

SI152 - Numerical Optimization homework 1

Deadline: 2022-03-18 23:59:00

1. You can use Word, Latex or handwriting to complete this assignment. If you want to submit a handwritten version, scan it clearly.
2. The **report** has to be submitted as a PDF file to Blackboard, other formats are not accepted.
3. The submitted file name is **student_id+your_student_name.pdf**.
4. Late policy: You have 4 free late days for the quarter and may use up to 2 late days per assignment with no penalty. Once you have exhausted your free late days, we will deduct a late penalty of 25% per additional late day. Note: The timeout period is recorded in days, even if you delay for 1 minute, it will still be counted as a 1 late day.
5. You are required to follow ShanghaiTech's academic honesty policies. You are not allowed to copy materials from other students or from online or published resources. Violating academic honesty can result in serious sanctions.

Any plagiarism will get Zero point.

1 Basic Rule

1. Explain an optimization problem encountered in real life, and establish a mathematical model, write it in a standard form (need to include variables, objective functions, constraints), and explain the scale and application scenarios of this model in detail. (20 pt)

2 Linear Programming

2. Reformulate the following problem into linear programming. Explain in detail the relation between the optimal solution of each problem and the solution of its equivalent LP. (20 pt)
 - a) minimize $\|Ax - b\|_\infty$
 - b) minimize $\sum_{i=1}^N c_i x_i$ subject to $|x_i - x_{i+1}| \leq r$
3. Solve the following linear programming problem:
 - a) Use graphical method (15 pt)

$$\begin{aligned} \min \quad & 3x_1 + x_2 \\ \text{s.t.} \quad & x_1 - x_2 \leq 0 \\ & x_1 + x_2 \leq 5 \\ & 6x_1 + 2x_2 \leq 21 \\ & x_1, x_2 \geq 0 \end{aligned} \tag{1}$$

(You can draw your graph by computer, ipad, or by handwriting and taking pictures. Note that you need to clearly draw the feasible region and the intersection between the straight line.)

- b) Use simplex (15 pt)

$$\begin{aligned} \min \quad & -3x_1 + 2x_2 - x_3 \\ \text{s.t.} \quad & 2x_1 + x_2 - x_3 \leq 5 \\ & 4x_1 + 3x_2 + x_3 \geq 3 \\ & -x_1 + x_2 + x_3 = 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned} \tag{2}$$

4. Suppose $S = \{x | Ax \geq b\}$, A is a $m \times n$ matrix, $m > n$ and $\text{rank}(A) = n$. Prove: $x^{(0)}$ is an extreme point of S iff A and b can be separated as:

$$A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix},$$

where A_1 is an $n \times n$ matrix, and $\text{rank}(A_1) = n$. $b_1 \in \mathbf{R}^n$ to satisfy $A_1 x^{(0)} = b_1$, $A_2 x^{(0)} \geq b_2$. (30 pt)