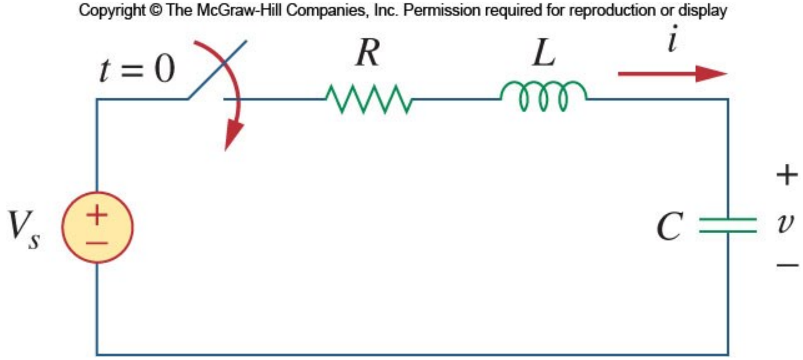
Step Response of a Series RLC Circuit



$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = \frac{V_s}{LC}$$

- The solution

$$v(t) = v_t(t) + v_{ss}(t)$$

- The complete solutions for the three conditions of damping are:

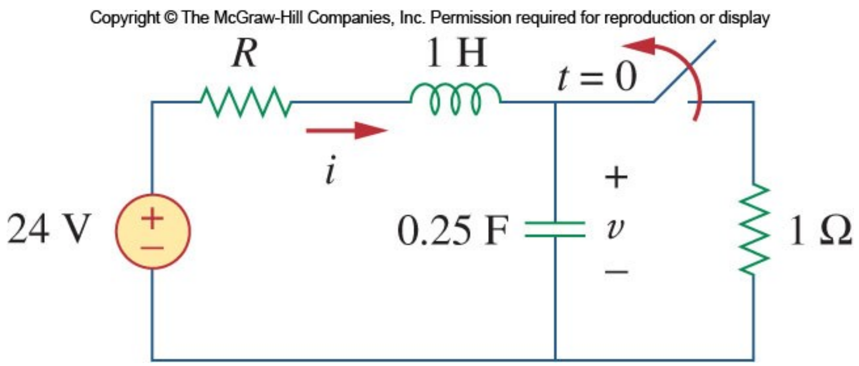
$$v(t) = V_s + (A_1 e^{s_1 t} + A_2 e^{s_2 t}) \text{ (Overdamped)}$$

$$v(t) = V_s + (A_1 + A_2 t)e^{-\alpha t} \text{ (Critically Damped)}$$

$$v(t) = V_s + (B_1 \cos\omega_d t + B_2 \sin\omega_d t)e^{-\alpha t} \text{ (Underdamped)}$$

Example

- Find $v(t)$ and $i(t)$ for $t > 0$.
  Consider three cases:
  - $R = 5\Omega$
  - $R = 4\Omega$
  - $R = 1\Omega$

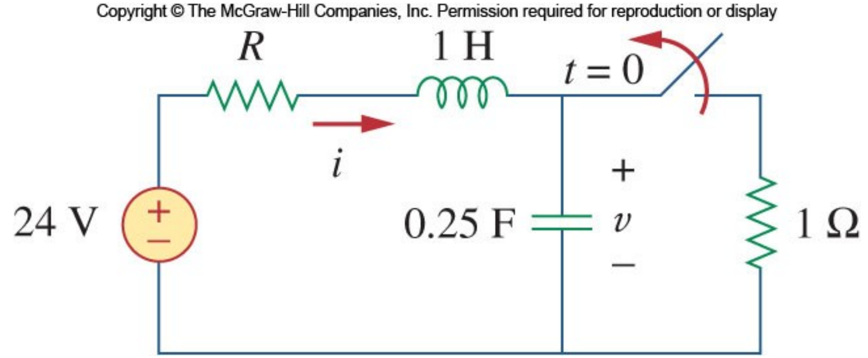


When $R = 5\Omega$,

- For $t < 0$, switch closed, capacitor open, inductor shorted.
- For $t > 0$, switch open, a series RLC network
- $v(0) = 4V \quad i(0) = 4A = C\dfrac{dv(0)}{dt}, \quad \dfrac{dv(0)}{dt} = 16$

$$\alpha = \frac{R}{2L} = 2.5,\ \omega_0 = \frac{1}{\sqrt{LC}} = 2,\ s_{1,2} = -1, -4$$ Overdamped.

$$v(t) = v_s + \left(A_1 e^{-t} + A_2 e^{-4t}\right)$$

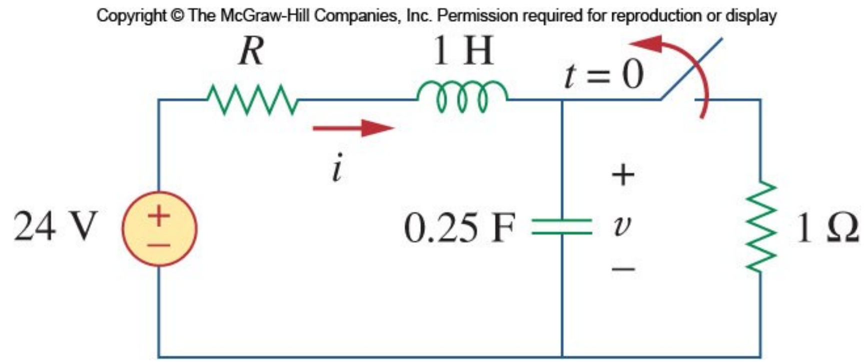


When $R = 4\Omega$,

- For $t < 0$, switch closed, capacitor open, inductor shorted.
- For $t > 0$, switch open, a series RLC network
- $v(0) = 4.8V, i(0) = 4.8A = C\frac{dv(0)}{dt}, \frac{dv(0)}{dt} = 19.2$

$$\alpha = \frac{R}{2L} = 2,\ \omega_0 = \frac{1}{\sqrt{LC}} = 2,\ s_{1,2} = -2$$ Critically damped

$$v(t) = v_s + (A_1 + A_2 t)e^{-2t}$$

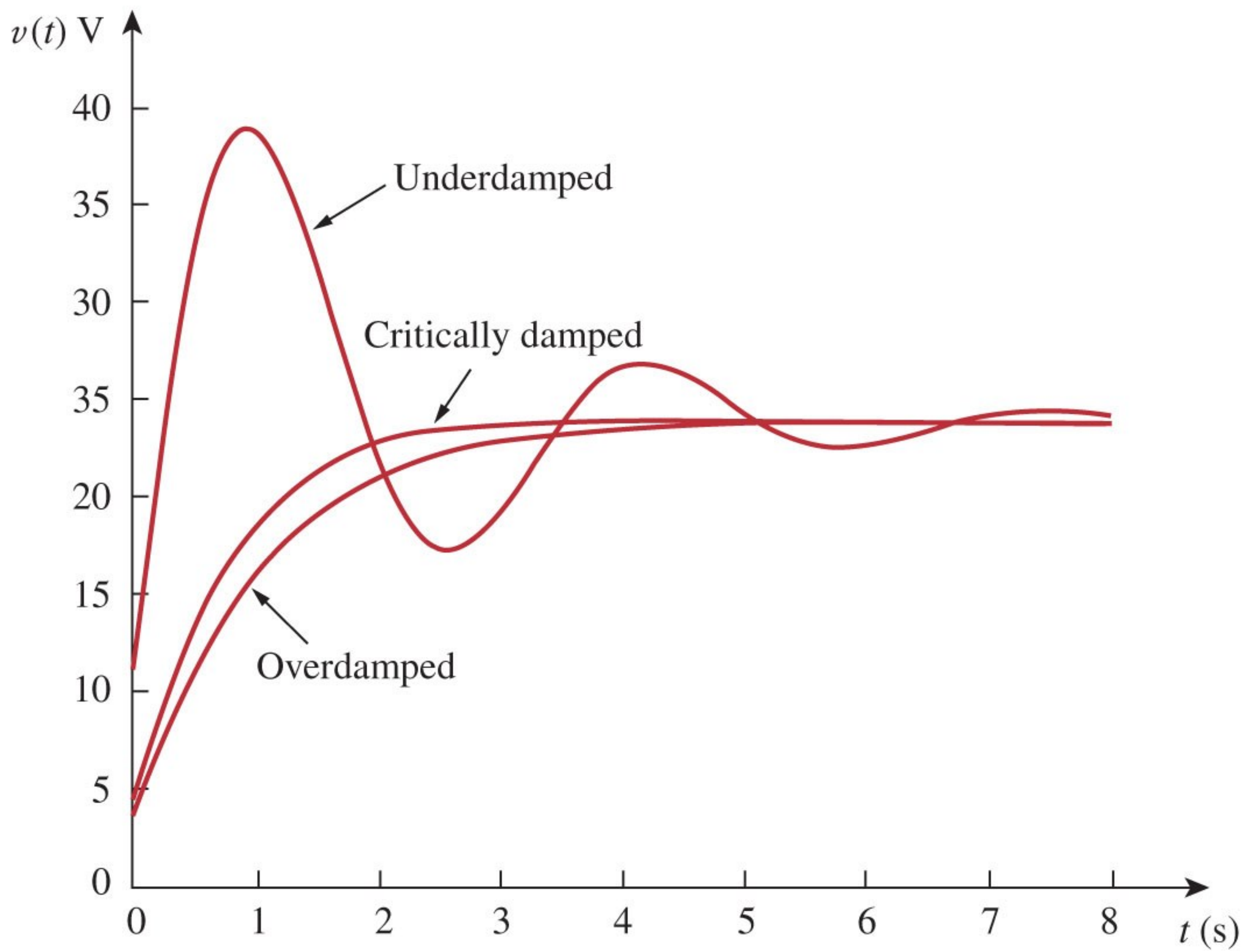


When $R = 1\Omega$,

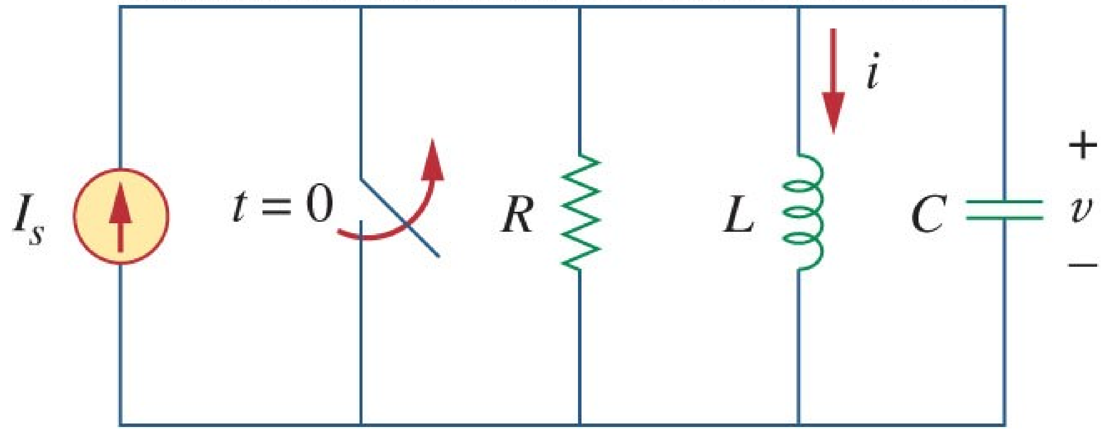
- For $t < 0$, switch closed, capacitor open, inductor shorted.
- For $t > 0$, switch open, a series RLC network
- $v(0) = 12V, \;\; i(0) = 12A = C\frac{dv(0)}{dt}, \frac{dv(0)}{dt} = 48$

$$\alpha = \frac{R}{2L} = 0.5, \;\omega_0 = \frac{1}{\sqrt{LC}} = 2, \;s_{1,2} = -0.5 \pm j1.936$$ Underdamped

$$v(t) = v_s + (B_1 \cos 1.936t + B_2 \sin 1.936t)e^{-0.5t}$$


v(t) V
40
35
30
35
20
15
10
5
0
Underdamped
Critically damped
Overdamped
0
1
2
3
4
5
6
7
8
t (s)

Step Response of a Parallel RLC Circuit



Apply KCL,

$$\frac{v}{R} + i + C\frac{dv}{dt} = I_s$$

& $$v = L\frac{di}{dt}$$

So we get

$$\frac{d^2 i}{d^2 t} + \frac{1}{RC}\frac{di}{dt} + \frac{i}{LC} = \frac{I_s}{LC}$$

Step Response of a Parallel RLC Circuit

$$\frac{d^2i}{d^2t}+\frac{1}{RC}\frac{di}{dt}+\frac{i}{LC}=\frac{I_s}{LC}$$

- The total response is a combination of steady state responses and transient response:

$$i(t)=I_s+(A_1e^{s_1t}+A_2e^{s_2t})\ \text{(Overdamped)}$$

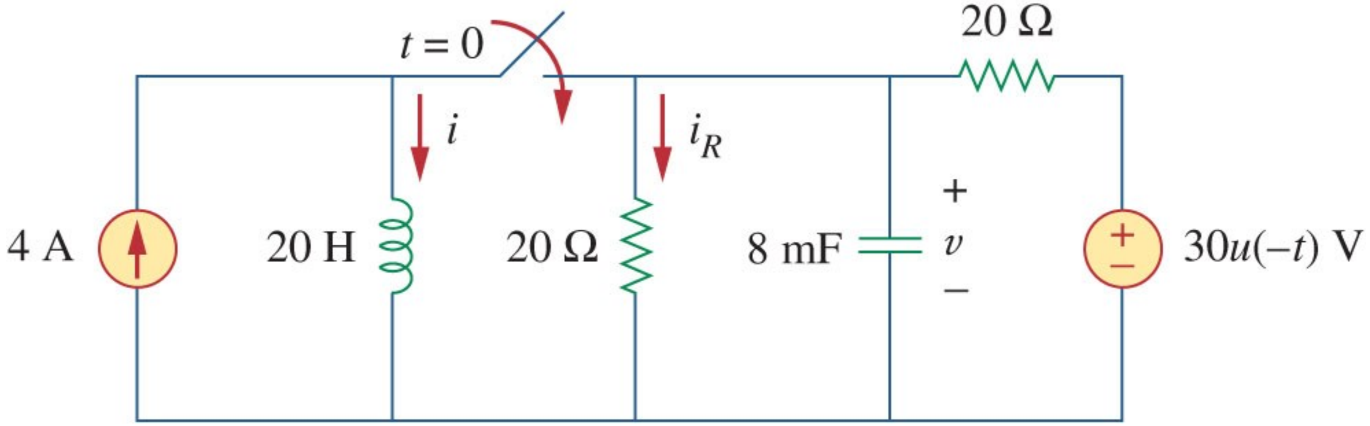
$$i(t)=I_s+(A_1+A_2t)e^{-\alpha t}\ \text{(Critically Damped)}$$

$$\mathrm{i}(t)=I_s+(B_1\cos\omega_d t+B_2\sin\omega_d t)e^{-\alpha t}\ \text{(Underdamped)}$$

Here the variables A_1/A_2 B_1/B_2 are obtained from the initial conditions, $i(0)$ and $di(0)/dt$.

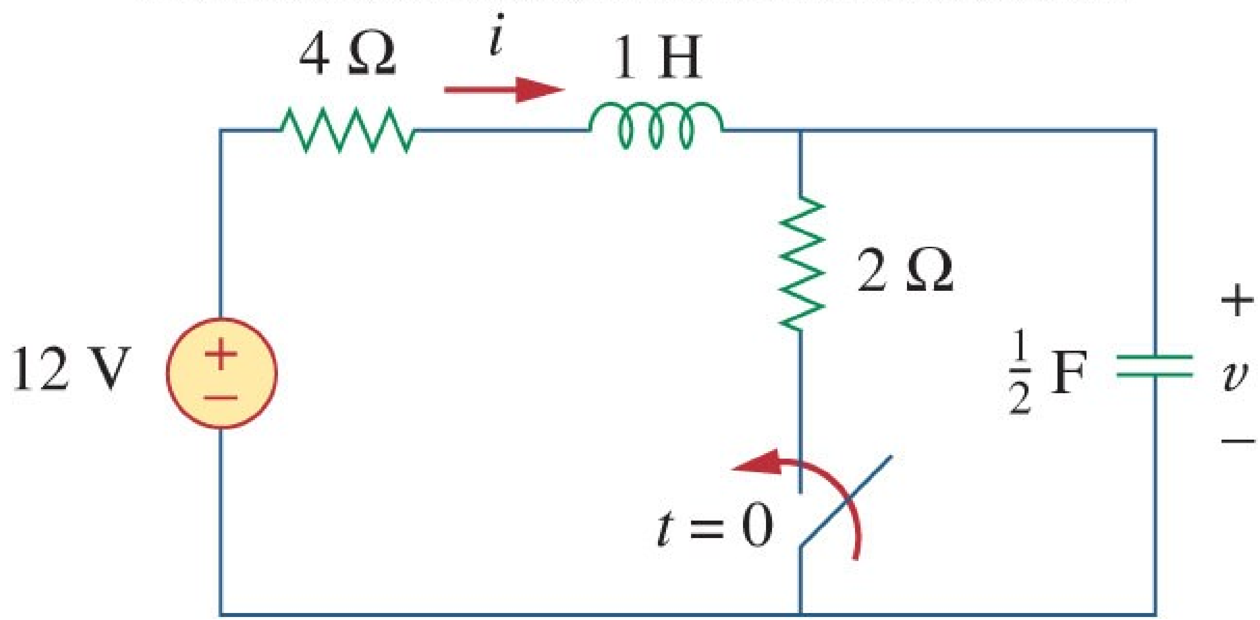
Example

- Find $i(t)$ and $i_R(t)$ for $t > 0$.



General Second-Order Circuits

- An example



General Second-Order Circuits

- The principles of solving the series/parallel forms of RLC circuits can **be applied** to **general second-order circuits**, by taking the following six steps:

  1. First determine the initial conditions, $x(0)$ and $dx(0)/dt$.

  2. Applying KVL and KCL, to find the general second-order differential equation to describe $x(t)$. 3.**Depending on the roots of C.E. , the form of the general solution $x_{g.s.}(t)$ (3 cases) of homogeneous equation can be determined.**

  4. We obtain the **particular solution** by observation/calculation, **specially** for a DC/step response

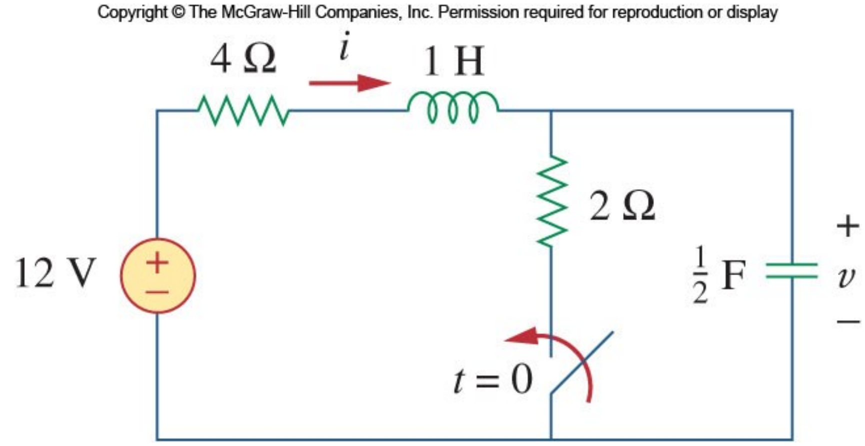
$$x_{p.s.}(t)=x(\infty)$$

  5. The total response = general solution + particular solution.

$$x(t)=x_{p.s.}(t)+x_{g.s.}(t)$$

  6. Using the initial conditions to determine the constants of $x(t)$.

General RLC Circuits

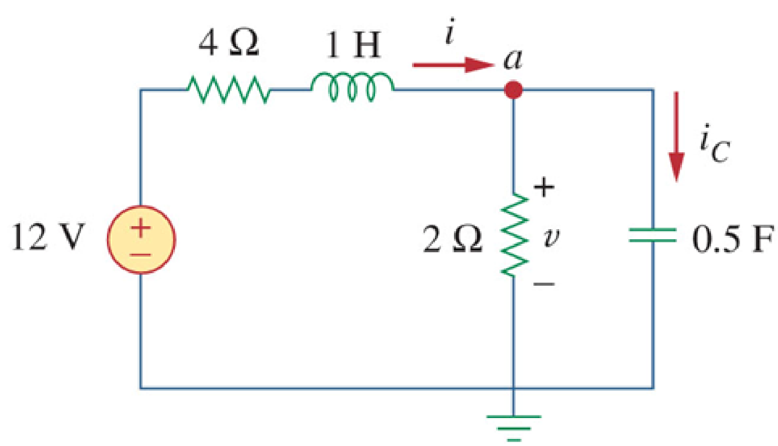


- Find the complete response $v(t)$ for $t > 0$ in the circuit.

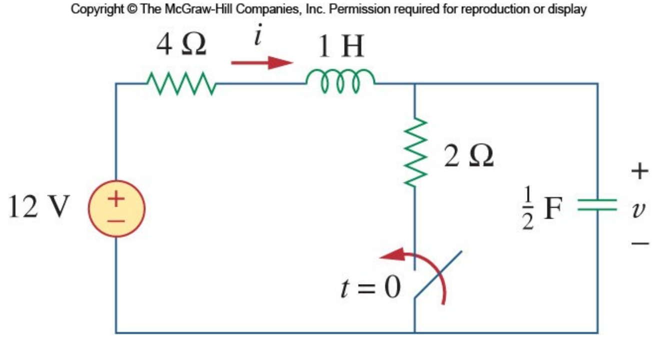
1. Initial conditions

$$v(0^+) = v(0^-) = 12V,\ i(0^+) = i(0^-) = 0$$

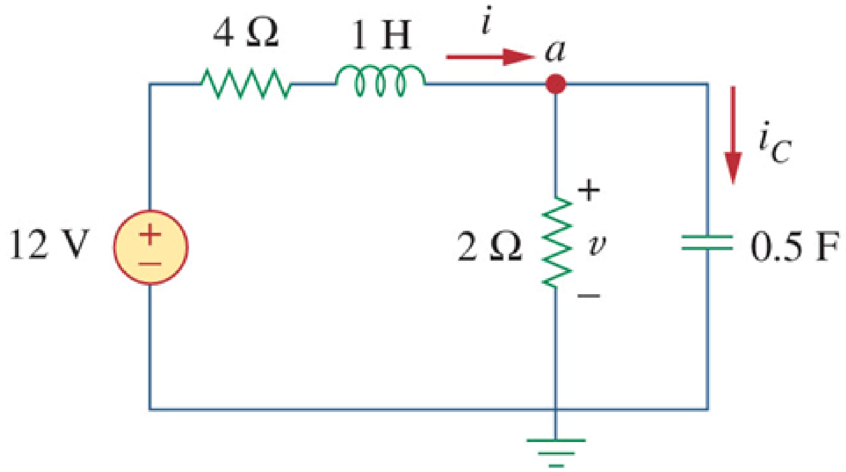
$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{-12/2}{0.5} = -12\ V/s$$



General RLC Circuits



- Find the complete response $v(t)$ for $t > 0$ in the circuit.



2. KCL at node *a*: $i = \frac{v}{2} + 0.5\frac{dv}{dt}$

   KVL on left mesh: $4i + 1\frac{di}{dt} + v = 12$

$$\Rightarrow \quad \frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 6v = 24$$

$$\frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 6v = 24$$

3. General Solution:

⇨ General Solution $v_t(t) = A_1e^{-2t} + A_2e^{-3t}$

4. Particular Solution : Steady-state response $v_{ss}(t) = 4V$

5. Put together : $v(t) = 4 + \mathrm{A}_1e^{-2t} - \mathrm{A}_2e^{-3t}$

6. Using initial conditions to determine A_1 , A_2

Self-test-General RLC Circuit

- Find the complete response $i(t)$ for $t > 0$ in the circuit.