

# CS244 Theory of Computation

## Homework 2

Due: October 19, 2020 at 11:59pm

Name - ID

You may discuss this assignment with other students and work on the problems together. However, your write-up should be your own individual work and you should indicate in your submission who you worked with, if applicable. You should use the L<sup>A</sup>T<sub>E</sub>X template provided by us to write your solution and submit the generated PDF file into Gradescope.

I worked with: (Name, ID), (Name, ID), ...

Let  $\Sigma = \{0, 1\}$  if not otherwise specified.

### Problem 1

Let  $\Sigma = \{0, 1\}$  and let  $C_2 = \{ztz \mid z \in 0^* \text{ and } t \in 0^*10^*10^*, \text{ where } |t| = |z|\}$ .

- (a) Show that  $C_2$  is not a CFL.
- (b) Is  $C_2 \cup (\Sigma\Sigma\Sigma)^*$  a CFL? Why or why not?
- (c) Is  $C_2 \cup \Sigma(\Sigma\Sigma\Sigma)^*$  a CFL? Why or why not?

### Problem 2

Let  $G = (V, \Sigma, R, \langle \text{STMT} \rangle)$  be the following grammar.  $\Sigma = \{\text{if}, \text{condition}, \text{then}, \text{else}, \text{a:=1}\}$ ,  $V = \{\langle \text{STMT} \rangle, \langle \text{IF-THEN} \rangle, \langle \text{IF-THEN-ELSE} \rangle, \langle \text{ASSIGN} \rangle\}$  and the rules are:

$\langle \text{STMT} \rangle \rightarrow \langle \text{ASSIGN} \rangle \mid \langle \text{IF-THEN} \rangle \mid \langle \text{IF-THEN-ELSE} \rangle$   
 $\langle \text{IF-THEN} \rangle \rightarrow \text{if condition then } \langle \text{STMT} \rangle$   
 $\langle \text{IF-THEN-ELSE} \rangle \rightarrow \text{if condition then } \langle \text{STMT} \rangle \text{ else } \langle \text{STMT} \rangle$   
 $\langle \text{ASSIGN} \rangle \rightarrow \text{a:=1}$

- (a) Show that  $G$  is ambiguous.
- (b) Give a new unambiguous grammar that generates  $L(G)$ .  
(You do not need to prove that your grammar works or that it is unambiguous, but please add a few comments about why it does work to help the grader.)

### Problem 3

Let the **rotational closure** of language  $A$  be  $\text{RC}(A) = \{yx \mid xy \in A \text{ where } x, y \in \Sigma^*\}$ . Show that the class of CFLs is closed under rotational closure.