

- **Quiz 1(1):**

f is continuous at x_0 means that: $\forall \varepsilon > 0, \exists \delta > 0$, such that $\forall |x - x_0| \leq \delta$, i.e., $x \in \mathbb{U}(x_0, \delta)$, we have $|f(x) - f(x_0)| \leq \varepsilon$.

While $\{x_n\}$ converging to x_0 means that: $\forall \varepsilon_1 > 0, \exists N > 0$ such that $\forall n \geq N, |x_n - x_0| \leq \varepsilon_1$.

Let $\varepsilon_1 = \varepsilon$, then for the ε and δ there is a positive integer N_δ such that $\forall n \geq N_\delta, |x_n - x_0| < \delta$, with the continuous of f at x_0 we get $|f(x_n) - f(x_0)| < \varepsilon$. That is to say $\lim_{n \rightarrow \infty} f(x_n) = f(x_0)$.

In short: $\lim_{n \rightarrow \infty} f(x_n) = f\left(\lim_{n \rightarrow \infty} x_n\right) = f(x_0)$.

- **Quiz 1(2):**Textbook page 114, example 4.

- **Quiz 2:**

suppose we apply the Composite Simpson's rule with n subintervals to a function f on $[a, b]$ and determine the maximum bound for the round-off error. Assume that $f(x_i)$ is approximated by $\tilde{f}(x_i)$ and that

$$f(x_i) = \tilde{f}(x_i) + e_i, \quad \text{for each } i = 0, 1, \dots, n,$$

where e_i denotes the round-off error associated with using $\tilde{f}(x_i)$ to approximate $f(x_i)$. Then the accumulated error, $e(h)$, in the Composite Simpson's rule is

$$\begin{aligned} e(h) &= \left| \frac{h}{3} \left[e_0 + 2 \sum_{j=1}^{(n/2)-1} e_{2j} + 4 \sum_{j=1}^{n/2} e_{2j-1} + e_n \right] \right| \\ &\leq \frac{h}{3} \left[|e_0| + 2 \sum_{j=1}^{(n/2)-1} |e_{2j}| + 4 \sum_{j=1}^{n/2} |e_{2j-1}| + |e_n| \right]. \end{aligned}$$

If the round-off errors are uniformly bounded by ϵ , then

$$e(h) \leq \frac{h}{3} \left[\epsilon + 2 \sum_{j=1}^{(n/2)-1} \epsilon + 4 \sum_{j=1}^{n/2} \epsilon + \epsilon \right] = \frac{h}{3} 3n\epsilon = nh\epsilon.$$

Here $nh = b - a$, so

$$e(h) \leq (b - a)\epsilon,$$

a bound independent of h and n . This means that, even though we may need to divide an interval into more parts to ensure accuracy, the increased computation that is required does not increase the round-off error.

- **Quiz 3:**Textbook page 271, Theorem 5.9.