ShanghaiTech University

CS240 Algorithm Design and Analysis

Fall 2022

Problem Set 1

Due: 23:59, Sept. 30, 2022

1. Submit your solutions to Gradescope (www.gradescope.com).
2. In "Account Settings" of Gradescope, set your FULL NAME to your Chinese name and enter your STUDENT ID correctly.
3. If you want to submit a handwritten version, scan it clearly. Camscanner is recommended.
4. When submitting your homework, match each of your solution to the corresponding problem number.

Problem 1:

Sort the following functions in ascending order of growth.

$f_1(n) = n^{\sqrt{n}}$

$f_2(n) = n^2$

$f_3(n) = (\log n)^{\log n}$

$f_4(n) = (\log n)^n$

$f_5(n) = n^{2/3}$

$f_6(n) = 666^{\sqrt{n}}$

$f_7(n) = 2^n$

$f_8(n) = 100^{100}$

Problem 2:

Give the time complexity of the following code and explain the reason.

(1)
```
for ( i=1; i < n; i *= 2 ) {
    for ( j = n; j > 0; j /= 2 ) {
        for ( k = j; k <= n; k += 2 ) {
            res += (i * j + j * k );
        }
    }
}
```

(2)
```
for ( i = n; i > 0; i -= 1 ) {
    for ( j = 1; j < n; j *= 2 ) {
        for ( k = 0; k < j; k += 1 ) {
            res += (i * j + j * k );
        }
    }
}
```

Problem 3:

Given a connected graph $G = (V, E)$ with at most $n + c$ edges where c is a constant and $n := |V|$. Find the MST of G in $O(n)$ running time. You should not only give a brief explanation of your algorithm but also prove the correctness and time complexity.

Problem 4:

In the new semester, SIST students have all enrolled in some courses. We asked n students "how many other students took the same course as you?" and collected the answers in an integer array $answer$ where $answer[i]$ is the answer of the i-th student.

Given an array $answer$, return the minimum number of students that could be in SIST.

Problem 5:

Suppose you are a security guard at ShanghaiTech University and now troubled by fraud detection. Since criminals may make some student cards to help thieves sneak into the campus, you have confiscated n student cards. Each card is a small plastic sheet with a magnetic stripe which encodes data, and belongs to a unique student. The cards have the same content on the surface because the student service department wants to save cost.

As a security guard, you have to find out whether there is someone colluding with the criminals and copying his student card. You has an equivalence tester, which can tell whether two cards belong to one person, but cannot tell who it is. (We say two cards are equivalent if they belong to one person.)

Assume that to invoke the equivalence tester, each time you can only take two cards and plug them into the equivalence tester. Please determine whether there is a set of more than $n/2$ cards that are equivalent in n cards, i.e. belonging to one person, with only $O(n \log n)$ invocations of the equivalence tester.