ShanghaiTech University

CS240 Algorithm Design and Analysis

Fall 2022

Problem Set 4

Due: 23:59, Dec. 18, 2022

1. Submit your solutions to Gradescope (www.gradescope.com).
2. In "Account Settings" of Gradescope, set your FULL NAME to your Chinese name and enter your STUDENT ID correctly.
3. If you want to submit a handwritten version, scan it clearly. Camscanner is recommended.
4. When submitting your homework, match each of your solution to the corresponding problem number.

Problem 1:

Suppose that there are n items need to be placed in some bins. The capacity of each bin is 1, and the volume of items are not same and all volumes are smaller than 1. We want to use the fewest number of bins to place all items. Please design a 2-approximation algorithm to solve this problem.

Problem 2:

In HW3, you've considered the problem that optimally assign m items to two players. Now suppose there are n players and each of them has a unique valuation and an upper bound. Provide a 2-approximation algorithm to solve the problem and prove it.

Problem 3:

Suppose to perform a sequence of n operations on a data structure. The i-th operation costs i if i is an exact power of 2, otherwise i-th operation costs 1.
a) Using accounting method to determine the amortized cost per operation.
b) Using potential method to determine the amortized cost per operation.

Problem 4:

Given a function rand2() that returns 0 or 1 with equal probability, implement rand3() using rand2() that returns 0, 1 or 2 with equal probability. Minimize the number of calls to rand2() method. Prove the correctness.

Problem 5:

Suppose that for some decision problem, we have an algorithm which on any instance computes the correct answer with probability at least 2/3. We wish to reduce the probability of error by running the algorithm n times on the same input using independent randomness between trials and taking the most common result. Using Chernoff bounds, give an upper bound on the probability that this new algorithm produces an incorrect result.