SIST, ShanghaiTech University

Electromagnetics, Spring 2022

Midterm Homework

Deadline: 2022/4/27 22:00

Problem 1. (**10 points**)

(a) (**5 points**) In Cartesian coordinates, a vector $\vec{A}$ points from $P_1 = (\sqrt{3}, 1, 2)$ to $P_2 = (1, \sqrt{3}, -1)$. Express the vector $\vec{A}$ in cylindrical coordinates. Express the unit vector $\hat{r}$ and $\hat{\varphi}$ at point P_1 in terms of the unit vectors $\hat{x}, \hat{y}, \hat{z}$ in Cartesian coordinates.

(b) (**5 points**) In Cartesian coordinates, a vector $\vec{A}$ points from $P_1 = (2, 2, 2\sqrt{3})$ to $P_2 = (1, 1, \sqrt{3})$. Express the vector $\vec{A}$ in spherical coordinates. Express the unit vector $\hat{\theta}$ and $\hat{\varphi}$ at point P_1 in terms of the unit vectors $\hat{x}, \hat{y}, \hat{z}$ in Cartesian coordinates.

Problem 2. (**10 points**)

(a) (**6 points**) Write out the four Maxwell's equations for dynamic fields in their differential form using phasors. You are supposed to use $\vec{E}$, $\vec{H}$, $\vec{D}$ and $\vec{B}$. For the quantities regarding charge density or current density, specify what kinds of density they are (linear, surface or volume).

(b) (**2 points**) One of Maxwell's greatest contribution is adding an extra item to the Ampere's law for static fields, what is that item called? Also write out its expression.

(c) (**2 points**) From which two equations can you derive the wave equation? Write out the wave equation that $\vec{H}$ satisfies for waves in a lossy medium.

Problem 3. (**42 points**)

(a) (**12 points**) An infinitely large uniform steady positive surface charge density $2\rho_s$ resides in the $y = 0$ plane in free space. Another infinitely large uniform steady negative surface charge density $-2\rho_s$ resides in the $y = d$ plane in free space ($d > 0$). First, obtain the corresponding electric fields due to the positive charge density in the $y < 0$ region, $0 < y < d$ region, and $y > d$ region, which should be respectively denoted as $\vec{E}_1$, $\vec{E}_2$ and $\vec{E}_3$. Second, obtain the corresponding electric fields due to the negative charge density in the $y < 0$ region, $0 < y < d$ region, and $y > d$ region, which should be respectively denoted as $\vec{E}_4$, $\vec{E}_5$ and $\vec{E}_6$. Third, obtain the total electric fields in the $y < 0$ region, $0 < y < d$ region, and $y > d$ region, which should be respectively denoted as $\vec{E}_7$, $\vec{E}_8$ and $\vec{E}_9$. The charge sources are abstract sources, which means no real objects are needed to carry them and the fields can pass them. A figure needs to be clearly drawn to show how you define all the variables.

(b) (**6 points**) For a steady surface current density $2\hat{y}J_s$ on an infinitely large sheet residing in the *yoz* plane, obtain the magnetic field intensity generated by this source in both the $x > 0$ and $x < 0$ regions. A figure needs to be clearly drawn to show how you define all the variables.

(c) (**12 points**) An alternating surface current density $2\hat{z}J_s$ resides on an infinitely large sheet in the *xoz* plane. The $y < 0$ region is filled with a lossless material with intrinsic impedance η_1 and the $y > 0$ region is filled with a lossless material with intrinsic impedance η_2. Obtain the electric and magnetic field intensity generated by this source in both the $y > 0$ and $y < 0$ regions. A figure needs to be clearly drawn to show how you define all the variables.

(d) (**12 points**) An alternating surface current density $2\hat{y}J_s$ resides on an infinitely large sheet at $z = d$ with $d > 0$. An infinitely large planar PEC plate is placed at $z = 0$. First, obtain an image current of the original current source after the PEC is removed using the image theory. Need to specify its location, direction and magnitude. Second, obtain the electric and magnetic field intensity in the $z > 0$ region using the original current and the image current. **DO NOT use the field reflection method to do it.**

Problem 4. (**15 points**) Consider a *y*-polarized 2-GHz uniform plane wave propagating in +z direction in a nonmagnetic lossy medium with attenuation constant of α = 40 m^{-1}. The permeability of this lossy medium is μ_0 and real part of the permittivity is $5\varepsilon_0$. The electric initial phase is 0.

(a) (**3 points**) Calculate the distance of propagation after which the power density of this wave is attenuated to only 5% of its original value. Where does the lost power go or what is the lost power converted to? Also calculate the skin depth of this lossy medium.

(b) (**4 points**) Calculate the complex propagation constant γ of this wave. **Hint**: by carefully comparing the expressions for the attenuation constant and phase constant, you may find a simple way to calculate the phase constant.

(c) (**4 points**) Calculate the wavelength and phase velocity of this plane wave.

(d) (**4 points**) Write out the time-domain electric field intensity, assuming its amplitude at $z = 0$ is 2 V/m.

Problem 5. (**17 points**) The electric field intensity of a uniform plane wave in vacuum is given by

$$\vec{E} = \left[\left(-1 + j\sqrt{3}\right)\hat{x} - 2\hat{z}e^{j7\pi/6}\right]e^{-jky}$$

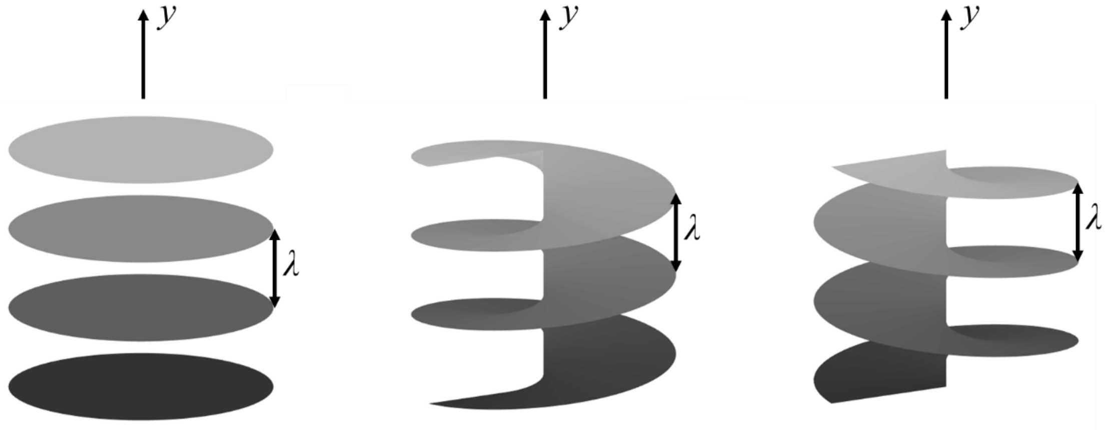
(a) (**3 points**) Determine the polarization of the wave. Detailed derivation is needed.

(b) (**4 points**) Write out the corresponding magnetic field intensity in phasor form. Just use η to represent the intrinsic impedance.

(c) (**3 points**) Does the magnitude of the electric field change with time and location? Show your reasoning.

(d) (**4 points**) Calculate the time-average power density of this wave in phasor form.

(e) (**3 points**) The wavefront of this wave should be which of the following ones.



(i) (ii) (iii)

Problem 6. (**6 points**) An infinitely long wire shown below carries a current I = 10 A along the z direction. A metal rod near the wire moves with a constant velocity $\vec{u} = \hat{z}10\text{m/s}$. Calculate the induced voltage V_{12}, where the footnotes 1 and 2 denotes the ends of the metal rod.