Lecture 14
-- Laplace Transform in Circuit Analysis

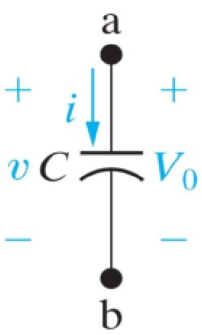
V-I relations of R,L,C

- R
$$U_R(s) = RI_R(s)$$
- C
$$\boxed{V(s) = \frac{1}{sC} I(s) + \frac{V_0}{s}}$$
$$\boxed{I(s) = sCV(s) - CV_0}$$
- L
$$\boxed{I(s) = \frac{1}{sL} V(s) + \frac{I_0}{s}}$$
$$\boxed{V(s) = sLI(s) - LI_0}$$

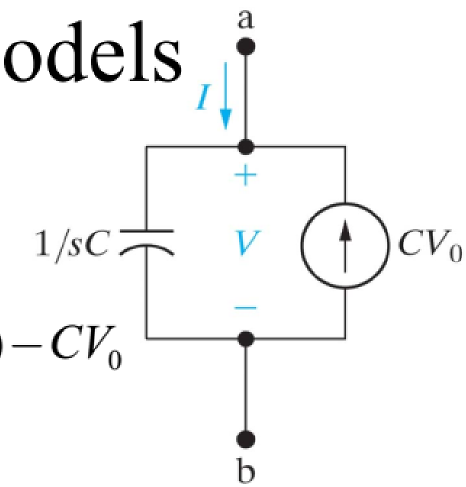
S-domain circuit models for a capacitor

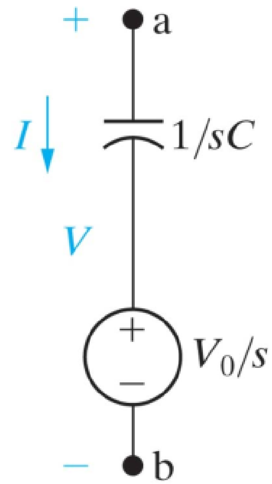
s-Domain Circuit Models

For a capacitor
(with initial conditions)



$$i(t) = C\frac{dv(t)}{dt} \Rightarrow I(s) = sCV(s) - CV_0$$





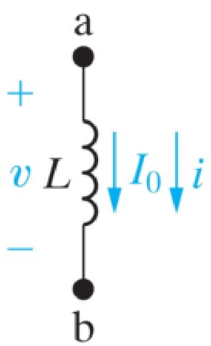
$$V(s) = \frac{1}{sC}I(s) + \frac{V_0}{s}$$

5

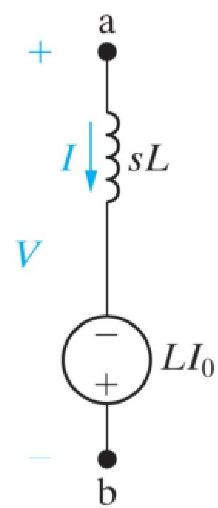
S-domain circuit models for an inductor

s-Domain Circuit Models

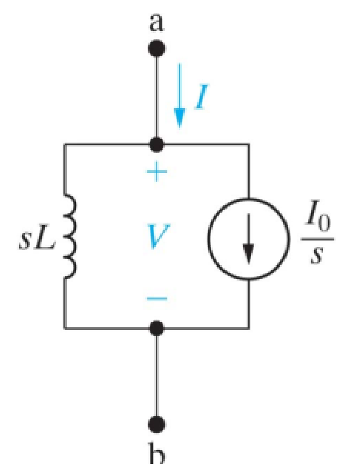
For an inductor
(with initial conditions)

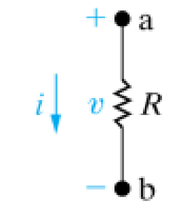
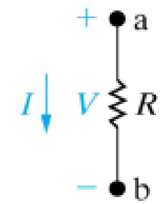
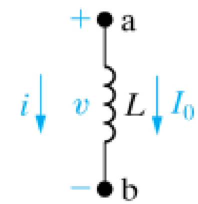
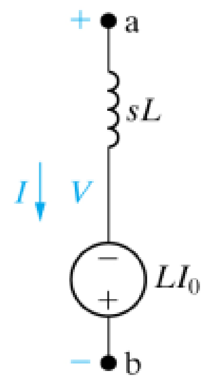
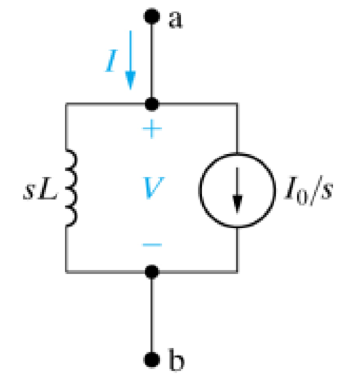
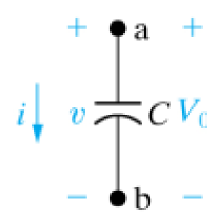
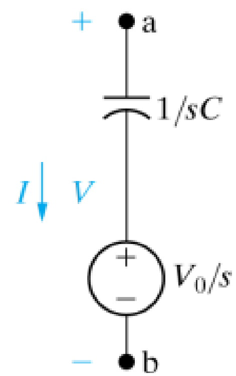
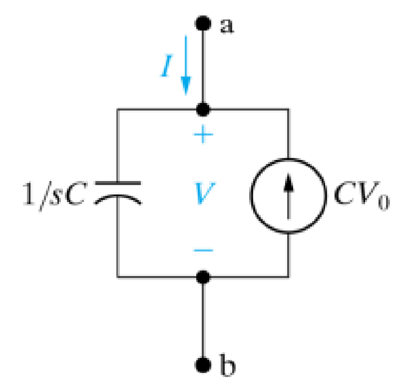


$$v(t) = L\frac{di(t)}{dt} \quad \Rightarrow \quad V(s) = sLI(s) - LI_0$$



$$I(s) = \frac{1}{sL}V(s) + \frac{I_0}{s}$$



Time domain	s-domain	
 $v = Ri$	 $V = RI$	
 $v = L\,di/dt,$ $i = \frac{1}{L}\int_{0^-}^{t} v\,dx + I_0$	 $V = sLI - LI_0$	 $I = \frac{V}{sL} + \frac{I_0}{s}$
 $i = C\,dv/dt,$ $v = \frac{1}{C}\int_{0^-}^{t} i\,dx + V_0$	 $V = \frac{I}{sC} + \frac{V_0}{s}$	 $I = sCV - CV_0$

D.C. sources and Dependent Sources

- The models for dependent sources are easy to develop, drawing from the simple fact that if the Laplace transform of *f(t)* is *F(s)*, then the Laplace transform of *af*(*t*) is *aF*(*s*) — the linearity property.

$$\mathcal{L}[av(t)] = aV(s)$$

$$\mathcal{L}[ai(t)] = aI(s)$$

Steps in Applying the Laplace transform

- **Transform the circuit from the time domain to the Laplace (*s*) domain, including initial conditions.**

--*The elegance of using the Laplace transform in circuit analysis lies* in (1) transforming the differential equation into an *algebraic* equation; and (2) automatic inclusion of initial conditions in the transformation process, thus providing a complete (**transient and steady-state**) solution.

- **Solve the circuit using nodal analysis, mesh analysis, source transformation, superposition, or any other analysis technique with which we are familiar.**

- Take the inverse transform of the solution and thus obtain the solution in the time domain.

Example 1

Assuming no initial energy storage, find $i_1(t)$ and $i_2(t)$ for $t > 0$.