

1. (8 points) True or False

For each statement, choose T if the statement is correct, otherwise, choose F.

Note: You should write down your answers in the box below.

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)

- (a) (1') The time complexity of graph traversal on a connected graph is $\Theta(V)$ where V is the number of vertices.
- (b) (1') A directed graph with n vertices has $n - 1$ edges may be strongly connected.
- (c) (1') In a simple undirected graph with n vertices and 3 connected components, the maximum number of edges in the graph is $\frac{(n-3)(n-2)}{2}$.
- (d) (1') It's convenient to use a stack to implement BFS.
- (e) (1') Union-Find/Disjoint-Set-Union cannot be used to determine reachability(whether a directed path from a given vertex to another vertex exists) on a directed graph.
- (f) (1') Union-Find/Disjoint-Set-Union can be used to solve connected component size problem (counting vertices in a connected component) on a undirected graph.
- (g) (1') A union tree generated by a union-by-rank DSU of n nodes has $\Theta(\log n)$ height.
- (h) (1') A union tree generated by a union-by-rank DSU of height h has $O(2^h)$ nodes.

The **Union-by-rank** strategy: When merging two sets A, B whose root is u and v respectively, if $\text{height}(u) < \text{height}(v)$, make u a child of v . That is: merge the tree with lower rank into the other one.

2. (5 points) Maze

Background(you can skip this:) After your friend gets lost in the dungeon for the 1234th time(in video game), he begs you to design a unified maze solver.

Goal: The maze is stored in a $(n \times m)$ -sized matrix. Each cell holds the information of whether its position has an obstacle (of course you can't pass the cell if there's an obstacle). You start from $(1, 1)$ and you want to go to (m, n) . Design an algorithm to find the shortest path from $(1, 1)$ to (m, n) .

Briefly explain your algorithm with **natural language**.

(hint: you should try build a graph first).

Example:

0	1	2	3	4	5
1	S				
2	1				
3	2				
4	3		7	8	9
5	4	5	6		T

The output of your graph should be shown in the picture. The length of the path is 10.