SI231b - Matrix Computations, 2020-21 Spring

Homework Set #3

Prof. Ziping Zhao

Acknowledgements:

1) Deadline: **2021-04-14 23:59:59**
2) Submit your homework in pdf format to Email: zhangzp1@shanghaitech.edu.cn.
3) You can write your homework using LaTeX/Word, or you can write in handwriting and submit the scanned pdf.

Problem 1. (20 points) Prove that a Vandermonde matrix has full rank if its roots are distinct.

Problem 2. (20 points) For $\mathbf{A} \in \mathbb{R}^{m \times n}$, $\mathbf{y}, \mathbf{b} \in \mathbb{R}^m$, and $\lambda \in \mathbb{R}^+$, derive the optimal solution of

$$\underset{\mathbf{x} \in \mathbb{R}^n}{\text{minimize}} \quad \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 + \lambda \|\mathbf{b} - \mathbf{x}\|_2^2.$$

Problem 3. (20 points) For a full-column rank matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$, prove that the Gram matrix $\mathbf{A}^T\mathbf{A}$ is nonsingular.

Problem 4. (20 points) For $\mathbf{A} \in \mathbb{R}^{m \times n}$, prove that $\mathbf{A}^T\mathbf{A}$ and $\mathbf{A}^T$ have the same range space.

Problem 5. (20 points) Given the orthogonal projector of $\mathbf{A} \in \mathbb{R}^{m \times n}$ as $\mathbf{P}_{\mathbf{A}} = \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T$, show that $\mathbf{I} - \mathbf{P}_{\mathbf{A}}$ is also a orthogonal projector and state which subspace $\mathbf{I} - \mathbf{P}_{\mathbf{A}}$ projects onto.