EE150 Signal and System
Homework 4

Due on 3 Nov 23:59 UTC+8

Note:

- Please provide enough calculation process to get full marks.
- Please submit your homework to Gradescope.
- It's highly recommended to wirte every exercise on single sheet of page.

Exercies 1. (20pt)

Determine the Fourier transform of each of the following signals:

(a) $\frac{d}{dt}\{u(-2-t)+u(t-2)\}$

(b) $2+\cos(6\pi t+\frac{\pi}{8})$

sketch and label the magnitude of each Fourier transform.

Exercies 2. (20pt)

Given that $x(t)$ has the Fourier transform $X(j\omega)$, express the Fourier transform of the signals listed below in terms of $X(j\omega)$.

(a) The inverse of $x^*(3t-6)$

(b) $\frac{d^2x(t-1)}{dt^2}$

Exercies 3. (15pt)

Use the duality property to solve the following problems:

(a) If the Fourier transform of $x(t)$ is $X(j\omega)$, find the Fourier transform of $X(t)$ and prove it.

(b) $x(t) = \sum_{k=-\infty}^{\infty} \frac{2\sin(k\frac{2\pi}{W}W_1)}{k}\delta(t - k\frac{2\pi}{W})$, sketch the Fourier transform without analysis equation.

Exercies 4. (20pt)

A causal and stable LTI system S has the frequency response

$$H(j\omega) = \frac{j\omega + 4}{6 - \omega^2 + 5j\omega}$$

(a) Determine the differential equation relating the input $x(t)$ and output $y(t)$ of the system S

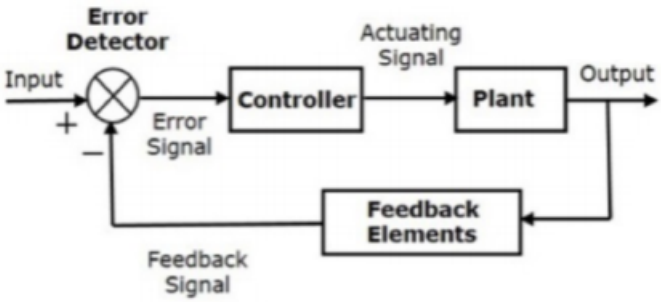
(b) Determine the impulse response $h(t)$ of the system S

(c) Determine the output of the system S when the input signal is

$$x(t) = e^{-4t}u(t) - te^{-4t}u(t)$$

Exercies 5. (25pt)

A general closed control system can be presented like this:



Input signal is $x(t)$, output signal is $y(t)$, error signal is e(t). Assume that the gain of Plant and Feedback Elements are "1"(which means you can think of them as wire).
For controller, there are three situation:

(i) PID controller: the output of controller is $K_p e(t) + K_i \int_{-\infty}^{t} e(\tau)d\tau + K_d \frac{de(t)}{dt}$. (when using intergation property, ignoring CD components)

(ii) PI contorller: it's similar to PID controller with $K_d = 0$.

(iii) PD contorller: it's similar to PID controller with $K_i = 0$.

(a) Determine the differential equation of PID controller closed system relating the input $x(t)$ and output $y(t)$, and calculate the Fourier transform of this system.

(b) Calculate the Fourier transform of PID, PI, PD controller.

(c) Let signal pass through the PI($K_p = 2, K_i = 1$), PD($K_p = 5, K_d = 2$) controllers in turn. Prove that this is equivalent to letting the signal pass through the PID($K_p = 12, K_i = 5, K_d = 4$) controller directly.