ShanghaiTech University

CS240 Algorithm Design and Analysis

Fall 2022

Problem Set 3

Due: 23:59, Nov. 24, 2022

1. Submit your solutions to Gradescope (www.gradescope.com).
2. In "Account Settings" of Gradescope, set your FULL NAME to your Chinese name and enter your STUDENT ID correctly.
3. If you want to submit a handwritten version, scan it clearly. Camscanner is recommended.
4. When submitting your homework, match each of your solution to the corresponding problem number.

Problem 1:

Suppose there is a finite set C and a collection of subsets of C. The SET-PACKING problem asks if some K subsets in the collection are pairwise disjoint(in other words, no two of them share an element). Show that SET-PACKING problem is NP-complete.
Hint: Reduction from Independent Set.

Problem 2:

Red and Xiaoyu are allocating a set of items $M = \{1, ..., m\}$ among themselves. Each of them evaluates the items respectively and gives each of the items a valuation to denote their preference on the item. Given a subset of items $S \subseteq M$, one's utility is defined as

$$u_i(S) = max\{\sum_{j \in S} v_i(j), b_i\}$$

where $i \in \{Red, Xiaoyu\}$, $v_i(j)$ is i's valuation on item j and b_i is the upper bound of i's utility. Their goal is to find the optimal allocation S_1, S_2 that maximizes $u_R(S_1) + u_X(S_2)$. Prove this problem is NP-hard with "Knapsack".

Here is an example for you to understand the problem. There are 3 items indexed by $1, 2, 3$. Red's valuation on the items are $10, 20, 30$ and Xiaoyu's valuation is $30, 15, 10$ while the upper bound of their total utilities are 40 and 50. Then the best choice is to allocate item 3 to Red and $1, 2$ to Xiaoyu which brings a $30 + (30 + 15) = 75$ utility in total.

Problem 3:

The problem: Suppose there is an undirected graph $G = (V, E)$ and a positive integer $M \leq |V|$. Does the gragh G contain a path which visits vertex at most once and has the number of path's edges $N \geq M$?
Prove this problem is NP-complete.

Problem 4:

STINGY SAT is the following problem: given a set of clauses (each a disjunction of literals) and an integer k, find a satisfying assignment in which at most k variables a true, if such an assignment exists. Prove that STINGY SAT is NP-complete.

Problem 5:

SIST allows students to work as TAs but would like to avoid TA cycles. A TA cycle is a list of TAs (A_1, A_2, . . . , A_k) such that A_1 works as a TA for A_2 in some course, A_2 works as a TA for A_3 in some course, $\cdots$, and finally A_k works as a TA for A_1 in some course. We say a TA cycle is simple if it does not contain the same TA more than once. Given the TA arrangements of SIST, we want to find out whether there is a simple TA cycle containing at least K TAs. Prove this problem is NP-complete.