

EE160 Introduction to Control: Homework 6

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Deadline: May 6, 2022

1. *Proportional-Differential (PD) Control.* Consider the linear multivariate control system

$$\begin{aligned}\dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= -x_1(t) + u(t) \\ y(t) &= x_1(t)\end{aligned}\tag{1}$$

- (a) Show that it's impossible to find a proportional controller of the form $u(t) = Ky(t)$ such that the closed-loop system is asymptotically stable.
- (b) Find all control gains $K, K_D \in \mathbb{R}$ for which the associated PD closed-loop control systems with

$$u(t) = Ky(t) + K_D \dot{y}(t)\tag{2}$$

is asymptotically stable.

2. *Proportional-Integral (PI) Control.* Consider scalar linear control system

$$\dot{x}(t) = x(t) + u(t) \quad \text{with} \quad x(0) = 0$$

Design a proportional integral controller such that the closed-loop system satisfies $\lim_{t \rightarrow \infty} x(t) = 1$.