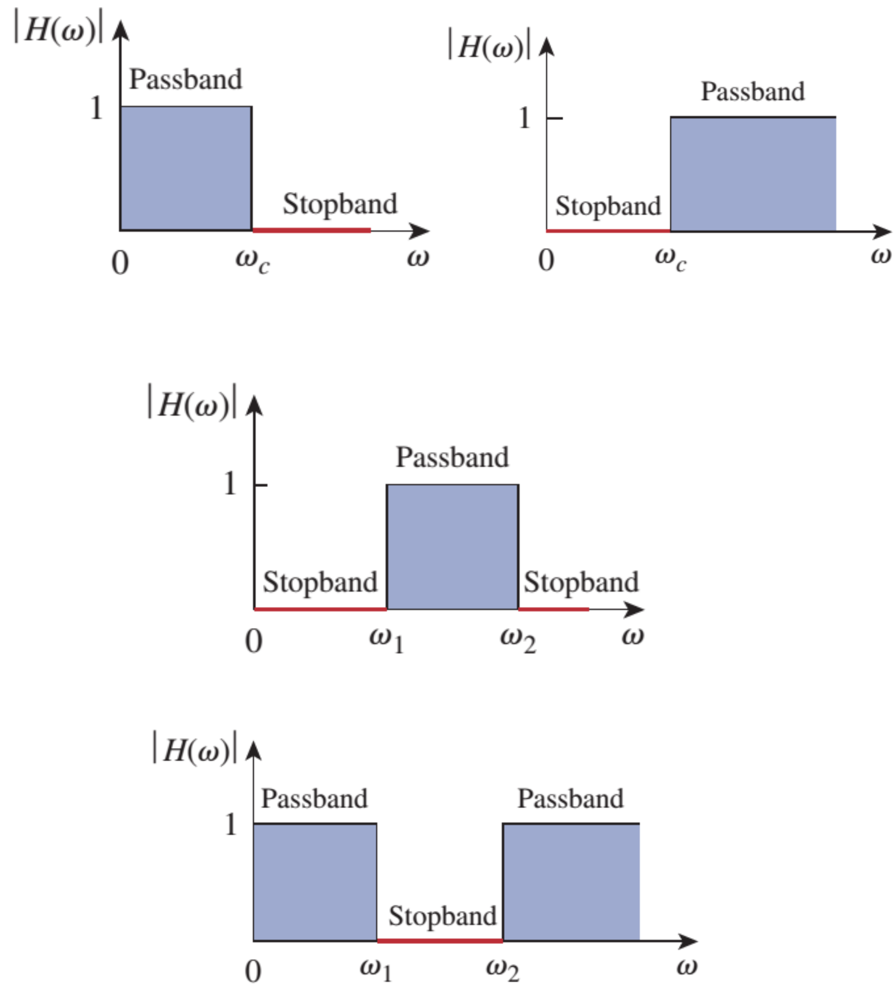
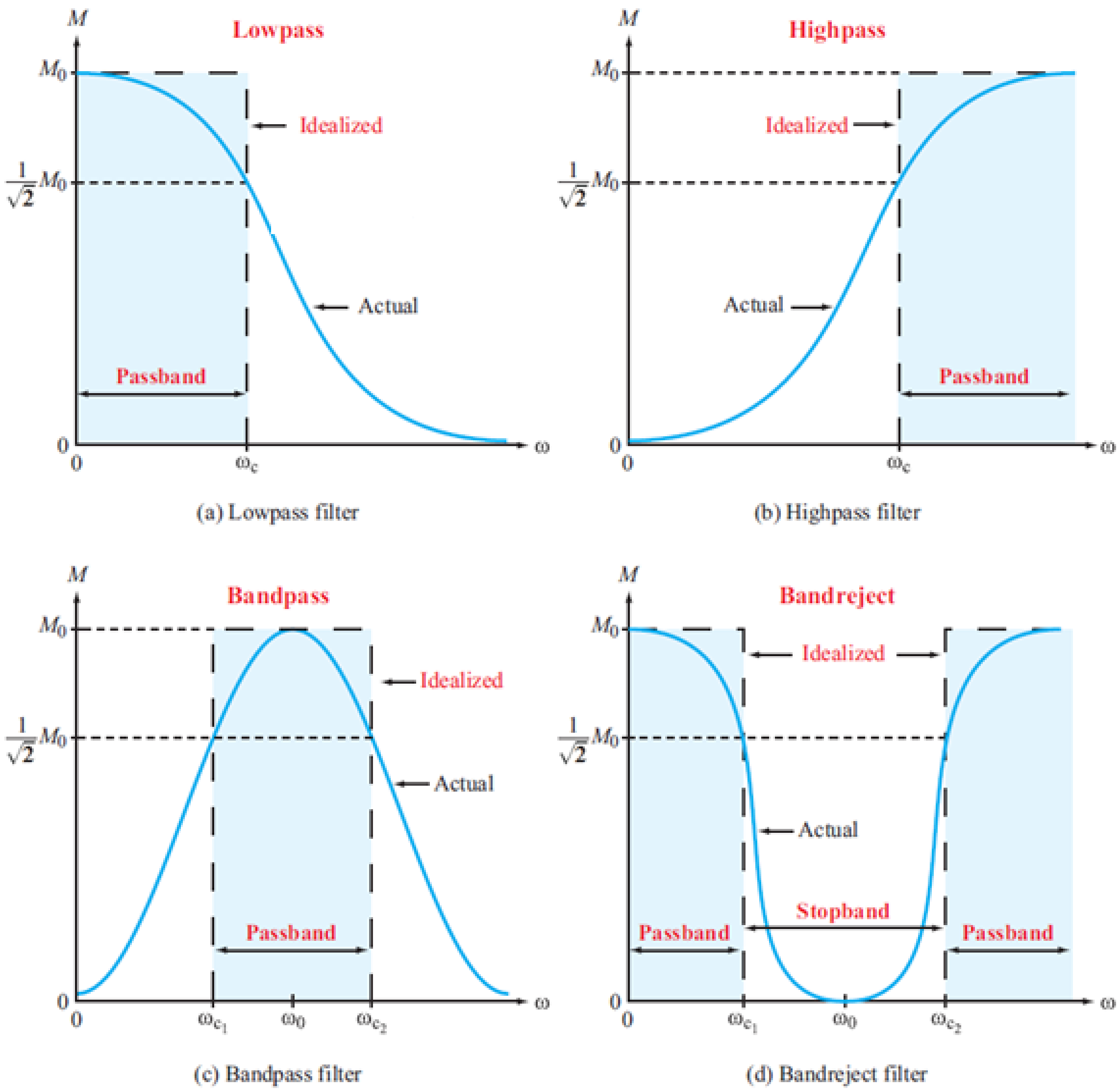
Lecture 14
- Filters

Filters

- Circuit designed to retain certain frequency range but discard or attenuate others
  - *Low-pass*: pass low frequencies and reject high frequencies
  - *High-pass*: pass high frequencies and reject low frequencies
  - *Band-pass*: pass some particular range of frequencies, reject other frequencies outside that band
  - *Band-reject (notch)*: reject a range of frequencies and pass all other frequencies



Filters – Realistic Curves



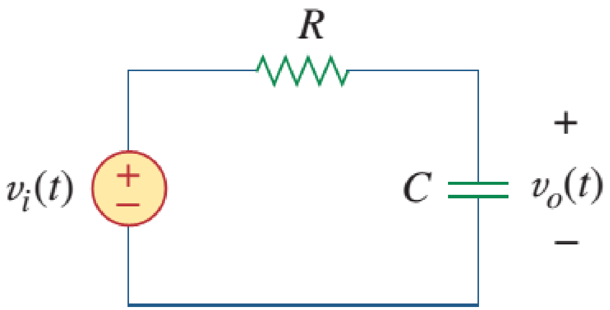
[Source: Berkeley]

Passive Filters

- A filter is passive if it consists only of passive elements
  - R, L and C

- *LC* filters have been used in practical applications for decades
  - Very important circuits
    - many technological advances would not have been possible without the development of filters
  - *LC* filter technology feeds many areas
    - equalizers, impedance-matching networks, transformers, shaping networks, power dividers, attenuators, and directional couplers …

First-Order RC Lowpass Filter



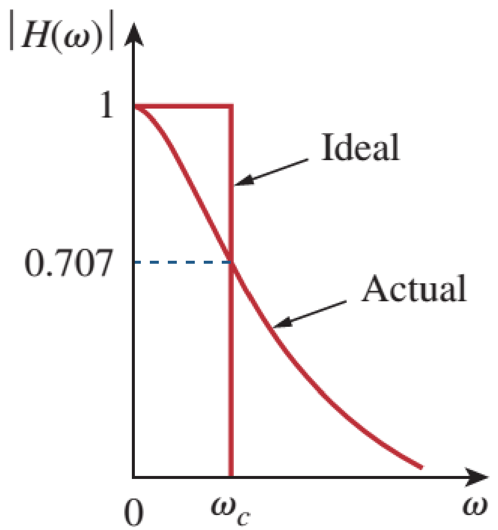
- A typical lowpass filter is formed when the output of a RC circuit is taken off the capacitor.

$$\mathbf{H}(\omega) = \frac{1}{1 + j\omega RC}$$

- The -3dB (half power) frequency is:

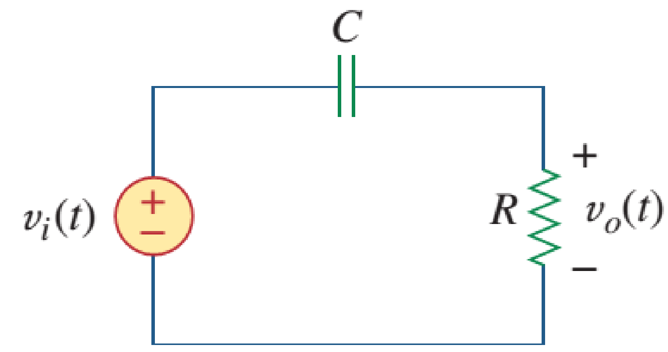
$$\omega_c = \frac{1}{RC}$$

  - Also referred as the <u>cutoff frequency</u>.
  - Filter is designed to pass from DC up to ω_c.



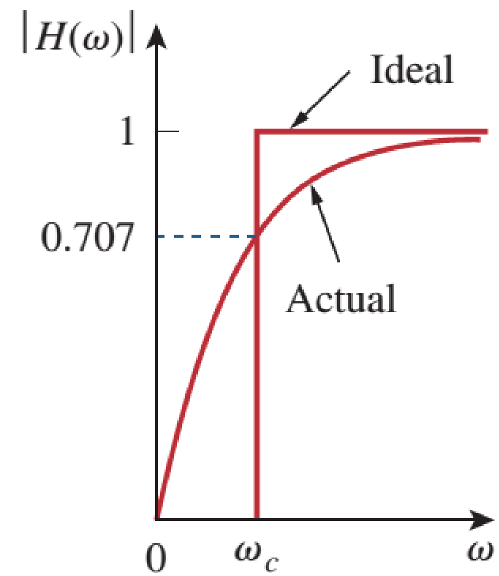
First-Order RC Highpass Filter

$$\mathbf{H}(\omega) = \frac{1}{1 + \frac{1}{j\omega RC}}$$



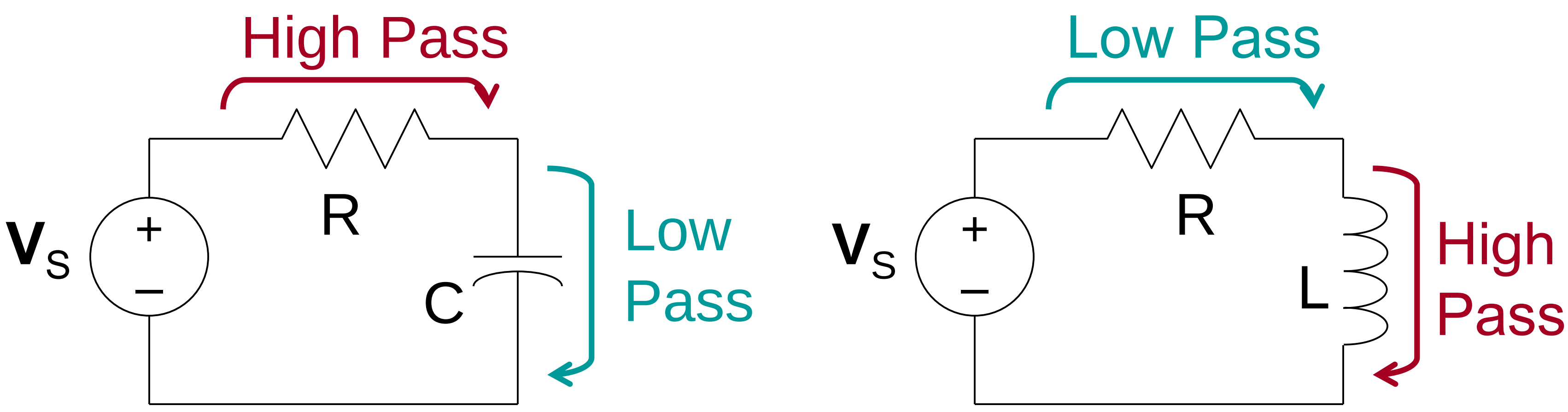
- The cutoff frequency will be the same as the lowpass filter.

$$\omega_c = \frac{1}{RC}$$

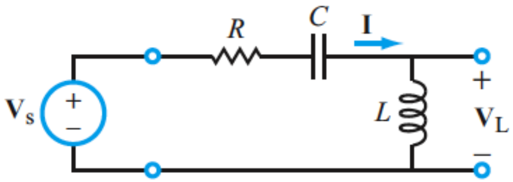


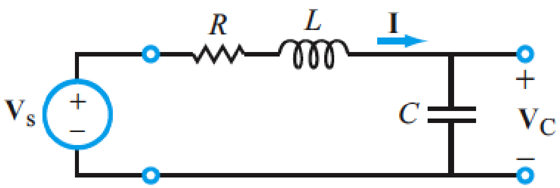
- The difference being that the frequencies passed go from ω_c to infinity.

How about RL Circuits?

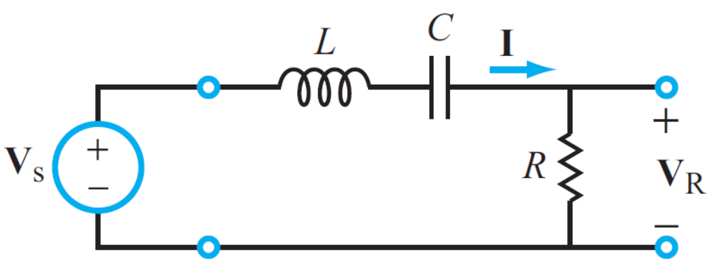


How about RLC Circuits?

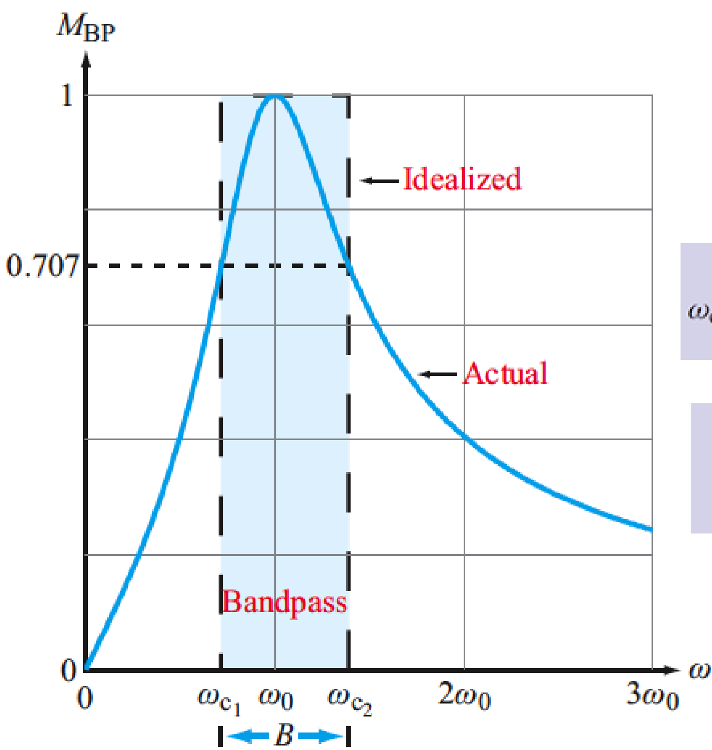




Bandpass RLC Filter



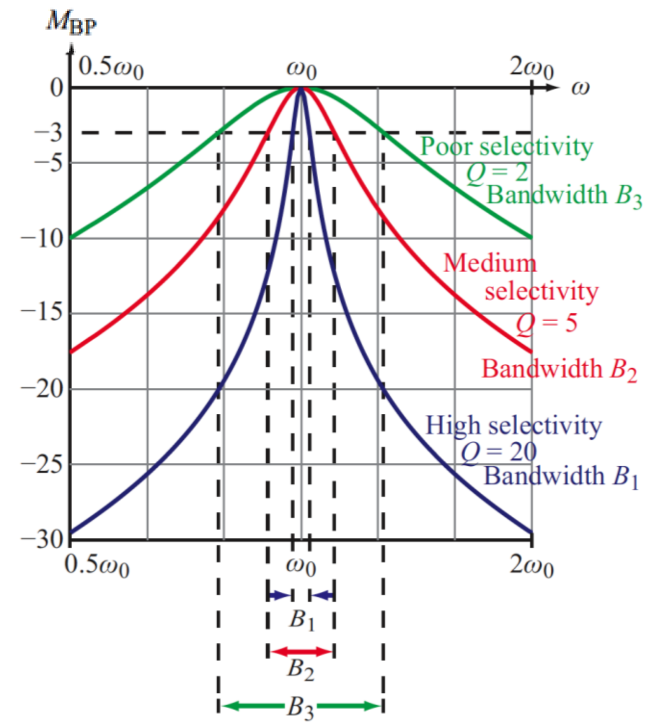
$$\mathbf{H}_{BP}(\omega) = \frac{\mathbf{V}_R}{\mathbf{V}_s} = \frac{R}{Z}$$



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

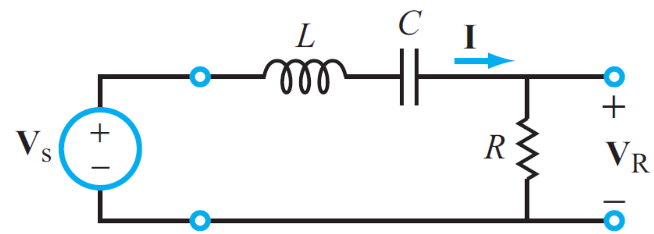
$$\omega_{c_1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}},$$

$$\omega_{c_2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}.$$

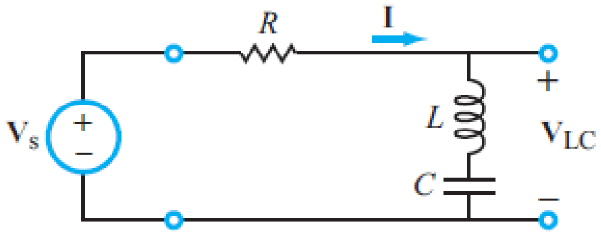


Example

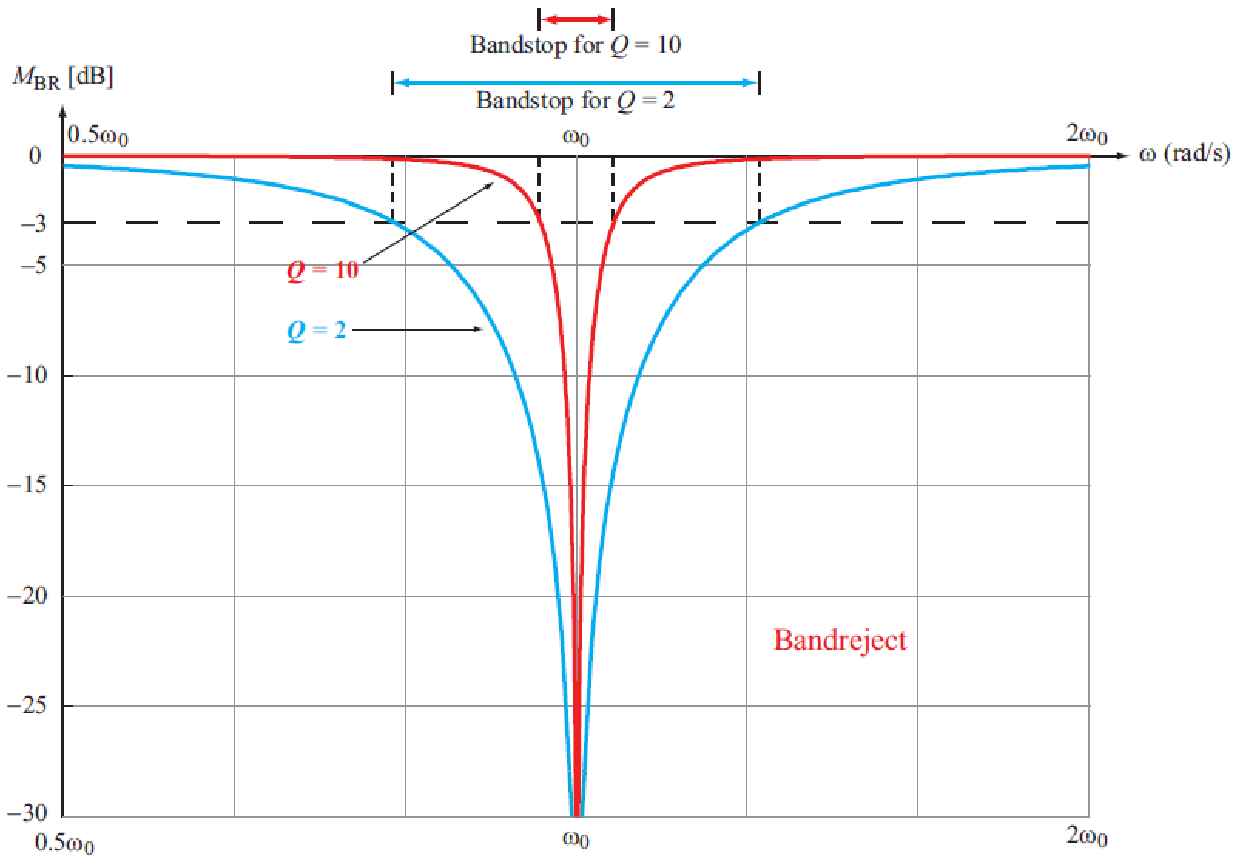
(a) Design a series RLC bandpass filter with a center frequency $f_0 = 1$ MHz and a quality factor $Q = 20$, given that $L = 0.1$ mH.



Bandstop (Bandreject) Filter



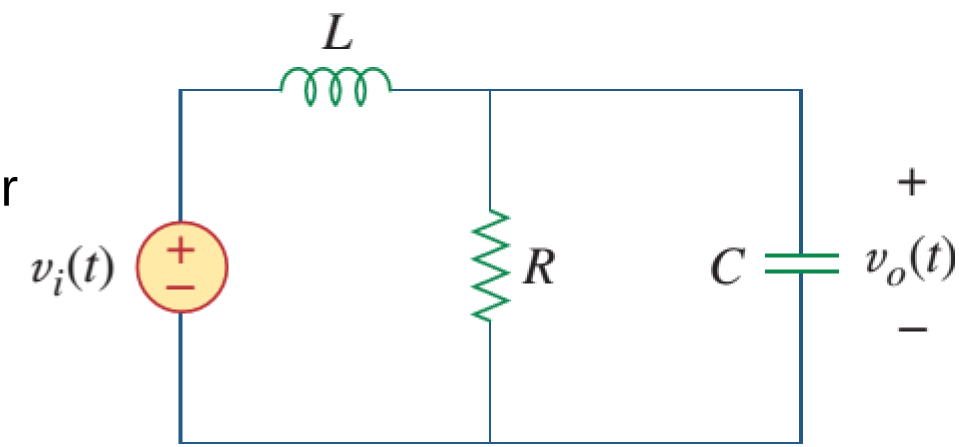
$$H_{BR} = V_{LC} / V_s$$



Example

- Determine what type of filter is shown below. Calculate the corner or cutoff frequency.

  $R = 2\text{k}\Omega$, $L = 2\text{H}$ and $C = 2\mu\text{F}$.

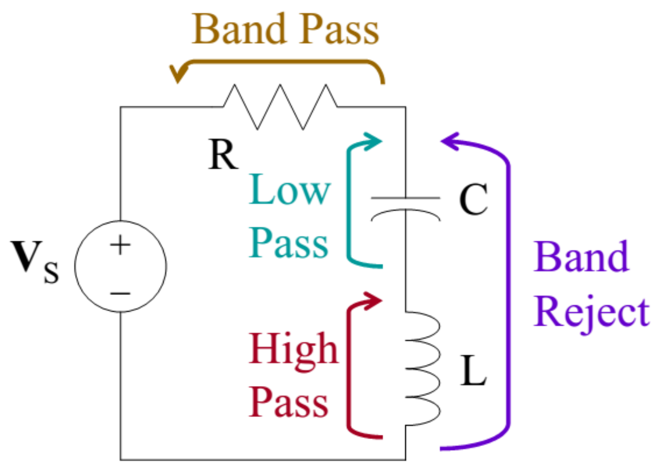


second-order lowpass filter

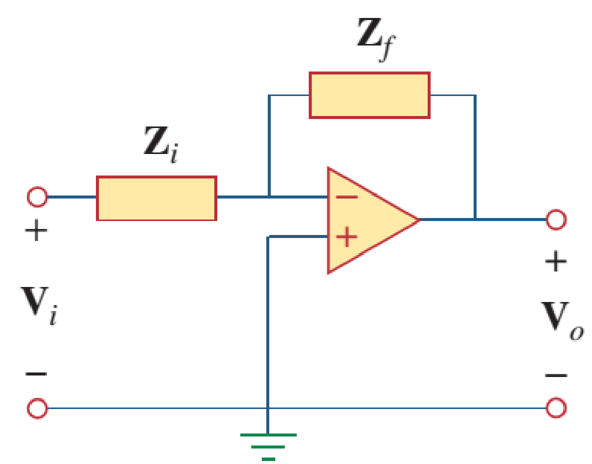
$$\omega_c = 0.742 \text{ krad/s} = 742 \text{ rad/s}$$

Active Filters

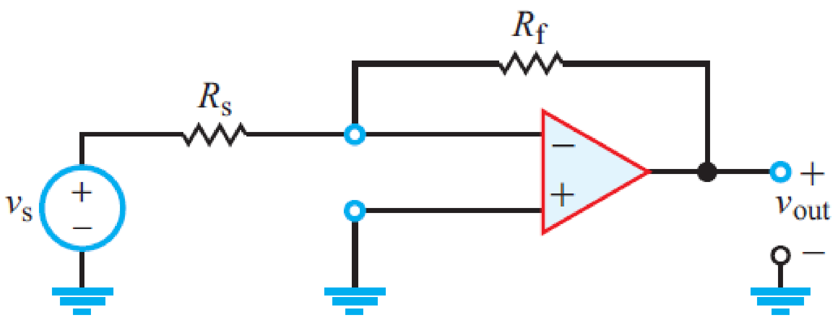
- Passive filters have a few drawbacks.
  - Gain.
  - Impedance.



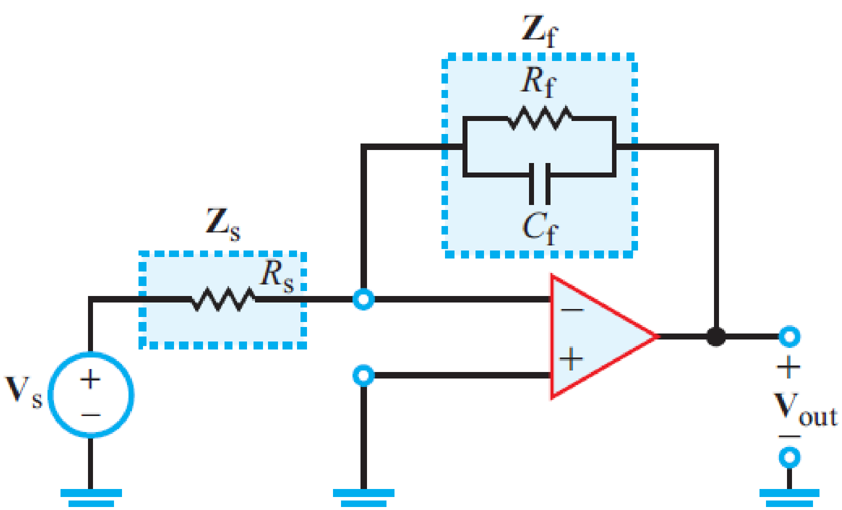
- It is possible, using <u>op-amps, together with resistors and capacitors</u>, to create all the common filters.
  - Their ability to isolate input and output also makes them very desirable.
  - Limited to frequency less than 1MHz.



Active Filters – Lowpass



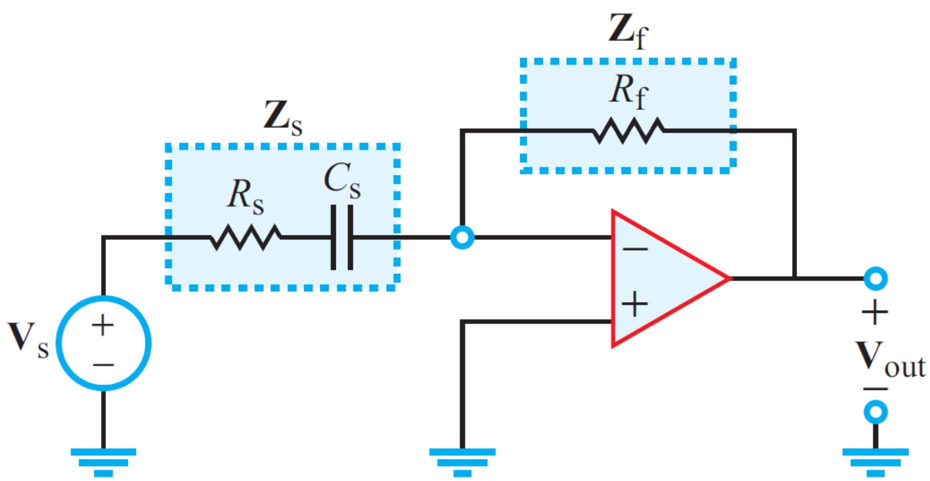
(a) Inverting amplifier



(b) Phasor domain with impedances

$$H(\omega) = -\frac{Z_f}{Z_s}$$

Active Filters – Highpass

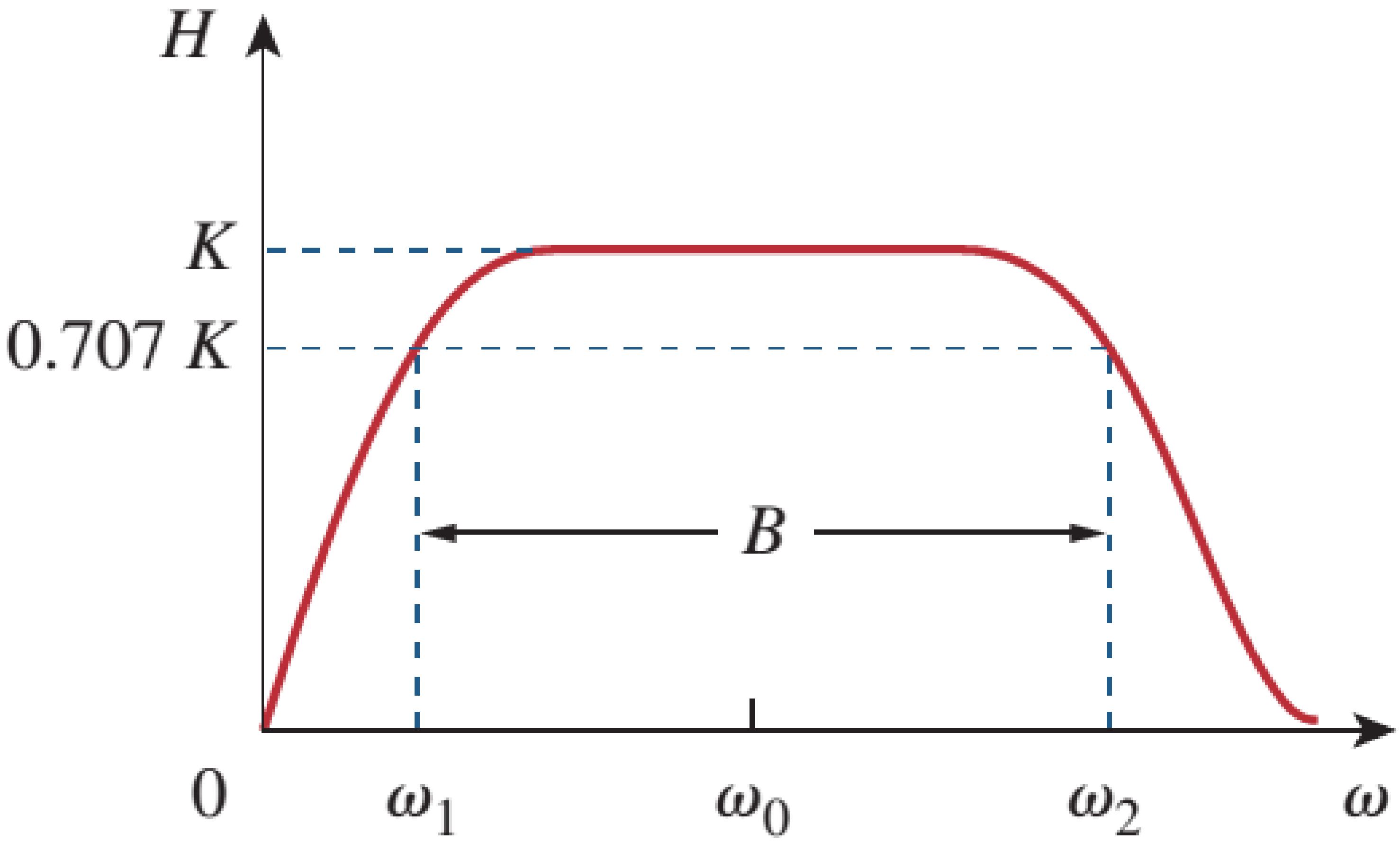


$$\mathbf{H}_{HP}(\omega) = \frac{\mathbf{V}_{out}}{\mathbf{V}_s} =$$

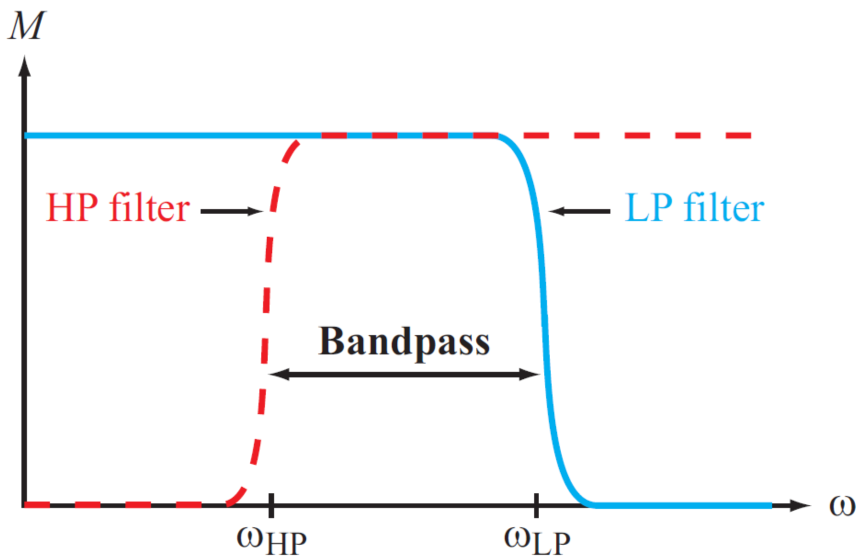
Exercise

- Design a 1^{st} order low-pass filter having a gain of 1 in the passband and a cutoff frequency 1 rad/s.

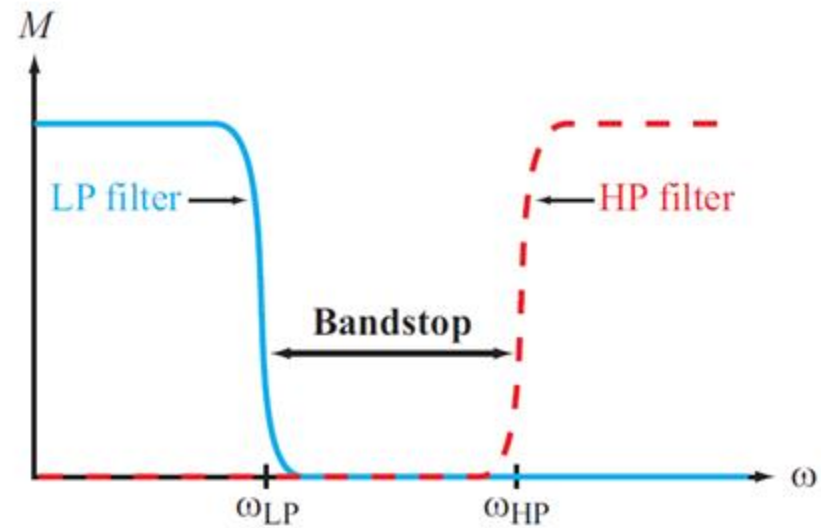
Bandpass



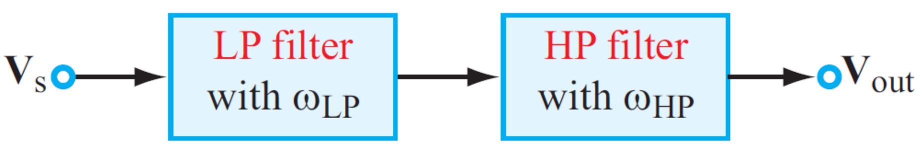
Cascading Active Filters

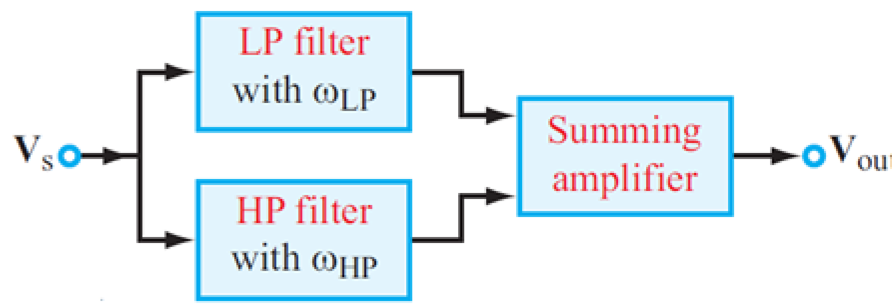


(a) Bandpass filter

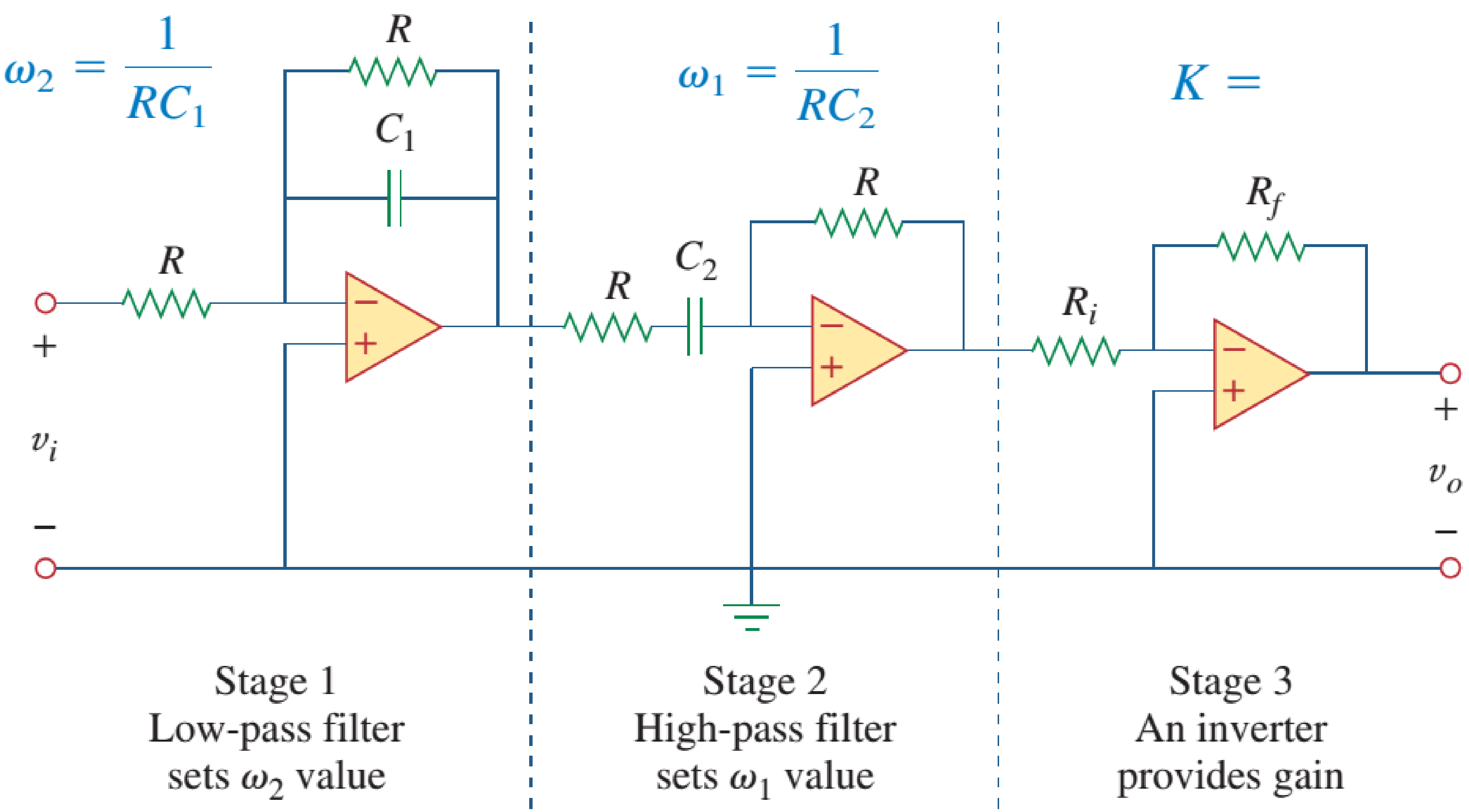


(b) Bandreject filter



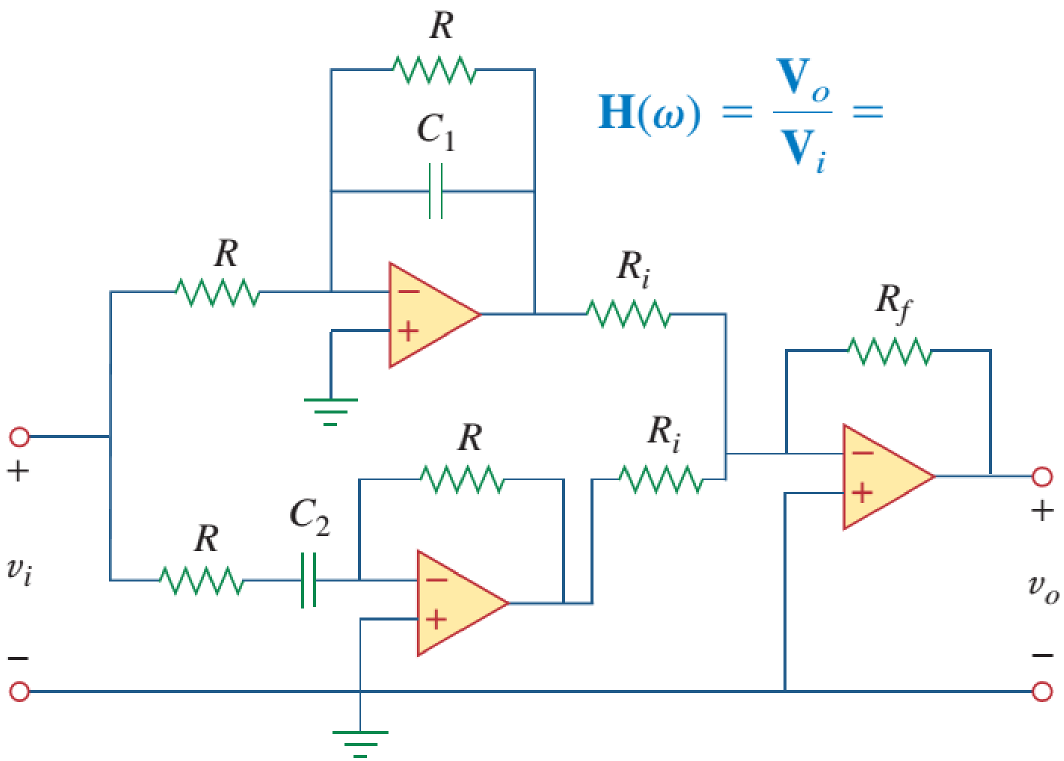


Active Bandpass Filter



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \left(-\frac{1}{1 + j\omega C_1 R}\right)\left(-\frac{j\omega C_2 R}{1 + j\omega C_2 R}\right)\left(-\frac{R_f}{R_i}\right)$$

Active Bandreject Filter



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} =$$