A JEE Notebook

Part 1: on Mathematics

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March 8, 2024

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Preface

§1 The Philosophy

First, before jumping to any content, I shall discuss the philosophy behind these notes. These notes are made because I did not like pen-paper notes, these have version control, can be easily rewritten are more accessible(on phone/pc and if needed, on print).

Another thing to note is that these notes follow a specific philosophy, in that they try to be expository. They are meant to explain things. I have noticed that books often talk circularly, create redundancies, and etc.

To combat this, I have tried to clearly lay out everything here. Also, because of how books are made, generally in various volumes, content often goes repeated. These notes will try to remove that.

Lastly, I have often added some sort of additional non-JEE relevant content in many places, because 1) JEE is not the only thing I care for, 2) I am particularly interested in Mathematics/Physics so I study extra stuff, 3) They often help in exposition and better understanding. These are, however, mentioned in the text, and you can easily skip them.

Happy learning!

CHAPTER 1

Sets and Manipulations

"You don't have to be great to start, but you have to start to be great."

Zig Ziglar

§1.1 Naive Set Theory

Motivation: There is little Mathematics that does not use the language of sets.

Sets are perhaps the most important structures in all of Mathematics. Although an indepth discussion of sets is beyond these notes, and their axiomation is complicated, we shall not let this affect us and discuss a less rigorous formalisation of sets. Namely, *Naive Set Theory*.

These definitions are superficious We have not and will not define what we mean by collection or elements. That is what makes this theory *naive*.

Definition 1.1.1

A set is any collection of elements.
It is denoted as $\{a, b, c\}$ where a, b, c are elements of the set.

Well, what do we mean by any collection? To give a few examples, the set of flowers would be {roses, lilies, sunflowers, ...} The trailing ...are used to show that the elements continue on *ad infinitum*. Hence, any collection of objects is a set.

Often, instead of manually writing out {roses, lilies, sunflowers, ...}, we will use a condition that each element in the set follows. Here, the condition is that all elements all flowers. We denoted this by saying that S is the set of all x such that x is a flower. We represent this in what is called *Set-Builder* notation. Thus,

$$S = \{x : P(x)\} \quad \text{or} \quad S = \{x \mid P(x)\}$$

Here, $P(x)$ represents the condition. We use the different symbols depending on the context, mostly for ensuring that the statements are clear.

Do not be overwhelmed by terms like "set builder notation". They're only formal names and not something to remember.

Remark 1.1.2. Notice the equality symbol between S and $\{x : P(x)\}$. Generally, we denote sets as capital latin letters $A, B, C, \ldots$, and their elements by little latin letters $a, b, c, \ldots$.

Example 1.1.3

The sets of all Natural numbers is denoted by $\mathbb{N}$. Generally, it is either used to represent $\{0, 1, 2, \ldots\}$ or $\{1, 2, 3, \ldots\}$. It matters little. We will let $\mathbb{N} = \{1, 2, 3, \ldots\}$ and $\mathbb{N}_0 = \{0, 1, 2, \ldots\}$ in this text

Example 1.1.4

The other common sets are similarly denoted by *blackboard bold* letters as well.

1. $\mathbb{Z}$ is the set of all integers.
2. $\mathbb{Q}$ is the set of all rationals.
3. $\mathbb{R}$ is the set of all reals.
4. $\mathbb{C}$ is set of all complex numbers.

The symbol $\in$ in $x \in S$ is used to show that x is an element of S. It may be read as 'in'.

Example 1.1.5

The cartesian plane is also a set. It defined as

$$\{(x, y) \mid x, y \in \mathbb{R}\}$$

The next section is skippable, and I advise you to skip it if you're reviewing or are not interested in extra stuff at all.

§1.1.1 Why Naive?

One interesting formulation possible in naive set-theory is the "Russel's Paradox". It goes as follows. Let S be a set $\{x \mid x \notin S\}$. Basically, S is the set of all elements not in S. This leads to a paradox because for x to be in S, it has to follow the condition that it is not in S! And any element not in S must be in S.

There is no particular solution to this using naive set-theory. This is one of the reasons why this discussion of sets is so naive.

§1.2 Subsets and Equality