CS244 Theory of Computation
Homework 1

Due: September 28, 2020 at 11:59pm

Name - ID

You may discuss this assignment with other students and work on the problems together. However, your write-up should be your own individual work and you should indicate in your submission who you worked with, if applicable. You should use the LaTeX template provided by us to write your solution and submit the generated PDF file into Gradescope.

I worked with: (Name, ID), (Name, ID), . . .

Let $\Sigma = \{0, 1\}$ if not otherwise specified.

Problem 1

Let A be the languages that every 1 has at least 2 zeros following immediately after. Show that A is regular in the following ways:

(a) by giving an NFA that recognizes A,

(b) by giving a DFA that recognizes A,

(c) by giving a regular expression that describes A, and

(d) by giving a right linear grammar that describes A.

Problem 2

(a) Let $A = \{0^k u 0^k \mid k \geq 1 \text{ and } u \in \Sigma^*\}$. Show that A is regular.

(b) Let $B = \{0^k 1 u 0^k \mid k \geq 1 \text{ and } u \in \Sigma^*\}$. Show that B is not regular.

Problem 3

Let B be the set of all strings with even length that contain at least one 1 in their first half.

(a) Is B regular? Please show your proof.

(b) Let $C = \{w \mid w \in B \text{ or } w \text{ has odd length}\}$. Is C regular? Please show your proof.

(c) Let $D = \{w \mid w \text{ has even length but } w \notin B\}$. Is D regular? Please show your proof.

Problem 4

(a) Let A be an infinite regular language. Prove that A can be split into two infinite disjoint regular subsets.

(b) Let B and D be two languages. Write $B \subsetneq D$ if $B \subseteq D$ and D contains infinitely many strings that are not in B. Show that if B and D are two regular languages where $B \subsetneq D$, then we can find a regular language C where $B \subsetneq C \subsetneq D$.