EE160 Introduction to Control: Homework 8

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Deadline: June 16, 2022

1. *(6 points) Infinite Horizon Optimal Control.* Solve the following optimal control problems explicitly. What are the optimal solution for x and u?

   (a)
   $$\min_{x,u} \int_0^\infty \left(2x(t)^2 + 3u(t)^2\right)\,\mathrm{d}t \quad \text{s.t.} \begin{cases} \dot{x}(t) = 2x(t) + u(t) \\ x(0) = 1 \end{cases}$$

   (b)
   $$\min_{x,u} \int_0^\infty \left((x(t)-1)^2 + u(t)^2\right)\,\mathrm{d}t \quad \text{s.t.} \begin{cases} \dot{x}(t) = x(t) + u(t) + 1 \\ x(0) = 1 \end{cases}$$

   (c)
   $$\min_{x,u} \int_0^\infty \left(x_1(t)^2 + u(t)^2\right)\,\mathrm{d}t \quad \text{s.t.} \begin{cases} \dot{x}_1(t) = x_2(t) \quad \text{with} \quad x_1(0) = 1 \\ \dot{x}_2(t) = x_3(t) \quad \text{with} \quad x_2(0) = -1 \\ \dot{x}_3(t) = u(t) \quad \text{with} \quad x_3(0) = 1 \end{cases}$$

   Hint: Always think first about whether the infinite horizon optimal control problem has a solution at all.

2. *(4 points) Finite Horizon Optimal Control.* Solve the following scalar finite horizon optimal control problem explicitly,
   $$\min_{x,u} \int_0^{10} \left(x(t)^2 + u(t)^2\right)\,\mathrm{d}t + 5x(10)^2 \quad \text{s.t.} \begin{cases} \dot{x}(t) = -x(t) + u(t) \\ x(0) = 1 \end{cases}$$
   What are the optimal solution for x and u?
   Hint: Use the "separation of variables" to solve the associated Riccati differential equation.