EE150 Signal and System
Homework 5

Due on 20 Nov 23:59 UTC+8

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Note:

- Please provide enough calculation process to get full marks.
- Please submit your homework to Gradescope.
- It's highly recommended to wirte every exercise on single sheet of page.

Exercies 1. (20pt)

Compute the Fourier transform of each of the following signals by definition:

(a)

$$x[n] = \left[(\frac{1}{2})^n \cos[\frac{\pi n}{2}]\right] u[n]$$

(b)

$$x[n] = \begin{cases} a, & |n| \leq N_1 \\ 0, & |n| > N_1 \end{cases} \quad \text{where } a \text{ and } N_1 \text{ are positive integers}$$

Exercies 2. (20pt)

Compute the Fourier transform of each of the following signals by properties:

(a) $x[n] = (\frac{1}{3})^{|n|}(n-1)$

(b) $x[n] = \frac{\sin(\pi n/5)}{\pi n} \cos(\frac{7}{2}\pi n)$

Exercies 3. (20pt)

The following four facts are given about a real signal $x[n]$ with Fourier transform $X(e^{j\omega})$:

1. $x[n] = 0$ for $n > 0$.
2. $x[0] < 0$.
3. $Im\{X(e^{j\omega})\} = \sin\omega - \sin 2\omega$
4. $\frac{1}{2\pi}\int_{-\pi}^{\pi}|X(e^{j\omega})|^2 d\omega = 6$

Determine $x[n]$ and give the reasons.

Exercies 4. (10pt)

Given the fact that

$$a^{|n|} \overset{\mathcal{F}}{\longleftrightarrow} \frac{1-a^2}{1-2a\cos\omega + a^2}, \quad |a| < 1,$$

use duality to determine the Fourier series coefficients of the following continuous-time signal with period $T = 1$:

$$x(t) = \frac{1}{5 - 4\cos(2\pi t)}$$

Exercies 5. (20pt)

A causal LTI system in described by the difference equation

$$y[n] - ay[n-1] = bx[n] + x[n-1],$$

where a is real and less than 1 in magnitude.

(a) Find a value of b such that the frequency response of the system satisfies

$$|H(e^{j\omega})| = 1, \quad \text{for all } \omega$$

This kind of system is called an all-pass system, as it does not attenuate the input $e^{j\omega n}$ for any value of ω. Use the value of b that you have found in the rest of the problem.

(b) Find and plot the output of this system with $a = -\frac{1}{2}$ when the input is $x[n] = (\frac{1}{2})^n u[n]$.

Exercies 6. (10pt)

(a) Consider a discrete-time system with unit sample response

$$h[n] = (\frac{1}{2})^n u[n] + \frac{1}{2}(\frac{1}{4})^n u[n]$$

Determine a linear constant-coefficient difference equation relating the input and output of the system.

(b) The frequency response is given by

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\frac{1}{4} + \frac{7}{8}e^{-j\omega} - \frac{1}{2}e^{-2j\omega}}{1 - \frac{1}{4}e^{-2j\omega}}$$

Find a difference equation relating $x[n]$ and $y[n]$ for this system, and determine the system's impulse response.