

**1. (1 points) Notes of discussion**

I promise that I will complete this QUIZ independently and will not use any electronic products or paper-based materials during the QUIZ, nor will I communicate with other students during this QUIZ.

- (a) (1') True or False: I have read and understood the notes.

problem 1

**2. (8 points) True or False**

Determine whether the following statements are true or false.

problem (a)	problem (b)	problem (c)	problem (d)	problem (e)	problem (f)	problem (g)

- (a) (1')  $f(n) = O(g(n)) \wedge f(n) = O(h(n)) \implies f(n) = O(h(n)g(n))$ .
- (b) (1') If  $f(n) = \log_2 n$  then for all  $0 < \alpha \leq 1$ , we have  $f(n) = O(n^\alpha)$ .
- (c) (1') For an algorithm **A**, it is *possible* that the worst-case running time is  $O(n)$  and the best-case running time is  $\Omega(n)$ .
- (d) (1') For an algorithm **B**, it is *impossible* that the worst-case running time is  $\Omega(n)$  while the best-case running time is  $O(\sqrt{n})$ .
- (e) (1') The number of collisions in a hash table solely(only) depends on the table capacity and the hash function.
- (f) (1') The worst-case running time for insertion in a hash table is  $O(n)$ .
- (g) (1') Hash tables using open addressing are better implemented with linked lists than with arrays because key values can be added or deleted quickly.
- (h) (1') Quadratic probing is equivalent to double hashing with a secondary hash function of  $h_2(k) = k^2$ .

**3. (8 points) Hash Table Insertions and Deletions**

Consider a empty hash table of capacity 7 where the hash function for a key  $k$  is  $h(k) = (2k + 5) \bmod 7$ . Collisions are resolved by quadratic probing where the  $i$ -th probe accesses  $H_i(k) = (h(k) + i^2) \bmod 7$  for  $i = 0, 1, 2, \dots$ , paired with lazy erasing. We will give 8 instructions (**Insert/Delete/Search key\_value**). For **Insert/Delete** instructions, you need to fill the hash table after each instruction. For **Search** instructions, write down probing sequence(index). Use 'D' to indicate that the bin has been marked as deleted.

- (a) (1') **Insert 13**

Index	0	1	2	3	4	5	6
Key Value							

(b) (1') **Insert 27**

Index	0	1	2	3	4	5	6
Key Value							

(c) (1') **Insert 34**

Index	0	1	2	3	4	5	6
Key Value							

(d) (1') **Delete 27**

Index	0	1	2	3	4	5	6
Key Value							

(e) (1') **Insert 8**

Index	0	1	2	3	4	5	6
Key Value							

(f) (1') **Delete 13**

Index	0	1	2	3	4	5	6
Key Value							

(g) (1') **Insert 24**

Index	0	1	2	3	4	5	6
Key Value							

(h) (1') **Search 34**

#### 4. (4 points) **Balancing the Running Time**

Suppose that the running time of an algorithm is  $T(n) = B^2 + \frac{n^2}{B^2}$ , where  $n$  is the size of the input and  $B$  is a unknown fixed parameter that might not be independent of  $n$ .

(a) (2') What is the asymptotic tight upper bound of  $T(n)$  if  $B = n$

(b) (2') Try to find a function  $g(n)$  such  $B = g(n)$  minimizes the order of growth of  $T(n)$ .