CS244 Theory of Computation
Homework 4

Due: Sunday, Dec 18, 2022 at 11:59pm

Name - ID

You may discuss this assignment with other students and work on the problems together. However, your write-up should be your own individual work and you should indicate in your submission who you worked with, if applicable. You should use the LaTeX template provided by us to write your solution and submit the generated PDF file into Gradescope.

I worked with: (Name, ID), (Name, ID), ...

Problem 1

(10 points)
Let $EQ_{\mathsf{BP}} = \{\langle B_1, B_2\rangle \mid B_1 \text{ and } B_2 \text{ are equivalent branching programs}\}$. Show that EQ_{BP} is coNP-complete.

Problem 2

(20 points)

(a) (10 points)Show that $A_{\mathsf{LBA}} = \{\langle B, w\rangle \mid B \text{ is an } \mathsf{LBA} \text{ that accepts input } w\}$ is PSPACE-complete.

(b) (10 points)Show that $E_{\mathsf{DFA}} = \{\langle A\rangle \mid A \text{ is a } \mathsf{DFA} \text{ and } L(A) = \emptyset\}$ is NL-complete.

Problem 3

(10 points)
Say that two Boolean formulas are ***equivalent*** if they have the same set of variables and are true on the same set of assignments to those variables (i.e., they describe the same Boolean function). A Boolean formula is ***minimal*** if no shorter Boolean formula is equivalent to it. (For definiteness, say that the length of a Boolean formula is the number of symbols it has.) Let $MIN\_FORMULA$ be the collection of minimal Boolean formulas.
Show that $MIN\_FORMULA \in$ PSPACE.

Problem 4

(20 points)
Describe a deterministic polynomial-time SAT-oracle Turing machine M^{SAT}

(a) (10 points)that takes as input a directed graph G and nodes s and t, and performs as follows

If a Hamiltonian path from s to t exists, outputs one.
If none exist, outputs **No Hamiltonian path**.

(b) (10 points)that takes as input a Boolean formula ϕ and performs as follows:

If ϕ is satisfiable, outputs a satisfying assignment of ϕ.
If ϕ is not satisfiable, outputs **Unsatisfiable**.

Problem 5

(20 points)
For any positive integer x, let $x^{\mathcal{R}}$ be the integer whose binary representation is the reverse of the binary representation of x. (Assume no leading `0`s in the binary representation of x.) Define the function $\mathcal{R}^+ : \mathcal{N} \to \mathcal{N}$ where $\mathcal{R}^+(x) = x + x^{\mathcal{R}}$.

(a) (10 points)Let $A_2 = \{\langle x, y\rangle \mid \mathcal{R}^+(x) = y\}$. Show $A_2 \in$ L.

(b) (10 points)Let $A_3 = \{\langle x, y\rangle \mid \mathcal{R}^+(\mathcal{R}^+(x)) = y\}$. Show $A_3 \in$ L.

Problem 6

(20 points)
Recall that $E_{\mathrm{CFG}} = \{\langle G\rangle \mid G \text{ is a CFG and } L(G) = \emptyset\}$. Say that a problem is NL-hard if all problems in NL are log-space reducible to it, even though it may not be in NL itself. (Similarly define NP-hard and PSPACE-hard.) It is not known whether $E_{\mathrm{CFG}} \in$ NL. Show that E_{CFG} is NL-hard.