EE160 Homework 3

Deadline: 2022-11-27, 23:59:59, Submit your homework on Blackboard
(Hint:You can use MATLAB to help you do the homework.)

1. Consider a unity feedback system with a loop transfer function (10')

$$L(s) = G_c(s)G(s) = \frac{10(1-s)}{(0.5s+3)(s+T)}$$

   (a) Sketch the root locus of the system by hand when T changes from 0 to $+\infty$. (4')

   (b) Determine the value of T when system is critically stable and critically damping. (3')

   (c) Calculate the unit step response in time domain of the system with $T = 20$. (3')

Solution:
(a): From $1 + G_c(s)G(s) = 0$, the characteristic equation of the system can be written as

$$\begin{aligned} (0.5s+3)(s+T) + 10(1-s) &= 0.5s^2 + T(0.5s+3) - 7s + 10 && (1)\\ &= 1 + T\frac{s+6}{s^2-14s+20} && (2)\\ &= 0 && (3) \end{aligned}$$

after preprocessing the above equation, the equivalent open-loop transfer function of the system can be written as

$$G_k(s) = T\frac{s+6}{s^2-14s+20} \tag{4}$$

The drawing steps are as follows:
1) Zero is -6 and Poles are $7 \pm \sqrt{29}$
2) Root locus on the real axis: $[-\infty, -6]; [7-\sqrt{29}, 7+\sqrt{29}]$
3) $\sigma_A = \frac{14-(-6)}{2-1} = 20$, $\phi_A = \frac{1}{2-1} \times 180^\circ = 180^\circ$
4) Determine where the locus crosses imagine axis. $s^2 + (T-14)s + 20 + 6T = 0 \rightarrow s^2 + 104 = 0$ $\rightarrow s = \pm 2\sqrt{26}j$
5) Determine the breakaway point on the real axis. $T\frac{s+6}{s^2-14s+20} = -1 \rightarrow T = \frac{-(s^2-14s+20)}{s+6} \rightarrow$ $\frac{dT}{ds} = -\frac{s^2+12s-104}{(s+6)^2} = 0 \rightarrow s^2 + 12s - 104 = 0 \rightarrow s_{1,2} = -6 \pm 2\sqrt{35}$
6) $\theta_{p_1} = \theta_{p_2} = 180^\circ$ and $\theta_{z_1} = 180^\circ$
The above equation shows that the root locus on the complex plane is a circle with the center of the open loop pole $p_0 = -6$ and the radius $2\sqrt{35}$. Based on the preceding information, you can draw the exact root path of the system, as shown in Figure 1.

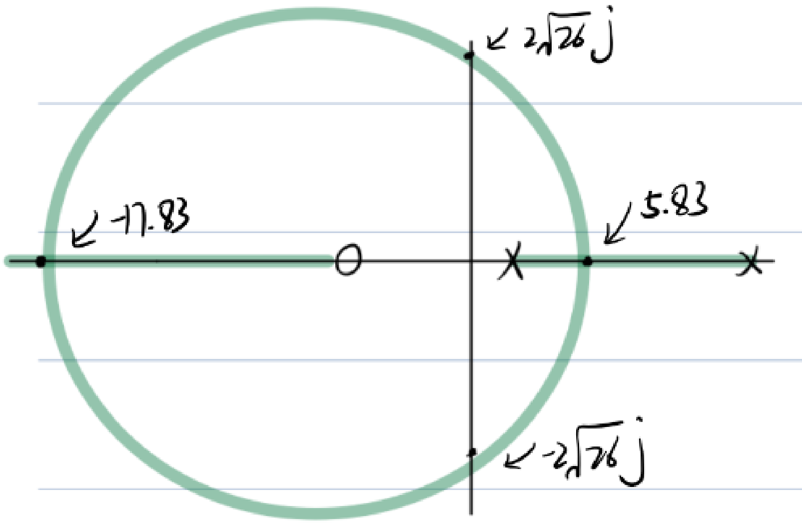

Figure 1: The root locus of the circuit system

(b) $s^2 + (T-14)s + 20 + 6T = 0$ When system is critically stable, let $D(jw) = 0$, $Re[D(jw)] = 6T + 20 - w^2 = 0$, $Im[D(jw)] = (T-14)w = 0$, one obtains $T = 14$ and $w = \pm 2\sqrt{26}$. When system is critically damping, $d_2 = -17.832$, one obtains $T = -\frac{(s-7-\sqrt{29})(s-7+\sqrt{29})}{s+6} = 49.66$.
Critical damping $s = -6 - 2\sqrt{35}$, $T = -\frac{s^2-14s+20}{s+6} = 49.6643$.
(c) The closed-loop transfer function of the system at $T = 20$

$$\Phi(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)} = \frac{20 - 20s}{s^2 + 6s + 140} \tag{5}$$

Therefore, the unit step response of the system is $Y(s) = \Phi(s)R(s) = \frac{20-20s}{s^2+6s+140}\frac{1}{s}$ or $y(t) = L^{-1}[Y(s)] = \frac{1}{7} - \frac{1}{7}e^{-3t}(\cos\sqrt{131}t + \frac{143\sqrt{131}}{131}\sin\sqrt{131}t)$.

2. A control system with a PI-controller is shown in Figure 2. (10')
   (a) Let $K_I/K_P = 0.5$ and determine K_P so that complex roots have the maximal damping ratio. (5')
   (b) Draw the step response of the system with K_P set to the value determined in part (a). (5')

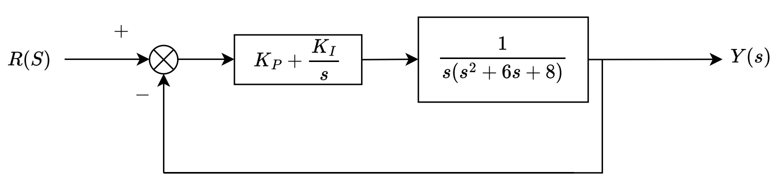

Figure 2: A control system with a PI controller

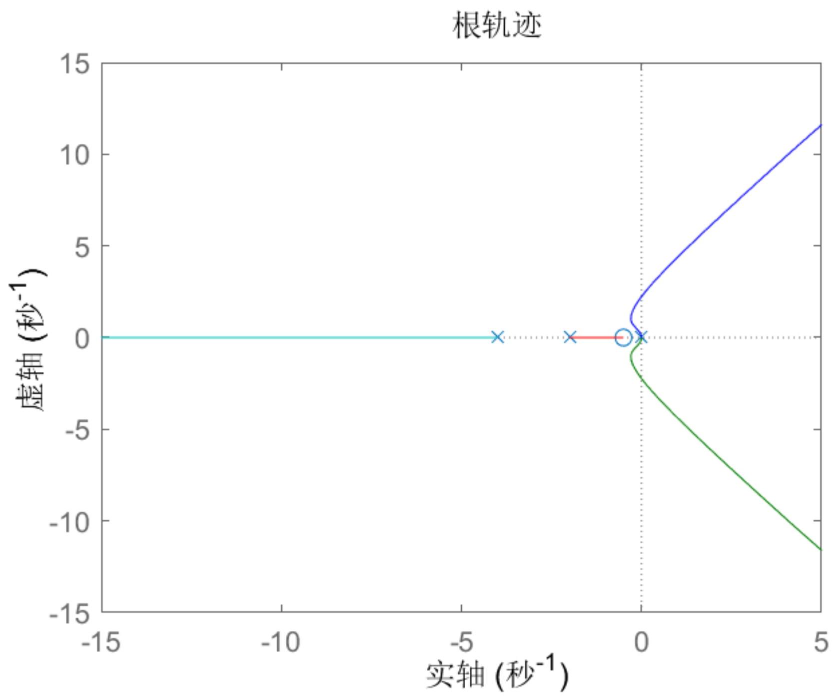

Figure 3: A control system with a PI controller

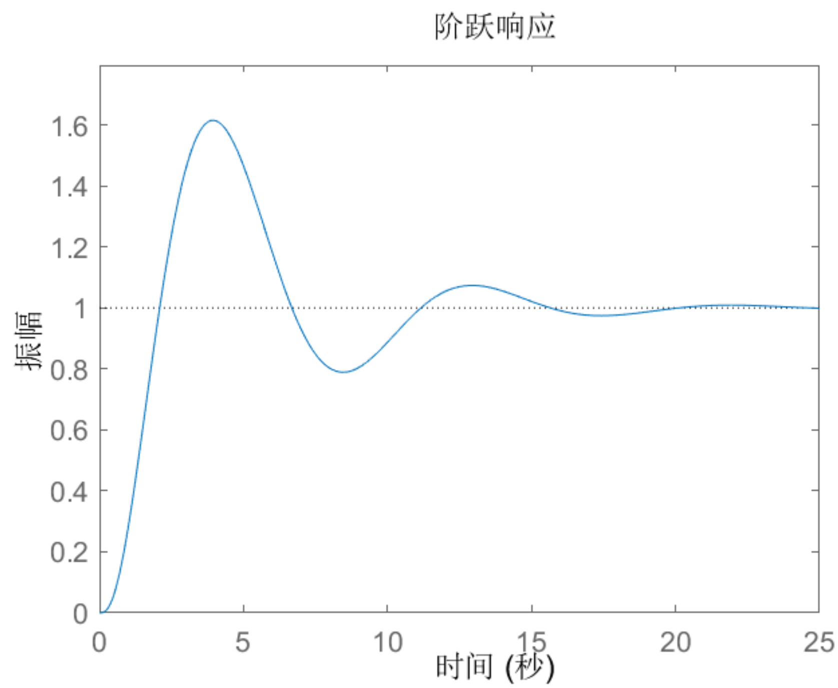

Figure 4: A control system with a PI controller

The root locus is shown in Figure 3. The PI controller can be written as

$$Gc(s) = \frac{K_P s + K_I}{s} \tag{6}$$

and setting $K_I = 0.5K_P$, the characteristic equation can be written as

$$1 + K_P \frac{s + 0.5}{s(s^2 + 6s + 8)} = 0 \tag{7}$$

$G_c(s)G(s) = (K_p + \frac{K_p}{2s})(\frac{1}{s(s^2+6s=8)}) = \frac{2sK_P+K_P}{2s^2(s+2)(s+4)}$. Then $2s^4+12s^3+16s^2 = 2sK_P+K_P = 0$, A suitable gain as $K_P = 5.55$. The step response is shown in Figure 4.

3. A control system with a PD-controller is shown in Figure 5. (20')

   (a) Given $K = 10$, if one wish the $P.O. \leq 16\%$, and the settling time is $T_s \leq 4s$ (2% of the steady-state error). Apply root locus to determine the control parameter K_P and K_D. (10')

   (b) Given the K_P and K_D obtained in question (a), sketch the root locus of the system when K changes from 0 to $-\infty$. (10')

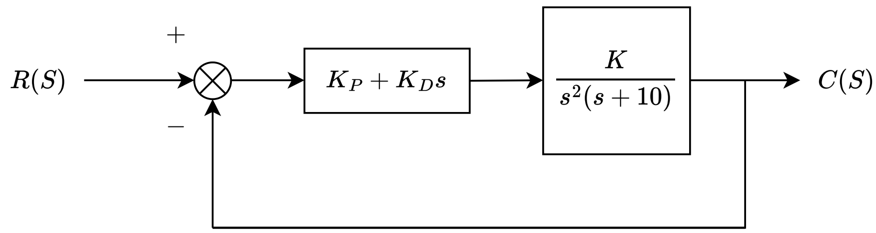

Figure 5: A control system with a PD controller

(a) Since $\sigma_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}100\% = 16\%$, one obtains $\xi = 0.5$. Since $t_s = \frac{4}{\xi\omega_n} = 4$, one obtains $\xi\omega_n = 1$ and $\omega_n = 2$. Since $-\xi\omega_n = -1, \omega_n\sqrt{1-\xi^2} = 1.73$, one obtains the leading pole of the closed loop $s_{1,2} = -1 + j1.73$, as shown in Figure 6. Since $\arctan\xi = 30°$, one obtains $\theta_1 = 120°$, $\theta_2 = \arctan\frac{1.73}{10-1} = 10.88°$ and the phase angle provided by the correction network $\phi = 2 \times 120° + 10.88° - 180° = 70.88°$. Since

$$\tan\phi = 2.88 = \frac{1.73}{\frac{K_P}{K_D} - 1} \tag{8}$$

one obtains $\frac{K_P}{K_D} = 1.6$. By the amplitude condition

$$\frac{s_1 + |\frac{K_P}{K_D}| \times 10K}{|s_1^2| \cdot |s_1 + 10|}|_{s_1=-1+j1.73} = 1 \tag{9}$$

one obtains $K_D = 2, K_P = 1.6K_D = 3.2$. After K_P and K_D are determined, it is necessary to check whether S_1 and S_2 are closed-loop dominant poles. Let the third root is s_3, which is given by the particular equation $s^3 + 10s^2 + 10K_Ds + K_P = 0$, one obtains $\Sigma_{i=1}^3 s_i = -10$. Substituting $s_{1,2} = -1 \pm 1.73$ into the above equation, we get $s_3 = -8$. Therefore, $s_{1,2}$ is the closed-loop dominant poles of the system.

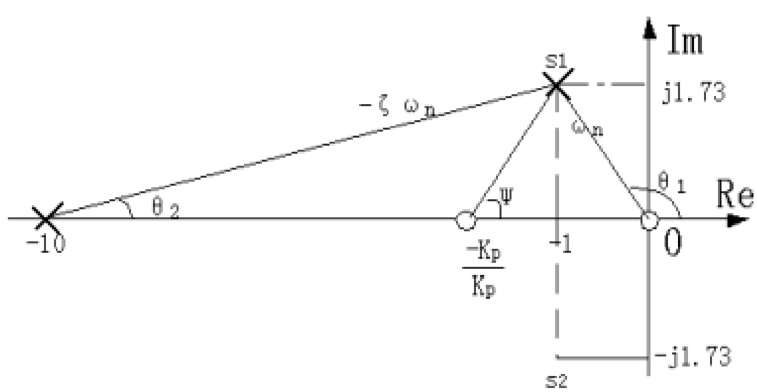

Figure 6: A control system with a PD controller

(b) The angle between the asymptote and the positive direction of the real axis is $\pm\frac{\pi}{2}$. Separation point and rendezvous point:

$$\frac{d}{ds}\left[\frac{s^2(s+10)}{s+\frac{K_P}{K_D}}\right] = \frac{2s^2+24.8s+32}{(s+1.6)^2} = 0 \tag{10}$$

i.e., $2s^2+24.8s+32=0$, this equation has no real roots, so there is no separation point and rendezvous point. The root locus is shown in Figure 7.

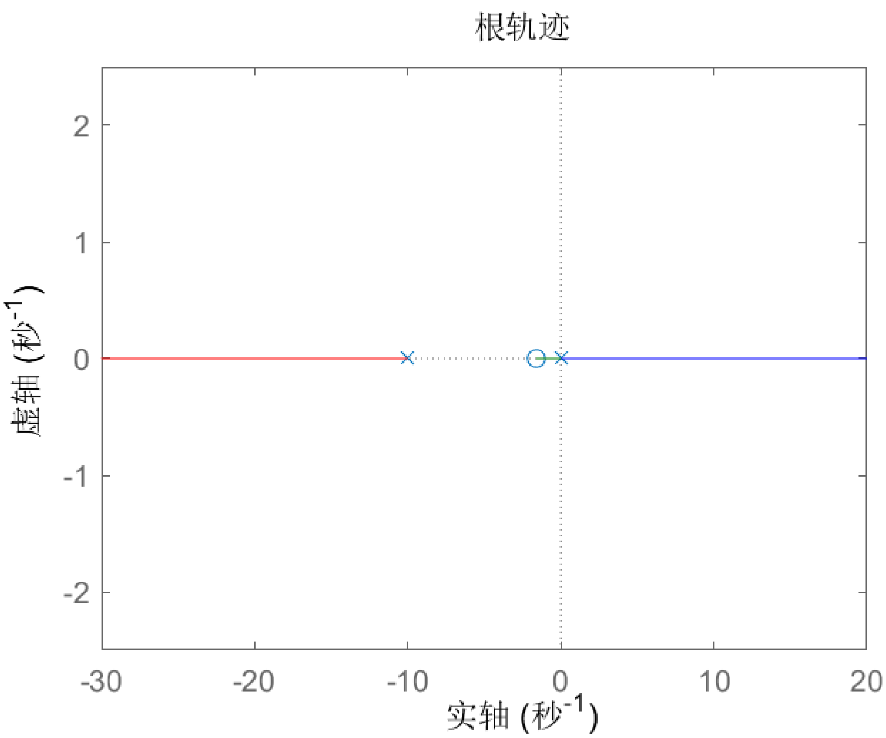

Figure 7: A control system with a PD controller

4. Consider a circuit system whose transfer function is $G(s)$, the high frequency gain of the system is $k_p = 1, (k_p = \lim_{s\to\infty} sG(s))$. We use a P-controller with proportional gain equals to K to control this system. The root locus of the system is given in in Figure 8. (15')

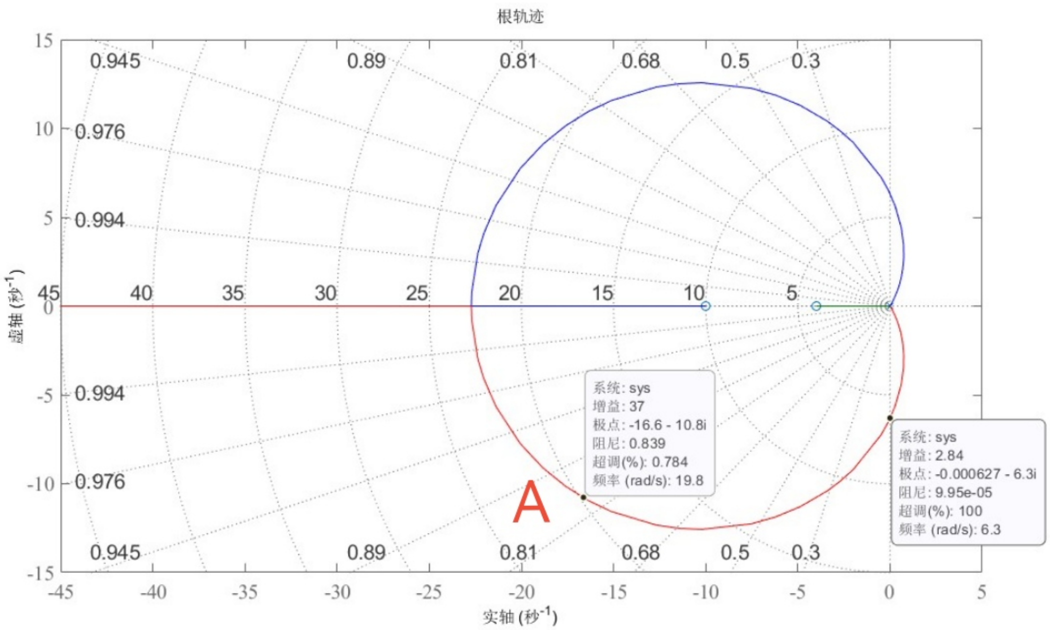

Figure 8: The root locus of the circuit system

(a) Determine the range of K which can stabilize the system. (4')

(b) Derive the transfer function of the system. (3')

(c) For point A in the graph, if there is a 10% change (including positive change and negative change) in K, determine the root sensitivity for point A. (8')

Solution:

(a) By the graph we can know that when K is greater than 2.84, the poles of the closed-loop are all in the right half plane. The range is $K \geq 2.84$.

(b) From the graph, we can know the poles are 0, and the zeros are -10 and -4. Since 0 breakaway into three trajectories, poles at 0 are triple roots. Therefore, The transfer function is

$$G(s) = K\frac{(s+4)(s+10)}{s^3}$$

Since $K = k_p = 1$, the transfer function is

$$G(s) = \frac{(s+4)(s+10)}{s^3}$$

(c) For point A, the gain is 37. The root is $r_1 = -16.6 - j10.8$. If there is a 10% change, $K = 40.7\&33.3$. When $K = 40.7$, the root: $r_1 = -18.4(\text{or } 18.5) - j9.43$, the change in the root is $\Delta r_1 = -2.2 + j1.38$. Thus, the root sensitivity for a positive change in K for r_1:

$$S_{K+}^{r_1} = \frac{\Delta r_1}{\Delta K/K} = \frac{-1.8(\text{or } -1.9) + j1.37}{+0.1} = 22.62\angle 142.72^\circ(\text{or } 23.42\angle 144.21^\circ)$$

For negative change in K, the root: $-14.8 - j11.7$, the change in the root is $\Delta r_1 = 1.8 - j0.9$. Thus, the root sensitivity for a negative change in K for r_1:

$$S_{K-}^{r_1} = \frac{\Delta r_1}{\Delta K/K} = \frac{1.8 - j0.9}{+0.1} = 25.97\angle 147.90^\circ = 20.12\angle -26.57^\circ$$

5. (a) Sketch the Bode plot for the following loop transfer function

$$G_c(s)G(s) = \frac{5(s^2 + 6s + 8)}{(s+2.5)^2}$$

(b) Use the Bode plot in Figure 9. Derive the transfer function for the open loop system.

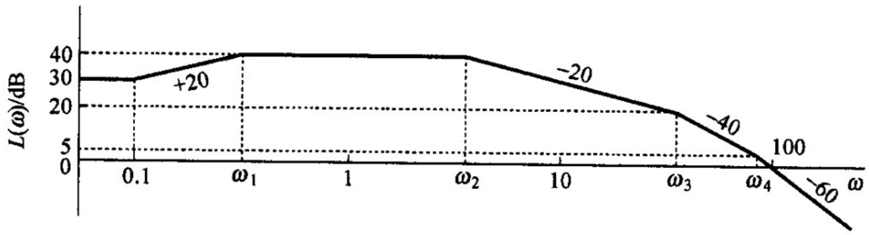

Figure 9: A control system

(c) Sketch the logarithmic-magnitude versus phase angle curve for (b)

(a) The gain is $\frac{5\times 4\times 2}{2.5^2} = 6.4$, thus, the starting point is $20\log_{10} 6.4 = 16.12dB$. The break frequencies are $\omega_1 = 2, \omega_2 = 4, \omega_3 = \omega_4 = 2.5$. The graph is shown below, and the break point is $(2, 16.12)$, $(2.5, 16.12 + 20\log_{10}^{(\frac{2.5}{2})}) = (2.5, 18.06)$, $(4, 16.12 + 20\log_{10}^{(\frac{4}{2})} - 40\log_{10}^{(\frac{4}{2.5})}) = (4, 13.98)$. The bode plot is

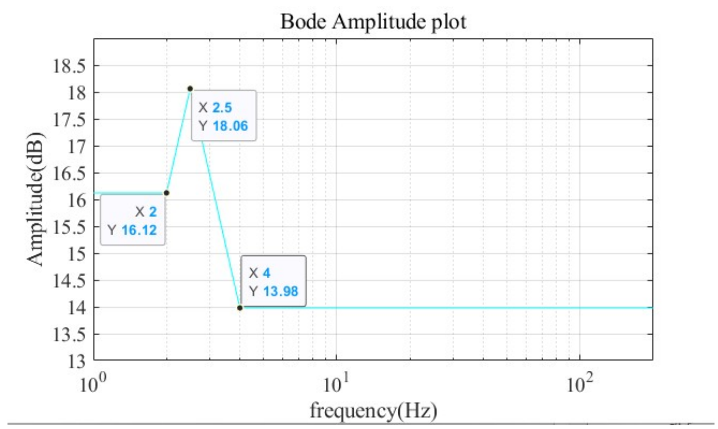

Figure 10: amplitude

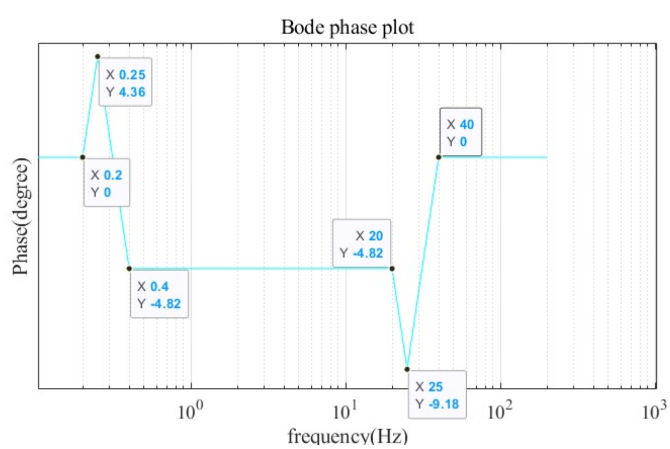

Figure 11: Phase

(b) from the graph, we can know the gain is $10^{\frac{30}{20}} = 31.6$. The break frequencies are $0.1, 0.32, 4.76, 50, 100$ The transfer function is $G(s) = \frac{31.6(10s+1)}{(3.13s+1)(0.21s+1)(0.02s+1)(0.01s+1)}$.

(c) the logarithmic-magnitude versus phase angle curve is

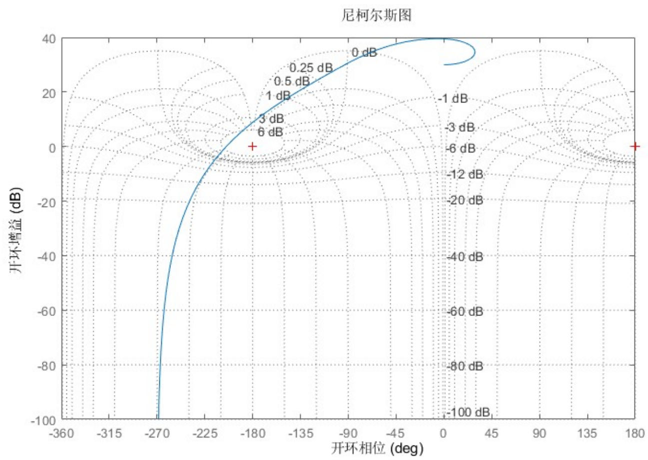

Figure 12: Logarithmic-magnitude vs phase

6. A model of an automobile course control system is shown in Figure 13, where $K = 5.6$. (15')

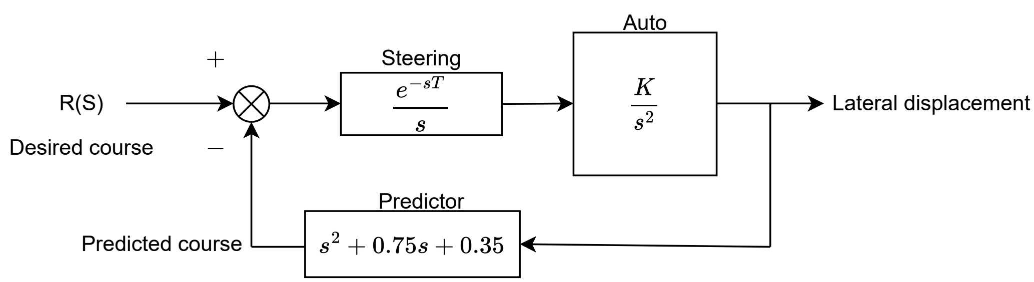

Figure 13: automobile and driver control

(a) Derive the transfer function, show the bode plot and find the gain and phase margin when the reaction time $T = 0$. (5')

(b) Find the phase margin when the reaction time is $T = 0.15s$. (5')

(c) Find the new gain that will let the system in (b) to be borderline stable ($P.M. = 0°$). (5')

(a) The transfer function is

$$T(s) = \frac{\frac{e^{-sT}k}{s^3}}{1 + (s^2 + 0.75s + 0.35)\frac{e^{-sT}k}{s^3}} \tag{11}$$

$$= \frac{5.6s^2 + 4.2s + 1.96}{s^3} \tag{12}$$

The phase margin is $P.M. = 82.3°$ at $\omega = 5.59$ when $T = 0$. The gain margin is $O.M. = -21.6dB$ at $\omega = 0.59$ when $T = 0$.
The bode plot is shown in Figure 14.

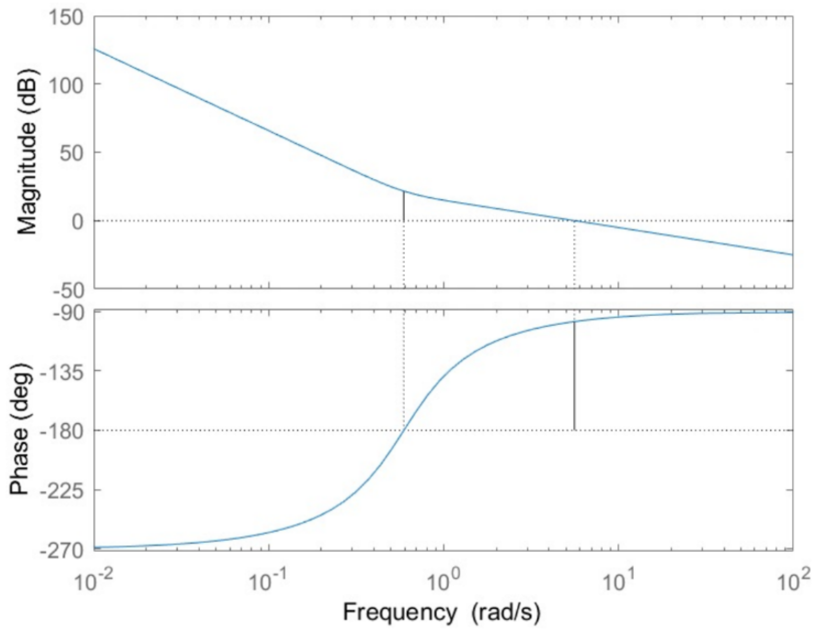

Figure 14: 6-a

(b) For $T = 0.15$, the added phase is $\phi = -T\omega$ (in radians). The phase margin is $P.M. = 82.3° - 48.15° = 34.2°$ at $\omega = 5.59$ when $T = 0.15$.
The bode plot is shown in Figure 15.

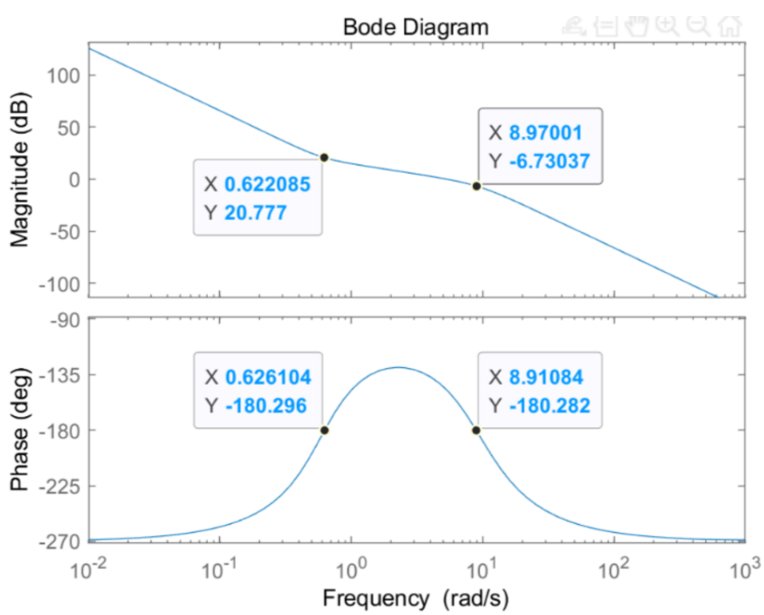

Figure 15: 6-b

(c) As for figure in (b), one obtains $\angle L(jw) = -180°$. Therefore, then $w_c' = 0.63$ rad/s or 8.91. 1) $w_c' = 0.63$ corresponds to 20.7 dB, $20log\alpha_1 = 20.7 \rightarrow \alpha_1 = 10^{\frac{20.7}{20}} \rightarrow K = \frac{5.6}{\alpha} = 0.512$. 2) $w_c' = 8.91$ corresponding to 6.73 dB, one obtains $20log\alpha = 6.73 \rightarrow \alpha_2 = 10^{\frac{6.73}{20}} \rightarrow K = 5.6 \times \alpha_2 = 12.15$.

7. Anesthesia is used in surgery to induce unconsciousness. One problem with drug-induced unconsciousness is differences in patient responsiveness. Furthermore, the patient response changes

during an operation. A model of drug-induced anesthesia control is formulated with a feedback control system when the open-loop transfer function $L(s) = G(s)H(s)$ and $F(s) = 1+G(s)H(s)$. If we use a PD-controller $G_c(s) = K(s+z), z > 0$ to control this system. Setting $K = 5$, the Nyquist plot of the feedback system is shown in Figure 16, in which the poles of the open-loop system are -1 and -3.

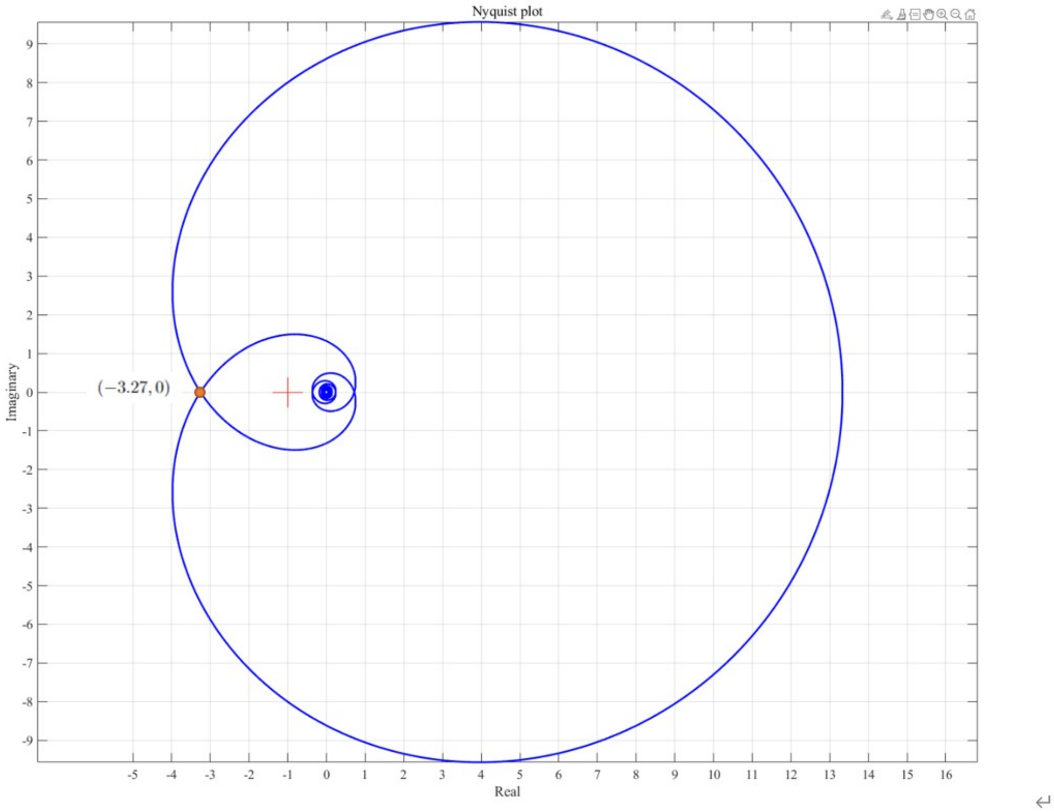

Figure 16: Nyquist plot for control of blood pressure with anesthesia

(a) The system shown in Figure 16 is stable? Give reasons to explain it.

(b) Determine the range of K such that the closed-loop system is to be stable.(You can neglect the inner loop)

(c) Assume that the system has a time-delay T. Setting $K = 5$, how to make this system stable by changing the delay time.

Solution:

(a) This system is not stable since the poles of the open-loop system are not in the right half plane. Thus, P=0 and the con tour encircles (-1,0) N=2 times. Therefore, by Nyquist criteria, the pole in the right half plane of the closed-loop system Z=2. The system is not stable.

(b) If the contour encircles (-1,0) N=0 times, by Nyquist criteria, the system would be stable. When $K = 1/(3.27/5) = 1.53$, the system is critically stable. Thus, the range is $0 < K < 1.53$.

(c) Since the time delay is greater, the system will become more unstable. In order to make the system stable, we have to decrease the time delay T. (By MATLAB, you can know that T=0.19, the system would be stable, but give the tendency is enough.)