EE150: Signals and Systems, Spring 2022
Homework 1
(Due Saturday, Mar. 5 at 11:59pm (CST))

1. [15 points] Sketch each of the following signals.

(a) $x[n] = \delta[n] + \delta[n-3]$
(b) $x[n] = u[n] - u[n-5]$
(c) $x[n] = \delta[n] + \frac{1}{2}\delta[n-1] + (\frac{1}{2})^2\delta[n-2] + (\frac{1}{2})^3\delta[n-3]$
(d) $x(t) = u(t+3) - u(t-3)$
(e) $x(t) = \delta(t+2)$
(f) $x(t) = e^{-t}u(t)$

2. [10 points] For x(t) indicated in Figure 1,sketch the following.

(a) $x(1-t)[u(t+1)-u(t-2)]$
(b) $x(1-t)[u(t+1)-u(2-3t)]$

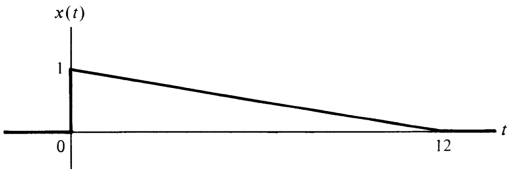

Figure 1: x(t)

3. [10 points] Determine whether each of the following signals is periodic.

$$\textbf{(a)} x(t) = 2e^{j(t+\frac{\pi}{4})}u(t)$$

$$\textbf{(b)} x[n] = \sum_{k=-\infty}^{\infty} (\delta[n-4k] - \delta[n-1-4k])$$

4. [15 points] Consider a discrete-time system with input x[n] and output y[n]

$$y[n] = \sum_{k=n-n_0}^{n+n_0} x[k]$$

where n_0 is a finite positive integer.

(a) Is this system linear?

(b) Is this system time-invariant?

(c) If $x[n]$ is known to be bounded by a finite integer B (i.e.,$|x[n]| <$ B for all n), it can be shown that $y[n]$ is bounded by a finite number C. We conclude that the given system is stable. Express C in terms of B and n_0.

5. [10 points] Consider the following systems

$$H : y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$
$$G : y(t) = x(2t),$$

where the input is x(t) and the output is y(t).

(a) What is H^{-1}? What is G^{-1}?

(b) Consider the system in Figure 2. Find the inverse F^{-1} and draw it in block diagram form in terms of H^{-1} and G^{-1}.

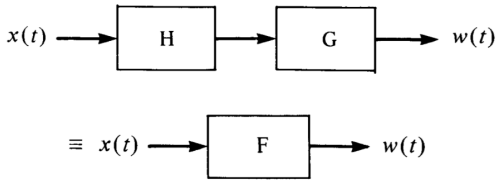

Figure 2: System of 3.(b)

6. [15 points] Determine whether or not each of the following discrete-time signals is periodic.If the signal is periodic, determine its fundamental period.

$$\textbf{(a)} x[n] = sin(\frac{6\pi}{7}n + 1)$$
$$\textbf{(b)} x[n] = cos(\frac{\pi}{8}n^2)$$
$$\textbf{(c)} x[n] = 2cos(\frac{\pi}{4}n) + sin(\frac{\pi}{8}n) - 2cos(\frac{\pi}{2}n + \frac{\pi}{6})$$

7. [10 points]

(a) Consider a system with input x(t) and with output y(t) given by

$$y(t) = \sum_{n=-\infty}^{+\infty} x(t)\delta(t - nT)$$

(i) Is this system linear?

(ii) Is this system time-invariant?

For each part, if your answer is yes, show your reason, else produce a counterexample.

(b) Suppose that the input to this system is $x(t) = \cos 2t$. Sketch and label carefully the output y(t) for each of the following values of T: T=1, $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{12}$. Make sure that all your sketches should have the same horizontal and vertical scales.

8. [15 points] In this chapter, we introduced a number of general properties of systems. In particular, a system may or may not be
(1) Memoryless
(2) Time invariant
(3) Linear
(4) Causal
(5) Stable
Determine which of these properties hold and which do not hold for each of the following continuous-time systems. Justify your answers. In each example, $y(t)$ denotes the system output and $x(t)$ is the system input.

$$\textbf{(a)} y(t) = cos(3t)x(t)$$

$$\textbf{(b)} y(t) = \begin{cases} 0, & x(t) < 0 \\ x(t) + x(t-2), & x(t) \geq 0 \end{cases}$$