 Total : 70

Homework 3

Due date: 25^{th}, Oct.

Turn in your homework in class

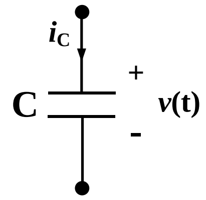
Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

15'

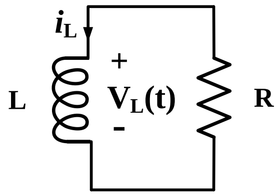
1. (a). A capacitor with 10mF capacitance has the terminal voltage:

$$v(t) \begin{cases} 15\text{ V} & t \leq 0 \\ Ae^{-100t} + Be^{-500t}\text{ V} & t \geq 0 \end{cases}$$

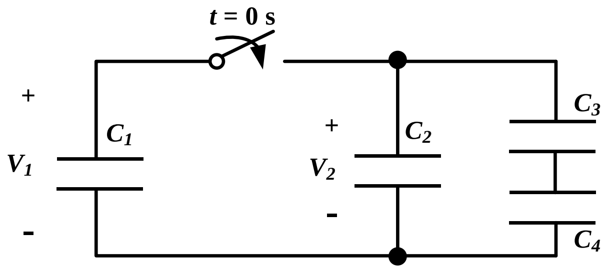


Assuming that the initial current (t=0s) on the capacitor is 5 A, please find: (1). constants A and B, (2). the capacitor current for t > 0, and (3). the energy of capacitor at t=1ms.

(b). A 200-mH inductor is connected in parallel with a resistor. The current through the inductor is $\boldsymbol{i_L}\mathbf{(t)} = 10e^{-800t}\,mA$. Please find the voltage $\mathbf{V_L}(t)$ on the inductor with respect to time, as well as the energy of the inductor at t = 1ms.



(c). For the capacitance circuit below, $\boldsymbol{C_1}$ =1 μF, $\boldsymbol{C_2}$ =0.125 μF, $\boldsymbol{C_3}$ = $\boldsymbol{C_4}$ =0.25 μF. Initially the switch is at "off state", and the voltage on the capacitor $\boldsymbol{C_1}$ is 10 V while other capacitors have the same voltage drop of 0 V. At $\boldsymbol{t}$ =0s, the switch is closed. Please find the voltage $\boldsymbol{V_2}$ after the circuit becomes stable. Note that no loss need to be considered.



1. (a) (1) $\because V(0^+) = V(0^-)$

$\therefore A + B = 15$ ① 1'

4 $\because i_c(t) = C\dfrac{dV(t)}{dt} = 0.01(-100Ae^{-100t} - 500Be^{-500t})$

$\therefore i_c(0) = -A - 5B = 5$ ② 1'

① ② $\Rightarrow$ $A = 20$ 1'

$B = -5$ 1'

2 (2) $i_c(t) = C\dfrac{dV}{dt} = -20e^{-100t} + 25e^{-500t}$ A

2 (3) $V(1ms) = 20e^{-0.1} - 5e^{-0.5} = 15.064$ V

$E = \frac{1}{2}CV^2 = \frac{1}{2} \times 0.01 \times 15.064^2 = 1.135$ J

(b) $V_L(t) = L\dfrac{di_L(t)}{dt} = 0.2 \times (-800) \times 10e^{-800t} \times 10^{-3} = -1.6e^{-800t}$ V 1'

3 $i_L(1ms) = 10e^{-0.8} = 4.493$ mA 1'

$E = \frac{1}{2}Li^2 = \frac{1}{2} \times 0.2 \times (4.493 \times 10^{-3})^2 = 2.019 \times 10^{-6}$ J 1'

(c) $C_{2+3+4} = (C_3 /\!/ C_4) + C_2 = 0.125 + 0.125 = 0.25\ \mu F$ 1'

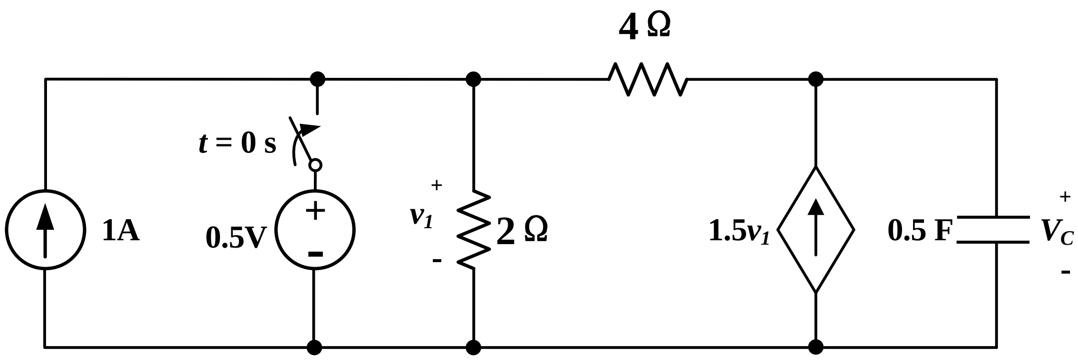
4 $\because Q = C_1V_1 = C_1V_1' + C_{2+3+4}V_2$ 1'

$V_1' = V_2$ 1'

$\Rightarrow V_2 = 8V$ 1'

10'

2. In the following circuit, the switch is open for a long time for t<0s. At t=0s, the switch closes immediately. Please find out the voltage on 0.5 F capacitor ($V_C(t)$) when $t \geq$ 0s.



$t<0$ $\quad \frac{u_1}{2} - 1.5u_1 = 1$ 1'

$u_1 = -1V$ 1'

$u_c(0_-) = 4\times 1.5u_1 + u_1 = -7V$ 1'

$t\to\infty$ $\quad u_{oc} = 1.5u_1 \times 4 + u_1 = 7u_1$ 1'

and $u_1 = 0.5V$

$\Rightarrow u_{oc} = 3.5V$ 1'

$\Rightarrow R_{eq} = 4\Omega$ 1'

$\tau = R_{eq}C = 4\times 0.5 = 2s$ 1'

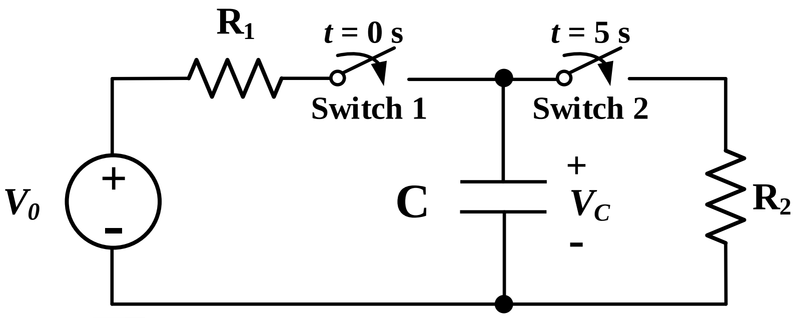
$u_c(t) = u_c(\infty) + [u_c(0_-) - u_c(\infty)]e^{-t/\tau}$

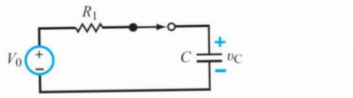
$= 3.5 - 10.5e^{-0.5t}$ V 2'

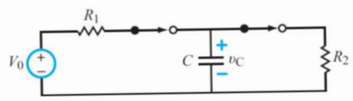
13'

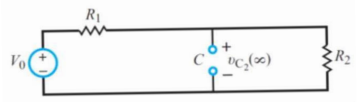
3. The circuit contains two switches, both of which have been open for a long time before t = 0. **Switch 1** closes at t = 0s, and **switch 2** closes at t = 5s. Determine $V_C(t)$ for $t \geq 0$, given that V_0=24V, R_1=R_2=16kΩ, and C=250μF. Assume $V_C(0)$=1 V.

Also, please sketch $V_C(t)$, capacitor current **$I_c(t)$**, current on **R_1** (**$I_{R1}(t)$**), and voltage on **R_1** (**$V_{R1}(t)$**) for t>0s, **respectively**.



$0 \leq t \leq 5$


$t \geq 5$


$t = \infty$


$0 \leq t \leq 5$ $\quad \tau_1 = R_1 C = 16 \times 10^3 \times 250 \times 10^{-6} = 4s$ 1'

3' $\quad V_{c_1}(t) = V_{c_1}(\infty) + (V_{c_1}(t) - V_{c_1}(\infty)) e^{-t/\tau_1}$ 1'

$= V_0 + (1 - V_0) e^{-0.25t}$

$= 24 - 23 e^{-0.25t}$ V 1'

$t \geq 5$ $\quad \tau_2 = \frac{R_1 R_2}{R_1 + R_2} C = 8 \times 10^3 \times 250 \times 10^{-6} = 2s$ 1'

4' $\quad V_{c_2}(\infty) = \frac{V_0 R_2}{R_1 + R_2} = 12V$ 1'

$V_{c_2}(5s) = V_{c_1}(5s) = 24 - 23 e^{-1.25} = 17.41V$ 1'

$V_{c_2}(t) = V_{c_2}(\infty) + (V_{c_2}(5s) - V_{c_2}(\infty)) e^{-\frac{t-5}{\tau_2}}$

$= 12 + 5.41 e^{-0.5(t-5)}$ V 1'

$i_c(t) = C \frac{dV_c(t)}{dt} = \begin{cases} \frac{23}{16} e^{-0.25t} \text{ mA} = 1.4375 e^{-0.25t} \text{ mA} \\ -0.676 e^{-0.5(t-5)} \text{ mA} \end{cases}$ 2'

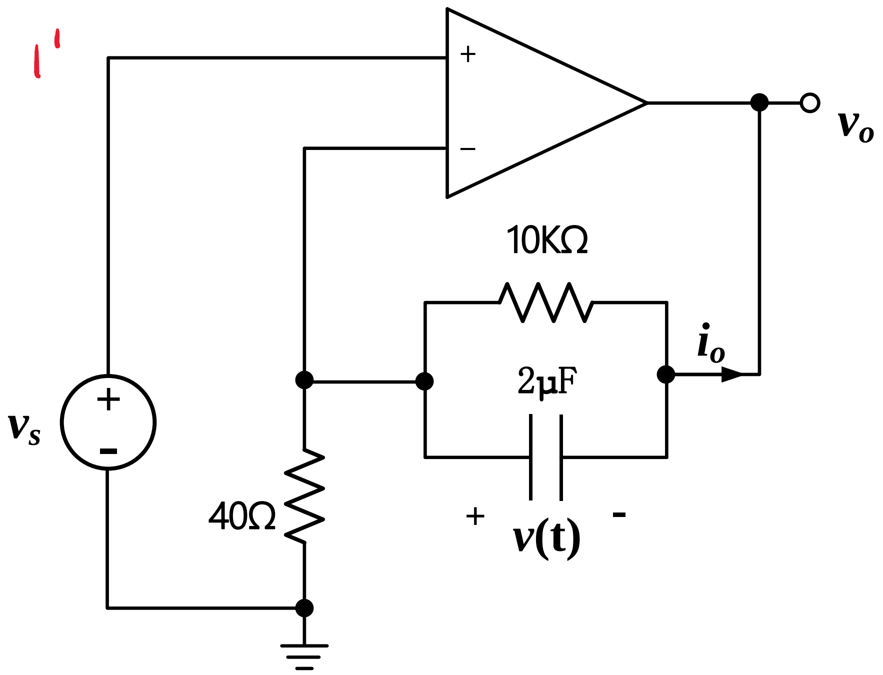
$I_{R_1}(t) = \frac{V_0 - V_c}{R_1} = \begin{cases} \frac{23}{16} e^{-0.25t} \text{ mA} \\ \frac{12 - 5.41 e^{-0.5(t-5)}}{16} \text{ mA} = 0.75 - 0.338 e^{-0.5(t-5)} \text{ mA} \end{cases}$ 2'

$V_{R_1}(t) = V_0 - V_c = \begin{cases} 23 e^{-0.25t} \text{ V} \\ 12 - 5.41 e^{-0.5(t-5)} \text{ V.} \end{cases}$ 2'

10

4. In the circuit below, find $v_o(t)$ and $i_o(t)$, given that $v_s = 4u(t)$ V and $v(0)$=1V. (Note that $u(t)$ is step function.)

unit 1'



Solution:

t->∞:

$$\frac{0 - v_{40\Omega}}{40} = \frac{v_{40\Omega} - v_0}{10k} \quad 1'$$

$$v_{40\Omega} = v_s = 4V \quad 1'$$

$$v_0 = 1004V \quad 1'$$

$$v_\infty = v_{40\Omega} - v_0 = -1000V \quad 1'$$

t≥0:

$$R_{eq} = 10k\Omega$$

$$\tau = R_{eq}C = 0.02s \quad 1'$$

$$v(t) = v(\infty) + (v(0) - v(\infty))e^{-\frac{t}{\tau}} = -1000 + 1001e^{-50t}V \quad 2'$$

$$v_0(t) = v_{40\Omega} - v(t) = 1004 - 1001e^{-50t}V \quad 1'$$

$$i_0(t) = \frac{0 - v_{40\Omega}}{40} = -0.1A \quad 1'$$

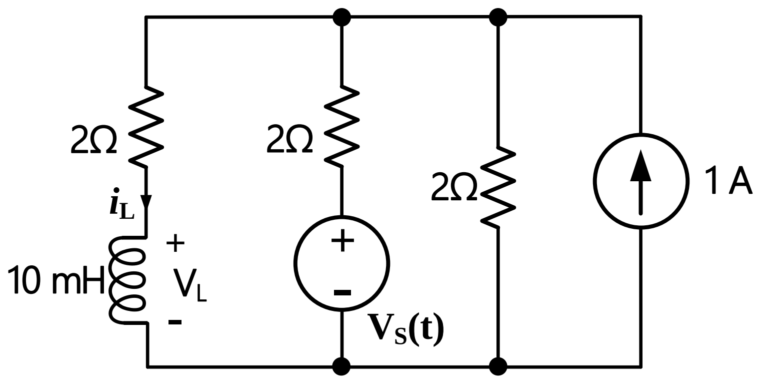
12'

5. For the circuit below, the independent voltage source has the voltage:

unit 1'

$$V_S(t) = \begin{cases} 0\text{ V} & t < 0\text{s} \\ e^{-100t}\text{ V} & t \geq 0\text{s} \end{cases}$$

Please find the current on the inductor in terms of time (i_L(t)) for t⩾0s.



$t<0s$, $i_L = \frac{1}{3}A$ 1'

$\Rightarrow i_L(0^-) = i_L(0^+) = \frac{1}{3}A$ 1'

t>0 s

$$\begin{cases} \frac{V_1}{2} + \frac{V_1 - V_s}{2} + \frac{V_1 - V_L}{2} = 1 \Rightarrow 3V_1 = 2 + V_s + V_L \\ \frac{V_1 - V_L}{2} = i_L \Rightarrow \frac{2+V_s+V_L}{3} - V_L = 2i_L \\ V_L = L\frac{di_L}{dt} \Rightarrow \frac{2}{3}L\frac{di_L}{dt} + 2i_L = \frac{2}{3} + \frac{V_s}{3} \end{cases}$$ 3'

$$\Rightarrow \frac{di_L}{dt} + 300\, i_L = 100 + 50e^{-100t}$$ 2'

$$i_L(t) = e^{-300t}\left[\int(100+50e^{-100t})e^{300t}dt + c\right]$$

$$= e^{-300t}\left[\frac{1}{3}e^{300t} + \frac{1}{4}e^{200t} + c\right]$$ 2'

$$= \frac{1}{3} + \frac{1}{4}e^{-100t} + ce^{-300t}$$

$$\because i_L(0^+) = \frac{1}{3} + \frac{1}{4} + c = \frac{1}{3}$$ 1'

$$c = -\frac{1}{4}$$

$$i_L(t) = \frac{1}{3} + \frac{1}{4}e^{-100t} - \frac{1}{4}e^{-300t}\ \text{A}.$$ 1'

10'

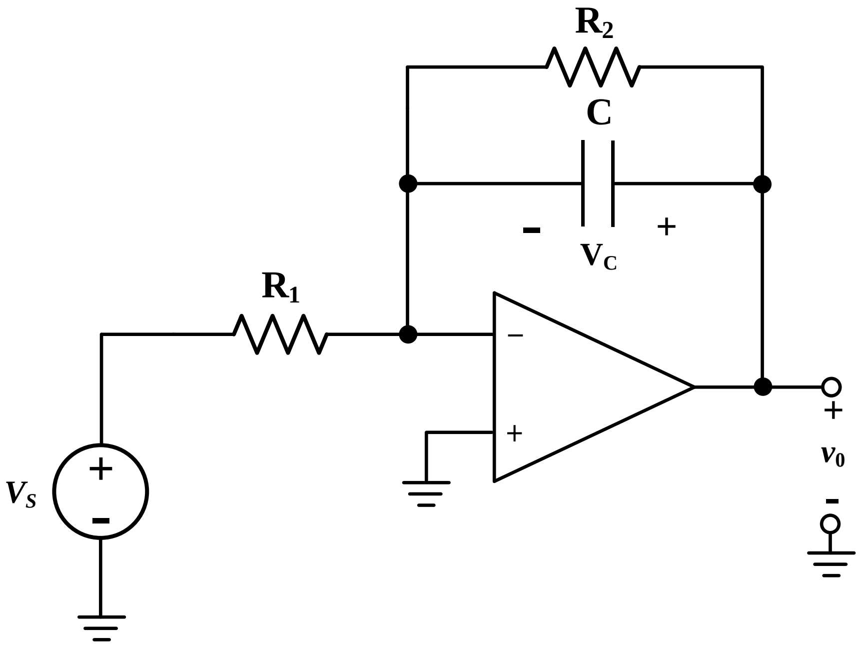
6. For the circuit below, assume the operational amplifier is always working in its linear mode, $\mathbf{V_C}(0^-) = 5V$, $\mathbf{R_1} = 5k\Omega$, $\mathbf{R_2} = 500\Omega$, $\mathbf{C}=5\mu F$, and

$$V_S(t) = \begin{cases} 0, & t \le 0 \\ e^{-200t}, & t > 0 \end{cases}$$

(unit for $V_S(t)$ is V)

Find the output voltage of the operational amplifier $v_0(t)$ for $t > 0$.

unit 1'



$i_c = C\dfrac{dV_c}{dt}$, $V_c = V_o$ 2'

Apply nodal analysis and KCL.

$\dfrac{V_s - 0}{R_1} + C\dfrac{dV_c}{dt} = \dfrac{0 - V_c}{R_2}$ 2'

$\dfrac{e^{-200t}}{5000} + 5\times10^{-6}\dfrac{dV_c}{dt} = -\dfrac{V_c}{500}$

$e^{-200t} + 0.025\dfrac{dV_c}{dt} = -10V_c$

$\dfrac{dV_c}{dt} + 400V_c = -40e^{-200t}$ 1'

$V_c = e^{-400t}\left[\int(-40e^{-200t}\cdot e^{400t})dt + C\right]$

$= -\frac{1}{5}e^{-200t} + Ce^{-400t}$ 2'

$V_c(0) = V_c(0^-) = -\frac{1}{5} + C = 5$ V 1'

$C = 5.2$

$V_0 = V_c = 5.2e^{-400t} - 0.2e^{-200t}$ V 1'