Homework 8

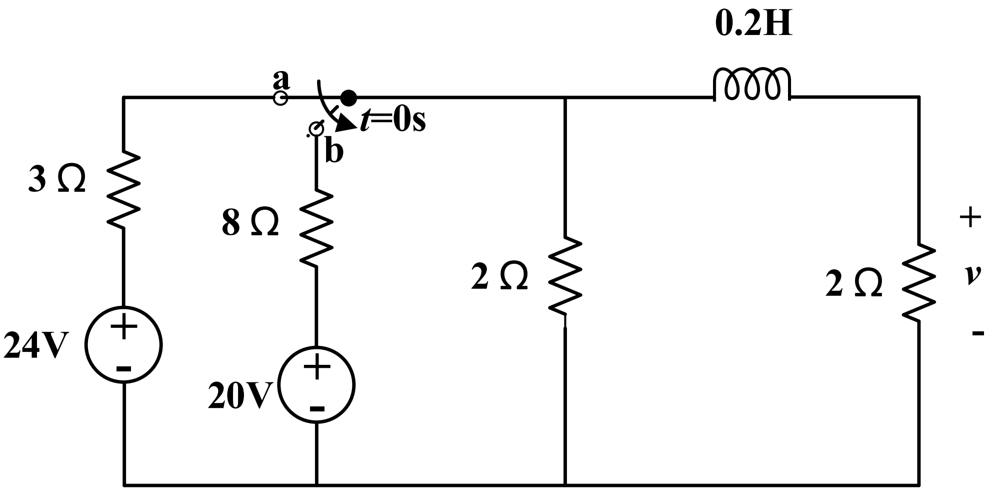
Due date: 18:00, 23rd, Dec.

Turn in your homework to room 3-305, SIST

Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

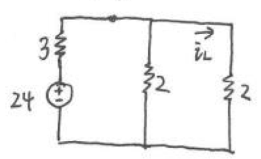
1. For the following circuit, the switch had been at node **a** for a long time before t=0s. At t=0s, the switch was turned to node **b** immediately. Please find the voltage on the 2Ω resistor for t>0s by using **time domain method AND Laplace domain method**, **respectively**.



$\boxed{\text{time domain}}$

略.

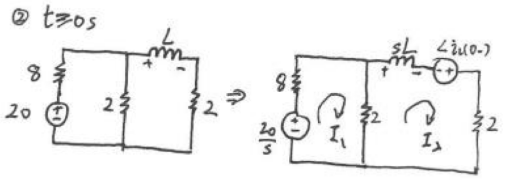
Q1:①初始条件:



$i_L(0-) = \frac{24}{3+2\|2} \cdot \frac{1}{2}$ A

$= 3$ A

② $t \geq 0s$



$$\begin{cases} 8I_1 + 2(I_1 - I_2) = \frac{20}{s} & \cdots ① \\ 2(I_2 - I_1) + sLI_2 - Li_L(0-) + 2I_2 = 0 & \cdots ② \end{cases}$$

①② ⇒

$$\begin{cases} 10I_1 - 2I_2 = \frac{20}{s} & \cdots ③ \\ -2I_1 + (4+sL)I_2 = Li_L(0-) & \cdots ④ \end{cases}$$

③ + ④×5

$$[5(4+sL)-2]I_2 = \frac{20}{s} + 5L\,i_L(0-)$$

$$(18+5sL)I_2 = \frac{20}{s} + 5L\,i_L(0-)$$

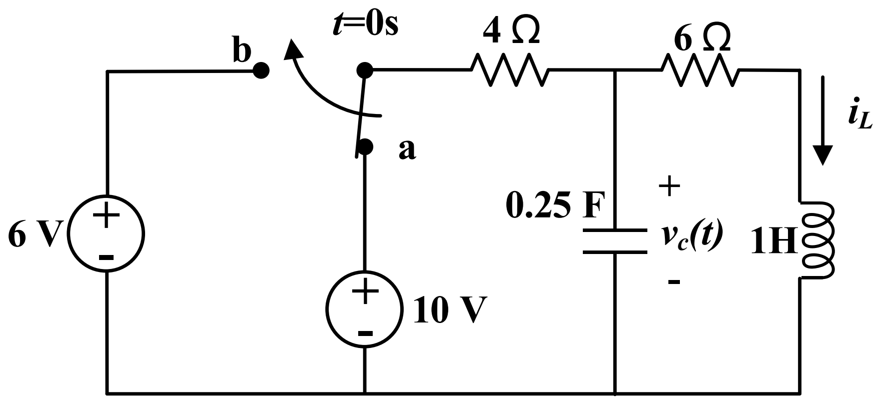
$$\Rightarrow I_2 = \frac{20}{(18+5sL)s} + \frac{15L}{18+5sL} \qquad (L = 0.2H)$$

$$= \frac{20}{(s+18)s} + \frac{3}{s+18} \quad A$$

$$i_2(t) = \frac{10}{9} + \frac{17}{9}e^{-18t} \quad (A)$$

$$V = 2i_2(t) = \frac{20}{9} + \frac{34}{9}e^{-18t} \quad (V)$$

2. For the following circuit, the switch had been at node ***a*** for a long time before ***t***=0s. When ***t***=0s, the switch was turned to node ***b*** immediately. Please use **Laplace domain method** to find $i_L(t)$ for ***t***>0s.



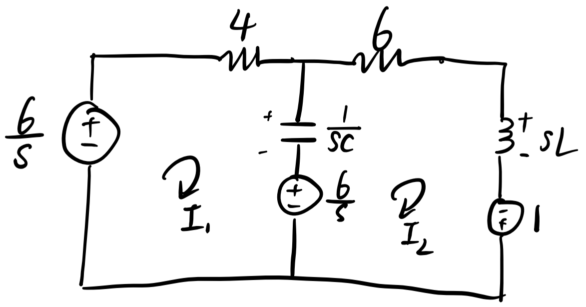
Solutions:

① Initial condition:

$$i_L(0_+) = i_L(0^-) = 1\,A$$

$$V_c(0^+) = V_c(0^-) = 6V$$

② $t>0$



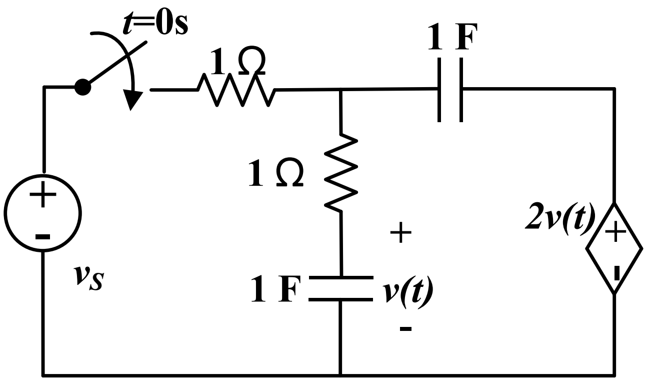
$$\begin{cases} \frac{1}{sC} = \frac{4}{s} \\ sL = s \end{cases}$$

$$\begin{cases} 4I_1 + (I_1 - I_2)\frac{4}{s} + \frac{6}{s} = \frac{6}{s} & \text{mesh 1} \\ 6I_2 + sI_2 + (I_2 - I_1)\frac{4}{s} - 1 - \frac{6}{s} = 0 & \text{mesh 2} \\ i_L = I_2 \end{cases}$$

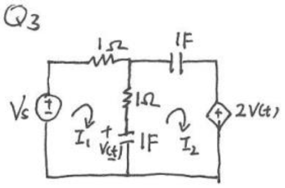
$$i_L = \frac{(s+6)(s+1)}{s(s+2)(s+5)} = \frac{3/5}{s} + \frac{2/3}{s+2} - \frac{4/15}{s+5} \quad V$$

$$i_L(t) = \frac{3}{5} + \frac{2}{3}e^{-2t} - \frac{4}{15}e^{-5t} \; A \quad (t \geq 0s)$$

3. For the circuit below, the switch closed immediately at t=0s, and $v_S(t)=e^{-t}sin(t)$ V. Please find the voltage $v(t)$ shown in the circuit for t>0s by using **Laplace domain method.** Note that there is no energy stored in this circuit before t=0s.



Q3



$V_s(t) = e^{-t}\sin t \quad (V)$

$\Rightarrow V_s(s) = \dfrac{1}{(s+1)^2+1} \quad (V)$

Mesh
~~KC~~ KVL:

$$\begin{cases} I_1 \cancel{(s)} + (I_1 - I_2) + \frac{1}{sC}(I_1 - I_2) = V_s(s) & \cdots ① \\ (I_2 - I_1)\frac{1}{sC} + (I_2 - I_1) + \frac{1}{sC} I_2 + 2V(s) = 0 & \cdots ② \end{cases}$$

①②, C=1F

$\Rightarrow (1 + 1 + \frac{1}{s}) I_1 - (1 + \frac{1}{s}) I_2 = V_s(s) \quad \cdots ③$

$-(1 + \frac{1}{s}) I_1 + (1 + \frac{1}{s} + \frac{1}{s}) I_2 = -2V(s) \quad \cdots ④$

$V(s) = \frac{1}{s}(I_1 - I_2)$

③, ④, ⑤ $\Rightarrow$

$$V(s) = \frac{1}{s^2+s+1} V(s)$$

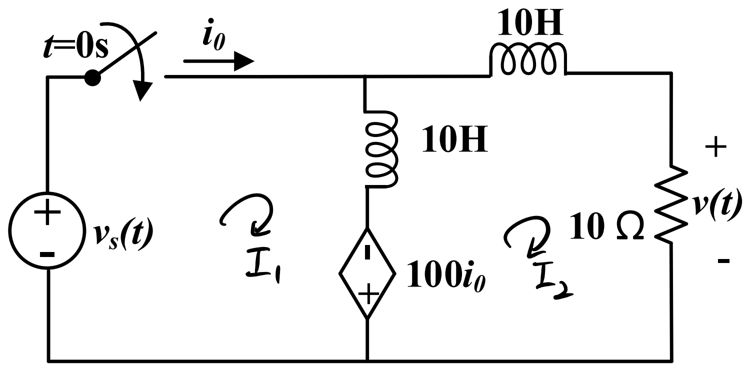
$$= \frac{1}{s^2+s+1} \cdot \frac{1}{(s+1)^2+1} \quad (V)$$

~~V(t) =~~

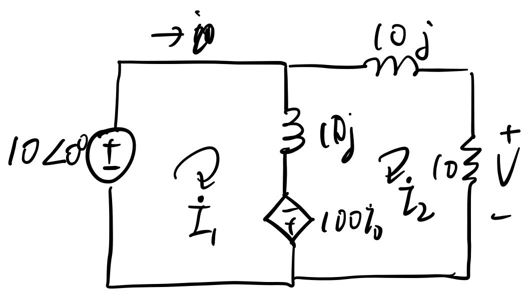
$$V(t) = e^{-t}\cos t + \frac{\sqrt{3}}{3} e^{-t/2} \sin(\frac{\sqrt{3}}{2} t) - e^{-t/2} \cos(\frac{\sqrt{3}}{2} t) \quad (V)$$

$$= e^{-t}\cos t + \frac{2\sqrt{3}}{3} e^{-t/2} \cos(\frac{\sqrt{3}}{2} t - 150°) \quad (V)$$

4. For the following circuit, $v_s(t)$=10cos*t* V, and the switch closed immediately at **t**=0s. There is no energy stored in the circuit before *t*=0s. Please
   (a) Use **phasor method** to find the **steady-state** for the voltage of ***v***(***t***).
   (b) Use **Laplace domain method** to find ***v(t)*** for *t*>0s and compare the results from (a).



Phasor



$$\begin{cases} (\dot{I}_1-\dot{I}_2)10j = 100\dot{I}_1 + 10\angle 0° \\ 100\dot{I}_1 + (\dot{I}_2-\dot{I}_1)10j + 10j\dot{I}_2 + 10\dot{I}_2 = 0. \end{cases}$$

$$(10+10j)\dot{I}_2 = 10\angle 0°$$

$$\dot{I}_2 = \frac{10\angle 0°}{10+10j} = \frac{\sqrt{2}}{2}\angle -45° \text{ A}$$

$$V = 10\cdot\frac{\sqrt{2}}{2}\angle -45° \text{ V}$$
$$= 5\sqrt{2}\angle -45° \text{ V}$$
$$= 5\sqrt{2}\cos(t-45°) \text{ V}$$

Laplace

$$V_s = 10\cos t = \frac{10s}{s^2+1} \text{ V}$$

$$\begin{cases} 10s\,(I_1(s) - I_2(s)) = 100I_1(s) + V_s \\ 100I_1(s) + 10s[I_2(s) - I_1(s)] + 10sI_2(s) + 10I_2(s) = 0. \end{cases}$$

$$(10s+10)I_2(s) = \frac{10s}{1+s^2}$$

$$I_2(s) = \frac{10s}{(s^2+1)(10+10s)}$$

$$i_2(t) = -\frac{1}{2}e^{-t} + \frac{\sqrt{2}}{2}\cos(t-45°) \text{ A}$$

$$V(t) = -5e^{-t} + 5\sqrt{2}\cos(t-45°) \text{ V } (t\geq 0s)$$

transient: $-5e^{-t}$ V

steady state: $5\sqrt{2}\cos(t-45°)$ V