EE150 Signal and System
Homework 8

Due on 18 Dec 23:59 UTC+8

Note:

- Please provide enough calculation process to get full marks.
- Please submit your homework to Gradescope.
- It's highly recommended to wirte every exercise on single sheet of page.

Exercise 1. (20pt)

Determine the Laplace transform by definition and the associated ROC and pole-zero plot for each of the following functions of time

(a) $x(t) = e^{-2t}u(t) + e^{-3t}u(t)$

(b) $x(t) = te^{-2|t|}$

(c) $x(t) = e^{-5t}(sin5t)u(t)$

(d) $x(t) = \delta(3t) + u(3t)$

Exercise 2. (20pt)

The following facts are given about a real signal $x(t)$ with Laplace transform $X(s)$

a. $X(s)$ has no zeros in the finite s-plane

b. $X(0) = 4$ and $X(1) = 1.6$

c. $X(s)$ has two poles

d. The real part of one pole of $X(s)$ is -1

e. $e^{2t}x(t)$ is not absolutely integrable

Determine $X(s)$ and its ROC

Exercise 3. (20pt)

In this problem, we consider the construction of various types of block diagram representations for a causal LTI system S with input $x(t)$, output $y(t)$ and system function

$$H(s) = \frac{2s^2 + 2s - 40}{s^2 + 4s + 3}$$

To derive the direct form block diagram representations of S, we first consider a causal LTI system S_1 that has the same input $x(t)$ as S,but whose system function is

$$H_1(s) = \frac{1}{s^2 + 4s + 3}$$

With the output denoted by $y_1(t)$, the direct form block diagram representation of S_1 is shown in the figure below, The signals $e(t)$ and $f(t)$ indicated in the figure represent respective inputs into two integrators

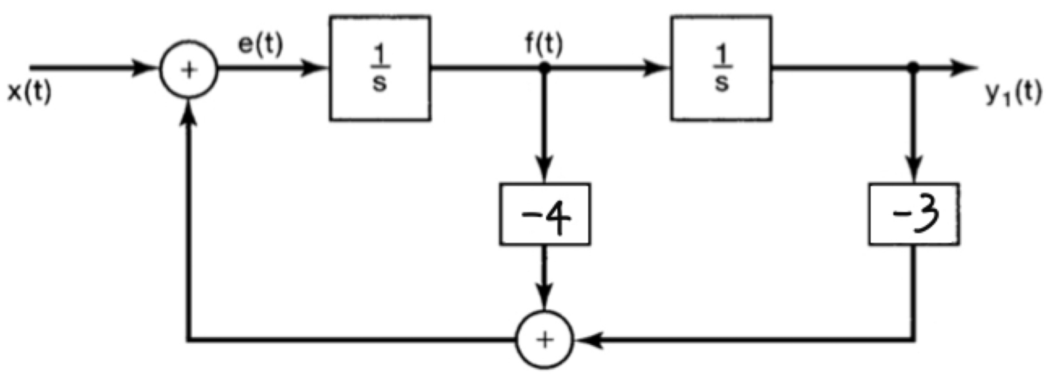

Figure 1: ex.3

(a) Express $y(t)$ (the output of S) as a linear combination of $y_1(t)$, $dy_1(t)/dt$ and $d^2y_1(t)/dt^2$,how is $dy_1(t)/dt$ related to $f(t)$ and how is $d^2y_1(t)/dt^2$ related to $e(t)$?

(b) Express $y(t)$ as a linear combination of $e(t)$,$f(t)$, and $y_1(t)$

(c) Use the result from the previous part to extend the direct form block diagram representation of S_1 and create a block diagram representation of S

(d) Observing that

$$H(s) = \left(\frac{2(s+5)}{s+3}\right)\left(\frac{s-4}{s+1}\right)$$

draw a block diagram representation for S as a cascade combination of two subsystems

Exercise 4. (20pt)

a. Let $H(s)$ denote the Laplace transform of $h(t)$, the system impulse response of a continuous time LTI system:

$$H(s) = \frac{1}{s^2 - s - 2}$$

Determine $h(t)$ for each of the following cases:

1. The system is stable
2. The system is causal
3. The system is neither stable nor causal

b. $y(t) = e^{-3t}u(t)$ is the output of a causal all pass system for which the system function is

$$H(s) = \frac{s-2}{s+2}$$

find and sketch at least two possible inputs $x(t)$ that could produce $y(t)$

Exercise 5. (20pt)

Consider the system characterized by the differential equation

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 2\frac{dx(t)}{dt} + 6x(t)$$

with initial condition $y(0_-) = c_0$ and $y'(0_-) = c_1$

a. When $x(t) = e^{-t}u(t)$,determine the zero-state response

b. Determine zero-input response

c. Determine the output of this system,when input is $x(t) = e^{-t}u(t)$ and initial conditions are the same in b