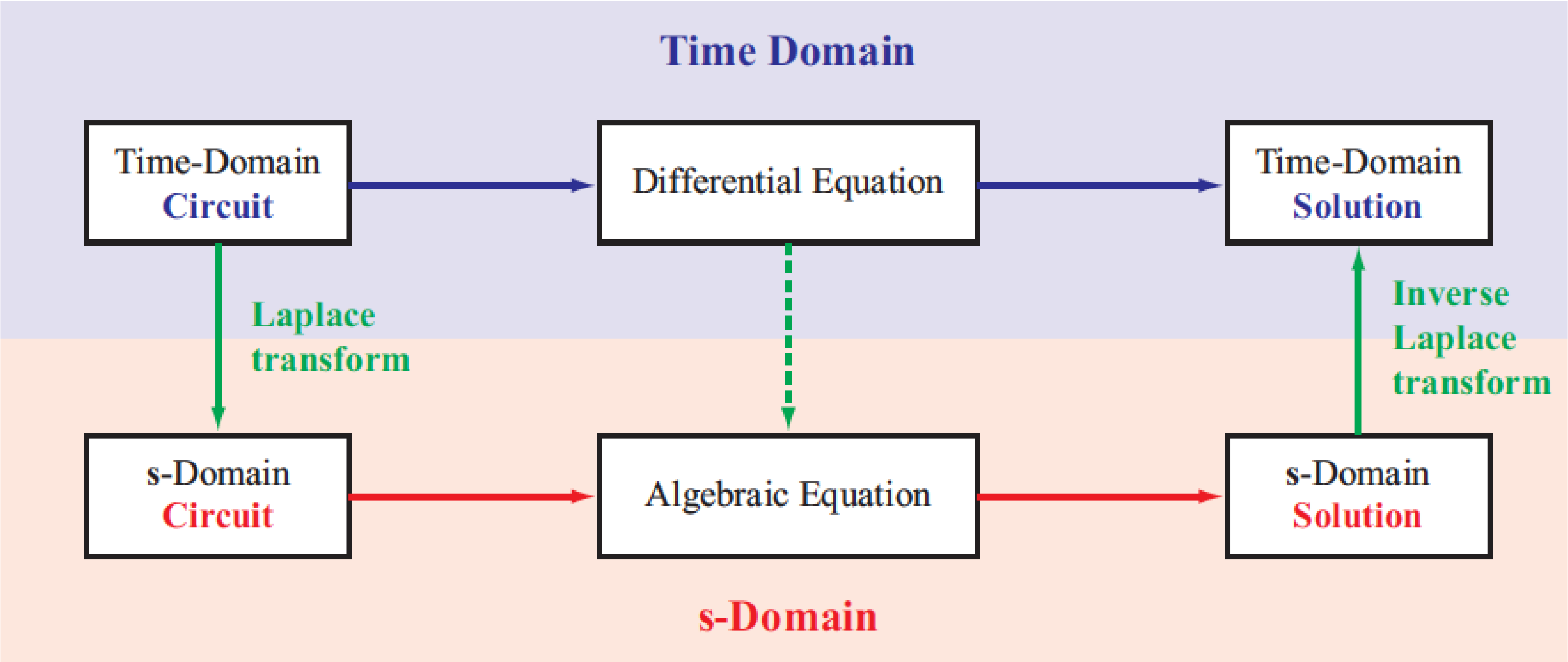
Lecture 16
- Laplace Transform Circuit Analysis

Laplace Transform Technique



[Source: Berkeley]

Analysis Techniques

Circuit Excitation	Method of Solution
dc	DC/Transient analysis
ac	Phasor-domain analysis (Steady state only)
Periodic waveform	Fourier series + Phasor-domain (Steady state only)
Waveform (single-sided)	Laplace transform (transient + steady state)

Single-sided: defined over $[0, \infty]$

[Source: Berkeley]

Application to Differential Equations

- The Laplace transform is useful in solving linear integrodifferential equations.
  - Initial conditions are automatically taken into account.

Use the Laplace transform to solve the differential equation

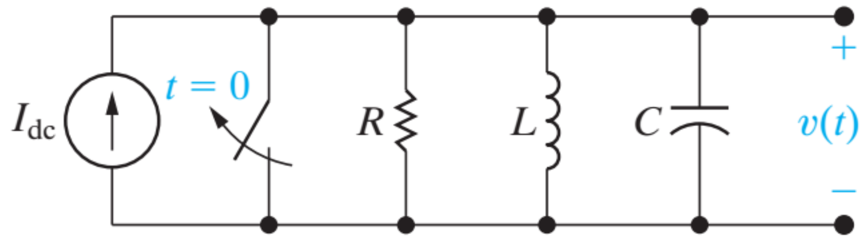
$$\frac{d^2v(t)}{dt^2} + 6\frac{dv(t)}{dt} + 8v(t) = 2u(t)$$

subject to $v(0) = 1, v'(0) = -2$.

$$[s^2V(s) - sv(0) - v'(0)] + 6[sV(s) - v(0)] + 8V(s) = \frac{2}{s}$$

$$V(s) = \frac{s^2 + 4s + 2}{s(s+2)(s+4)} = \frac{\frac{1}{4}}{s} + \frac{\frac{1}{2}}{s+2} + \frac{\frac{1}{4}}{s+4} \qquad v(t) = \frac{1}{4}(1 + 2e^{-2t} + e^{-4t})u(t)$$

Applying the Laplace Transform



$$\frac{v(t)}{R} + \frac{1}{L}\int_0^t v(x)\,dx + C\frac{dv(t)}{dt} = I_{\mathrm{dc}}u(t).$$

$$\frac{V(s)}{R} + \frac{1}{L}\frac{V(s)}{s} + C[sV(s) - v(0^-)] = I_{\mathrm{dc}}\left(\frac{1}{s}\right)$$

$$V(s)\left(\frac{1}{R} + \frac{1}{sL} + sC\right) = \frac{I_{\mathrm{dc}}}{s}$$

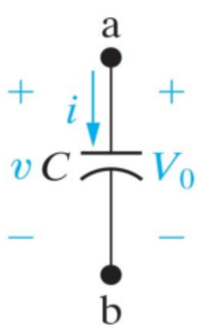
$$V(s) = \frac{I_{\mathrm{dc}}/C}{s^2 + (1/RC)s + (1/LC)}. \longrightarrow \boxed{v(t) = \mathscr{L}^{-1}\{V(s)\}.}$$

V-I relations of R

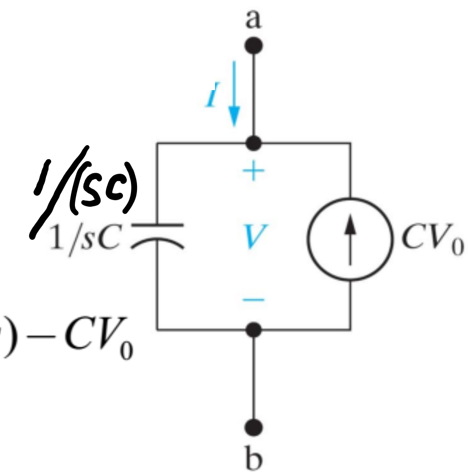
$$U_R(s) = RI_R(s)$$

S-domain circuit models for a capacitor

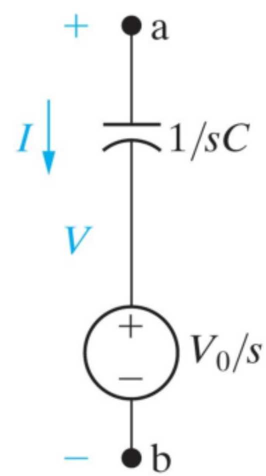
$$I_C(s) = sCU_C(s) - Cu_C(0_-)$$



$$i(t) = C\frac{dv(t)}{dt} \Rightarrow I(s) = sCV(s) - CV_0$$



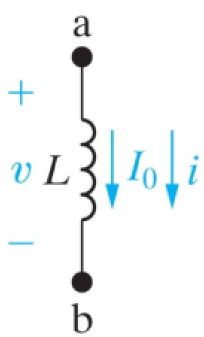
$$V(s) = \frac{1}{sC}I(s) + \frac{V_0}{s}$$



1/(sc)

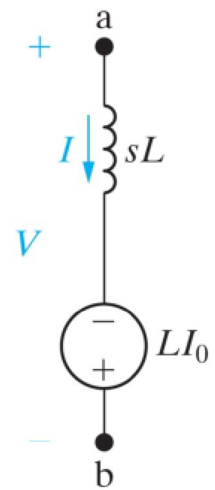
S-domain circuit models for an inductor

$$I_L(s) = \frac{i_L(0_-)}{s} + \frac{1}{sL} U_L(s)$$

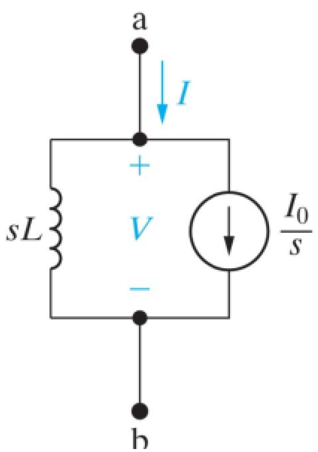


$$v(t) = L\frac{di(t)}{dt}$$

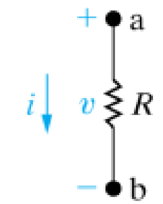
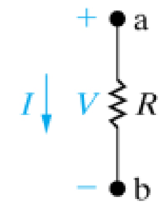
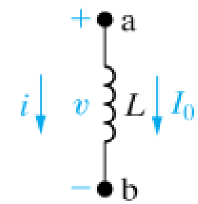
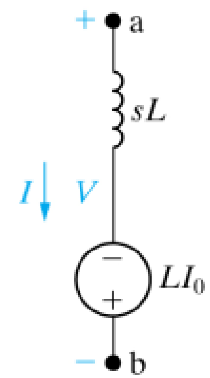
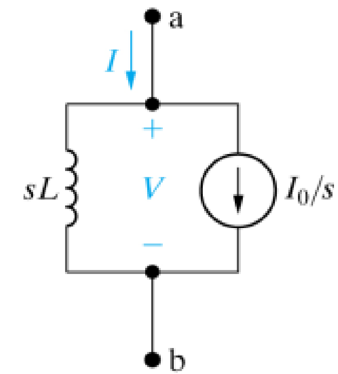
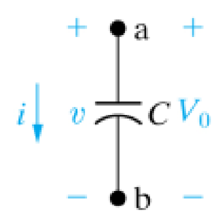
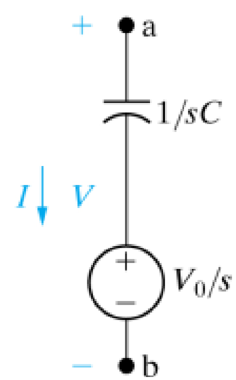
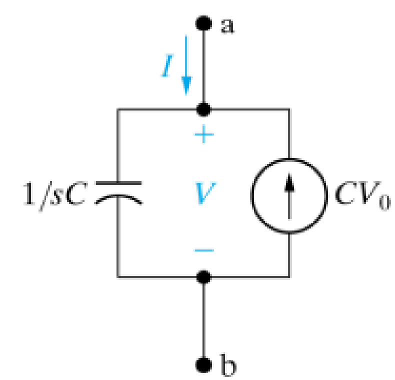
$$V(s) = sLI(s) - LI_0$$



$$I(s) = \frac{1}{sL}V(s) + \frac{I_0}{s}$$



Summary

Time domain	s-domain	
 $v = Ri$	 $V = RI$	
 $v = L\,di/dt,$ $i = \frac{1}{L}\int_{0^-}^{t} v\,dx + I_0$	 $V = sLI - LI_0$	 $I = \frac{V}{sL} + \frac{I_0}{s}$
 $i = C\,dv/dt,$ $v = \frac{1}{C}\int_{0^-}^{t} i\,dx + V_0$	 $V = \frac{I}{sC} + \frac{V_0}{s}$	 $I = sCV - CV_0$

Dependent Sources

- The models for dependent sources are easy to develop, drawing from the simple fact that if the Laplace transform of *f(t)* is *F(s)*, then the Laplace transform of *af*(*t*) is *aF*(*s*) — the linearity property.

$$\mathcal{L}[av(t)] = aV(s)$$

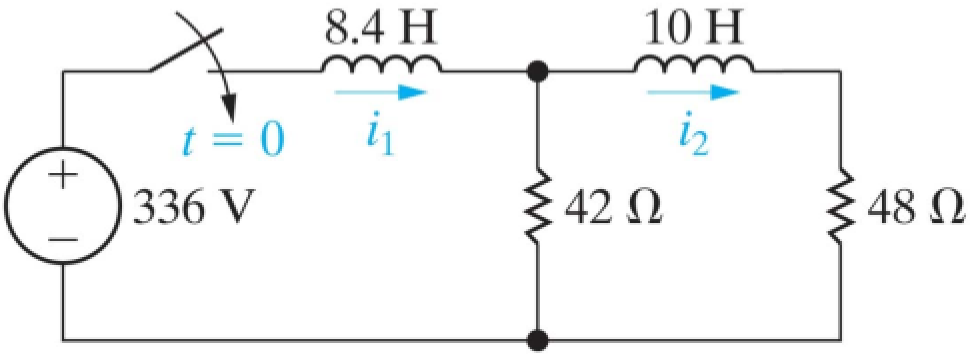
$$\mathcal{L}[ai(t)] = aI(s)$$

Steps in Applying the Laplace transform

- Transform the circuit from the time domain to the Laplace (*s*) domain, including possible initial conditions.
- Solve the circuit using nodal analysis, mesh analysis, source transformation, superposition, or any other analysis technique with which we are familiar.
- Take the inverse transform of the solution and thus obtain the solution in the time domain.

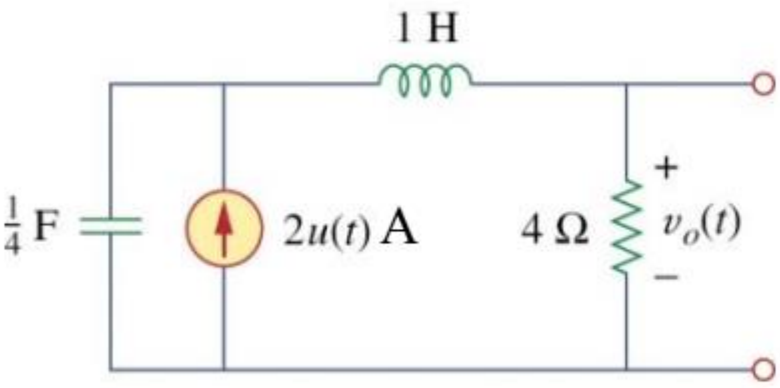
Example 1

Assuming no initial energy storage, find $i_1(t)$ and $i_2(t)$ for $t > 0$.



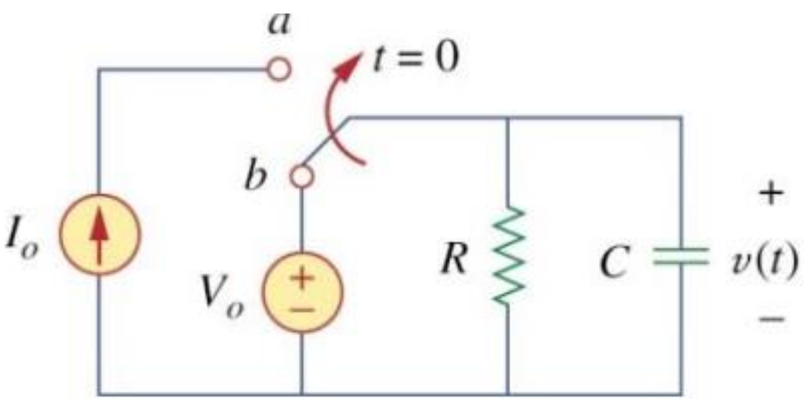
Example 2

Determine $v_0(t)$ *for* $t>0$ *assuming zero initial conditions:*

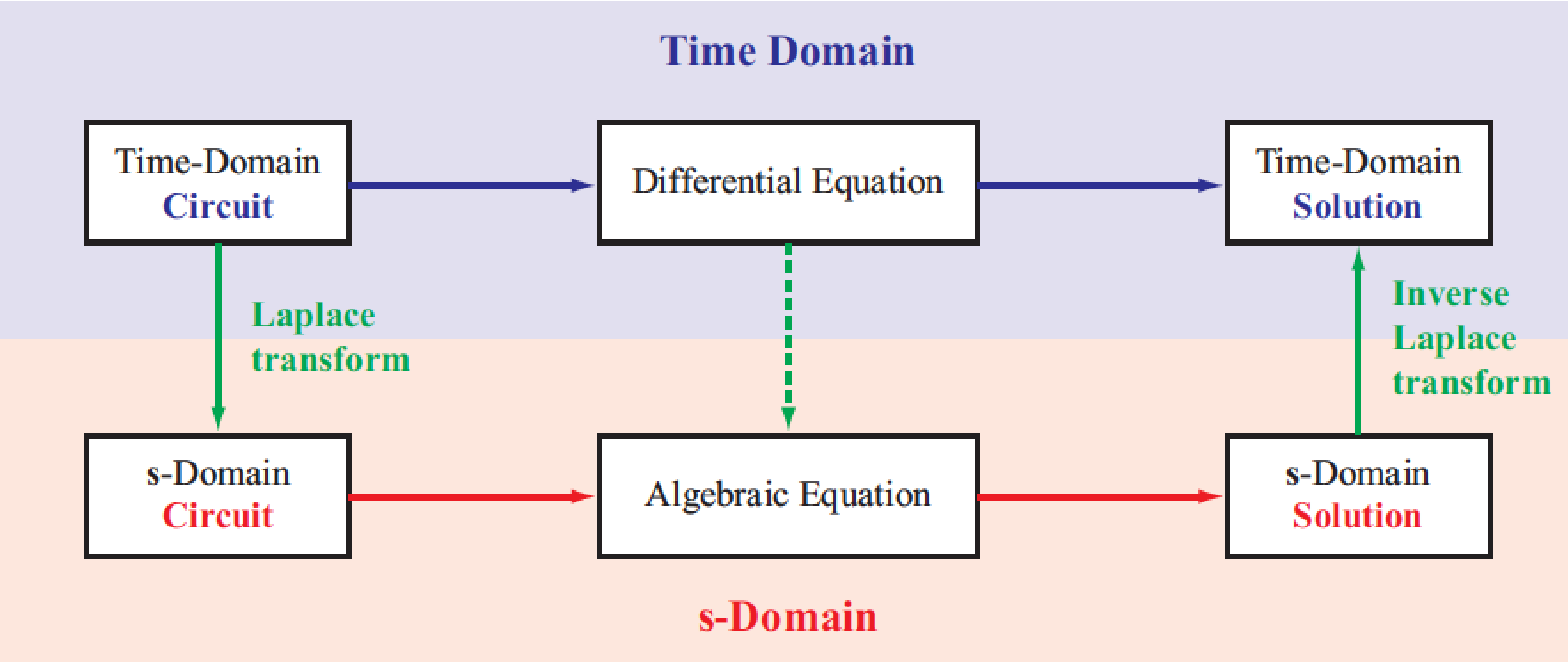


Example 3

- The switch has been in position *b* for a long time. It is moved to position *a* at $t = 0$. Determine $v(t)$ for $t > 0$.



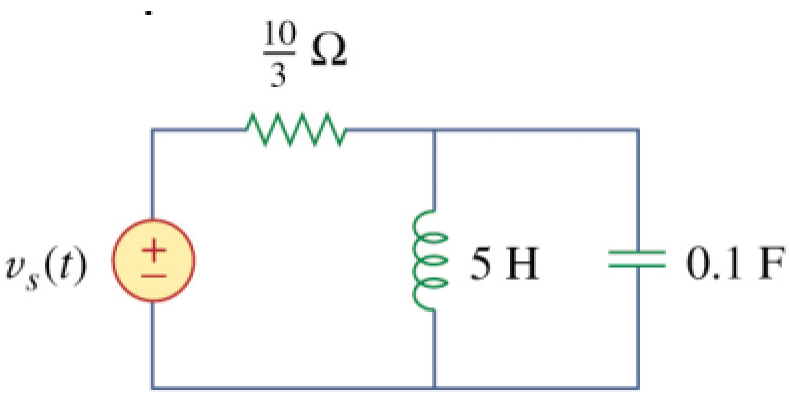
Laplace Transform Technique

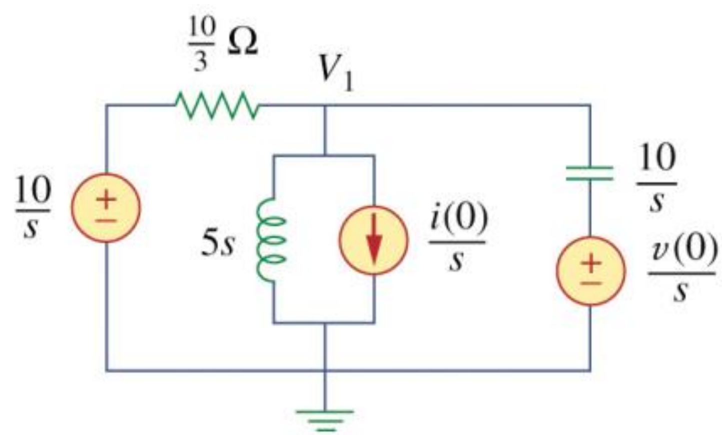


[Source: Berkeley]

Example 4

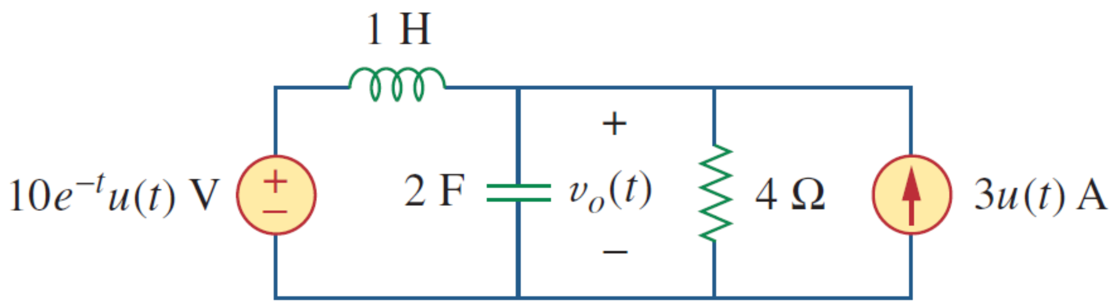
- Find the voltage across the capacitor assuming that *vs*(*t*) = 10*u*(*t*) V, and assume that at *t* = 0, *-1 A* flows through the inductor and +5 V is across the capacitor.





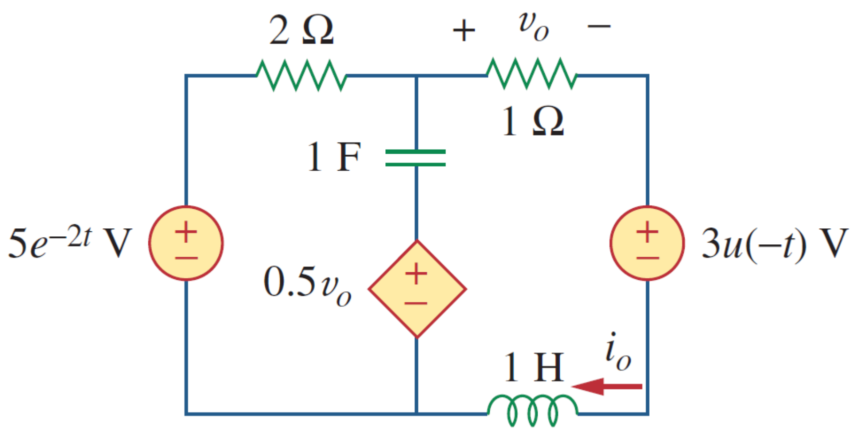
Example 5

16.35 Find $v_o(t)$ in the circuit of Fig. 16.58.



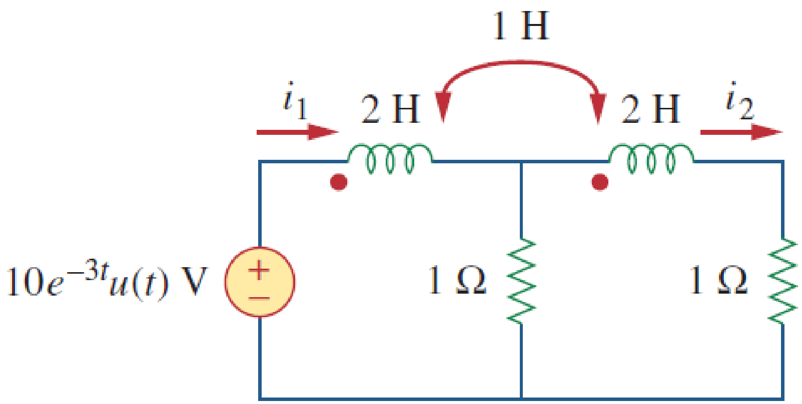
Example 6

16.49 Find $i_0(t)$ for $t > 0$ in the circuit in Fig. 16.72.



Example 7

16.69 Find $I_1(s)$ and $I_2(s)$ in the circuit of Fig. 16.92.



Example 8

16.81 For the op-amp circuit in Fig. 16.99, find the transfer function, $T(s) = I(s)/V_s(s)$. Assume all initial conditions are zero.