EE150: Signals and Systems, Spring 2022
Homework 6

(Due Saturday, Jun. 4 at 11:59pm (CST))

1. [15 points] Determine $x(t)$ for the following conditions if $X(s)$ is given by

$$X(s) = \frac{1}{(s+1)(s+2)}$$

   (a) $x(t)$ is right-sided
   (b) $x(t)$ is left-sided
   (c) $x(t)$ is two-sided

2. [20 points] An LTI system has an impulse response $h(t)$ for which the Laplace transform $H(s)$ is

$$H(s) = \int_{-\infty}^{\infty} h(t)e^{-st}dt = \frac{1}{s+1}, \quad Re\{s\} > -1$$

Determine the system output $y(t)$ for all t if the input $x(t)$ is given by

$$x(t) = e^{\frac{-t}{2}} + 2e^{\frac{-t}{3}} \quad \forall t$$

3. [15 points] Determine the time function x(t) for each Laplace transform X(s).

   (a) $\frac{s}{s^2+4}, \quad Re\{s\} > 0$

   (b) $\frac{s^2-s+1}{(s+1)^2}, \quad Re\{s\} > -1$

   (c) $\frac{s+1}{(s+1)^2+4}, \quad Re\{s\} > -1$

4. [20 points] Let

$$g(t) = x(t) + \alpha x(-t)$$

where

$$x(t) = \beta e^{-t} u(t)$$

and the Laplace transform of $g(t)$ is

$$G(s) = \frac{s}{s^2 - 1}, \quad -1 < Re\{s\} < 1$$

Determine the values of the constants α and β.

5. [20 points] Consider a signal $y(t)$ which is related to two signals $x_1(t)$ and $x_2(t)$ by

$$y(t) = x_1(t-2) * x_2(-t+3)$$

where

$$x_1(t) = e^{-2t}u(t) \quad and \quad x_2(t) = e^{-3t}u(t)$$

Given that

$$e^{-at}u(t) \xrightarrow{\mathcal{L}} \frac{1}{s+a}, \quad Re\{s\} > -a$$

use properties of the Laplace transform to determine the Laplace transform $Y(s)$ of $y(t)$.

6. [10 points] Draw a direct-form representation for the causal LTI systems with the following system functions:

   (a) $H_1(s) = \frac{s+1}{s^2+5s+6}$

   (b) $H_2(s) = \frac{s}{(s+2)^2}$