Homework 6 Solution

1. Calculate the distance at which the intensity of a 1 MHz and 5 MHz ultrasound beam will be reduced by half traveling through (a) bone, (b) air, and (c) muscle. (The attenuation coefficient for muscle, bone and air are 1, 8.7 and 45 dB cm^{-1} MHz^{-1}, respectively.dB $= 10\log_{10}(\frac{I_x}{I_0})$) (20Point)

Solution.

a. Use dB.

As stated in the text the values of μ for muscle, bone and air are 1, 8.7 and 45 dB cm^{-1} MHz^{-1}, respectively. For the intensity to be reduced by half, the value of (μx) must be ~3 dB. Therefore, at 1 MHz, the half value distance is 0.34 cm for bone (3/8.7), 0.067 cm for air and 3 cm for muscle (3/45, 3/1). At 5 MHz, the distances are one-fifth those at 1 MHz, i.e. 0.068 cm for bone, 0.013 cm for air and 0.6 cm for muscle.

b. Use formula.

$$\mathrm{I} = \mathrm{I}_0 e^{-\mu z}$$

The intensity of ultrasound beam is reduced by half means that:

$$\frac{1}{2} = \frac{I}{I_0} = e^{-\mu z}$$

1MHz, For muscle,

$$\frac{1}{2} = \frac{I}{I_0} = e^{-\frac{1*z}{4.343}} => \mathrm{z} \approx 3\mathrm{cm}$$

1MHz, For Bone,

$$\frac{1}{2} = \frac{I}{I_0} = e^{-\frac{8.7*z}{4.343}} => \mathrm{z} \approx 0.34\mathrm{cm}$$

1MHz, For Air,

$$\frac{1}{2} = \frac{I}{I_0} = e^{-\frac{45*z}{4.343}} => \mathrm{z} \approx 0.067\mathrm{cm}$$

Here, using conversion factor to convert units

$$\frac{1}{4.343}\mu(\mathrm{dBcm}^{-1}) = \mu(\mathrm{cm}^{-1}))$$

2. Use the following data to sketch the A-mode scan from Figure 1 (a) to Figure 1 (b). The amplitude axis should be on a dB scale, and the time axis in microseconds. Ignore any reflected signal from the transducer/fat interface, and assume that a signal of 0 dB enters the body. At a transducer frequency of 5 MHz, the linear attenuation coefficient for muscle and liver is 5 dB cm^{-1}, and for fat is 4dBcm^{-1}. Relevant values of the characteristic acoustic impedance and speed of sound can be found in Table 1. (35poin)

Hint:

1. Only the first reflection between layers need to be considered, multi-reflection can be neglected

2. $\mathrm{dB} = 10\log_{10}(\frac{I_r}{I_i})$.

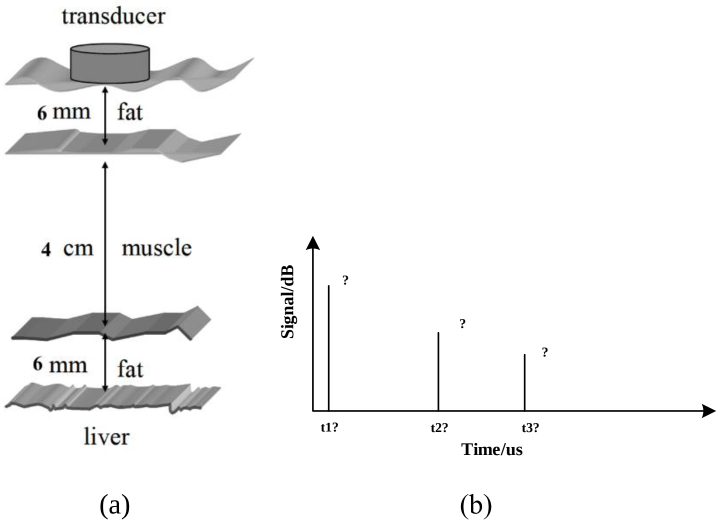


Figure 1(a) and (b)

	$Z \times 10^5$ ($g\ cm^{-2}\ s^{-1}$)	Speed of sound ($m\ s^{-1}$)	Density (gm^{-3})	Compressibility $x10^{11}$ ($cm\ g^{-1}\ s^2$)
Air	0.00043	330	1.3	70 000
Blood	1.59	1570	1060	4.0
Bone	7.8	4000	1908	0.3
Fat	1.38	1450	925	5.0
Brain	1.58	1540	1025	4.2
Muscle	1.7	1590	1075	3.7
Liver	1.65	1570	1050	3.9
Kidney	1.62	1560	1040	4.0

Table 1: Acoustic properties of biological tissues

Solution.

a. Transmitted attenuation is not considered

The ultrasound beam is first attenuated through the fat layer, and then partially backscattered from the fat/muscle boundary. The backscattered signal is further attenuated by the fat layer during its transit to the transducer, where it produces the first echo. The fraction of the beam that is transmitted through the fat/muscle boundary is then attenuated by the muscle layer before it reaches the muscle/fat boundary, where a second process of partial reflection and partial transmission happens. The same analysis applies to the entire path. Due to high loss, second order reflection, i.e. waves bouncing back and forth between layers can be neglected because of the very low signal intensity.

The intensity of the signal at the fat/muscle boundary due to attenuation is (-0.6x4) = -2.4 dB. The intensity reflection coefficient at this boundary is given by:

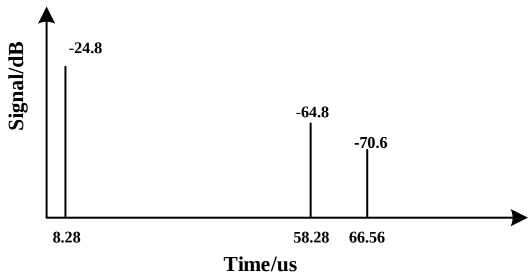
$$R_I = \frac{\left(1.38x10^5 - 1.7x10^5\right)^2}{\left(1.38x10^5 + 1.7x10^5\right)^2} \approx 0.01$$

This is equivalent to a reflection "loss" of R(dB) $= 10\log 0.01 = -20$dB. Another attenuation of -2.4dB occurs on the return path through the fat layer. Therefore, the transducer detects a signal with an intensity of -24.8dB. It occurs at a time 8.28μs (round trip path 0.012m, speed 1450m-s-1) after the pulse is transmitted.

The other signal strengths are calculated using the same method. One-way attenuation within the fat layer is -2.4 dB; One-way attenuation within the muscle layer is -20 dB; Intensity reflection coefficient at the fat/muscle boundary is -20 dB; Intensity reflection coefficient at the fat/liver boundary is -21 dB. One-way traveling time of signal within the fat layer is $0.006/1450 = 4.14$ μs; One-way traveling time of signal within the muscle layer is $0.04/1590 \sim 25.16$ μs;

The second echo occurs at $(4.14 + 25.16)$ x $2 = 58.28$ μs. Its intensity is $0 - 2.4 - 20 - 20 - 20 - 2.4 = -64.8$ dB.

The third echo occurs at $(4.14 + 25 + 4.14)$x$2 = 66.56$μs. Its intensity is $0 - 2.4 - 20 - 2.4 - 21 - 2.4 - 20 - 2.4 = -70.6$ dB.



b. Transmitted attenuation is considered

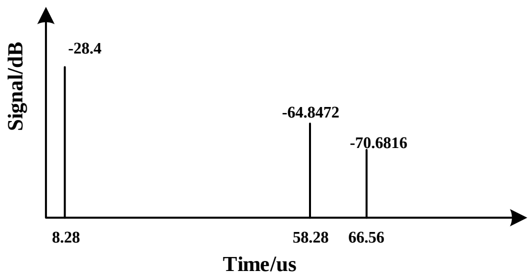
For 8.28us, 0-2.4-2.4-20 = -28.4;

For 58.28us, $T_I = 0.9892$ => 10log10(0.9892) = -0.0472dB

0-2.4-0.0472-20-20-20-2.4 = -64.8472dB

For 66.56us, $T_I = 0.9921$ => 10log10(0.9921) = -0.0344dB

0-2.4-0.0472-20-0.0344-2.4-21-2.4-20-2.4 = -70.6816dB



3. The three ultrasound images in Figure 2 are of the same object. Explain which single operating parameter changes from image (a) to image (b) to image (c). (20 Point)

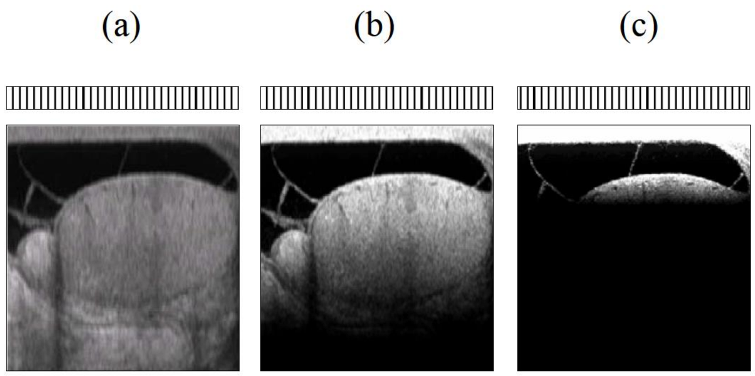


Figure 2

Solution.
There are three image features to note: the penetration depth decreases from A to B to C, the signal-to-noise decreases from A to B to C, and the axial spatial resolution increases from A to B to C. The only operating parameter that could give rise to all three of these effects is an increase in the ultrasound frequency.

4. Sketch the Doppler spectral patterns at points 1, 2, and 3 below in a stenotic artery, shown in Figure 3. (25 Point)

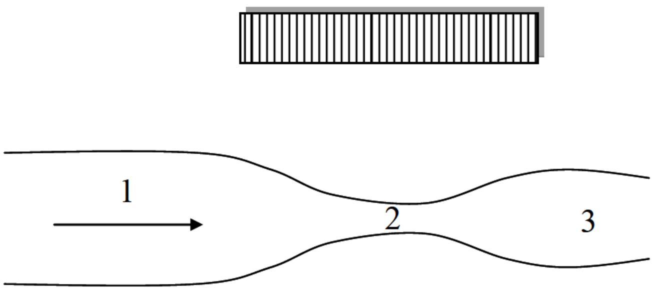
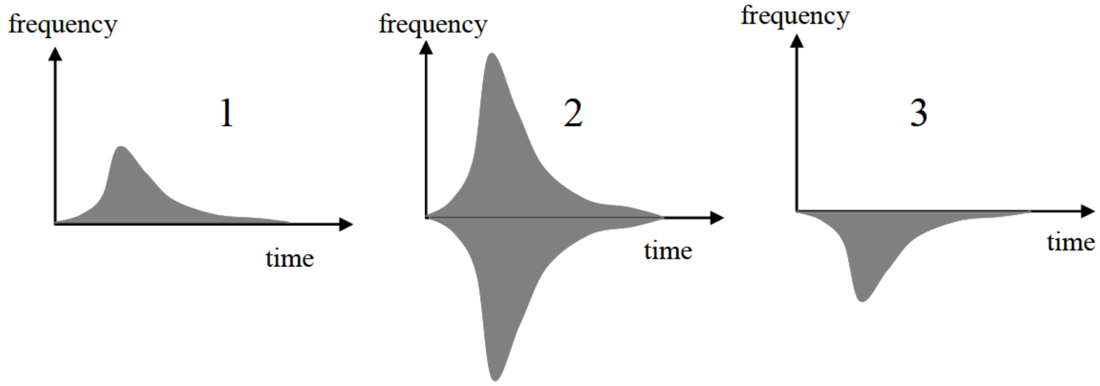


Figure 3

Solution. *All of the plots are made over one cardiac cycle.* At position 1 the flow is characterized by a range of relatively low velocities predominantly flowing towards the transducer. At 2, since the vessel narrows, the velocities become much higher. However, there are equal contributions from flow towards and away from the transducer, and so equal positive and negative frequencies. At 3, there will be a broad range of velocities, probably including turbulent flow. The Doppler spectral patterns will have the general appearance shown below.


frequency
1
time
frequency
2
time
frequency
3
time