EE160 Homework 4 Solution

1. *(3 points) Explicit solution of nonlinear differential equations.*

   Solution:

   (a) $x(t) = \exp(1 - \cos(t))$

   (b) $x(t) = \dfrac{3e^{2t} + 1}{3e^{2t} - 1}$

   (c) $x(t) = \sqrt{-\ln\left(2t + \dfrac{1}{e}\right)}, \quad \forall t \in \left(-\dfrac{1}{2e}, \dfrac{1}{2} - \dfrac{1}{2e}\right)$

2. *(3 points) Picard Iteration.*

   Solution: Picard iteration:

$$\begin{aligned} y_0(t) &= 0\,, \\ y_1(t) &= \int_0^t 2s\mathrm{d}s = t^2\,, \\ y_2(t) &= \int_0^t 2s(1+s^2)\mathrm{d}s = t^2 + \frac{t^4}{2}\,, \\ y_3(t) &= \int_0^t 2s(1+s^2+\frac{s^4}{2!})\mathrm{d}s = t^2 + \frac{t^4}{2} + \frac{t^6}{6}\,, \\ &\cdots \end{aligned}$$

   Take the limit, we get an analytic expression for solution of this ODE,

$$x(t) = t^2 + \frac{t^4}{2} + \frac{t^6}{3!} + \cdots = -1 + \sum_{n=0}^{\infty} \frac{t^{2n}}{n!} = -1 + e^{t^2}.$$