Numerical Optimization, 2022 Fall
Homework 1 Solution

1 AMPL实现

考虑如下 Haverly pooling 问题, 如图 1 示, 请使用AMPL实现并求解.

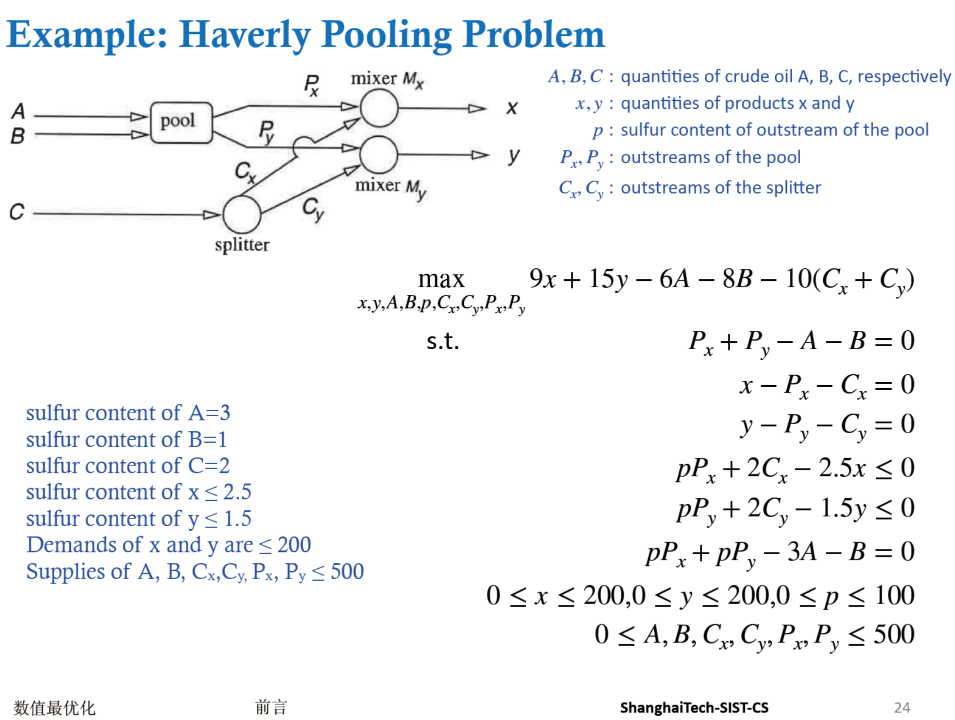

图 1: Example: Haverly Pooling Problem.

```
var x >= 0, <= 200;
var y >= 0, <= 200;
var p >= 0, <= 100;
```

```
var A >= 0, <= 500;
var B >= 0, <= 500;
var Cx >= 0, <= 500;
var Cy >= 0, <= 500;
var Px >= 0, <= 500;
var Py >= 0, <= 500;

maximize res: 9 * x + 15 * y - 6 * A - 8 * B - 10 * (Cx + Cy);

subject to cons1: Px + Py - A - B = 0;
subject to cons2: x - Px - Cx = 0;
subject to cons3: y - Py - Cy = 0;
subject to cons4: p * Px + 2 * Cx - 2.5 * x <= 0;
subject to cons5: p * Py + 2 * Cy - 1.5 * y <= 0;
subject to cons6: p * Px + p * Py - 3 * A - B = 0;
```

Command:

```
solve;
display A, B, x, y, p, Px, Py, Cx, Cy;
```

Result by BARON:

```
Objective 1800
A = 100
B = 300
x = 200
y = 200
p = 1.5
Px = 200
Py = 200
Cx = 0
Cy = 0
```

Alternatively, if you specify initial parameters and/or use a different solver, you may obtain different solutions. For example,

```
var x := 100;
var y := 0;
var p := 7;
var A := 75;
```

```
var B := 25;
var Cx := 500;
var Cy := 11;
var Px := 0;
var Py := 1;

maximize res: 9 * x + 15 * y - 6 * A - 8 * B - 10 * (Cx + Cy);

subject to cons1: Px + Py - A - B = 0;
subject to cons2: x - Px - Cx = 0;
subject to cons3: y - Py - Cy = 0;
subject to cons4: p * Px + 2 * Cx - 2.5 * x <= 0;
subject to cons5: p * Py + 2 * Cy - 1.5 * y <= 0;
subject to cons6: p * Px + p * Py - 3 * A - B = 0;

subject to cons7: 0 <= x <= 200;
subject to cons8: 0 <= y <= 200;
subject to cons9: 0 <= p <= 100;
subject to cons10: 0 <= A <= 500;
subject to cons11: 0 <= B <= 500;
subject to cons12: 0 <= Cx <= 500;
subject to cons13: 0 <= Cy <= 500;
subject to cons14: 0 <= Px <= 500;
subject to cons15: 0 <= Py <= 500;
```

Command:

```
solve;
display A, B, x, y, p, Px, Py, Cx, Cy;
```

Result by LGO:

```
Objective 1600.044484
A = 66.666
B = 266.69
x = 200
y = 200
p = 1.39997
Px = 133.356
Py = 200
Cx = 66.6438
Cy = 0
```

P2

(1) Equivalent to the LP

$$\begin{array}{cc} \min & 1^T s \\ \text{subject to} & Ax - b \leq s \\ & Ax - b \geq -s \end{array}$$

Assume x is fixed in this problem, and we optimize only over s. The constraints say that

$$-s_k \leq a_k^T x - bk \leq s_k$$

for each k, i.e., $sk \geq |a_k^T x - b_k|$. The objective function of the LP is separable, so we achieve the optimum over s by choosing

$$s_k = |a_k^T x - b_k|$$

and obtain the optimal value $p(x) = \| Ax - b \|_1$. Therefore optimizing over t and s simultaneously is equivalent to the original problem

(2) Equivalent to the LP

$$\begin{array}{cc} \min & t \\ \text{subject to} & Ax - b \leq t\mathbf{1} \\ & Ax - b \geq -t\mathbf{1} \end{array}$$

Assume x is fixed in this problem, and we optimize only over t. The constraints say that

$$-t \leq a_k^T x - bk \leq t$$

for each k, i.e., $t \geq |a_k^T x - b_k|$.

i.e.

$$t \geq \max_k |a_k^T x - b_k| = ||Ax - b||_\infty$$

Clearly, if x is fixed, the optimal value of the LP is $p^*(x) = \| Ax - b \|_\infty$. Therefore optimizing over t and x simultaneously is equivalent to the original problem