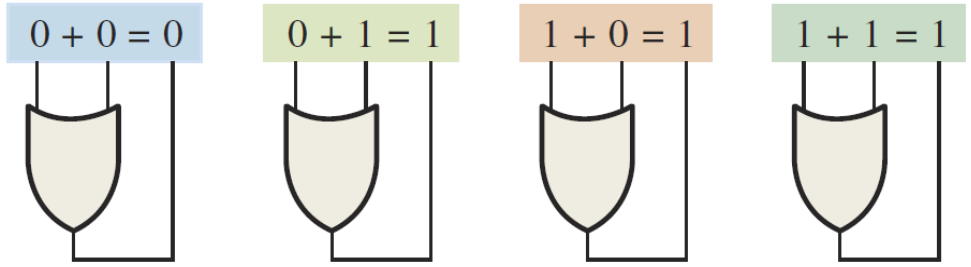
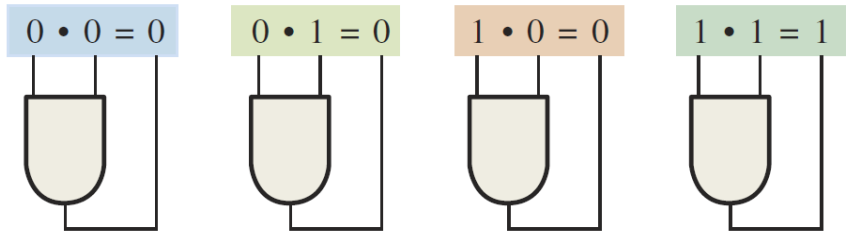
Boolean Addition and Multiplication



Addition



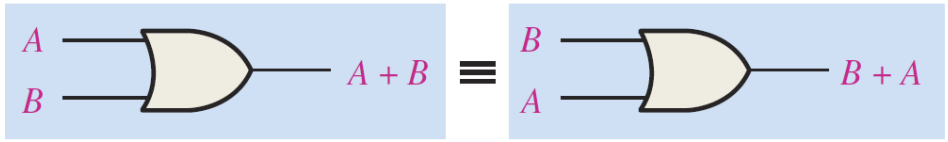
Multiplication

Laws of Boolean Algebra

- Commutative Laws

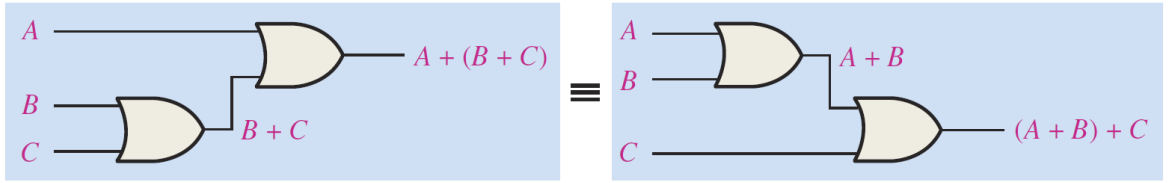
$$A + B = B + A$$

$$AB = BA$$



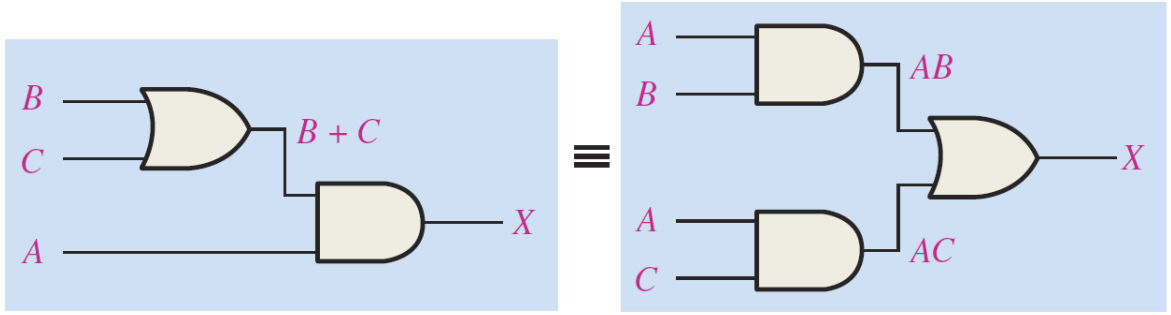
- Associative Laws

$$A + (B + C) = (A + B) + C$$



- Distributive Law

$$A(B + C) = AB + AC$$



Rules of Boolean Algebra

Basic rules of Boolean algebra.

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$ *
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$ *

A, B, or C can represent a single variable or a combination of variables.

13. $AB + A'C + BC = AB + A'C$

$AB + A'C + BCD = AB + A'C$ *

Question:

- Use truth table to proof eq. 12
- Proof eq. 12 and 13

Rules of Boolean Algebra

- Rule 1: A + 0 = A, A variable ORed with 0 is always equal to the variable

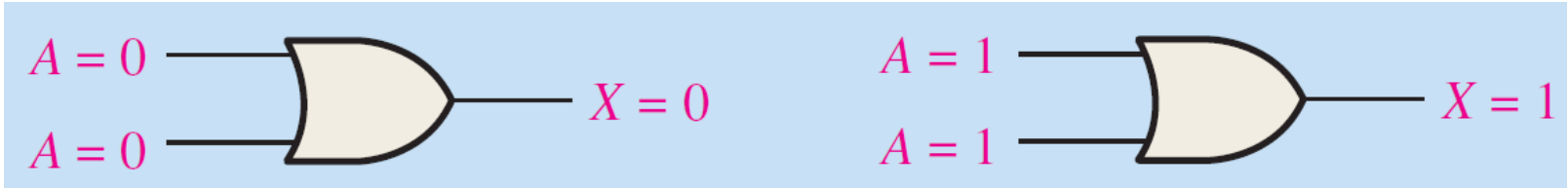
- Rule 2: A + 1 = 1, A variable ORed with 1 is always equal to 1.

- Rule 3: A · 0 = 0, A variable ANDed with 0 is always equal to 0.

- Rule 4: A · 1 = A, A variable ANDed with 1 is always equal to the variable.

Rules of Boolean Algebra

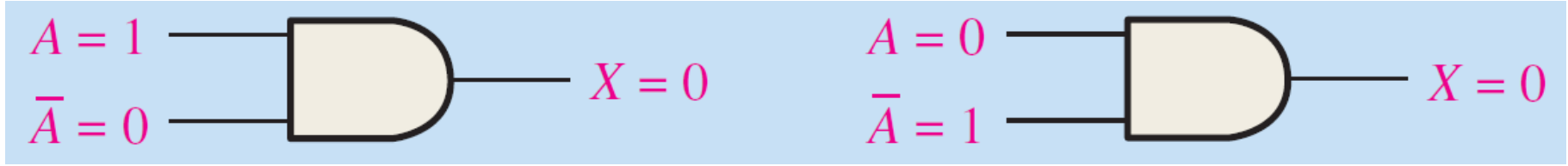
- Rule 5: A + A = A, A variable ORed with itself is always equal to the variable.



- Rule 6: A + Abar = 1, A variable ORed with its complement is always equal to 1.

- Rule 7: A · A = A, A variable ANDed with itself is always equal to the variable.

- Rule 8: A · Abar = 0, A variable ANDed with its complement is always equal to 0.



Rules of Boolean Algebra

- Rule 9: The double complement of a variable is always equal to the variable.

- Rule 10: A + AB = A

$$\begin{aligned} A + AB &= A \cdot 1 + AB = A(1 + B) \\ &= A \cdot 1 \\ &= A \end{aligned}$$

Rules of Boolean Algebra

- Rule 11: A + A'B = A + B

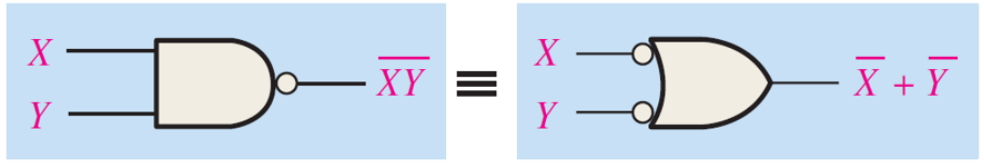
$$
\begin{aligned}
A + \overline{A}B &= (A + AB) + \overline{A}B \\
&= (AA + AB) + \overline{A}B \\
&= AA + AB + A\overline{A} + \overline{A}B \\
&= (A + \overline{A})(A + B) \\
&= 1 \cdot (A + B) \\
&= A + B
\end{aligned}
$$

- Rule 12: (A + B)(A + C) = A + BC

$$
\begin{aligned}
(A + B)(A + C) &= AA + AC + AB + BC \\
&= A + AC + AB + BC \\
&= A(1 + C) + AB + BC \\
&= A \cdot 1 + AB + BC \\
&= A(1 + B) + BC \\
&= A \cdot 1 + BC \\
&= A + BC
\end{aligned}
$$

DeMorgan's Theorems

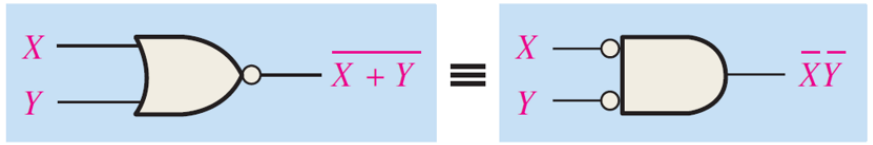
- DeMorgan's first theorem $\overline{XY} = \overline{X} + \overline{Y}$



NAND Negative-OR

Inputs		Output	
X	Y	$\overline{XY}$	$\overline{X} + \overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

- DeMorgan's second theorem $\overline{X + Y} = \overline{X}\,\overline{Y}$



NOR Negative-AND

Inputs		Output	
X	Y	$\overline{X + Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Think about it:

- 3 variable DeMorgan's Theorems?

- DeMorgan's theorems provide mathematical equivalency of the NAND and negative-OR gates and the equivalency of the NOR and negative-AND gates

Simplify Boolean expression

$$\overline{AB + AC} + \overline{A}\,\overline{B}C$$

DeMorgan's theorem

$$(\overline{AB})(\overline{AC}) + \overline{A}\,\overline{B}C$$

DeMorgan's theorem

$$(\overline{A} + \overline{B})(\overline{A} + \overline{C}) + \overline{A}\,\overline{B}C$$

Distributive law

$$\overline{A}\,\overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C} + \overline{A}\,\overline{B}C$$

Rule 7 and Rule 10

$$\overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C}$$

Rule 10

$$\overline{A} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C}$$

Rule 10

$$\overline{A} + \overline{B}\,\overline{C}$$

- Use the basic laws, rules, and theorems to simplify an expression depends on a thorough knowledge and considerable practice, not to mention a little ingenuity and cleverness.

Simplify Boolean expression

1. ABC+B’C+ACD
2. A(B’CD)’+AB’CD
3. AB’+ACD+A’B’+A’CD
4. A’BC’+AC’+B’C’
5. BC’D+BCD’+BC’D’+BCD
6. ((A’B)’+C)ABD+AD
7. AB+ABC’+ABD+AB(C’+D’)
8. A+(A’(BC)’)’(A’+(B’C’+D)’)+BC
9. AC+AB’+(B+C)’
10. AB’CD’+(AB’)’E+A’CD’E
11. A’B’C+ABC+A’BD’+AB’D’+A’BCD’+BCD’E’
12. B’+ABC
13. AB’+B+A’B
14. AC+A’D+C’D
15. A’BC’+A’BC+ABC
16. AB’+A’B+BC’+B’C
17. AC+B’C+BD’+CD’+A(B+C’)+A’BCD’+AB’DE

Reading materials

- Chapter 4 of Floyd book
- Chapter 2 of 阎石 book