EE150: Signals and Systems, Spring 2022
Homework 4

(Due Thursday, May. 5 at 11:59pm (CST))

1. [15 points] Find the Fourier transform of each of the following signals, derive and sketch their magnitude and phase as a function of frequency, both positive and negative frequency required.

   (a) $\delta(t-5)$

   (b) $e^{-at}u(t)$, a real and positive

   (c) $e^{(-1+j2)t}u(t)$

2. [15 points] Find the signal corresponding to the following Fourier transforms.

$$\textbf{(a)} X(\omega) = \frac{1}{7 + j\omega}$$

(b) The signal sketched in Figure 1

$$\textbf{(c)} X(\omega) = \frac{1}{9 + \omega^2}$$

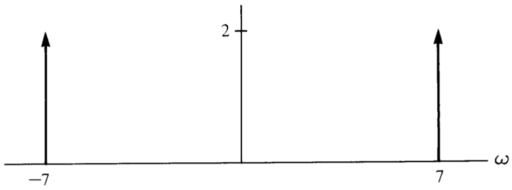

Figure 1: Problem 2.b

3. [20 points]

(a) Let $x(t) = e^{-at}u(t)$, Using the linearity and scaling properties, derive the Fourier transform $e^{-a|t|} = x(t) + x(-t)$.

(b) Using part (a) and the duality property, determine the Fourier transform of $x(t) = \frac{1}{1+t^2}$

(c) If

$$r(t) = \frac{1}{1 + (3t)^2}$$

find $R(\omega)$

(d) $x(t)$ is sketched in Figure 2, if $y(t) = x(\frac{t}{2})$, sketch $y(t), Y(\omega), X(\omega)$.

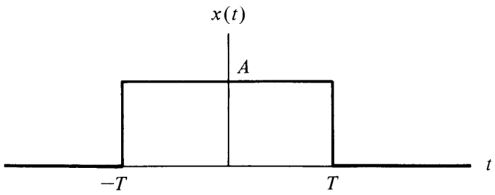

Figure 2: Problem 3.d

4. [6 points] If $X(\omega)$ is the Fourier transform of $x(t)$, prove the following equations.

(a) $x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)d\omega$

(b) $X(0) = \int_{-\infty}^{\infty} x(t)dt$

5. [4 points] Consider a causal LTI system with frequency response

$$H(\omega) = \frac{1}{j\omega + 3}$$

Derive the input $x(t)$ so that the output of the system is

$$y(t) = e^{-3t}u(t) - e^{-4t}u(t)$$

6. [15 points] Compute the convolution of each of the following pairs of signals $x(t)$ and $h(t)$ using the convolution property of the Fourier transform.

   (a) $x(t) = e^{-2t}u(t), h(t) = te^{-4t}u(t)$

   (b) $x(t) = te^{-2t}u(t), h(t) = te^{-4t}u(t)$

   (c) $x(t) = e^{-t}u(t), h(t) = e^{t}u(-t)$

7. [5 points] Suppose $g(t) = x(t)cos(t)$ and the Fourier transform of $g(t)$ is

$$G(\omega) = \begin{cases} 1, & |\omega| \leq 2 \\ 0, & otherwise \end{cases}$$

Determine $x(t)$.

8. [20 points] Consider the signal $x(t)$ in Figure 3

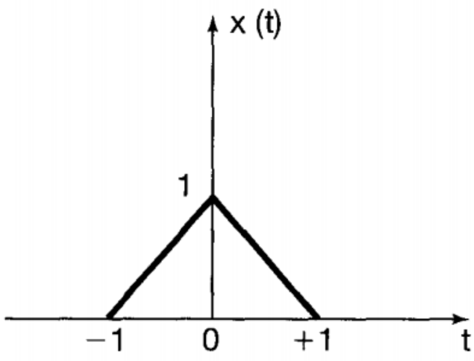

Figure 3: Problem 8

(a) Find the Fourier transform $X(j\omega)$ of $x(t)$.

(b) Sketch the signal

$$\widetilde{x}(t) = x(t) * \sum_{k=-\infty}^{\infty} \delta(t-4k).$$

(c) Find another signal $g(t)$ such that $g(t) \neq x(t)$ and

$$\widetilde{x}(t) = g(t) * \sum_{k=-\infty}^{\infty} \delta(t-4k).$$

(d) Prove that $G(j\frac{\pi k}{2}) = X(j\frac{\pi k}{2})$ for all integers k. You should not explicitly evaluate $G(j\omega)$ to answer this question.