Final Review: A TA's Perspective

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Outline

Outline of Topics

Part I

- Probability & Counting
- Conditional Probability
- Random Variables
- Expectations
- Continuous Random Variables

Part II

- Joint Distributions
- Transformations
- Monto Carlo Methods & Concentration Inequalities
- Statistical Inference
- Markov Chains

Key Pages of Lecture Slides (Before Midterm)

1. Pages 8, 22, 25, 40, 41, 46, 51, 64, 65, 66, 68, 70
2. Pages 5, 6, 8, 11, 12, 14, 15, 17, 18, 20, 21, 26, 59, 61, 73
3. Pages 6, 7, 10, 12, 13, 15, 16, 25, 26, 27, 35, 38, 41, 46, 47, 48, 49, 53, 59, 60, 63, 65
4. Pages 4, 6, 8, 10, 11, 12, 15, 16, 17, 18, 19, 20, 22, 23, 24, 32, 38, 39, 45, 48, 50, 53, 54, 55, 59, 60, 65, 66, 68, 73
5. Pages 5, 6, 10, 12, 15, 16, 17, 19, 20, 23, 26, 29, 37, 39, 41, 44, 46, 47, 48, 51, 54, 56, 59, 60, 63, 64, 65, 67, 68, 77, 81, 82, 83, 85

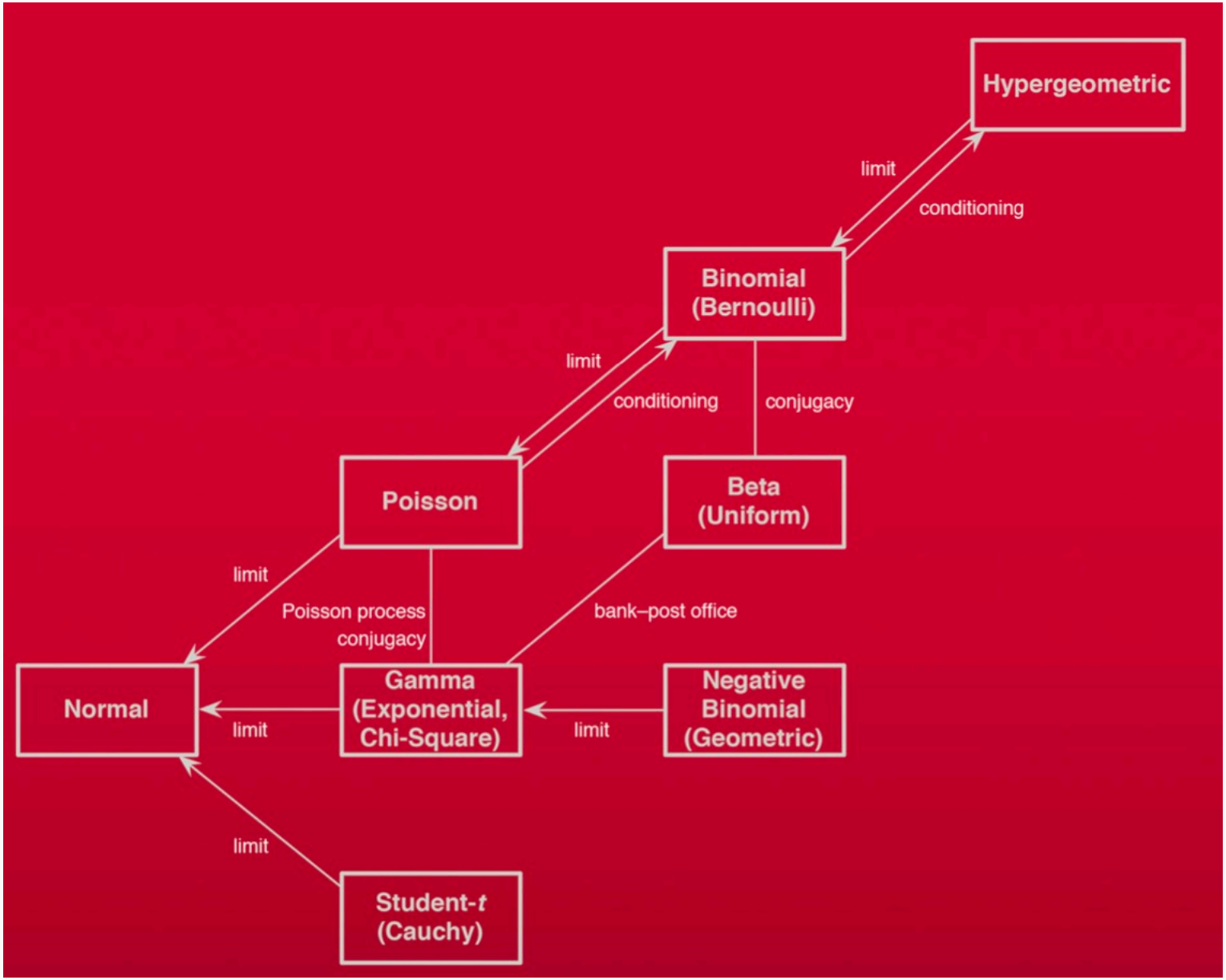
Key Pages of Lecture Slides (After Midterm)

6. Pages 5, 6, 8, 11, 13, 14, 17, 18, 22, 24, 25, 30, 34, 37, 40, 41, 42, 44, 45, 46, 48, 49, 51, 52, 57, 61, 63, 64, 66, 67, 70, 73, 74, 75, 77, 78, 80, 81
7. Pages 4, 7, 9, 14, 16, 18, 20, 23, 25, 29, 32, 35, 39, 44, 45, 48, 49, 50, 53, 55, 56
8. Pages 68, 69, 70, 74, 76, 78, 80, 82, 86, 87
9. Pages 19, 20, 21, 22, 23, 24, 25, 28, 29, 30, 39, 40, 44, 45, 46, 47, 48, 49, 53, 55, 59, 63, 64, 65, 66, 67, 69, 70, 71, 74, 77, 78, 81, 87, 88, 89, 90
10. Pages 4, 10, 12, 13, 15, 16, 17, 22, 23, 24, 26, 28, 30, 36, 38, 39, 43, 44, 48, 49, 51, 55, 56, 57, 58

Random Variables

- First success & Geometric
- Exponential & Poisson & Gamma
- Bernoullis
- Uniform
- Normal & MVN
- Beta & Binomial

Random Variables



Random Variables

- Ordering
  - Order statistics
  - Max & Min operators
- Jointness
  - Independence: pairwise & conditional
  - Correlation & Covariance
- Transformation
  - Change of variables & convolution
  - Inverse transform method
- Relationship
  - Discrete to continuous with δ-step method
  - Conjugacy: Beta-Binomial, Normal-Normal, Gamma-Pois
  - Connection: Beta-Gamma, Uniform-Beta, Binomial-Gamma

Independence

- $P(X, Y) = P(X)P(Y)$
- $P(X|Y) = P(X)$ with $P(Y) \neq 0$
- Factorization of PDF $f_{X,Y}(x, y)$ and MGF $M_{X,Y}(t)$
- $E(XY) = E(X)E(Y)$
- $\text{Corr}(X, Y) = \text{Cov}(X, Y) = 0$

Tools

- Bayes' rule, LOTP & LOTE, LOTUS: 1D & 2D
- Indicator & linearity of expectation
- First-step analysis & recursive equations
- Conditional expectation: Adam's & Eve's law
- Generating functions: PGF & MGF
- Symmetry
  - $X+Y, XY, |X-Y|, \frac{X}{Y}, \frac{X}{X+Y}, \frac{Y}{X+Y}$
  - The property of i.i.d. continuous random variables
  - Normal distributions

Model-Based Problems

- Birthday problem: static & dynamic
- Sequence of coin tosses: biased or not
- Gambler's ruin & Random walk
- Coupon collector: given total number or not
- Pattern matching: coin or dice
- Chicken-egg problem & Poisson process
- Bank-post office

Model-Free Problems

- Computation via definitions
  - PMF, PDF, CDF, Joint distribution
  - Expectation, PGF, MGF
  - Markov chains
- Approximation
  - CLT & Law of Large Number
  - Poisson approximation & Law of Small Number
  - Non-asymptotic inequalities
- Estimation
  - MLE & MAP
  - Confidence interval
  - MMSE & LLSE

Outline

HW1

1. Counting (story proof)
2. Counting (matrix)
3. Counting (matrix)
4. Graphical proof
5. Birthday problem
6. Coupon collector

HW2

1. Bayes rule + LTOP
2. Bayes rule + LTOP, conditional independence
3. Bayes rule + LTOP, conditional independence
4. Monty Hall (graphical proof)
5. Monty Hall (conditional probability)
6. First step analysis + Bayes rule + LOTP

HW3

1. Story proof
2. First step analysis
3. First step analysis
4. Interpretation
5. Induction
6. First step analysis

HW4

1. Expectation (definition)
2. Interpretation
3. Expectation (indicator)
4. Expectation (definition)
5. Birthday problem (indicator)
6. Pattern matching + PGF
7. First step analysis

HW5

1. Distribution (definition & validation)
2. Story proof
3. Expectation (definition)
4. Interpretation + Expectation (indicator) + Symmetry
5. Expectation (definition + counting)
6. First step analysis + induction

HW6

1. Distribution + Expectation (definition)
2. Distribution + Expectation (definition + ordering)
3. Distribution (definition)
4. Distribution (definition + ordering)
5. Expectation (definition)
6. Distribution + Expectation (definition + transformation)

HW7

1. Bayes rule + LOTP
2. Chicken egg
3. Distribution (definition + Bayes rule)
4. Distribution (definition + ordering)
5. Distribution (definition)
6. Interpretation + Expectation (definition)
7. Independence (factorization)

HW8

1. Distribution (definition)
2. Distribution (definition) + Symmetry
3. MVN (definition)
4. MVN (transformation)
5. MVN (definition + transformation)

HW9

1. Distribution (transformation via Jacobian)
2. Distribution (transformation via convolution)
3. MVN (definition)
4. Order statistics (story proof)
5. Conjugacy + Indicator
6. Distributional relationship (story proof)

HW11

1. Expectation (definition) + Inequality
2. Conditional expectation (definition)
3. Conditional expectation (memorylessness)
4. Distribution + Conditional expectation (definition)
5. Inequality
6. Pattern matching (first step analysis + LOTE)

HW12

1. MLE + MAP + MMSE
2. Conditional expectation (definition)
3. Conditional expectation (definition) + Conjugacy
4. Markov chains (definition)
5. Markov chains (computation)
6. Pattern matching

A Final Note

Please, Check Out the Previous Example Papers.

Please, Check Out the Slides.

Please, Check Out the HWs.

Please, Check Out the Textbooks.