Numerical Optimization, 2023 Fall
Homework 2

Due 23:59 (CST), Nov. 2, 2023

1 Standard Form

Convert the following problem to a linear program in standard form. [20pts]

$$
\begin{aligned}
\max_{\boldsymbol{x}\in\mathbb{R}^4} \quad & 2x_1 - x_3 + x_4 \\
\text{s.t.} \quad & x_1 + x_2 \geq 5 \\
& x_1 - x_3 \leq 2 \\
& 4x_2 + 3x_3 - x_4 \leq 10 \\
& x_1 \geq 0
\end{aligned}
\tag{1}
$$

2 Two-Phase Simplex

Use the two-phase simplex procedure to solve the following problem. [40pts]

$$
\begin{aligned}
\min_{\boldsymbol{x}\in\mathbb{R}^4} \quad & -3x_1 + x_2 + 3x_3 - x_4 \\
\text{s.t.} \quad & x_1 + 2x_2 - x_3 + x_4 = 0 \\
& 2x_1 - 2x_2 + 3x_3 + 3x_4 = 9 \\
& x_1 - x_2 + 2x_3 - x_4 = 6 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}
\tag{2}
$$

3 Extreme Point

3.1 Q1

Prove that the extreme points of the following two sets are in one-to-one correspondence. [20pts]

$$
\begin{aligned}
S_1 &= \{\boldsymbol{x} \in \mathbb{R}^n : \boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{b}, \boldsymbol{x} \geq 0\} \\
S_2 &= \{(\boldsymbol{x}, \boldsymbol{y}) \in \mathbb{R}^n \times \mathbb{R}^m : \boldsymbol{A}\boldsymbol{x} + \boldsymbol{y} = \boldsymbol{b}, \boldsymbol{x} \geq 0, \boldsymbol{y} \geq 0\},
\end{aligned}
\tag{3}
$$

where $\boldsymbol{A} \in \mathbb{R}^{m\times n}$, $\boldsymbol{b} \in \mathbb{R}^m$.

3.2 Q2

Does the set $P = \{\boldsymbol{x} \in \mathbb{R}^2 : 0 \leq x_1 \leq 1\}$ have extreme points? What is its standard form? Does it have extreme points in its standard form? If so, give a extreme point and explain why it is a extreme point. [20pts]