CS244 Theory of Computation
Homework 1

Due: October 7, 2022 at 11:59pm

Name - ID

You may discuss this assignment with other students and work on the problems together. However, your write-up should be your own individual work and you should indicate in your submission who you worked with, if applicable. You should use the LaTeX template provided by us to write your solution and submit the generated PDF file into Gradescope.

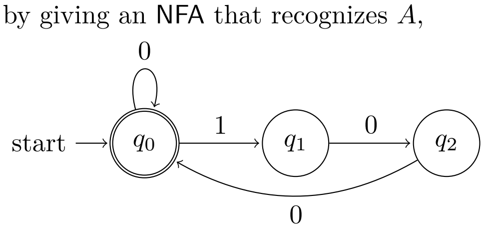
I worked with: (Name, ID), (Name, ID), ...

Let $\Sigma = \{0, 1\}$ if not otherwise specified.

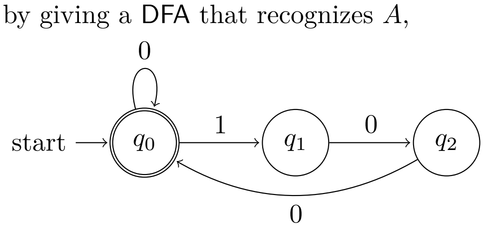
Problem 0

(0 points)
This is an example for you. You can prove that an language is regular only by the following four methods and disprove by pumping lemma or Myhill-Nerode Theorem in this homework.
Let A be the languages that every 1 has at least 2 zeros following immediately after. You can show that A is regular in the following ways:

(1) by giving an NFA that recognizes A,



(2) by giving a DFA that recognizes A,



(3) by giving a regular expression that describes A, and
$(0^*100)^*0^*$

(4) by giving a right linear grammar that describes A.

$$\begin{aligned} S &\rightarrow 0S \mid 1A \mid \epsilon \\ A &\rightarrow 0B \\ B &\rightarrow 0S \end{aligned}$$

Problem 1

Show if the following languages are regular or not. ($5 \times 10 = 50$ points)

(a) $A = \{$all strings containing at least three 1's$\}$.

(b) $B = \{$all strings containing at most two 0's$\}$.

(c) $C = \{$all strings containing at most two 0's and at least three 1's$\}$.

(d) $D = \{$all strings containing at least three 1's, but no two 1's appear consecutively$\}$.

(e) $E = \{0^k u 0^k \mid k \geq 1 \text{ and } u \in \Sigma^*\}$.

(f) $F = \{0^k 1 u 0^k \mid k \geq 1 \text{ and } u \in \Sigma^*\}$.

(g) $G = \{$binary sequences that can be divided by 5$\}$. For example, 00/5=0, 1010/5=2 and 00001010/5=2, thus $00, 1010, 00001010 \in G$.

(h) $H = \{$all strings with even length that contain at least one 1 in their first half$\}$.

(i) $I = \{w \mid w \in H \text{ or } w \text{ has odd length}\}$.

(j) $J = \{w \mid w \text{ has even length but } w \notin H\}$

Problem 2

(10 points)

(a) Prove that every NFA can be converted to an equivalent one that has a single accept state.

Problem 3

($20 \times 2 = 40$ points)

(a) Let A be an infinite regular language. Prove that A can be split into two infinite disjoint regular subsets.

(b) Let B and D be two languages. Write $B \subsetneq D$ if $B \subseteq D$ and D contains infinitely many strings that are not in B. Show that if B and D are two regular languages where $B \subsetneq D$, then we can find a regular language C where $B \subsetneq C \subsetneq D$.