Lecture 21 – Magnetic Resonance Imaging

This lecture will cover: *(CH5.8-5.10)*

– Gradient magnetic field

– Image acquisition

- Slice selection
- Phase encoding
- Frequency encoding

– The K-space and image reconstruction

(Supplementary reading: The Essential Physics of Medical Imaging CH12.6-12.7, 13.1)

Gradient magnetic field

- The resonant frequencies vary spatially with the magnetic field;
- The three separate magnetic field gradients are produced;
- No additional contribution at the isocenter where x,y,z=0.

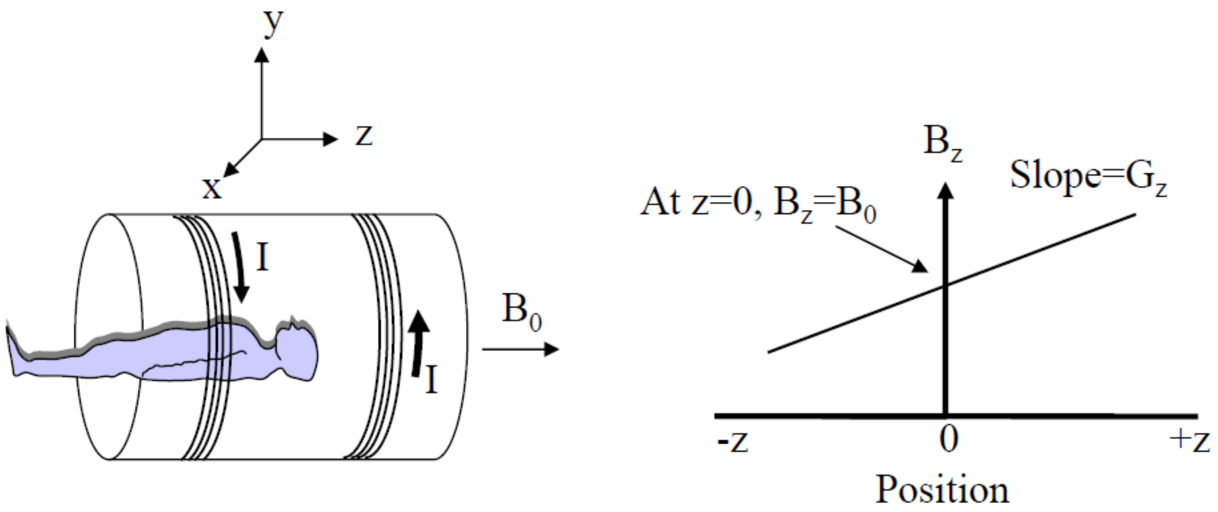

Fig. (left) A Maxwell-pair gradient coil in which an equal and opposite current is passed through a set of wires which are wound around a cylinder with its major axis in the z-direction. (right) The resulting magnetic field, B_z, is a function of position in z. The slope of the graph is G_z, the z-gradient. Since the currents in the two halves of the gradient coil are equal, the total magnetic field at the center of the gradient coil is B_0. Applying Fleming's left-hand rule, given the current directions indicated, the total magnetic field is less than B_0 for z_0 and greater than B0 for z_0.

Gradient magnetic field

- The linear spatial variation in magnetic field

$$\frac{\partial B}{\partial x} = G_x \qquad \frac{\partial B}{\partial y} = G_y \qquad \frac{\partial B}{\partial z} = G_z$$

- The magnetic field B experienced by protons in x, y, z direction respectively:

$$B_x = B_0 + xG_x \qquad B_y = B_0 + yG_y \qquad B_z = B_0 + zG_z$$

- The precession frequency of the protons

$$\omega_x = \gamma(B_0 + xG_x) \qquad \omega_y = \gamma\left(B_0 + yG_y\right) \qquad \omega_z = \gamma(B_0 + zG_z)$$

In the rotating reference frame,

$$\omega_x = \gamma x G_x \qquad \omega_y = \gamma y G_y \qquad \omega_z = \gamma z G_z$$

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- **Image acquisition**
  - **Slice selection**
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Image acquisition

➢ The process of image formation can be broken down into three independent components: slice selection, phase encoding, frequency encoding.

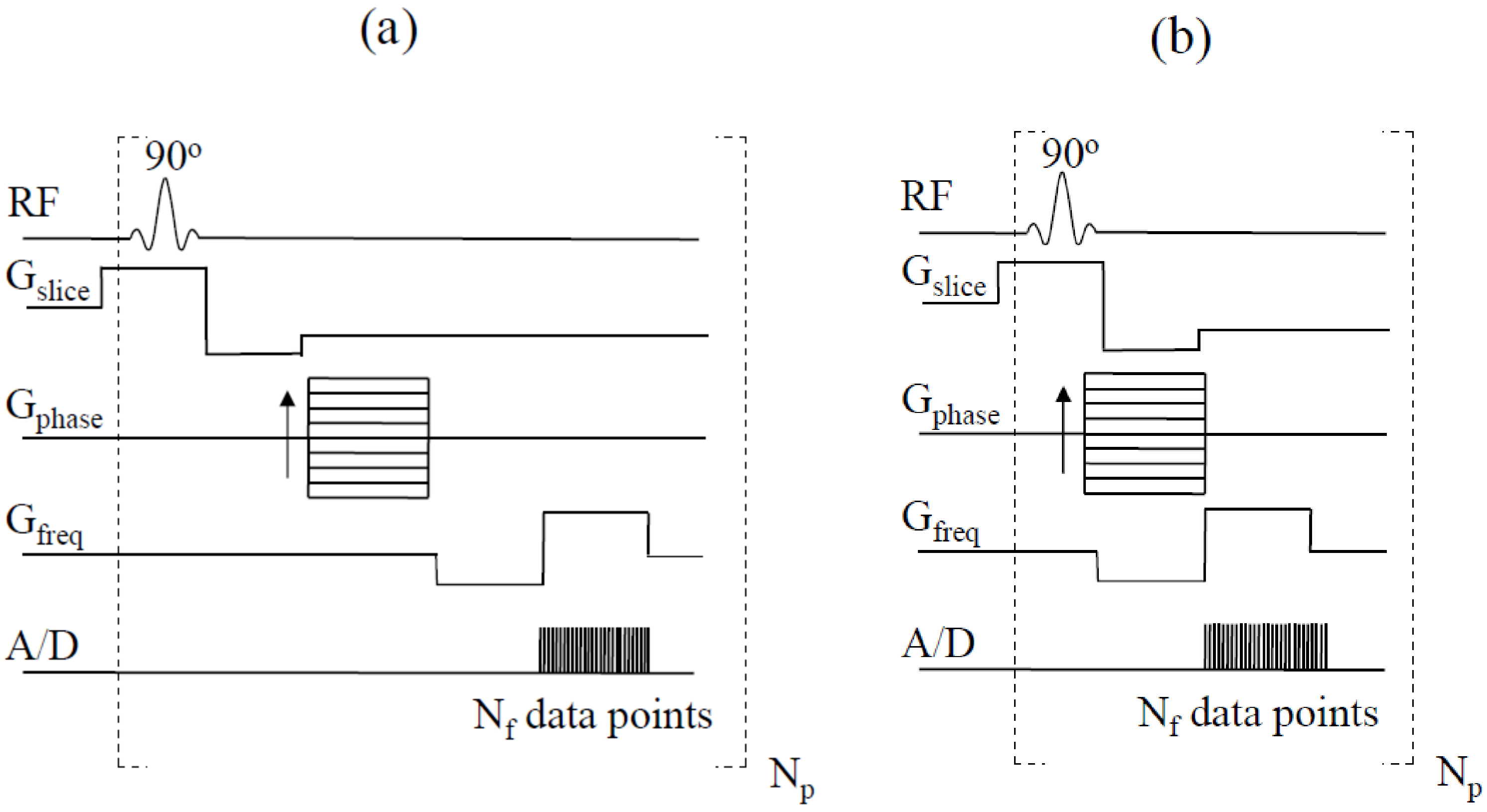
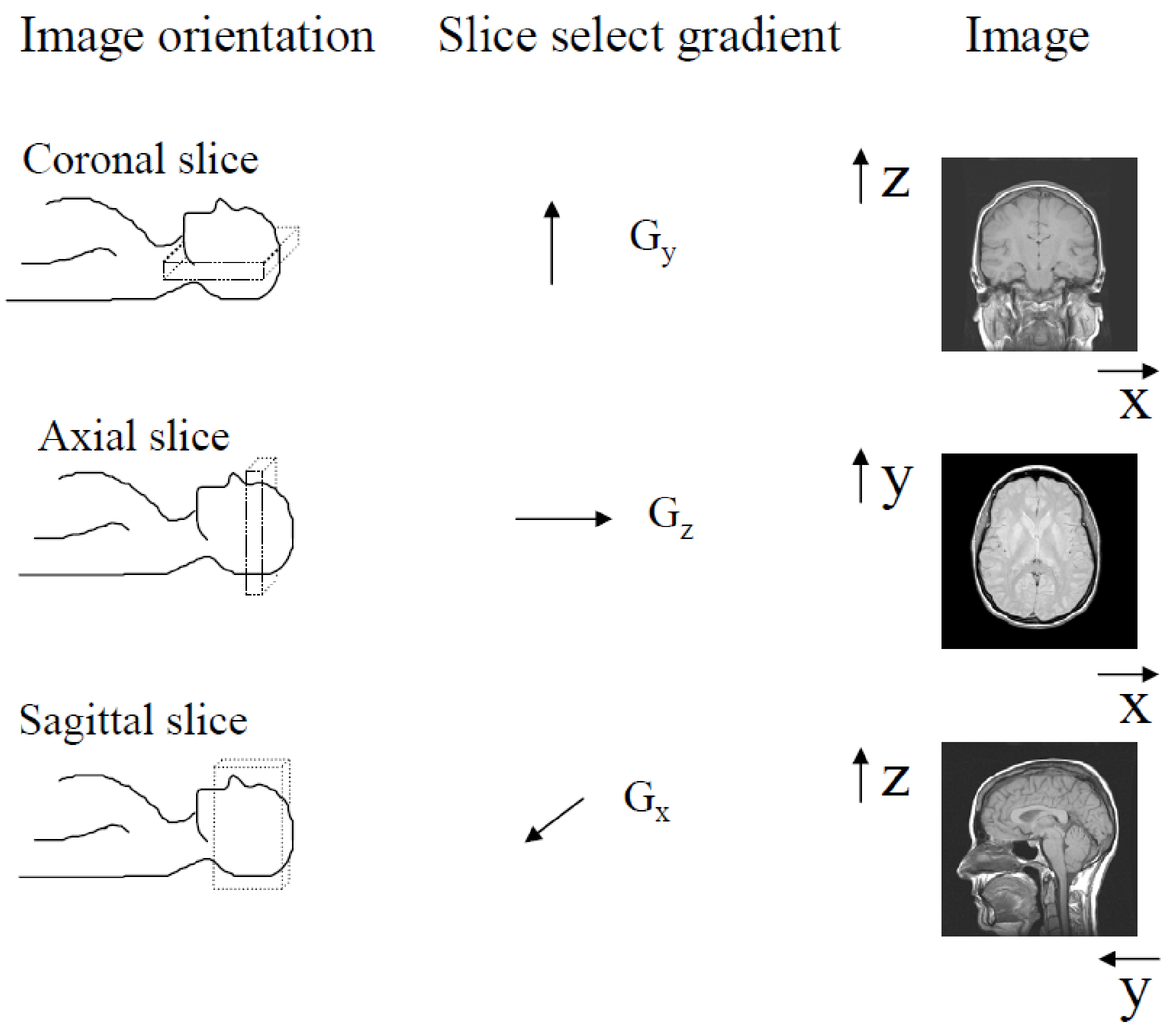


Fig. Pulse sequence diagrams for imaging sequences. An RF pulse is applied, various gradients are turned on and off, and the analogue-to-digital (A/D) converter is gated on to acquire data. (a) Individual steps in image formation can be considered independently in terms of slice selection (RF and G_{slice}), phase encoding (G_{phase}) and frequency encoding (G_{freq} and the A/D on). (b) In practice, the gradients are applied simultaneously where appropriate in order to minimize the time between RF excitation and signal acquisition.

Slice selection



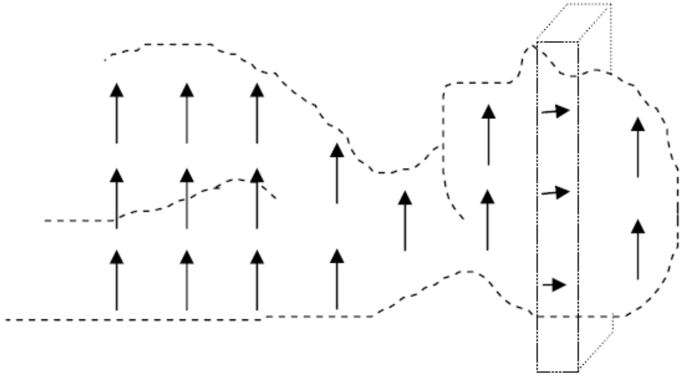
Slice selection (层面选择):

- MRI can acquire the image in any given orientation;
- Different parts of the body call for different image orientation.

Fig. Showing the different orientations for image acquisition. Coronal (top), axial (middle) or sagittal (bottom) slices can be produced by turning on the y, z, or x gradients, respectively, while the RF pulse is being applied

Slice selection

- A frequency-selective RF pulse (specific frequency ω_s with a bandwidth of $\pm\Delta\omega_s$) applied simultaneously with G_{slice};
- Protons with a precession frequency within the bandwidth ($\omega_s \pm\Delta\omega_s$) are rotated into the transverse plane by the RF pulse;
- The slice thickness is given by $T = \dfrac{2\Delta\omega_s}{\gamma G_{\text{slice}}}$



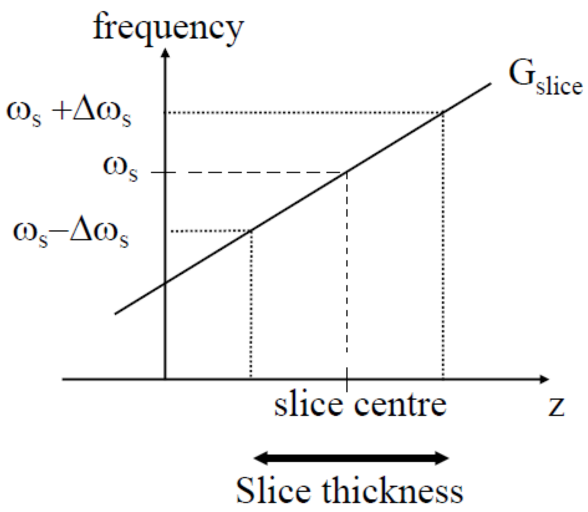


Fig. (left) The effects of a 90° RF pulse applied simultaneously with the z-gradient. Only protons within the axial slice are tipped by 90° into the transverse plane, and so give a measureable MR signal. Protons outside the chosen slice have their net magnetization remain in the z-direction. (right) Showing the gradient strength (the slope of the graph) as a function of position in z.

RF pulse

- A sinc or similarly shaped pulse for a rectangular frequency excitation.
- The slice position is moved by changing ω_s;
- Phase difference:

$$\varphi_{\mathrm{sl}} = \gamma G_z z \frac{\tau}{2}$$

- Reverse the dephasing by applying a negative rephrasing gradient

$$G_{\mathrm{slice}}^{\mathrm{ref}} \tau^{\mathrm{ref}} = \frac{\tau}{2} G_z$$

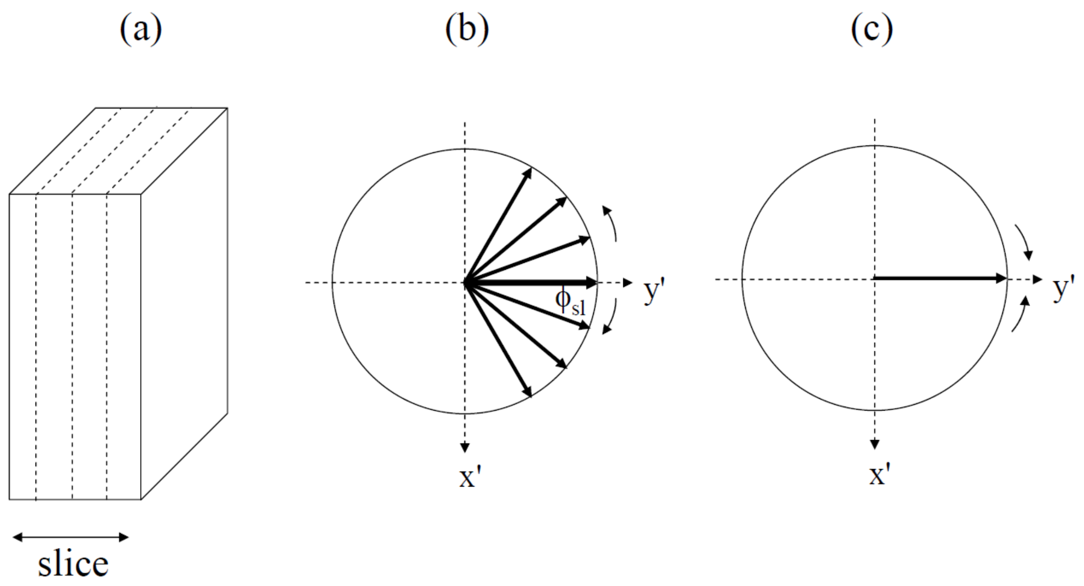


Fig. (a) Since the slice has a finite thickness, protons at different z-positions within the slice precess at different frequencies during the RF pulse. (b) At the end of the RF pulse protons at each z-position have acquired a phase φ_{sl}, and the net magnetization vector is reduced significantly due to the lack of phase coherence. (c) Application of a negative rephasing slice gradient of the appropriate duration and strength refocuses all the magnetization to give the maximum net magnetization.

Phase encoding (相位编码)

- Spatially dependent phase shift: $\varphi_{\mathrm{pe}}(G_y, \tau_{\mathrm{pe}}) = \gamma G_y y \tau_{\mathrm{pe}}$
- Generally the phase encoding gradient changes from the maximum negative value to the maximum positive value with gradient step of Δg_{pe} and N_{pe} data points

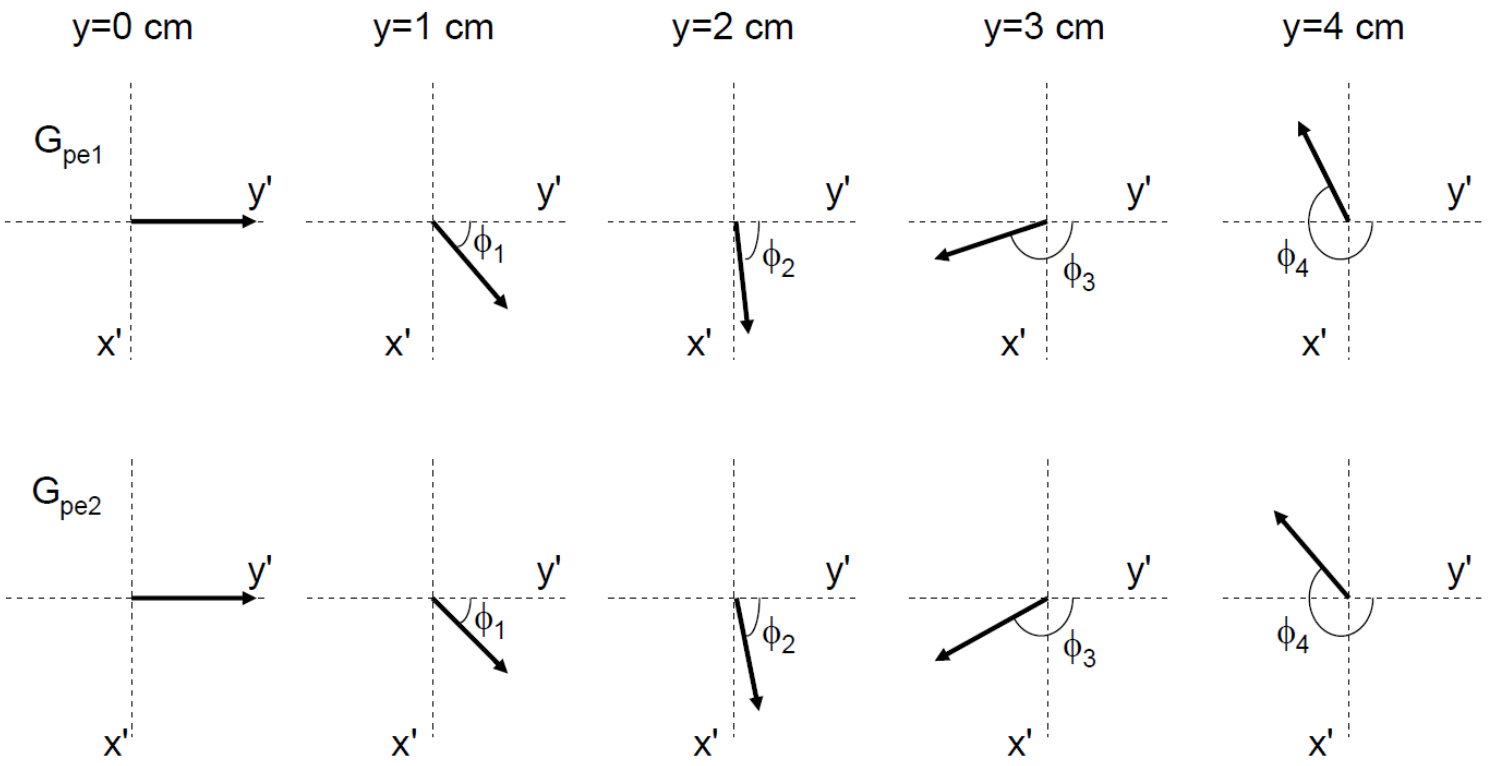
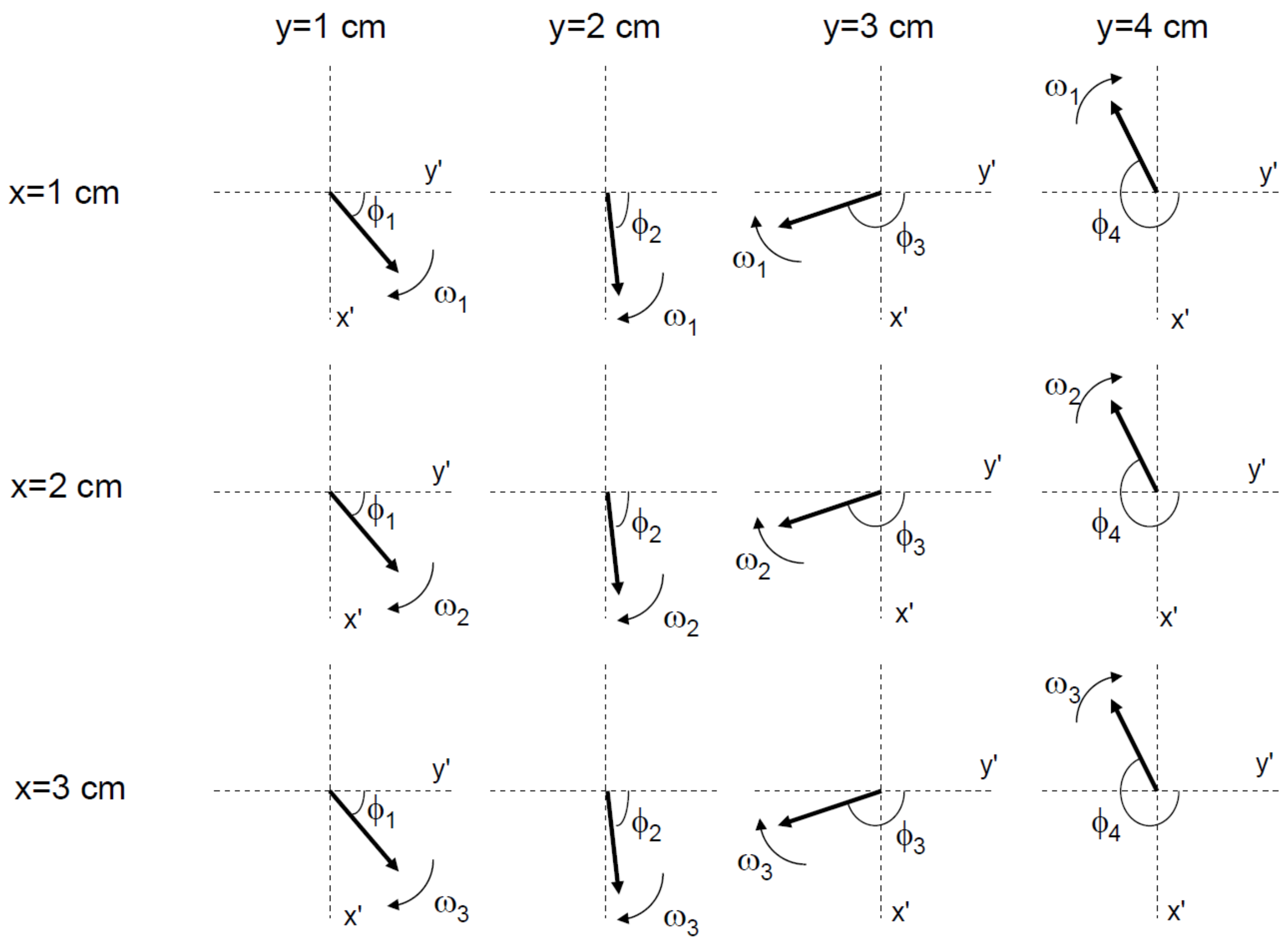


Fig. Phase encoding for protons at four different vertical positions with respect to the center of the y-gradient. (upper) a gradient G_{pe1} is applied. Protons at the very center of the y-gradient experience no additional magnetic field and so accumulate no phase in the rotating reference frame. The larger the offset in the y-dimension, the larger the phase shift accumulated. (lower) the next value of the phase encoding gradient is applied, with G_{pe2} being slightly less negative than G_{pe1}. Again, protons at y=0 accumulate no phase, and protons at different y-positions accumulate phases slightly lower in value than when G_{pe1} was applied. Typically between 128 and 512 different values of G_{pe} are used to acquire the full image.

Frequency encoding (频率编码)



- Apply frequency-encoding gradient while the receiver is gated on and data are being acquired and $\omega_x = \gamma G_x x$;
- A total of N_f data points are acquired while the receiver is on.

Fig. Combined effect of phase and frequency encoding gradients. The phases accumulated by protons at a given y-position are identical to those shown in the previous figure. During data acquisition the frequencies at which the protons precess are linearly dependent upon their position in the x-dimension. In this case $\omega_2 = 2\omega_1$ and $\omega_3 = 3\omega_1$.

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The k-space formalism

The $N_f * N_{pe}$ signals at the same slice can be mathematically expressed as:

$$s(G_y, \tau_{pe}, G_x, t) \propto \iint \rho(x, y)\, e^{-j\gamma G_y y \tau_{pe}} e^{-j\gamma G_x x t} dxdy$$

Where $\rho(x, y)$ is the number of protons at each position (x, y) , called the proton density

Two variables are defined as spatial frequencies:

$$k_x = \frac{\gamma}{2\pi} G_x t \qquad k_y = \frac{\gamma}{2\pi} G_y \tau_{pe}$$

Then

$$S(k_x, k_y) \propto \iint \rho(x, y)\, e^{-j2\pi k_x x} e^{-j2\pi k_y y} dxdy$$

K_x filling

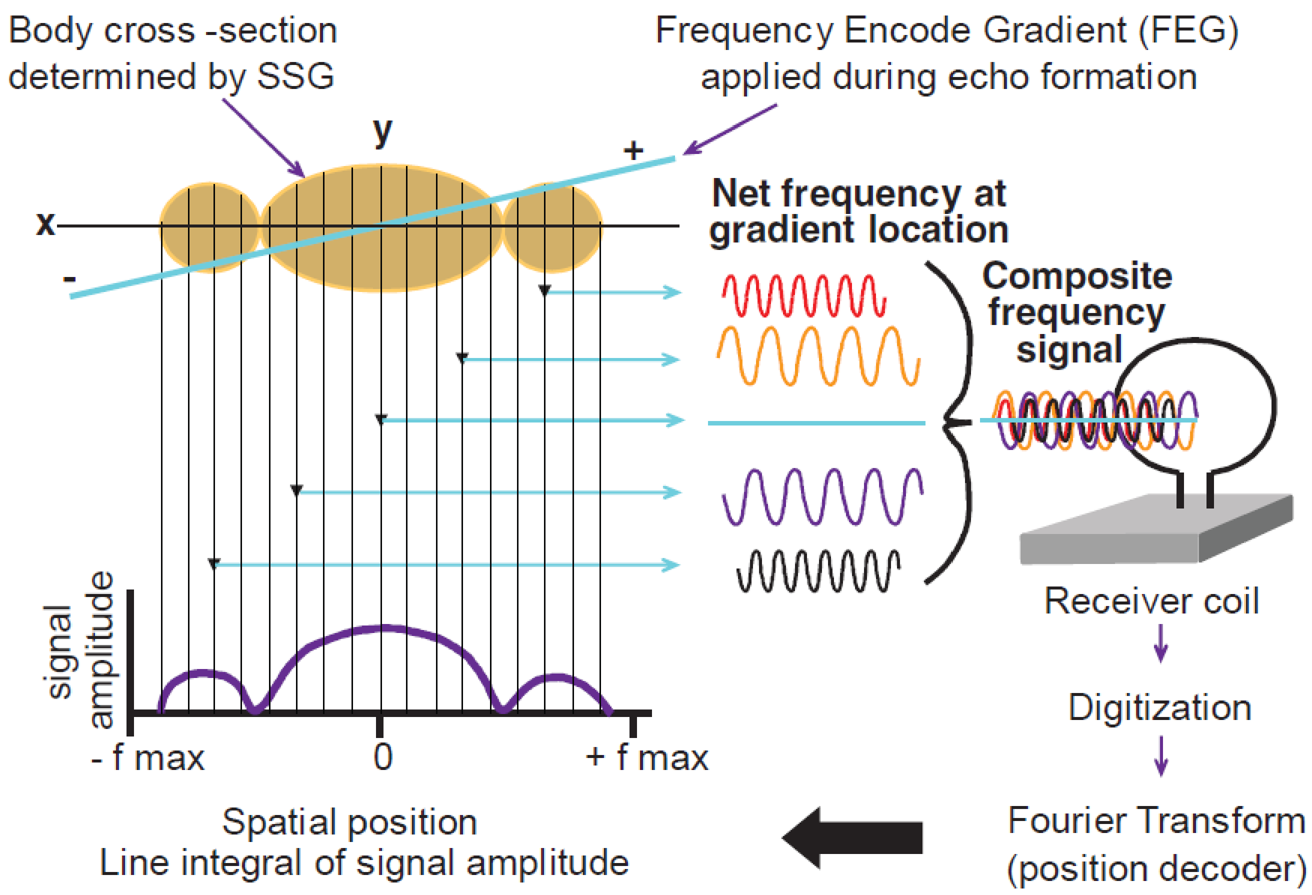


Fig. The FEG is applied in an orthogonal direction to the SSG (Slice Selection Gradient), and confers a spatially dependent variation in the precessional frequencies of the protons. Acting only on those protons in a slice determined by the SSG excitation, the composite signal is acquired, digitized, demodulated (Larmor frequency removed), and Fourier transformed into frequency and amplitude information. A one-dimensional array represents a projection of the slice of tissue (amplitude and position) at a specific angle. (Demodulation into net frequencies occurs after detection by the receiver coil; this is shown in the figure for clarity only.).

K_y filling

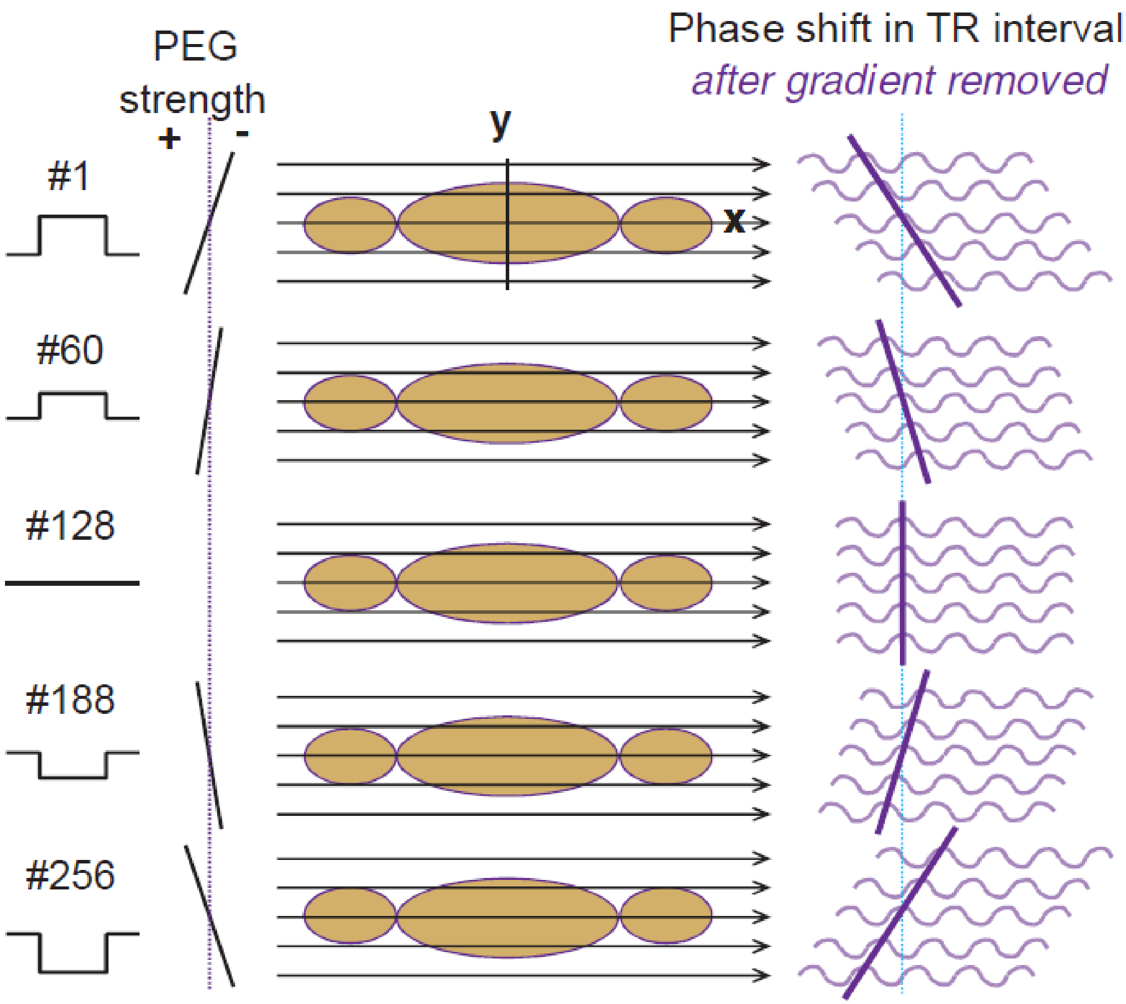
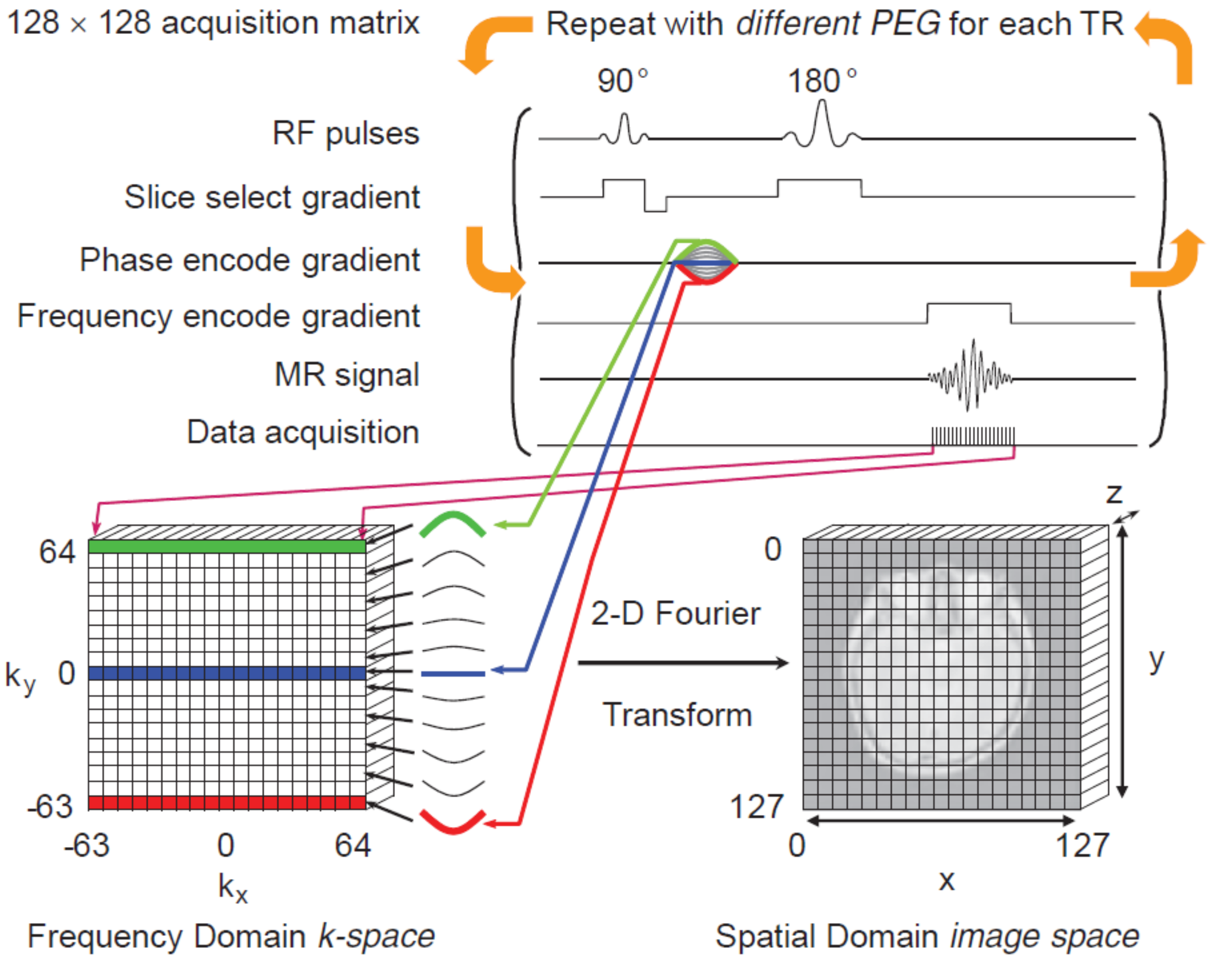


Fig. The PEG (Phase Encoding Gradient) is applied before the FEG and after the SSG. The PEG produces a spatially dependent variation in angular frequency of the excited spins for a brief duration, and generates a spatially dependent variation in phase when the spins return to the Larmor frequency. Incremental changes in the PEG strength for each TR (time of repetition) interval spatially encodes the phase variations: protons at the null of the PEG do not experience any phase change, while protons in the periphery experience a large phase change dependent on their distance from the null. The incremental variation of the PEG strength can be thought of as providing specific “views” of the volume because the SSG and FEG remain fixed throughout the acquisition.

2D data acquisition



$$\rho(x,y) = \iint_{-\infty}^{\infty} S(k_x,k_y)e^{j2\pi(k_x x+k_y y)}dk_x dk_y$$

Fig. MR data are acquired into k-space matrix, where each row in k-space represents spatially dependent frequency variations under a fixed FEG strength, and each column represents spatially dependent phase shift variations under an incrementally varied PEG strength. Data are placed in a specific row determined by the PEG strength for each TR interval. The grayscale image is constructed from the two-dimensional Fourier transformation of the k-space matrix by sequential application of one-dimensional transforms along each row, and then along each column of the intermediate transformed data. The output image matrix is arranged with the image coordinate pair, x = 0, y = 0 at the upper left of the image matrix.

2D image

- MR images are represented in magnitude mode;
- Higher amplitude at the low frequency;
- The higher the number of phase encoding steps, the higher the spatial resolution.

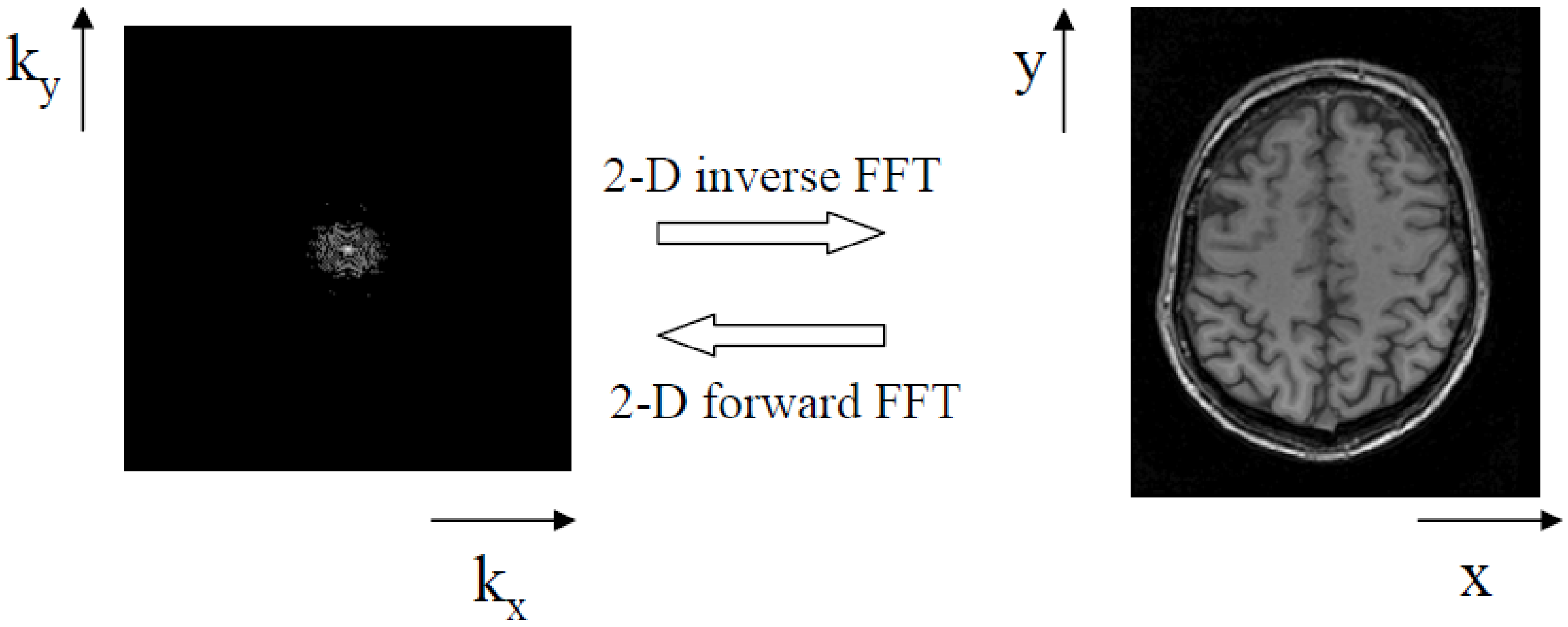


Fig. The mathematical relationship between the acquired k-space data on the left and the image on the right is a two-dimensional Fourier-transform. Although both the k-space data and image data are complex with real and imaginary components, they both typically illustrated in magnitude mode.