EE160 Homework 4

Deadline: 2022-12-24, 23:59:59, Submit your homework on Blackboard
(Hint:You can use MATLAB to help you do the homework.)

1. A block diagram of a turret lathe control system is shown in Figure 1. The parameters in the system are $n = 0.2$, $J = 10^{-3}$ and $b = 2.0 \times 10^{-2}$. It is necessary to attain an accuracy of 4.7×10^{-4} inches. To satisfy this condition, a steady-state position accuracy of 2% is specified for a ramp input. Design a cascade compensator to be inserted before the controller in order to provide a response to a step command with a percent overshoot of $P.O \leq 4.5\%$. A suitable damping ratio for this system is $\zeta \geq 0.7$. The gain of the controller is $K_R = 5$. Design a suitable phase-lag compensator with the following two methods.

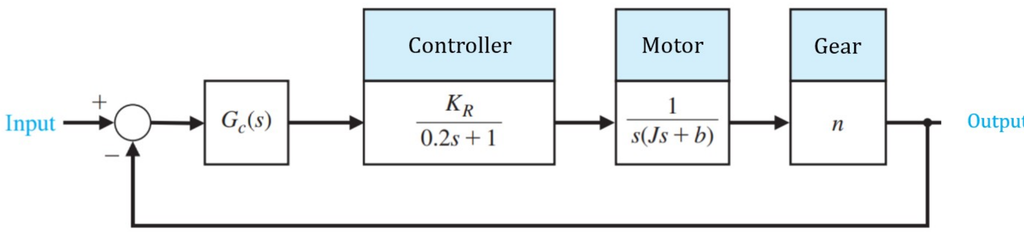

Figure 1: A feedback control system

   (a) Use bode plot.(10')
   (b) Use root locus.(10')

2. Consider a unity feedback system in Figure 2. We want the step response of the system to have a percent overshoot of $P.O. \leq 9\%$ and a settling time (with a 2% criterion) of $T_s \leq 4$ s.

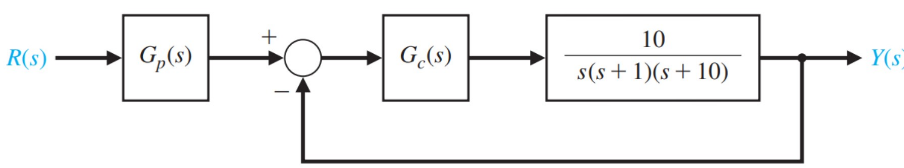

Figure 2: A feedback control system

   (a) Design a phase-lead compensator $G_c(s)$ to achieve the dominant roots desired.(15')
   (b) If we add a PD controller between the system and the compensator, and change the system to $\frac{1}{s(s+1)(s+2)}$. Design a first-order compensator($\frac{s+z}{s+p}$) and a first-order prefilter($\frac{z}{s+z}$), and determine the coefficients that yield the optimal deadbeat response.(15')

3. Prove the following propositions:

   (a) If two state-space models share the same controllable canonical form, the two models are consistent in controllability. (5')

(b) Consider a general system

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned} \tag{1}$$

Define the dual system of (1)

$$\begin{aligned}\dot{x} &= A^\top x + C^\top u \\ y &= B^\top x\end{aligned} \tag{2}$$

Show that system (1) is observable if and only if system (2) is controllable. (5')

4. Consider the third-order system

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -3 & -1 \end{bmatrix} \quad B = \begin{bmatrix} -1 \\ 2 \\ -6 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 8 & 10 \end{bmatrix}$$

(a) Sketch a block diagram model of the system. (5')

(b) Write the transfer function of the system $G(s) = Y(s)/U(s)$. (5')

(c) Check the controllability and observability of the system. (5')

(d) Design a full-state observer for the system with an expected settling time of less than 1 second. (5')

(e) Suppose system state $x(t)$ is available. Design a full-state feedback controller for the system. The desired poles of the closed-loop system are $\begin{bmatrix} -4 + j3 & -4 - j3 & -8 \end{bmatrix}$. (7')

(f) Prove that if (A, B) are controllable, (A, C) are observable, the closed-loop system with full-state observer-based feedback controller is stable. Verify the proposition with the control scheme designed above. (8')

(g) Consider a piece-wise constant reference signal $r(t)$. Design a compensator such that the tracking error $y(t) - r(t)$ asymptotically converges to zero. (5')

5. Consider a LRC circuit with input voltage $v_i(t)$ and output voltage $v_o(t)$. The system model is given as

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{LCs^2 + RCs + 1}$$

where $L = 0.1$, $R = 0.5$, $C = 20$.

(a) Write the above system into the state space model

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t)\end{aligned}$$

where $x(t) = [v_o(t)\ \dot{v}_o(t)]^\top$ (3')

(b) Suppose input $u(t)$ is unknown. Design a system state observer $\hat{x}(t)$ such that the amplitude of the frequency response of observation error $e(t) = C\hat{x}(t) - y(t)$ is smaller than -15 dB. (7')

(c) Design the infinite LQR controller law $u(t) = -Kx(t)$ that minimizes the infinite horizon cost

$$\int_0^\infty 3v_o^2(t) + \dot{v}_o^2 + v_i^2(t)dt$$

Write the corresponding optimal control problem in standard form, indicate Q matrix and R matrix, then solve the control gain K explicitly. (5')

(d) Compare the performance and control effort of (c) and $K = [-0.5 \quad -2]$ using MATLAB. The initial state is given as $x_0 = [3, 2]^\top$, simulation time $T = 100s$. (5')