EE160 Introduction to Control: Homework 3

Prof. Boris Houska

Deadline: Mar 18, 2022

1. *Set points.* Consider the linear control system

$$\dot{x}(t) = 3x(t) + 4u(t)$$

Design an affine feedback law of the form $\mu(x) = k(x - x_{\rm s}) + u_{\rm s}$ such that the closed-loop system state converges to $x_{\rm s} = 5$. How would you choose k and $u_{\rm s}$? Write out the expression of $\dot{x}(t)$ in dependence on $(x - x_{\rm s})$?

2. *Uncertain control system with bounded noise.* Consider a linear control system with time-varying bounded noise $w(t)$, $|w(t)| \leq \bar{w}$

$$\begin{aligned}\dot{x}(t) &= ax(t) + bu(t) + cw(t)\\ x(0) &= x_0 + w_0\end{aligned}$$

Show that if we apply suitable feedback law $\mu(x) = k(x - x_{\rm s}) + u_{\rm s}$, the trajectory of $x(t)$ is bounded.

3. *Proportional control of an RC-circuit.* Let us revisit the standard RC-circuit with input voltage $V(t)$ recalling that the voltage $V_C(t)$ at the capacitor satisfies

$$\dot{V}_C(t) = \frac{V(t) - V_C(t)}{RC}$$

Assume that $R = 10\Omega$ and $C = 10^{-3}$F. Find a steady-state voltage u_s such that $V_C(t)$ converges to 10V for $t \to \infty$. Also discuss how to design a proportional feedback law, which ensures that $V_C(t)$ converges to 10V as quickly as possible while satisfying the control input constraints

$$-220V \leq V(t) \leq 220V.$$