#### Homework 8

Due data: Jun.15  $^{th}\,$ 

Turn in your homework before  $8{:}15~\mathrm{AM}$ 

#### Rules:

- Please work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism!
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

[8 points] Calculate the Laplace transform of the following functions. You can use definitions or properties of Laplace transform. Hint: $cosh(t) = \frac{e^{t} + e^{-t}}{2}$ (a).  $f(t) = ((1 - t)e^{-t} - te^{-t}sint)u(t)$ (b).  $g(t) = 8e^{-3t}cosh(t)u(t - 2)$ 

(a). 
$$f(t)=((1-t)e^{-t}-te^{-t}sint)u(t)$$

(b). 
$$g(t)=8e^{-3t}cosh(t)u(t-2)$$

# $\mathbf{2}$

[12 points] Calculate the inverse Laplace transform of the following functions. (a).  $F(s) = \frac{s^2 - 2s + 1}{4(s - 2)(s^2 + 2s + 4)}$  (b).  $G(s) = \frac{4(s - 1)}{s^4 - 1} + \frac{7}{s(s + 1)^3} + 1$  (c).  $P(s) = \frac{se^{-\pi s}}{s^2 + 5}$ 

(a). 
$$F(s) = \frac{s^2 - 2s + 1}{4(s - 2)(s^2 + 2s + 4)}$$

(b). 
$$G(s) = \frac{4(s-1)}{s^4-1} + \frac{7}{s(s+1)^3} + 1$$

(c). 
$$P(s) = \frac{se^{-\pi s}}{s^2+5}$$

[18 points] The function f(t) is shown in Fig.3. f(t)=0 for  $t\leq 0$ .

- (a). Calculate the Laplace transform of f(t).
- (b). Verify the initial value thereom for f(t).
- (c). Verify the final value theorem for f(t).

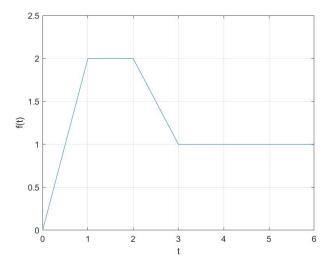


Figure 3:

[18 points] The circuit is shown in **Fig.4**. Assume the circuit has reached steady state before t=0s. Given that  $C=\frac{1}{9}F$ ,  $R=3\Omega$ , L=0.6H,  $i(t)=te^{-t}u(t)+5u(-t)$  A, determine U1(t) for t>0s using Laplace domain method.

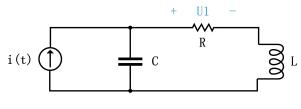


Figure 4:

[18 points] The circuit is shown in **Fig.5**. The ideal operator amplifier is working at its linear region and  $v_1(0-) = 3V$ ,  $v_2(0-) = 2V$ . And a voltage source is applied to the circuit with  $v_s(t) = e^{-t}u(t)V$ . Determine the  $v_o(t)$  for t>0 s with Laplace transform.

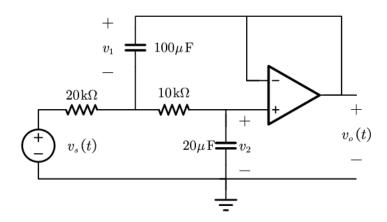


Figure 5:

[26 points] The circuit is shown in **Fig.6**. Given that  $U_c(0)=1V$ ,  $i_c(0)=2A$ .

- (a). Use the time domain method to determine  $i_L(t)$  for t > 0s.
- (b). Use the Laplace transform to determine  $i_L(t)$  for t > 0s.
- (c). Use the phasor domain method to determine i<sub>L</sub>(t) if the circuit has reached the sinusoidal steady state.
- (d). Determine the value of  $i_L(t)$  at t = 1s and t = 100s using the three expressions you obtained above respectively. Explain why the values are the same or different.

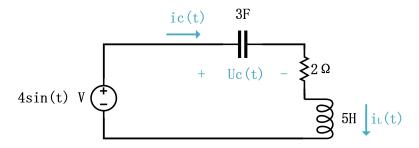


Figure 6: