

## SI211 Homework 2

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Deadline: March 17, 2022

1. *Rational function interpolation.* The function  $f(x)$  satisfies

$$f(-2) = -11, f(-1) = -6, f(1) = 70, f(2) = 27, f(3) = 14.$$

Construct a polynomial

$$p(x) = \frac{a_{-2}}{x^2} + \frac{a_{-1}}{x} + a_0 + a_1x + a_2x^2$$

that interpolates  $f$  at all given points. Write down the divided difference table. Find  $a_{-2}, a_{-1}, a_0, a_1, a_2$ .

2. *Polynomial approximation.* Write computer program that constructs polynomial approximation of the following functions.

- (a)  $f(x) = \cos(x)$ . Sample  $f(x)$  at  $x = \text{range}(-4, \text{stop}=4, \text{length}=n)$  to obtain its polynomial approximation  $p_{n-1}(x)$ , where  $n \in \{5, 9, 13\}$ . Plot  $f(x)$ ,  $p_4(x)$ ,  $p_8(x)$  and  $p_{12}(x)$  over  $[-4.5, 4.5]$ .
- (b)  $f(x) = \frac{1}{1+e^{-x}}$ . Sample  $f(x)$  at  $x = \text{range}(-10, \text{stop}=10, \text{length}=n)$  to obtain its polynomial approximation  $p_{n-1}(x)$ , where  $n \in \{5, 9, 13\}$ . Plot  $f(x)$ ,  $p_4(x)$ ,  $p_8(x)$  and  $p_{12}(x)$  over  $[-10.1, 10.1]$ .

Attach your figures and code. Interpret your results.

3. *Hermite interpolation.* Consider a smooth function  $f(x)$  that satisfies

$$\begin{aligned} f(0) &= 0, f'(0) = -1, f''(0) = 2, f'''(0) = -24 \\ f(1) &= -1, f'(1) = -1, f''(1) = 0 \end{aligned}$$

Find a polynomial  $p(x)$  of degree 6 that interpolates  $f$ . Work out the complete Hermite interpolation table, and write your result in the form of  $p(x) = \sum_{i=0}^6 a_i x^i$ .