SI231b - Matrix Computations, 2020-21 Spring

Homework Set #2

Prof. Ziping Zhao

Acknowledgements:

1) Deadline: **2021-03-28 23:59:59**
2) Submit your homework in pdf format to Email: zhangzp1@shanghaitech.edu.cn.
3) You can write your homework using latex/word or you can write in handwriting and submit the scanned pdf.
4) You need submit the code of Problem 2 as well.

Problem 1. (20 points) For $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 7 & 11 \\ 4 & 10 & 19 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ -i \\ 3-5i \end{bmatrix}$, solve $\mathbf{A}\mathbf{x} = \mathbf{b}$ with LU decompisition.

Problem 2. (20 points) Write MATLAB codes to calculate LU decomposition and LDL decomposition of $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 6 & 10 \\ 3 & 10 & 20 \end{bmatrix}$, then discuss the complexity of these two approaches. (built-in function like lu can not be used.)

Problem 3. (20 points) Prove that if $\mathbf{A}$ is a positive definite matrix if and only if its Cholesky decomposition exists.

Problem 4. (20 points) Prove that if $\mathbf{A}$ is a positive definite matrix then its Cholesky decomposition is uniquely determined.

Problem 5. (20 points) For $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 3 & 7 & 11 & 5 \\ 4 & 10 & 19 & 11 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}$, solve $\mathbf{A}\mathbf{x} = \mathbf{b}$ with rectangular LU decompisition.