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4. (1). $y'' - 2y' - y = 0$ $\quad \lambda^2 - 2\lambda - 1 = 0$ $\quad \lambda_1 = \frac{2+\sqrt{4+4}}{2} = 1+\sqrt{2}$ $\quad \lambda_2 = 1-\sqrt{2}$

$y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$.

(2). $y'' + 2y' + 2y = 0$ $\quad \lambda^2 + 2\lambda + 2 = 0$ $\quad \lambda_1 = \frac{-2+\sqrt{4-8}}{2} = -1+i$ $\quad \lambda_2 = -1-i$

$y = e^{-x}(C_1 \cos x + C_2 \sin x)$

(3). $y'' + y' - 6y = 0$ $\quad \lambda^2 + \lambda - 6 = 0. \Rightarrow (\lambda+3)(\lambda-2) = 0 \Rightarrow \lambda_1 = -3, \lambda_2 = 2$

$y = C_1 e^{-3x} + C_2 e^{2x}$

5. (1). $y'' + y = 2\sin\frac{x}{2}$ $\quad \lambda^2 + 1 = 0. \quad \lambda_1 = i. \lambda_2 = -i$

设特解. $y = C \cdot \sin\frac{x}{2}$, $\quad y' = \frac{C}{2}\cdot\cos\frac{x}{2}$ $\quad y'' = -\frac{C}{4}\cdot\sin\frac{x}{2}$

$\Rightarrow. \left(C - \frac{C}{4}\right)\sin\frac{x}{2} = 2\sin\frac{x}{2}$ $\quad \Rightarrow. \ C = \frac{8}{3}$

$\Rightarrow. \ y = \frac{8}{3}\sin\frac{x}{2}$

(2). $y'' - 6y' + 9y = (x+1)e^{2x}$

$\lambda^2 - 6\lambda + 9 = 0 \Rightarrow. \ \lambda_1 = \lambda_2 = 3.$

设 $y = (ax+b)e^{2x}$ $\quad y' = e^{2x}(2ax + 2b + a) = e^{2x}(2ax + a + 2b)$

$y'' = e^{2x}(4ax + 2a + 4b + 2a) = e^{2x}(4ax + 4a + 4b).$

$y'' - 6y' + 9y = e^{2x}(4ax + 4a + 4b - 12ax - 6a - 12b + 9ax + 9b).$

$= e^{2x}(ax + b - 2a) = e^{2x}(x+1)$

$\Rightarrow a = 1, \ b = 3.$

8.

when $x \in [0,1]$, $\quad y_1(x) = (x-1)^2$. $\quad y_2(x) = 0.$ $\quad k_1 y_1(x) + k_2 y_2(x) = 0 \Rightarrow k_1 = 0.$

$x \in [1,2]$. $\quad y_1(x) = 0$, $\quad y_2(x) = (x-1)^2$, $\quad k_1 y_1(x) + k_2 y_2(x) = 0 \Rightarrow k_2 = 0.$

$\Rightarrow. \ y_1, y_2$ are linear independent.

$x \in [0,1]$. $W(x) = \begin{vmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{vmatrix} = \begin{vmatrix} (x-1)^2 & 0 \\ 2(x-1) & 0 \end{vmatrix} = 0.$ $\quad x \in [1,2]$. it is the same.

9. (1). $x''' + 3x'' + 3x' + x = 0$. $\lambda^3 + 3\lambda^2 + 3\lambda + 1 = 0 \Rightarrow (\lambda+1)^3 = 0 \Rightarrow \lambda_1 = \lambda_2 = \lambda_3 = -1$

$x = (C_1 + C_2 t + C_3 t^2) e^{-t}$

(3). $x^{(4)} - 8x'' + 18x = 0$. $\lambda^4 - 8\lambda^2 + 18 = 0$ $\lambda^2 = 4 + \sqrt{2} i$, $\lambda^2 = 4 - \sqrt{2} i$

设 $\lambda = \alpha + \beta i$ $\lambda^2 = \alpha^2 - \beta^2 + 2\alpha\beta i \Rightarrow \begin{cases} \alpha^2 - \beta^2 = 4 \\ 2\alpha\beta = \sqrt{2} \end{cases}$

$\Rightarrow \begin{cases} \alpha = \sqrt{\frac{4+3\sqrt{2}}{2}} \\ \beta = \sqrt{\frac{-4+3\sqrt{2}}{2}} \end{cases}$

$x = C_1 e^{\alpha x} \sin\beta x + C_2 e^{\alpha x} \cos\beta x + C_3 e^{-\alpha x} \sin\beta x + C_4 e^{-\alpha x} \cos\beta x$.

Chapter 8

6. $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=\frac{1}{2}\left[(\vec{a}+\vec{b}+\vec{c})^2-a^2-b^2-c^2\right]$

$=\frac{1}{2}[-1-1-1]=-\frac{3}{2}$

7. $(a+3b)(7a-5b)=7a^2-5ab+21ab-15b^2=7a^2+16ab-15b^2=0.$ ①

$(a-4b)(7a-2b)=7a^2-2ab-28ab+8b^2=7a^2-30ab+8b^2=0.$ ②

① - ② $\quad 46ab-23b^2=0 \Rightarrow 2ab=b^2 \quad\Rightarrow a^2=b^2.$

$7a^2-30ab+16ab=0 \Rightarrow a^2=2ab.$

$\Rightarrow a^2=2a^2\cos\theta \Rightarrow \cos\theta=\frac{1}{2}. \Rightarrow \theta=\frac{\pi}{6}$

8 (1) $|(a+b)\times(a-b)|=|a\times a-a\times b+b\times a-b\times b|$

$=2|a\times b|=2ab\sin\theta=2\times3\times4=12$

(2) $|(3a-b)\times(a-2b)|=|3a\times a-6a\times b-b\times a+2b\times b|.$

$=|-6a\times b+a\times b|=|5a\times b|=5|a\times b|=5\cdot3\cdot4=60.$

9. (2). $|(a+3b)\times(3a-b)|=|3a\times a-a\times b+3b\times3a-3b\times b|=10|a\times b|=10\cdot1\cdot2\cdot\frac{\sqrt{3}}{2}=10\sqrt{3}$

$\Rightarrow |(a+3b)\times(3a-b)|^2=300.$

14. $\vec{a}-\vec{b}=(4,-6,12).$

$|\vec{a}-\vec{b}|=\sqrt{4^2+6^2+12^2}=\sqrt{196}=14$

$e_{\vec{a}-\vec{b}}=\left(\frac{4}{14},\frac{-6}{14},\frac{12}{14}\right)=\left(\frac{2}{7},-\frac{3}{7},\frac{6}{7}\right)$

$\cos\alpha=\frac{2}{7}.\quad\cos\beta=-\frac{3}{7},\quad\cos\gamma=\frac{6}{7}$

18 (2). $(3a-2b)\cdot(a+2b)=3a^2+6ab-2ba-4b^2=3a^2+4ab-4b^2$

$=3\cdot3^2+4\cdot3\cdot4\cdot\cos(\frac{2\pi}{3})-4\cdot16.$

$=27-24-64=27-88=-61$

21. $a \cdot e_b = \frac{a \cdot b}{|b|} = \frac{10 - 2 + 10}{\sqrt{2^2 + 1^2 + 2^2}} = \frac{18}{3} = 6$