EE150: Signals and Systems, Spring 2022
Homework 5

(Due Tuesday, May. 24 at 11:59pm (CST))

1. [10 points]

   (a) Consider a system with impulse response

   $$h[n] = [(\frac{1}{2})^n cos(\frac{\pi n}{2})]u[n]$$

   Determine the system transfer function $H(e^{j\omega})$.

   (b) Suppose that $x[n] = cos(\frac{\pi n}{2})$. Determine the system output $y[n]$ using the system function in (a).

2. [15 points] In the system in Figure 1, $x(t)$ is sampled with a periodic impulse train, and a reconstructed signal $x_r(t)$ is obtained from the samples by lowpass filtering. The sampling period T is 1 ms, and $x(t)$ is a sinusoidal signal of the form $x(t) = cos(2\pi f_0 t + \theta)$.
For each of the following choices of f_0 and θ, determine $x_r(t)$.

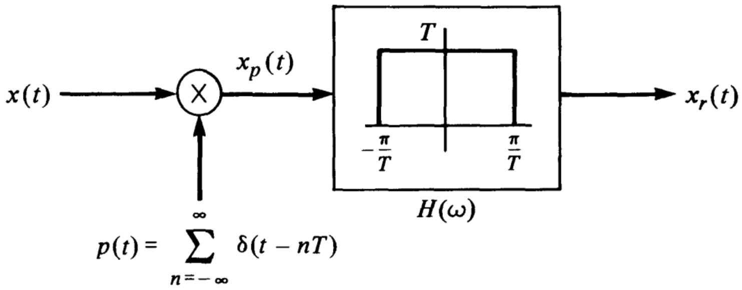


Figure 1: Problem 2

(a) $f_0 = 250Hz, \theta = \frac{\pi}{4}$.
(b) $f_0 = 750Hz, \theta = \frac{\pi}{2}$.
(c) $f_0 = 500Hz, \theta = \frac{\pi}{2}$.

3. [20 points] Figure 2 gives a system in which the sampling signal is an impulse train with alternating sign. The Fourier transform of the input signal is as indicated in the figure.

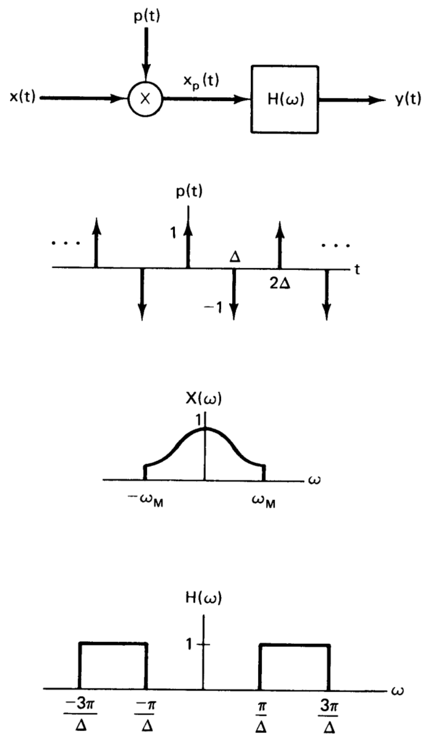


Figure 2: Problem 3

(a) For $\Delta < \frac{\pi}{2\omega_M}$, sketch the Fourier transform of $x_p(t)$ and $y(t)$.

(b) For $\Delta < \frac{\pi}{2\omega_M}$, determine a system that will recover $x(t)$ from $x_p(t)$.

(c) For $\Delta < \frac{\pi}{2\omega_M}$, determine a system that will recover $x(t)$ from $y(t)$.

(d) What is the maximum value of Δ in relation to ω_M for which $x(t)$ can be recovered from either $x_p(t)$ or $y(t)$.

4. [5 points] Let

$$y[n] = (\frac{sin(\frac{\pi}{4}n)}{\pi n})^2 * (\frac{sin\omega_c n}{\pi n})$$

where * denotes convolution and $|\omega_c| \leq \pi$. Determine a stricter constraint on ω_c which ensures that

$$y[n] = (\frac{sin(\frac{\pi}{4}n)}{\pi n})^2$$

5. [5 points] Given the fact that

$$a^{|n|} \xrightarrow{F} \frac{1-a^2}{1-2acos\omega+a^2}, |a|<1.$$

use duality to determine the Fourier series coefficients of the following continuous time signal with period $T = 1$:

$$x(t) = \frac{1}{5-4cos(2\pi t)}$$

6. [10 points] Consider a discrete-time LTI system with impulse response

$$h[n] = [(\frac{1}{2})^n cos(\frac{\pi n}{2})]u[n].$$

Determine the response to each of the following input signals:

(a) $x[n] = (\frac{1}{2})^n u[n]$

(b) $x[n] = cos(\frac{\pi n}{2})$

7. [20 points] Figure 3(a) shows the overall system for filtering a continuous-time signal using a discrete-time filter. If $X_c(j\omega)$ and $H(e^{j\omega})$ are as shown in Figure 3(b), with $\frac{1}{T} = 20kHz$, sketch $X_p(j\omega)$, $X(e^{j\omega})$, $Y(e^{j\omega})$, $Y_p(j\omega)$ and $Y_c(j\omega)$.

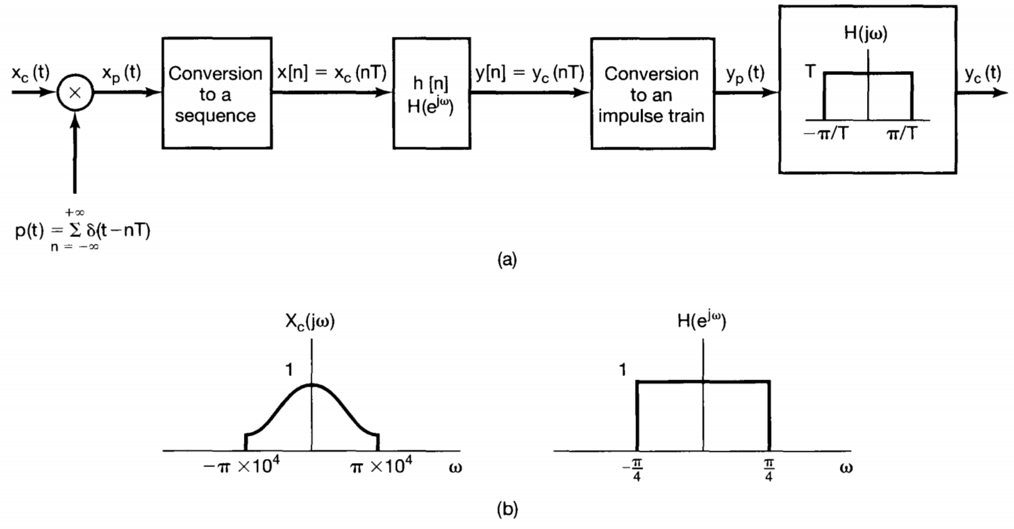


Figure 3: Problem 7

8. [15 points] Consider a discrete-time sequence $x[n]$ from which we form two new sequences, $x_p[n]$ and $x_d[n]$, where $x_p[n]$ corresponds to sampling $x[n]$ with a sampling period of 2 and $x_d[n]$ corresponds to decimating $x[n]$ by a factor of 2, so that

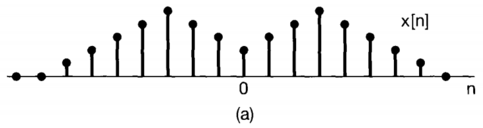
$$x_p[n] = \begin{cases} x[n], & n = 0, \pm 2, \pm 4 ... \\ 0, & n = \pm 1, \pm 3 ... \end{cases}$$

and

$$x_d[n] = x[2n].$$

(a) If $x[n]$ is as illustrated in Figure 4(a), sketch the sequences $x_p[n]$ and $x_d[n]$.

(b) If $X(e^{j\omega})$ is as shown in Figure 4(b), sketch $X_p(e^{j\omega})$ and $X_d(e^{j\omega})$



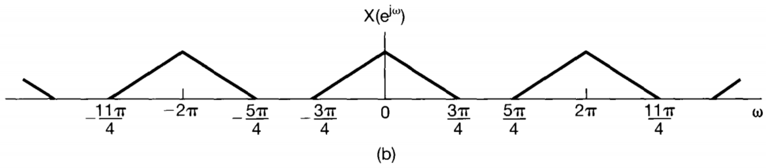


Figure 4: Problem 8