First part

$$A = \begin{bmatrix} -1 & 23 & 10 \\ 0 & -2 & -11 \end{bmatrix}, \quad B = \begin{bmatrix} -6 & 2 & 10 \\ -3 & -3 & 4 \\ -5 & -11 & 9 \\ 1 & -1 & 9 \end{bmatrix}, \quad C = \begin{bmatrix} -3 & 2 & 9 & -5 & 7 \end{bmatrix}$$

$$D = \begin{bmatrix} -2 & 6 \\ -5 & 2 \end{bmatrix}, \quad E = \begin{bmatrix} 3 \end{bmatrix}, \quad F = \begin{bmatrix} 3 \\ 5 \\ -11 \\ 7 \end{bmatrix}, \quad G = \begin{bmatrix} -6 & -4 & 23 \\ -4 & -3 & 4 \\ 23 & 4 & 1 \end{bmatrix}$$

(a) Dimensions of the matrices:

$$\begin{aligned} &A : 2 \times 3, \quad B : 4 \times 3, \quad C : 1 \times 5, \\ &D : 2 \times 2, \quad E : 1 \times 1, \quad F : 4 \times 1, \quad G : 3 \times 3. \end{aligned}$$

(b) Square matrices: D, E, G

(c) Symmetric matrices: D, G

(d) The matrix with the entry at row 3 and column 2 equal to -11 is B.

(e) The matrix with the entry at row 1 and column 3 equal to 10 is A.

(f) Column matrices: F

(g) Row matrices: C

(h) Transposes:

$$A^T = \begin{bmatrix} -1 & 0 \\ 23 & -2 \\ 10 & -11 \end{bmatrix}$$

$$C^T = \begin{bmatrix} -3 \\ 2 \\ 9 \\ -5 \\ 7 \end{bmatrix}$$

$$E^T = \begin{bmatrix} 3 \end{bmatrix}$$

$$G^T = \begin{bmatrix} -6 & -4 & 23 \\ -4 & -3 & 4 \\ 23 & 4 & 1 \end{bmatrix} = G$$

Second part

$$A = \begin{bmatrix} -1 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 & 0 \\ 0 & -3 & 4 \\ -1 & -2 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} -3 & 2 & 9 & -5 & 7 \end{bmatrix}$$

$$D = \begin{bmatrix} -2 & 6 \\ -5 & 2 \end{bmatrix}$$

(a) **AB**: Possible if the number of columns of A matches the number of rows of B. A is 2×3 and B is 3×3, so AB is possible and results in a 2×3 matrix.

$$AB = \begin{bmatrix} 1+0+2 & -2-3+4 & 0+4-6 \\ 0+0-1 & 0+6-2 & 0-8+3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & -1 & -2 \\ -1 & 4 & -5 \end{bmatrix}$$

(b) **BC**: B is 3×3 and C is 1×5. Since the number of columns in B does not match the number of rows in C, BC is not possible.

(c) **AD**: A is 2×3 and D is 2×2. Since the number of columns in A does not match the number of rows in D, AD is not possible.

Third part

Determinant Of M (2×2)

$$M = \begin{bmatrix} 15 & 10 \\ 3 & 2 \end{bmatrix}$$

$$\det(M) = (15 \times 2) - (10 \times 3) = 30 - 30 = 0.$$

Determinant Of M (3×3)

$$M = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 2 & 3 \\ 3 & 2 & -1 \end{bmatrix}$$

$$\det(M) = 2 \begin{vmatrix} 2 & 3 \\ 2 & -1 \end{vmatrix} - 3 \begin{vmatrix} -1 & 3 \\ 3 & -1 \end{vmatrix} + 1 \begin{vmatrix} -1 & 2 \\ 3 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 3 \\ 2 & -1 \end{vmatrix} = (2 \times -1) - (3 \times 2) = -2 - 6 = -8$$

$$\begin{vmatrix} -1 & 3 \\ 3 & -1 \end{vmatrix} = (-1 \times -1) - (3 \times 3) = 1 - 9 = -8$$

$$\begin{vmatrix} -1 & 2 \\ 3 & 2 \end{vmatrix} = (-1 \times 2) - (2 \times 3) = -2 - 6 = -8$$

$$\det(M) = (2 \times -8) - (3 \times -8) + (1 \times -8) = -16 + 24 - 8 = 0.$$

Fourth part

$$A = \begin{bmatrix} -3 & -2 \\ 3 & 3 \end{bmatrix}$$

The inverse A^{-1} exists if $\det(A) \neq 0$:

$$\det(A) = (-3 \times 3) - (-2 \times 3) = -9 + 6 = -3 \neq 0$$

The inverse is given by:

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{-3} \begin{bmatrix} 3 & 2 \\ -3 & -3 \end{bmatrix} = \begin{bmatrix} -1 & -\frac{2}{3} \\ 1 & 1 \end{bmatrix}$$

Fifth part

Three equations are linearly independent if:

(b) There is no way to express one equation as a linear combination of the others.

(c) The graphical representations of the equations are lines that do not intersect.

Sixth part

I am too lazy to do it, but would it not be valid if we make all the matrices A, B, C, D of size (1×1)?