Homework 3

Submit: Blackboard/Paper Due: Nov. 7th

Please write down Your Name & Student ID

1. In a nuclear medicine scan using $^{99m}Tc(\lambda = 3.22 * 10^{-5} s^{-1})$, the image SNR for a 30-minute scan was 25:1 for an injected radioactive dose of 1 mCi. Imaging began immediately after injection.
(a) If the injected dose were increased to 5 mCi, what would be the image SNR for a 30-minute scan?
(b) If the scan time were doubled to 60 minutes with an initial dose of 1 mCi, what would be the image SNR compared to that of 30 minutes?

Solution: (14 points = 7 + 7)

(a) Since the SNR is proportional to the square root of the number of counts, tripling the injected dose increases the SNR by the square root of 5 to give a value of 55.9/$25\sqrt{5}$:1.

(b) After 30 minutes the number of nuclei is reduced to ($e^{-3.22x10^{-5}*30*60}$)= 94.4% of the original number. After 60 minutes, the number is reduced to 89.1%. The total number of disintegrations during the second 30 minutes is approximately 94% (89.1/94.4) more than that of the first 30 minutes. Therefore, the S/N is 25*$\sqrt{1.94}$=34.8:1.

2. A dose of 1 mCi of ^{99m}Tc (Half-Life: 6 hours) is administered to a patient at 9 am. Calculate the total dose to the patient at 12 am on the same day if the biological half-life of the radiotracer in the body is:

(a) 4 years,

(b) 6 hours,

(c) 2 minutes.

Solution: (15 points = 5*3)

Using equation

$$\tau_{1/2,\mathrm{eff}} = \frac{\tau_{1/2}\tau_{1/2,bio}}{\tau_{1/2} + \tau_{1/2,bio}}$$

Therefore, the effective half-lives are 6 hours, 3 hours and 2 minutes, respectively, which gives λ_{eff}=3.2x10^{-5}, 6.4x10^{-5} and 5.8x10^{-3} s^{-1}.

If you think of dose as N, then:

The overall amount of radioactivity is given by the time integral:

$$\int_{\mathrm{t=9am}}^{\mathrm{t=12am}} Qdt = \lambda_{\mathrm{eff}}\mathrm{N}_0 \int_{\mathrm{t=9am}}^{\mathrm{t=12am}} e^{-\lambda t}dt = \frac{\mathrm{Q}_0}{\lambda_{\mathrm{eff}}}[1 - \mathrm{e}^{-\lambda_{\mathrm{eff}}*3\mathrm{hrs}}]$$

Take λ_{eff} into this equation, we get:

(a) $9.131 * 10^3 mCi * s$

(b) $7.797 * 10^3 mCi * s$

(c) $172 mCi * s$

If you think of dose as Q, then:

$$Q = \lambda N = \lambda N_0 e^{-\lambda t} = Q_0 e^{-\lambda t}$$

Then:

(a) $0.708 mCi$

(b) $0.501 mCi$

(c) 0mCi

3. In the technetium generator, show mathematically that if $\lambda_2 \gg \lambda_1$, the radio activities of the parent and daughter nuclei become equal in value at long times.

Solution: (12 points)

Since $\lambda_2 \gg \lambda_1$, Equation (3.11)

$$Q_2 = \frac{\lambda_1 \lambda_2 N_0}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

can be rewritten as

$$Q_2 \xrightarrow{\lambda_2 \gg \lambda_1} \lambda_1 N_0\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) \xrightarrow{\lambda_2 \gg \lambda_1} \lambda_1 N_0 e^{-\lambda_1 t} = \lambda_1 N_1 = Q_1$$

4. (i) The thickness of the lead septa is chosen to ensure that only 5% of the γ-rays penetrate from one collimator hole to the adjacent one. Suppose the linear attenuation coefficient of lead septa is μ, using Figure 4 show that the thickness is given by $[6d/\mu]/[L-3/\mu]$ with *appropriate approximation*.

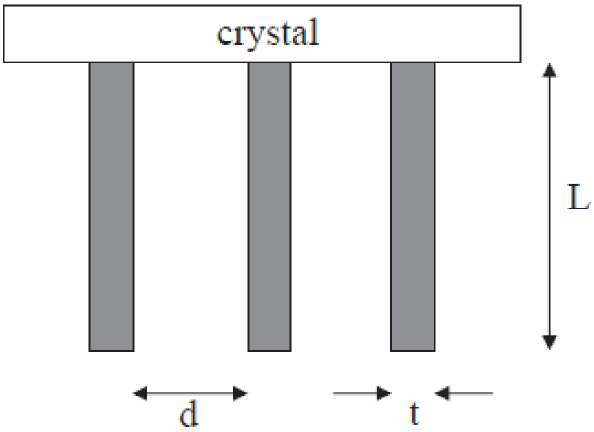


Figure 4

Solution. (18 points = 9 + 9)

From the figure below, the minimum path distance (x) for a γ-ray to pass through the collimator and be detected is related to l, t and d by (*Considering in tiny triangles, hypotenus (斜边) can be approximated by the longer right-angle side (直角边)*):

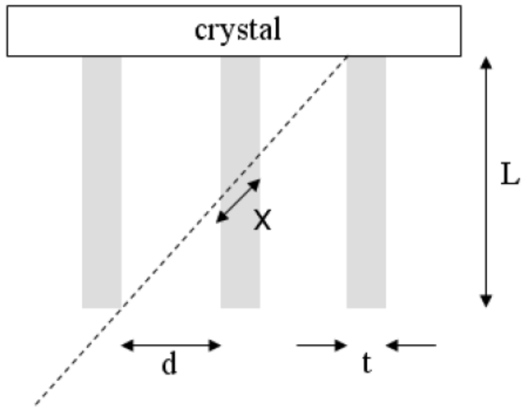
$$x = \frac{tL}{2d + t}$$

Since only 5% of the γ-rays can penetrate:

$$e^{-\mu x} = 0.05 \rightarrow x = \frac{3}{\mu}$$

Rearranging the above two equations gives:

$$t = \frac{\frac{6d}{\mu}}{L - \frac{3}{\mu}}$$

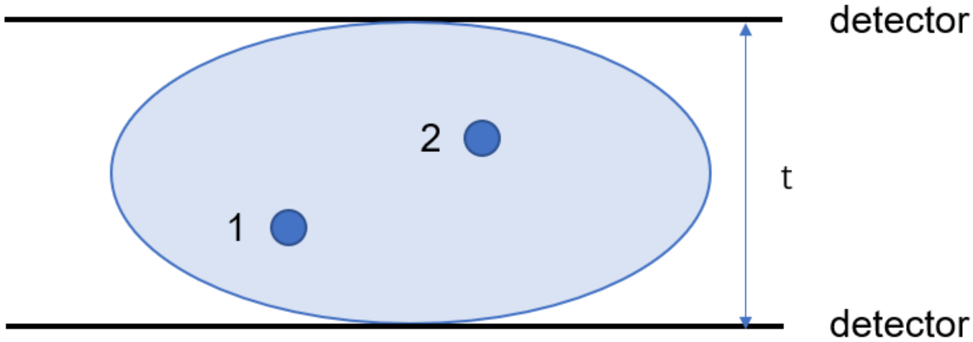


(ii) Calculate the septal thickness required for γ-rays of 140 keV for lead collimators with a hole diameter of 0.1 cm and a length of 2.5 cm. The attenuation coefficient for lead is 30 cm^{-1} at 140 keV.

Solution:

$$t = \frac{6\left(\frac{0.1}{30}\right)}{2.5 - \frac{3}{30}} = 0.008cm$$

5. Isosensitive imaging is a technique that acquires nuclear medicine scans from opposite sides of the patient, and then combines the signals to remove the depth dependence of the signal intensity. By considering the attenuation of γ-rays in the patient, show how this technique works, and what mathematical processing of the two scans is necessary.



Solution: (15 points)

Suppose that there are two sources of radioactivity, labeled 1 and 2 in the figure below. If only one scan is taken, shown on the left, then the intensity of γ-rays detected are:

$$I_1 = I_{1,0}e^{-\mu x_1}, I_2 = I_{2,0}e^{-\mu x_2}$$

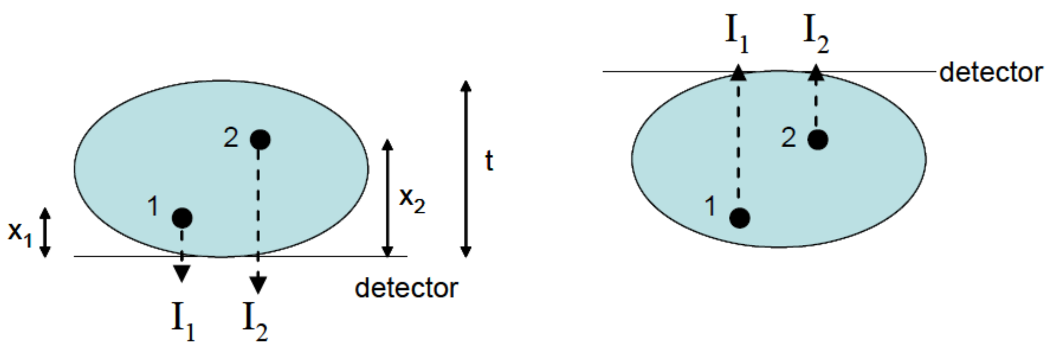
If we denote the thickness of the body as t, then the intensities from the second scan shown on the right:

$$I_1 = I_{1,0}e^{-\mu(t-x_1)}, I_2 = I_{2,0}e^{-\mu(t-x_2)}$$

If the signals are multiplied together, and the square root of the product taken, to give I1' and I2' then these have values:

$$I_1' = I_{1,0}e^{-\mu t/2}, I_2' = I_{2,0}e^{-\mu t/2}$$

This shows that the depth-dependence of the signals has been eliminated



6. What timing resolution would be necessary to obtain a position resolution of 10 mm in TOF PET based only upon time-of-flight considerations?

Solution. (10 points)

Applying equation

$$\Delta x = \frac{c\Delta t}{2} \rightarrow 0.01 = \frac{3 * 10^{8}\Delta t}{2} \rightarrow \Delta t = 67\text{ps}$$

This is well below the resolving power of any current PET detectors.

7. Suggest why a PET/CT scanner operating in 2D mode has a relatively uniform axial sensitivity profile, whereas in 3D mode the sensitivity is much higher at the center of the scanner.

Solution:(16 points)

In two-dimensional mode (with the septa in place) the PET signal at each crystal ring comes only from a very well-defined region in the z-direction which is adjacent to that ring. In three-dimensional mode, on the other hand, as shown in Figure 7, the PET signal detected by each crystal ring can originate from anywhere within the body. Since the central ring in the z-direction detects γ-rays that have been attenuated in the body to a lower degree than for the outer rings, due to a shorter pathway through tissue, the sensitivity will be higher. The central ring also receives signal from tissue lying either side of it, whereas the outer rings only receive signal from one side.

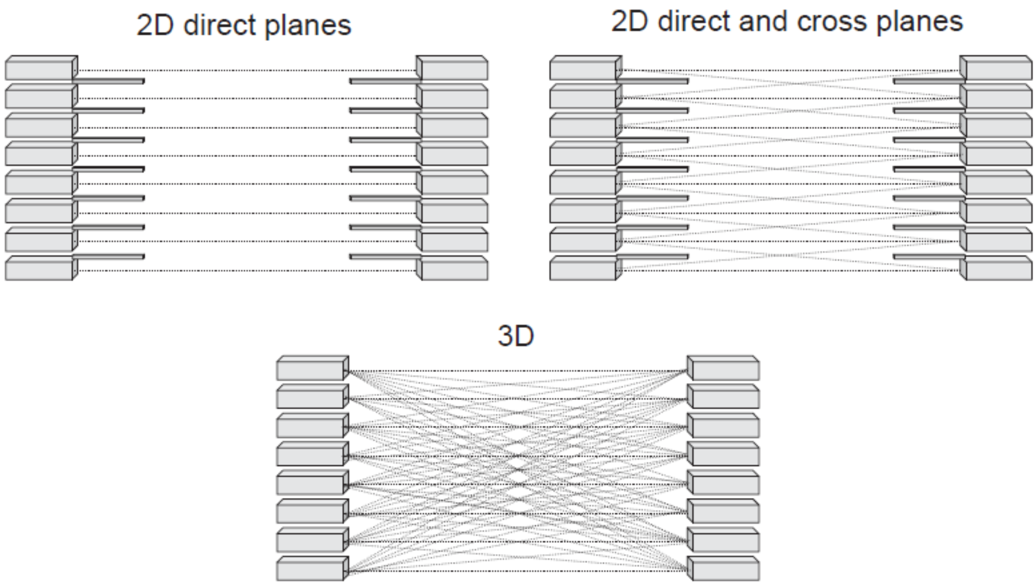


Figure 7