

# CS244 Theory of Computation

## Homework 3

Due: October 28, 2020 at 11:59pm

Name - ID

You may discuss this assignment with other students and work on the problems together. However, your write-up should be your own individual work and you should indicate in your submission who you worked with, if applicable. You should use the L<sup>A</sup>T<sub>E</sub>X template provided by us to write your solution and submit the generated PDF file into Gradescope.

I worked with: (Name, ID), (Name, ID), ...

Let  $\Sigma = \{0, 1\}$  if not otherwise specified.

### Problem 1

Consider the problem of testing whether a pushdown automaton ever uses its stack. Formally, let  $PUSHER = \{\langle P \rangle \mid P \text{ is a PDA that pushes a symbol on its stack on some (possibly non-accepting) branch of its computation at some point on some input } w \in \Sigma^*\}$ . Show that  $PUSHER$  is decidable.

### Problem 2

Let  $C$  be a language. Prove that  $C$  is Turing-recognizable iff a decidable language  $D$  exists such that  $C = \{x \mid \exists y \in \{0, 1\}^* (\langle x, y \rangle \in D)\}$ .

(Hint: You must prove both directions of the “iff”. The  $(\leftarrow)$  direction is easier. For the  $(\rightarrow)$  direction, think of  $y$  as providing additional information that allows you to confirm when  $x \in C$ , but without the possibility of looping.)

### Problem 3

Consider the problem of determining whether a single-tape Turing machine ever writes a blank symbol over a nonblank symbol during the course of its computation on any input string. Formulate this problem as a language and show that it is undecidable.

### Problem 4

A *2-way pushdown automaton* (2WAY-PDA) is a nondeterministic pushdown automaton that has a single stack and that can move its input head in both directions on the input tape. In addition we assume that a 2WAY-PDA is capable of detecting when its input head is at either end of its input tape. A 2WAY-PDA accepts its input by entering an accept state.

(a) Give a 2WAY-PDA that recognizes the language  $\{a^m b^m c^m \mid m \geq 0\}$ .

- (b) Let  $E_{2\text{WAY-PDA}} = \{\langle P \rangle \mid P \text{ is a 2WAY-PDA which recognizes the empty language}\}$ .  
Show that  $E_{2\text{WAY-PDA}}$  is not decidable.

## Problem 5

Let  $A$  be a language.

- (a) Show that  $A$  is Turing-recognizable iff  $A \leq_m A_{\text{TM}}$ .  
(b) Show that  $A$  is decidable iff  $A \leq_m 0^*1^*$ .