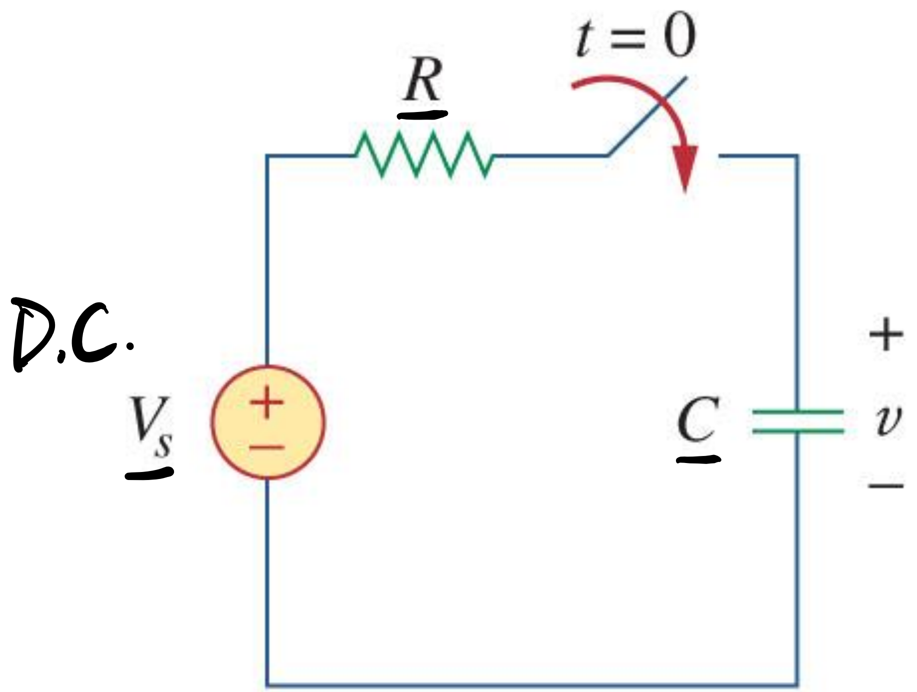
Outline

- Capacitors and inductors
- Natural response of RC/RL circuits
- Step response of RC/RL circuits

  阶越
- Others

Step Response of RC Circuit

- When a DC source is suddenly applied to a RC circuit, the circuit response is known as the step response.

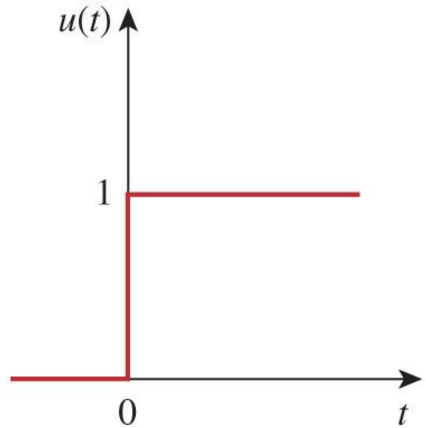


$v(0^-) = v(0^+) = V_0$

(a)

The Unit Step *u(t)* function $u(t)$

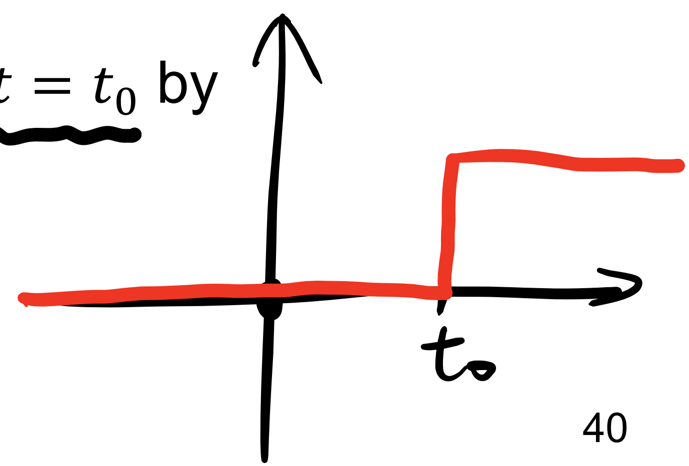
- A step function is one that maintains a constant value before a certain time and then changes to another constant afterwards.



$$u(t)=\begin{cases}0, & t<0\\ 1, & t>0\end{cases}$$

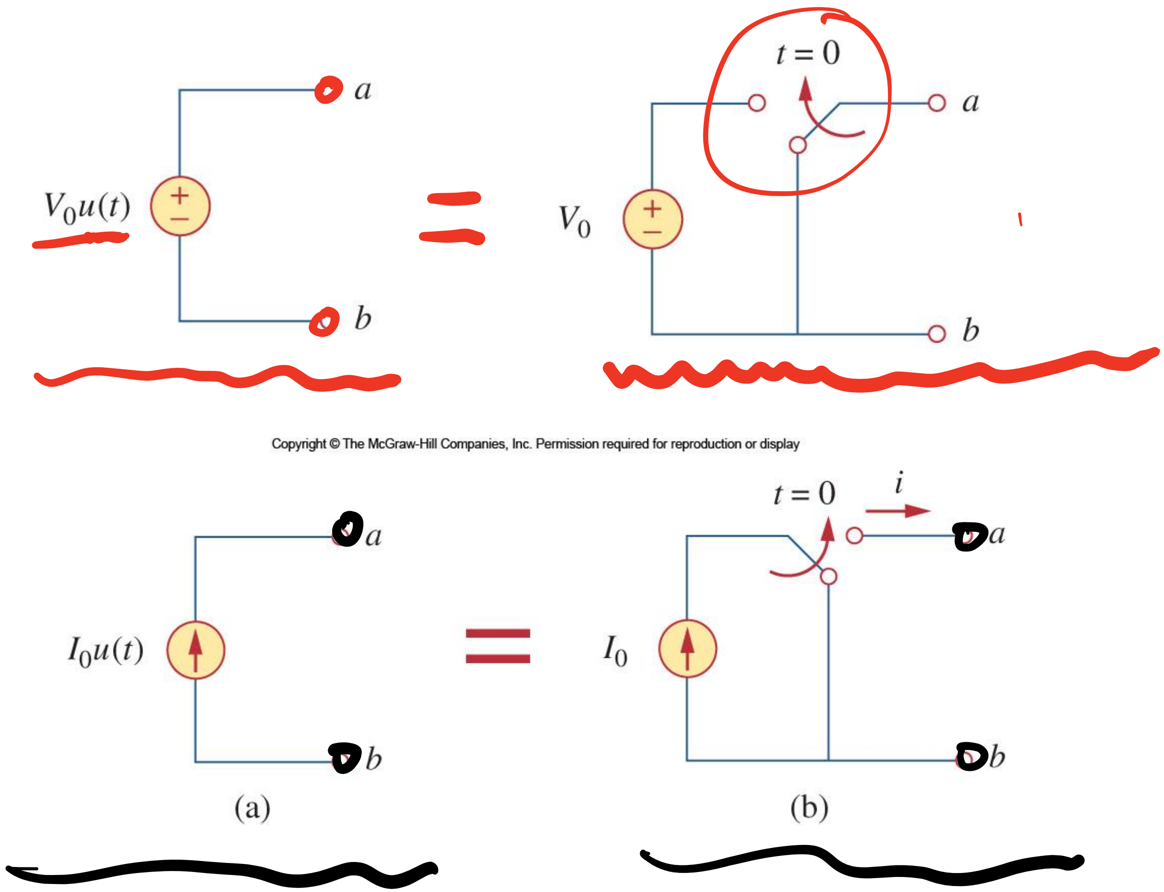
switching time may be shifted to $t = t_0$ by

$$u(t-t_0)=\begin{cases}0, & t<t_0\\ 1, & t>t_0\end{cases}$$

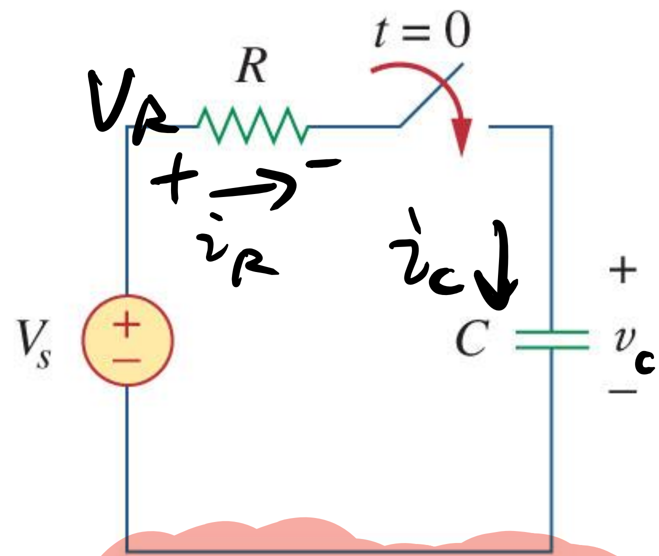


Equivalent Circuit of Unit Step

- The unit step function has an equivalent circuit to represent when it is used to switch on a source.



Step Response of the RC Circuit



$v(0^-) = v(0^+) = V_0$

$V_c(t>0)$

$V_s > V_0$ $t \to \infty$

$V_c(t\to\infty)$? =
<1> 0
<2> V_0
<3> V_s ✓

$$C\frac{dV_c}{dt} = i_C = i_R = \frac{V_R}{R} = \frac{V_s - V_c}{R}$$

$$\frac{dV_c}{dt} + \frac{1}{RC} \cdot V_c = \frac{V_s}{RC}$$

G.S. : $V_c' = A \cdot e^{-\frac{1}{RC} \cdot t}$

P.S. : $V_c'' = V_s$

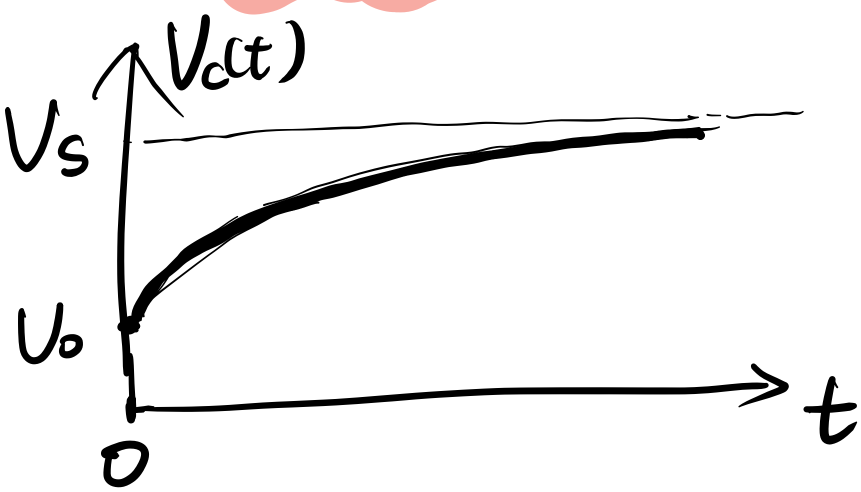
$\therefore V_c(t) = A \cdot e^{-\frac{1}{RC} t} + V_s$ ①

$$\to V_c(t=0) = V_0 = A + V_s$$

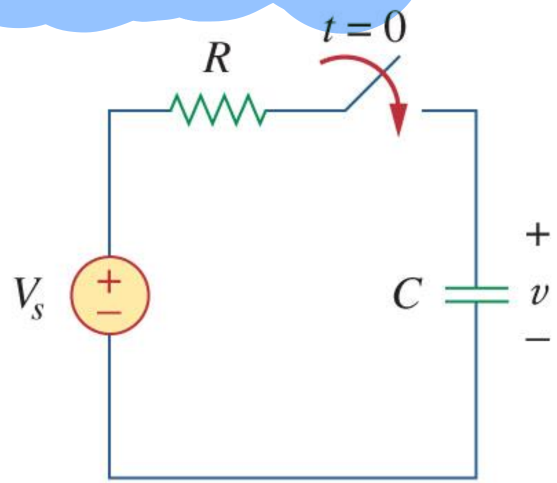
$$\Rightarrow A = V_0 - V_s$$

$$V_c(t) = V_s + (V_0 - V_s) \cdot e^{-t/RC}, \quad t > 0 \quad ②$$

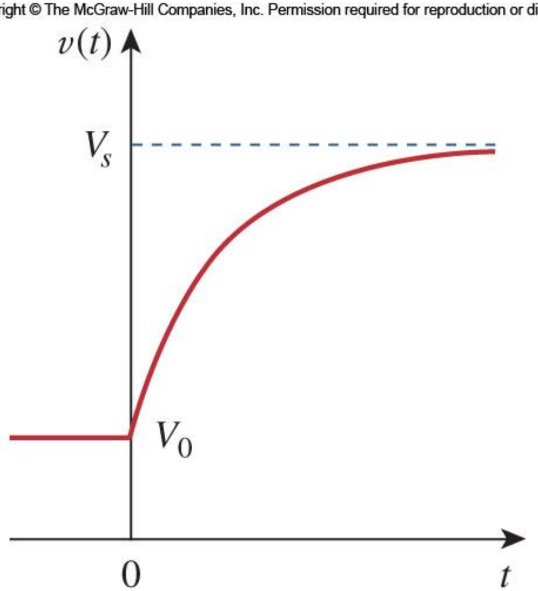
$V_s > V_0$



Step Response of the RC Circuit



$$v(0^-) = v(0^+) = V_0$$

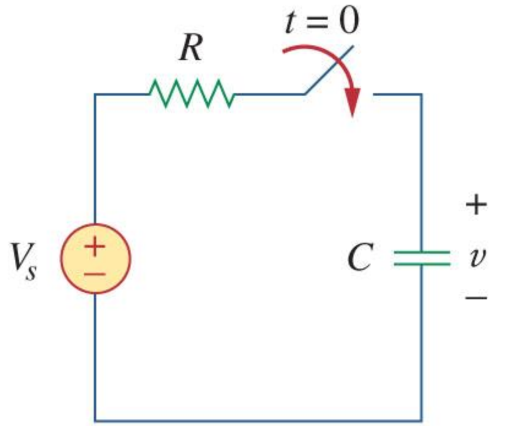


$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau} & t > 0 \end{cases}$$

$\tau = RC$

- This is known as the complete response, or total response.

Complete response



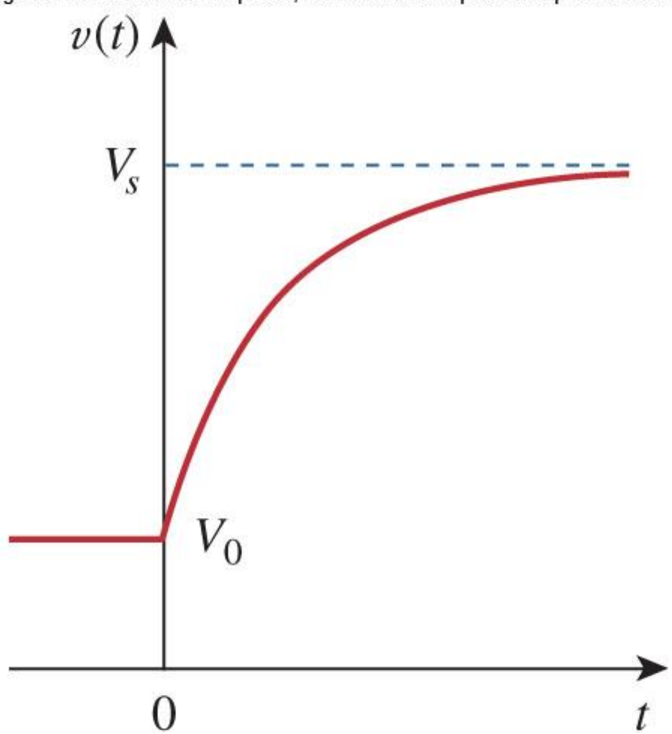
$$v(0^-) = v(0^+) = V_0$$

- The complete response

$$v(t)=\begin{cases} V_0, & t<0 \\ V_s+(V_0-V_s)e^{-t/\tau} & t>0 \end{cases}$$

can be written as:

$$\boxed{v = v_n + v_f}$$



$$V_s - V_s \cdot e^{-t/\tau}$$

$$V_s + (V_0 - V_s)e^{-t/\tau} = \underbrace{V_0 \cdot e^{-t/\tau}} + \text{aka: } \underline{V_s(1-e^{-t/\tau})}$$

$$\text{Complete response} = \text{natural response} + \text{forced response}$$

stored energy — independent source

or

$$v = v_n + v_f$$

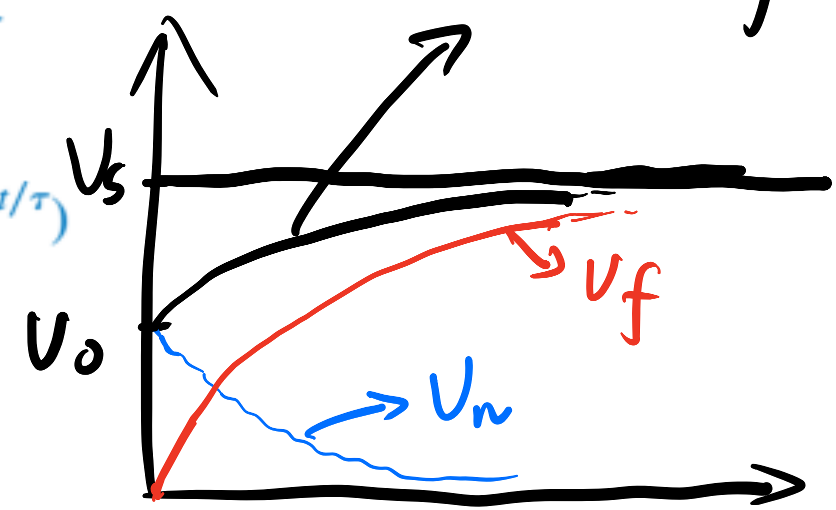
where

$$v_n = V_o e^{-t/\tau}$$

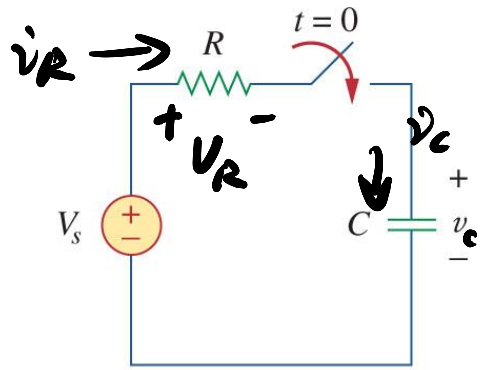
and

$$v_f = V_s(1 - e^{-t/\tau})$$

$$V = V_n + V_f.$$



Another Perspective

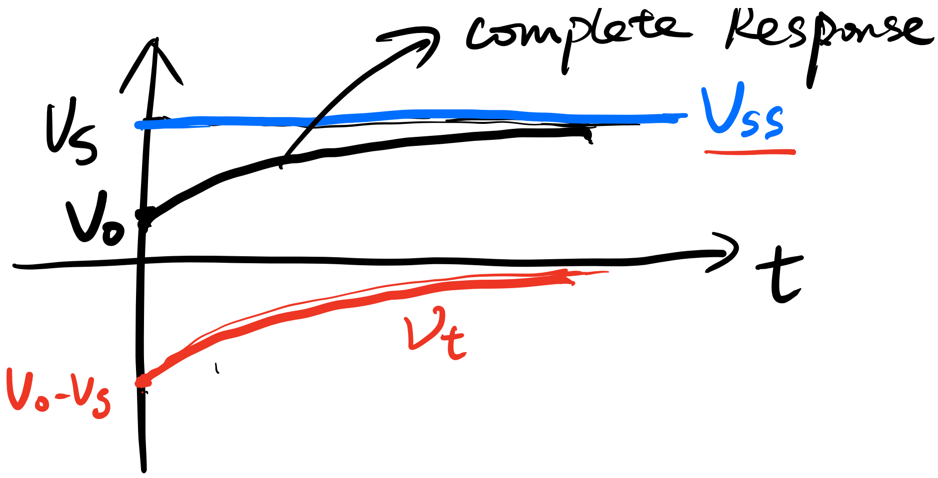


$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau} & t > 0 \end{cases}$$

- Another way to look at the response is to break it up into the transient response and the steady state response:

$$\rightarrow \quad v(t) = \underbrace{v(\infty)}_{\text{steady } v_{ss}} + \underbrace{[v(0) - v(\infty)]e^{-t/\tau}}_{\text{transient } v_t} \qquad \tau = RC$$

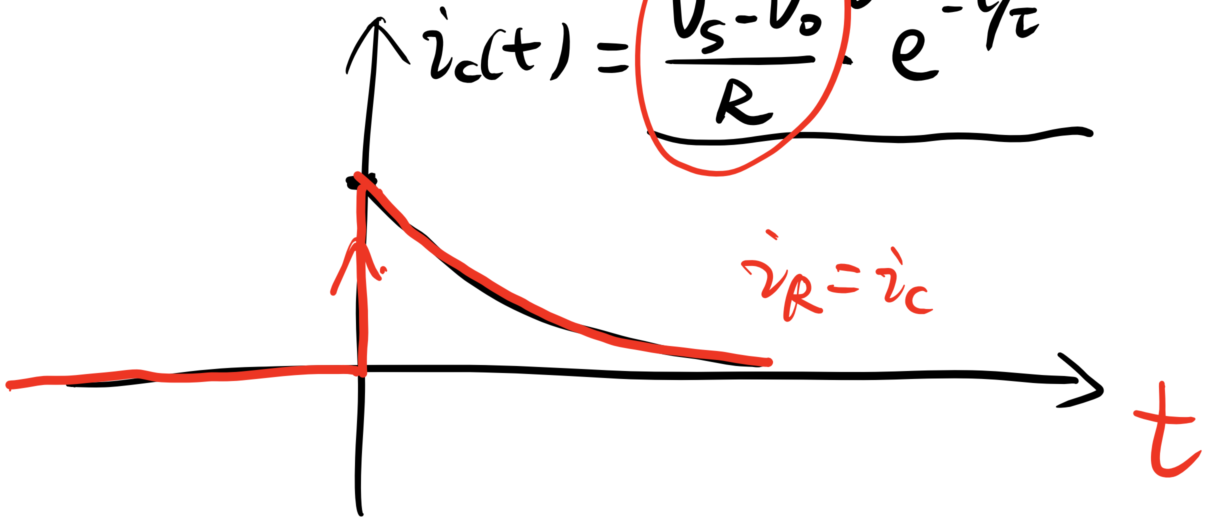
$v(\infty) = V_s$, $v(0) = V_0$, $v(\infty) = V_s$



$$V_c(t) \longrightarrow \underline{i_c(t)} = C \cdot \frac{dV_c(t)}{dt}$$

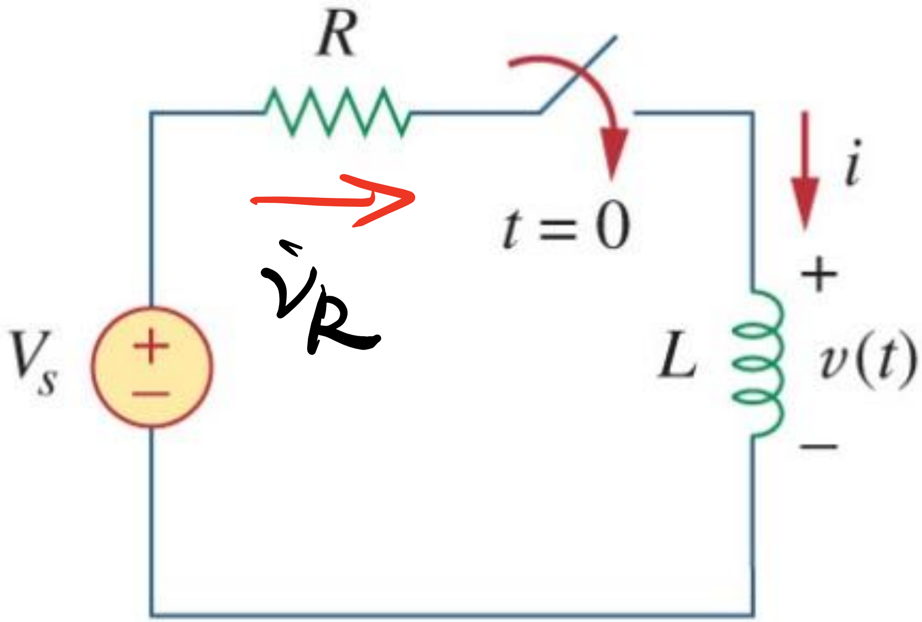
$$\underline{i_R(t)} = \underline{i_c(t)}$$

$$\underline{i_c(t) = \frac{V_s - V_0}{R} \cdot e^{-t/\tau}}$$



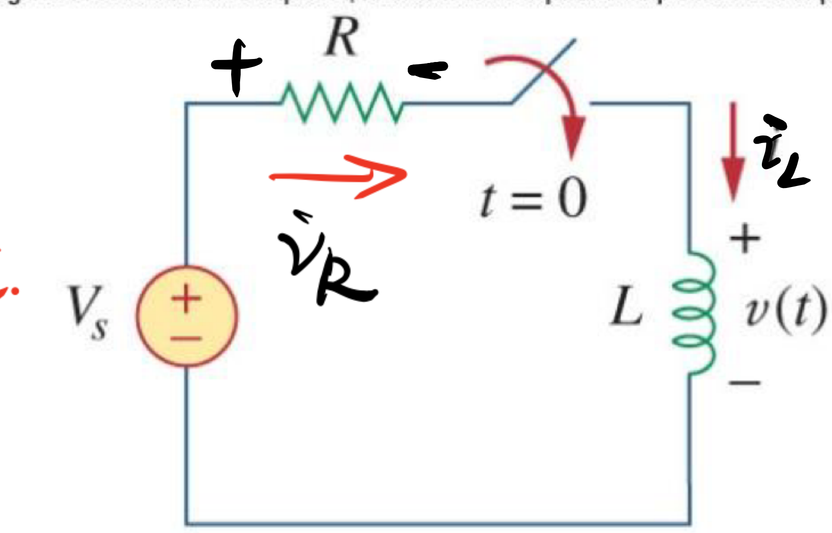
Step Response of the RL Circuit

D.C.



$$i(0^+) = i(0^-) = I_0$$

$$t \to \infty \quad i_L(t) = \begin{cases} 0 \\ I_0 \\ ?? = \dfrac{V_s}{R} \end{cases}$$



$$i(0^+) = i(0^-) = I_0$$

$$L\frac{di_L}{dt} = V_L = V_s - V_R = V_s - i_L \cdot R$$

$$\frac{di_L}{dt} + \frac{R}{L} \cdot i_L = \frac{V_s}{L} \quad (1)$$

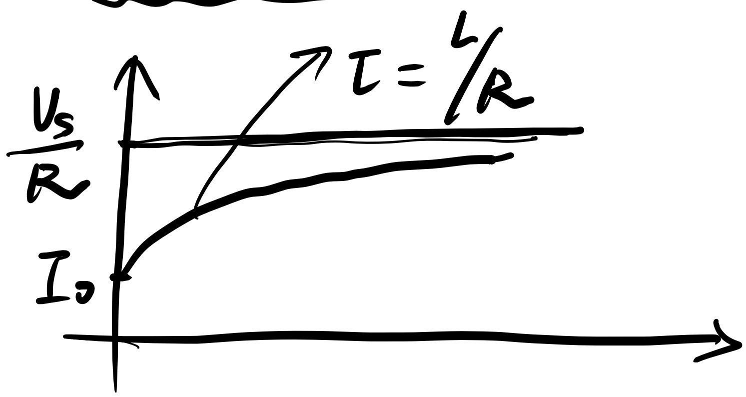
G.S. $\underline{\dfrac{di_L}{dt} + \dfrac{R}{L} \cdot i_L = 0}$ $\quad \therefore\ i_L' = A \cdot e^{-\frac{R}{L} \cdot t}$

P.S. $\quad \underline{i_L'' = \dfrac{V_s}{R}}$

$$\underline{i_L(t) = A \cdot e^{-\frac{R}{L} \cdot t} + \frac{V_s}{R}} \quad (2)$$

$$i_L(t=0) = I_0 = A + \frac{V_s}{R} \Rightarrow A = I_0 - \frac{V_s}{R}$$

$$\underline{i_L(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{R}{L} \cdot t}, \quad t > 0}$$



$$i_L(t) = \underline{i_n} + \underline{i_f}$$

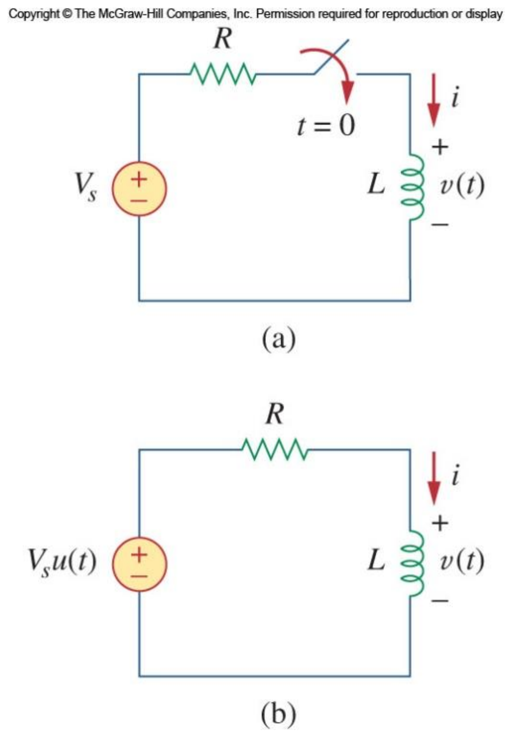
$$= I_0 \cdot e^{-t/\tau} + \underline{\frac{V_s}{R}\left(1 - e^{-t/\tau}\right)}$$

$$i_L(t) = \underbrace{\frac{V_s}{R}}_{\text{s.s.}} + \underbrace{\left(I_0 - \frac{V_s}{R}\right) e^{-\frac{R}{L}\cdot t}}_{\text{transient}}, \quad t > 0$$

$$= \underline{I(\infty)} + \left[\underline{I(0) - I(\infty)}\right] e^{-t/\tau}, \quad t > 0$$

$$\underline{\tau = L/R}$$

Step Response of the RL Circuit



- We will use the transient and steady state response approach.
- We know that the transient response will be an exponential:

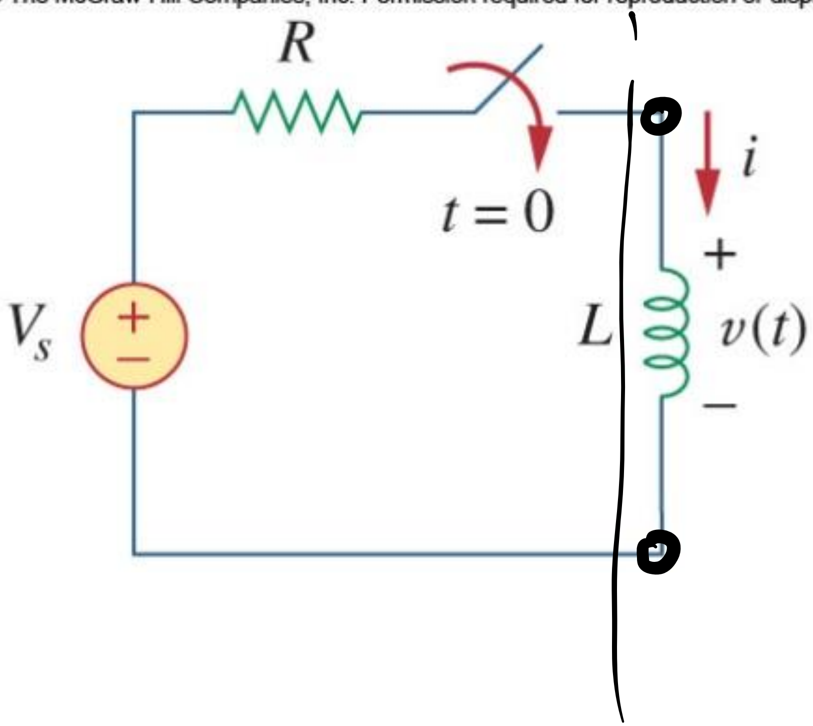
$$i_t = Ae^{-t/\tau}$$

- After a sufficiently long time, the current will reach the steady state:

$$i_{ss} = \frac{V_s}{R}$$

Step Response of RL Circuit

- This yields an overall response of:



$$i = Ae^{-t/\tau} + \frac{V_s}{R}$$

$$i\left(0^+\right) = i\left(0^-\right) = I_0 \qquad A = I_0 - \frac{V_s}{R}$$

$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-t/\tau}$$

General Procedure of Finding RC/RL Response with D.C. sources

1. **Identify the variable of interest**
   - For RL circuits, it is usually the inductor current $i_L(t)$.
   - For RC circuits, it is usually the capacitor voltage $v_c(t)$.

2. **Determine the initial value of the variable at T_0**
   - Recall that $i_L(t)$ and $v_c(t)$ are continuous variables:

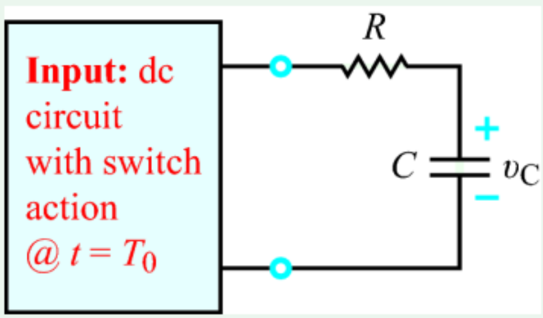
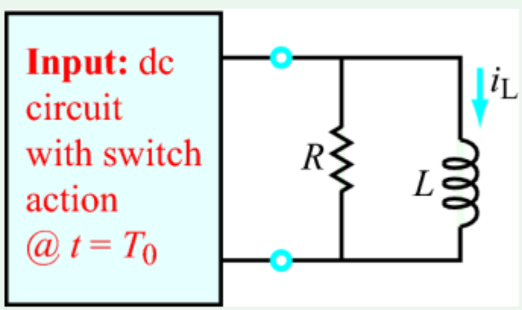
$$i_L(T_0^+) = i_L(T_0^-) \quad \text{and} \quad v_c(T_0^+) = v_c(T_0^-)$$

3. **Determine the final value of the variable** (as $t \to \infty$)

If needed, recall that an inductor behaves like a short circuit in steady state ($t \to \infty$) & that a capacitor behaves like an open circuit in steady state ($t \to \infty$). e.g. e.g.

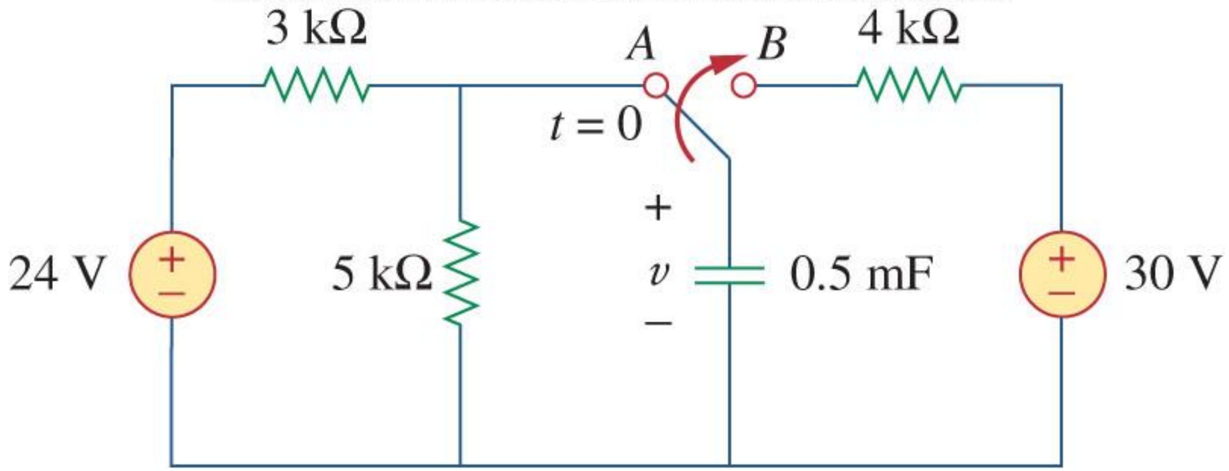
4. **Calculate the time constant for the circuit**
   - $\tau = CR$ for an RC circuit where R is the Thévenin equivalent resistance "seen" by the capacitor.
   - $\tau = L/R$ for an RL circuit, where R is the Thévenin equivalent resistance "seen" by the inductor.

Response Form of Basic First-Order Circuits

Circuit	Diagram	Response
RC		$v_C(t) = \left\{ \overset{2}{v_C(\infty)} + [\overset{1}{v_C(T_0)} - \overset{2}{v_C(\infty)}]e^{-\overset{\text{time shift}}{(t-T_0)}/\overset{3}{\tau}} \right\}$ $(\tau = RC)$
RL		$i_L(t) = \left\{ i_L(\infty) + [i_L(T_0) - i_L(\infty)]e^{-(t-T_0)/\tau} \right\}$ $(\tau = L/R)$

Example

- The switch has been in position A for a long time. At $t = 0$, the switch moves to B. Find $v(t)$.



Example

- Find $i(t)$ in the circuit for $t > 0$. Assume that the switch has been closed for a long time.