Homework 2

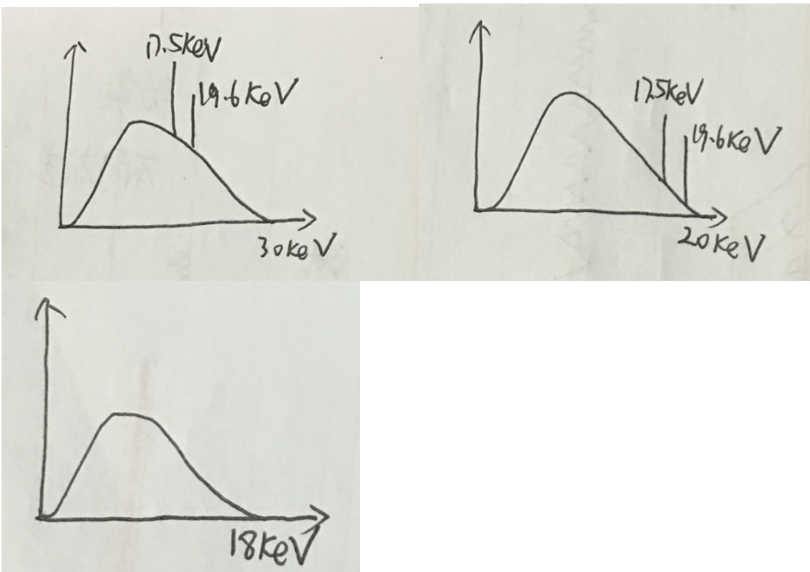
Due: Mar.29th   Submit: Blackboard

1. For Molybdenum, the binding energy for K, L, M-shell are -20 keV, -2.5keV, and -0.4keV respectively. (15')
   (1) Please calculate the characteristic X-ray for Molybdenum which can be used for the X-ray imaging;
   (2) Plot the energy spectra from a Molybdenum tube with the following kVp values: 30 keV, 20 keV, 18 keV.

   SOLUTION:

   (1) -2.5 - (-20) = 17.5 keV; -0.4 – (-20) = 19.6 keV.

   (2)



2. A digital radiograph is acquired but the CNR in the region of interest is insufficient. Calculate the relative CNR of the following options and explain: (20')
   (1) Double the mAs (6')
   (2) Take four radiographs and calculate the average image; (6')
   (3) Change the kVp. (8')

   SOLUTION:

   (1) The X-ray intensity is proportional to the mAs, the CNR is proportional to the square root of the mAs. Doubling the mAs doubles the intensity of X-ray and increases the CNR by a factor $\sqrt{2}$.

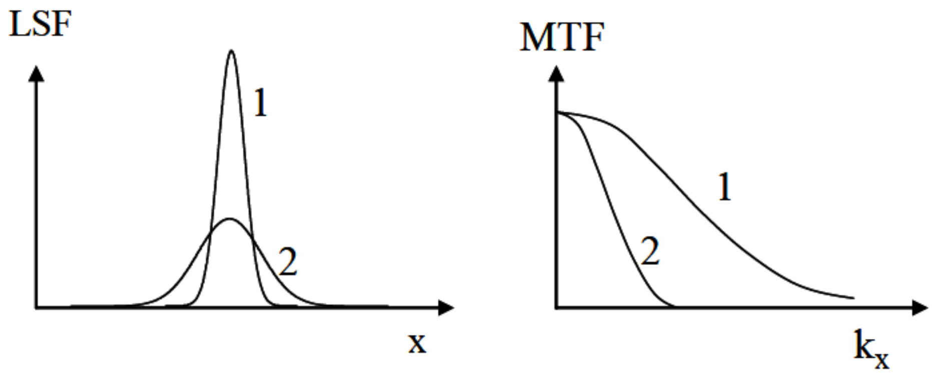
   (2) By taking multiple radiographs with the same mAs and kV, the contrast and noise in the images are expected to be the same. By averaging, it does not affect the contrast, but reduces the noise by factor of square root of number of images. The CNR thus increases by a factor of 2.

   (3) Increasing the kVp (without changing the mAs) increases the energy of X-ray, resulting in the smaller difference of attenuation coefficients between tissues, therefore the contrast tends to decrease with increasing kVp. But the SNR improves due to the larger beam intensity at higher kVp. The effect on the CNR is difficult to predict a priori. At lower kVp, the scattering becomes more pronounced, which also reduce the CNR.

3. Sketch the LSF and MTF for the computed radiography (15’)
   (1) with a thick or a thin phosphor layer of CR plate respectively, and explain (7’)
   (2) with a small or large X-ray focal spot respectively and explain (8’)

   SOLUTION:

   (1) A thick phosphor layer has high SNR but poor spatial resolution due to longer travelling distance of photon, therefore corresponds to a broad LSF and a narrow MTF, namely number 2. The thin phosphor layer corresponds to number 1.
   (2) A small X-ray focal spot has a high spatial resolution since it reduces the geometric unsharpness, and therefore has a narrow LSF and a broad MTF as number 1. A large X-ray focal spot corresponds to number 2.



4. When wide X-ray beams transmit through a matter, part of scattered x-ray photons will be received by the detectors. Therefore we can modify the equation to calculate number of transmitted X-ray photons as $N = BN_0e^{-\mu x}$, where B is the accumulation factor. The HVL of aluminum is 2.1mm. In the case of a wide X-ray beam with intensity I_0 passed through an aluminum sheet with thickness of HVL (2.1mm), the intensity of the scattered x-ray photons received by the detectors is equivalent to $0.1I_0$. Please calculate the HVL for a wide x-ray beam. (15’)

   SOLUTION:

   According to definition of HVL: $\frac{1}{2}I_0 = I_0e^{-\mu*2.1} \Longrightarrow \mu = \frac{ln2}{2.1}$

   For wide beam through the thickness of HVL, the detected intensity

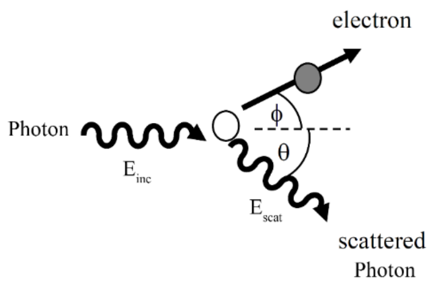
   $\frac{1}{2}I_0 + 0.1I_0 = BI_0e^{-\mu*2.1} \Rightarrow B = 1.2$

   the HVL for wide beam $\frac{1}{2}I_0 = BI_0e^{-\mu*HVLW} \Rightarrow HVLW = 2.65mm$

5. Figure shows the Compton scattering effect. In the case of incident photons with low energy, please prove that the wavelength change of between the incident and scattered photon is

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta)$$

where $\mathbf{m_e}$ is the mass of electron, **c** is the speed of light, and **h** is the plank's constant (**Tip**: the energy loss of photon during scattering is very small comparing to the energy of incident photon, you may use approximation to simplify your deduction) (20')



SOLUTION:

$$E = \frac{hc}{\lambda}, \qquad \frac{E}{c} = \frac{h}{\lambda}$$

$$\frac{hc}{\lambda} = \frac{hc}{\lambda+\Delta\lambda} + \frac{1}{2}mv^2 \qquad (1)$$

$$\frac{h}{\lambda} = \frac{h}{\lambda+\Delta\lambda}cos\theta + mv\,cos\varphi \qquad (2)$$

$$\frac{h}{\lambda+\Delta\lambda}sin\theta = mv\,sin\varphi \qquad (3)$$

$$(2)+(3):\ \left(\frac{h}{\lambda} - \frac{h}{\lambda+\Delta\lambda}cos\theta\right)^2 + \left(\frac{h}{\lambda+\Delta\lambda}sin\theta\right)^2 = m^2v^2 \qquad (4)$$

$$(1)+(4):\ \left(\frac{h}{\lambda} - \frac{h}{\lambda+\Delta\lambda}cos\theta\right)^2 + \left(\frac{h}{\lambda+\Delta\lambda}sin\theta\right)^2 = 2m\left(\frac{hc}{\lambda} - \frac{hc}{\lambda+\Delta\lambda}\right)$$

$$\frac{h^2}{\lambda^2} + \frac{h^2}{(\lambda+\Delta\lambda)^2} - \frac{2h^2cos\theta}{\lambda(\lambda+\Delta\lambda)} = 2mhc\frac{\Delta\lambda}{\lambda(\lambda+\Delta\lambda)}$$

$$\lambda \approx \lambda+\Delta\lambda \ \rightarrow \ \Delta\lambda = \frac{h}{mc}(1-cos\theta)$$

6. In digital subtraction angiography by combination of sequencing and energy, four images are acquired, the images before injection of the contrast agent under high and low energy, I_{0H} and I_{0L}, the images after injection of the contrast agent under high and low energy, I_{1H} and I_{1L}.Given the mass attenuation coefficient for bone and soft tissue under high and low energy, μ_{BL}, μ_{BH}, μ_{TL}, and μ_{TH}. Derive the expression for the final subtraction image. (15')

SOLUTION:

I_{0L} : regular image from low energy

I_{0H} : regular image from high energy

I_{1L} : contrast image from low energy

I_{1H} : contrast image from high energy

S_{0T} : regular tissue subtraction image derived from I_{0L} and I_{0H}

S_{1T} : contrast tissue subtraction image derived from I_{1L} and I_{1H}

S : final subtraction image where $S = S_{0T} - S_{1T}$

According to the Dual-energy subtraction formula, the tissue-subtracted image:

$$S_{0T} = \mu_{TL} lnI_{0H} - \mu_{TH} lnI_{0L}$$
$$S_{1T} = \mu_{TL} lnI_{1H} - \mu_{TH} lnI_{1L}$$

Final subtraction image:

$$S = S_{0T} - S_{1T} = \mu_{TL} lnI_{0H} - \mu_{TH} lnI_{0L} - (\mu_{TL} lnI_{1H} - \mu_{TH} lnI_{1L})$$