CS 101 Fall 2022 - Quiz 10-B

13/12/2022 - 20 Minutes **Name:** **ID number:**

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1. (4 points) True or False

The following questions are True or False questions, you should judge whether each statement is true or false. You should write down your answers in the box below.

(a)	(b)	(c)	(d)
T	T	F	F

(a) (1') Since the solutions of DP problems depend on other subproblems, one need to at least give out the solutions of some subproblems to solve the problem.

(b) (1') Relatively to recursion, using DP in computing Fibonacci sequence is more time efficient.

(c) (1') When the number of different states(subproblems) is $\Theta(n^2)$, then the space complexity is sure to be $\Theta(n^2)$ when running our DP algorithm.

(d) (1') When the weight of items knapsack problem can be any positive rational number, we can use Greedy to find the optimal solution since it is more time efficient.

2. (8 points) Different Trees

We have counted distinct shapes of binary trees with n nodes in Homework5. Now let's review this question in a dynamic programming view. Slightly more formally, we denote the number of distinct shapes of 0 node tree as **1**.

Note: No need for explanation or proof

(a) (2') Define your subproblems.

Solution:

$Cnt(i)$ =the number of distinct shapes of binary trees with i nodes.

(b) (4') Give your Bellman equation to solve the subproblems.

Solution:

$$Cnt(i) = \begin{cases} 1 & i = 0 \\ \sum_{k=1}^{i} Cnt(k-1)Cnt(i-1-k) & otherwise \end{cases}$$

(c) (1') What is the answer to this question in terms of the subproblems?

Solution: $\mathtt{Cnt}(\mathtt{n})$

(d) (1') How many subproblems are there?

Solution: n subproblems: since $0 \leq i \leq n$.