Homework 5

Due date: May.4^{th}

Turn in your homework online before the class

Rules:

- Please work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism!
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

1

[16 points]

(a). The circuit is shown in **Fig 1:a**. Assume steady state of the circuit. Known that $u_S(t) = 1.5\sqrt{2}cos(10^5t + 60°)$V. Express $i_R(t), i_L(t), i_C(t), i(t)$, in phasor domain.

(b). The circuit is shown in **Fig 1:b**. Assume steady state of the circuit. Known that $i(t) = 1cos(10^7t+90°)$A. Express $u_R(t), u_L(t), u_C(t), u_S(t)$ in phasor domain..

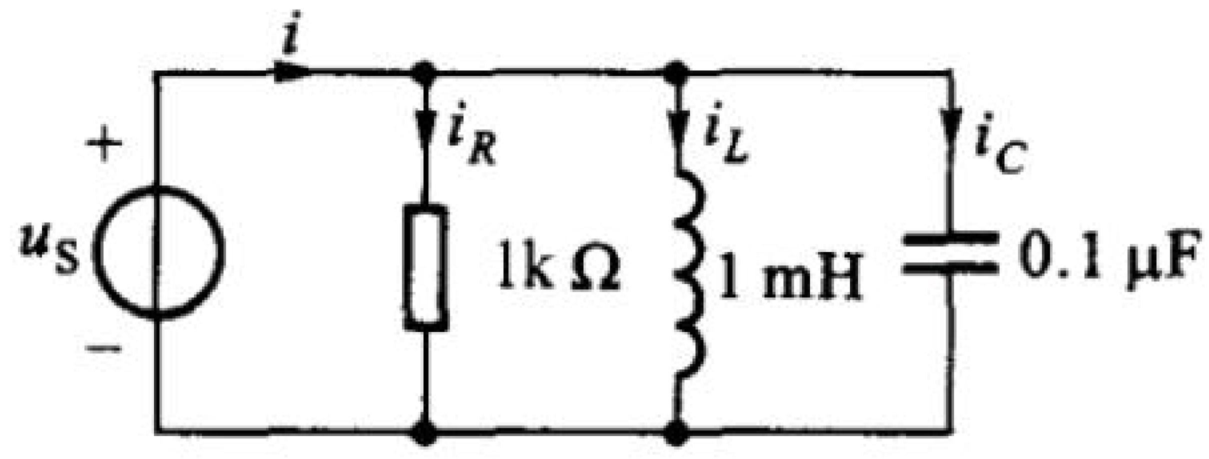


Figure 1: a

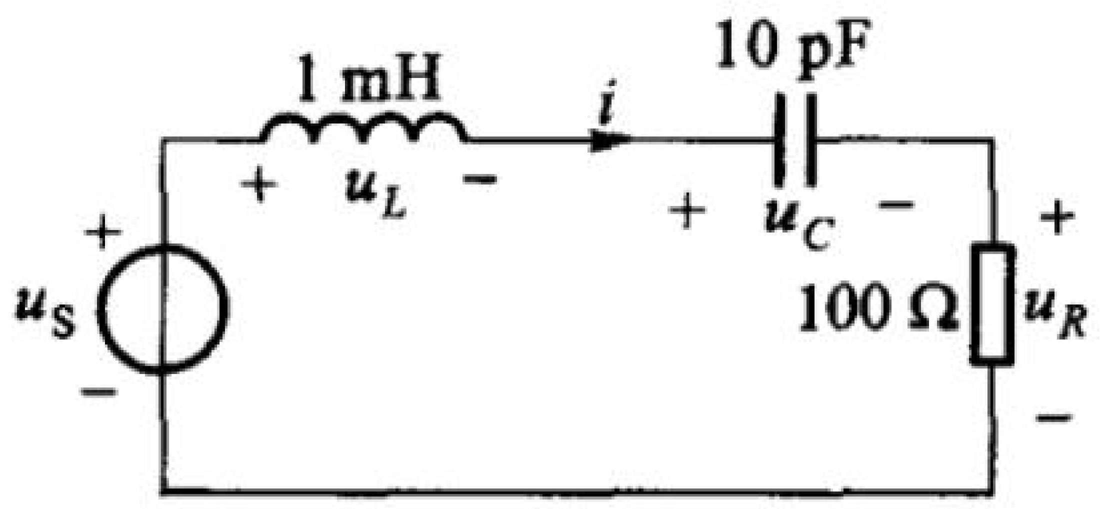


Figure 1: b

(a) $u_S(t) \to \dot{U}_S = 1.5\sqrt{2}\angle 60^\circ V$

$\dot{I}_R = \frac{\dot{U}_S}{R} = \frac{1.5\angle 60^\circ}{1000} = \cancel{\sqrt{2}}1.5\times 10^{-3}\angle 60^\circ A = 1.5\sqrt{2}\angle 60^\circ mA = 2.12\angle 60^\circ mA$ 2'

$\dot{I}_L = \frac{\dot{U}_S}{j\omega L} = \frac{1.5\angle 60^\circ}{j10^5\times 10^{-3}} A = \cancel{\sqrt{2}}15\times 10^{-3}\angle -30^\circ A = 15\sqrt{2}\angle -30^\circ mA = 21.21\angle -30^\circ mA$ 2'

$\dot{I}_C = j\omega C\dot{U}_S = \cancel{\sqrt{2}}j10^5\times 10^{-7}\times 1.5\angle 60^\circ A = 15\sqrt{2}\angle 150^\circ mA = 21.21\angle 150^\circ mA$ 2'

$\dot{I} = \dot{I}_R + \dot{I}_L + \dot{I}_C = 2.12\angle 60^\circ mA$ 2'

(b) $i(t) \to \dot{I} = 1\angle 90^\circ A = j1A$

$\dot{U}_R = R\dot{I} = j100 V$ 2'

$\dot{U}_L = j\omega L\dot{I} = j10^7\times 10^{-3}\times 1\angle 90^\circ V = 10^4\angle 180^\circ V$ 2'

$\dot{U}_C = \frac{1}{j\omega C}\dot{I} = 10^4\angle 0^\circ V$ 2'

$\dot{U}_S = \dot{U}_R + \dot{U}_L + \dot{U}_C = j100 V$ 2'

2

[12 points]The circuit is shown in **Fig 2**. Use phasor approach to calculate the currents $\dot{I}_R$, $\dot{I}_C$, $\dot{I}_L$, and draw the phasor diagram of the above three currents.

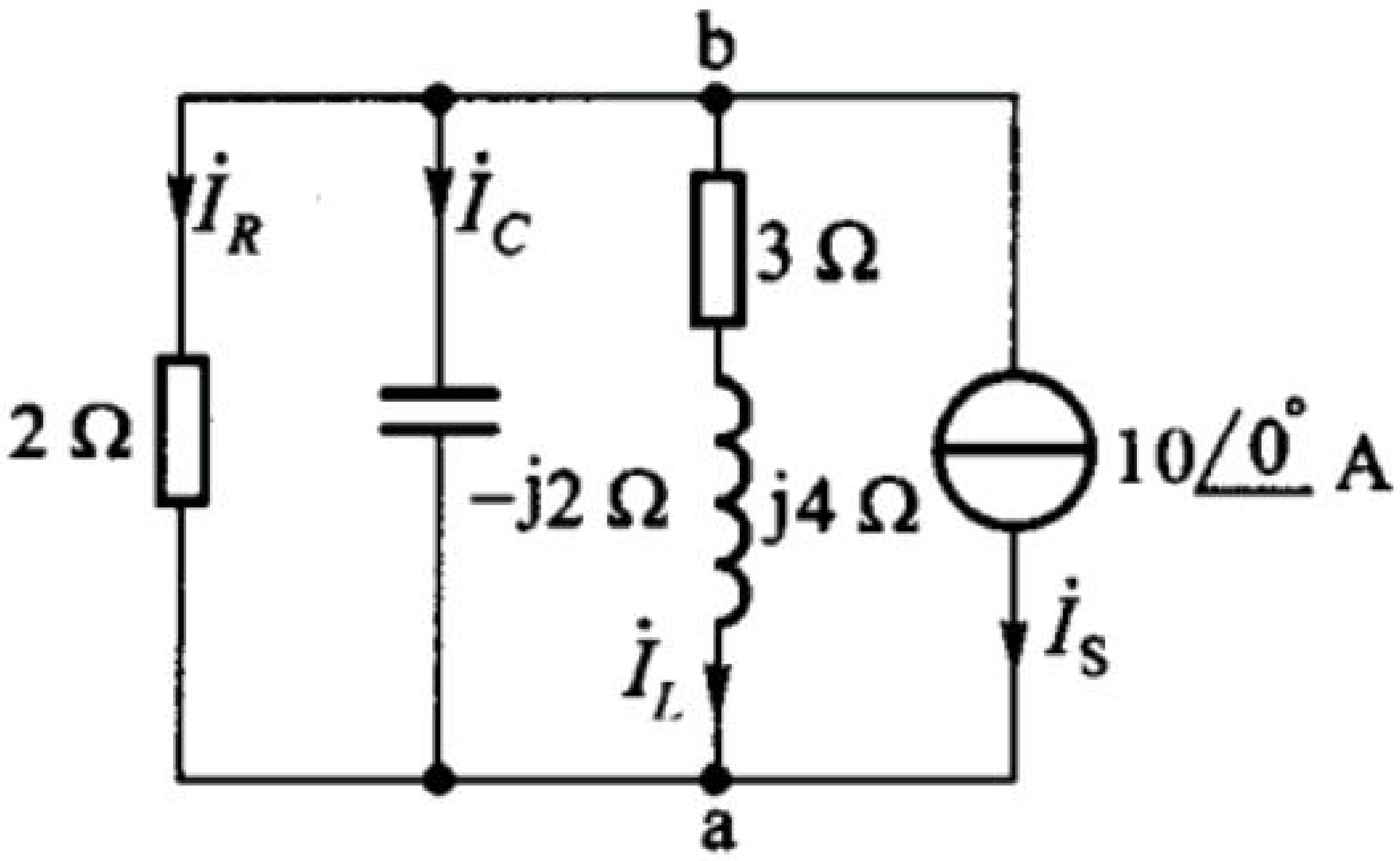
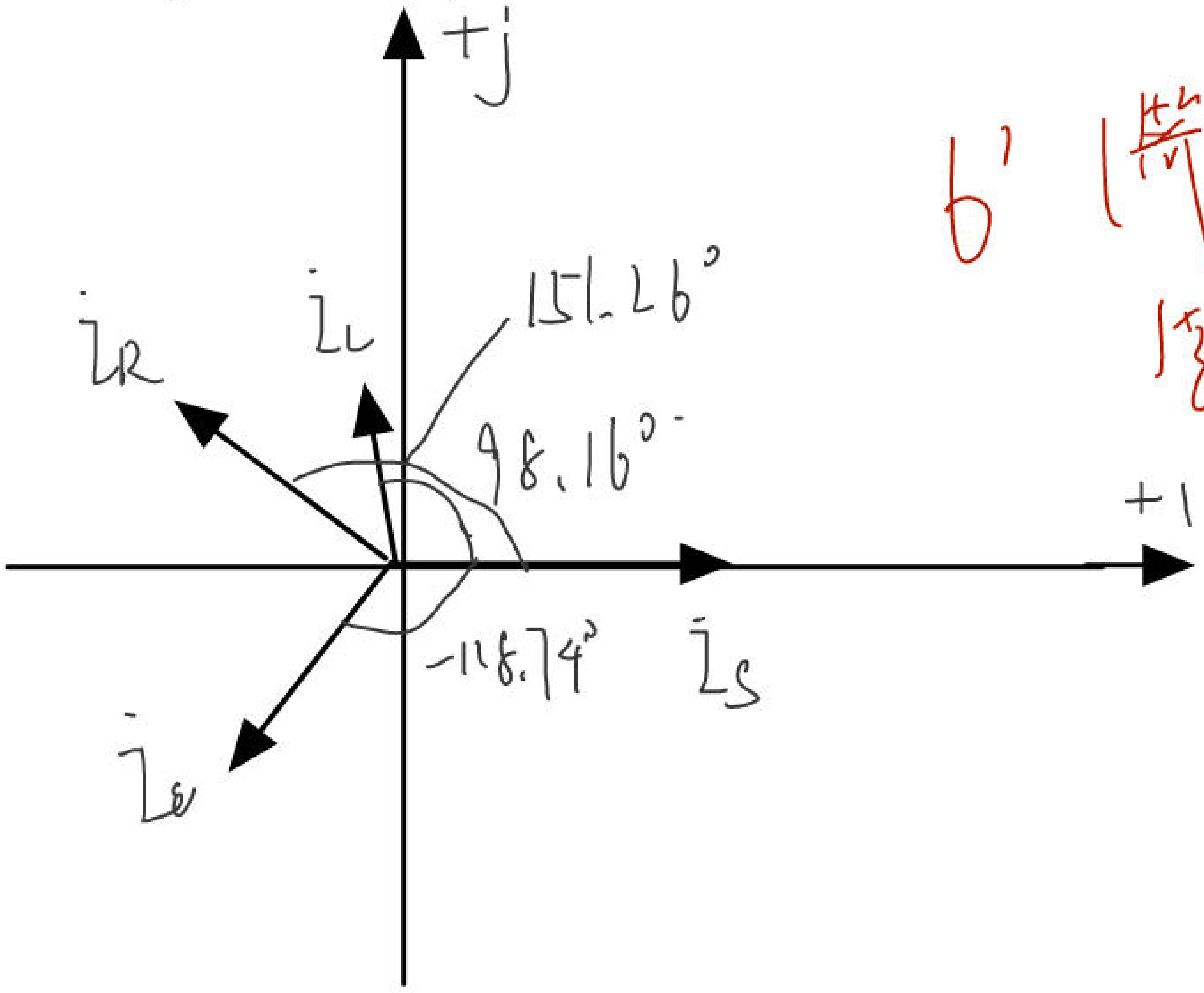


Figure 2

$$\dot{I}_R = \frac{0.5\times(-10)}{0.5+j0.5+\frac{1}{3+j4}} = \frac{-5}{0.62+j0.34} = \frac{-5}{0.707\angle 28.74^\circ} = 7.07\angle 151.26^\circ A$$ 2'

$$\dot{I}_C = \frac{j0.5\times(-10)}{0.5+j0.5+\frac{1}{3+j4}} = \frac{-j5}{0.707\angle 28.74^\circ} = 7.07\angle -118.74^\circ A.$$ 2'

$$\dot{I}_L = \frac{\frac{1}{3+j4}\times(-10)}{0.5+j0.5+\frac{1}{3+j4}} = \frac{2\angle 126.9^\circ}{0.707\angle 28.74^\circ} = 2.823\angle 98.16^\circ A.$$ 2'



6' (箭头长度、角度、相对位置 各调节酌情扣分)

3

[10 points]The circuit is shown in **Fig 3**. $\dot{U}_S = 24\angle 60°$V, $\dot{I}_S = 6\angle 0°$A. Use mesh analysis to calculate $\dot{I}_1$ and $\dot{I}_2$.

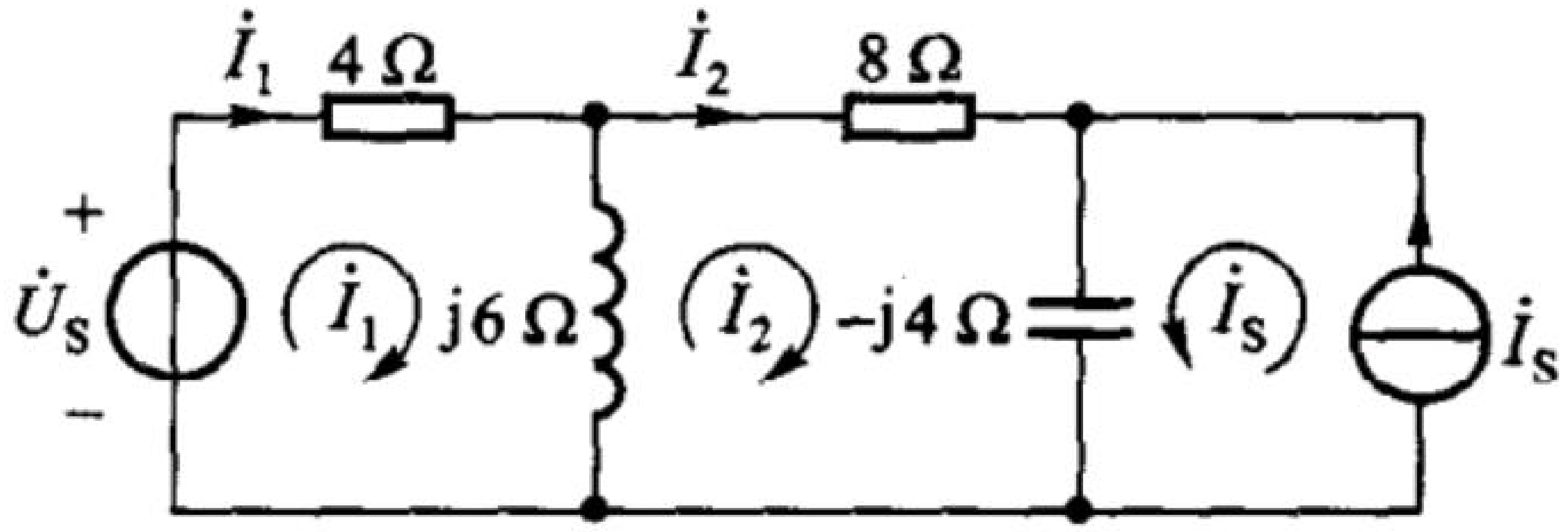


Figure 3

$$\begin{cases} (4+j6)\dot{I}_1 - j6\dot{I}_2 = 24\angle 60° \\ -j6\dot{I}_1 + (8+j2)\dot{I}_2 - j4\dot{I}_S = 0 \\ \dot{I}_S = 6\angle 0° A \end{cases} \Rightarrow \begin{cases} (4+j6)\dot{I}_1 - j6\dot{I}_2 = 12 + j20.8 \\ -j6\dot{I}_1 + (8+j2)\dot{I}_2 = j24 \end{cases}$$

4'

$$\dot{I}_1 = \frac{(12+j20.8)(8+j2) - 144}{(4+j6)(8+j2)+36} A$$

$$= \frac{-89.6 + j190.4}{56+j56} A$$

$$= \frac{210.4\angle 115.2}{79.2\angle 45°} A = 2.66\angle 70.2° A$$

3'

$$\dot{I}_2 = \frac{j24 \times (4+j6) - (12+j20.8)(-j6)}{(4+j6)(8+j2)+36} A$$

$$= \frac{317\angle 48°}{79.2\angle 45°} A = 4\angle 03° A.$$

3'

4

[10 points]The circuit is shown in **Fig 4**. $\dot{U}_S = 10\angle 0°$V, μ=0.5. Use nodal analysis to calculate $\dot{U}_2$.

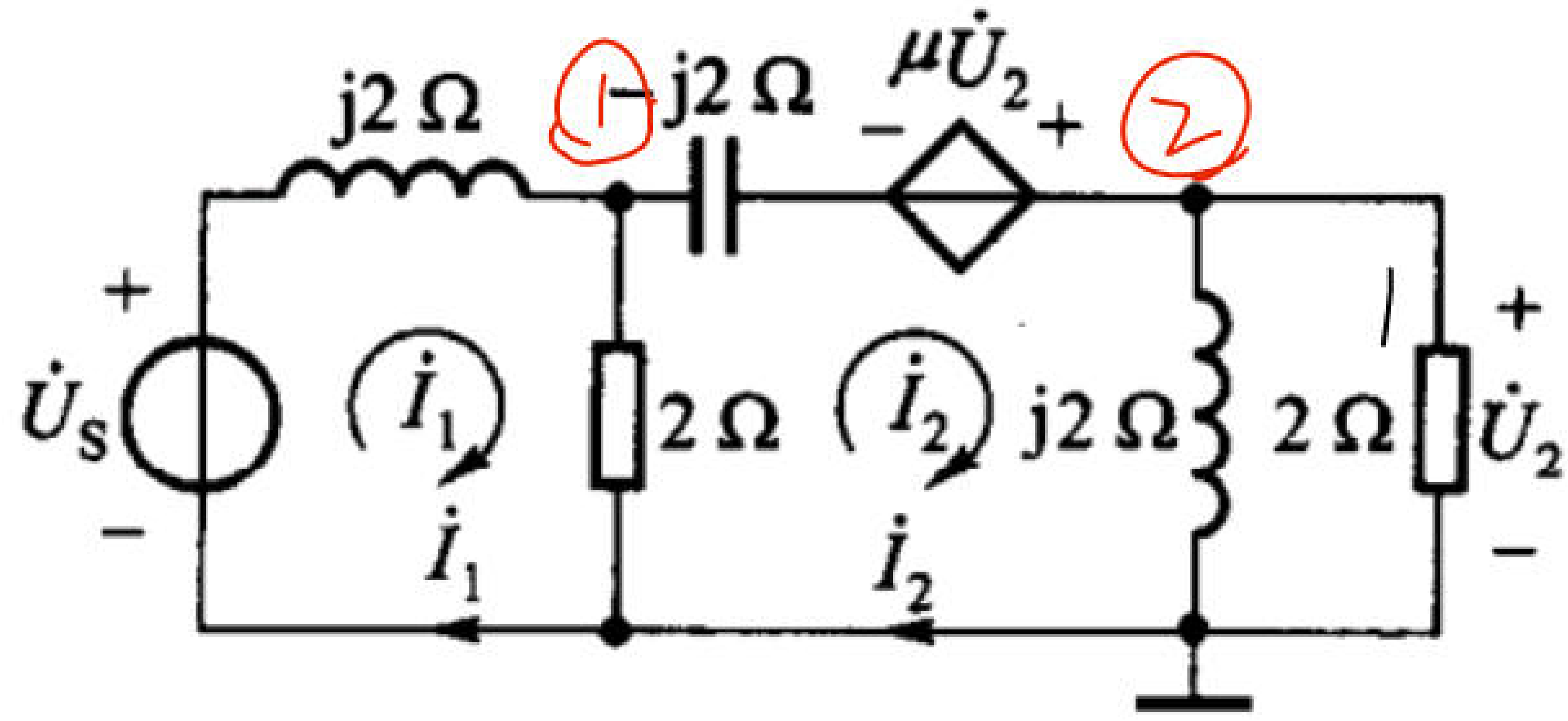


Figure 4

$$\begin{cases}(0.5-j0.5+j0.5)\dot{U}_1 - j0.5\dot{U}_2 = -j5 - j0.25\dot{U}_2 \\ -j0.5\dot{U}_1 + (0.5+j0.5-j0.5)\dot{U}_2 = j0.25\dot{U}_2\end{cases}$$

4'

$$\Rightarrow \begin{cases}2\dot{U}_1 - j1\dot{U}_2 = -j20 \\ -j2\dot{U}_1 + (2-j1)\dot{U}_2 = 0\end{cases}$$

$$\Rightarrow \dot{U}_1 = \frac{-j20\times(2-j1)}{2\times(2-j1)+2}V = \frac{-20-j40}{6-j2}V = (-1-j7)V = 7.07\angle -98.1° V$$ 3'

$$\Rightarrow \dot{U}_2 = \frac{-j20\times j2}{2\times(2-j1)+2}V = \frac{40}{6-j2}V = 6+j2\ V = 6.32\angle 18.43° V$$ 3'

5

[12 points]The circuit is shown in **Fig 5**. The circuit is under sinusoidal steady state. Known that $i_S(t) = 30\sqrt{2}cos20t$ A. For the circuit excluding the 30Ω resistance, find the Thevenin equivalance(phasor domain) at the terminals a-b. Afterwards, use the Thevenin equivalence to calculate $u_k(t)$.

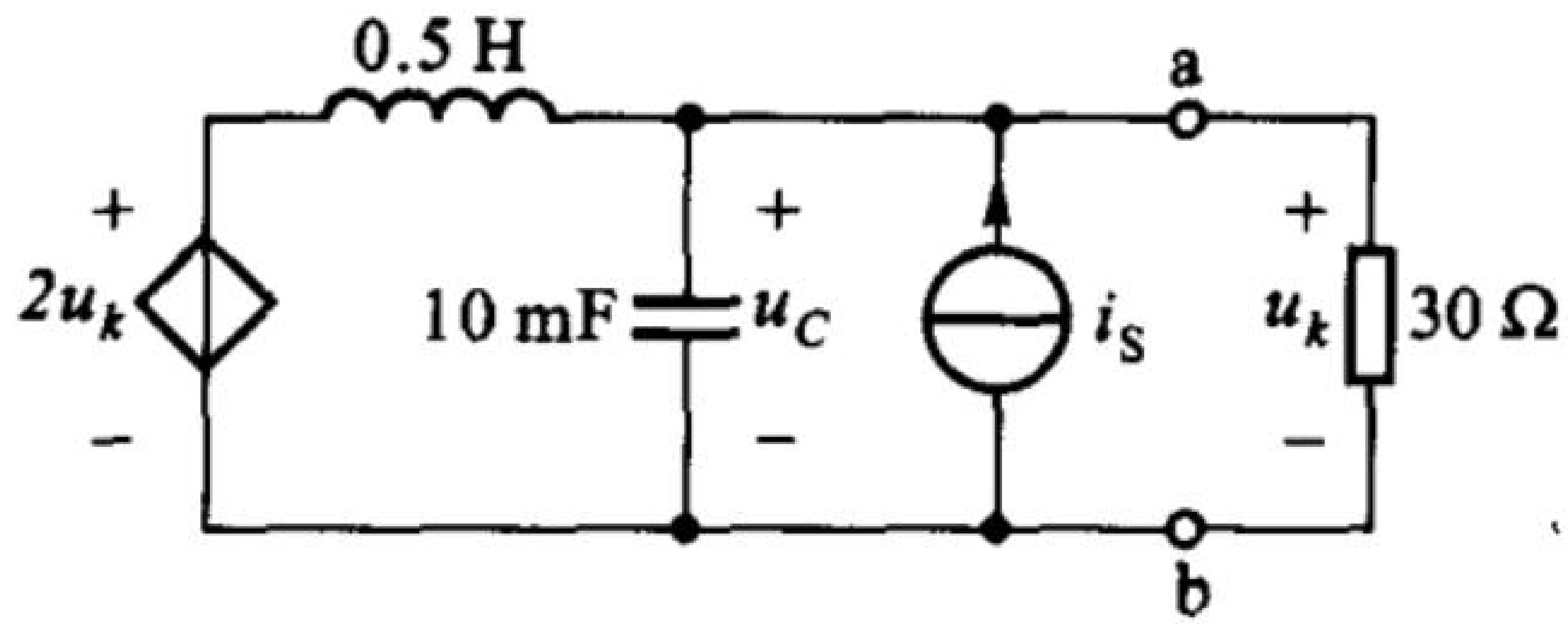
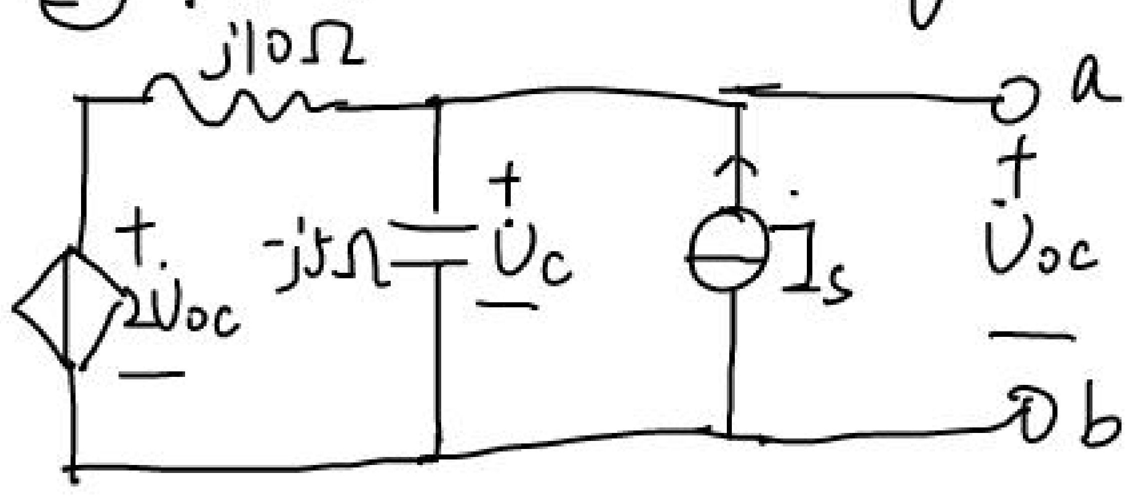


Figure 5

① find thevenin equivalence circuit:



$\left(\frac{1}{-j5}+\frac{1}{j10}\right)\dot{U}_{oc} = 30\sqrt{2}\angle 0^\circ - j0.2\dot{U}_{oc} \Rightarrow \dot{U}_{oc} = 100\sqrt{2}\angle -90^\circ V$ 2'

$\dot{I}_{sc} = \dot{I}_s = 30\sqrt{2}\angle 0^\circ A$. 2'

$Z_{Th} = \frac{\dot{U}_{oc}}{\dot{I}_{sc}} = 3.33\angle -90^\circ \Omega$ 2'

② find $U_k(t)$

$\dot{U}_k = \frac{30}{30+Z_{Th}}\dot{U}_{oc} = \frac{30}{30-j\frac{10}{3}} \times 100\sqrt{2}\angle -90^\circ = 140.56\angle -83.66^\circ$ V 2'

$U_k(t) = 140.56\cos(20t - 83.66^\circ)(V)$. 4'

6

[14 points]The circuit is shown in **Fig 6**. The circuit is under sinusoidal steady state. Known that $i_S(t) = 10\sqrt{2}cos100t$ A, $u_S(t) = 100\sqrt{2}cos1000t$ V. Find $i_L(t)$. (Note: you should be careful about the operating frequency of the system when applying phasor domain equivalence)

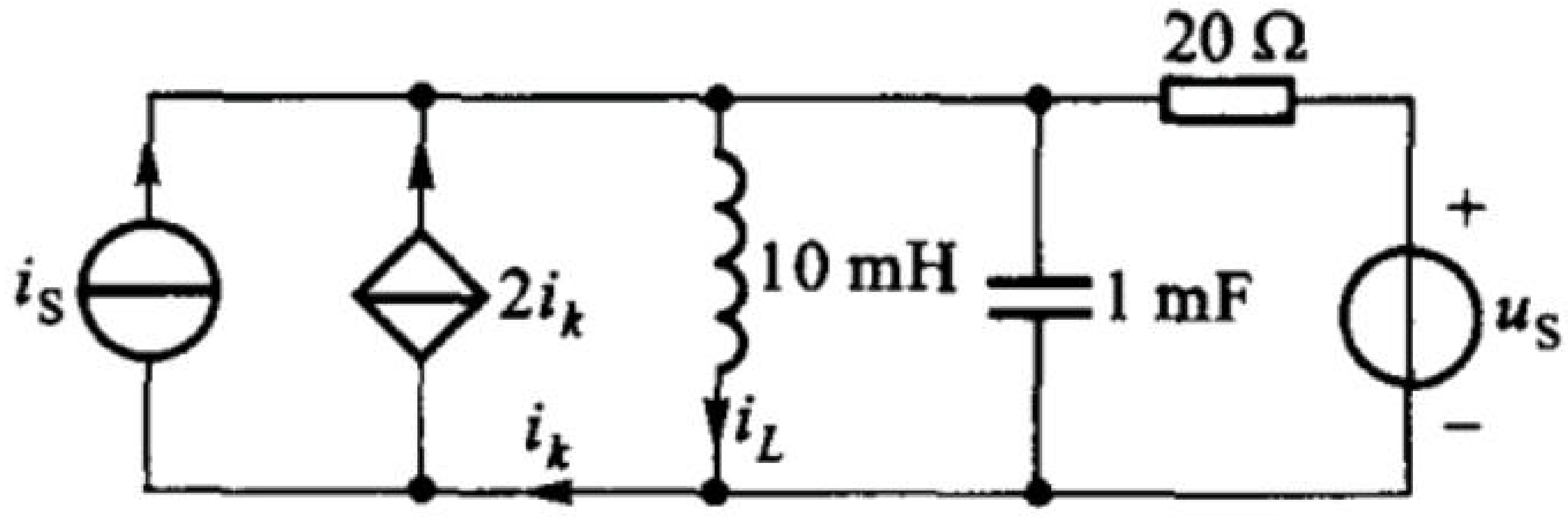
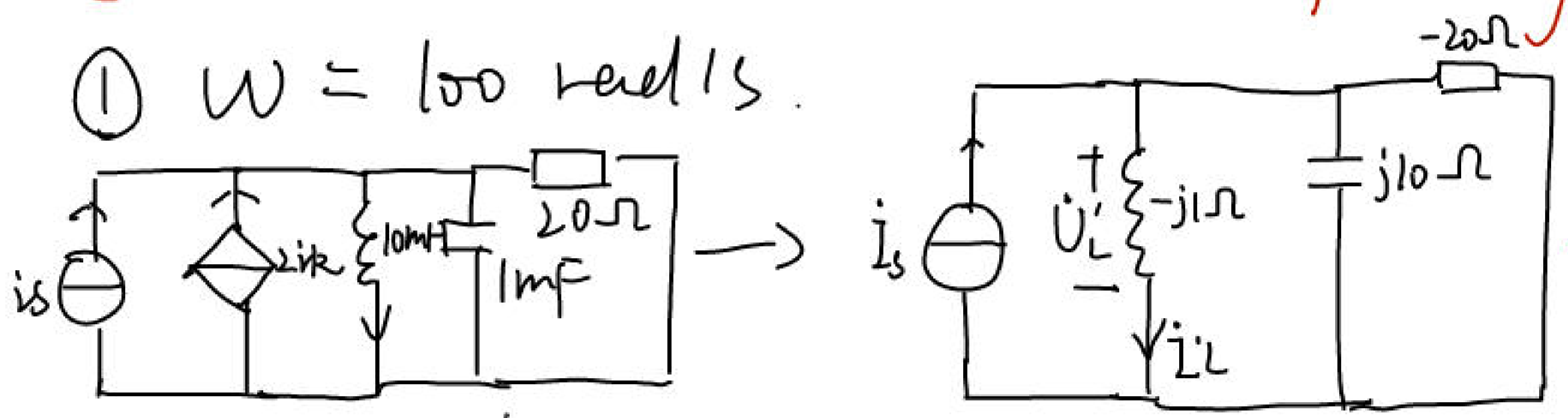


Figure 6

Let the two sources work desperately.

① $\omega = 100$ rad/s

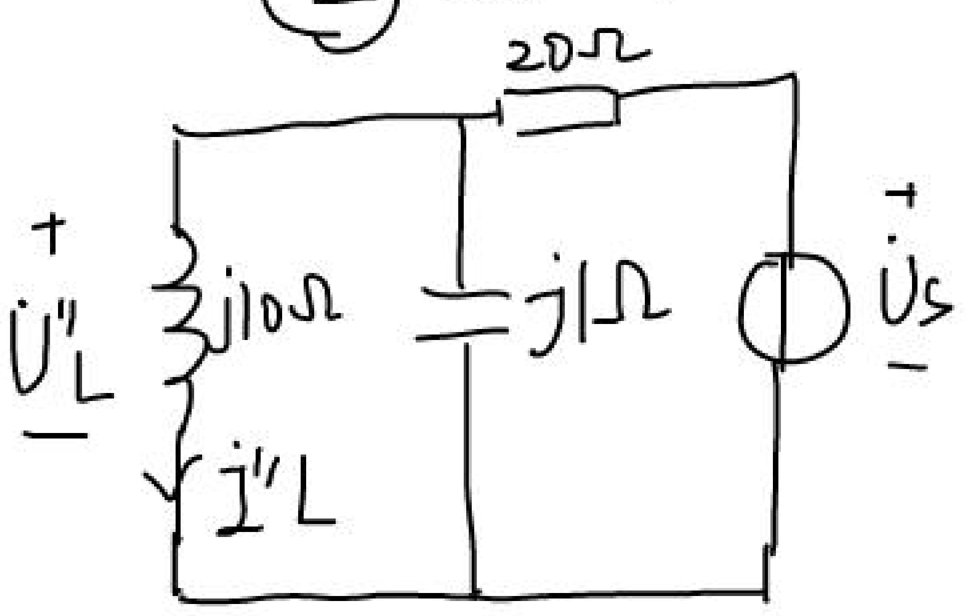


The controlled source causes the impedance of each element to be negative of its original value.

$$\dot{I}_L = \frac{10\sqrt{2}\angle 0^\circ \times \frac{1}{-j1}}{\frac{1}{-20}+\frac{1}{-j1}+\frac{1}{j10}} = \frac{10\sqrt{2}\angle 0^\circ \times \frac{1}{-j1}}{-0.05+j0.9} = 15.69\angle 176.82^\circ A \text{ — } 2'$$

$$I_L'(t) = 15.69\cos(100t+176.82^\circ)A \text{ — } 2'$$

② $\omega = 1000$ rad/s



$$\dot{U}_L'' = \frac{5\sqrt{2}\angle 0^\circ}{\frac{1}{20}+\frac{1}{j10}+\frac{1}{-j1}} = 7.84\angle -86.82^\circ V \text{ — } 2'$$

$$\dot{I}_L'' = \frac{\dot{U}_L''}{j10} = 0.784\angle -176.82^\circ A \text{ — } 2'$$

$$i_L''(t) = 0.784\cos(1000t-176.82^\circ)A \text{ — } 2'$$

$$i_L(t) = 15.69\cos(100t+176.82^\circ) + 0.78\cos(1000t-176.82^\circ)A \text{ — } 4'$$

7

[12 points]

(a). The circuit is shown in **Fig 7:a**. All of the elements are working in Sinusoidal Steady-state. Find out the the relationship that the values of the elements satisfy to make the equivalent impedance(between a and b) pure resistive at any frequency. (Please make sure that you write down all of the conditions).

(b). The circuit is shown in **Fig 7:b**. All of the elements are working in Sinusoidal Steady-state. Try to figure out what conditions the values of the elements and ω have to meet to make sure that $\frac{\dot{U}_1}{\dot{U}_2}$ has nothing to do with Z (Z is the value of the impedence of the element), and write down the ratio.

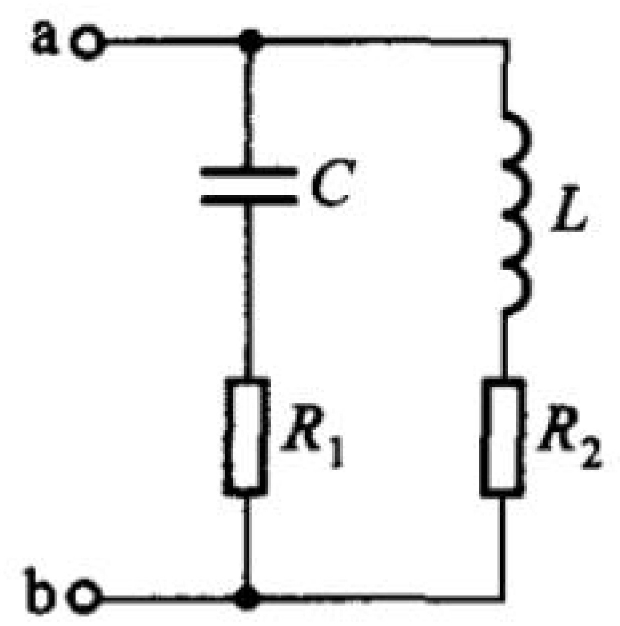


Figure 7: a

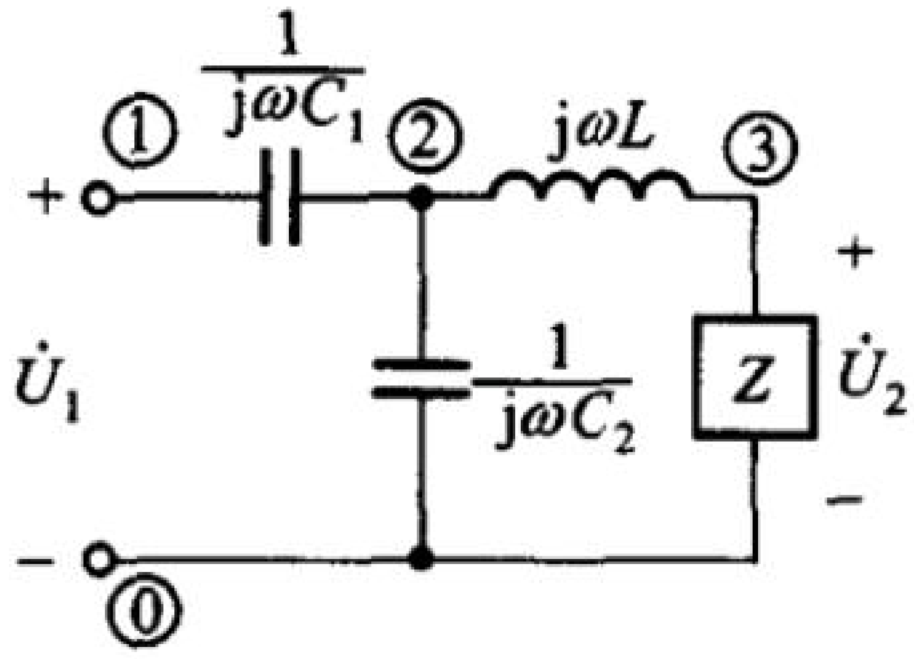


Figure 7: b

(a) $Z_{ab} = \dfrac{(R_2+j\omega L)(R_1+\frac{1}{j\omega C})}{R_2+j\omega L+R_1+\frac{1}{j\omega C}} = R_1 \times \dfrac{1+j\omega(\frac{L}{R_2}+R_1C)-\frac{\omega^2 R_1 LC}{R_2}}{1+j\omega C(R_1+R_2)-\omega^2 LC}$ 2'

$$\begin{cases}(R_1+R_2)C = \frac{L}{R_2}+R_1C \\ LC = \frac{R_1 LC}{R_2}\end{cases} \Rightarrow \begin{cases}R_1 = R_2 = R \text{ 1'} \\ L = R^2 C \text{ 1'}\end{cases} \text{ or } L = C = 0 \text{ 2'}$$

(b) $\dfrac{\dot{U}_1}{\dot{U}_2} = Z \cdot \dfrac{\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_2}+j\omega L+Z} \cdot \left(\dfrac{1}{j\omega C_1}+\dfrac{(j\omega L+Z)\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_2}+j\omega L+Z}\right) = \dfrac{1+j\omega C_2 Z-\omega^2 C_2 L}{j\omega C_1 Z}+\dfrac{(j\omega L+Z)(1+j\omega C_2 Z-\omega^2 C_2 L)}{Z(1-\omega^2 C_2 L+j\omega C_2 Z)}$

$$= \frac{C_2}{C_1}+\frac{1-\omega^2 C_2 L}{j\omega C_1 Z}+\frac{j\omega L}{Z}+1$$

$$= \frac{C_1+C_2}{C_1}+\frac{1-\omega^2(C_1+C_2)L}{j\omega C_1 Z} \text{ 2'}$$

Then we can get that: $\underline{\omega^2 L(C_1+C_2) = 1.}$ 2'

The ratio is $\dfrac{C_1+C_2}{C_1}$ 2'

8

[14 points]

(a). Known that the sinusoidal voltage has an amplitude of 100V. The instantaneous value of the voltage at t=0 is 10V, and the period is 1ms. Please write down the expression of the voltage as a cosine function.

(b). $i_1(t) = 4cos(\omega t - 80°)A$, $i_2(t) = 10cos(\omega t + 20°)A$, $i_3(t) = 8sin(\omega t - 20°)A$. Please express these currents in phasor domain.

(a) $100\cos\phi_0 = 10 \Rightarrow \phi_0 = \arccos 0.1 = \pm 84.26°$ 2'

$\omega = \frac{2\pi}{T} = 2\pi \times 10^3$ rad/s 2'

$u(t) = 100\cos(2\pi \times 10^3 t \pm 84.26°)V$

$= 100\cos(6.28 \times 10^3 t \pm 84.26°)V$ 4'

rad

angle

$= 100\cos(3.60 \times 10^5 t \pm 84.26°)V$

(b) $\dot{I}_1 = 4\angle -80° A$ — 2'

$\dot{I}_2 = 10\angle 20° A$ — 2'

$\dot{I}_3 = 8\angle -110° A$ — 2'