EE160 Introduction to Control: Homework 5

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1. *Properties of Matrix Exponientials* Let $A \in \mathbb{R}^{n \times n}$ be a given matrix. Prove that

   (a) the equation
   $$\det(e^A) = e^{\mathrm{Tr}(A)}$$
   holds with $\mathrm{Tr}(A) = \sum_{i=1}^{n} A_{ii}$, and

   (b) if e^{At} is a polynomial function in t, then A is nilpotent.

2. *Explicit solution of linear time-invariant differential equations.* Write the following differential equation system in standard form and solve it explicitly:
   $$\begin{aligned} \dot{x}_1(t) &= -x_2(t) \quad \text{with} \quad x_1(0) = 0 \\ \dot{x}_2(t) &= x_1(t) \quad \text{with} \quad x_2(0) = 1 \, . \end{aligned}$$

3. *Third order time-invariant differential equation.* Find all solutions of the differential equation
   $$\dddot{x}(t) = x(t) \, .$$