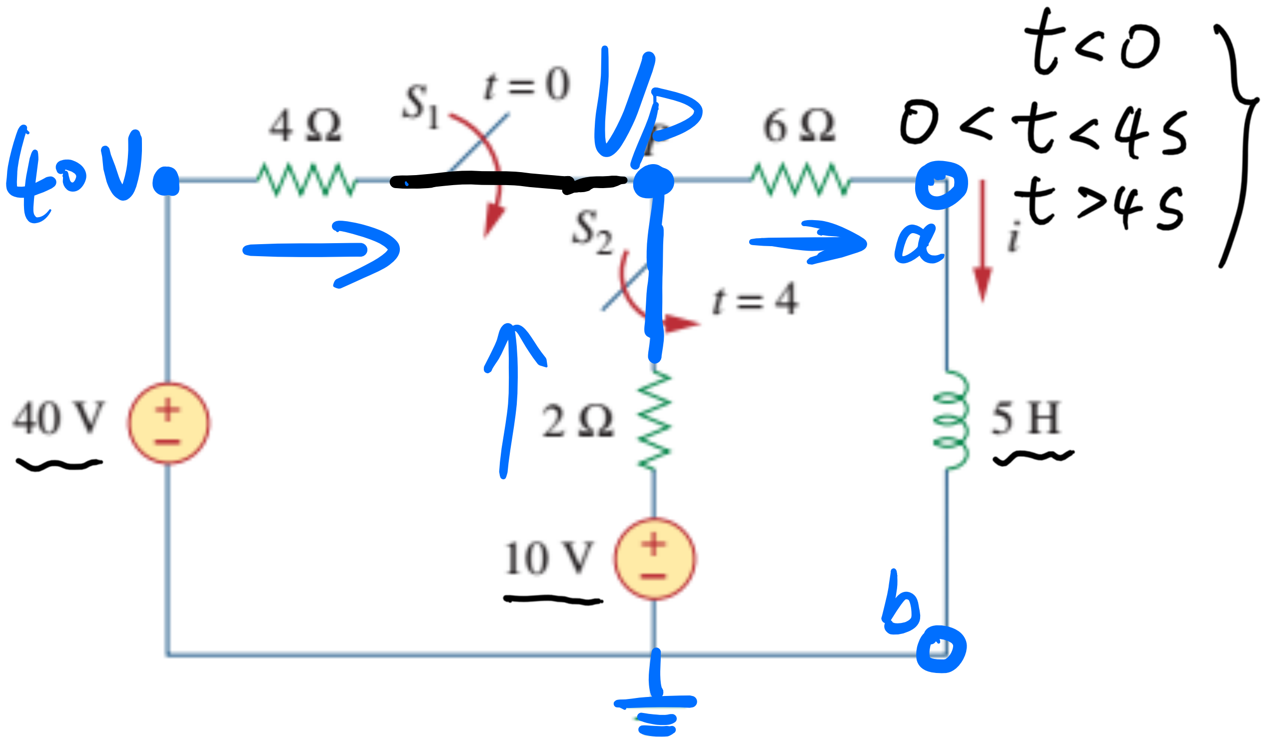
Sequential switch

At $t = 0$, switch 1 in Fig. 7.53 is closed, and switch 2 is closed 4 s later. Find $i(t)$ for $t > 0$. Calculate i for $t = 2$ s and $t = 5$ s.



We need to consider the three time intervals $t \leq 0$, $0 \leq t \leq 4$, and $t \geq 4$ separately. For $t < 0$, switches S_1 and S_2 are open so that $i = 0$. Since the inductor current cannot change instantly,

$$i(0^-) = i(0) = i(0^+) = 0$$

$$i(\infty): \frac{40-V_p}{4} + \frac{10-V_p}{2} = \frac{V_p - 0}{6}$$

$$\Rightarrow V_p = \frac{180}{11} V, \quad i(\infty) = \frac{V_p}{6} = \frac{30}{11} A$$

$$R_{TH} = (4 \parallel 2) + 6 = \frac{22}{3} \Omega$$

$t<0$: $i_L(0^+) = i_L(0^-) = 0$

$0<t<4s$:

$$i_L(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

$$i(\infty) = \frac{40}{4+6} = 4A, \quad \tau = \frac{L}{R} = \frac{1}{2}s$$

$$i_L(t)\Big|_{0<t<4s} = 4 + (-4)e^{-2t} \ A.$$

$t>4s$:

$$i_L(t)\Big|_{t>4} = i(\infty) + [i(4s) - i(\infty)] \cdot e^{-(t-4)/\tau}$$

$$i(4s^+) = i(4s^-) = 4 + (-4)\cdot e^{-8} \simeq 4A$$

$$i(\infty) = \frac{30}{11} A.$$

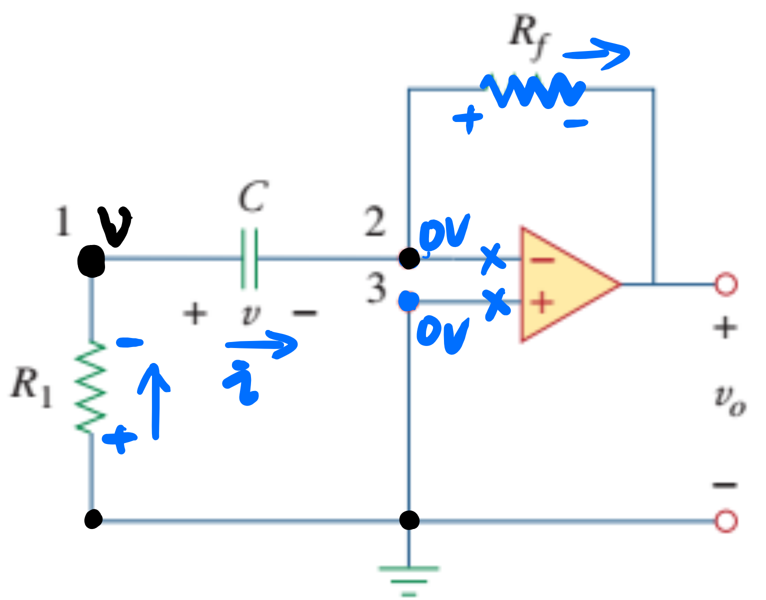
$$\tau = L/R_{TH} = 5/22 = \frac{15}{22} s$$

$t>4s$

$$i_L(t) = 2.73 + 1.27 e^{-1.47(t-4)}, \quad t \geq 4$$

First order op-amp circuit

For the op amp circuit in Fig. 7.55(a), find v_o for $t > 0$, given that $v(0) = 3$ V. Let $R_f = 80$ kΩ, $R_1 = 20$ kΩ, and $C = 5$ μF.



(a)

$V(0) = 3V$

NI:

$$\frac{0 - v}{R_1} = C \cdot \frac{dv}{dt} \Rightarrow \frac{dv}{dt} + \frac{1}{R_1 C} \cdot v = 0$$

$$V(t) = A \cdot e^{-\frac{1}{R_1 C} \cdot t}$$

$\therefore \; A = 3 \quad \therefore \; V(t) = 3 \cdot e^{-t/R_1 C}$

N2: $$C \cdot \frac{dV}{dt} = \frac{0 - V_o}{R_f}$$

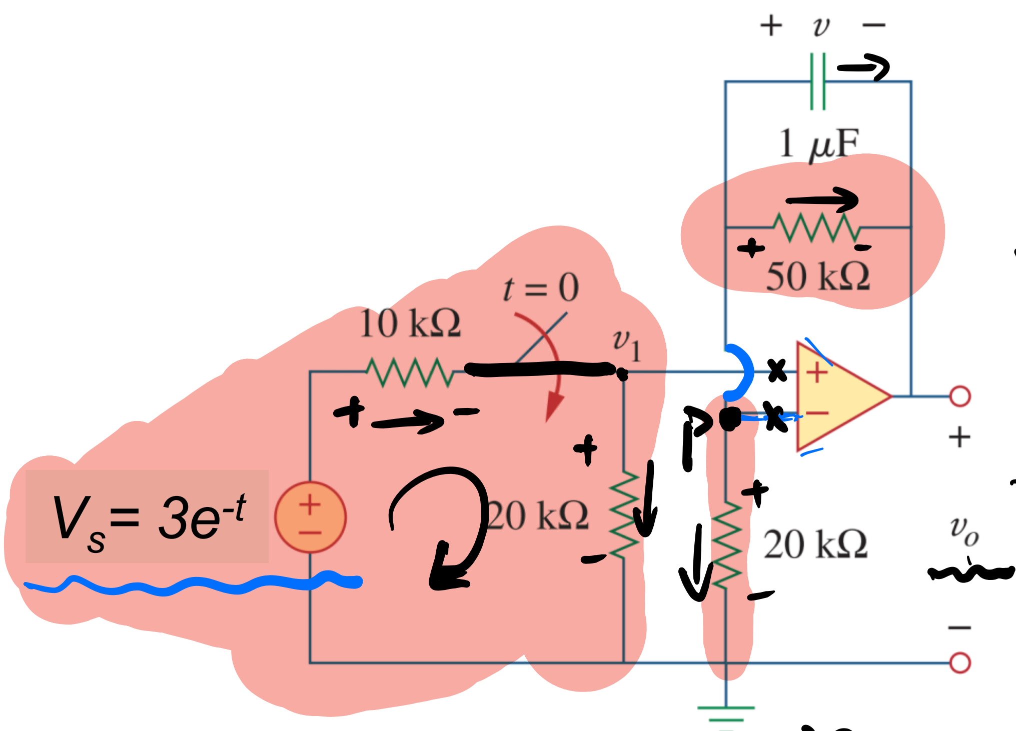
$$\therefore \; V_o = -R_f \cdot C \cdot \frac{dV}{dt}$$

$$= -R_f \cdot \cancel{C} \cdot 3 \cdot -\frac{1}{R_1 \cancel{C}} \cdot e^{-t/R_1 C}$$

$$= \frac{R_f}{R_1} \cdot 3 \cdot e^{-t/R_1 C}$$

$$= 12 \cdot e^{-10t} \; (V), \; t > 0$$

find $v(t)$ and $V_o(t)$



$t<0$

$V(0^+) = V(0^-) = 0V$

$V_o = V_p - V$

$t=0$, $V_1 = \frac{20}{10+20} \times 3 = 2V$

$V_p = V_1 = 2V$

Node P → KCL: $C \cdot \frac{dv}{dt} + \frac{v}{50k} + \frac{2}{20k} = 0$

$\underline{t>0}$ $V_s = 3 \cdot e^{-t}$, $V_1 = \frac{20}{10+20} \cdot 3 \cdot e^{-t} = 2 \cdot e^{-t}$

$V_p = V_1 = \underline{2 \cdot e^{-t}}$

$P \rightarrow$ KCL: $C \cdot \frac{dv}{dt} + \frac{v}{50k} + \frac{2 \cdot e^{-t}}{20k} = 0$

$$\frac{dv}{dt} + \frac{v}{C \cdot 50k} = - \frac{2 \cdot e^{-t}}{C \cdot 20k} \quad ①$$

$$\frac{1}{C \cdot 50k} = 20$$

G.S. $\underline{v' = A \cdot e^{-20t}}$

P.S. $v'' = \underline{B} \cdot e^{-t}$

$$-B \cdot \cancel{e^{-t}} + \frac{B \cdot \cancel{e^{-t}}}{C \cdot 50k} = - \frac{2 \cdot \cancel{e^{-t}}}{C \cdot 20k}$$

$\therefore$ $B = -\frac{100}{19}$

$$\therefore V'' = -\frac{100}{19} \cdot e^{-t}$$

$$V(t) = V' + V''$$

$$= A \cdot e^{-20t} - \frac{100}{19} e^{-t}$$

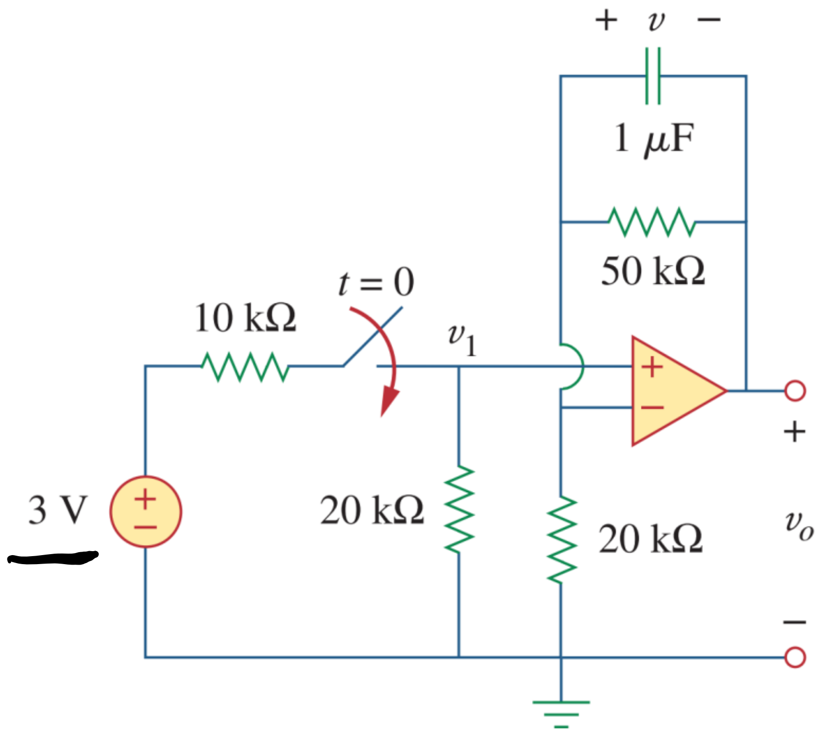
$$V(0^+) = V(0^-)$$

$$A: \quad A = \frac{100}{19}$$

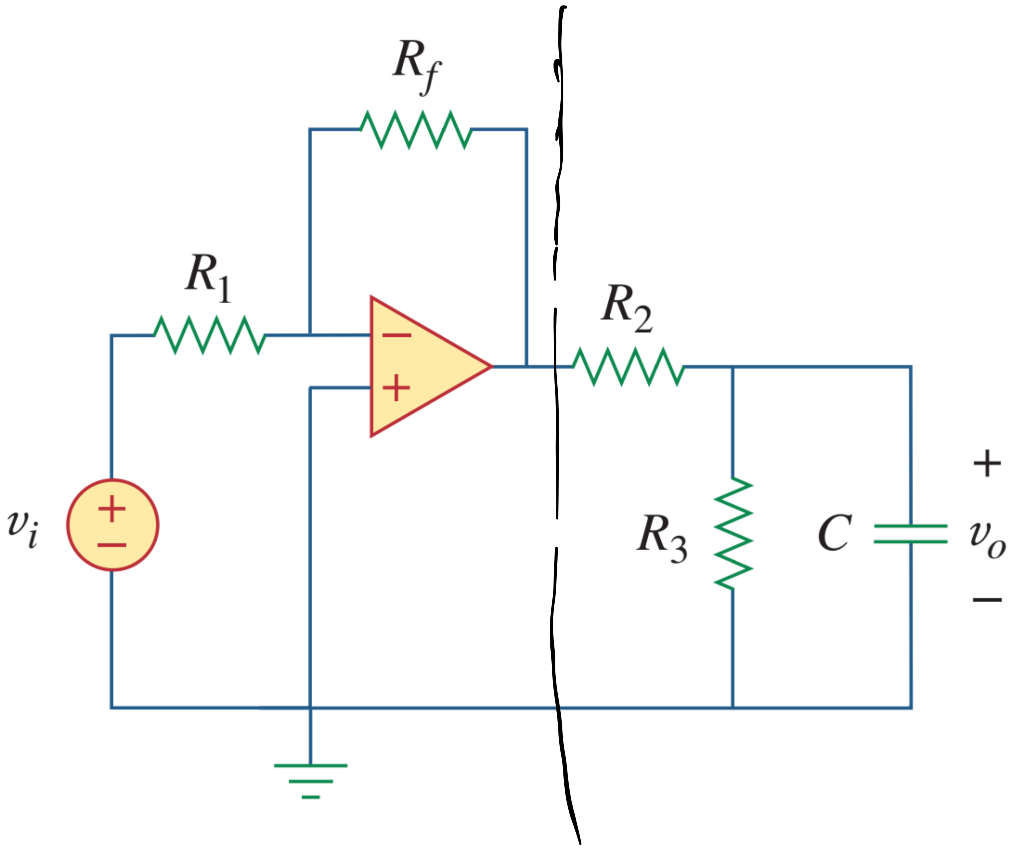
$$V(t) = \frac{100}{19} \cdot e^{-20t} - \frac{100}{19} e^{-t} \quad ②$$

$$V_o(t) = V_p - V(t) = 2 \cdot e^{-t} -$$

$$=$$


+ v −
1 μF
50 kΩ
$t = 0$
10 kΩ
v_1
+
−
3 V
20 kΩ
20 kΩ
+
v_o
−

Find the step response $v_o(t)$ for $t > 0$ in the op amp circuit of Fig. 7.59. Let $v_i = 2u(t)$ V, $R_1 = 20$ kΩ, $R_f = 50$ kΩ, $R_2 = R_3 = 10$ kΩ, $C = 2\ \mu$F.



Practice