation[16] to vision[24], machine learning has transformed many domains, and is increasingly used in science. For example, in clinical and cognitive neuroscience, classifiers have been used as key evidence in identifying disease[34, 37], and to further scientific understanding of cognitive[39] and mental states [23] from complex patterns of brain activation. It is vital that underlying code be correct.

In most programming domains, preventing and catching programming errors is aided by supportive tooling. Tools also aid programming by suggesting appropriate completions while writing code, and refactoring code[1] to improve design and reuse. These tools usually are part of IDEs, of which Eclipse and IntelliJ are well known for Java.

Machine learning code is commonly written in Python, in part because of its wealth of machine learning libraries, but also reflecting a more-general trend toward dynamic programming languages. For example, witness the dominance of JavaScript in Web development[14]. While these languages are often felt to make development faster, the price is having less error detection at code creation time.

This is especially problematic for machine learning: due to its statistical nature, code with subtle errors may run and produce results that look plausible but are meaningless. Meaningless results can occur when logical errors cause code to find spurious correlations that arise in big data. In science, this could appear to support erroneous hypotheses.

Supportive tooling could ameliorate such problems, but it is less sophisticated for dynamic languages, in large part because such tooling is usually driven by static types that these languages lack. However, without static types, information can still be gleaned using static analysis; this has been done in tools like IBM’s AppScan[4] security analysis for JavaScript.

In this paper, we apply static analysis to machine learning code that uses Tensorflow[2]. We extend WALA to support Python, build a type system for tracking tensors—one of machine learning’s core data structures—and an analysis of tensor usage. Our contributions are the following:

1. Application of WALA program analysis to Python
2. Type system capturing semantic properties of tensors
3. An open-source implementation

We start with a running example in Section 2, and then discuss how we analyze the code in Section 3. We define our type system for tensors in Section 4, and present how it is used to analyze an example of tensor usage in Section 5. We then survey related work and conclude.

2 Example

Figure 1 [22] is an example that uses well-known MNIST [21] data to train image recognition. It reads training data on line 3. To illustrate the kind of checking we want, we focus on line 22, which is a reshape operation, along with a comment describing it. The comment is detailed, describing aspects of the original MNIST data and the input data format expected by the subsequent conv2d operation. The reshape operation transforms a tensor, using a new shape to access the same underlying data. A reshape can transform dimensions, but the total size must be the same since it is accessing the same underlying data. Thus, this piece of code depends on the structure of the MNIST data, which is only documented in comments; hence, there is nothing in the program to help the programmer get it right.

Our hypothesis is that operations such as this one could be checked for correctness, and perhaps even generated automatically, using type system and program analysis technology and minimal declarations by the programmer to specify input formats. This would check correctness in terms of both dimensions and types of data. If the types of MNIST and its dimensions were declared when the data is read, this information could be used to verify the reshape.

Assume that MNIST, at line 3, were declared to be of a type indicating the semantics of its dimensions,

*[batch, x(28) * y(28) of channel]*

in our syntax; note that *batch*, *x* and *y* have no particular meaning, but are simply names meaningful to the user for describing the tensor dimensions. The declaration of the second dimension specifies it to be composed from two logical dimensions, *x* and *y*. Further assume that type inference could track that declaration to line 18, and we had a declaration for what conv2d expects. In that case, we would be able to observe that the reshape does the right thing, even observing that $28^2 \cdot 1 = 784$, so those dimensions evenly match the two logical dimensions of the second dimension in MNIST. We would end up with a type like

[batch, x(28), y(28), 1 of channel]

where the *x*, *y* and *channel* come from the conv2d operation.

3 WALA

Existing program analysis frameworks (e.g. Doop[7], Soot[38], Safe[18], WALA[40]) provide the whole-program analysis that would enable analyzing examples like in Section 2; they can build call graphs describing how functions interact and analysis of how objects behave. But none of these, to our

knowledge, support Python. Fortunately, WALA does have a front end designed to make adding new languages as easy as possible. WALA supports Java and JavaScript, and has been shown to scale on substantial Java enterprise [11] applications. The Java applications also demonstrated thorough-going support for modeling frameworks[35], vital for the many frameworks employed in Java enterprise code. The JavaScript applications showed that WALA can be applied to dynamic languages and get results on security analysis[13]. After highlighting the challenges faced by analysis, we describe how WALA allows us to model Python.

3.1 Challenges

In general, machine learning code uses all the normal features of programming languages, such as classes, methods, heap state; hence, program analysis must handle them. This requires the kind of program analysis that has been used on a wide range of programming languages. Three necessary aspects are the following, illustrated by Figure 1:

Call graph construction is required even to understand calls like the one on line 46 to the function *conv_net* at line 14. This is a direct call that could be handled easily, but there are more-complex cases that require higher-order functions. The call to *model_fn* at line 42 is indirect from line 77, necessitating analysis of first-class functions; the function is stored by the constructor at line 71 and that callback is ultimately invoked by the call at line 77.

Pointer analysis is needed to know that the read of the *images* property at line 18 is reading what is written by the store of *images* at line 74; it requires pointer analysis of some kind to understand that the objects at those two points might be aliased. Unlike Java, there are not even any types to use for a coarse approximation, and the same goes for functions, as well. The function called indirectly at line 77 is written at line 71, and the connection is through state in the heap.

Library modeling is what makes the connection between the call at line 71 and the corresponding call to that function at line 77. There is no such connection in the source code being analyzed, and the tensorflow source is not typically available when analyzing a program, as is common with other frameworks, especially in the enterprise space. Analysis has used models for frameworks such as J2EE and Apache Struts, and this rich modeling machinery can be reused to model how Tensorflow behaves with respect to an application.

The pieces are all connected, with pointer analysis being needed to understand function pointers, which is in turn needed to include them in the call graph, which in turn adds new code with pointers to analyze. It all is supported by rich models of the library. Fortunately, these challenges are similar to what analysis frameworks have long had to do for

```

1 # Import MNIST data
2 from tensorflow.examples.tutorials.mnist import input_data
3 mnist = input_data.read_data_sets("/tmp/data/", one_hot=False)
4 import tensorflow as tf
5 # Training Parameters
6 learning_rate = 0.001
7 num_steps = 2000
8 batch_size = 128
9 # Network Parameters
10 num_input = 784 # MNIST data input (img shape: 28*28)
11 num_classes = 10 # MNIST total classes (0-9 digits)
12 dropout = 0.75 # Dropout, probability to keep units
13 # Create the neural network
14 ## (x_dict : {images : [batch, y(28)*x(28) of channel]}) → [batch, n_classes of channel]
15 def conv_net(x_dict, n_classes, dropout, reuse, is_training):
16     # Define a scope for reusing the variables
17     with tf.variable_scope('ConvNet', reuse=reuse):
18         # TF Estimator input is a dict, in case of multiple inputs
19         x = x_dict['images']
20         # MNIST data input is a 1-D vector of 784 features (28*28 pixels)
21         # Reshape to match picture format [Height x Width x Channel]
22         # Tensor input become 4-D: [Batch Size, Height, Width, Channel]
23         x = tf.reshape(x, shape=[-1, 28, 28, 1])
24         # Convolution Layer with 32 filters and a kernel size of 5
25         conv1 = tf.layers.conv2d(x, 32, 5, activation=tf.nn.relu)
26         # Max Pooling (down-sampling) with strides of 2 and kernel size of 2
27         conv1 = tf.layers.max_pooling2d(conv1, 2, 2)
28         # Convolution Layer with 64 filters and a kernel size of 3
29         conv2 = tf.layers.conv2d(conv1, 64, 3, activation=tf.nn.relu)
30         # Max Pooling (down-sampling) with strides of 2 and kernel size of 2
31         conv2 = tf.layers.max_pooling2d(conv2, 2, 2)
32         # Flatten the data to a 1-D vector for the fully connected layer
33         fc1 = tf.contrib.layers.flatten(conv2)
34         # Fully connected layer (in tf contrib folder for now)
35         fc1 = tf.layers.dense(fc1, 1024)
36         # Apply Dropout (if is_training is False, dropout is not applied)
37         fc1 = tf.layers.dropout(fc1, rate=dropout, training=is_training)
38         # Output layer, class prediction
39         out = tf.layers.dense(fc1, n_classes)
40     return out
41
42 # Define the model function (following TF Estimator Template)
43 def model_fn(features, labels, mode): ## (features:{images:[batch, y(28)*x(28) of channel]},labels:[10 of label])→T
44     # Build the neural network
45     # Because Dropout have different behavior at training and prediction time, we
46     # need to create 2 distinct computation graphs that still share the same weights.
47     logits_train = conv_net(features, num_classes, dropout, reuse=False, is_training=True)           ## [batch, 10 of channel]
48     logits_test = conv_net(features, num_classes, dropout, reuse=True, is_training=False)             ## [batch, 10 of channel]
49     # Predictions
50     pred_classes = tf.argmax(logits_test, axis=1)
51     pred_probas = tf.nn.softmax(logits_test)
52     # If prediction mode, early return
53     if mode == tf.estimator.ModeKeys.PREDICT:
54         return tf.estimator.EstimatorSpec(mode, predictions=pred_classes)
55     # Define loss and optimizer
56     loss_op = tf.reduce_mean(tf.nn.sparse_softmax_cross_entropy_with_logits(logits=logits_train, labels=tf.cast(labels, dtype=tf.int32)))
57     optimizer = tf.train.AdamOptimizer(learning_rate=learning_rate)
58     train_op = optimizer.minimize(loss_op, global_step=tf.train.get_global_step())
59     # Evaluate the accuracy of the model
60     acc_op = tf.metrics.accuracy(labels=labels, predictions=pred_classes)
61     # TF Estimators requires to return a EstimatorSpec, that specify
62     # the different ops for training, evaluating, ...
63     estim_specs = tf.estimator.EstimatorSpec(
64         mode=mode,
65         predictions=pred_classes,
66         loss=loss_op,
67         train_op=train_op,
68         eval_metric_ops={'accuracy': acc_op})
69     return estim_specs
70
71 # Build the Estimator
72 model = tf.estimator.Estimator(model_fn)
73 # Define the input function for training
74 input_fn = tf.estimator.inputs.numpy_input_fn(
75     x={'images': mnist.train.images}, y=mnist.train.labels,
76     batch_size=batch_size, num_epochs=None, shuffle=True)
77 # Train the Model
78 model.train(input_fn, steps=num_steps)

```

Figure 1. Example using MNIST data

other languages, especially Java and JavaScript, and we are able to leverage that technology.

3.2 Front End

WALA provides a framework for handling new languages, the Common Abstract Syntax Tree (CAst) system. The supported way to handle new languages is to translate a normal language AST into CAst, and use built in support for translating CAst into WALA's Internal Representation (IR) for analysis; this IR is a fairly traditional three-address code in Static Single Assignment (SSA) form. We use the existing Jython functionality to create Python ASTs for translation to CAst. WALA translation support takes three major forms; see the classes (<https://github.com/wala/WALA>) and JavaDoc (<https://github.io/wala/javadoc>) of WALA for full details. We focus here on aspects most relevant to our translation; we illustrate with concrete examples showing how Python is translated using methods from a visitor class over the Python AST, provided by Jython. The full translation code is at <https://github.com/wala/ML>.

- Some constructs are standard enough across languages that CAst provides a construct with translation to IR; e.g., if, while, goto (including break and continue). For such constructs, simply creating the appropriate CAst construct suffices. Figure 2 shows translation of an if statement; many forms such as assignments, arithmetic operations and constants are similar.
- Some common constructs differ significantly in details; e.g. field accesses, where notion of what constitutes a field name vary. Java has fixed constants, whereas dynamic languages like JavaScript and Python allow variables to name fields as well. For such constructs, WALA provides CAst constructs with hooks in the IR translator to specialize it. We illustrate the two pieces with CAst construction in Figure 3. and then the hook in IR translation from CAst in Figure 4.
- Idiosyncratic constructs are easiest to model in a custom way. For this, WALA provides the ability to add new CAst node types and also the generic primitive CAst node type with custom translation to IR. For example, we currently model the import construct of Python as a static function call. We illustrate with CAst generation of import as primitive in Figure 5, and show IR translation in Figure 6.

WALA uses this machinery for multiple languages, JavaScript from Rhino and Java source from Eclipse. It has been public for several years now, used by researchers for a range of work, such as hybrid app analysis[19], correlation tracking[36], a special form of context sensitivity, tunable analysis techniques [17], and approximate analysis to handle real-world JavaScript code[10]. It has been robust and scalable enough to support products, and the Python analysis for machine learning leverages that technology.

3.3 IR extensions

Beyond customizing IR generation from the source code, WALA also provides ways to create synthetic IR to represent aspects of the code not directly present in the source. This machinery involves writing explicit models of functions as IR using something analogous to an assembler for IR. There are two way of constructing such models that we employ in this work, both of which have been used extensively in the past to model Java and JavaScript.

Programmatic Creating IR with analysis code is the most flexible way to create IR, and WALA provide many ways to integrate such IR into analysis. Python has relatively straightforward semantics in many cases, and needs relatively little such modeling; however, the precise semantics of method calls are a little tricky due to how self is handled. In effect, all normal method calls in Python act like calls to a normal function closed over the self object. Consider the following snippet of code:

```

1 class Foo(object):
2     def foo(self, a):
3         return self.contents+a
4
5 x = Foo()
6 y = x.foo
7
8 x.foo(3)
9 Foo.foo(x, 3)
10 y(3)
11
12 x.foo = Foo.foo
13 x.foo(x, 3)

```

The calls at lines 8, 9, 10 and 13 are all equivalent, but the latter are unusual in object-oriented languages. When the method foo is read from the class Foo, the result is an unbound method that needs to be called with an explicit self parameter, but when it is read from the object x, the result is a method bound to that object. That is like a closure, since the call on line 10 carries the self object along with the method. And finally observe that methods are writable just like normal fields, resulting in the call at line 13.

We model this with synthetic IR that, for every class, creates a constructor method that initializes each method field with a trampoline that captures its self object to pass as part of a call to the real method from the trampoline. This has the effect of making calls like line 8 and 10 work and allows the field to be mutated as required.

XML We choose to model popular machine learning frameworks in order to explicitly capture their semantic properties, using an XML representation of the IR. We focus in Figures 7 and 8 on aspects of the popular Tensorflow framework from our example. Figure 7 shows the model of importing Tensorflow, which creates the key objects in the Tensorflow model.

```

1 public CAstNode visitIf(If arg0) throws Exception {
2     return Ast.makeNode(CAstNode.IF_STMT,
3         arg0.getInternalTest().accept(this),
4         block(arg0.getInternalBody()),
5         block(arg0.getInternalOrElse()));
6 }

```

Figure 2. CAst generation for ordinary constructs. Ast is a factory for CAst nodes, and CAstNode holds constants for different kinds of nodes. A Python if statement, arg0, is being translated, with accessors for its test (getInternalTest), then block (getInternalBody) and else block (getInternalOrElse). The accept method invokes the visitor recursively on children; the block method is a helper that does the same for multiple statements.

```

1 public CAstNode visitSubscript(Subscript arg0) throws Exception {
2     return notePosition(Ast.makeNode(CAstNode.OBJECT_REF,
3         notePosition(arg0.getInternalValue().accept(this), arg0.getInternalValue()),
4         notePosition(arg0.getInternalSlice().accept(this), arg0.getInternalSlice()), arg0));
5 }

```

Figure 3. CAst generation for customizable constructs. Ast is a factory for CAst nodes, and CAstNode holds constants for different kinds of nodes. A Python field access, arg0, is being translated, with accessors for the object (getInternalValue) and field (getInternalSlice). The access could be a read or write. notePosition records source locations.

```

1 protected void doFieldRead(WalkContext context, int result, int receiver, CAstNode elt, CAstNode parent) {
2     if (elt.getKind() == CAstNode.CONSTANT && elt.getValue() instanceof String) {
3         FieldReference f = FieldReference.findOrCreate(PythonTypes.Root, Atom.findOrCreateUnicodeAtom((String)elt.getValue()), PythonTypes.Root);
4         context.cfg().addInstruction(Python.instructionFactory().GetInstruction(context.cfg().getCurrentInstruction(), result, receiver, f));
5     } else ...
6 }

```

Figure 4. IR generation for customizable constructs. IR generation for a field read, showing the case where the field name is a constant. result is the value number for the result, receiver denotes the object, elt is the CAst node of the field. getValue retrieves the constant value of a node; findOrCreate makes an IR field reference for the field. Line 4 creates a field get instruction and adds it to the IR.

```

1 public CAstNode visitImport(Import arg0) throws Exception {
2     int i = 0;
3     CAstNode[] elts = new CAstNode[ arg0.getInternalNames().size()*2 ];
4     for(alias n : arg0.getInternalNames()) {
5         elts[i++] = Ast.makeNode(CAstNode.DECL_STMT,
6             Ast.makeConstant(new CastSymbolImpl(name(n), PythonCastToIRTranslator.Any)));
7         elts[i++] = Ast.makeNode(CAstNode.ASSIGN,
8             Ast.makeNode(CAstNode.VAR, Ast.makeConstant(name(n))),
9             Ast.makeNode(CAstNode.PRIMITIVE, Ast.makeConstant("import"), Ast.makeConstant(n.getInternalName().replaceAll("[.]", "/"))));
10    }
11    return Ast.makeNode(CAstNode.BLOCK_STMT, elts);
12 }

```

Figure 5. CAst generation for idiosyncratic constructs. Ast is a factory for CAst nodes, and CAstNode holds constants for different kinds of nodes. The Jython AST has multiple imports as one node, so this code loops over them, making each import the result of a primitive import, with the first argument being the module name. The DECL_STMT create a new variable, and the ASSIGN sets its value. These statements are grouped together into a block with a BLOCK_STMT.

```

1 protected void doPrimitive(int resultVal, WalkContext context, CAstNode primitiveCall) {
2     if (primitiveCall.getChildCount() == 2 && "import".equals(primitiveCall.getChild(0).getValue())) {
3         TypeReference imprt = TypeReference.findOrCreate(PythonTypes.pythonLoader, "L" + primitiveCall.getChild(1).getValue());
4         MethodReference call = MethodReference.findOrCreate(imprt, "import", "()L" + primitiveCall.getChild(1).getValue());
5         int idx = context.cfg().getCurrentInstruction();
6         context.cfg().addInstruction(
7             Python.instructionFactory().InvokeInstruction(
8                 idx, resultVal, new int[0], context.currentScope().allocateTempValue(), CallSiteReference.make(idx, call, Dispatch.STATIC), null));
9     }
10 }

```

Figure 6. IR generation for idiosyncratic constructs. IR translation of imports as function calls. TypeReference and MethodReference create a reference to method corresponding to a module (only built-in modules are handled for now). Lines 6 to 8 create a call to that function and insert it into the IR.

Figure 8 shows models of functions of the Tensorflow objects. For example, on line 71, the estimator object is created, which we model in two pieces: The Estimator object is allocated at line 14 of Figure 7 and the call is modeled by the IR at line 3 of Figure 8. The reshape call at line 22 of Figure 1 is analogous, with the reshape object defined at 37 in Figure 7.

4 Types

Definition 4.1 (Types for ML).

$$\begin{aligned} \text{(Python Type)} \quad \pi &::= \{f_1 : \pi_1, \dots, f_n : \pi_n\} \\ &\quad | \quad (f_1 : \pi_1, \dots, f_n : \pi_n) \rightarrow \pi \\ &\quad | \quad \tau \mid l \mid \top \\ \text{(Tensor Type)} \quad \tau &::= [d_1, \dots, d_n \text{ of } \pi] \\ \text{(Dimension)} \quad d &::= l \mid n \mid l(n) \mid d * d \end{aligned}$$

where f are record and parameter names (strings), l are labels (strings), and n are non-negative integers.

We ignore the ordering of fields in a record and consider the $*$ operator to be associative.

Defintion 4.1 presents a simple type system for tracking tensors in a Python program. Python types, π , track values having tensor type. Additionally, since records may contain tensors, and functions may consume and return tensors, these are also modelled. These types can be easily extended to handle other structures in python (e.g. lists). We are deliberately simplifying these features to emphasize the contribution of this paper, tensor types.

To fucs on tensors, it also includes the top type, \top , which to indicate irrelevant types. It also allows labels to identify the category/type of the actual data. By convention, function and record arguments of type \top are omitted.

Tensor types τ , track the contents and dimensions of a tensor/matrix/vector. The second part of a tensor type describes the data that it contains. The first part is a list of the dimensions. This list notation denotes nested tensor types, each with a single dimension. In particular, these are equivalent:

$$[d_1, d_2, \dots, d_n \text{ of } \pi] = [d_1 \text{ of } [d_2, \dots, d_n \text{ of } \pi]]$$

4.1 Example Walkthrough

We illustrate the types using Figure 1; comments (starting with `##`) specify type information for function definitions and relevant variable assignments.

MNIST data is read on line 3, using `read_data_sets`; the returned data is modelled with type:

```
{train : {images : [batch, y(28) * x(28) of channel]},  
 test : {images : [batch, y(28) * x(28) of channel]}}
```

It has (at least) two fields, `train` and `test`, each of which contain a nested structure with an `images` field that contains a tensor. This tensor has two dimensions: the outer dimension denotes images, and the inner dimension is a `channel` (number) for each pixel in the image. The inner dimension has size 784, which represents a flattened 28x28 matrix of

channels. The dimension type is uses a combined dimension, $28 * 28$, to capture this; using labels, it captures their order: the first part of this dimension represents the height and the second the width. The type $y(28) * x(28)$ preserves this.

Around Line 74, the mnist training data, which has the type

`{images : [batch, y(28) * x(28) of channel]}`

and the training labels, with type `[[10 of label]]` are used to create the input for the estimator.

These arguments will be passed into the estimator model, which, as shown on Line 71, is `model_fn` method defined on Line 42. This call is validated by our system, as the types of the arguments agree with their expected types, shown in the comments. The first thing this method does is call the `conv_net` method defined on Line 14. Again, it is easy to see that the types of the data passed in to the method are compatible with the parameters' expected types.

Finally, we the `conv_net` method, which creates the neural network. On Line 18, the method first gets the tensor with the actual image data, of type `[batch, y(28) * x(28) of channel]`. It then preforms a reshape operation, turning it into a 4-dimensional vector of channels, with type

`[batch, y(28), x(28), 1 of channel]`

Note how the (common) free-form comments about the input and output dimensionality are captured by the types.

This reshape operation highlights the advantage of keeping structured combined dimensional information. In the serialized mnist data, each image is represented by a flattened vector of 784. However, logically, it is a 28x28 matrix of channels. That information allows checking that requested reshape is reasonable. Had the code tried to reshape the data into a 56x14 matrix,that is not a valid factorization.

Next, on Line 24, the reshaped tensor is passed into the `conv2d` method. This method requires that the input tensor has four dimensions, a constraint that our type for the input easily be seen to satisfy. It also requires that the contents of the dimension be a number. In our example, the `channel` label does indeed represent a numeric value. Finally, it requires that the middle two dimensions represent `height` and `width` (or `y` and `x`, or similar). This can be checked by observing the labels of the dimensions of the input tensor.

Checking the meaning of a dimension, provides a new way to find common bugs. In particular, it would be easy to accidentally swap the `y` and `x` dimensions in our example. To prevent this kind of bug, multiple machine learning experts have independently stumbled upon a common technique: ensuring that dimensions are all of different sizes. In our example, this could be done by artificially changing the image size to, e.g. 28x29. The type system presented here allows this information to instead be captured symbolically in the type system.

Continuing on in the `conv_net` method, Line 26 down-samples the given tensor, resulting in a tensor of type `[batch, y(14), x(14), 1 of channel]`. Line 30 down-samples it again, and Line 32 flattens the resulting tensor, yielding a tensor with type `[batch, y(7) * x(7) of channel]`.

5 Analyzing Tensors

WALA produces a dataflow graph as part of pointer analysis and call graph construction. The dataflow graph summarizes the flow of objects and values in the program; this graph is an abstraction of possible program behavior, and is defined as follows:

Definition 5.1 (Dataflow Graph). A *dataflow graph* $\mathfrak{G} = \langle V, S, \prec \rangle$ where V is the set of program variables, $S(v)$ is the set of objects or values possibly held by $v \in V$ and $x \prec y$ iff there is potential dataflow from $y \in V$ to $x \in V$, e.g. through an assignment or function call.

Given a dataflow graph \mathfrak{G} , we defined a *tensor estimate* $T(v)$ as the set of possible tensor types that tensors held by v may have. We use \mathfrak{I} to denote the declared tensor type of input; `reshape` takes a tensor and object representing a tensor type (a list of numbers in Python) and produces a reshaped tensor if the input tensor can be reshaped to the given size, denoted \doteq . This is implemented directly using WALA dataflow analysis, and is defined as follows:

Definition 5.2 (Tensor Estimate). Given a dataflow graph \mathfrak{G} , a *tensor estimate* $\mathfrak{T}(\mathfrak{G}) = \langle T \rangle$ defines the set of tensor types a variable may take on. This is defined as either the given input type, dataflow in the program or the result of a `reshape` or other Tensorflow API:

$$T(y) \subseteq \begin{cases} \{\mathfrak{I}\} & y \text{ is input} \\ T(x) & y \prec x \\ z & y \prec \text{reshape}(x, z) \wedge \exists_{z_i \in S(z)} T(x) \doteq z_i \\ \dots & y \prec \text{other Tensorflow APIs} \end{cases}$$

6 Results

We evaluated our prototype implementation by analyzing 5 Tensorflow examples based on MNIST and 1 Tensorflow-based code that analyzes neural imaging data. The codes are chosen because tutorials based on MNIST are common, and neuroscience is an area of focus for this project. Use of Tensorflow to analyze neural imaging data is an emerging area, and we know of few public codes that we can test. Our model of Tensorflow is not complete at this point, and so we focus on a subset of its API to illustrate our analysis.

`reshape(tensor, shape)` is part of the API illustrated in Figure 1; it reshapes a tensor to the dimensions specified in its second argument, which is a Python list. Our analysis checks that the tensor argument is the right size to be transformed into the desired shape. `conv2d(tensor, ...)` is part of the API illustrated in Figure 1; it performs a 2D convolution on images, and

it requires the images to be in a format of height, width, layer, where layers represent different colors. Our analysis checks that the argument has that structure.

`conv3d(tensor, ...)` is analogous to `conv2d`, but for 3D images. Since neural images are fMRI scans, they are 3D images of the brain, and hence `conv3d` is used. `placeholder(shape)` represents an unspecified tensor. It is used for operations that are later applied to data; for instance, it can be used to create a neural network which is then trained by feeding multiple dataset batches into the network. This is called `place` in Table 1

The six programs on which we evaluate are the following:

conv_network¹ builds and trains a convolutional neural network to classify MNIST data. Reshaping and convolving the images is part of the network.

mnist_deep² is an MNIST classifier using convolutional layers from the Tensorflow github site.

mnist_max³ is a simple MNIST classifier from the Tensorflow github site. It does not reshape, but it uses placeholders to allow training in batches.

mnist_max_xla⁴ is a simple MNIST classifier from the Tensorflow github site. It has options for optimization, but analysis is similar to `mnist_softmax`.

mnist_sum⁵ is a simple MNIST classifier from the Tensorflow github site. It reshapes the input images to generate a viewable summary.

neuroimage⁶ classifies 3D images of brain data. It resembles MNIST as image classification, but the images are 3D and so use different Tensorflow APIs.

Although our implementation is still preliminary, it is sufficient to parse these programs and to analyze them, at least partially. Table 1 shows the APIs that were analyzed in each program; for each API, our analysis was able to verify that these programs used them in correct ways. Given the preliminary nature of our implementation so far, we make no claims about false negatives; that is, there may be uses of the these constructs that are not analyzed fully in addition to the ones we did analyze. The ✓ notation in Table 1 denotes constructs that we found in the codes and were able to verify were used properly in at least some cases; our analysis so far found no false positives in these programs. The ✗ denotes that we verified that the program does not contain

¹https://github.com/aymericdamien/TensorFlow-Examples/blob/master/examples/3_NeuralNetworks/convolutional_network.py

²https://github.com/tensorflow/tensorflow/blob/master/tensorflow/examples/tutorials/mnist/mnist_deep.py

³https://github.com/tensorflow/tensorflow/blob/master/tensorflow/examples/tutorials/mnist/mnist_softmax.py

⁴https://github.com/tensorflow/tensorflow/blob/master/tensorflow/examples/tutorials/mnist/mnist_softmax_xla.py

⁵https://github.com/tensorflow/tensorflow/blob/master/tensorflow/examples/tutorials/mnist/mnist_with_summaries.py

⁶<https://raw.githubusercontent.com/corticometrics/neuroimage-tensorflow/master/train.py>

the construct; thus, since there are no empty spots in Table 1, any false negatives can only be from incomplete analyses of constructs that we found. Furthermore, the analysis of each program took just a few seconds on a normal laptop. All of these results were generated by our code on Github⁷ and the analysis can be seen in our tests on Travis CI⁸.

program	reshape	conv2d	conv3d	place
conv_network	✓	✓	✗	✗
mnist_deep	✓	✓	✗	✓
mnist_max	✗	✗	✗	✓
mnist_max_xla	✗	✗	✗	✓
mnist_sum	✓	✗	✗	✓
neuroimage	✓	✗	✓	✗

Table 1. Tensorflow APIs analyzed

7 Related Work

There has been some work in program analysis for Python, and we'll discuss the most-related tools and summarize the rest of the work.

Python Taint [32] is a static analysis tool for detecting security vulnerabilities in Python. It uses standard dataflow techniques, and can do some interprocedural analysis. However, its interprocedural analysis is limited: it looks for a definition of a function for a call using its name, rather than handling function pointers and object semantics as required by the language and needed for even our simple example. These sorts of features depend on the sort of rich analysis infrastructure provided by WALA.

The rest of the work breaks down into two broad classes, code quality checkers and dynamic analysis. Code quality checkers are static analyzers for code quality metrics or lint tools; examples are Pylint[30], pycodestyle[27], pyflakes[29], Flake8[12], QuantifiedCode CE[33], pydocstyle[28]. There are so many static analysis tools for code quality checking, that there are sites like Awesome Static Analysis[5] which keep up to date with the latest developments in the tools, for Python and for many other languages. Many of these tools are conveniently packaged in a wrapper like Prospector[25], which combine several tools. Prospector gives the ability to turn particular tools, features, or behaviors on and off as needed, and starts with easy-to-use defaults. Others include AST-based tools like jedi[15], bandit[6] and mccabe[20]. These tools are all local analyses, for instance, mccabe focuses on the syntactic code complexity of single functions and many focus on code style issues. These tools are not for whole-program static analysis based on dataflow, as WALA is.

⁷<https://github.com/wala/ML>

⁸<https://travis-ci.org/wala/ML>

Dynamic analyses do sometimes build call graphs, of which a notable example is simply called “Python CallGraph”[31]. Another dynamic analysis tool, PyChecker[26], does things one would expect a compiler to do for many other languages. For example, it ensures variables are set before they are used, it checks that class methods are used properly, and ensures that the number of arguments passed into a function is correct. These tools are very different from WALA, as our goal is to statically approximate all possible runs, rather than dynamic tools that focus on one or a few executions. There is a library-based taint analysis tool for Python [9]; unlike many other dynamic analysis tools, it is implemented purely in Python, meaning that it can be used without any modifications to the interpreter, and adapted easily.

There has been other work in types for tensors [8] from the SCALA'17 workshop. This work uses heterogeneous lists to represent the semantics of dimensions of tensors; however, this work is in Scala and so it is not integrated as ours is to to e.g. check reshape operations in Python.

There is also a lot of interesting work in the area of using machine learning to do analysis of code [3]. However, this work remains technically distinct, in that it uses machine learning to do analysis rather than applying code analysis to aid machine learning.

8 Conclusions

We have presented our application of WALA to analyzing the behavior of tensors in Tensorflow programs, using types to track their shapes. This involved adapting WALA to Python, which we believe is the first application of traditional whole-program analysis technology to Python.

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```

1 <class name="tensorflow" allocatable="true">
2   <method name="import"
3     static="true"
4     descriptor="()Ltensorflow;"
5     <new def="x" class="Ltensorflow"/>
6
7   <new def="estimator" class="LObject"/>
8   <putfield class="LRoot"
9     field="estimator"
10    fieldType="LRoot"
11    ref="x"
12    value="estimator"/>
13
14   <new def="Estimator"
15     class="Ltensorflow/estimator/Estimator"/>
16   <putfield class="LRoot"
17     field="Estimator"
18     fieldType="LRoot"
19     ref="estimator"
20     value="Estimator"/>
21
22   <new def="inputs" class="LObject"/>
23   <putfield class="LRoot"
24     field="inputs"
25     fieldType="LRoot"
26     ref="estimator"
27     value="inputs"/>
28
29   <new def="numpy_input_fn"
30     class="Ltensorflow/estimator/numpy_input_fn"/>
31   <putfield class="LRoot"
32     field="numpy_input_fn"
33     fieldType="LRoot"
34     ref="inputs"
35     value="numpy_input_fn"/>
36
37   <new def="reshape" class="Ltensorflow/functions/reshape"/>
38   <putfield class="LRoot"
39     field="reshape"
40     fieldType="LRoot"
41     ref="x"
42     value="reshape"/>
43
44   <return value="x"/>
45 </method>
46 </class>

```

Figure 7. Excerpt of TensorFlow model in WALA. This excerpt represents the initialization of the objects constituting the model of TensorFlow used by WALA. The TensorFlow object itself is created, its Estimator and reshape elements are created, and these are linked with fields that represent the TensorFlow API exposed to applications. The elements of the model are a direct representation of WALA IR in an XML format. The method import is invoked by the translation of import statements in Python. The new and putfield elements represent WALA IR object creations and field writes respectively. The def properties are names of local variables defined, and value and ref represent reads of those variables corresponding to values and objects of field writes.

```

1 <package name="tensorflow/estimator">
2   <class name="Estimator" allocatable="true">
3     <method name="do" descriptor="()LRoot;" numArgs="2">
4       <new def="x" class="Ltensorflow/estimator/train/train"/>
5       <putfield class="LRoot"
6         field="train"
7         fieldType="LRoot"
8         ref="arg0"
9         value="x"/>
10      <putfield class="LRoot"
11        field="callback"
12        fieldType="LRoot"
13        ref="x"
14        value="2"/>
15      <return value="arg0"/>
16    </method>
17  </class>
18
19 <class name="numpy_input_fn" allocatable="true">
20   <method name="do" descriptor="()LRoot;" numArgs="3">
21     <new def="x" class="LObject"/>
22     <putfield class="LRoot"
23       field="data"
24       fieldType="LRoot"
25       ref="x"
26       value="2"/>
27     <putfield class="LRoot"
28       field="labels"
29       fieldType="LRoot"
30       ref="x"
31       value="3"/>
32     <return value="x"/>
33   </method>
34 </class>
35 </package>
36
37 <package name="tensorflow/estimator/train">
38   <class name="train" allocatable="true">
39     <method name="do" descriptor="()LRoot;" numArgs="3">
40       <getfield class="LRoot"
41         field="$callback"
42         fieldType="LRoot"
43         ref="arg0"
44         def="x"/>
45       <call class="LRoot"
46         name="do"
47         descriptor="()LRoot;"
48         type="virtual"
49         arg0="x"
50         arg1="2"
51         arg2="3"
52         numArgs="3"
53         def="v"/>
54       <return value="v"/>
55     </method>
56   </class>
57 </package>

```

Figure 8. Excerpt of TensorFlow model in WALA. The functions of the TensorFlow. The method name and descriptor at line 3 is how Python functions are represented in WALA IR, so this portion of the model describes the functions of Estimator, numpy_input_fn and train. Estimator is called with 2 arguments, the Estimator object itself and a callback function; it allocates and stores the train object for the Estimator, and stores the callback function in it. The numpy_input_fn function stores the input data, which is its argument. The train function invokes the callback given to the Estimator.

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