Homework 5

Due date: 17th, Nov.

Turn in your homework in class

Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

20pts

1. Given that the voltage and current of two-terminal elements adopt the passive sign convention, the instantaneous values are expressed as:

(1) $\boldsymbol{v(t)} = 15\cos(400t + 30°)\ V$, $\boldsymbol{i(t)} = 3\sin(400t + 30°)\ A$;

(2) $\boldsymbol{v(t)} = 8\sin(500t + 50°)\ V$, $\boldsymbol{i(t)} = 2\sin(500t + 140°)\ A$;

(3) $\boldsymbol{v(t)} = 8\cos(250t + 60°)\ V$, $\boldsymbol{i(t)} = 5\sin(250t + 150°)\ A$;

(a) Transform the three voltage & current pairs into phasors.

(b) Try to determine whether the element is a resistor, inductor or capacitor, and determine its value (R=?,C=?, L=?) for **(1), (2), and (3), respectively**.

Solutions:

(a) (1) $\dot{V} = 15\angle 30°$; $\dot{I} = 3\angle -60°$ at $\omega = 400$ rad/s

(2) $\dot{V} = 8\angle -40°$; $\dot{I} = 2\angle 50°$ at $\omega = 500$ rad/s

(3) $\dot{V} = 8\angle 60°$; $\dot{I} = 5\angle 60°$ at $\omega = 250$ rad/s

(b) ① $Z_{(1)} = \frac{\dot{V}}{\dot{I}} = \frac{15\angle 30°}{3\angle -60°} = 5\angle 90°$ (Inductor)

$j\omega L = 5j \Rightarrow L = \frac{5}{400} H = 0.0125 H$

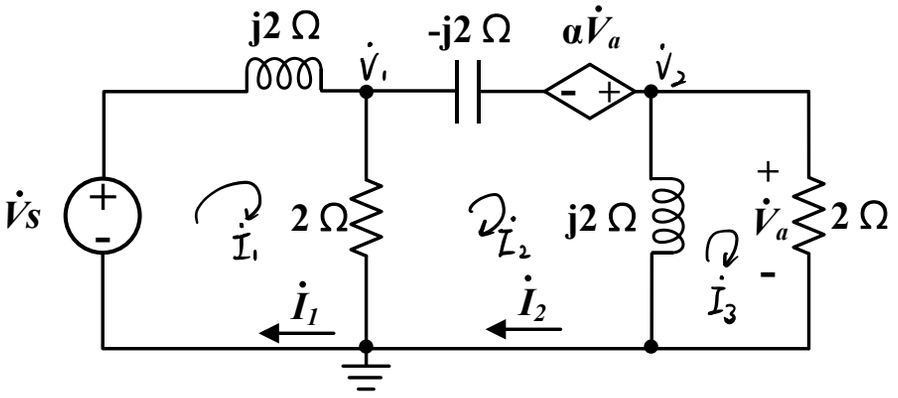
② $Z_{(2)} = \frac{\dot{V}}{\dot{I}} = \frac{8\angle -40°}{2\angle 50°} = 4\angle -90°$ (Capacitor)

$-\frac{j}{\omega C} = -4j \Rightarrow C = \frac{1}{4\times 500} = 0.5\ mF$

③ $Z_{(3)} = \frac{\dot{V}}{\dot{I}} = \frac{8\angle 60°}{5\angle 60°} = \frac{8}{5}\Omega = 1.6\Omega$ (resistor)

20 pts

2. Use nodal **AND** mesh analyze method to find $\dot{V}_a$, $\dot{I}_1$ and $\dot{I}_2$ for the circuit below, assuming that $\dot{V}_S = 10\angle 0°\ V$, $\alpha = 0.5$.



Mesh:

$$\begin{cases} j2\cdot\dot{I}_1 + (\dot{I}_1 - \dot{I}_2)\cdot 2 = \dot{V}_S & \cdots ① \\ (\dot{I}_2 - \dot{I}_1)2 - j2\cdot\dot{I}_2 - \alpha\dot{V}_a + j2\cdot(\dot{I}_2 - \dot{I}_3) = 0 & \cdots ② \\ j2\cdot(\dot{I}_3 - \dot{I}_2) + \dot{I}_3 2 = 0 & \cdots ③ \\ \dot{V}_a = 2\dot{I}_3 & \cdots ④ \end{cases}$$

$$\begin{cases} (2+2j)\dot{I}_1 - 2\dot{I}_2 = 10\angle 0° \\ -2\dot{I}_1 + 2\dot{I}_2 - (1+2j)\dot{I}_3 = 0 \\ -2j\dot{I}_2 + (2+2j)\dot{I}_3 = 0 \end{cases} \Rightarrow \begin{aligned} \dot{I}_1 &= 6.52\angle -57.53° \ A \\ \dot{I}_2 &= 4.47\angle -26.57° \ A \\ \dot{I}_3 &= 3.16\angle 18.43° \ A \end{aligned}$$

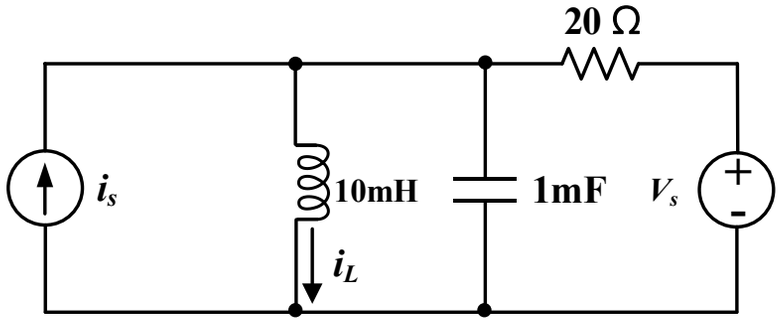
$$\dot{V}_a = 2\dot{I}_3 = 6.32\angle 18.43°\ V$$

Nodal:

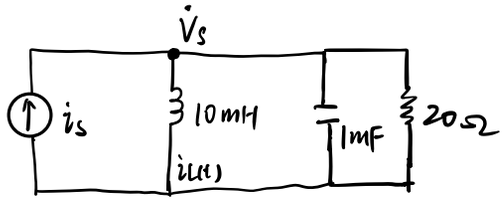
$$\begin{cases} \dfrac{\dot{V}_S - \dot{V}_1}{j2} + \dfrac{0 - \dot{V}_1}{2} + \dfrac{\dot{V}_2 - \alpha\dot{V}_a - \dot{V}_1}{-j2} = 0 \\ \dfrac{\dot{V}_2 - \alpha\dot{V}_a - \dot{V}_1}{-j2} + \dfrac{\dot{V}_2}{j2} + \dfrac{\dot{V}_2}{2} = 0 \\ \dot{V}_2 = \dot{V}_a \end{cases}$$

20pts 3. Using superposition method to find $i_L(t)$, assuming that $i_S(t) = 10\sqrt{2}\cos 100t$ A,

$V_S(t) = 100\sqrt{2}\cos 1000t$ V



Solution:

① 

$i_s = 10\sqrt{2}\cos 100t$

$\dot{I}_S = 10\sqrt{2}\angle 0°$ $\quad Z_R = 20\,\Omega$ $\quad (\omega = 100)$

$Z_C = \frac{1}{j\omega C} = -\frac{1}{10^{-3}\times 100}j\,\Omega$

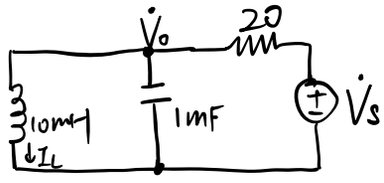
$= -10j\,\Omega$

$Z_L = j\omega L = j\cdot 100\times 10\times 10^{-3}\,\Omega = j\,\Omega$

$$\begin{cases} \frac{\dot{V}_S}{20} + \frac{\dot{V}_S}{j} - \frac{\dot{V}_S}{10j} = \dot{I}_S \\ \dot{I}_{L1} = \dot{V}_S / j \text{ A} \end{cases}$$

$\Rightarrow \dot{I}_{L1} = 15.69\angle -3.18°$ A

$i_{L1}(t) = 15.69\cos(100t - 3.18°)$ A

② 

$\dot{V}_S = 100\sqrt{2}\angle 0°$

$Z_C = -\frac{j}{\omega C} = -j\,\Omega$

$Z_L = j\omega L = 10j\,\Omega$

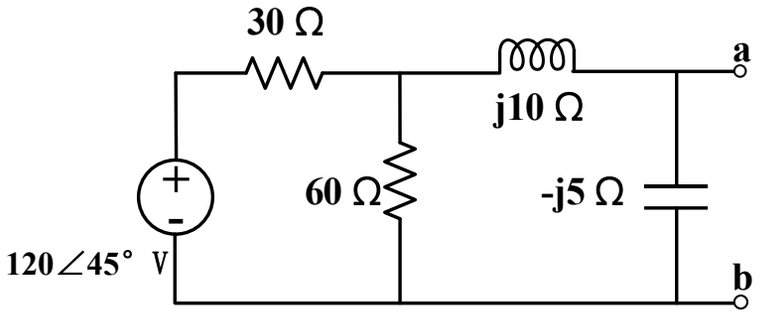
$\frac{\dot{V}_S - \dot{V}_0}{20} = \frac{\dot{V}_0}{-j} + \frac{\dot{V}_0}{10j}$

$\dot{I}_{L2} = \frac{\dot{V}_0}{10j} \simeq 0.78\angle -176.82°$

$i_{L2}(t) = 0.78\cos(1000t - 176.82°)$ A

$\Rightarrow i_L(t) = i_{L1}(t) + i_{L2}(t) = 15.69\cos(100t - 3.18°) + 0.78\cos(1000t - 176.82°)$ A

4. For the circuit below, please find the Thevenin equivalent circuit with respect to node **a** and node **b**.

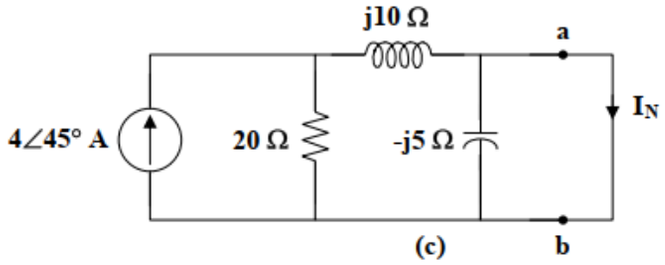


$R_{TH} = -j5\Omega \,||\, (j10\Omega + 30\Omega || 60\Omega)$

$= -j5\Omega \,||\, (20 + 10j)\Omega$

$= 5.42 \angle -77.47° \Omega$ $\quad 1.17 - 5.29j\Omega$

To find $\mathbf{V}_{th}$ and $\mathbf{I}_N$, we transform the voltage source and combine the 30 Ω and 60 Ω resistors. The result is shown in Fig. (c).

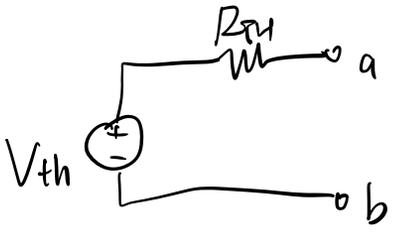


$$\mathbf{I}_N = \frac{20}{20 + j10}(4\angle 45°) = \frac{2}{5}(2 - j)(4\angle 45°)$$

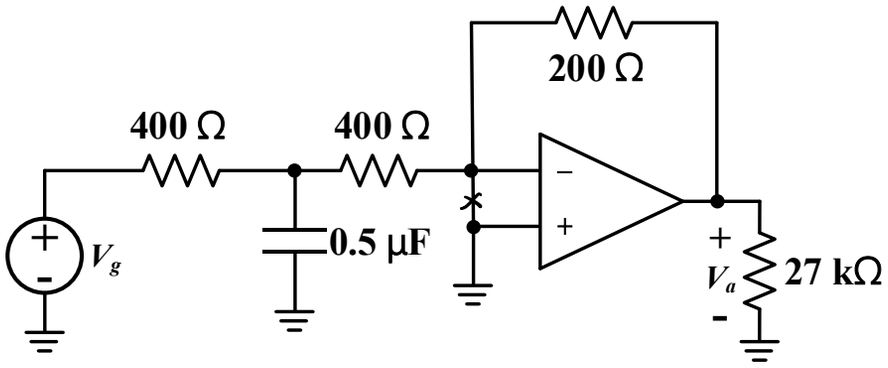
$$= \mathbf{3.578\angle 18.43°\ A}$$

$$\mathbf{V}_{th} = \mathbf{Z}_{th}\,\mathbf{I}_N = (5.423\angle -77.47°)(3.578\angle 18.43°)$$

$$= \mathbf{19.4\angle -59°\ V}$$



5. As shown in the figure, V_g = **20cos(5000t)** V. Please find the steady state expression for $V_a(t)$. The operational amplifier works in its linear region.



$$\mathbf{V}_g = 20\angle 0^\circ \text{ V}; \qquad \frac{1}{j\omega C} = -j400\,\Omega$$

Let $\mathbf{V}_a$ = voltage across the capacitor, positive at upper terminal
Then:

$$\frac{\mathbf{V}_a - 20\angle 0^\circ}{400} + \frac{\mathbf{V}_a}{-j400} + \frac{\mathbf{V}_a}{400} = 0; \qquad \therefore \quad \mathbf{V}_a = (8 - j4)\text{ V}$$

$$\frac{0 - \mathbf{V}_a}{400} + \frac{0 - \mathbf{V}_o}{200} = 0; \qquad \mathbf{V}_o = -\frac{\mathbf{V}_a}{2}$$

$$\therefore \quad \mathbf{V}_o = -4 + j2 = 4.47\angle 153.43^\circ \text{ V}$$

$$v_o = 4.47\cos(5000t + 153.43^\circ)\text{ V}$$