Online Lecture Notes

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1 Summary of Material for Mid-Term Exam

The mid-term exam will be on May 5 (this Thursday!). This will be an 24h take-home exam. We'll be starting at the beginning of next lecture (14:40).

1. Lecture 1: Error Analysis eps $= 2^{-52}$, Numerical and Algorithmic Differentiation

2. Lecture 2: Polynomial Interpolation: existence and uniqueness, Lagrange basis, Newton basis, Divided Difference Tables, Approximation Error, Hermite Interpolation

3. Lecture 3: Polynomial Extrapolation, Splines, Natural Splines and their properties

4. Lecture 4: Analysis in a Nutshell: Vector Spaces, Norms, Hilbert Spaces, Cauchy Schwarz inequality, Gram-Schmidt Algorithm, Legendre Polynomials, Gradients, Hessians, Jacobians, Forward Driectional Derivatives

5. Lecture 5: Gauss Approximation: optimality conditions for Gauss' approximation problem, orthogonal polynomials, solution by projection onto an orthonormal basis.

6. Lecture 6: Numerical Integration: Newton Cotes formulas and their coefficients, Simpson's rule and its integration error, Gauss Quadrature which uses "smart" evaluation points (namely the roots of the Legendre polynomials), indefinite integration: if your integration interval is large, eventually break the horizon into smaller intervals (Rhomberg quadrature) or use a suitable variable transformation, numerical differentiation in higher dimensions: apply the integration multiple times in a nested way (see our online lecture notes)

Numerical Analysis in a Nutshell: Slide 56: directional derivative of a function $f : \mathbb{R}^n \to \mathbb{R}^m$ in the direction $\lambda \in \mathbb{R}^n$:

$$\lim_{h \to 0} \frac{f(x + h\lambda) - f(x)}{h} = \underbrace{\frac{\mathrm{d}f(x)}{\mathrm{d}x}}_{\in \mathbb{R}^{m \times n}} \underbrace{\lambda}_{\in \mathbb{R}^n}$$

2 Question: How to do Algorithmic Differentiation in Forward Mode?

Let us look at an example, say

$$f(x) = x_1 * x_2$$

In order to write this as code list, we write

1. `a1 = x1`
2. `a2 = x2`
3. `a3 = a1*a2`
4. `return a3`

This is the code list for the first nominal evaluation of f. Let us try to develop a code that evaluates the directional derivative in the direction

$$\lambda = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

We want to have a code list for evaluating $f'(x)\lambda$, where $f'(x) = [x_2,\ x_1]$ the Jacobian f at x. The explicit answer would in this case be

$$f'(x)\lambda = [x_2,\ x_1] \cdot \begin{pmatrix} 3 \\ 4 \end{pmatrix} = 3x_2 + 4x_1$$

This function can be found by differentiating the above code list

1. `b1 = 3`
2. `b2 = 4`
3. `a3 = a1*b2+b1*a2`
4. `return a3`