CS 101 Fall 2022 - Quiz 10-A

12/12/2022 - 20 Minutes **Name:** **ID number:**

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1. (4 points) True or False

The following questions are True or False questions, you should judge whether each statement is true or false. You should write down your answers in the box below.

(a)	(b)	(c)	(d)
T	F	T	T

(a) (1') Since the solutions of DP problems depend on other subproblems, one need to at least give out the solutions of some subproblems to solve the problem.

(b) (1') We could always use recursion in DP and hence do not need to allocate extra space to store solutions of subproblems, which could save the memory use.

(c) (1') Even when the number of different states is exactly n^3, in some case, we may only use $O(n^2)$ extra space when running our DP algorithm.

(d) (1') When the amount of items in knapsack problem can be any positive rational number, we prefer to use Greedy because although DP is available Greedy is more time and space efficient.

2. (8 points) Increasing Subsequence

An increasing subsequence is a sequence $a_{i_1}, a_{i_1}...a_{i_k}$ where $i_j < i_{j+1}$ and $a_{i_j} < a_{i_j+1}$. Given a sequence $a_1, a_2...a_n$, we want to design a DP algorithm to find its Longest Increasing Subsequence.

Note: No need for explanation or proof

(a) (2') Define your subproblems.

Solution: L(i) =the length of the longest increasing subsequence ending at i.

(b) (4') Give your Bellman equation to solve the subproblems.

Solution:

$$L(i) = \begin{cases} 1 & i = 1 \\ \max_{j<i, a_j<a_i} L(j) + 1 & otherwise \end{cases}$$

(c) (1') What is the answer to this question in terms of the subproblems?

Solution: L(n)

(d) (1') How many subproblems are there?

Solution: n subproblems: since $0 \leq i \leq n$.