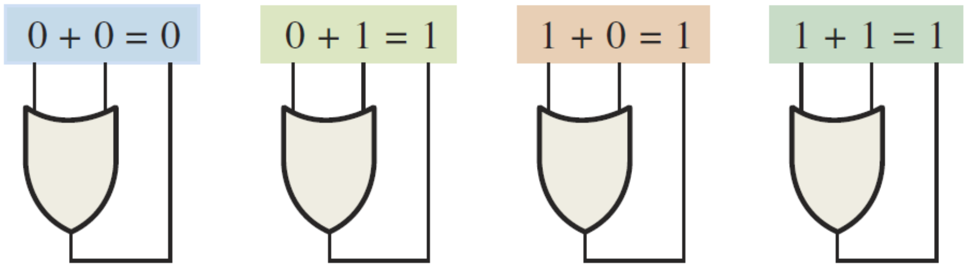
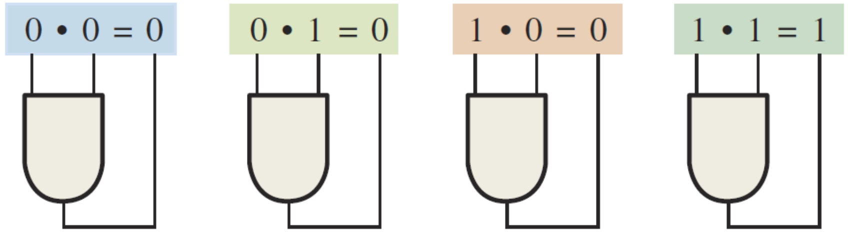
Boolean Addition and Multiplication



Addition



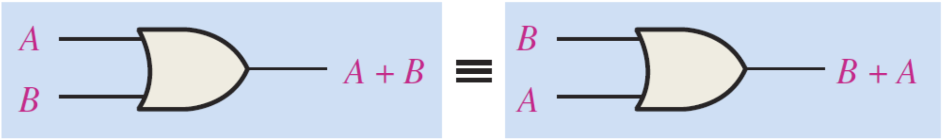
Multiplication

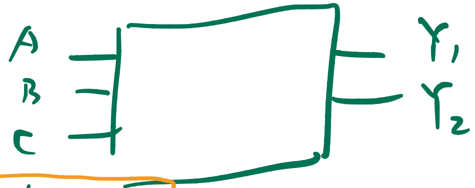
Laws of Boolean Algebra

- Commutative Laws

$$A + B = B + A$$

$$AB = BA$$





- Associative Laws

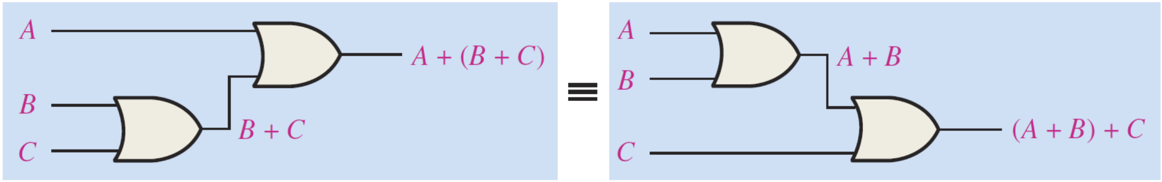
1. $Y_1 = A + B + C$

$\boxed{A + (B + C)}$

$(A + B) + C$

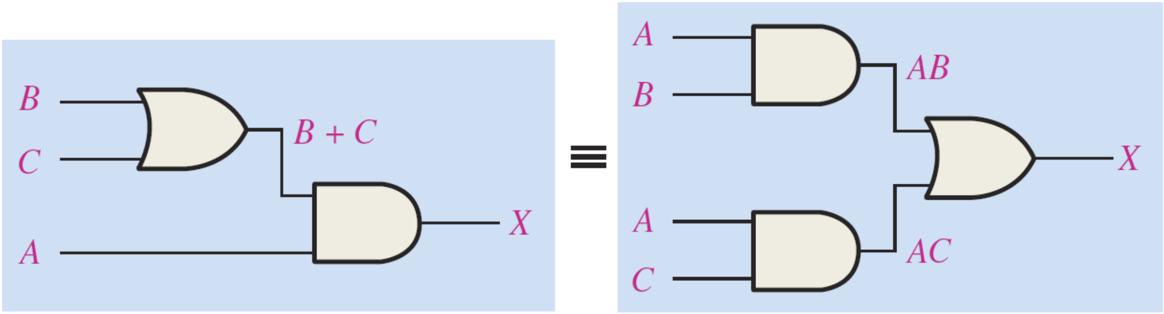
$$A + (B + C) = (A + B) + C$$

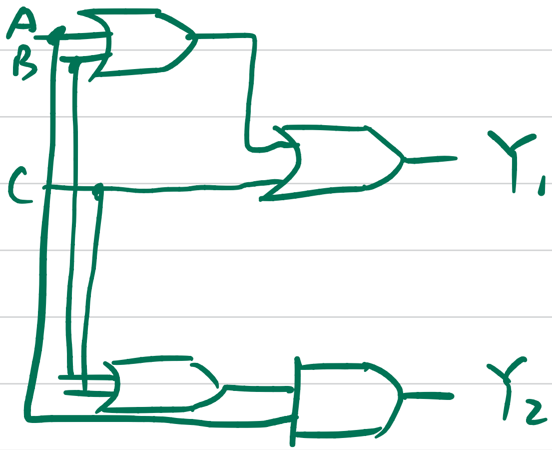
2. $Y_2 = A \cdot (B + C)$

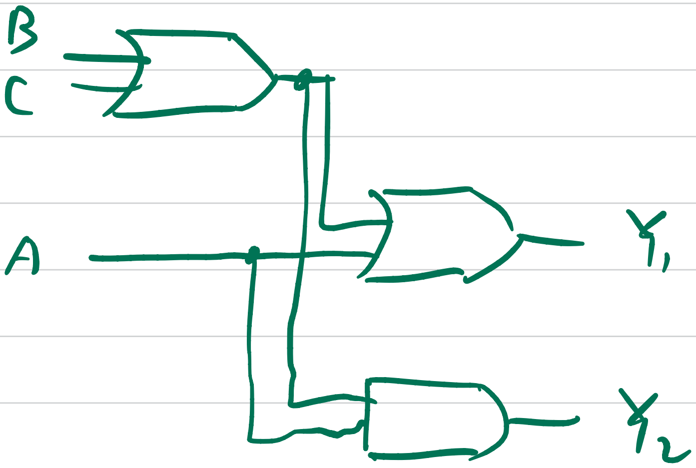


- Distributive Law

$$A(B + C) = AB + AC$$




A
B
C
Y_1
Y_2


B
C
A
Y_1
Y_2

Rules of Boolean Algebra

Basic rules of Boolean algebra.

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$ $\quad A(1+B) = A$
11. $A + \overline{A}B = A + B$ * $\quad A(1+B)+\overline{A}B = A+AB+\overline{A}B = A+B$
12. $(A + B)(A + C) = A + BC$ *

A, B, or C can represent a single variable or a combination of variables.

$$(A+B)(A+C) = AA+AC+AB+BC$$
$$= \underline{A}+AC+\underline{AB}+BC$$
$$= \underline{A}+AC+BC$$
$$= A+BC$$

13. $AB + A'C + BC = AB + A'C$
$AB + A'C + BCD = AB + A'C$ (EFG) *

$$(A+B)(A+C) = AA+AC+AB+BC$$
$$= A+AC+AB+BC$$
$$= A(1+C+B)+BC$$
$$= A+BC$$

Question:

- Use truth table to proof eq. 12
- Proof eq. 12 and 13

$$AB + A'C + BC = AB + A'C + BC(A + A')$$
$$= AB + A'C + ABC + A'BC$$
$$= AB(1 + C) + A'C(1 + B)$$
$$= AB + A'C$$

$$AB + A'C + BC = AB(1+C) + A'C(1+B) + (A'+A)BC$$
$$= AB + \underline{ABC} + A'C + \underline{A'BC} + \underline{A'\cancel{B}C} + \underline{A\cancel{B}C}$$
$$= ABC(1+C) + A'C(1+B)$$
$$= AB + A'C$$

$$AB + A'C + BC + B - B$$
$$= AB + A'C + B(C+1) - B$$
$$= AB + A'C + B - B$$
$$= AB + A'C$$

$$AB + A - A$$
$$= A(B+1) - A$$
$$= 0$$

✓ $AB + A'C + BCD = AB + A'C$

$AB + A'C + BCD + BC = AB + A'C + BC$

$AB + A'C + BC = AB + A'C + BC$

$$AB + A'C + BCD = AB + A'C + BC + BCD$$
$$= AB + A'C + BC(1+D)$$
$$= AB + A'C + BC$$
$$= AB + A'C$$

Rules of Boolean Algebra

- Rule 1: A + 0 = A, A variable ORed with 0 is always equal to the variable

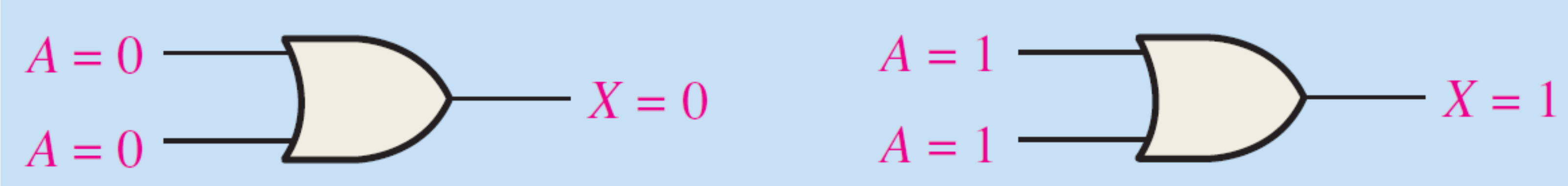
- Rule 2: A + 1 = 1, A variable ORed with 1 is always equal to 1.

- Rule 3: A · 0 = 0, A variable ANDed with 0 is always equal to 0.

- Rule 4: A · 1 = A, A variable ANDed with 1 is always equal to the variable.

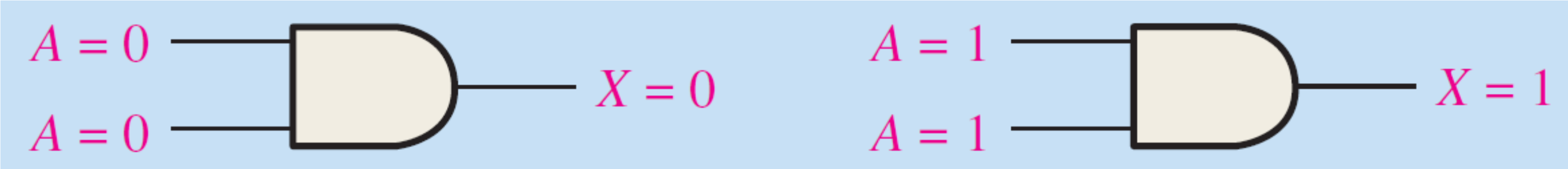
Rules of Boolean Algebra

- Rule 5: A + A = A, A variable ORed with itself is always equal to the variable.



- Rule 6: A + Abar = 1, A variable ORed with its complement is always equal to 1.

- Rule 7: A · A = A, A variable ANDed with itself is always equal to the variable.



- Rule 8: A · Abar = 0, A variable ANDed with its complement is always equal to 0.

Rules of Boolean Algebra

- Rule 9: The double complement of a variable is always equal to the variable.

- Rule 10: A + AB = A

$$
\begin{aligned}
A + AB &= A \cdot 1 + AB = A(1 + B) \\
&= A \cdot 1 \\
&= A
\end{aligned}
$$

Rules of Boolean Algebra

- Rule 11: A + A’B = A + B

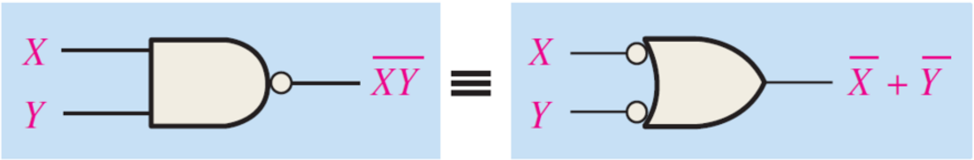
$$
\begin{aligned}
A + \overline{A}B &= (A + AB) + \overline{A}B \\
&= (AA + AB) + \overline{A}B \\
&= AA + AB + A\overline{A} + \overline{A}B \\
&= (A + \overline{A})(A + B) \\
&= 1 \cdot (A + B) \\
&= A + B
\end{aligned}
$$

- Rule 12: (A + B)(A + C) = A + BC

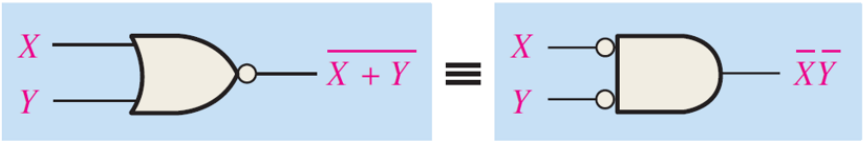
$$
\begin{aligned}
(A + B)(A + C) &= AA + AC + AB + BC \\
&= A + AC + AB + BC \\
&= A(1 + C) + AB + BC \\
&= A \cdot 1 + AB + BC \\
&= A(1 + B) + BC \\
&= A \cdot 1 + BC \\
&= A + BC
\end{aligned}
$$

DeMorgan’s Theorems

- DeMorgan’s first theorem $\overline{XY} = \overline{X} + \overline{Y}$
- DeMorgan’s second theorem $\overline{X + Y} = \overline{X}\,\overline{Y}$



NAND   Negative-OR



NOR   Negative-AND

Inputs		Output	
X	Y	$\overline{XY}$	$\overline{X} + \overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

$$(ABC)' = (AB)' + C' = A' + B' + C'$$

$$(A+B+C)' = (A+B)'C' = A'B'C'$$

Inputs		Output	
X	Y	$\overline{X + Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Think about it:

- 3 variable DeMorgan’s Theorems?

- DeMorgan’s theorems provide mathematical equivalency of the NAND and negative-OR gates and the equivalency of the NOR and negative-AND gates

Simplify Boolean expression

1 $\overline{\overline{AB} + \overline{AC}} + \overline{A}\,\overline{B}C$

DeMorgan's theorem ↓

$(\overline{\overline{AB}})(\overline{\overline{AC}}) + \overline{A}\,\overline{B}C$

DeMorgan's theorem ↓

$= (\overline{A} + \overline{B})(\overline{A} + \overline{C}) + \overline{A}\,\overline{B}C$

Distributive law ↓

$= A' + B'C' + A'B'C$

$= A' + B'(C' + A'C)$

$\overline{A}\,\overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C} + \overline{A}\,\overline{B}C$

$= A' + B'(C' + A')$

Rule 7 and Rule 10 ↓

$= A' + A'B' + B'C'$

$\overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C}$

$= A' + B'C'$

Rule 10 ↓

$\overline{A} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C}$

Rule 10 ↓

$\overline{A} + \overline{B}\,\overline{C}$

- Use the basic laws, rules, and theorems to simplify an expression depends on a thorough knowledge and considerable practice, not to mention a little ingenuity and cleverness.

Simplify Boolean expression

1. ABC+B’C+ACD
2. A(B’CD)’+AB’CD
3. AB’+ACD+A’B’+A’CD
4. A’BC’+AC’+B’C’
5. BC’D+BCD’+BC’D’+BCD
6. ((A’B)’+C)ABD+AD
7. AB+ABC’+ABD+AB(C’+D’)
8. A+(A’(BC)’)’(A’+(B’C’+D)’)+BC
9. AC+AB’+(B+C)’
10. AB’CD’+(AB’)’E+A’CD’E
11. ✓ A’B’C+ABC+A’BD’+AB’D’+A’BCD’+BCD’E’
12. B’+ABC
13. AB’+B+A’B
14. AC+A’D+C’D
15. A’BC’+A’BC+ABC
16. AB’+A’B+BC’+B’C
17. AC+B’C+BD’+CD’+A(B+C’)+A’BCD’+AB’DE

$$A'B'C+ABC+A'BD'+AB'D'+A'BCD'+BCD'E'$$

$$= C(A'B'+AB)+D'(A'B+AB')+CD'(A'B+BE')$$

$$=$$

Reading materials

- Chapter 4 of Floyd book
- Chapter 2 of 阎石 book