EE 160: Introduction to Control
Project

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Deadline for the proposal report: Nov.25

1 Introduction

The EE160 Project is about the modeling, simulation, and control of a nonlinear system of your choice(including but not limited to mechanical, circuit, thermodynamic system). Here, you have two options:

1. Pick at least one of the application examples (with given physical models) from the appendix of this project sheet; or
2. Propose a control problem of your own choice. If you want to go for this second option, please contact prof.Wang or one of the TAs. If you have a good idea, we will accept this as a project topic, too, as long as the project is related to the methods that are discussed in the lecture. The deadline for the first proposing report is Nov.25, 2020. Independent of whether you go for one of the default project or come up with your own project, you will need to complete a final report (≥ 4 pages) and a presentation.

You should form your team of no more than 3 member and submit your proposal report before Nov.25 and your presentation is in Dec.23 or Dec.25 and it should consist of several slides as well as on-site parameter adjustment display for $10 - 15$ minutes per group.

2 Main Requirements on the Project

The goal of this project is to design a closed-loop controller for at least one application that leads to a nonlinear control system with 2 or more differential states. The main tasks of the project are as follows:

1. Introduce your model and write in the form of a nonlinear control system in standard form
$$\dot{x}(t) = f(t, x(t), u(t)). \tag{1}$$
Explain what x, u, and f are in your application and define your preliminary control goal.
2. Linearize your model at a suitable set point and write the linear approximation in the form
$$\dot{z}(t) = Az(t) + Bv(t) + b. \tag{2}$$
What are A, B, b, z, and v in your application?
3. Design a open loop control input $u(t)$ intuitively and show the performance in both linear system(2) and nonlinear system(1) via simulation.

4. Design a feedback control law of the form

$$u(t) = Kx(t) + u_{ref}$$

by using at least one of the methods that has been introduced in the lecture. Alternatively, you may also use a time-varying reference, $u(t) = K(x(t) - x_{ref}(t)) + u_{ref}(t)$, if this is needed for your particular application.

5. Implement a Runge-Kutta integrator and simulate the nonlinear closed-loop dynamic for different initial values. Plot the closed-loop state trajectory and discuss your results.

6. Analyze the input of turning parameters on the performance of your system via simulation.

7. Analyze the robustness of your controller via simulation and find a case where your controller fails to stabilize the system due to one of the following reasons:

   (a) measure noise

   (b) model uncertainty

   (c) different operating point

Notice that if you work on especially creative and innovative applications, you might get full points even if your report is not perfect.

3 Project proposal

Write a short report (preferably in Latex) containing the following sections:

1. *Title and Authors* (find a good title + name of the author)

2. *Introduction* (describe the problem that you want to solve and cite relevant literature)

3. *Problem Formulation* (introduce a suitable mathematical notation do define the problem that you are trying to solve)

The proposal report only consists problem formulation (modeling), control objective setting and linearization as well as open loop simulation.

Appendix

This appendix collects a few suggestions for possible project topics and some hints on how to derive a differential equation model.

A Quadrotor Control

This work is of more practical significance. You are asked to design a controller that will allow the vehicle to eventually be stationary at a fixed altitude under the initial conditions that are set.

Figure 1: UAV

In this model, we have 12 states. $l_x(t),\ l_y(t),\ l_z(t)$ donates the three dimensions of world coordinates respectively, $v_x(t),\ v_y(t),\ v_z(t)$ are the velocity of each direction which a part of the status.

$$\begin{bmatrix} v_x(t) \\ v_y(t) \\ v_z(t) \end{bmatrix} = \begin{bmatrix} \dot{l_x}(t) \\ \dot{l_y}(t) \\ \dot{l_z}(t) \end{bmatrix}$$

indicates the velocity on each dimension of the UAV.

Each propeller has an input which are $u_1(t),\ u_2(t),\ u_3(t),\ u_4(t)$, the torque and force are defined as the following equations:

$$U(t) = \begin{bmatrix} \tau_x(t) \\ \tau_y(t) \\ \tau_z(t) \\ f(t) \end{bmatrix} = \begin{bmatrix} k_1\ k_1\ k_2\ k_3 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \end{bmatrix}$$

Moreover, we have 3 more states for angular velocity of the quadrotor which are $\omega_x,\ \omega_y,\ \omega_z$ and $\phi(t),\ \theta(t),\ \psi(t)$ are the additional 3 states for the Euler angle.

Reference:

1. Quadcopter Dynamics, Simulation, and Control. http://andrew.gibiansky.com/downloads/pdf/Quadcopter%20Dynamics,%20Simulation,%20and%20Control.pdf

B Spherical Robot Arm

Robot Arm manipulators are driven by electric, hydraulic, or pneumatic actuators, which apply torques at the joints of the robot. The dynamics of a robot manipulator describes how the robot moves in response to these actuator forces. The figure shows a robot arm equipped for spherical coordinate geometry with two movement, rotation ϕ, elevation θ.

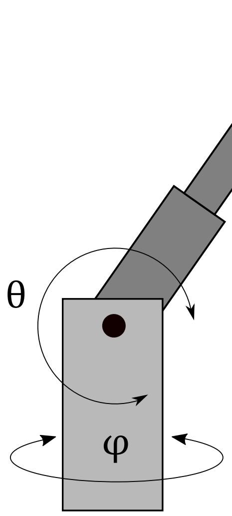

Figure 2: Spherical Robot Arm

Use Lagrange derivation to describe the dynamics of a robot arm in form of nonlinear differential equations which depend on the inertial, Coriolis, centrifugal, actuator acting on the robot's links.

Reference:

1. A mathematical introduction to robotic manipulation.
   https://www.cds.caltech.edu/~murray/books/MLS/pdf/mls94-complete.pdf

C Inverted Pendulum Control

There is an inverted pendulum on the trolley whose pole pivots on a horizontally moving base as shown in the image below. In order to stabilize the pendulum in this inverted position, a feedback control force F can be designed. The motion equation of state angle $\theta(t)$ and cart position $s(t)$ can be derived by using Lagrange's equations.

Reference:

1. The Acrobot and Cart-Pole.
   https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-832-underactuated-
   readings/MIT6_832s09_read_ch03.pdf

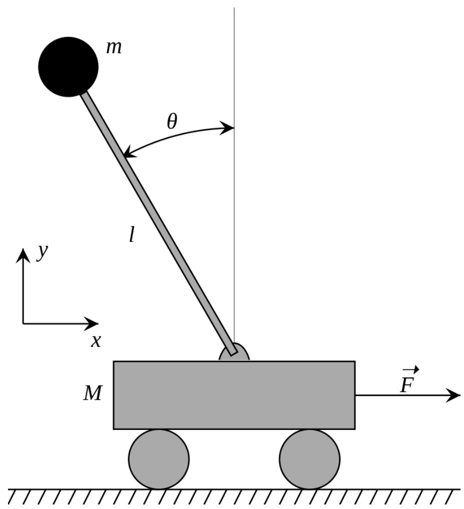

Figure 3: Cart Pendulum Control System

D Thermodynamic System

In this model, θ_i θ_o represent the liquid temperature(^{o}C) of the input and output port respectively. G denotes the liquid flow (kg/s), M represents the mass(kg) of liquid in the container and c is the specific heat capacity of the liquid. The last three components are the resister R (^{o}C s/kcal), the heat capacity (kcal/^{o}C) and steady-state heat input rate $\overline{H}$ (kcal/sec).

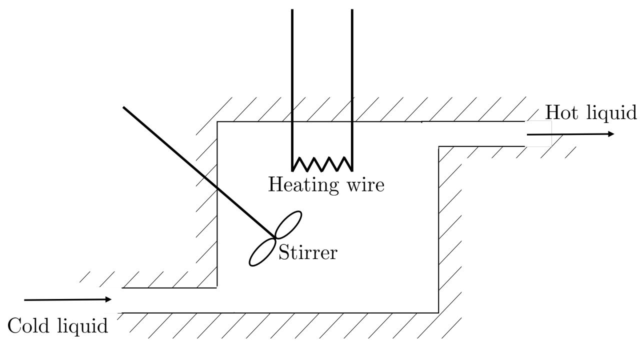

Figure 4: Thermodynamic System

Assume that the temperature of the inflowing liquid is kept constant and that the heat input rate to the system (heat supplied by the heater) is suddenly changed from $\overline{H}$ to $\overline{H} + h_i$ where h_i represents a small change in the heat input rate. The heat outflow rate will then change gradually from $\overline{H}$ to $\overline{H}+h_o$. The temperature of the outflowing liquid will also be changed from θ_o to $\theta_o+\theta$. For this case, h_o, C, and R are obtained, respectively, as

$$h_0 = Gc\theta$$
$$C = Mc$$
$$R = \frac{\theta}{h_0} = \frac{1}{Gc}$$

A heat balance equation related to h_0 C R can be modeled as a differential equation (please check).

Reference:

1. Modern Control Engineering.
http://sharif.edu/~salarieh/Downloads/Modern%20Control%20Engineering%205th%20Edition.pdf

E Circuit System

In the following circuit, the voltage on each component and the current on the wire can be regarded as the selected state. $e_i(t)$ and $e_0(t)$ are the input and output voltages respectively. Design and analyze the circuit model to select the final control law.

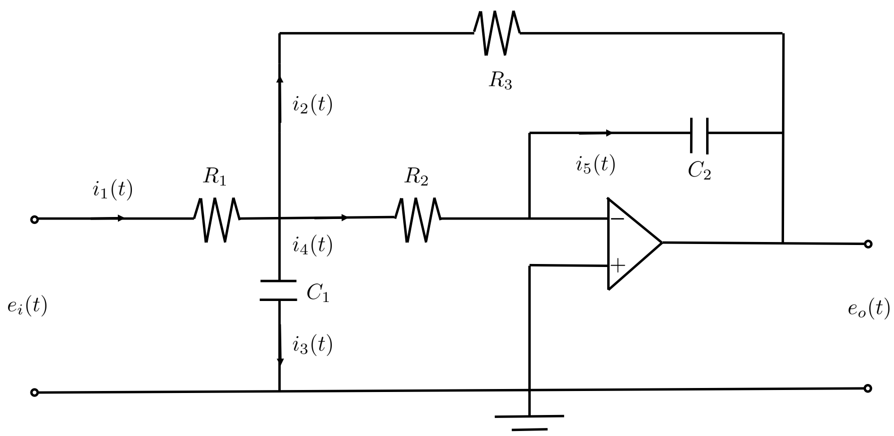

Figure 5: Circuit

Reference:

1. Modern Control Engineering.
http://sharif.edu/~salarieh/Downloads/Modern%20Control%20Engineering%205th%20Edition.pdf