ShanghaiTech University

EE 115B: Digital Circuits

Fall 2022

Lecture 3

Hengzhao Yang

September 13, 2022

Binary Conversions (1)

The decimal equivalent of a binary number can be determined by adding the column values of all of the bits that are 1 and discarding all of the bits that are 0.

Example Convert the binary number 100101.01 to decimal.

Solution Start by writing the column weights; then add the weights that correspond to each 1 in the number.

2^5	2^4	2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}
32	16	8	4	2	1	.	½	¼
1	0	0	1	0	1	.	0	1
32			+4		+1			+¼ = 37¼

Binary Conversions (2)

You can convert a decimal whole number to binary by reversing the procedure. Write the decimal weight of each column and place 1's in the columns that sum to the decimal number.

Example Convert the decimal number 49 to binary.

Solution The column weights double in each position to the right. Write down column weights until the last number is larger than the one you want to convert.

2^6	2^5	2^4	2^3	2^2	2^1	2^0.
64	32	16	8	4	2	1.
0	1	1	0	0	0	1.

Binary Conversions (3)

You can convert a decimal fraction to binary by repeatedly multiplying the fractional results of successive multiplications by 2. The carries form the binary number.

Example Convert the decimal fraction 0.188 to binary by repeatedly multiplying the fractional results by 2.

Solution

0.188 x 2 = 0.376	carry = 0	MSB
0.376 x 2 = 0.752	carry = 0	
0.752 x 2 = 1.504	carry = 1	
0.504 x 2 = 1.008	carry = 1	
0.008 x 2 = 0.016	carry = 0	

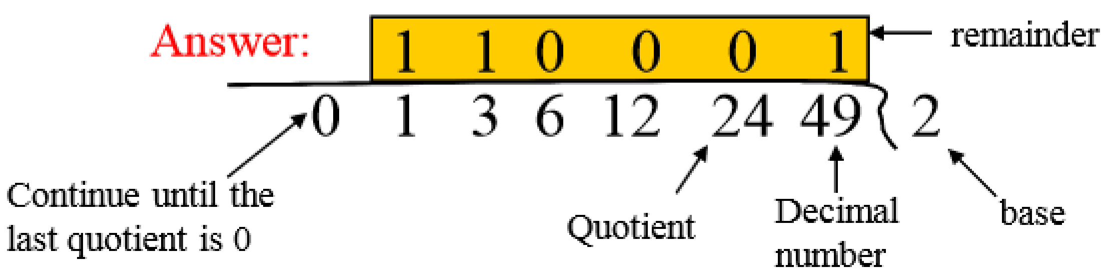
Answer = .00110 (for five significant digits)

Binary Conversions (4)

You can convert decimal to any other base by repeatedly dividing by the base. For binary, repeatedly divide by 2:

Example Convert the decimal number 49 to binary by repeatedly dividing by 2.

Solution You can do this by “reverse division” and the answer will read from left to right. Put quotients to the left and remainders on top.



Hexadecimal Numbers (1)

Hexadecimal uses sixteen characters to represent numbers: the numbers 0 through 9 and the alphabetic characters A through F.

Large binary number can easily be converted to hexadecimal by grouping bits 4 at a time and writing the equivalent hexadecimal character.

Example Express 1001 0110 0000 1110_2 in hexadecimal:

Solution Group the binary number by 4-bits starting from the right. Thus, 960E

Decimal	Hexadecimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

Hexadecimal Numbers (2)

Hexadecimal is a weighted number system. The column weights are powers of 16, which increase from right to left.

Column weights $\begin{cases} 16^3 & 16^2 & 16^1 & 16^0. \\ 4096 & 256 & 16 & 1. \end{cases}$

Example Express $1A2F_{16}$ in decimal.

Solution Start by writing the column weights:

4096 256 16 1

1 A 2 F_{16}

$1(4096) + 10(256) + 2(16) + 15(1) = 6703_{10}$

Decimal	Hexadecimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

Octal Numbers (1)

Octal Numbers

Octal uses eight characters the numbers 0 through 7 to represent numbers. There is no 8 or 9 character in octal.

Binary number can easily be converted to octal by grouping bits 3 at a time and writing the equivalent octal character for each group.

Example Express $1\,001\,011\,000\,001\,110_2$ in octal:

Solution Group the binary number by 3-bits starting from the right. Thus, 113016_8

Decimal	Octal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	10	1000
9	11	1001
10	12	1010
11	13	1011
12	14	1100
13	15	1101
14	16	1110
15	17	1111

Octal Numbers (2)

Octal is also a weighted number system. The column weights are powers of 8, which increase from right to left.

Column weights $\begin{cases} 8^3 & 8^2 & 8^1 & 8^0 \\ 512 & 64 & 8 & 1 \end{cases}$.

Decimal	Octal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	10	1000
9	11	1001
10	12	1010
11	13	1011
12	14	1100
13	15	1101
14	16	1110
15	17	1111

Example Express 3702_8 in decimal.

Solution Start by writing the column weights:

512 64 8 1

3 7 0 2_8

$$3(512) + 7(64) + 0(8) + 2(1) = 1986_{10}$$

BCD (1)

Binary coded decimal (BCD) is a weighted code that is commonly used in digital systems when it is necessary to show decimal numbers such as in clock displays.

The table illustrates the difference between straight binary and BCD. BCD represents each decimal digit with a 4-bit code. Notice that the codes 1010 through 1111 are not used in BCD.

Decimal	Binary	BCD
0	0000	0000
1	0001	0001
2	0010	0010
3	0011	0011
4	0100	0100
5	0101	0101
6	0110	0110
7	0111	0111
8	1000	1000
9	1001	1001
10	1010	00010000
11	1011	00010001
12	1100	00010010
13	1101	00010011
14	1110	00010100
15	1111	00010101

BCD (2)

You can think of BCD in terms of column weights in groups of four bits. For an 8-bit BCD number, the column weights are: 80 40 20 10 8 4 2 1.

Question: What are the column weights for the BCD number 1000 0011 0101 1001?

Answer:

8000 4000 2000 1000 800 400 200 100 80 40 20 10 8 4 2 1

Note that you could add the column weights where there is a 1 to obtain the decimal number. For this case:

$$8000 + 200 + 100 + 40 + 10 + 8 + 1 = 8359_{10}$$

BCD (3)

表 1.5.1 几种常见的十进制代码

编码种类 / 十进制数	8421 码（BCD 代码）	余 3 码	2421 码	5211 码	余 3 循环码
0	0 0 0 0	0 0 1 1	0 0 0 0	0 0 0 0	0 0 1 0
1	0 0 0 1	0 1 0 0	0 0 0 1	0 0 0 1	0 1 1 0
2	0 0 1 0	0 1 0 1	0 0 1 0	0 1 0 0	0 1 1 1
3	0 0 1 1	0 1 1 0	0 0 1 1	0 1 0 1	0 1 0 1
4	0 1 0 0	0 1 1 1	0 1 0 0	0 1 1 1	0 1 0 0
5	0 1 0 1	1 0 0 0	1 0 1 1	1 0 0 0	1 1 0 0
6	0 1 1 0	1 0 0 1	1 1 0 0	1 0 0 1	1 1 0 1
7	0 1 1 1	1 0 1 0	1 1 0 1	1 1 0 0	1 1 1 1
8	1 0 0 0	1 0 1 1	1 1 1 0	1 1 0 1	1 1 1 0
9	1 0 0 1	1 1 0 0	1 1 1 1	1 1 1 1	1 0 1 0
权	8 4 2 1		2 4 2 1	5 2 1 1	

Gray Code (1)

Gray code is an unweighted code that has a single bit change between one code word and the next in a sequence. Gray code is used to avoid problems in systems where an error can occur if more than one bit changes at a time.

Decimal	Binary	Gray code
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000

Gray Code (2)

格雷码(Gray Code)又称循环码。从表 1.5.2 的 4 位格雷码编码表中可以看出格雷码的构成方法,这就是每一位的状态变化都按一定的顺序循环。如果从 **0000** 开始,最右边一位的状态按 **0110** 顺序循环变化,右边第二位的状态按 **00111100** 顺序循环变化,右边第三位按 **0000111111110000** 顺序循环变化。可见,自右向左,每一位状态循环中连续的 **0**、**1** 数目增加一倍。由于 4 位格雷码只有 16 个,所以最左边一位的状态只有半个循环,即 **0000000011111111**。按照上述原则,我们就很容易得到更多位数的格雷码。

与普通的二进制代码相比,格雷码的最大优点就在于当它按照表 1.5.2 的编码顺序依次变化时,相邻两个代码之间只有一位发生变化。这样在代码转换的过程中就不会产生过渡"噪声"。而在普通二进制代码的转换过程中,则有时会产生过渡噪声。例如,第四行的二进制代码 **0011** 转换为第五行的 **0100** 过程中,如果最右边一位的变化比其他两位的变化慢,就会在一个极短的瞬间出现 **0101** 状态,这个状态将成为转换过程中出现的噪声。而在第四行的格雷码 **0010** 向第五行的 **0110** 转换过程中则不会出现过渡噪声。这种过渡噪声在有些情况下甚至会影响电路的正常工作,这时就必须采取措施加以避免。在第 4.9 节中我们还将进一步讨论这个问题。

Race Condition (竞争-冒险现象)

都是在输入、输出处于稳定的逻辑……下当输入信号逻辑电平发生变化的瞬间电路的工作情况。

首先让我们看两个最简单的例子。在图 4.9.1(a)所示的与门电路中，稳态下无论 $A=1$、$B=0$ 还是 $A=0$、$B=1$，输出皆为 $Y=0$。但是在输入信号 A 从 1 跳变为 0 时，如果 B 从 0 跳变为 1，而且 B 首先上升到 $V_{IL(max)}$ 以上，这样在极短的时间 Δt 内将出现 A、B 同时高于 $V_{IL(max)}$ 的状态，于是便在门电路的输出端产生了极窄的 $Y=1$ 的尖峰脉冲，或称为电压毛刺，如图中所示（在画波形时考虑了门电路的传输延迟时间）。显然，这个尖峰脉冲不符合门电路稳态下的逻辑功能，因而它是系统内部的一种噪声。

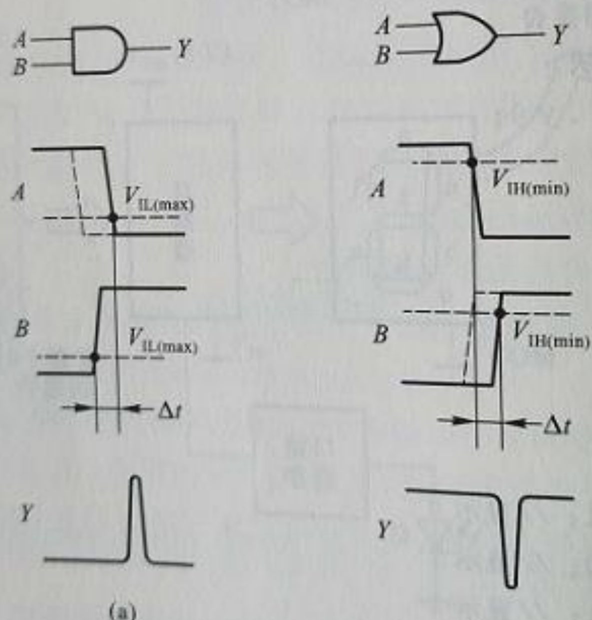


图 4.9.1 由于竞争而产生的尖峰脉冲

ASCII

ASCII is a code for alphanumeric characters and control characters. In its original form, ASCII encoded 128 characters and symbols using 7-bits. The first 32 characters are control characters, that are based on obsolete teletype requirements, so these characters are generally assigned to other functions in modern usage.

In 1981, IBM introduced extended ASCII, which is an 8-bit code and increased the character set to 256. Other extended sets (such as Unicode) have been introduced to handle characters in languages other than English.

TABLE 2–7

American Standard Code for Information Interchange (ASCII).

Control Characters				Graphic Symbols											
Name	**Dec**	**Binary**	**Hex**	**Symbol**	**Dec**	**Binary**	**Hex**	**Symbol**	**Dec**	**Binary**	**Hex**	**Symbol**	**Dec**	**Binary**	**Hex**
NUL	0	0000000	00	space	32	0100000	20	@	64	1000000	40	′	96	1100000	60
SOH	1	0000001	01	!	33	0100001	21	A	65	1000001	41	a	97	1100001	61
STX	2	0000010	02	”	34	0100010	22	B	66	1000010	42	b	98	1100010	62
ETX	3	0000011	03	#	35	0100011	23	C	67	1000011	43	c	99	1100011	63
EOT	4	0000100	04	$	36	0100100	24	D	68	1000100	44	d	100	1100100	64
ENQ	5	0000101	05	%	37	0100101	25	E	69	1000101	45	e	101	1100101	65
ACK	6	0000110	06	&	38	0100110	26	F	70	1000110	46	f	102	1100110	66
BEL	7	0000111	07	’	39	0100111	27	G	71	1000111	47	g	103	1100111	67
BS	8	0001000	08	(	40	0101000	28	H	72	1001000	48	h	104	1101000	68
HT	9	0001001	09	)	41	0101001	29	I	73	1001001	49	i	105	1101001	69
LF	10	0001010	0A	*	42	0101010	2A	J	74	1001010	4A	j	106	1101010	6A
VT	11	0001011	0B	+	43	0101011	2B	K	75	1001011	4B	k	107	1101011	6B
FF	12	0001100	0C	,	44	0101100	2C	L	76	1001100	4C	l	108	1101100	6C
CR	13	0001101	0D	−	45	0101101	2D	M	77	1001101	4D	m	109	1101101	6D
SO	14	0001110	0E	.	46	0101110	2E	N	78	1001110	4E	n	110	1101110	6E
SI	15	0001111	0F	/	47	0101111	2F	O	79	1001111	4F	o	111	1101111	6F
DLE	16	0010000	10	0	48	0110000	30	P	80	1010000	50	p	112	1110000	70
DC1	17	0010001	11	1	49	0110001	31	Q	81	1010001	51	q	113	1110001	71
DC2	18	0010010	12	2	50	0110010	32	R	82	1010010	52	r	114	1110010	72
DC3	19	0010011	13	3	51	0110011	33	S	83	1010011	53	s	115	1110011	73
DC4	20	0010100	14	4	52	0110100	34	T	84	1010100	54	t	116	1110100	74
NAK	21	0010101	15	5	53	0110101	35	U	85	1010101	55	u	117	1110101	75
SYN	22	0010110	16	6	54	0110110	36	V	86	1010110	56	v	118	1110110	76
ETB	23	0010111	17	7	55	0110111	37	W	87	1010111	57	w	119	1110111	77
CAN	24	0011000	18	8	56	0111000	38	X	88	1011000	58	x	120	1111000	78
EM	25	0011001	19	9	57	0111001	39	Y	89	1011001	59	y	121	1111001	79
SUB	26	0011010	1A	:	58	0111010	3A	Z	90	1011010	5A	z	122	1111010	7A
ESC	27	0011011	1B	;	59	0111011	3B	[	91	1011011	5B	{	123	1111011	7B
FS	28	0011100	1C	<	60	0111100	3C	\	92	1011100	5C	|	124	1111100	7C
GS	29	0011101	1D	=	61	0111101	3D	]	93	1011101	5D	}	125	1111101	7D
RS	30	0011110	1E	>	62	0111110	3E	^	94	1011110	5E	~	126	1111110	7E
US	31	0011111	1F	?	63	0111111	3F	_	95	1011111	5F	Del	127	1111111	7F

Parity Method

The parity method is a method of error detection for simple transmission errors involving one bit (or an odd number of bits). A parity bit is an "extra" bit attached to a group of bits to force the number of 1's to be either even (even parity) or odd (odd parity).

Example The ASCII character for "a" is 1100001 and for "A" is 1000001. What is the correct bit to append to make both of these have odd parity?

Solution The ASCII "a" has an odd number of bits that are equal to 1; therefore the parity bit is 0. The ASCII "A" has an even number of bits that are equal to 1; therefore the parity bit is 1.

Cyclic Redundancy Check

The cyclic redundancy check (CRC) is an error detection method that can detect multiple errors in larger blocks of data. At the sending end, a checksum is appended to a block of data. At the receiving end, the check sum is generated and compared to the sent checksum. If the check sums are the same, no error is detected.