EE101 Homework 1 Solution

Question 1 :

The sinc function, denoted by $\mathrm{Sinc}(t) = \frac{\sin(\pi t)}{\pi t}$ occurs often in Fourier analysis. Fig.1a,1b shows the function graph and spectrogram of $\mathrm{Sinc}(40t)$, its cut-off frequency is 20Hz. Fig1c shows an example of sampling $\mathrm{sinc}(40t)$ at 50Hz. Given the sampling frequency 25Hz\ 40Hz\100Hz\200Hz, please plot each spectrogram after sampling.

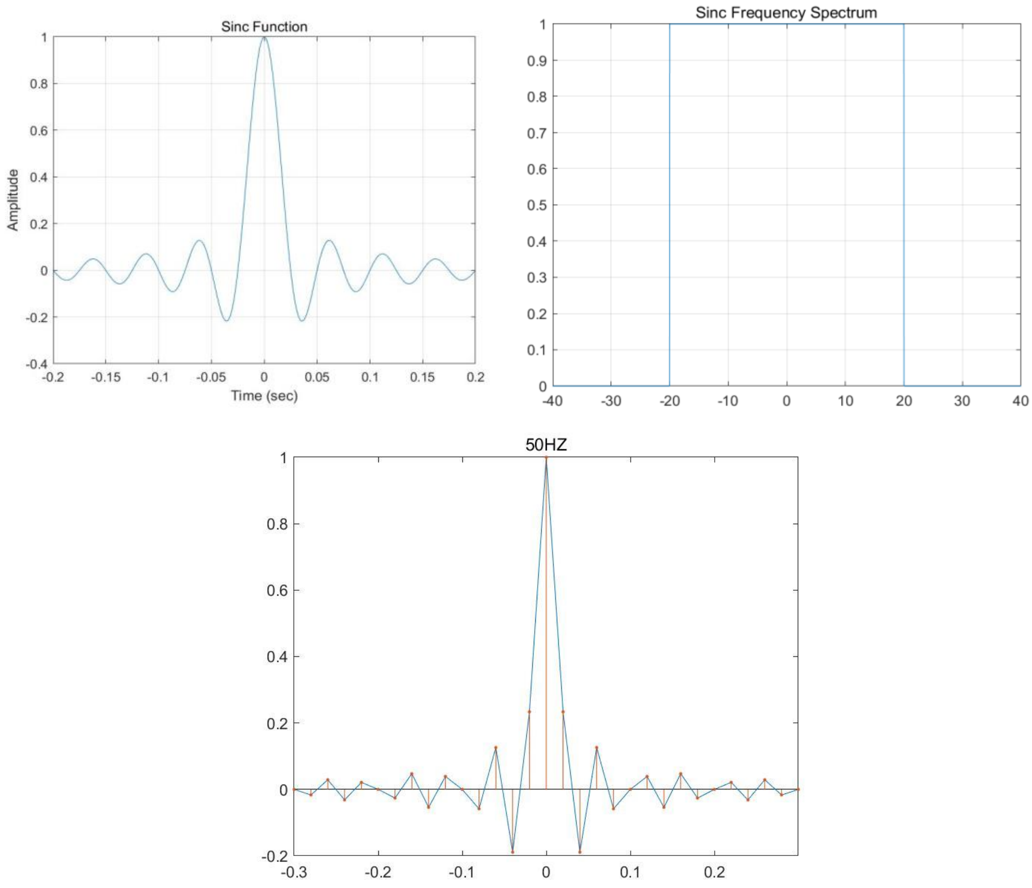
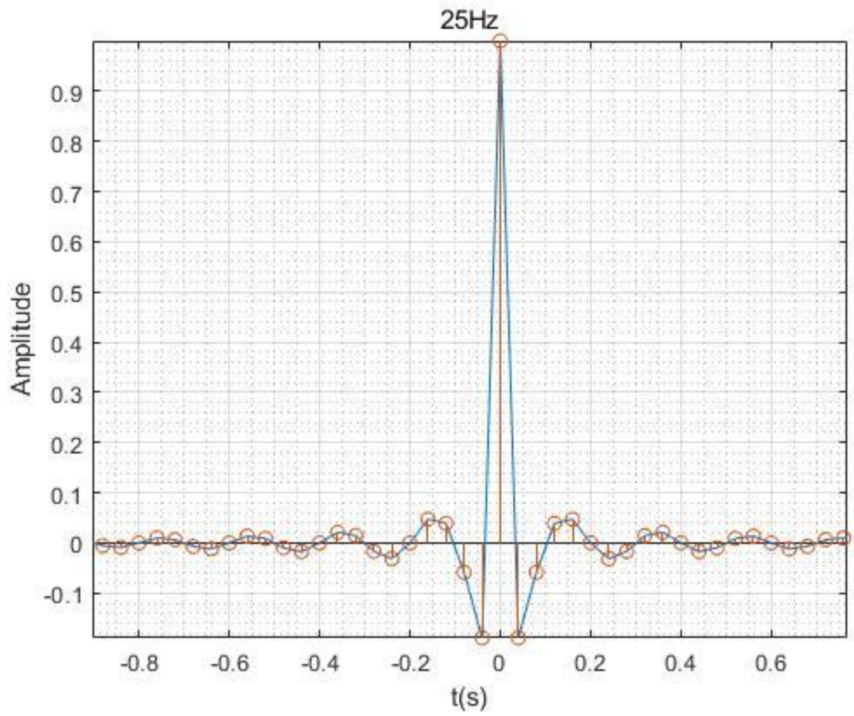


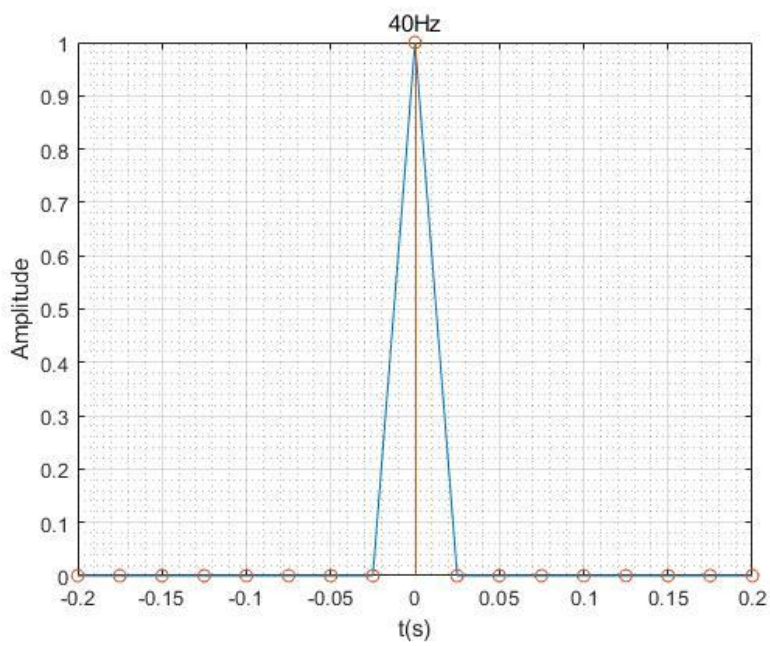
Figure 1 **a)** Function $\mathrm{sinc}(40t)$ **b)** Frequency Spectrum of $\mathrm{sinc}(40t)$ **c)** Sampling at 50Hz

Solution：

（1）Sampling at frequency = 25Hz, the sampled spectrogram is as follows: (10 points)

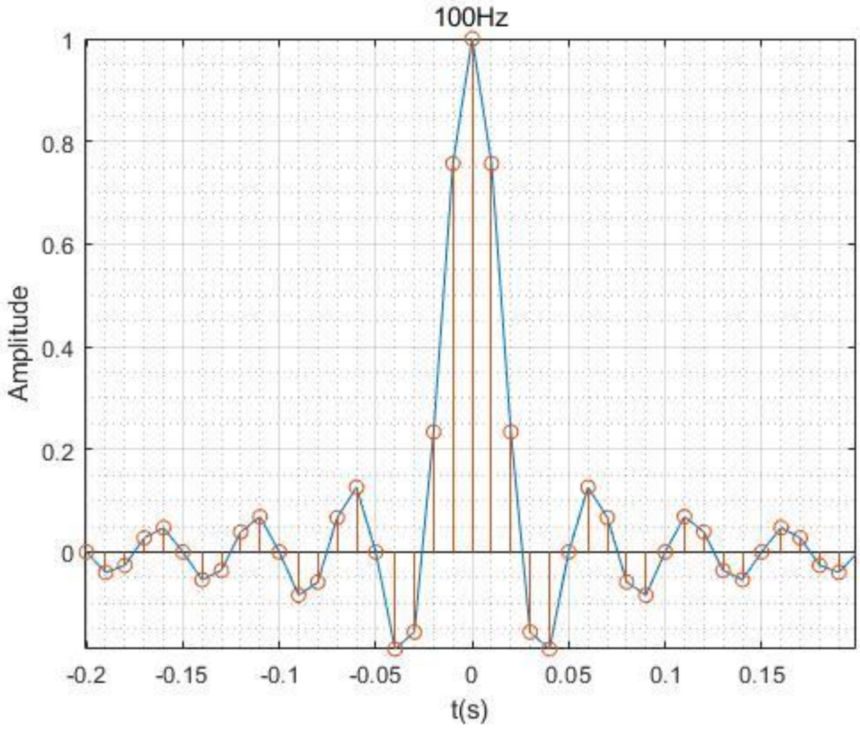


(2) Sampling at frequency = 50Hz, the sampled spectrogram is as follows: (10 points)



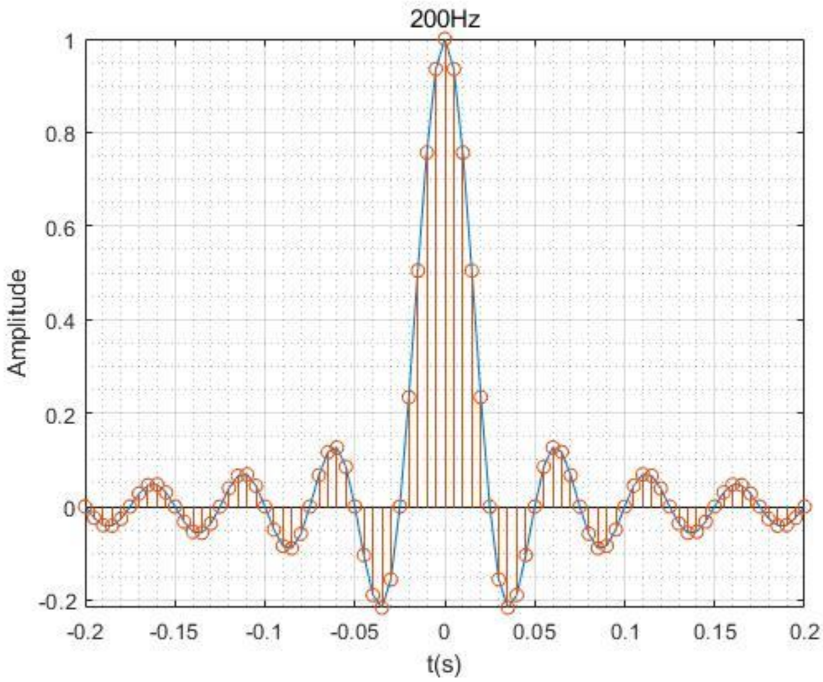
(3) Sampling at frequency = 100Hz, the sampled spectrogram is as follows:

(10 points)



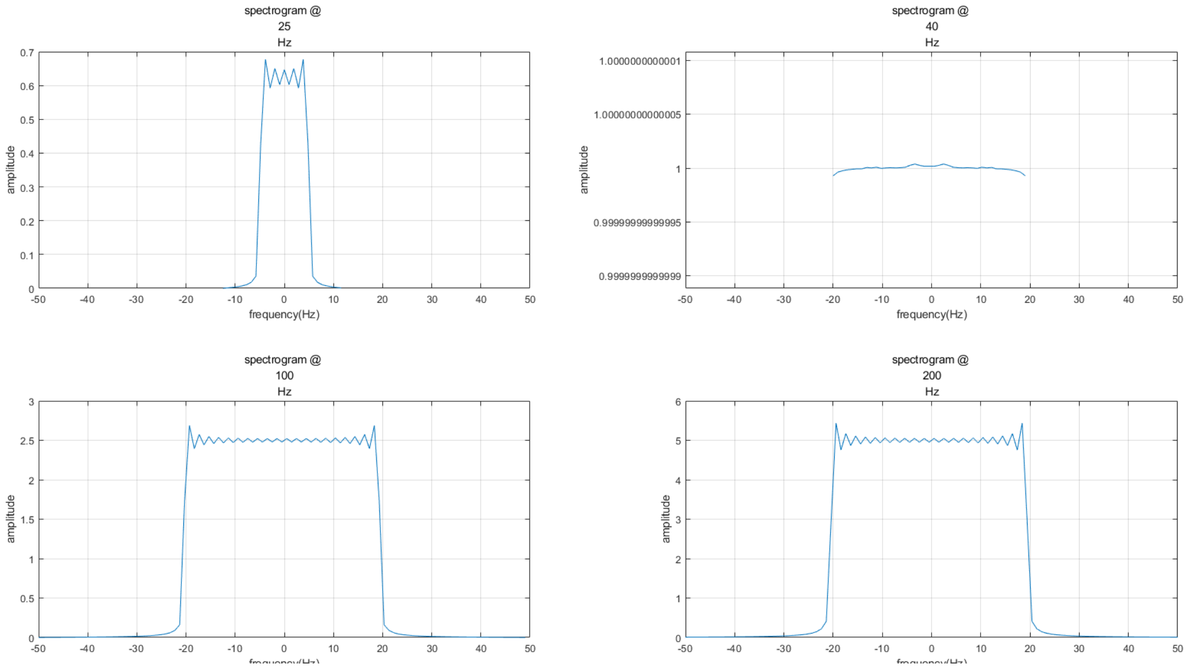
(4) Sampling at frequency = 100Hz, the sampled spectrogram is as follows:

(10 points)



Many students have also plotted the frequency spectrum but actually it is not needed. Anyway, plotting them will not get your points reduced~

For reference, frequency spectrums are attached below (Plot from *Yang Jing*)



Question 2:

In a study for testing a new form of cancer among 1000 suspected patients, multiple medical methods had been considered for auxiliary diagnosis. For each method, the number of positive and negative people corresponding to the different threshold values was given in the Table.1

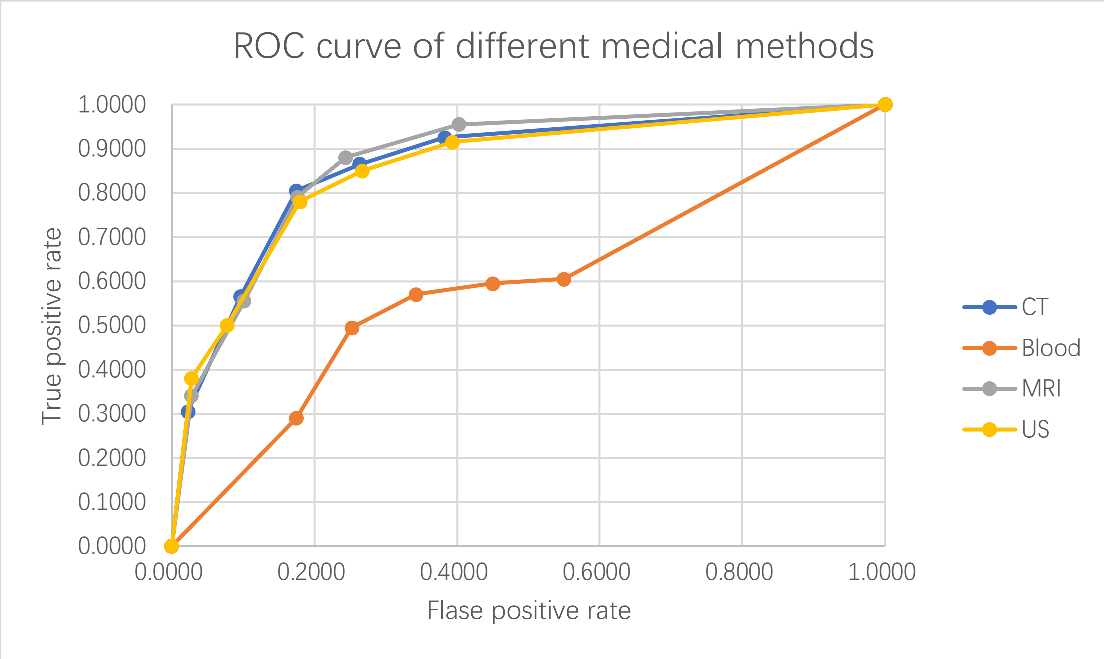
Table1 Diagnosis Results

Biopsy							
Positive Negative				200 800			
Blood(Immune cell concentration 10^9/L)							
Threshold	3	7.5	8.5	9.5	10.5	11.5	15
Positive(>Thr)	1000	561	479	388	301	198	0
TP	200	121	119	114	99	58	0
CT(length(cm))							
Threshold	0.5	2	2.5	3	3.5	4	8
Positive(>Thr)	1000	491	384	301	190	79	0
TP	200	185	173	161	113	61	0
MRI(length(cm))							
Threshold	0.5	2	2.5	3	3.5	4	8
Positive(>Thr)	1000	513	371	299	192	90	0
TP	200	191	176	158	111	68	0
Ultrasound(length(cm))							
Threshold	0.5	2	2.5	3	3.5	4	8
Positive(>Thr)	1000	498	384	300	162	98	0
TP	200	183	170	156	100	76	0

(1) Plot the ROC curve.
(2) Determine a standard Threshold for each method
(3) Consider which method is best for auxiliary diagnosis and explain.

Solution:

(1) The ROC curve is as follows: (either excel or MATLAB is okay) (10 points)



(2) (10 points)

A standard threshold must optimize both sensitivity and specificity. Since sensitivity = true positive rate and specificity = 1 – false positive rate, the nearer the distance between the threshold point to the upper-left corner point, the better the threshold point is.

	threshold	TP	positive	False negative	False positive	True negative	TP rate	FP rate	distance^2	distance	min distance	AUC
	0.5	200	1000	0	800	0	1.0000	1.0000	1.0000	1.0000		
	2	185	491	15	306	494	0.9250	0.3825	0.1519	0.3898		
	2.5	173	384	27	211	589	0.8650	0.2638	0.0878	0.2963		
CT	3	161	301	39	140	660	0.8050	0.1750	0.0687	0.2620	0.2620	0.8282
	3.5	113	190	87	77	723	0.5650	0.0963	0.1985	0.4455		
	4	61	79	139	18	782	0.3050	0.0225	0.4835	0.6954		
	8	0	0	200	0	800	0.0000	0.0000	1.0000	1.0000		
	3	200	1000	0	800	0	1.0000	1.0000	1.0000	1.0000		
	7.5	121	561	79	440	360	0.6050	0.5500	0.4585	0.6771		
	8.5	119	479	81	360	440	0.5950	0.4500	0.3665	0.6054		
Blood	9.5	114	388	86	274	526	0.5700	0.3425	0.3022	0.5497	0.5497	0.5522
	10.5	99	301	101	202	598	0.4950	0.2525	0.3188	0.5646		
	11.5	58	198	142	140	660	0.2900	0.1750	0.5347	0.7312		
	15	0	0	200	0	800	0.0000	0.0000	1.0000	1.0000		
	0.5	200	1000	0	800	0	1.0000	1.0000	1.0000	1.0000		
	2	191	513	9	322	478	0.9550	0.4025	0.1640	0.4050		
	2.5	176	371	24	195	605	0.8800	0.2438	0.0738	0.2717		
MRI	3	158	299	42	141	659	0.7900	0.1763	0.0752	0.2742	0.2717	0.8402
	3.5	111	192	89	81	719	0.5550	0.1013	0.2083	0.4564		
	4	68	90	132	22	778	0.3400	0.0275	0.4364	0.6606		
	8	0	0	200	0	800	0.0000	0.0000	1.0000	1.0000		
	0.5	200	1000	0	800	0	1.0000	1.0000	1.0000	1.0000		
	2	183	498	17	315	485	0.9150	0.3938	0.1623	0.4028		
	2.5	170	384	30	214	586	0.8500	0.2675	0.0941	0.3067		
US	3	156	300	44	144	656	0.7800	0.1800	0.0808	0.2843	0.2843	0.8104
	3.5	100	162	100	62	738	0.5000	0.0775	0.2560	0.5060		
	4	76	98	124	22	778	0.3800	0.0275	0.3852	0.6206		
	8	0	0	200	0	800	0.0000	0.0000	1.0000	1.0000		

After the calculation, the threshold with the minimum distance can be chosen as the standard threshold. The result is as follows:

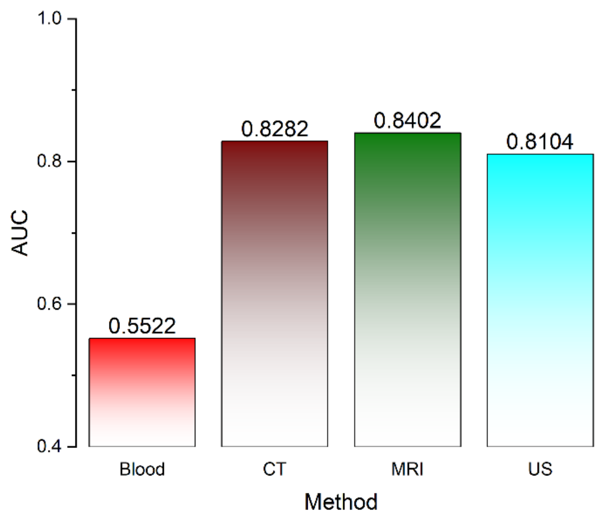
Standard threshold for each method:

method	standard threshold
Blood	9.5
CT	3
MRI	2.5
US	3

(3) (10 points)

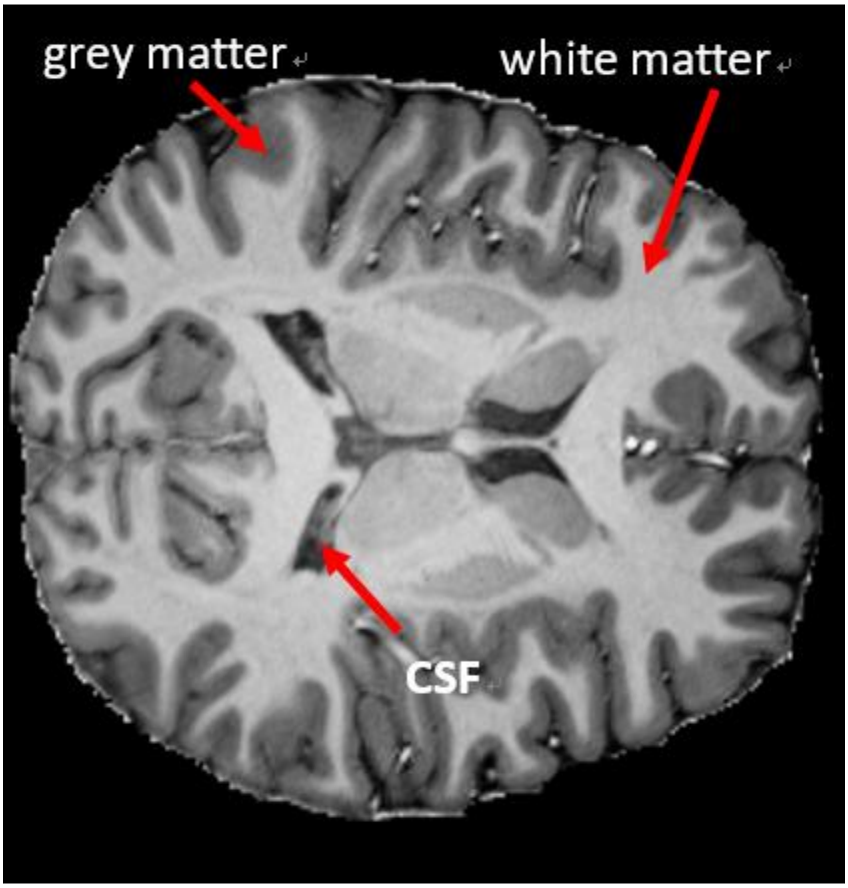
To determine the overall performance of a specific method, it is advisable to calculate the performance (considering sensitivity and specificity) of every threshold. Quantitatively speaking, AUC (area under curve) proves to be an appropriate measure. Comparing AUC of different methods, we will jump to the conclusion that MRI is the best for auxiliary diagnosis.

Method	Blood	CT	MRI	US
AUC	0.5522	0.8282	0.8402	0.8104



Question 3 MRI Brain Image Segmentation

An MRI brain image is provided. Please mark the different tissues with colors for brain using thresholding method



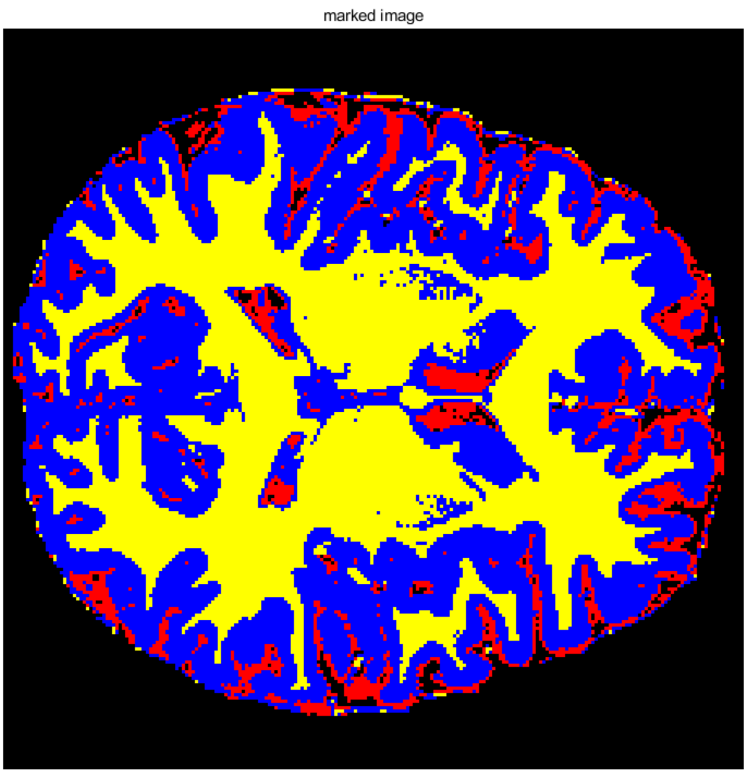
Grading rule:

10 points for marking grey matter, 10 points for marking white matter and 10 points for marking CSF.

There are various ways to mark different tissues, hence if the result is satisfactory, it will be acceptable.

A relatively easy but rough approach is to use the MATLAB ***grayslice*** function, which converts grayscale image to indexed image using multilevel thresholding. Here I will simply post the result from *Bai Zihuan.*

```
%Question 3
clear;clf;
I = imread("Tissue_Segment_for_q3.png");
figure
imhist(I);
figure
imshow(I);title('original image');
figure
thresholds = [40 80 160];
newImg = grayslice(I,thresholds);
cmap = [0 0 0
    1 0 0
    0 0 1
    1 1 0
    ];
imshow(newImg,colormap(cmap));title('marked image');
```

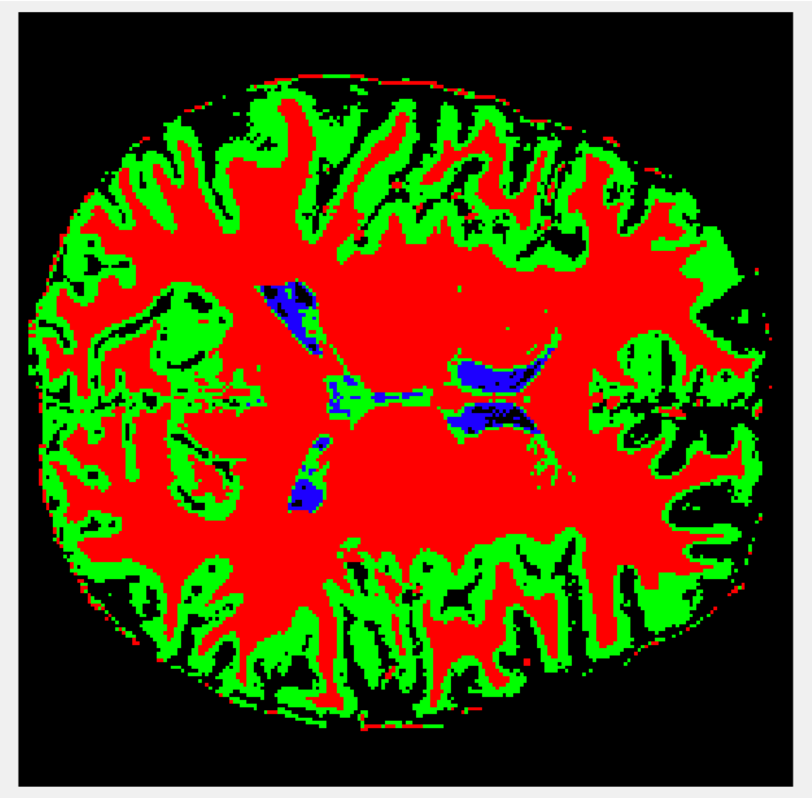


This method is easy to apply but to achieve better result, we need more operation

Or we can convert the image to indexed image and mark different tissues with different indexes, then form an RGB image, as *Jin Zheyu* did in the homework.

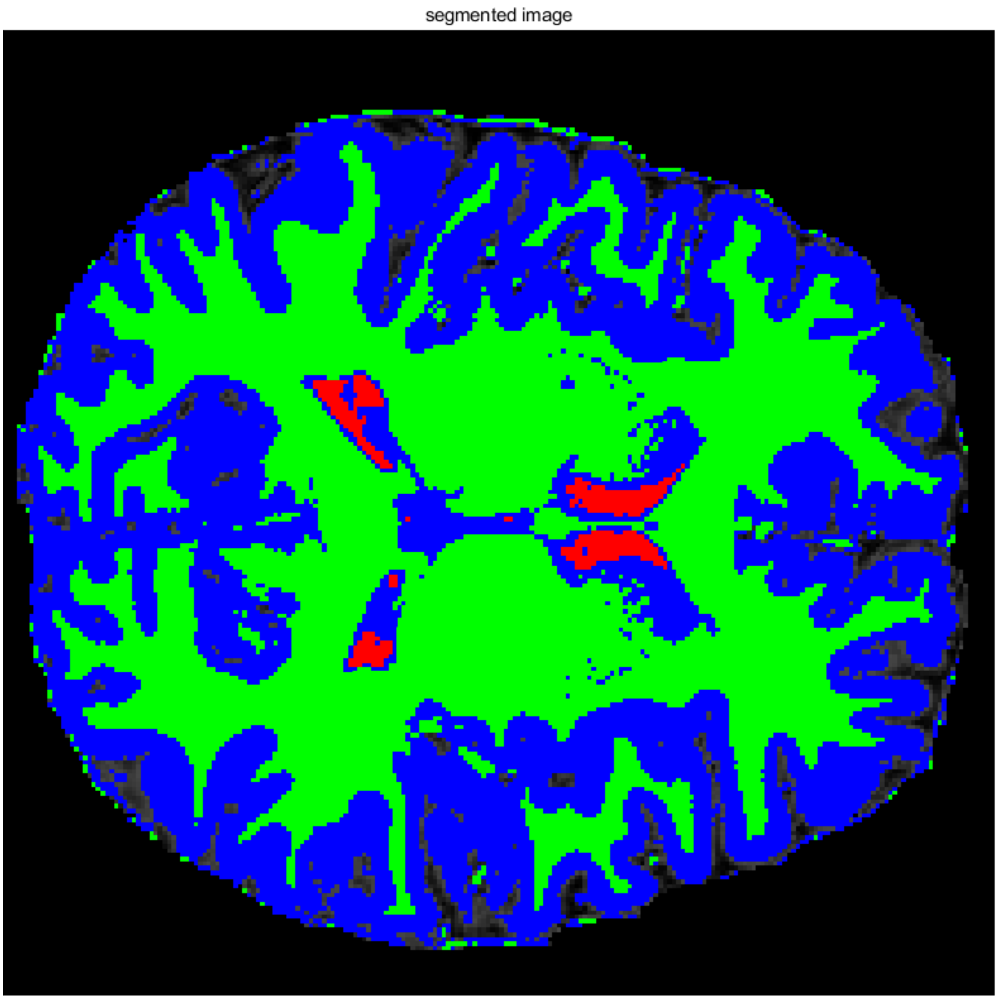
```
%% determine the proper thresholds
BW1 = im2bw(Image, 135/256);
BW2 = im2bw(Image, 90/256);
BW3 = im2bw(Image, 45/256);
%% mark different tissues with different indexs
matrix = zeros(224,224);
for i = 1:224
    for j = 1:224
        if BW1(i,j) == 1
            matrix(i,j) = 1;
        elseif BW2(i,j) == 1
            matrix(i,j) = 2;
        elseif BW3(i,j) == 1 && ...
                (66 < j) && (j < 157) && ...
                (78 < i) && (i < 148)
            matrix(i,j) = 3;
        else
            matrix(i,j) = 4;
        end
    end
end

%% convert the index to corresponding RGB value
map = [[1,0,0];[0,1,0];[0,0,1];[0,0,0]];
RGB = ind2rgb(matrix, map);
imshow(RGB);
```



Or you can set the threshold by looking at the histogram and apply gray matter, white matter and CSF into three RGB channels, as *Dou Yulong* did in the homework.

```
threshold1 = 150; %represent the threshold for 'white matter' and 'grey
matter'
threshold2 = 70;  %represent the threshold for 'grey matter' and 'CSF'
for i = [1:224]
    for j = [1:224]
        if((new_im(i,j,1) > threshold1) && (new_im(i,j,1) <= 255))
            new_im(i,j,1) = 0;
            new_im(i,j,2) = 255;
            new_im(i,j,3) = 0;
        end
        if((new_im(i,j,1) > threshold2) && (new_im(i,j,1) <= threshold1))
            new_im(i,j,1) = 0;
            new_im(i,j,2) = 0;
            new_im(i,j,3) = 255;
        end
        if((j >= 65 && j <= 163) && (i >= 74 && i <= 148)) %operate at the
center of imgae
            if((new_im(i,j,1) > 0) && (new_im(i,j,1) <= threshold2))
                new_im(i,j,1) = 255;
                new_im(i,j,2) = 0;
                new_im(i,j,3) = 0;
            end
        end
    end
end
imshow(new_im),title("segmented image");
```



There are also methods that auto-generate the threshold like Ostu method, but this is beyond the command of this homework. If you are interested, you can try but we do not have a requirement for this. Looking at the histogram is enough to achieve satisfactory effect for us.