

Project Presentation

- Plan

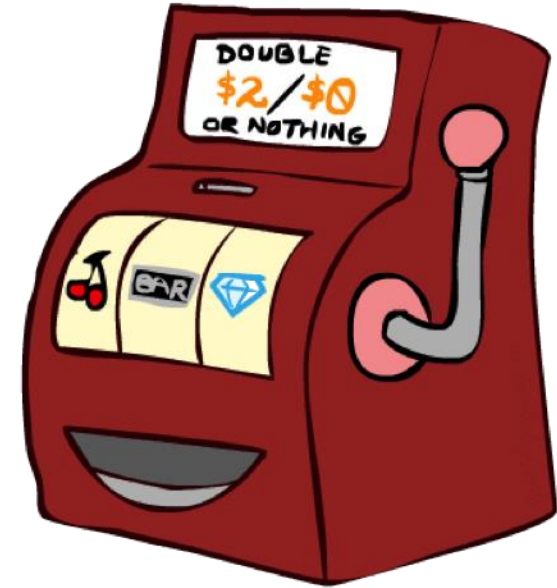
- The final project presentation will be on 7-June(in class) & 9-June(in class).

Reinforcement Learning



AIMA Chapter 21

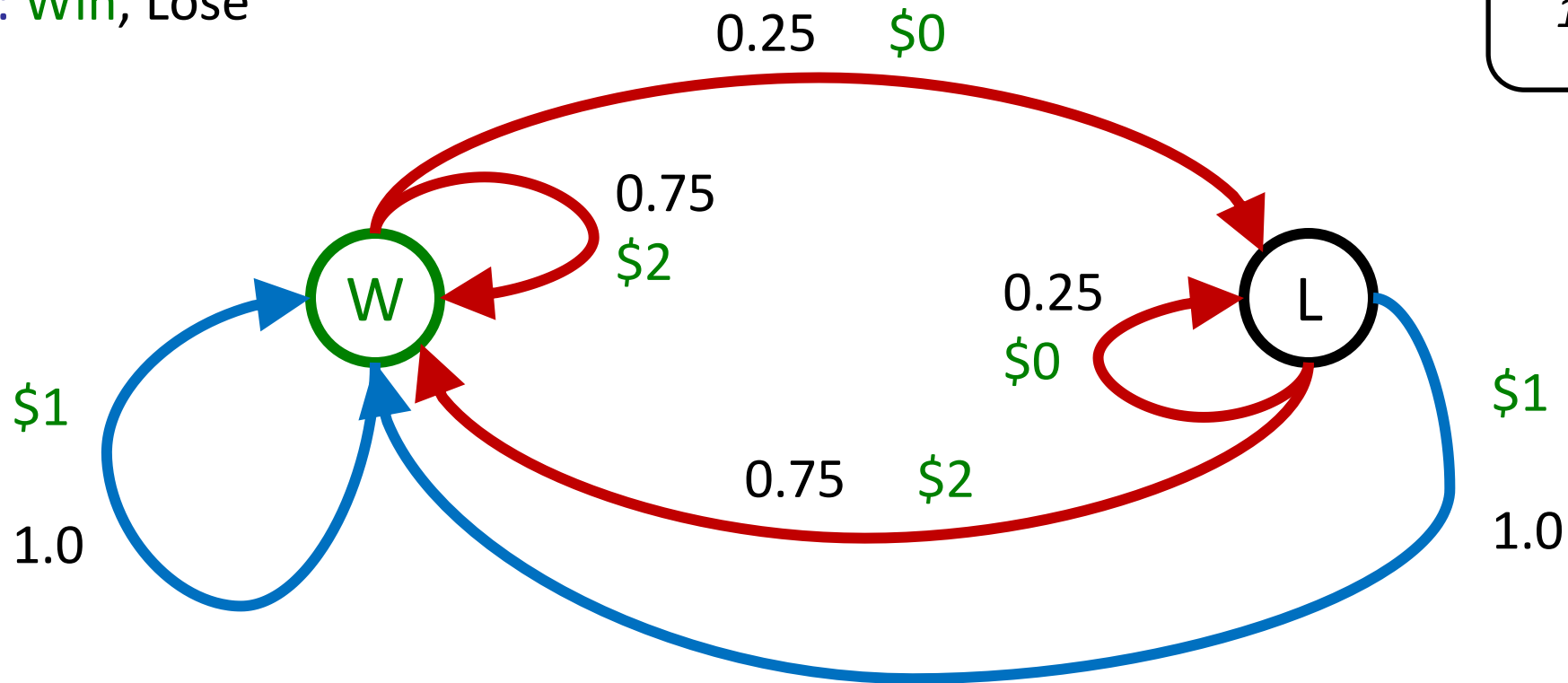
Double Bandits



Double-Bandit MDP

- Actions: *Blue*, *Red*
- States: *Win*, Lose

No discount
100 time steps

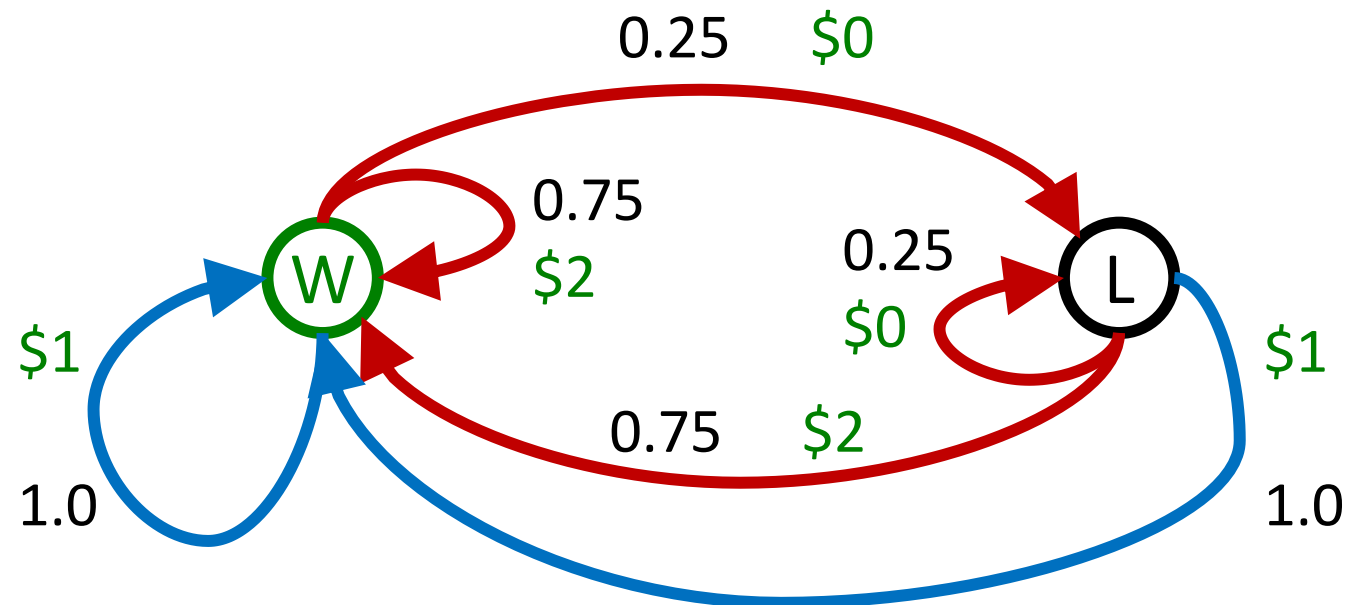


Offline Planning

- Solving MDPs is offline planning
 - You determine all quantities through computation
 - You need to know the details of the MDP
 - You do not actually play the game!

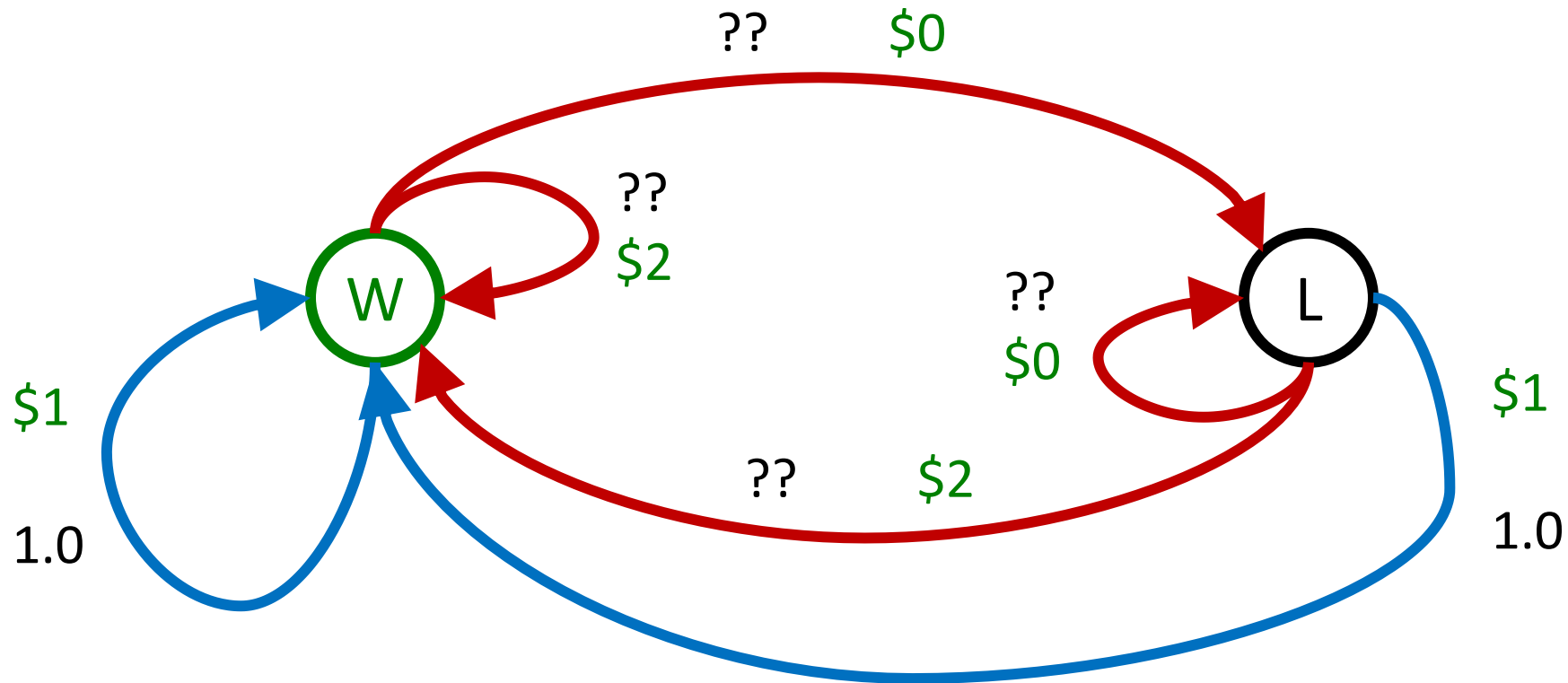
No discount
100 time steps

	Value
Play Red	150
Play Blue	100

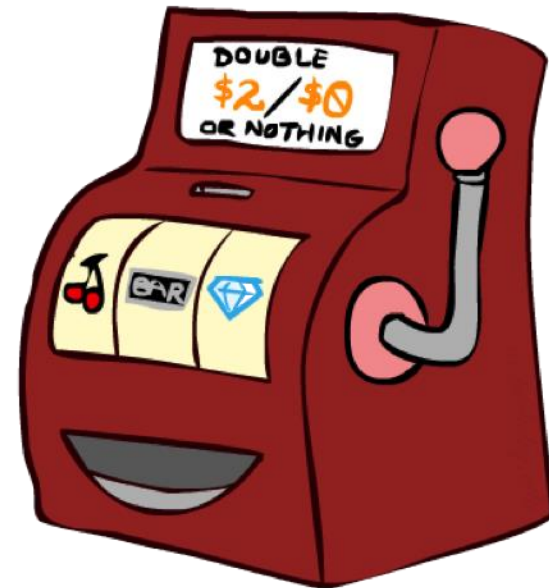


Online Planning

- Rules changed! Red's win chance is different.



Let's Play!



\$0 \$0 \$0 \$2 \$0
\$2 \$0 \$0 \$0 \$0

What Just Happened?

- That wasn't planning, it was learning!
 - Specifically, reinforcement learning
 - There was an MDP, but you couldn't solve it with just computation
 - You needed to actually act to figure it out
- Important ideas in reinforcement learning that came up
 - Exploration: you have to try unknown actions to get information
 - Exploitation: eventually, you have to use what you know
 - Regret: even if you learn intelligently, you make mistakes
 - Sampling: because of chance, you have to try things repeatedly
 - Difficulty: learning can be much harder than solving a known MDP

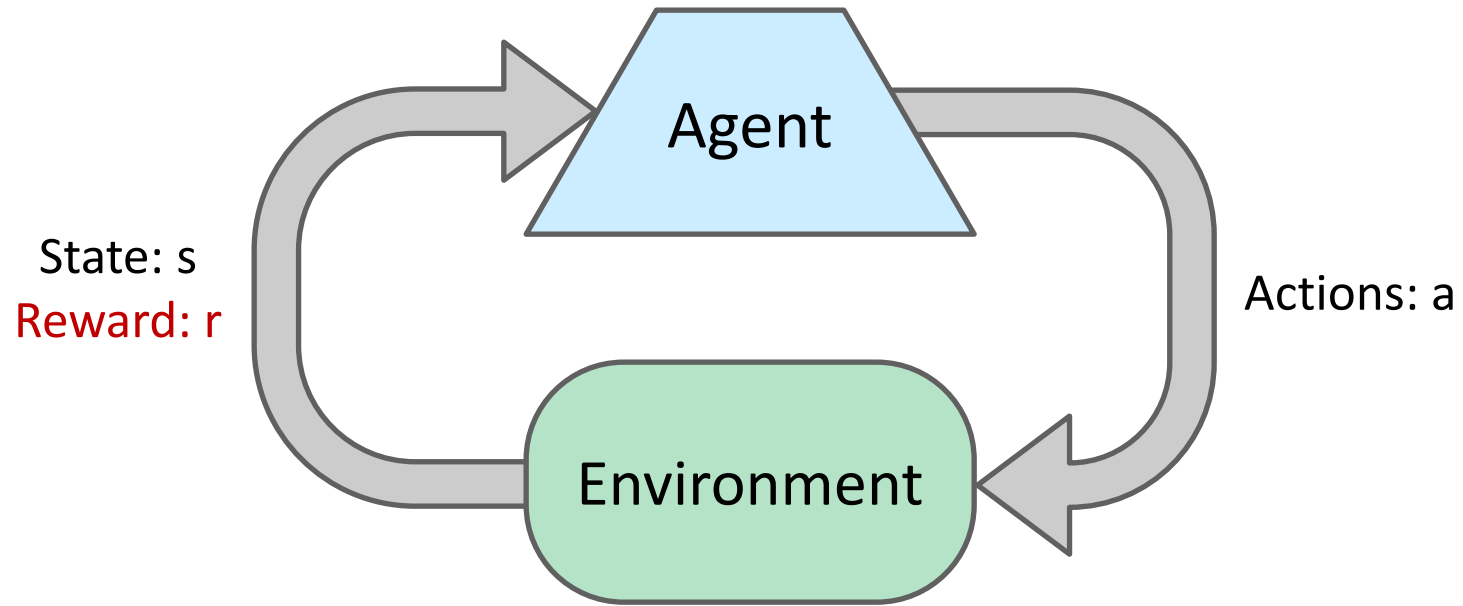


Reinforcement Learning

- Still assume a Markov decision process (MDP):
 - A set of states $s \in S$
 - A set of actions (per state) A
 - A model $T(s,a,s')$
 - A reward function $R(s,a,s')$
- Still looking for a policy $\pi(s)$
- New twist: don't know T or R
 - I.e. we don't know which states are good or what the actions do
 - Must actually try actions and states out to learn



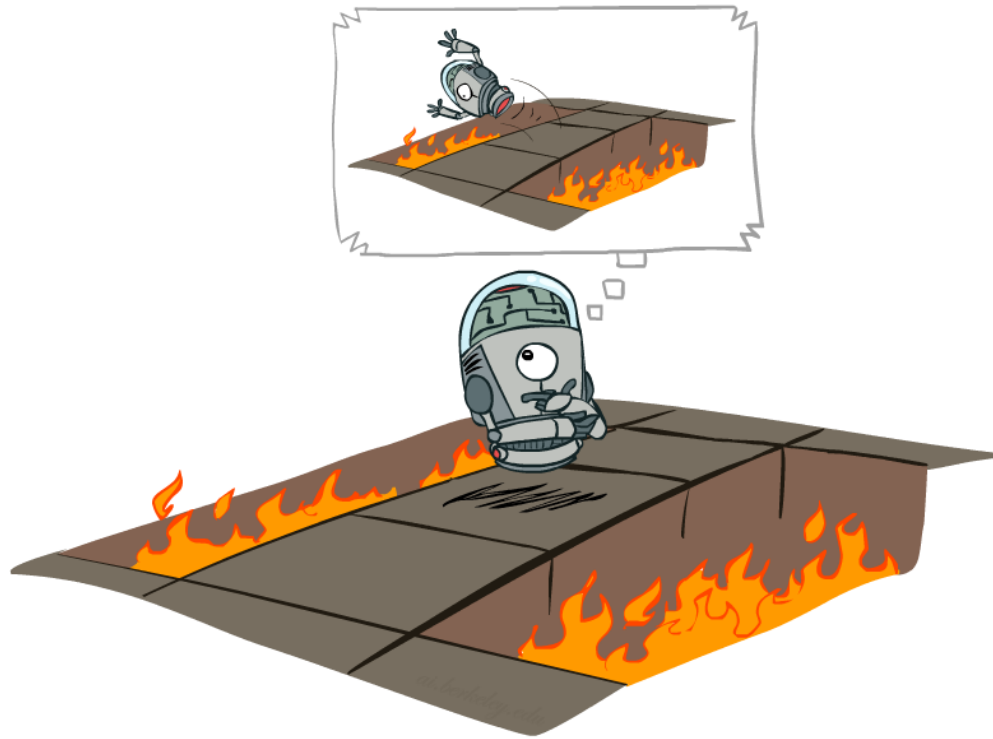
Reinforcement Learning



- **Basic idea:**

- Take actions and observe outcomes (new states, rewards)
- Learning is based on observed samples of outcomes
- Must (learn to) act so as to **maximize expected rewards**

Offline (MDPs) vs. Online (RL)



Offline Solution

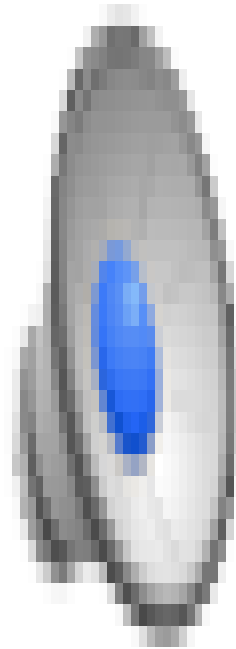


Online Learning

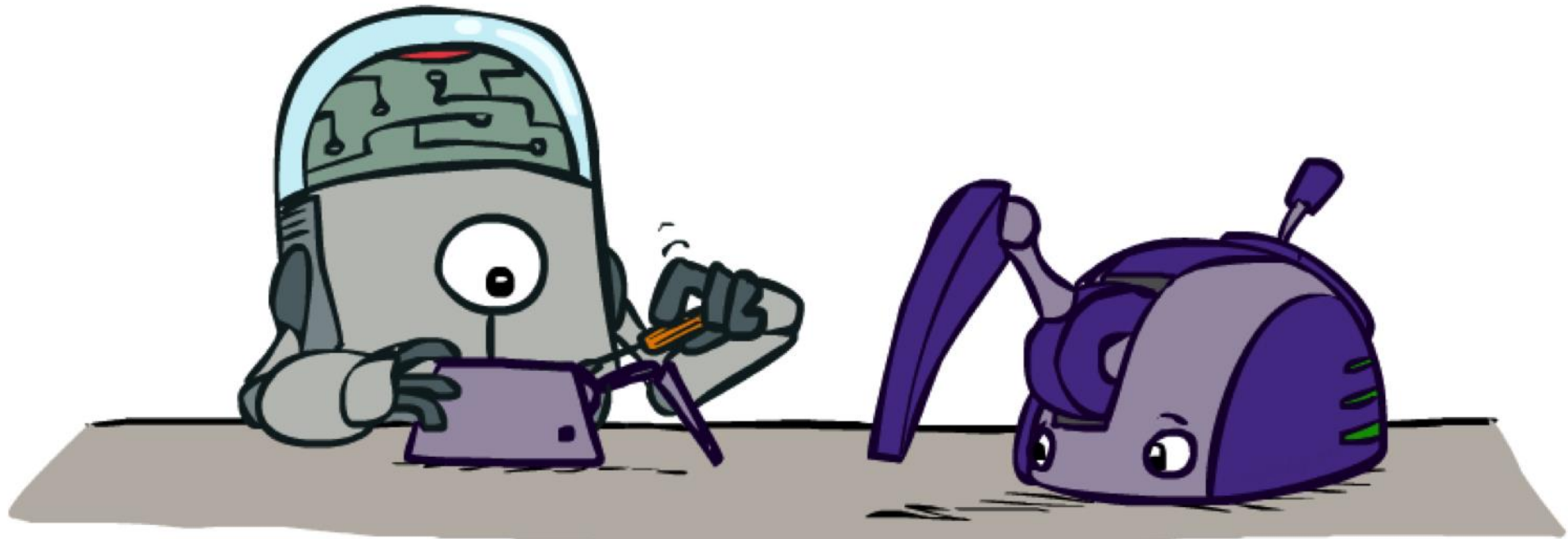
Cheetah



Atari

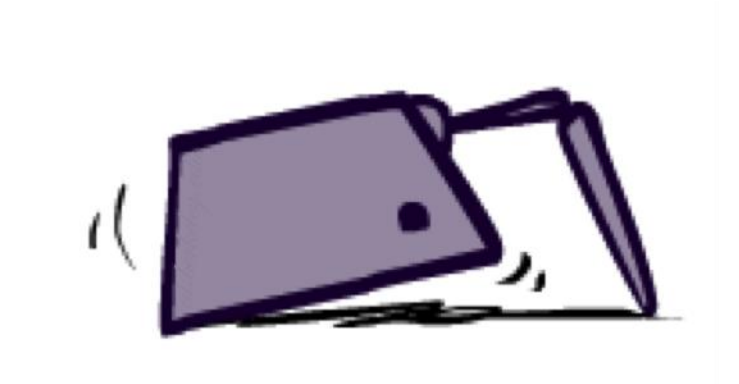
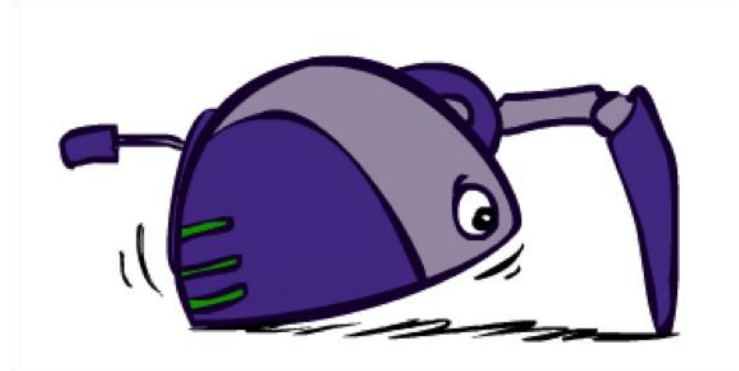


Model-Based Learning

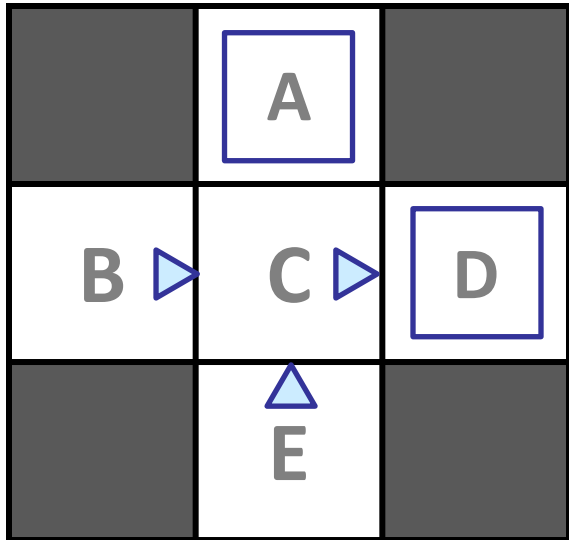


Model-Based Learning

- **Model-Based Idea:**
 - Learn an approximate model based on experiences
 - Solve for values as if the learned model was correct
- **Step 1: Learn empirical MDP model**
 - Count outcomes s' for each s, a
 - Normalize to give an estimate of $\hat{T}(s, a, s')$
 - Discover each $\hat{R}(s, a, s')$ when we experience (s, a, s')
- **Step 2: Solve the learned MDP**
 - For example, use value iteration, as before



Example: Model-Based Learning



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Learned Model

$$\hat{T}(s, a, s')$$

T(B, east, C) = 1.00
T(C, east, D) = 0.75
T(C, east, A) = 0.25
...

$$\hat{R}(s, a, s')$$

R(B, east, C) = -1
R(C, east, D) = -1
R(D, exit, x) = +10
...

Model-Based vs. Model-Free

Goal: Compute expected age of ShanghaiTech students

Known $P(A)$

$$E[A] = \sum_a P(a) \cdot a = 0.35 \times 20 + \dots$$

Without $P(A)$, instead collect samples $[a_1, a_2, \dots, a_N]$

Unknown $P(A)$: “Model Based”

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$
$$E[A] \approx \sum_a \hat{P}(a) \cdot a$$

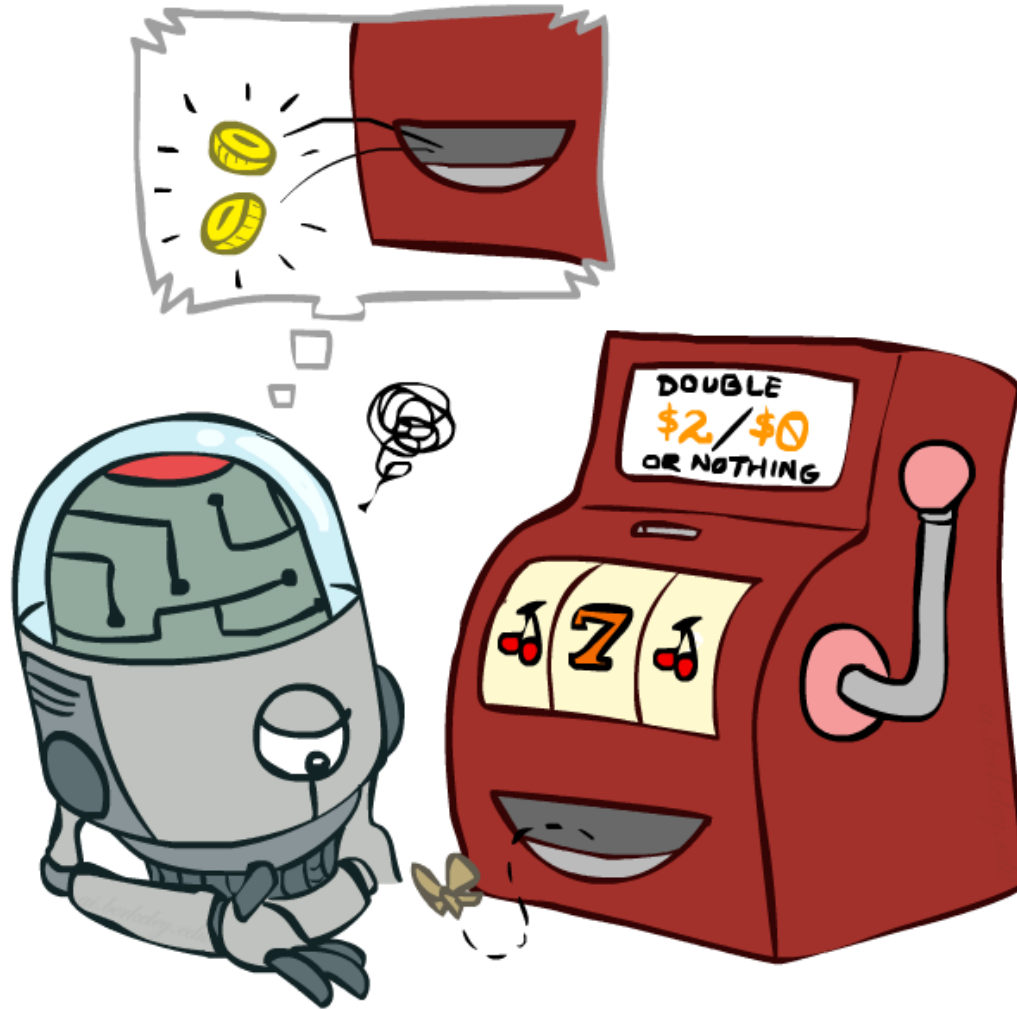
Why does this work? Because eventually you learn the right model.

Unknown $P(A)$: “Model Free”

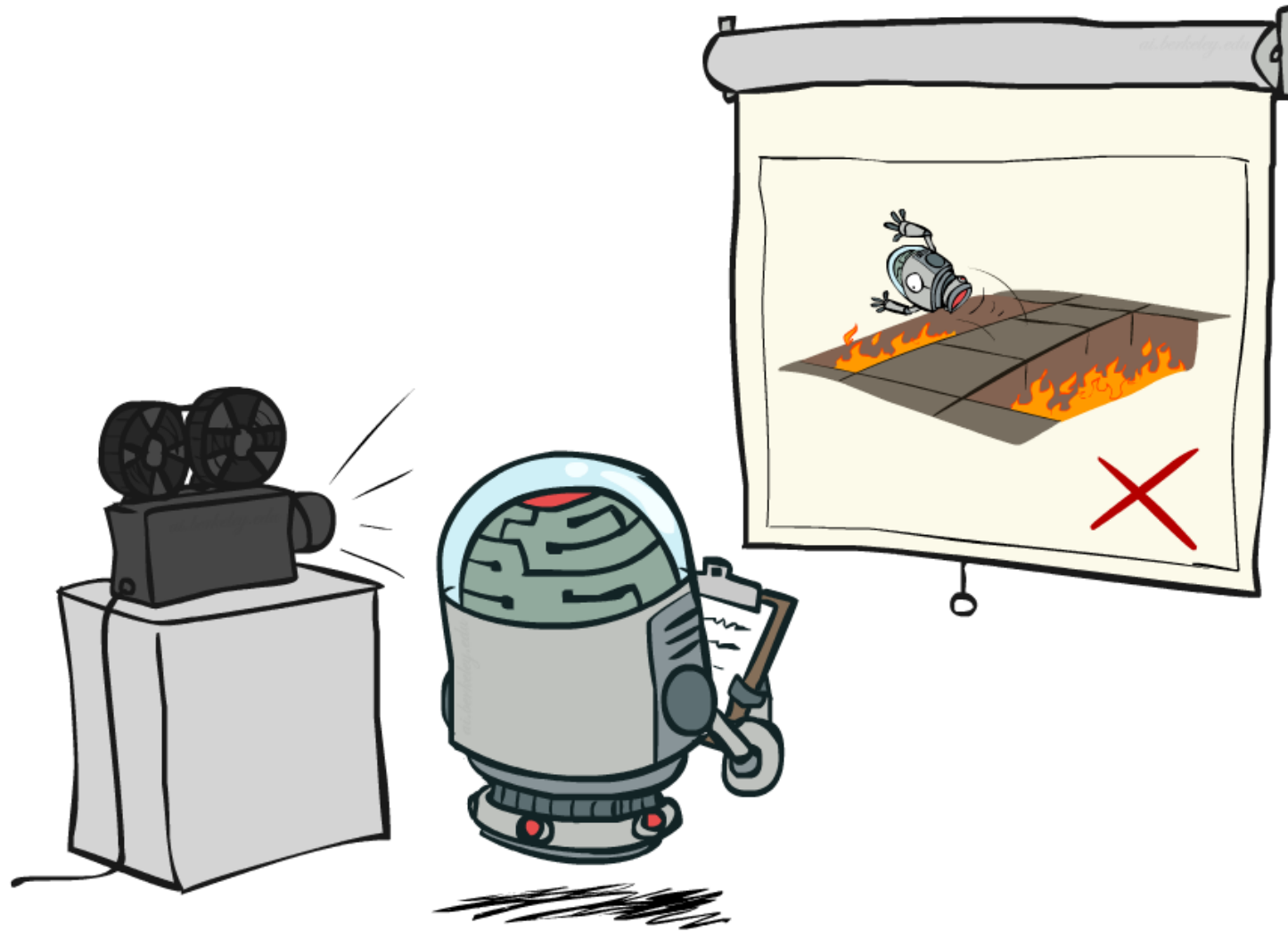
$$E[A] \approx \frac{1}{N} \sum_i a_i$$

Why does this work? Because samples appear with the right frequencies.

Model-Free Learning



Passive Reinforcement Learning



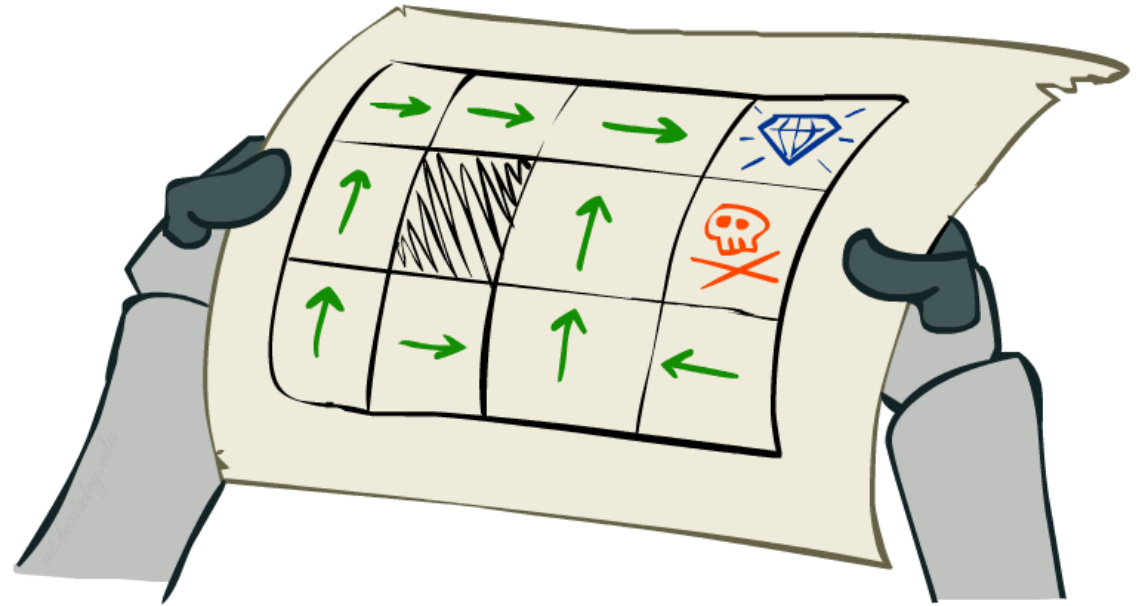
Passive Reinforcement Learning

- Simplified task: policy evaluation

- Input: a fixed policy $\pi(s)$
- You don't know the transitions $T(s,a,s')$
- You don't know the rewards $R(s,a,s')$
- Goal: learn the state values

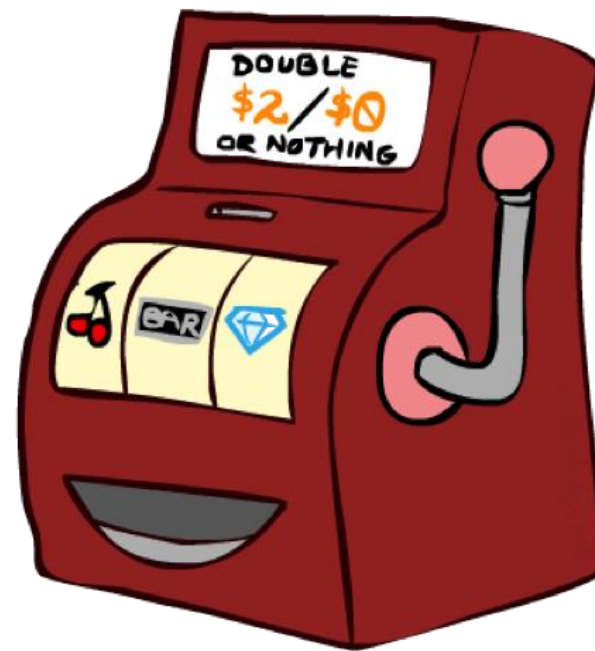
- In this case:

- No choice about what actions to take
- Just execute the policy and learn from experience
- This is NOT offline planning! You actually take actions in the world.



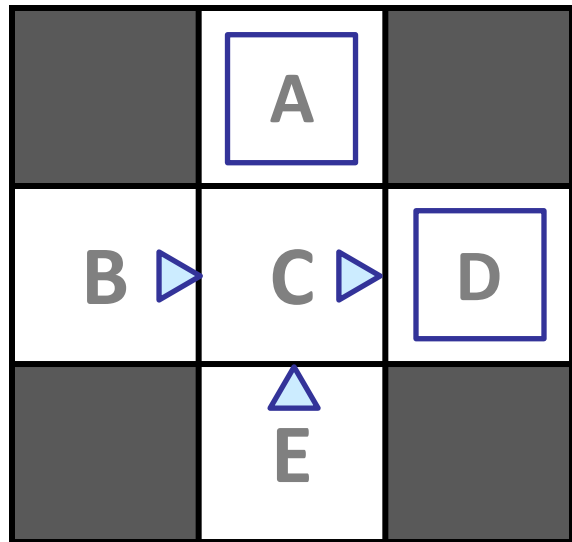
Direct Evaluation

- Goal: Compute values for each state under π
- Idea: Average together observed sample values
 - Act according to π
 - Every time you visit a state, write down what the sum of discounted rewards turned out to be
 - Average those samples
- This is called direct evaluation



Example: Direct Evaluation

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

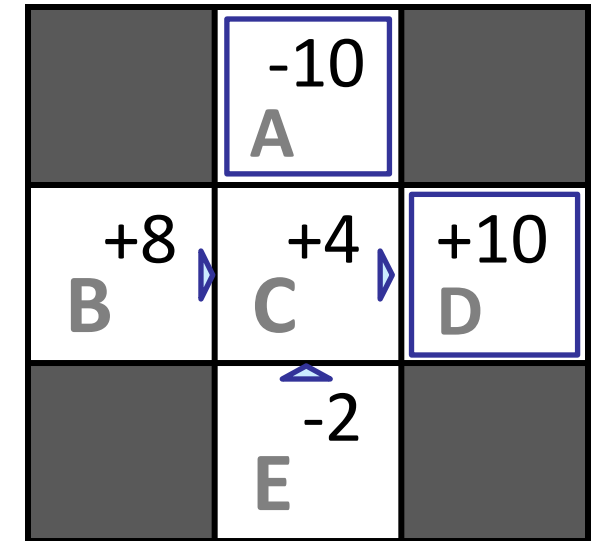
Output Values

	-10 A	
+8 B	+4 C	+10 D
	-2 E	

Problems with Direct Evaluation

- What's good about direct evaluation?
 - It's easy to understand
 - It doesn't require any knowledge of T , R
 - It eventually computes the correct average values, using just sample transitions
- What bad about it?
 - It wastes information about state connections
 - Each state must be learned separately
 - So, it takes a long time to learn

Output Values



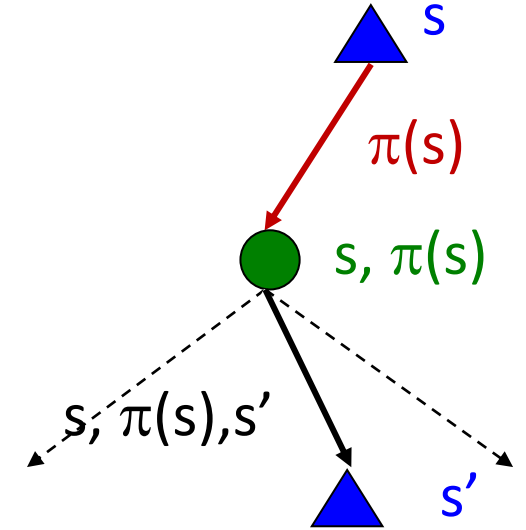
B goes to C, so we may use Bellman equation

Why Not Use Policy Evaluation?

- Simplified Bellman updates calculate V for a fixed policy:

$$V_0^\pi(s) = 0$$

$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} T(s, \pi(s), s') [R(s, \pi(s), s') + \gamma V_k^\pi(s')]$$



- This approach fully exploits the connections between the states
 - Unfortunately, we need T and R to do it!
- Key question: how can we do this update to V without knowing T and R ?
 - In other words, how do we take a weighted average without knowing the weights?

Sample-Based Policy Evaluation?

- We want to compute these averages:

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s') [R(s, \pi(s), s') + \gamma V_k^{\pi}(s')]$$

- Idea: Take samples of outcomes s' (by doing the action!) and average

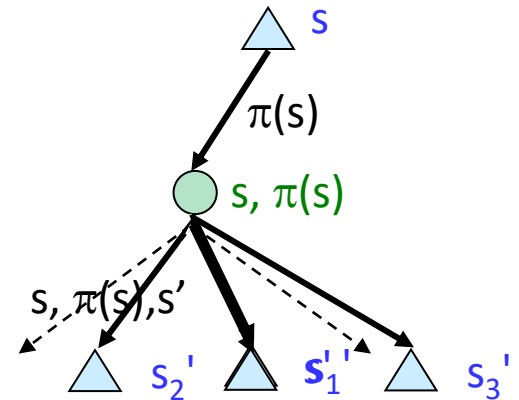
$$sample_1 = R(s, \pi(s), s'_1) + \gamma V_k^{\pi}(s'_1)$$

$$sample_2 = R(s, \pi(s), s'_2) + \gamma V_k^{\pi}(s'_2)$$

...

$$sample_n = R(s, \pi(s), s'_n) + \gamma V_k^{\pi}(s'_n)$$

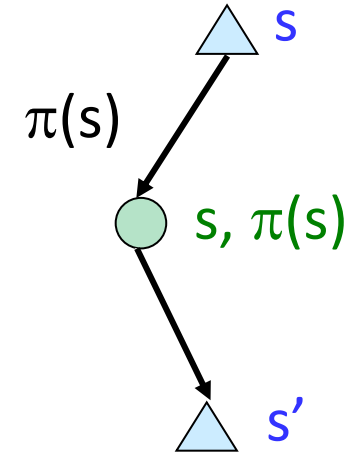
$$V_{k+1}^{\pi}(s) \leftarrow \frac{1}{n} \sum_i sample_i$$



*But we can't rewind time
to get sample after
sample from state s !*

Temporal Difference Learning

- Big idea: learn immediately from every experience!
 - Update $V(s)$ each time we experience a transition (s, a, s', r)
- Temporal difference learning of values
 - (Policy still fixed, still doing evaluation!)
 - Move the value towards the sample



Sample of $V(s)$: $sample = R(s, \pi(s), s') + \gamma V^\pi(s')$

Update to $V(s)$: $V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + (\alpha)sample$

Same update: $V^\pi(s) \leftarrow V^\pi(s) + \alpha(sample - V^\pi(s))$

Exponential Moving Average

- Exponential moving average

- The running interpolation update: $\bar{x}_n = (1 - \alpha) \cdot \bar{x}_{n-1} + \alpha \cdot x_n$
- Makes recent samples more important
- Forgets about the past (distant past values were wrong anyway)

$$\bar{x}_n = \frac{x_n + (1 - \alpha) \cdot x_{n-1} + (1 - \alpha)^2 \cdot x_{n-2} + \dots}{1 + (1 - \alpha) + (1 - \alpha)^2 + \dots}$$

- Decreasing learning rate (alpha) can give converging averages

Example: Temporal Difference Learning

States

	A	
B	C	D
	E	

Assume: $\gamma = 1$, $\alpha = 1/2$

Observed Transitions

B, east, C, -2

	0	
0	0	8
	0	

C, east, D, -2

	0	
-1	0	8
	0	

	0	
-1	3	8
	0	

$$V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + \alpha [R(s, \pi(s), s') + \gamma V^\pi(s')]$$

Limitations of TD Value Learning

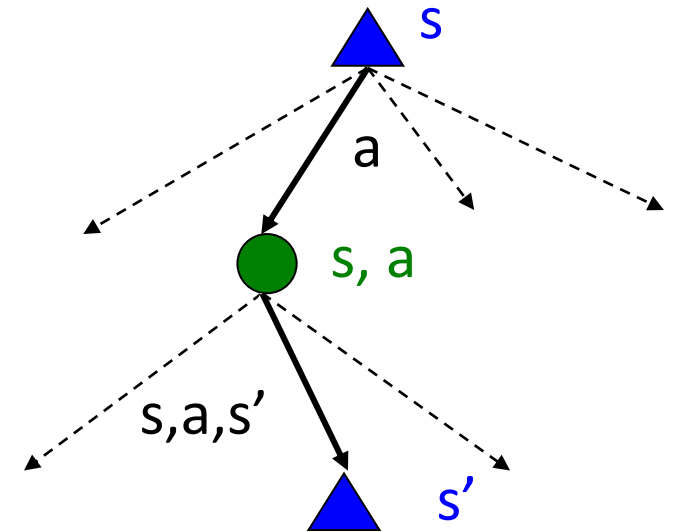
- TD value learning is a model-free way to do policy evaluation, mimicking Bellman updates with running sample averages
- However, if we want to turn values into a (new) policy...

$$\pi(s) = \arg \max_a Q(s, a)$$

$$Q(s, a) = \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V(s')]$$


Unknown!

- Idea: learn Q-values, not values
- Makes action selection model-free too!



Q-Learning

- Q-value iteration

$$Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$$

- Q-Learning: learn $Q(s,a)$ values as you go

- Receive a sample (s,a,s',r)
- Consider your old estimate: $Q(s, a)$
- Consider your new sample estimate:

$$sample = R(s, a, s') + \gamma \max_{a'} Q(s', a')$$

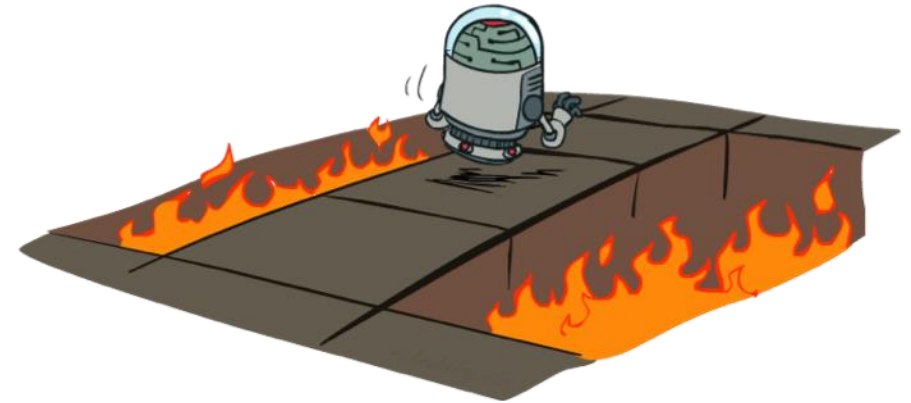
- Incorporate the new estimate into a running average:

$$Q(s, a) \leftarrow (1 - \alpha)Q(s, a) + (\alpha) [sample]$$

Active Reinforcement Learning

- Full reinforcement learning

- You don't know the transitions $T(s,a,s')$
- You don't know the rewards $R(s,a,s')$
- You choose the actions now
- Goal: learn the optimal policy / values

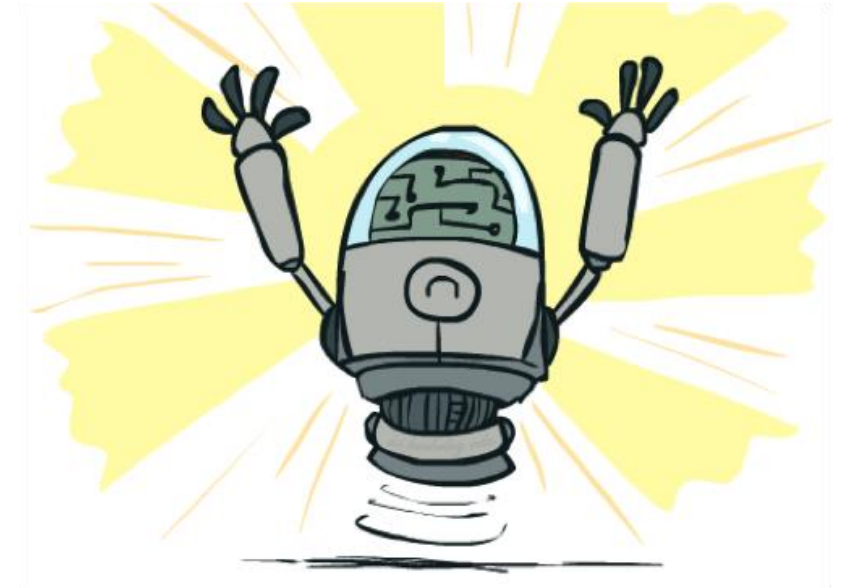


- Q-learning:

- Learner makes choices (according to current values / policy, and also explore...)
 - Fundamental tradeoff: exploration vs. exploitation
- This is NOT offline planning! You actually take actions in the world and find out what happens...

Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy -- even if you're acting suboptimally!
- This is called **off-policy learning**
- Caveats:
 - You have to explore enough
 - You have to eventually make the learning rate small enough
 - ... but not decrease it too quickly



The Story So Far: MDPs and RL

Known MDP: Offline Solution

Goal

Compute V^* , Q^* , π^*

Evaluate a fixed policy π

Technique

Value / policy iteration

Policy evaluation

Unknown MDP: Model-Based

Goal

Compute V^* , Q^* , π^*

Evaluate a fixed policy π

Technique

VI/PI on approx. MDP

PE on approx. MDP

Unknown MDP: Model-Free

Goal

Compute V^* , Q^* , π^*

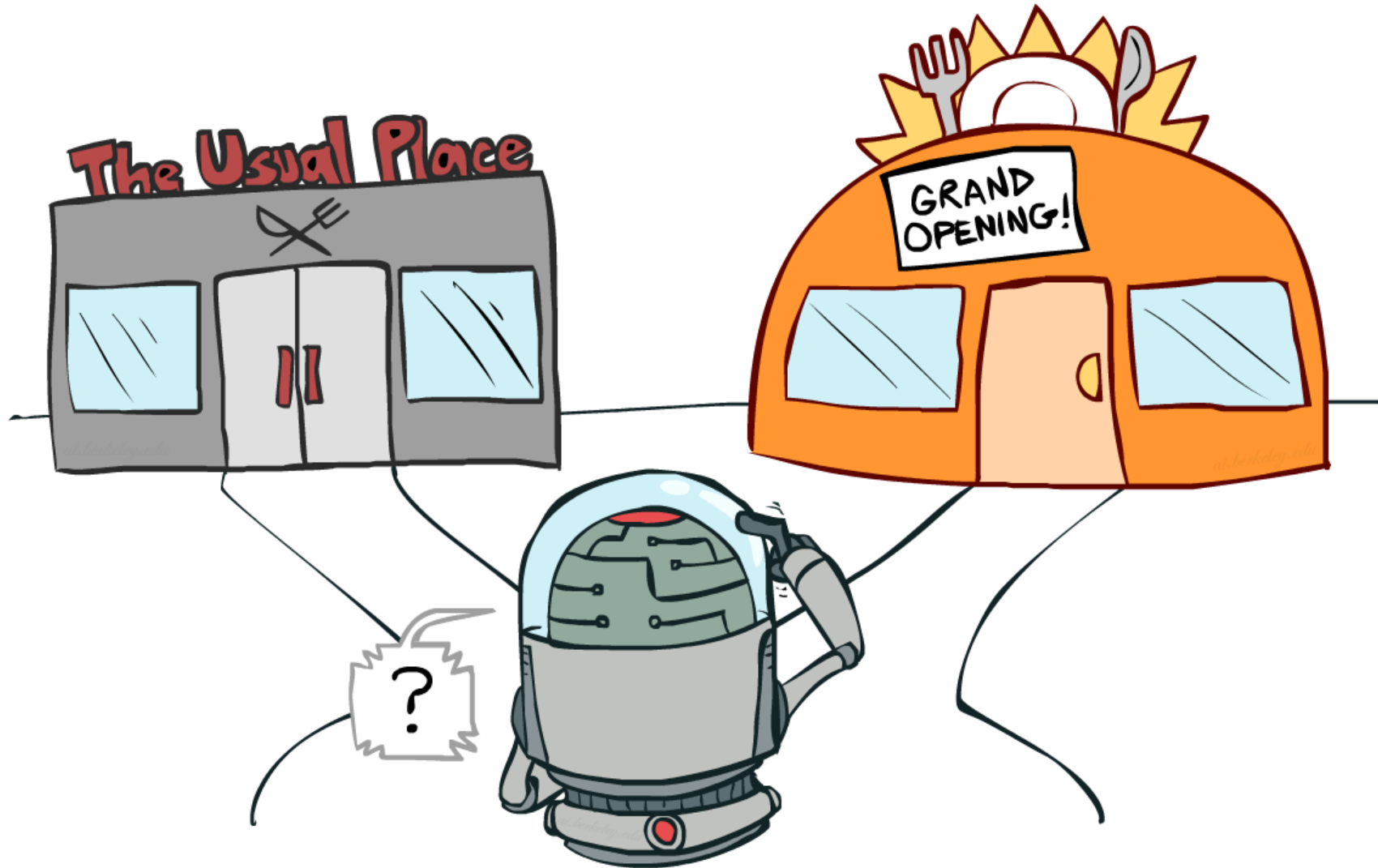
Evaluate a fixed policy π

Technique

Q-learning

TD Value Learning

Exploration vs. Exploitation



How to Explore?

- Several schemes for forcing exploration
 - Simplest: random actions (ϵ -greedy)
 - Every time step, flip a coin
 - With (small) probability ϵ , act randomly
 - With (large) probability $1-\epsilon$, act on current policy



How to Explore?

- Several schemes for forcing exploration
 - Problems with random actions?
 - You do eventually explore the space, but keep thrashing around once learning is done
 - One solution: lower ϵ over time
 - Another solution: exploration functions



Exploration Functions

- When to explore?

- Explore states that haven't been sufficiently explored
- Eventually stop exploring

- Idea: select actions based on modified Q-value

- Exploration function: takes a Q-value estimate u and a visit count n , and returns an optimistic utility, e.g.

$$f(u, n) = u + k/n$$

- Q-Update

Regular Update: $Q(s, a) \leftarrow_{\alpha} R(s, a, s') + \gamma \max_{a'} Q(s', a')$

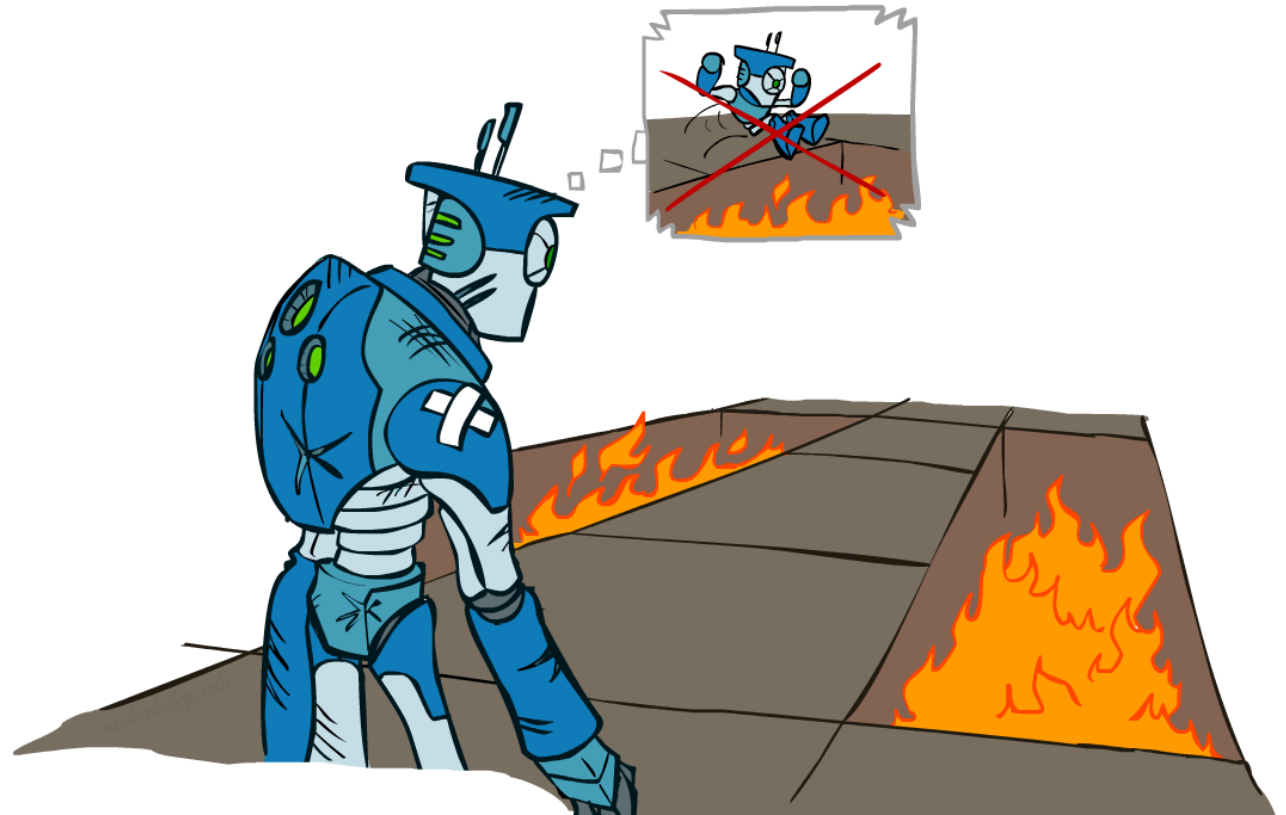
Modified Update: $Q(s, a) \leftarrow_{\alpha} R(s, a, s') + \gamma \max_{a'} f(Q(s', a'), N(s', a'))$

This propagates the “bonus” back to states that lead to under-explored states

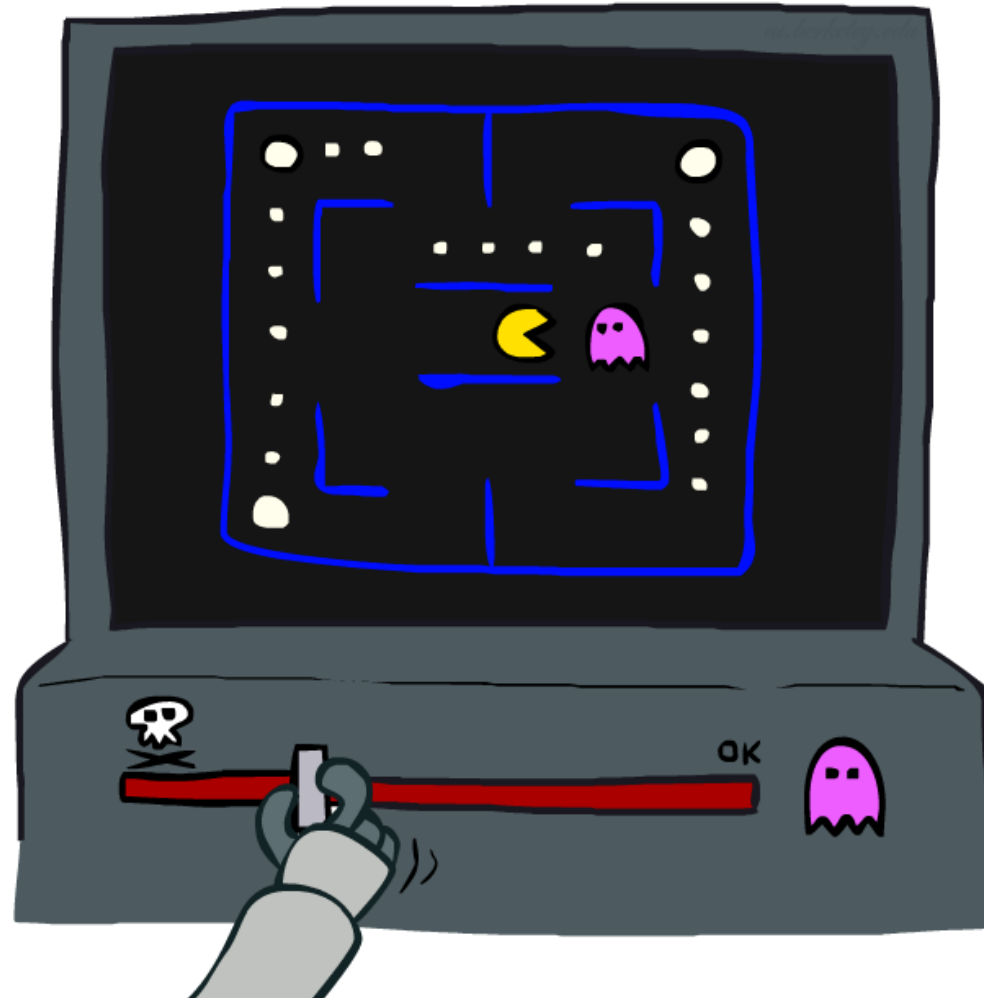


Regret

- Even if you learn the optimal policy, you still make mistakes along the way!
- Regret is a measure of your total mistake cost: the difference between your (expected) rewards, including youthful suboptimality, and optimal (expected) rewards
- Minimizing regret goes beyond learning to be optimal – it requires optimally learning to be optimal
- Example: random exploration and exploration functions both end up optimal, but random exploration has higher regret

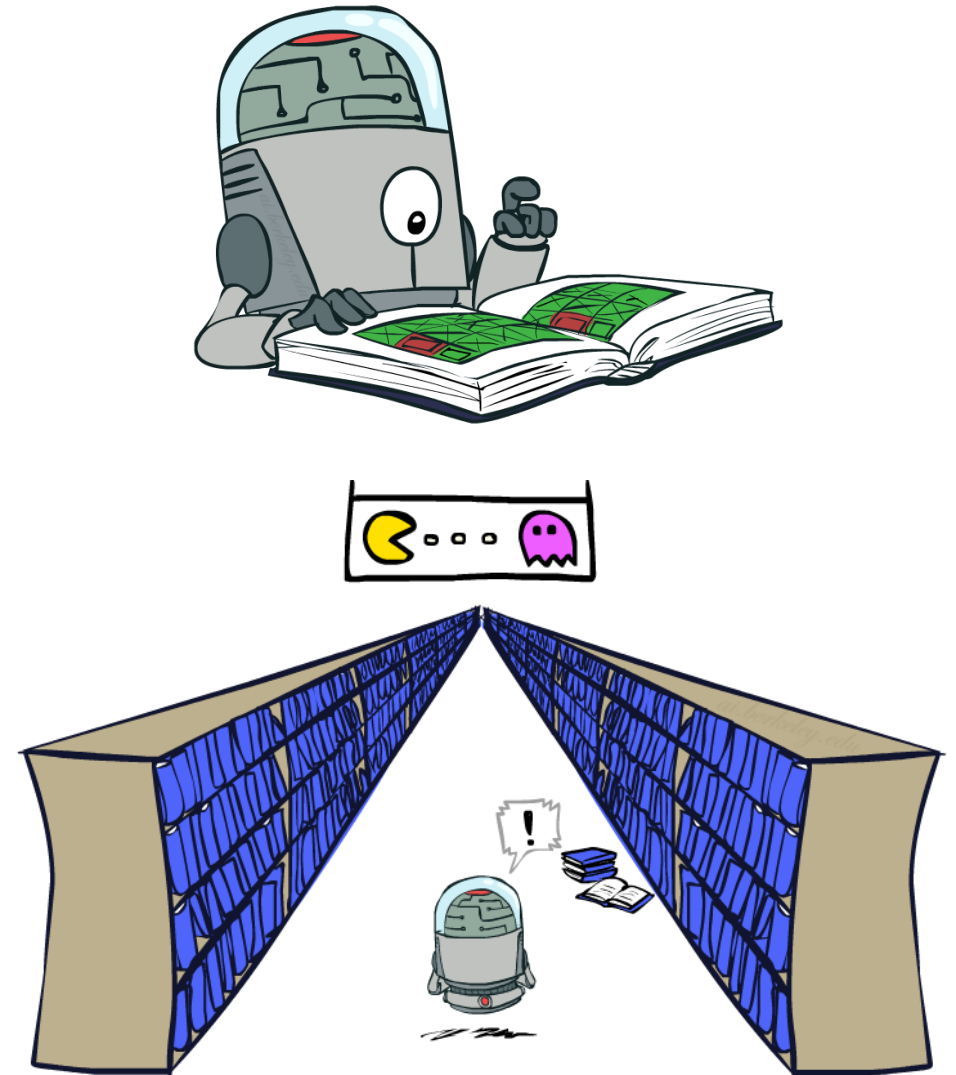


Approximate Q-Learning



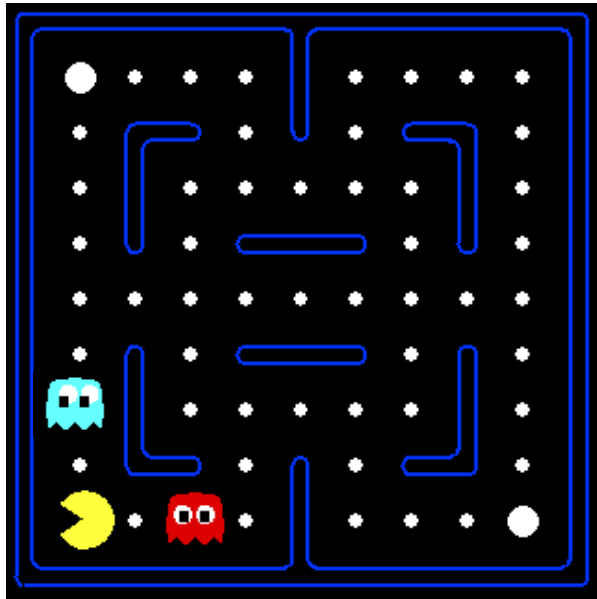
Generalizing Across States

- Basic Q-Learning keeps a table of all q-values
- In realistic situations, we cannot possibly learn about every single state!
 - Too many states to visit them all in training
 - Too many states to hold the q-tables in memory

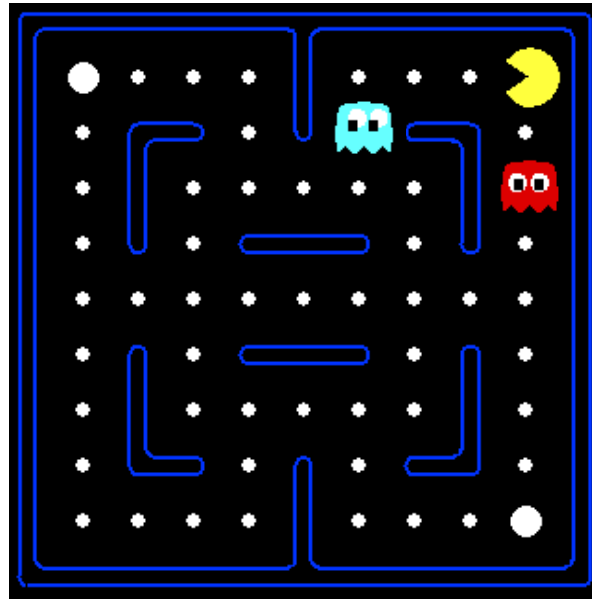


Example: Pacman

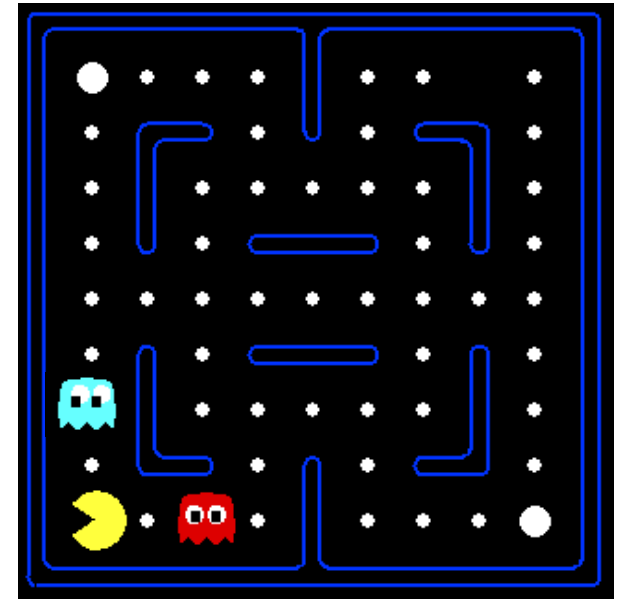
Let's say we discover through experience that this state is bad:



In naïve q-learning, we know nothing about this state:

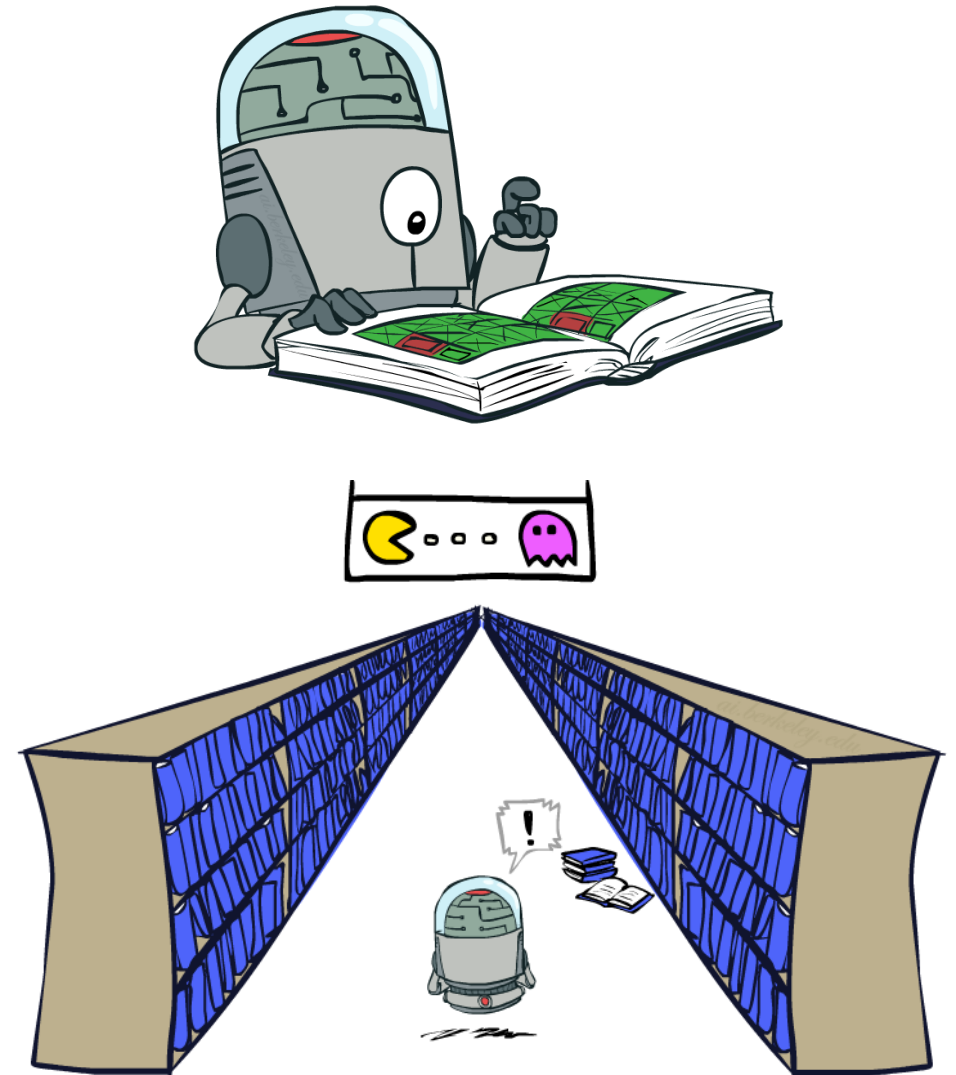


Or even this one!



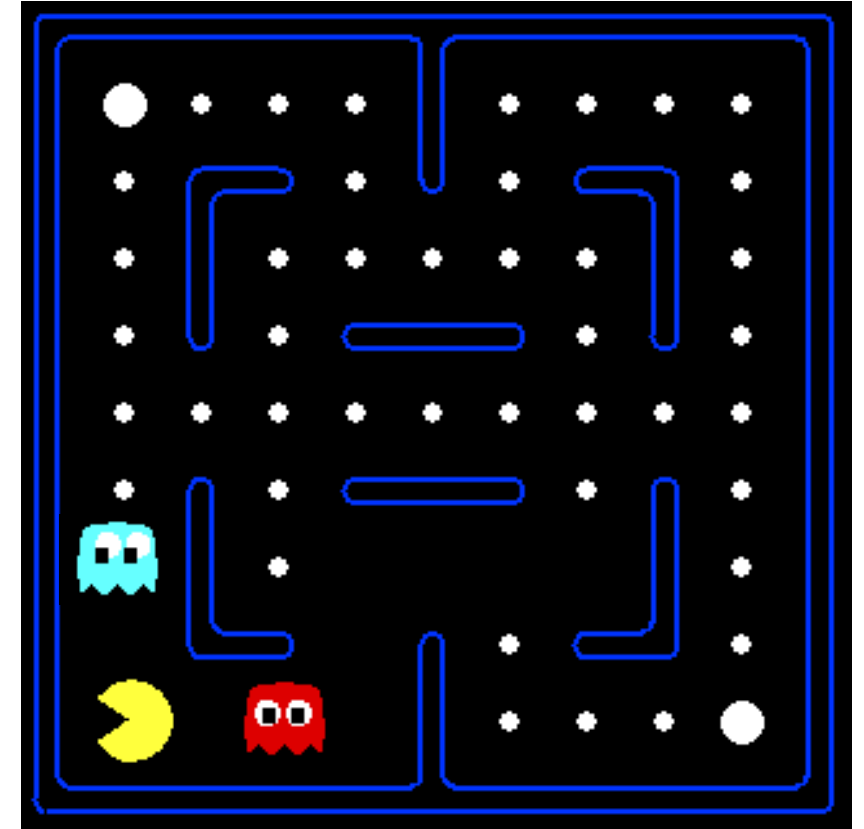
Generalizing Across States

- We want to generalize:
 - Learn about some small number of training states from experience
 - Generalize that experience to new, similar situations
 - This is a fundamental idea in machine learning, and we'll see it again later



Feature-Based Representations

- Solution: describe a state using a vector of features (properties)
 - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
 - Example features:
 - Distance to closest ghost
 - Distance to closest dot
 - Number of ghosts
 - $1 / (\text{dist to dot})^2$
 - Is Pacman in a tunnel? (0/1)
 - etc.
 - Is it the exact state on this slide?
 - Can also describe a q-state (s, a) with features (e.g. action moves closer to food)



Linear Value Functions

- Using a feature representation, we can write a q function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \dots + w_n f_n(s)$$

$$Q(s, a) = w_1 f_1(s, a) + w_2 f_2(s, a) + \dots + w_n f_n(s, a)$$

- Advantage: our experience is summed up in a few powerful numbers
- Disadvantage: states may share features but actually be very different in value!

Approximate Q-Learning

$$Q(s, a) = w_1 f_1(s, a) + w_2 f_2(s, a) + \dots + w_n f_n(s, a)$$

- Q-learning with linear Q-functions:

transition = (s, a, r, s')

difference = $\left[r + \gamma \max_{a'} Q(s', a') \right] - Q(s, a)$

$Q(s, a) \leftarrow Q(s, a) + \alpha [\text{difference}]$

Exact Q's

$w_i \leftarrow w_i + \alpha [\text{difference}] f_i(s, a)$

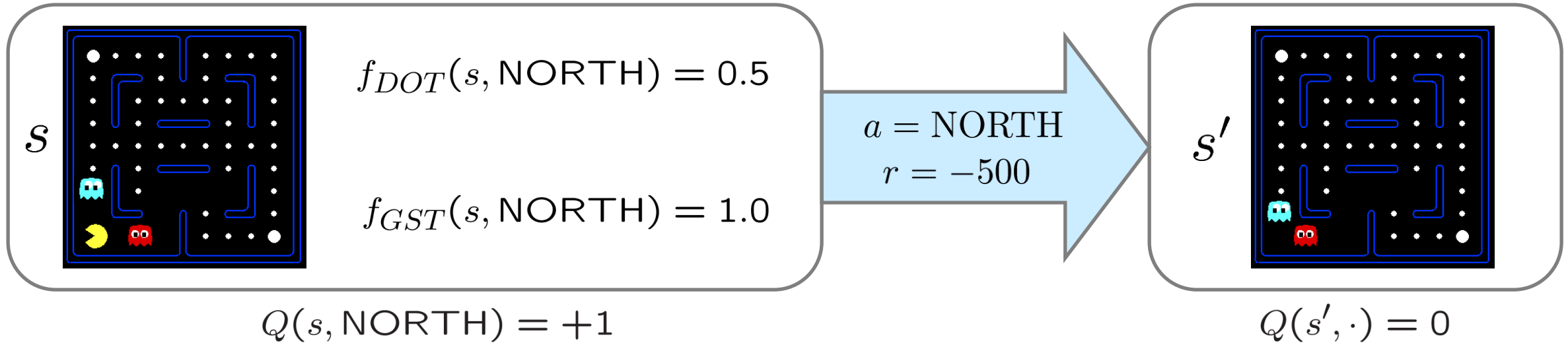
Approximate Q's
(based on online least squares)

- Intuitive interpretation:

- Adjust weights of active features
- E.g., if something unexpectedly bad happens, blame the features that were on: disprefer all states with that state's features

Example: Q-Pacman

$$Q(s, a) = 4.0 f_{DOT}(s, a) - 1.0 f_{GST}(s, a)$$



$$r + \gamma \max_{a'} Q(s', a') = -500 + 0$$

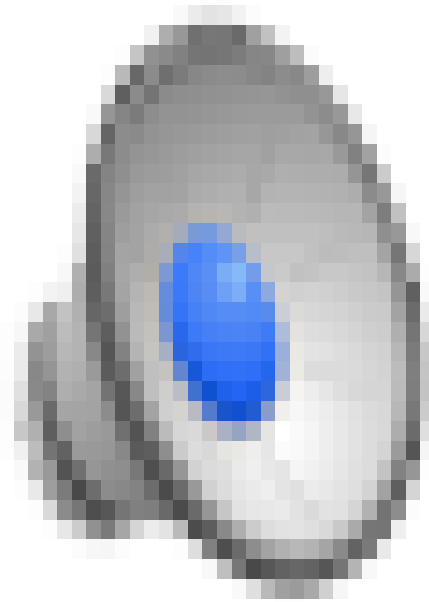
difference = -501

$$w_{DOT} \leftarrow 4.0 + \alpha [-501] 0.5$$
$$w_{GST} \leftarrow -1.0 + \alpha [-501] 1.0$$

If $\alpha = 0.004$:

$$Q(s, a) = 3.0 f_{DOT}(s, a) - 3.0 f_{GST}(s, a)$$

Video of Demo Approximate Q-Learning -- Pacman

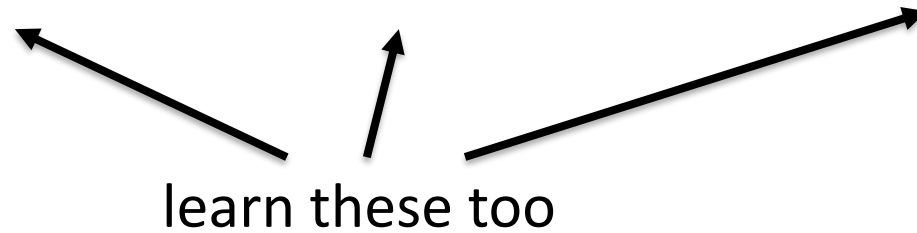


More Powerful Functions

Linear: $Q(s, a) = w_1 f_1(s, a) + w_2 f_2(s, a) + \dots + w_n f_n(s, a)$

Polynomial: $Q(s, a) = w_{11} f_1(s, a) + w_{12} f_1(s, a)^2 + w_{13} f_1(s, a)^3 + \dots$

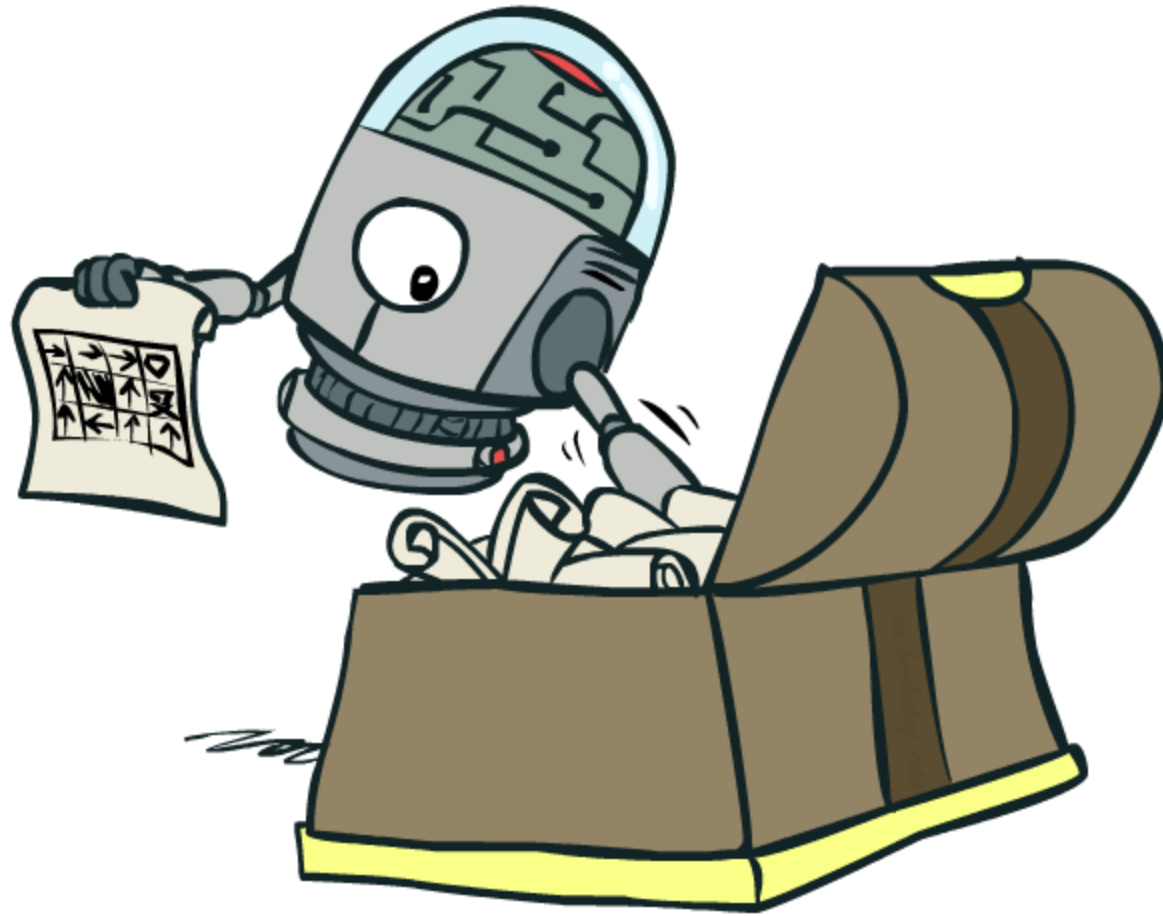
Neural network: $Q(s, a) = w_1 f_1(s, a) + w_2 f_2(s, a) + \dots + w_n f_n(s, a)$



$$w_m \leftarrow w_m + \alpha \left[r + \gamma \max_a Q(s', a') - Q(s, a) \right] \frac{dQ}{dw_m}(s, a)$$

\uparrow
 $= f_m(s, a)$ in linear case

Policy Search



Policy Search

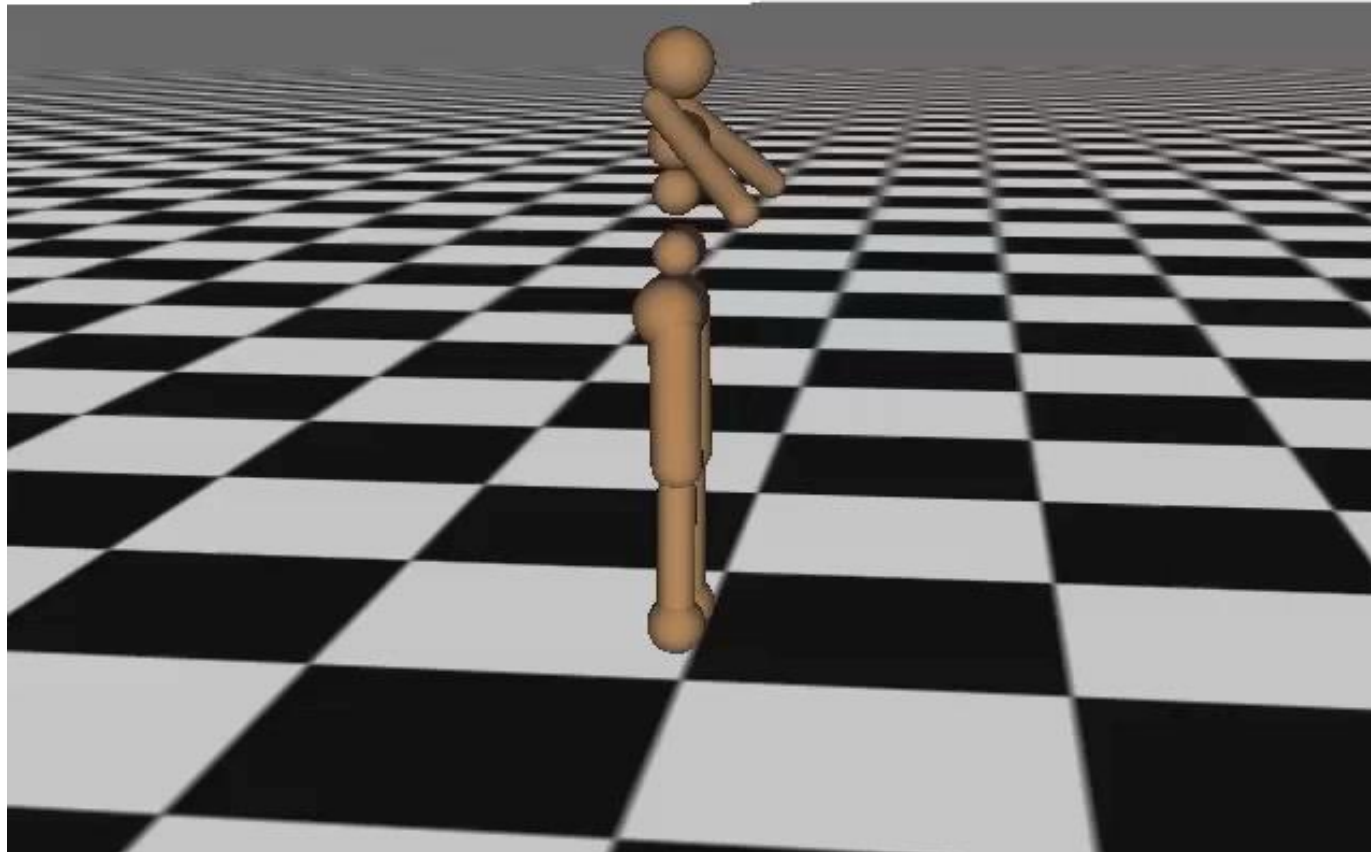
- Q-learning's priority: get Q-values close
- Observation: often the feature-based policies that work well (win games, maximize utilities) aren't the ones that approximate V / Q best
 - E.g. your value functions from project 1b were probably horrible estimates of future rewards, but they still produced good decisions
 - The real priority: get ordering of Q-values right (action prediction)
- Idea: learn policies that maximize rewards, not the values that predict them
- Policy search: start with an OK solution (e.g., approximate Q-learning), then fine-tune feature weights to find a better policy

Policy Search

- Simplest policy search:
 - Start with an initial linear value function or Q-function
 - Change each feature weight up and down and see if your policy is better than before
- Problems:
 - How do we tell the policy got better?
 - Need to run many sample episodes!
 - If there are a lot of features, this can be impractical
- Better methods exploit lookahead structure, sample wisely, change multiple parameters...

Policy Search

Iteration 0



Summary

- Reinforcement learning
 - MDP without knowing T and R
- Model-based learning
- Model-free learning
 - Policy evaluation: TD Learning
 - Computing q -values/policy: Q-Learning
- Exploration vs. Exploitation
 - Random exploration, exploration function
- Feature-based representation of states
- Policy Search

