Homework 2

Total points : 90

Due date: 11th Oct, 2022

Turn in your homework in class

Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

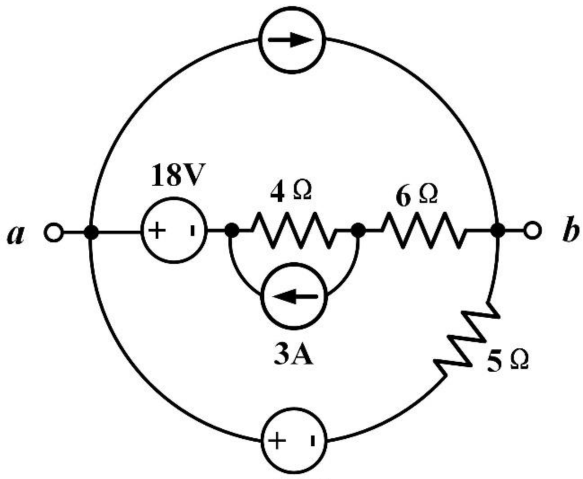
10'

1. For the circuit below,

(a) Use superposition theorem to find the voltage drop between ***a*** and ***b*,** namely V_{ab}

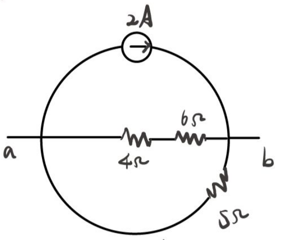
(b) Use source transformation (at least twice) to find V_{ab}

(c) Find the Norton equivalent circuit at terminals ***a*** and ***b***



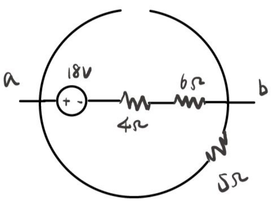
a)

1'



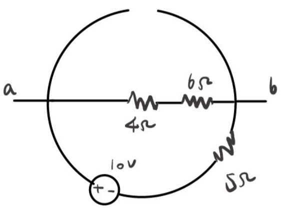
$V_1 = -2 \times \frac{4+6}{4+6+5} \times 5 = -\frac{20}{3}\,V$

1'



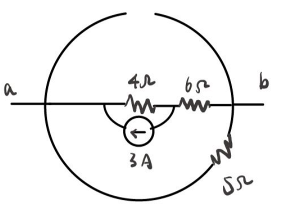
$V_2 = 18 \times \frac{5}{4+6+5} = 6\,V$

1'



$V_3 = 10 \times \frac{10}{4+6+5} = \frac{20}{3}\,V$

1'

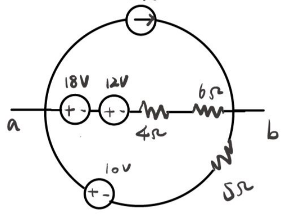


$V_4 = 3 \times \frac{4}{4+5+6} \times 5 = 4\,V$

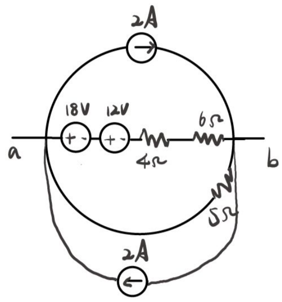
1'

$V_{ab} = V_1 + V_2 + V_3 + V_4 = 10\,V$

b)

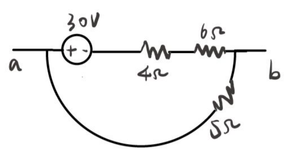


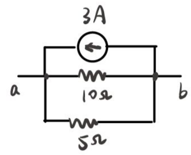
1.5'

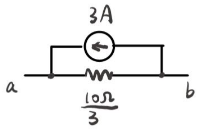


1.5'

twice

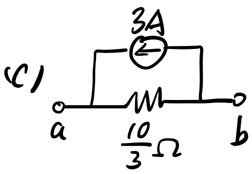






1'

$V_{ab} = 3 \cdot \frac{10\,V}{3} = 10\,V$



1'

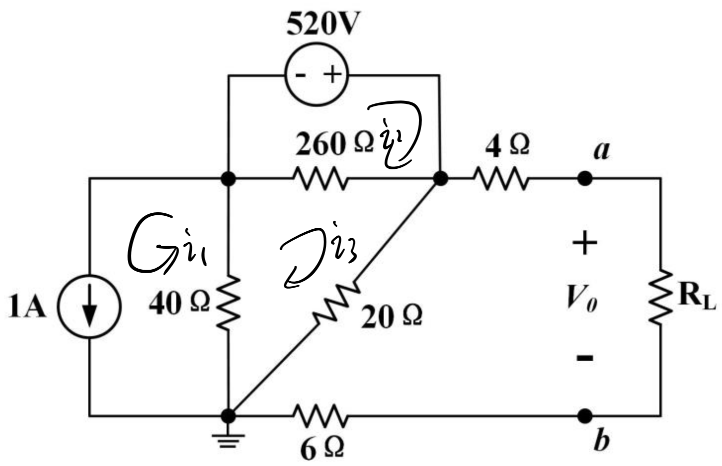
20 pts

2. (a) Apply superposition to find ***Vo*** in the circuit below when R_L=250Ω.

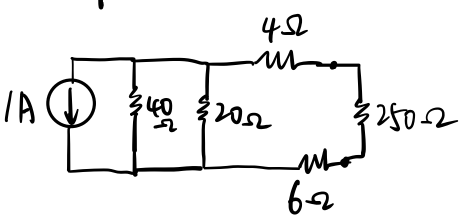
(b) Find the Thevenin equivalent circuit for the left hand side circuit of node ***a*** and node ***b***.

(c) Determine the value of R_L when maximum power could be transferred to it.

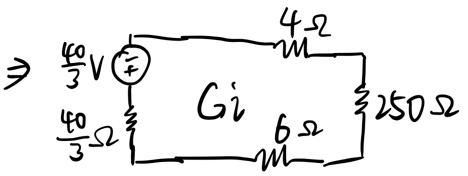
(d) for the situation in (c), find the power absorbed by the 520V voltage source and the 1A current source, **respectively**.



(a) Keep 1A current source:



⇒

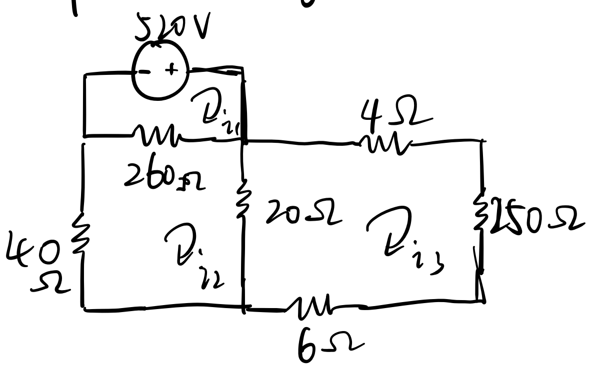


$i = \frac{40}{3} / (250+10+\frac{40}{3})$

$= \frac{2}{41}$ A 2'

$V_{o1} = -\frac{2}{41} \times 250\text{V} = -\frac{500}{41}$ V 1'

Keep 520V Voltage source

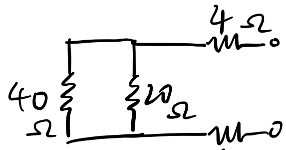


$$\begin{cases} -520+(i_1-i_2)260=0 \\ 260(i_2-i_1)+20(i_2-i_3)+40i_2=0 \\ 20(i_3-i_2)+260i_3=0 \end{cases}$$ 3'

$i_3 = \frac{26}{41}$ A $\Rightarrow V_{o2} = i_3 R_L = \frac{6500}{41}$ V

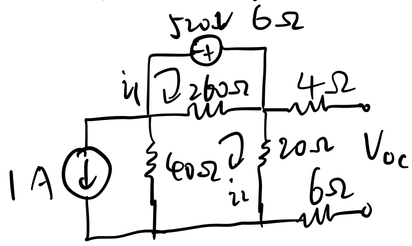
⇒ Superposition: $V_o = V_{o1}+V_{o2} = \frac{6000}{41}$ V 1'

(b)



$\Rightarrow R_{TH} = 4\Omega+6\Omega+40\Omega \| 20\Omega$

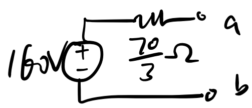
$= \frac{70}{3}\Omega$ 1'



$$\begin{cases} -520+(i_1-i_2)260=0 \\ (i_2-i_1)260+20i_2+(i_2+1)40=0 \end{cases}$$ 2'

$\Rightarrow -520+60i_2+40=0 \Rightarrow i_2 = 8A$

1' $\Rightarrow V_{oc} = 8 \cdot 20 = 160V$

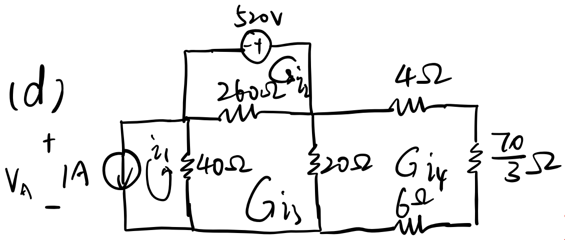


1'

(c) $R_L = R_{TH} = 70/3\ \Omega$

1'

(d)



2'

$$\begin{cases} i_1 = 1\ A \\ (i_2 - i_3)260 + 520 = 0 \\ 260\ (i_3 - i_2) + 40(i_3 - i_1) + 20(i_3 - i_4) = 0 \\ i_4\,(10 + \frac{70}{3}) + 20(i_4 - i_3) = 0 \end{cases}$$

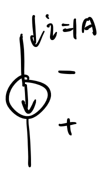
$$\Rightarrow \begin{cases} i_1 = 1\ A \\ i_2 = -\frac{78}{7} A \\ i_3 = -\frac{64}{7} A \\ i_4 = -\frac{24}{7} A \end{cases}$$

$\Rightarrow$ $P_{520V.absor.} = -\frac{78}{7} \cdot 520 = -\frac{40560}{7}\ W$ 2'

$$(i_1 - i_3)40 + V_A = 0 \Rightarrow V_A = (i_3 - i_1)40$$

$$= -\frac{71}{7} \times 40\ V$$

$$= -\frac{2840}{7}\ V$$

$\Rightarrow$ 

$\frac{2840}{7}$ V $\quad P_{1A.absor.} = -\frac{2840}{7} V \times 1\ A = -\frac{2840}{7}\ W$

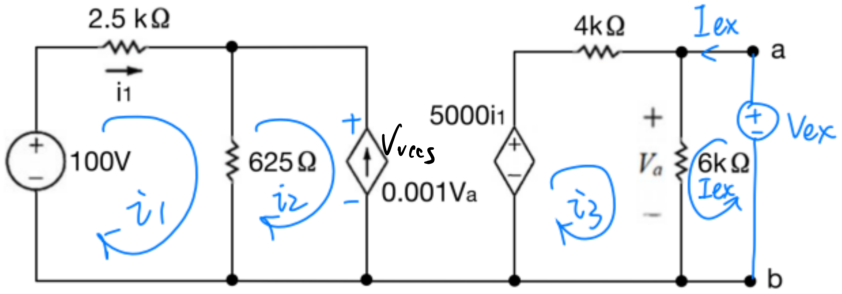
2'

$P < 0$ means the source generates power.

20pts

3. (a) Find the Thevenin equivalent circuit at the terminals **a** and **b**.

(b) If a load resistor **R**L is connected at terminals **a** and **b**, what is the maximum power that could be transferred to it by this circuit, and what is the value of **R**L in this situation? Also, find the power absorbed by the three sources, **respectively**.



V_{oc}: $\begin{cases} 100 = 2500 i_1 + 625(i_1 - i_2) \\ i_2 = -0.001 V_a \\ 5000 i_1 = 4000 i_3 + 6000 i_3 \\ V_a = 6000 i_3 \end{cases} \Rightarrow \begin{cases} i_1 = 0.02\,A \\ i_2 = -0.06\,A \\ i_3 = 0.01\,A \\ V_a = 60\,V = V_{ex} \end{cases}$ 1'

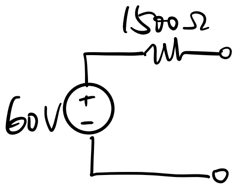
4'

R_{Th}: $\begin{cases} 2500 i_1' + 625(i_1' - i_2') = 0 \\ i_2' = -0.001 \times 60 \\ 5000 i_1' = 4000 i_3' + 60 \\ 60 = 6000(I_{ex} + i_3') \end{cases} \Rightarrow \begin{array}{l} 3125 i_1' - 625 i_2' = 0 \\ 5 i_1' = i_2' = -0.06\,A \\ i_1' = -0.012\,A \\ i_3' = -0.03\,A \end{array}$

4'

$I_{ex} = 0.04\,A$

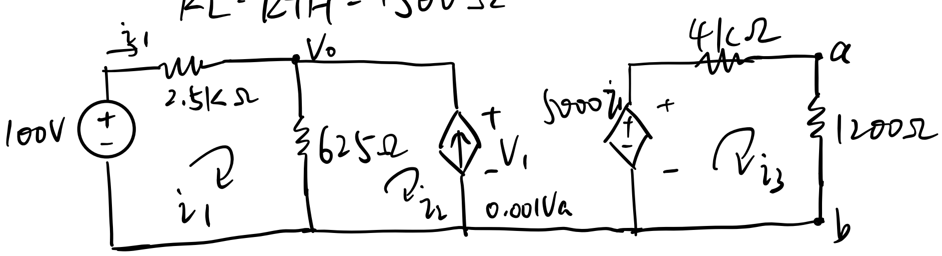
$R_{Th} = \frac{V_{ex}}{I_{ex}} = 1500\,\Omega$ 1'



1'

(b) $P_{max} = \frac{60^2}{4 \times 1500}\,W = 0.6\,W$ 1'

$R_L = R_{TH} = 1500\,\Omega$



$V_{ab} = 60V \cdot \frac{1}{2} = 30\,V$ 2'

$i_1 = \frac{100 - 35}{2500}\,A = \frac{65}{2500}\,A$

$\frac{100 - V_o}{2500} + 0.001 V_a = V_o / 625$ 2'

$P_{100V.absorbe} = \frac{-65}{2500} \times 100 = -2.6\,W$ 1'

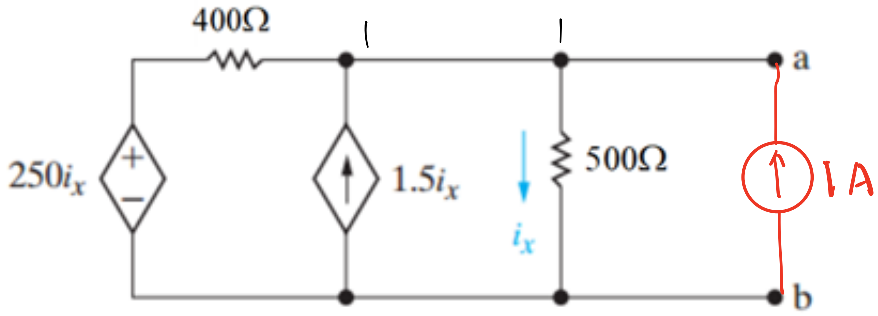
$\Rightarrow V_o = 35V = V_1$ 1'

$P_{CCVS.absorbe} = -5000 i \cdot \frac{5000 i - V_a}{4000} = -3.25\,W$ 1'

$\Rightarrow P_{VCCS.absorbe} = -0.001 V_a \cdot 35 = -1.05\,W$ 1'

10 pts

4. For the circuit below, find the Norton equivalent with respect to the terminals **a, b**.



Solution:

Because there are no independent sources in the circuit,

$$i_N = 0$$ 2'

To calculate R_N,we write no de-voltage equation, and place an independent 1A current source between nodes a and b with the current going from node b to node a.

$$\frac{v - 250i_x}{400} - 1.5i_x + i_x = 1$$ 2'

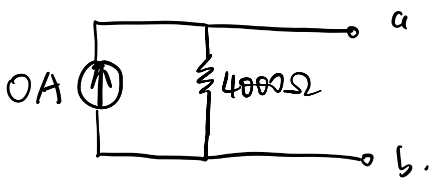
$$v=500*i_x$$

Now we can calculate

$$i_x=8A$$ 2'

$$v=4000V$$

Now we can calculate R_N =4000/1=4000Ω 2'



2'

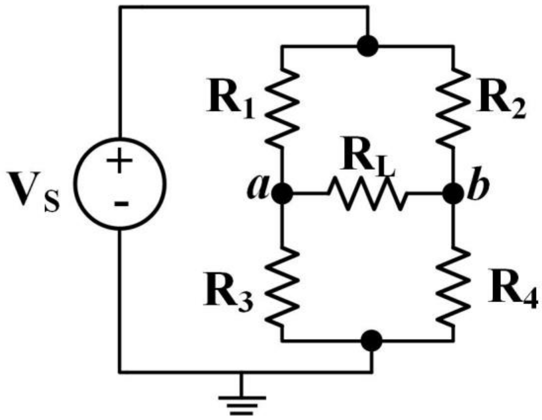
10pts

5. The values of **Vs, R1, R2, R3, R4** are known positive constants.

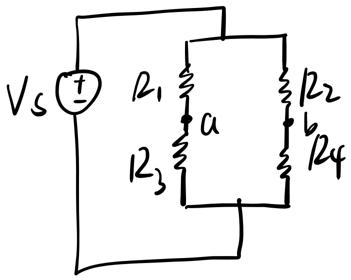
Use them to find:

(a) The Thevenin equivalent circuit between node ***a*** and ***b***.

(b) The resistance of **R_L** that absorbs the maximum power, and the maximum power **P_{max}** absorbed by the load **R_L**



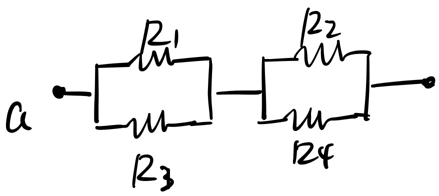
(a)



$$V_a = V_s \cdot \frac{R_3}{R_1+R_3} \Rightarrow V_{ab} = V_a - V_b$$

$$V_b = V_s \cdot \frac{R_4}{R_2+R_4} \quad V_{ab} = V_{oc} = V_s\left(\frac{R_3}{R_1+R_3} - \frac{R_4}{R_2+R_4}\right)$$ 2'

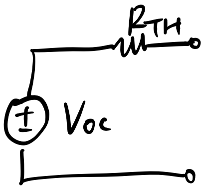
When finding R_{TH}, short circuit V_s, the equivalent circuit is



$$R_{TH} = R_1 \| R_3 + R_2 \| R_4$$

$$= \frac{R_1 R_3}{R_1+R_3} + \frac{R_2 R_4}{R_2+R_4}$$ 2'

$\Rightarrow$



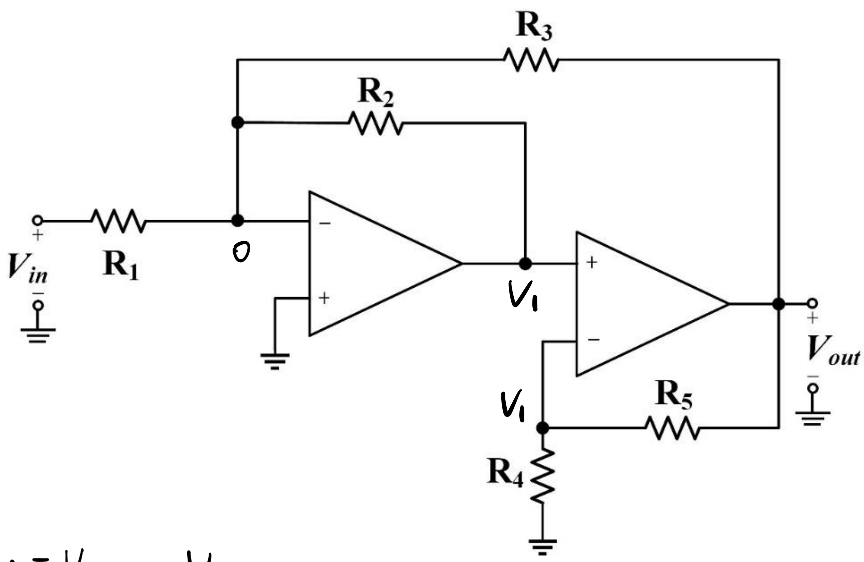
2'

(b) $R_L = R_{TH} = R_1 \| R_3 + R_2 \| R_4$ 2'

$$P_{max} = \frac{V_{oc}^2}{4R_L} = \frac{V_s^2\left(\frac{R_3}{R_1+R_3} - \frac{R_4}{R_2+R_4}\right)^2}{4\left(R_1\|R_3 + R_2\|R_4\right)}$$ 2'

10pts.

6. Find the voltage gain V_{out}/V_{in} of the following circuit, if R_1=1kΩ, R_2=R_3=2kΩ, R_4=R_5=4kΩ.



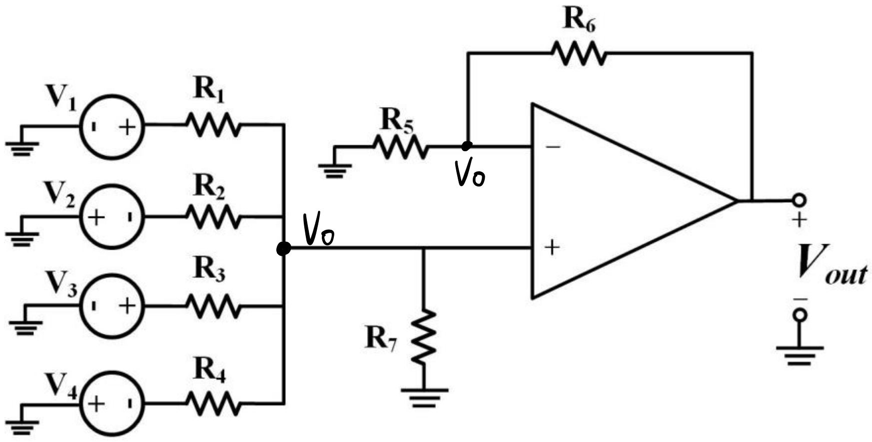
Solutions:

$$\begin{cases} \dfrac{V_{out} - V_1}{R_5} = \dfrac{V_1}{R_4} & 4' \\[2ex] \dfrac{V_{in}}{R_1} + \dfrac{V_1}{R_2} + \dfrac{V_{out}}{R_3} = 0 & 4' \end{cases}$$

$$\frac{V_{out}}{V_{in}} = -\frac{4}{3}. \quad 2'$$

10pts.

7. For the following circuit, find the output voltage V_{out} in terms of $\mathbf{V_1}$ to $\mathbf{V_4}$. Note that all the resistors in the circuit have the same resistance of 1kΩ. (Also, please pay attention to the given reference direction of the independent voltage sources)



$$\begin{cases} \dfrac{V_{out} - V_o}{R_6} = \dfrac{V_o}{R_5} \quad \cdots ① \\ \dfrac{V_1 - V_o}{R_1} + \dfrac{-V_2 - V_o}{R_2} + \dfrac{V_3 - V_o}{R_3} + \dfrac{-V_4 - V_o}{R_4} = \dfrac{V_o}{R_7} \quad \cdots ② \end{cases}$$

4'

4'

$$\Rightarrow V_{out} = \frac{2}{5}(V_1 - V_2 + V_3 - V_4)$$

2'