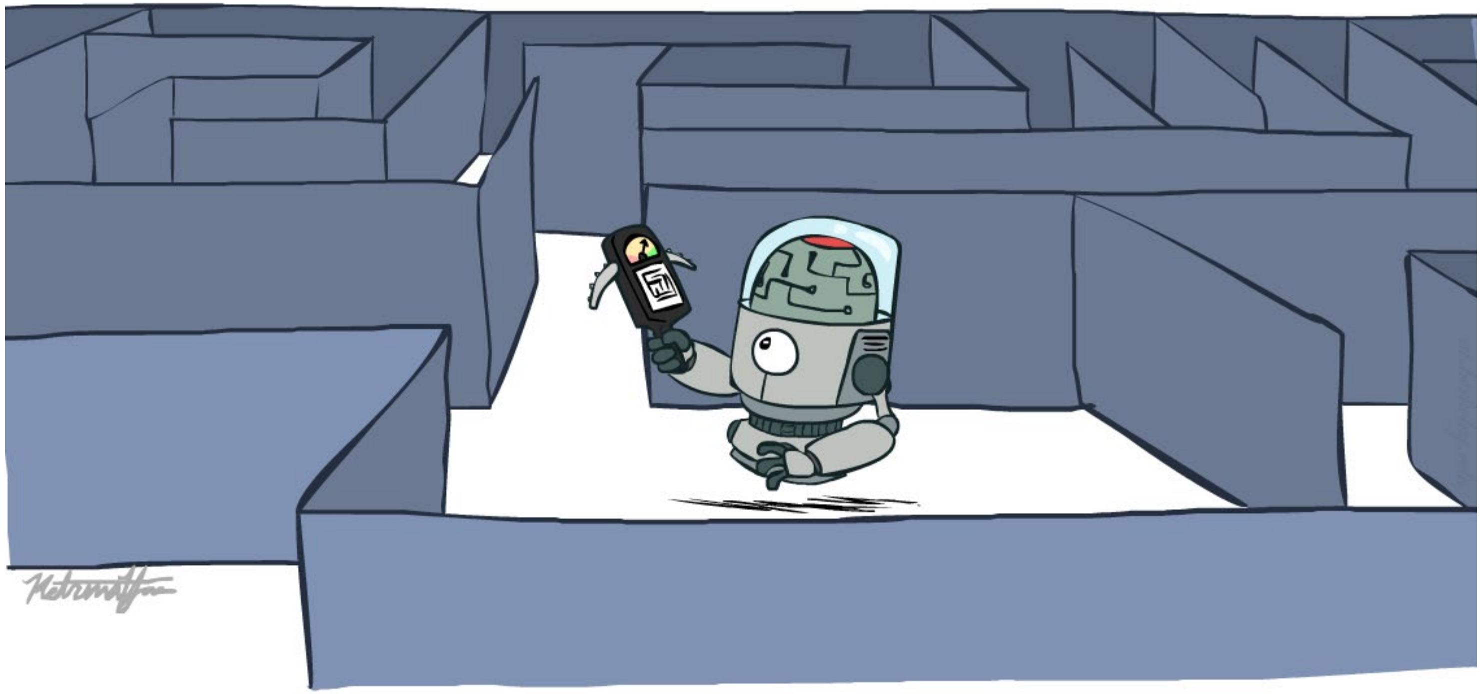
Announcement

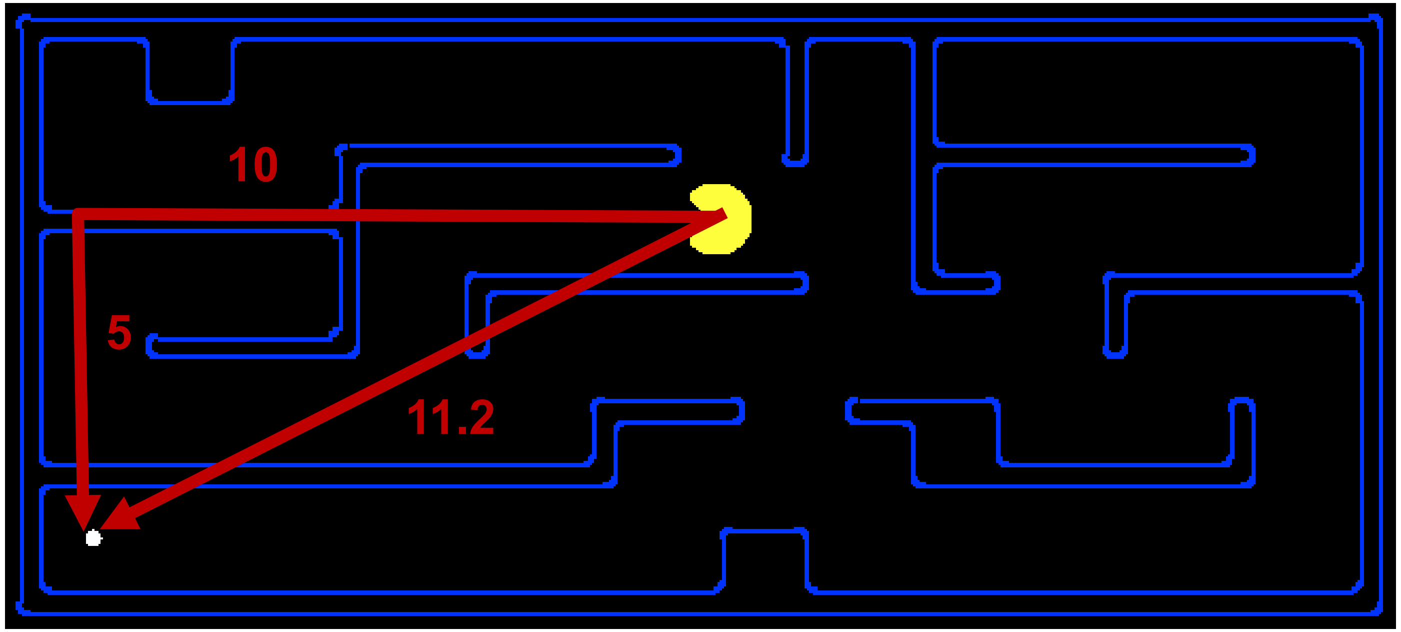
- TA office hour
  - Wed evening: 20:00-21:00
  - Location: SIST 1B-101
- Programming Assignment 1:
  - Released on Thursday
- Autolab registration
  - See email or BB announcement

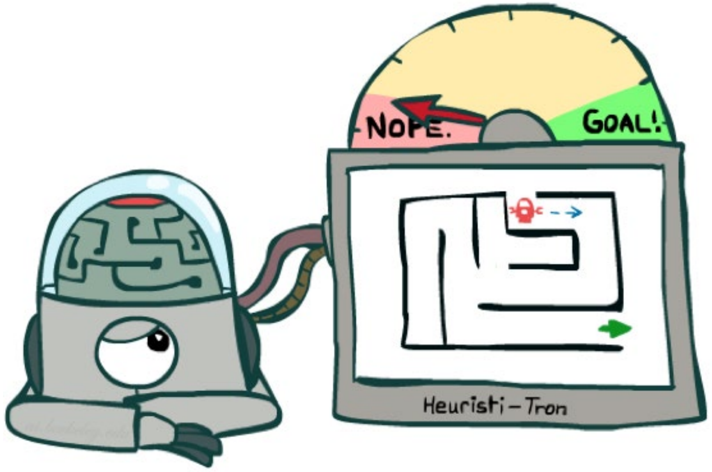
Informed Search

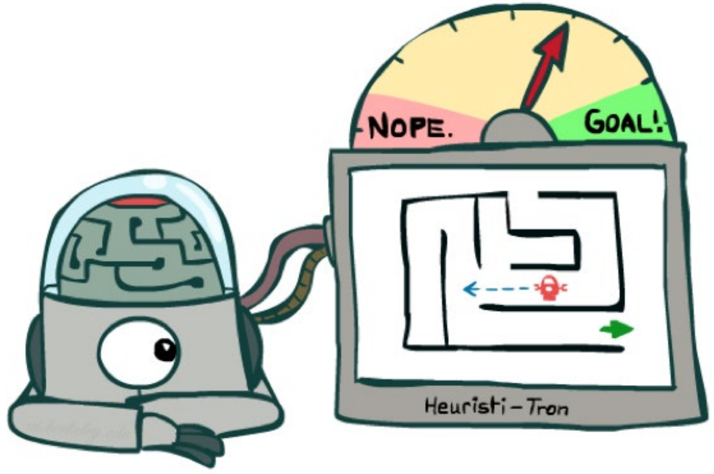


Search Heuristics

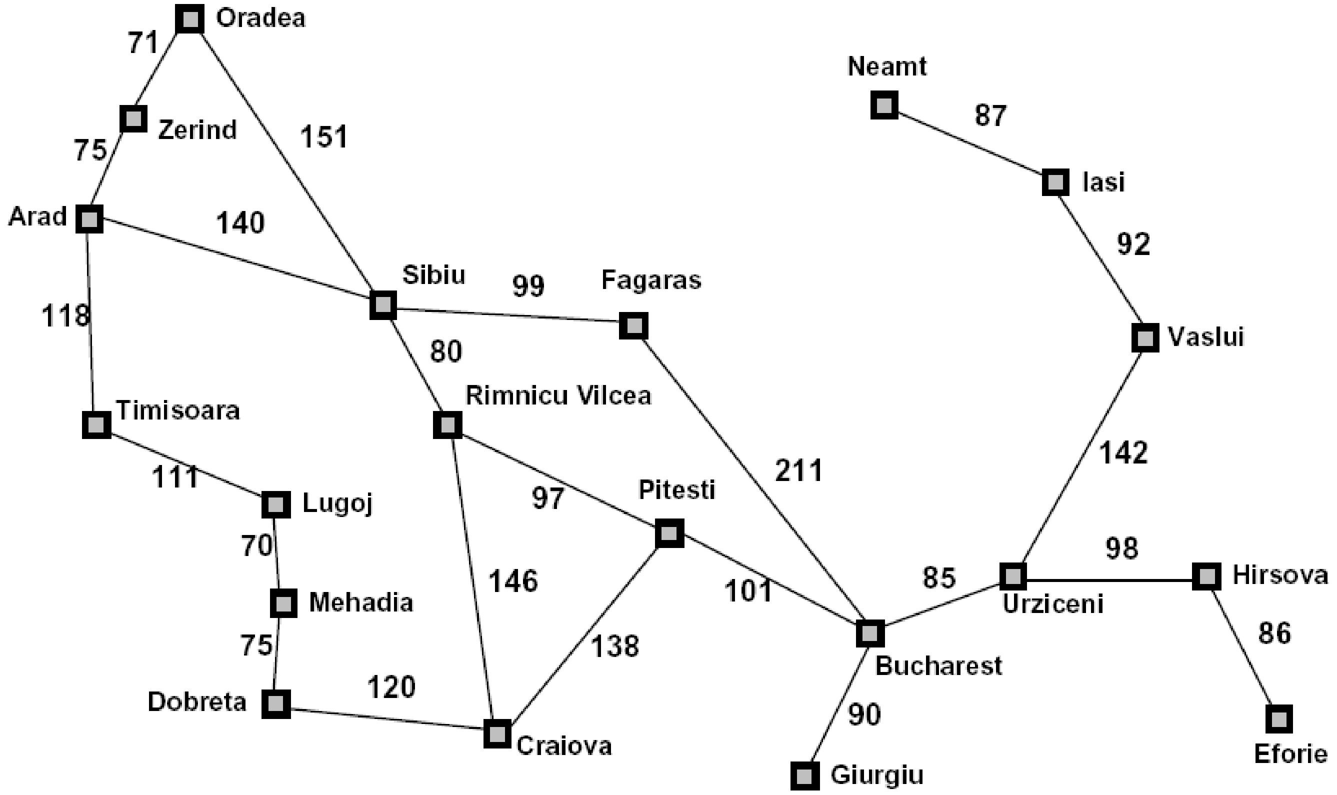
- A heuristic h is:
  - A function that *estimates* how close a state is to a goal
    - h(goal)=0
  - Designed for a particular search problem
  - Examples: Manhattan distance, Euclidean distance for pathing







Example: Heuristic Function

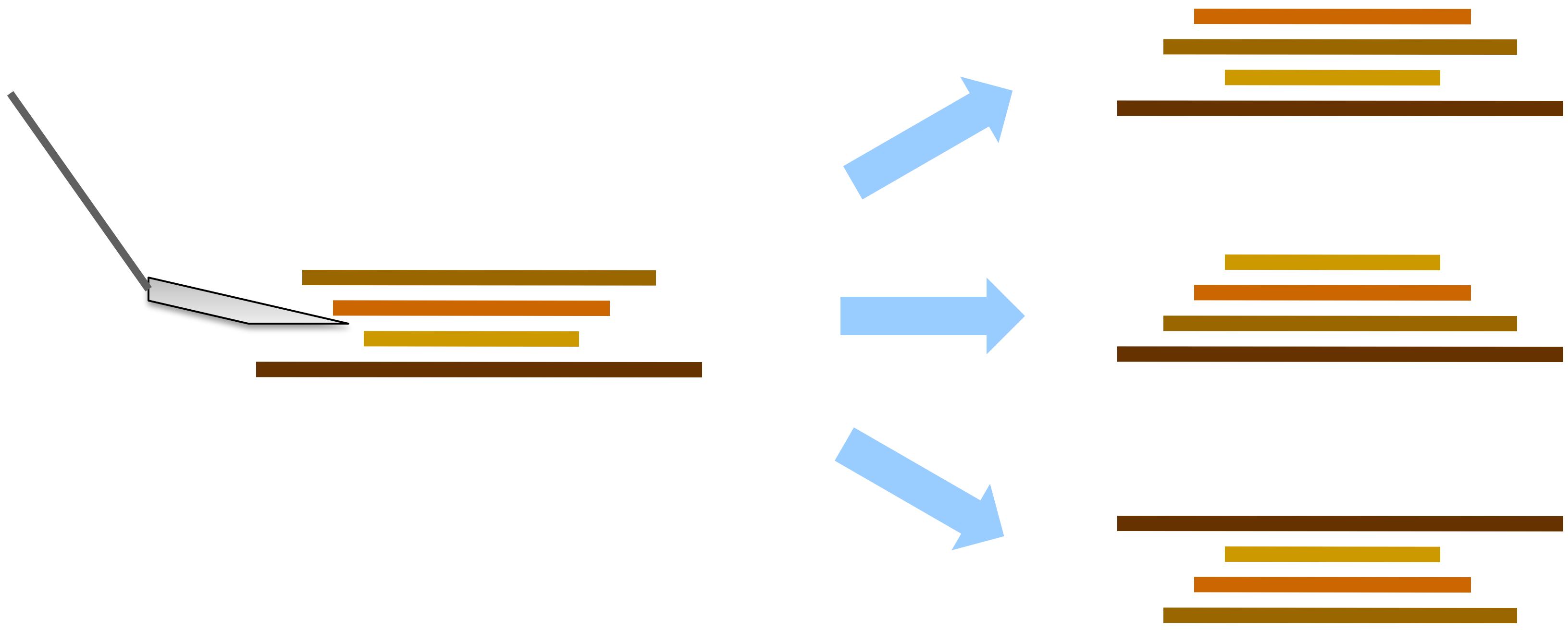


Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

h(x)

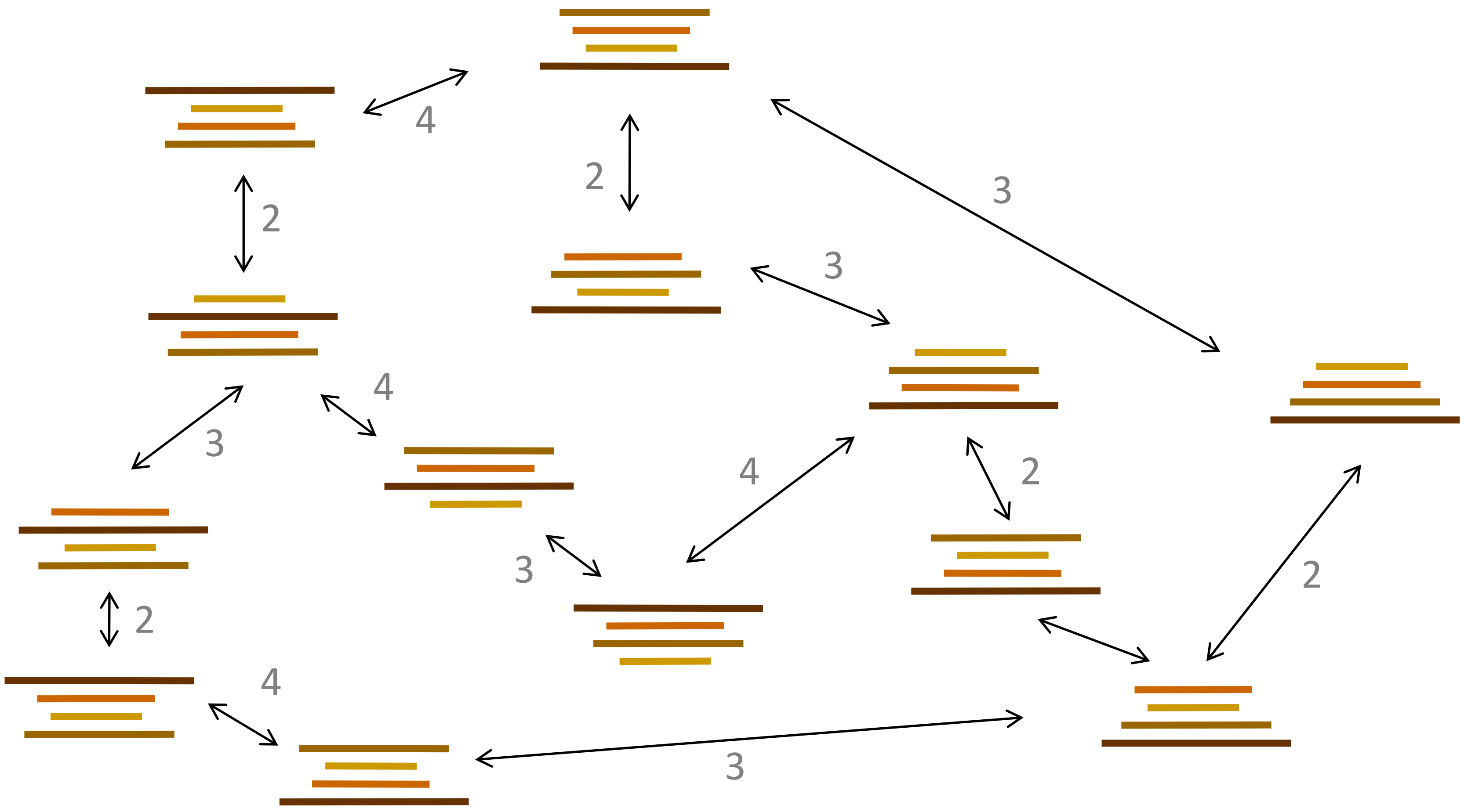
Example: Pancake Problem



Cost: Number of pancakes flipped

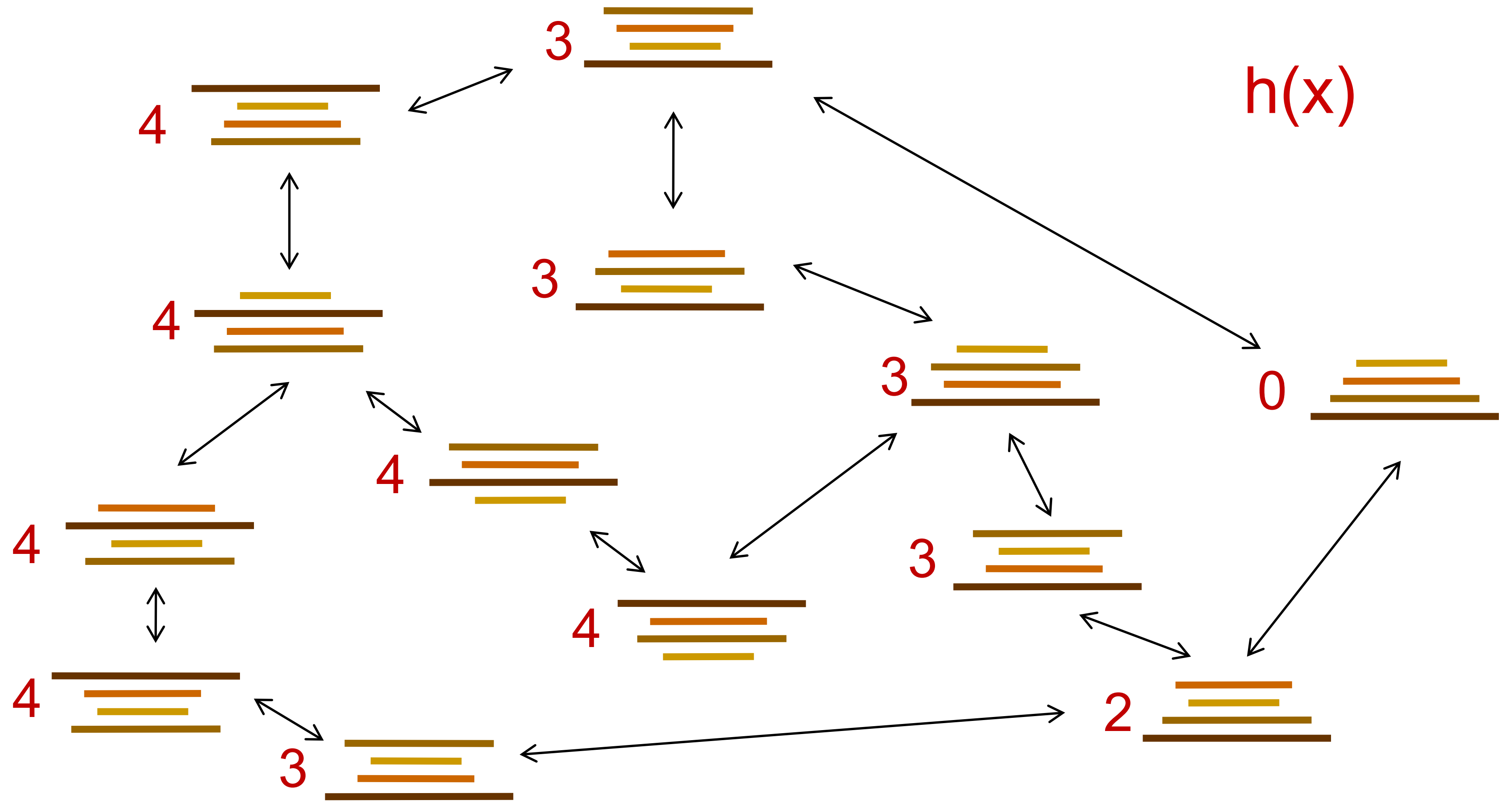
Example: Pancake Problem

State space graph with costs as weights



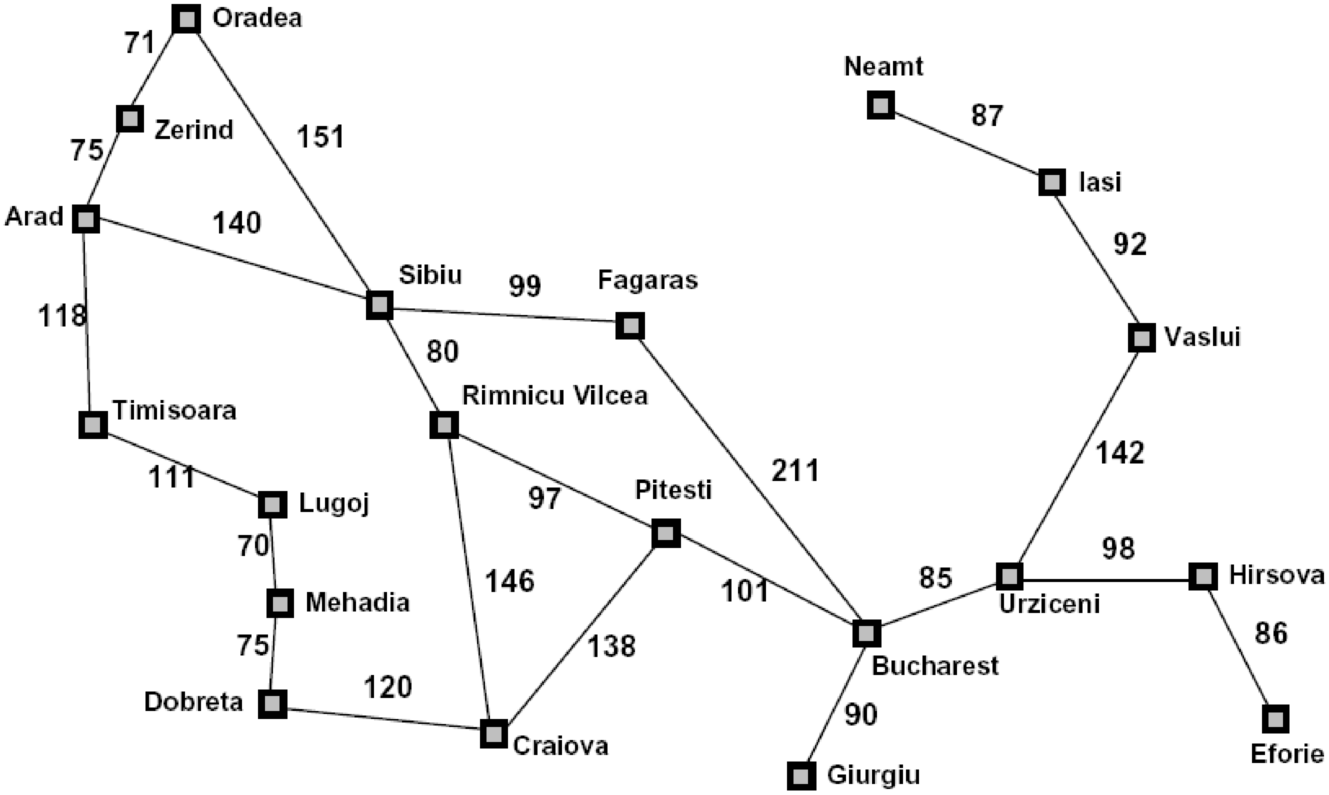
Example: Heuristic Function

Heuristic: the number of the largest pancake that is still out of place



Greedy Search

Example: Heuristic Function



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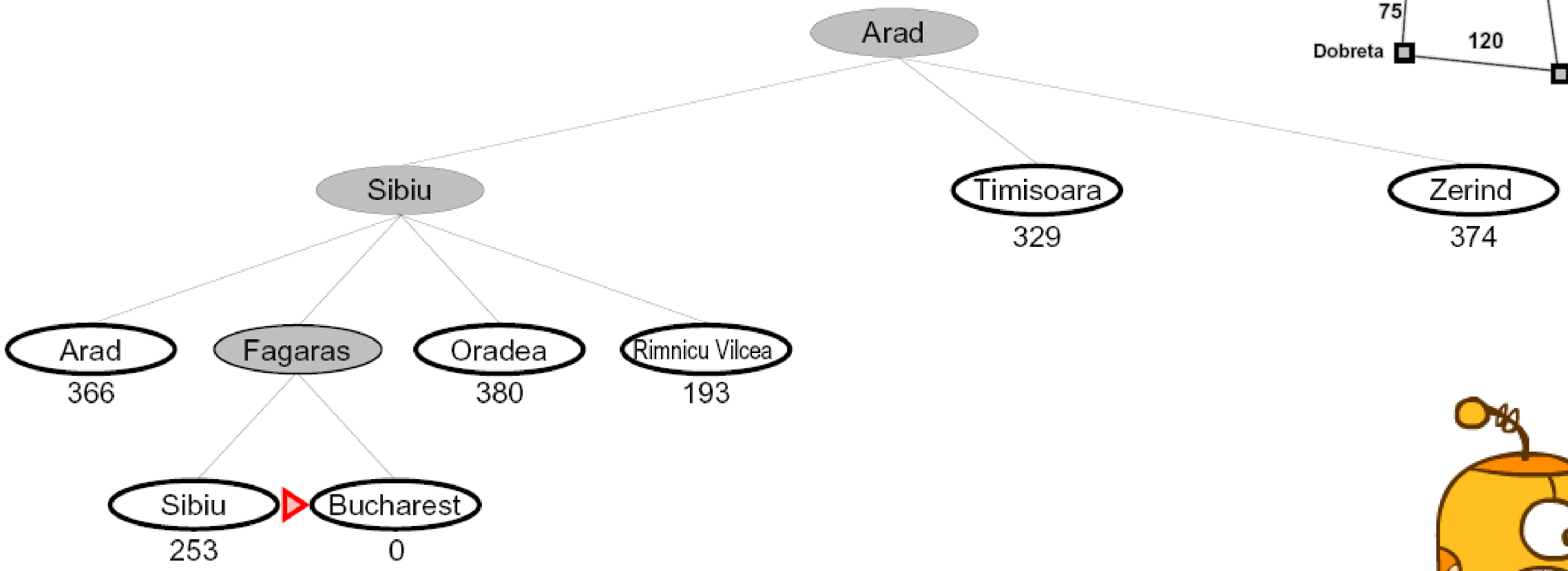
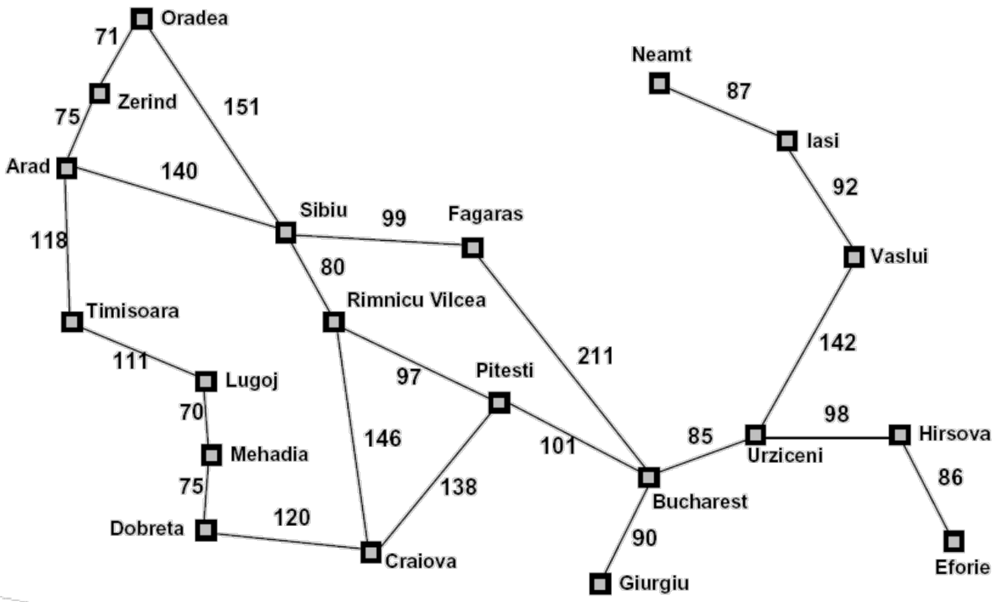
h(x)

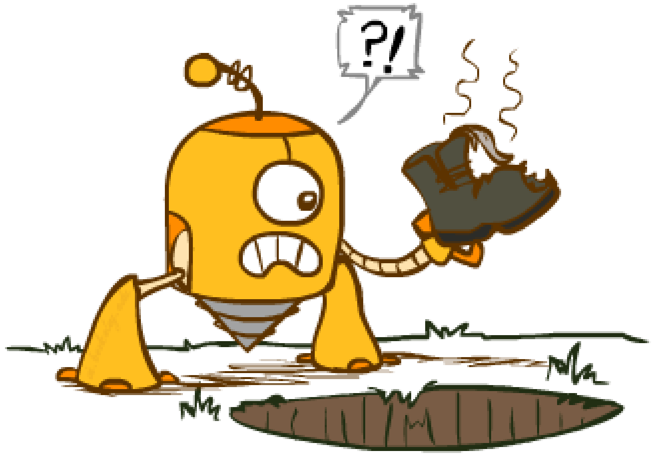
Greedy Search

- Expand the node that seems closest…





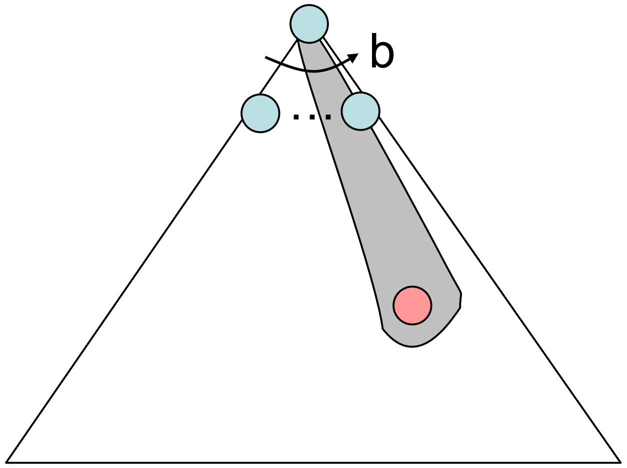
- What can go wrong?



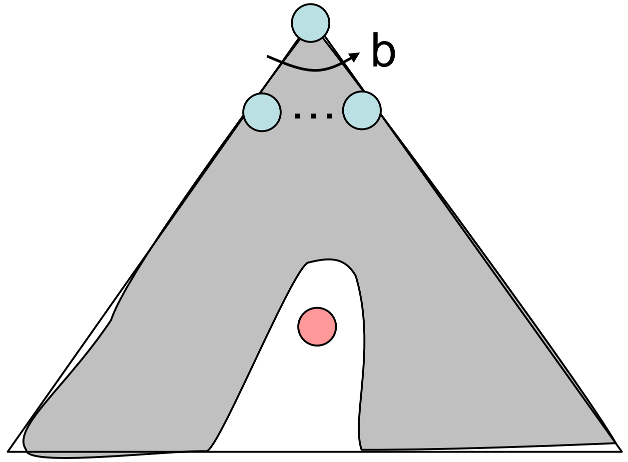
Greedy Search

- Strategy: expand a node that you think is closest to a goal state

- The ideal scenario:
  - Best-first takes you straight to the goal



- Worst-case: like a badly-guided DFS



Video of Demo Contours Greedy (Pacman Small Maze)

A* Search

A* Search

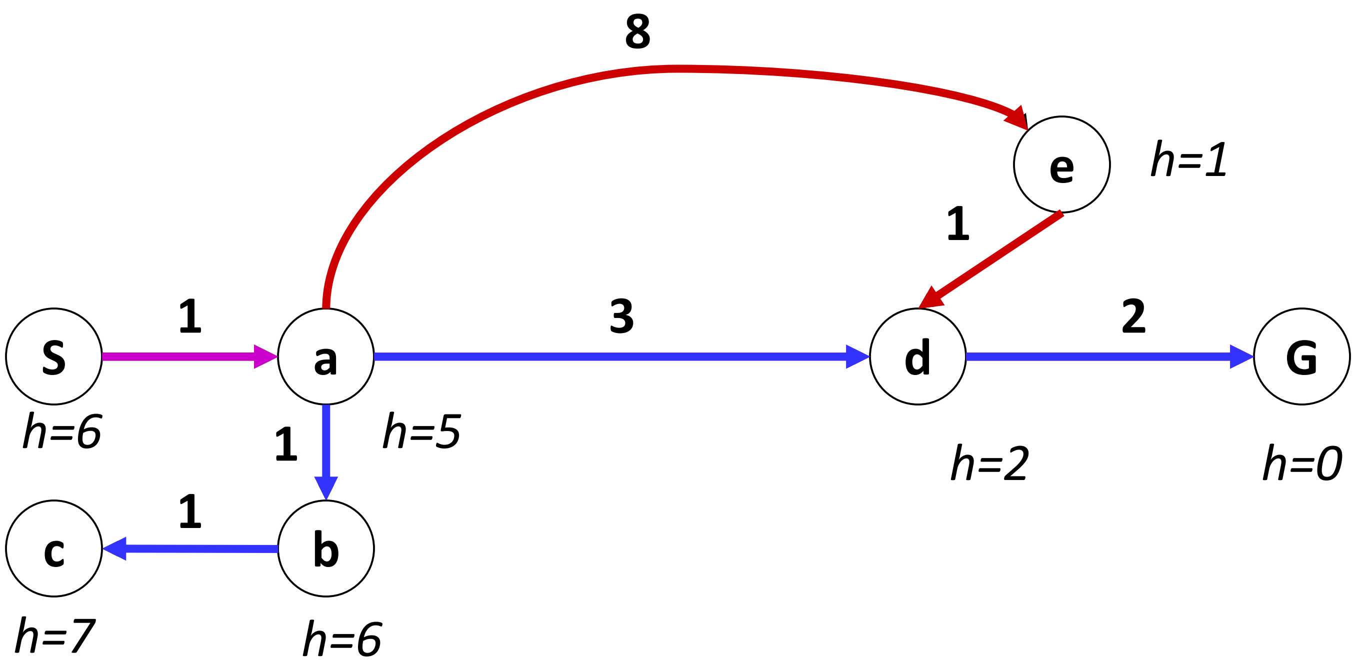
UCS

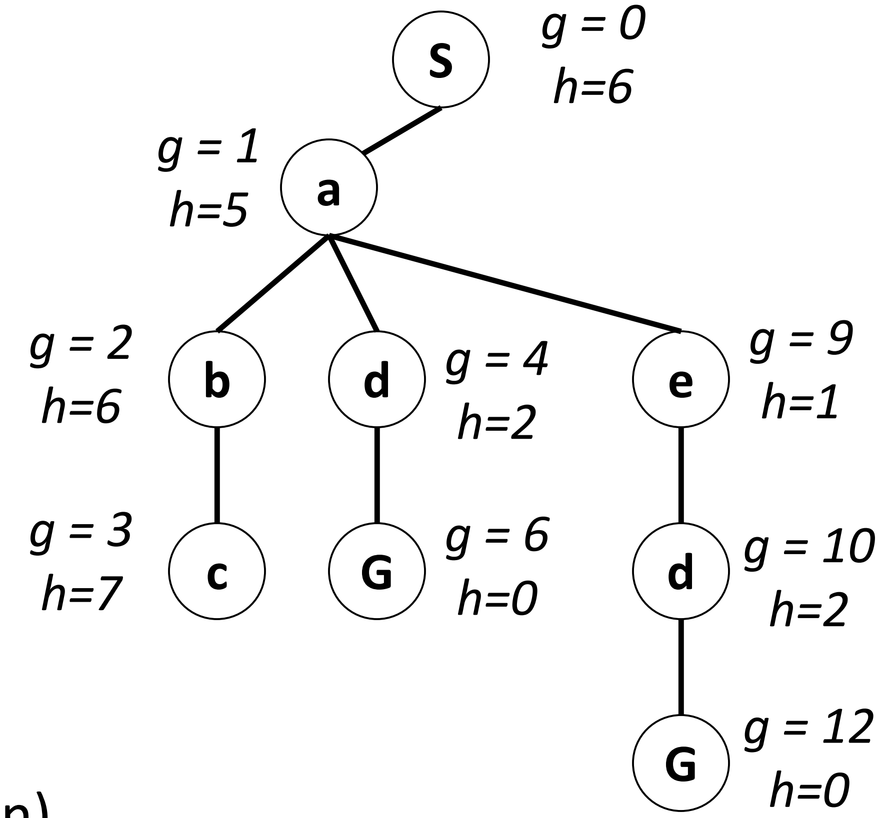
Greedy

A*

Combining UCS and Greedy

- Uniform-cost orders by path cost, or *backward cost* g(n)
- Greedy orders by goal proximity, or *forward cost* h(n)



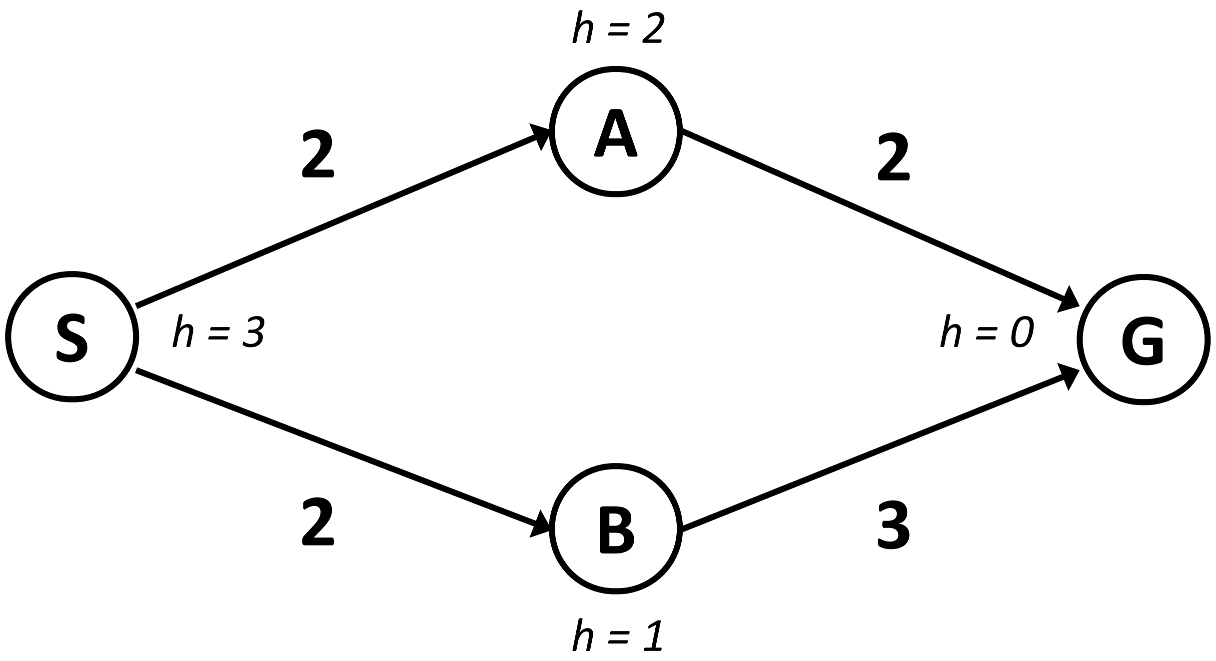


- A* Search orders by the sum: f(n) = g(n) + h(n)

Example: Teg Grenager

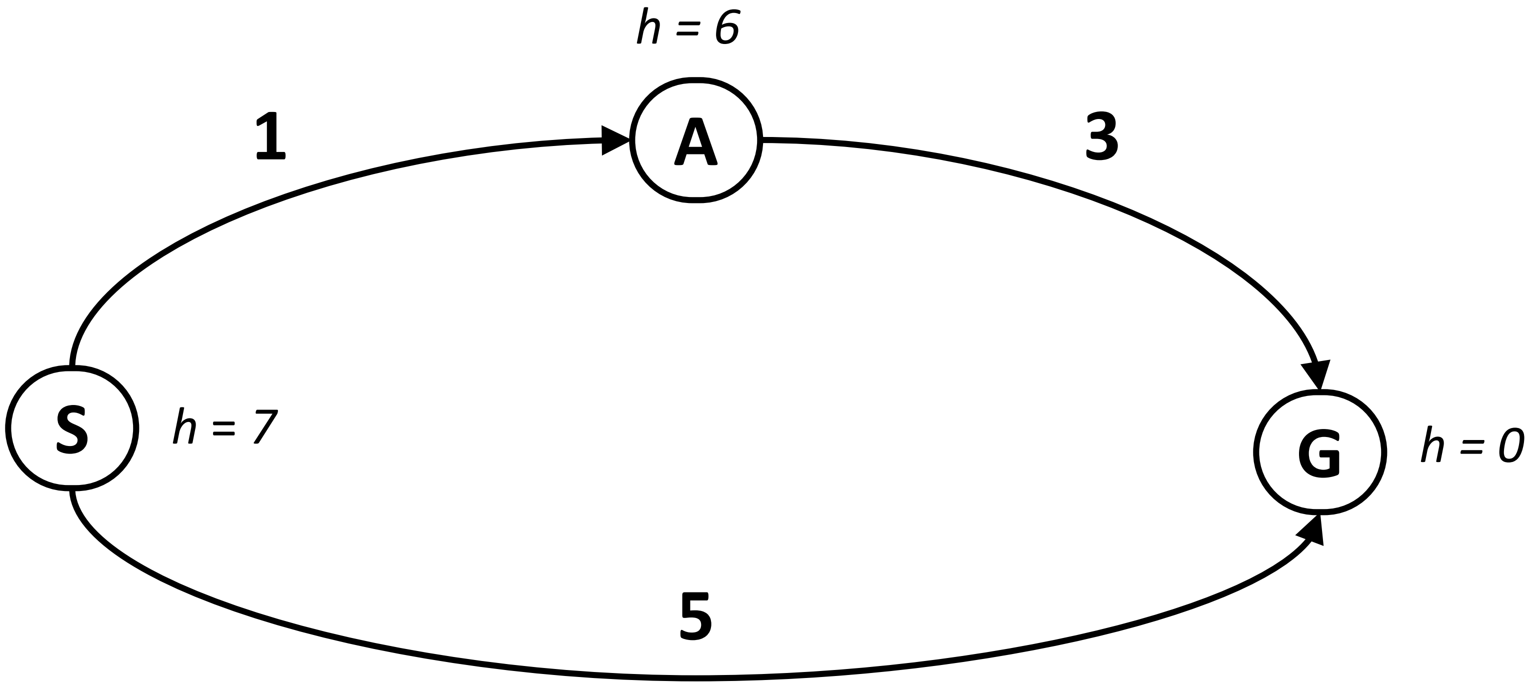
When should A* terminate?

- Should we stop when we enqueue a goal?



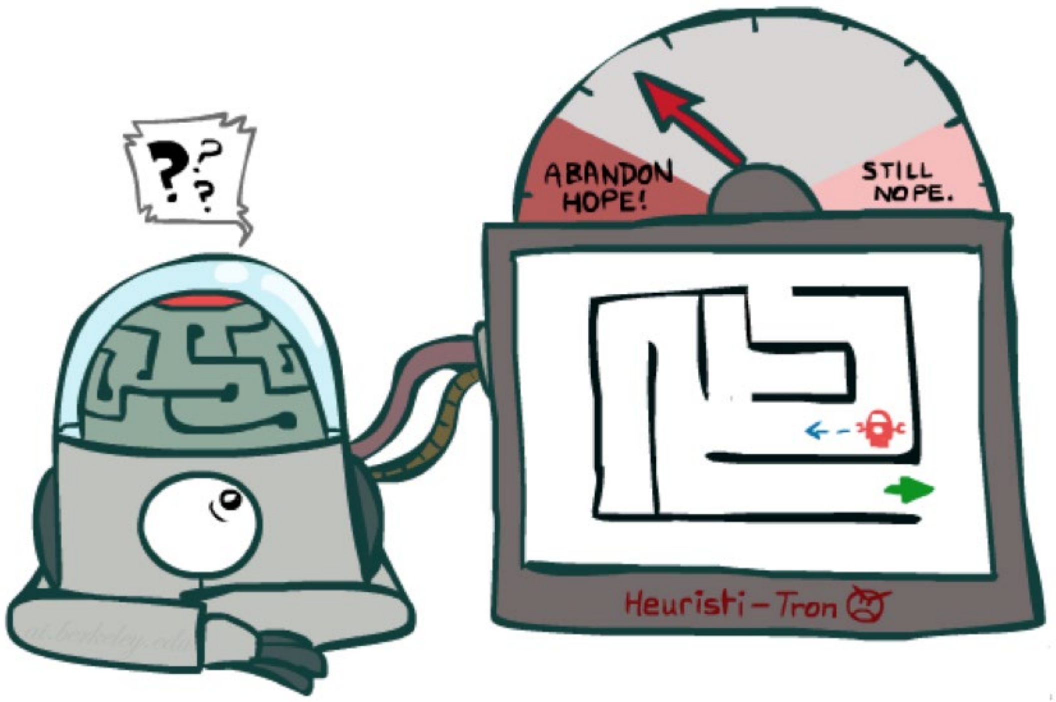
- No: only stop when we dequeue a goal

Is A* Optimal?

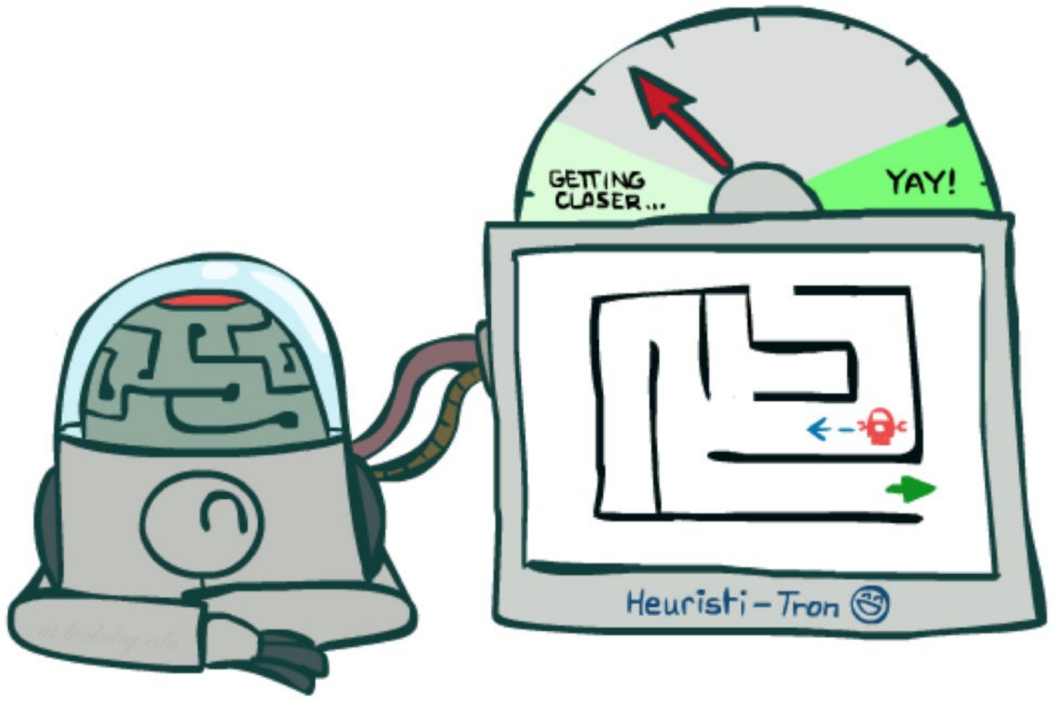


- What went wrong?
- Over-estimated goal cost

Idea: Admissibility



Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe



Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

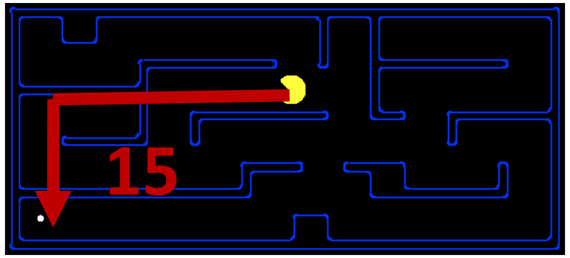
Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

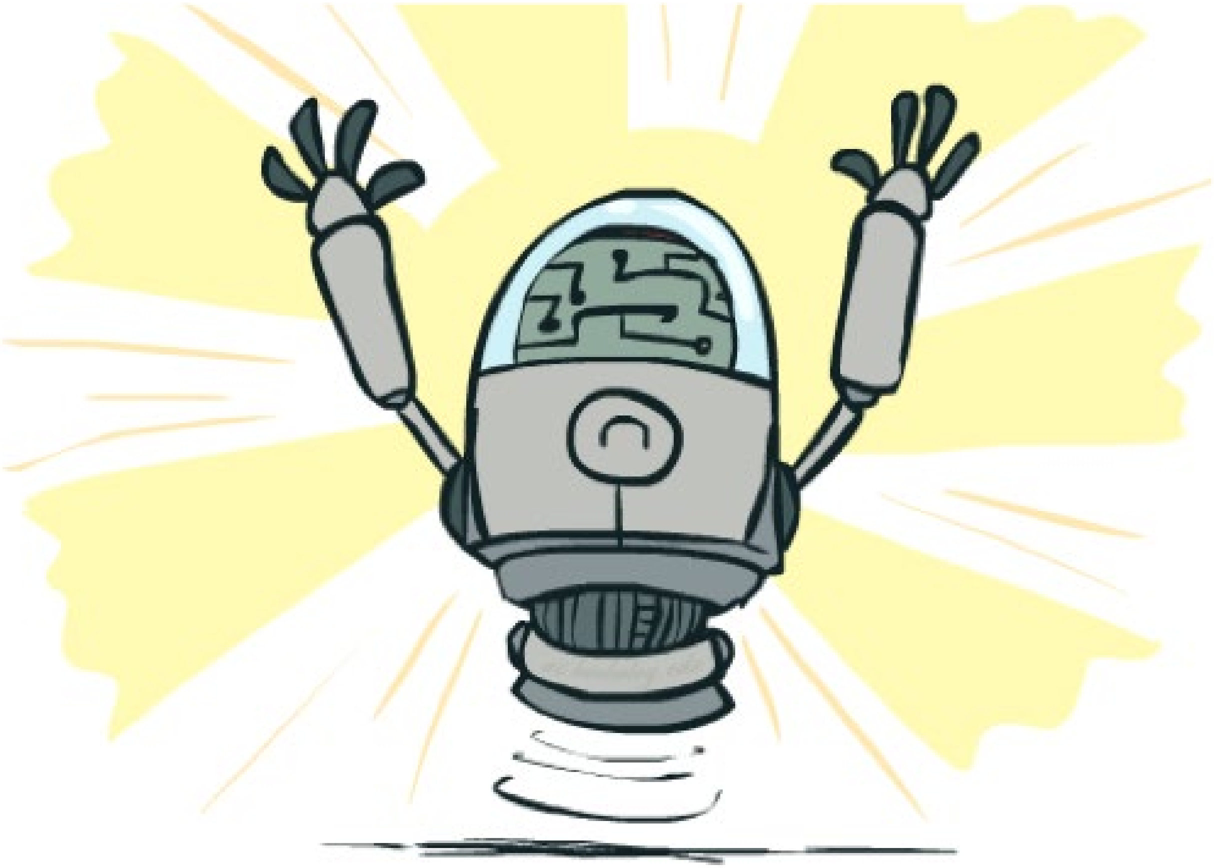
where $h^*(n)$ is the true cost to a nearest goal

- Examples:



- Coming up with admissible heuristics is most of what's involved in using A* in practice.

Optimality of A* Tree Search



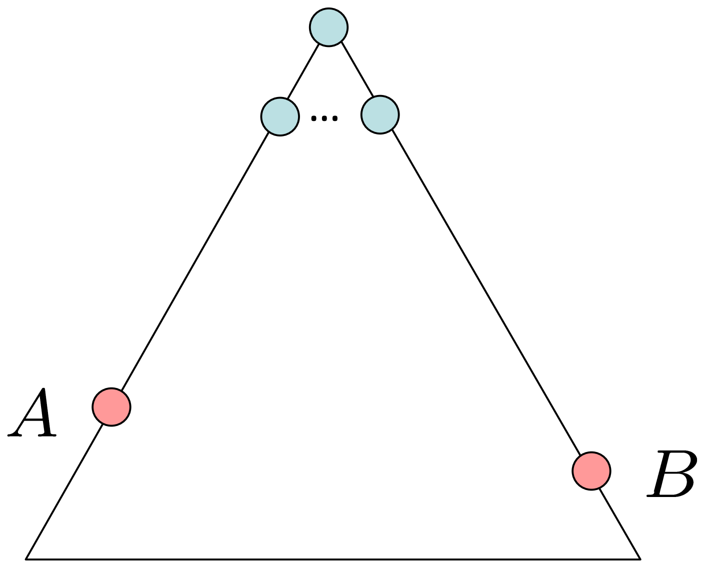
Optimality of A* Tree Search

Assume:

- A is an optimal goal node
- B is a suboptimal goal node
- h is admissible

Claim:

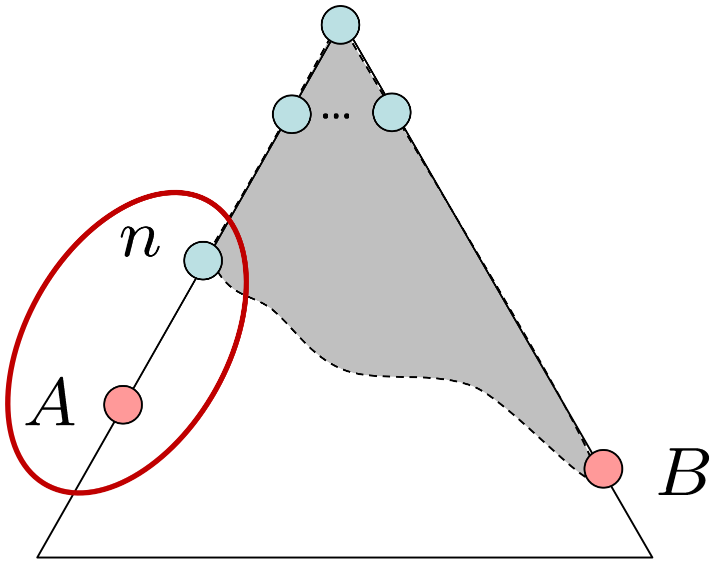
- A will exit the fringe before B



Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)

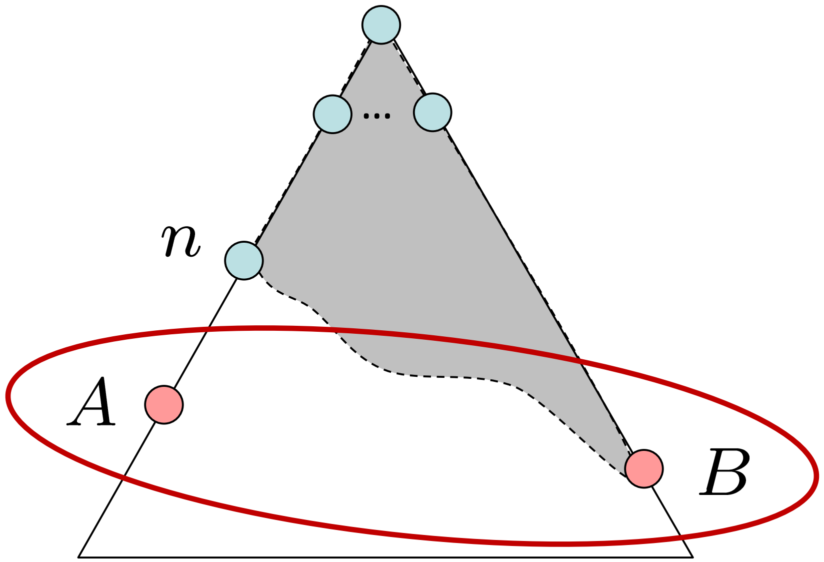


$f(n) = g(n) + h(n)$	Definition of f-cost
$f(n) \leq g(A)$	Admissibility of h
$g(A) = f(A)$	h = 0 at a goal

Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)
  2. f(A) is less than f(B)

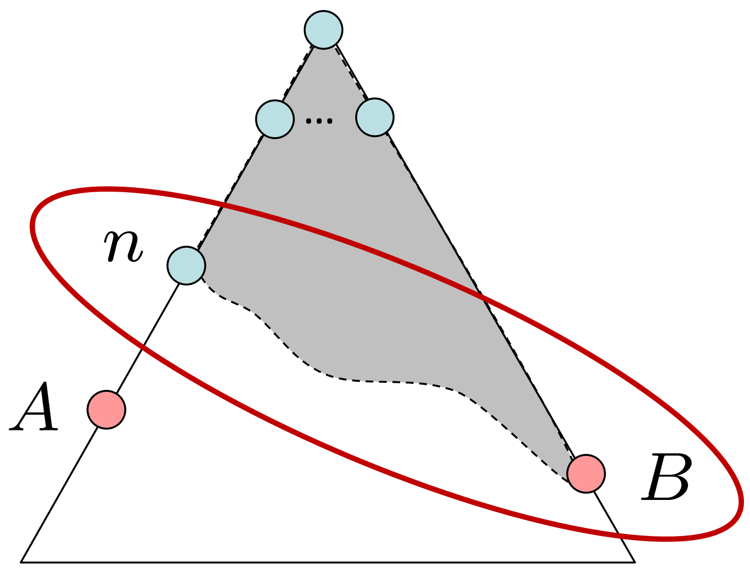


$g(A) < g(B)$	B is suboptimal
$f(A) < f(B)$	h = 0 at a goal

Optimality of A* Tree Search: Blocking

Proof:

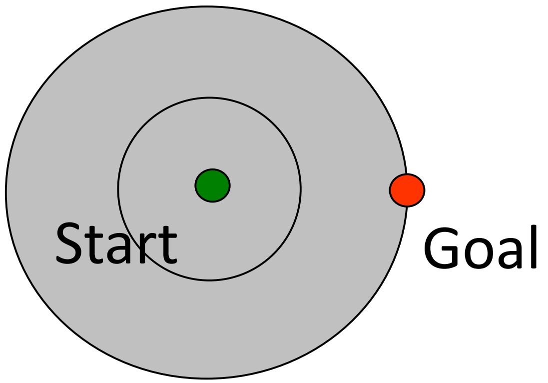
- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)
  2. f(A) is less than f(B)
  3. *n* expands before B
- All ancestors of A expand before B
- A expands before B
- A* search is optimal



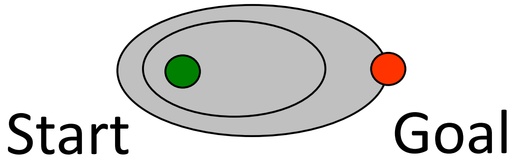
$$f(n) \leq f(A) < f(B)$$

UCS vs A* Contours

- Uniform-cost expands equally in all "directions"



- A* expands mainly toward the goal, but does hedge its bets to ensure optimality



Video of Demo Maze with Deep/Shallow Water --- UCS, Greedy, A*

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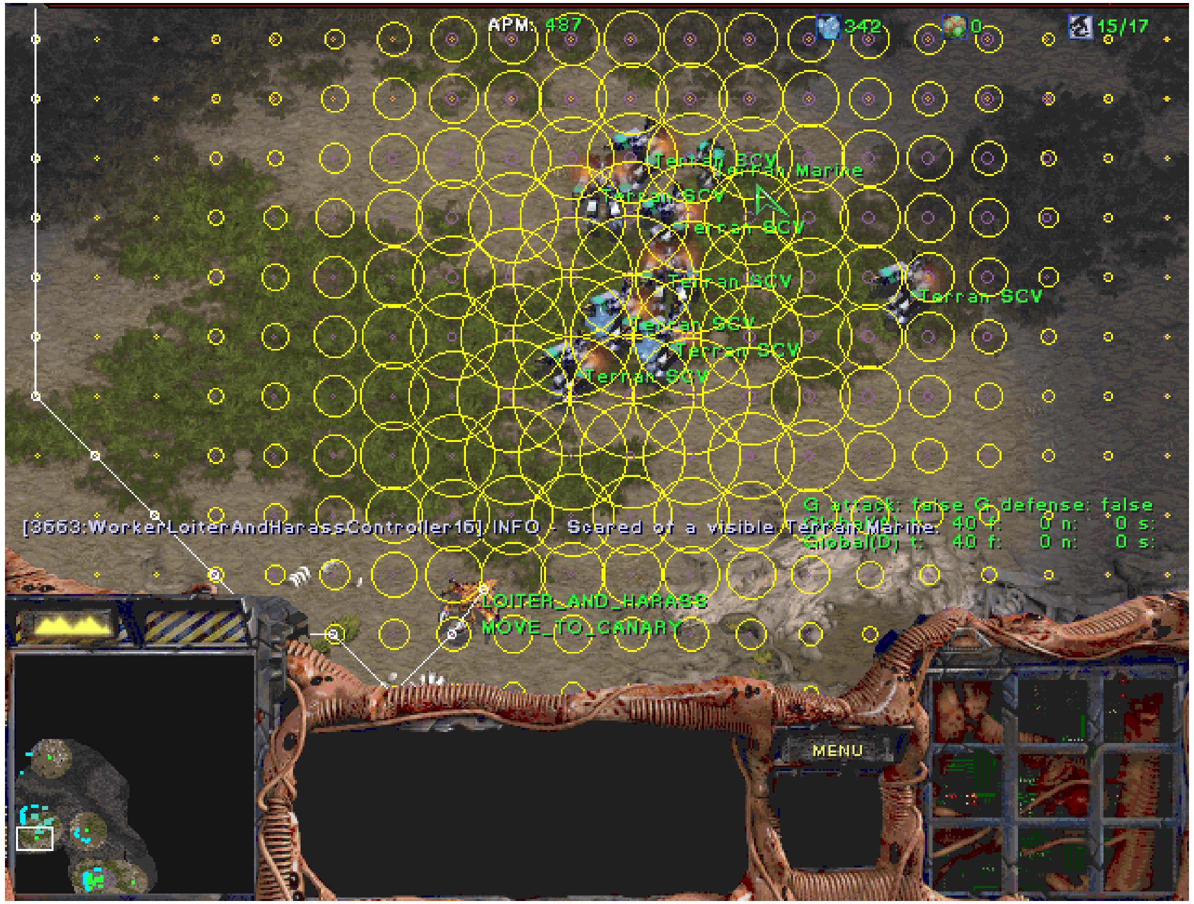
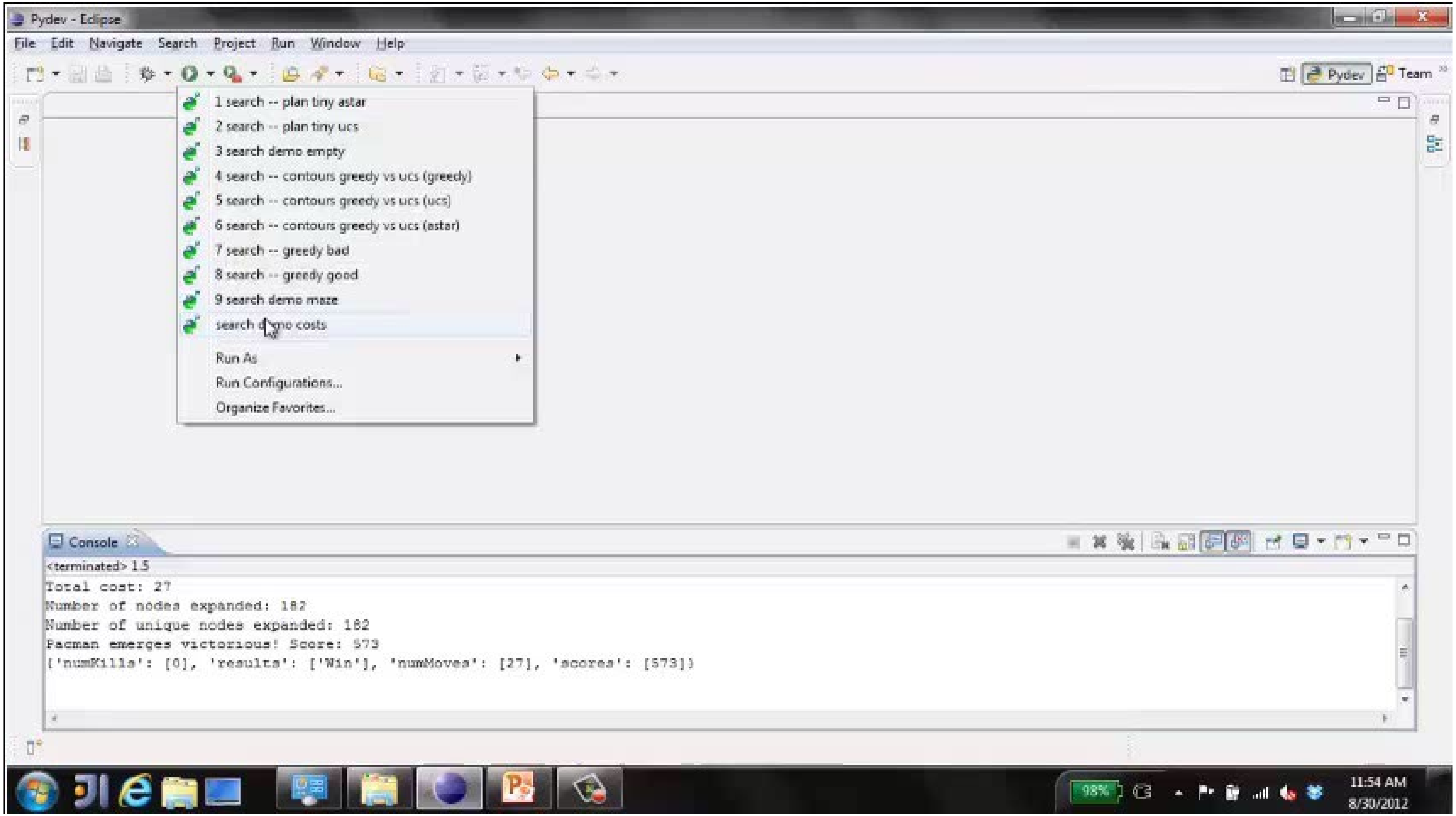
A* Applications

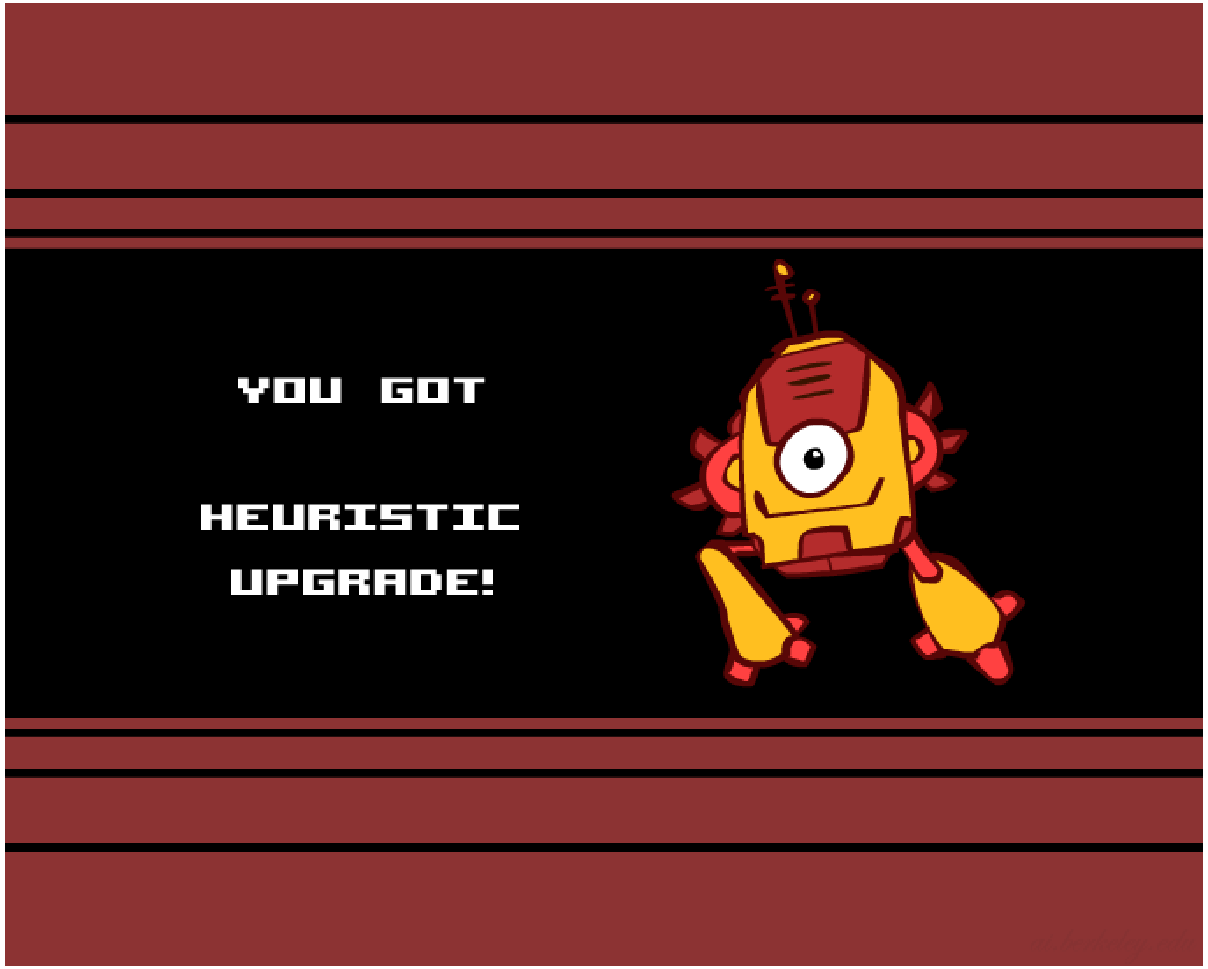
- Video games
- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- …



Video of Demo Empty Water Shallow/Deep – Guess Algorithm



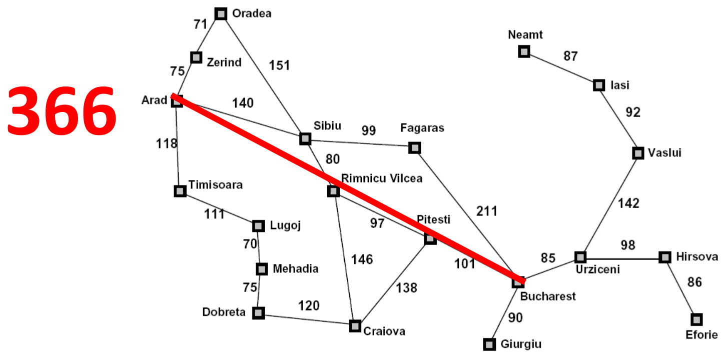
Creating Heuristics

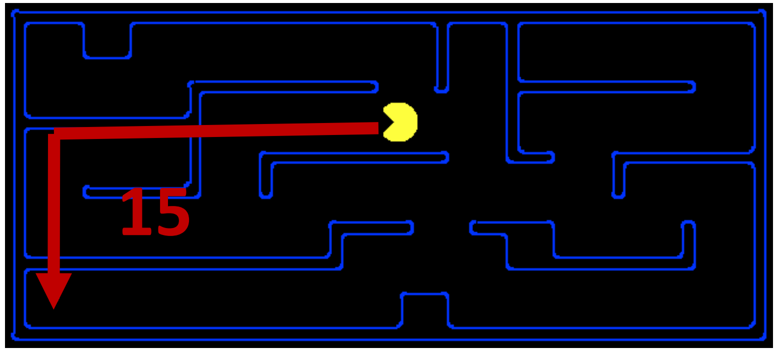


Creating Admissible Heuristics

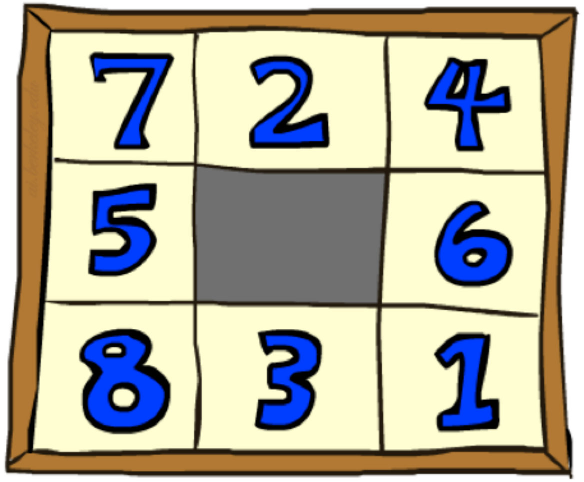
- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics
  - Inadmissible heuristics are often useful too

- Often, admissible heuristics are solutions to *relaxed problems*, where new actions are available

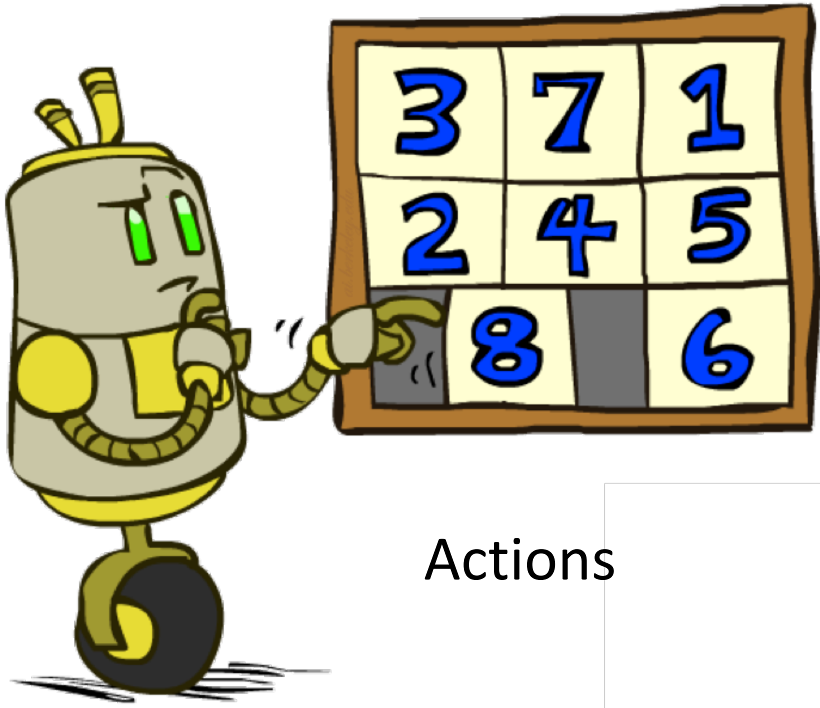




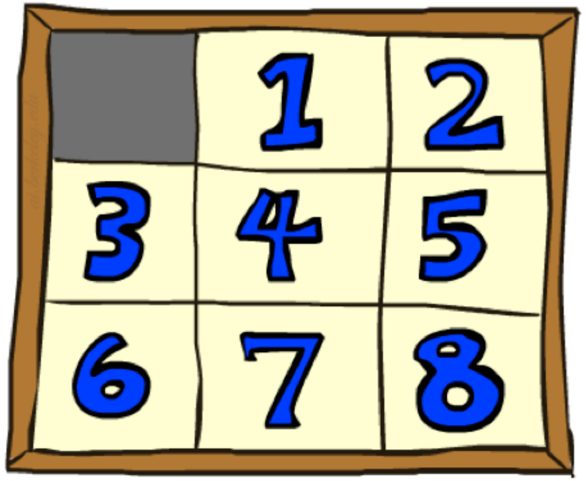
Example: 8 Puzzle



Start State



Actions

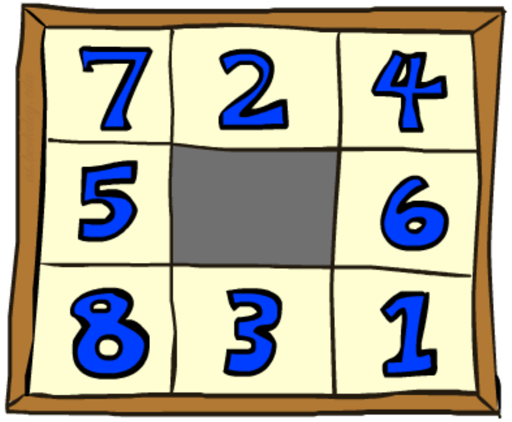


Goal State

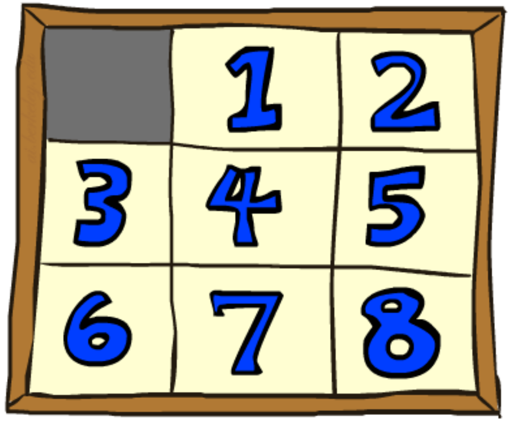
- What are the states?
- How many states?
- What are the actions?
- How many successors from the start state?
- What should the costs be?

8 Puzzle I

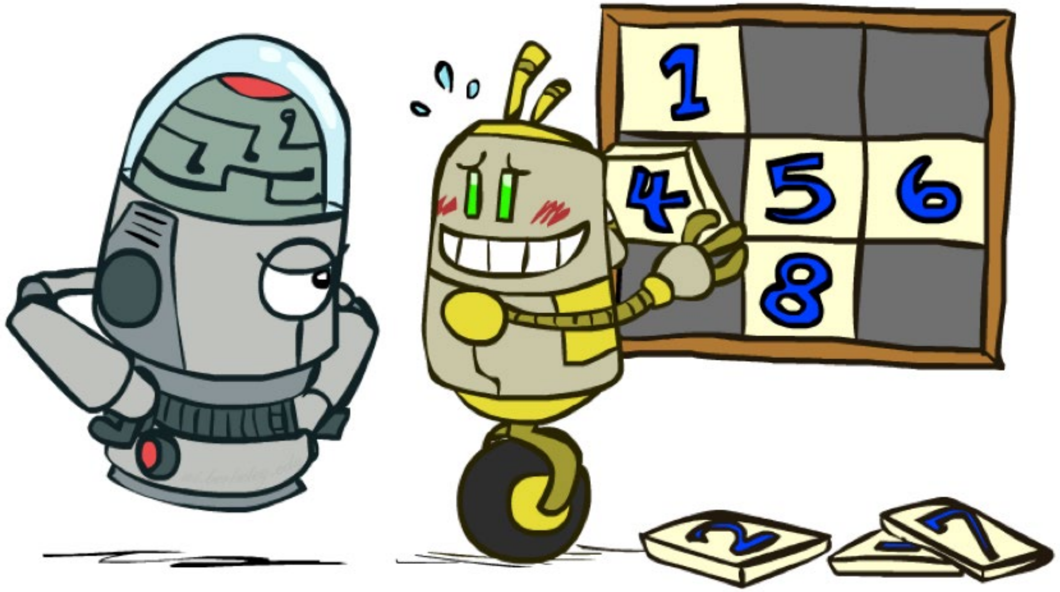
- Heuristic: Number of tiles misplaced
- h(start) = 8
- Is it admissible?
- This is a *relaxed-problem* heuristic



Start State



Goal State

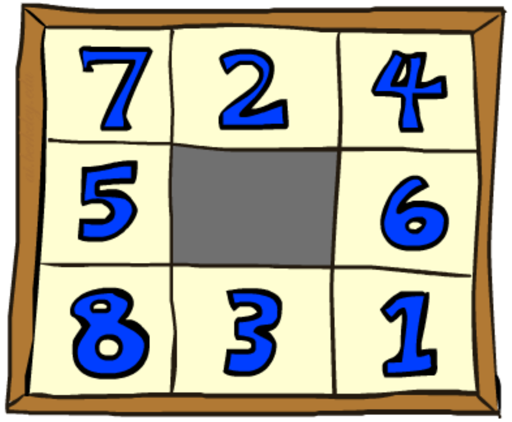


	Average nodes expanded when the optimal path has…		
	…4 steps	…8 steps	…12 steps
UCS	112	6,300	3.6×10^6
TILES	13	39	227

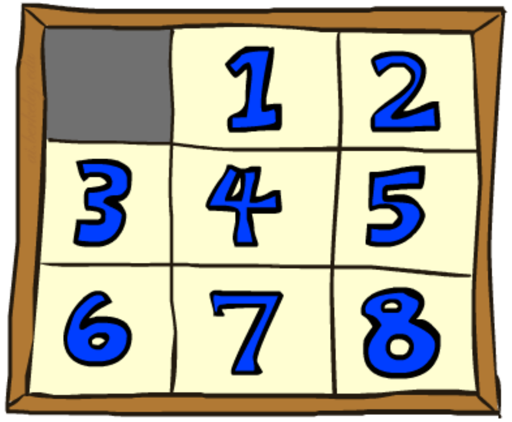
Statistics from Andrew Moore

8 Puzzle II

- Heuristc: total *Manhattan* distance
- h(start) = 3 + 1 + 2 + … = 18
- Is it admissible?
- *Relaxed-problem:* any tile could slide in any direction at any time, ignoring other tiles



Start State

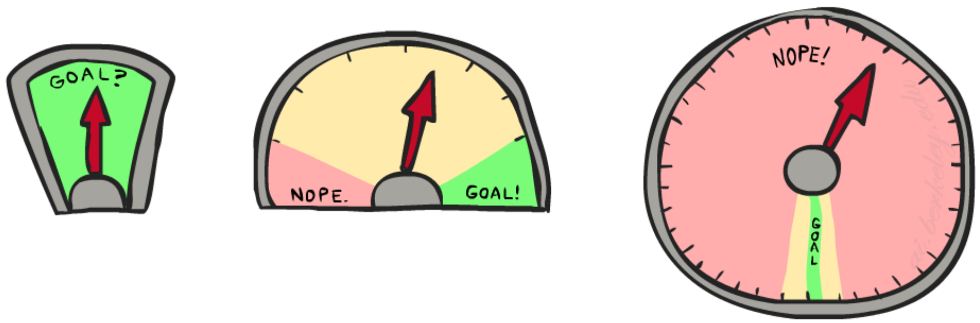


Goal State

	Average nodes expanded when the optimal path has…		
	…4 steps	…8 steps	…12 steps
TILES	13	39	227
MANHATTAN	12	25	73

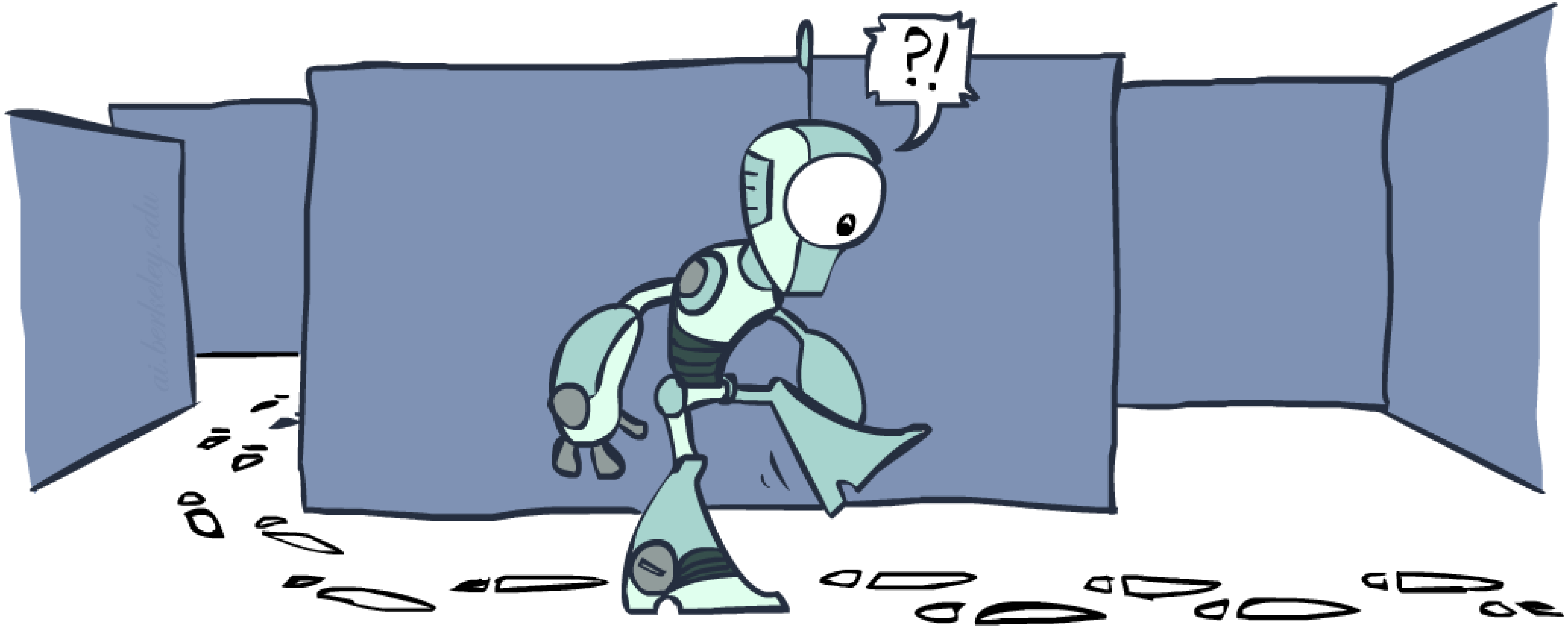
8 Puzzle III

- How about using the *actual cost* as a heuristic?
  - Would it be admissible?
  - Would we save on nodes expanded?
  - What's wrong with it?



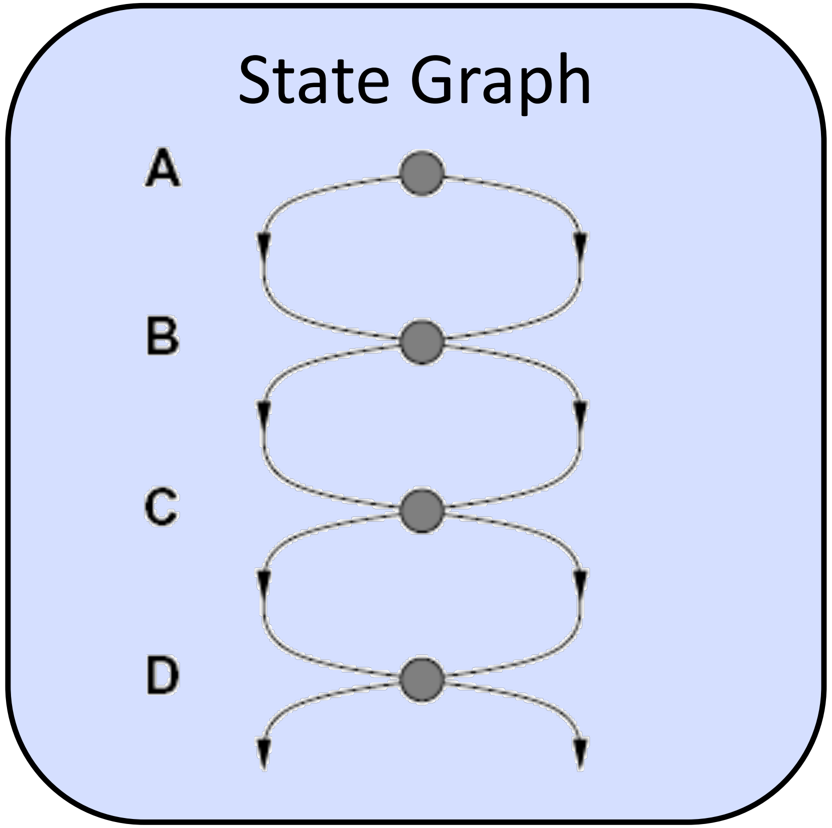
- With A*: a trade-off between quality of estimate and work per node
  - As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself

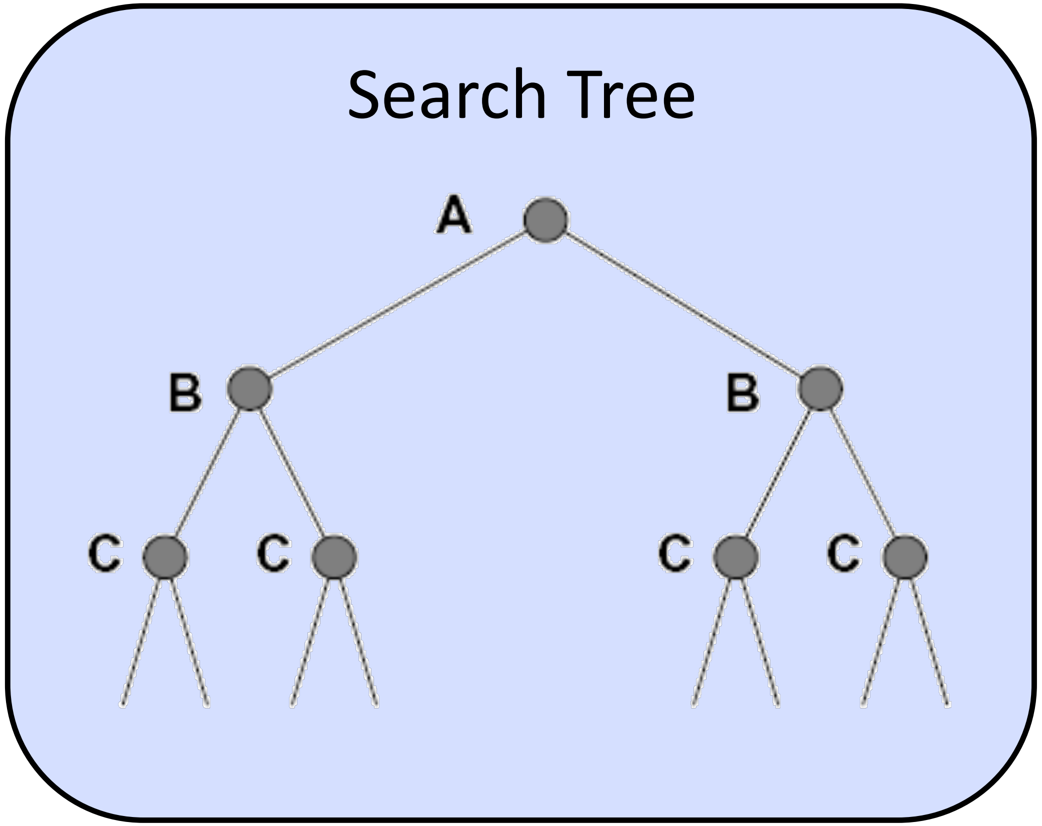
Graph Search



Tree Search: Extra Work!

- Failure to detect repeated states can cause exponentially more work.



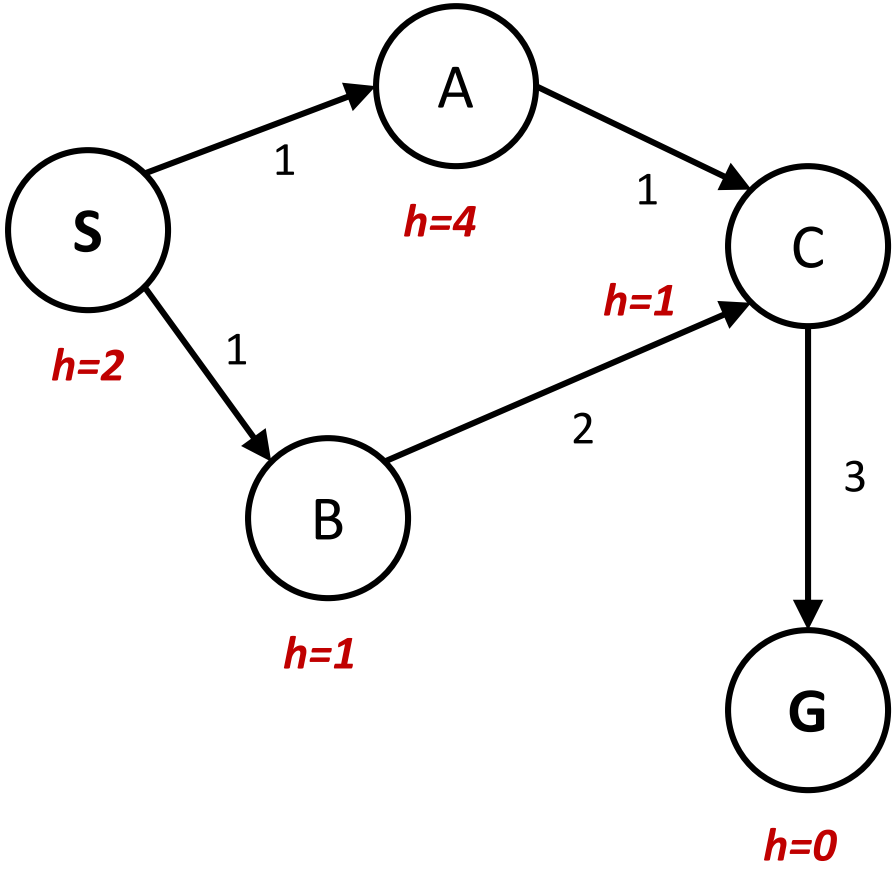


Graph Search

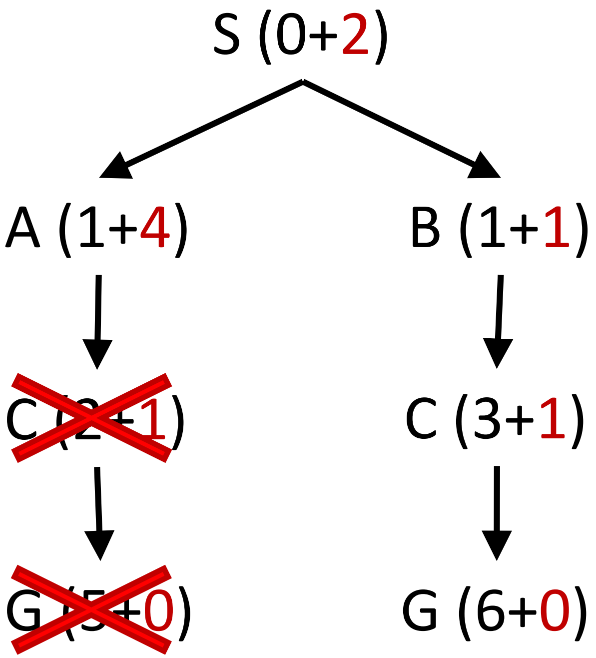
- Idea: never expand a state twice
- How to implement:
  - Tree search + set of expanded states (“closed set”)
  - Expand the search tree node-by-node, but…
  - Before expanding a node, check to make sure its state has never been expanded before
  - If not new, skip it, if new add to closed set
- Can graph search wreck completeness? Why/why not?
- How about optimality?

A* Graph Search Gone Wrong?

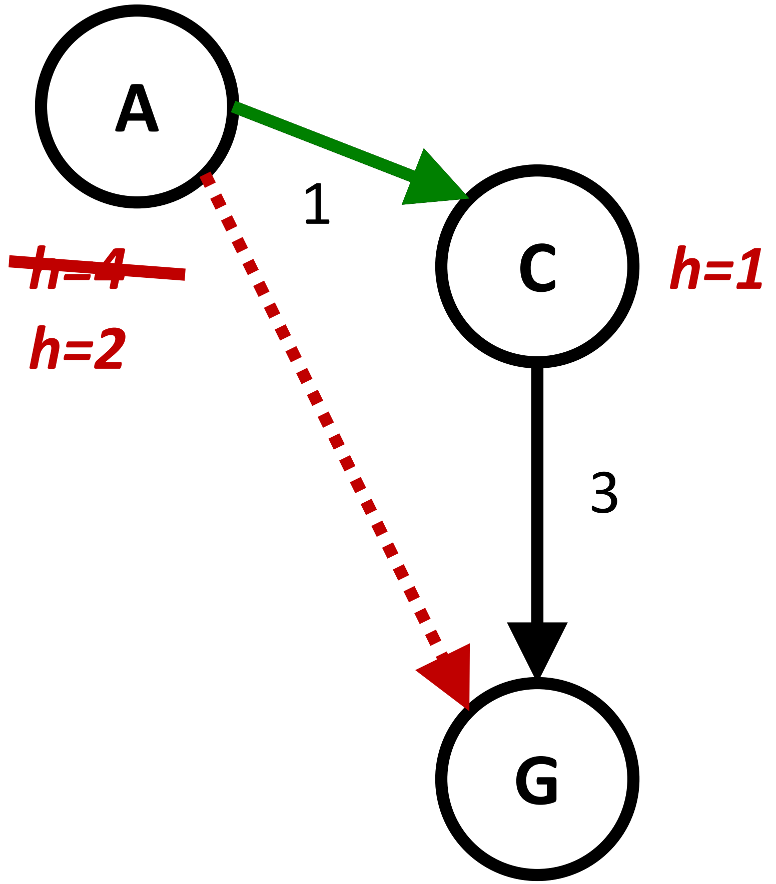
State space graph



Search tree



Consistency of Heuristics



- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal

    h(A) ≤ actual cost from A to G

  - Consistency: heuristic “arc” cost ≤ actual cost for each arc

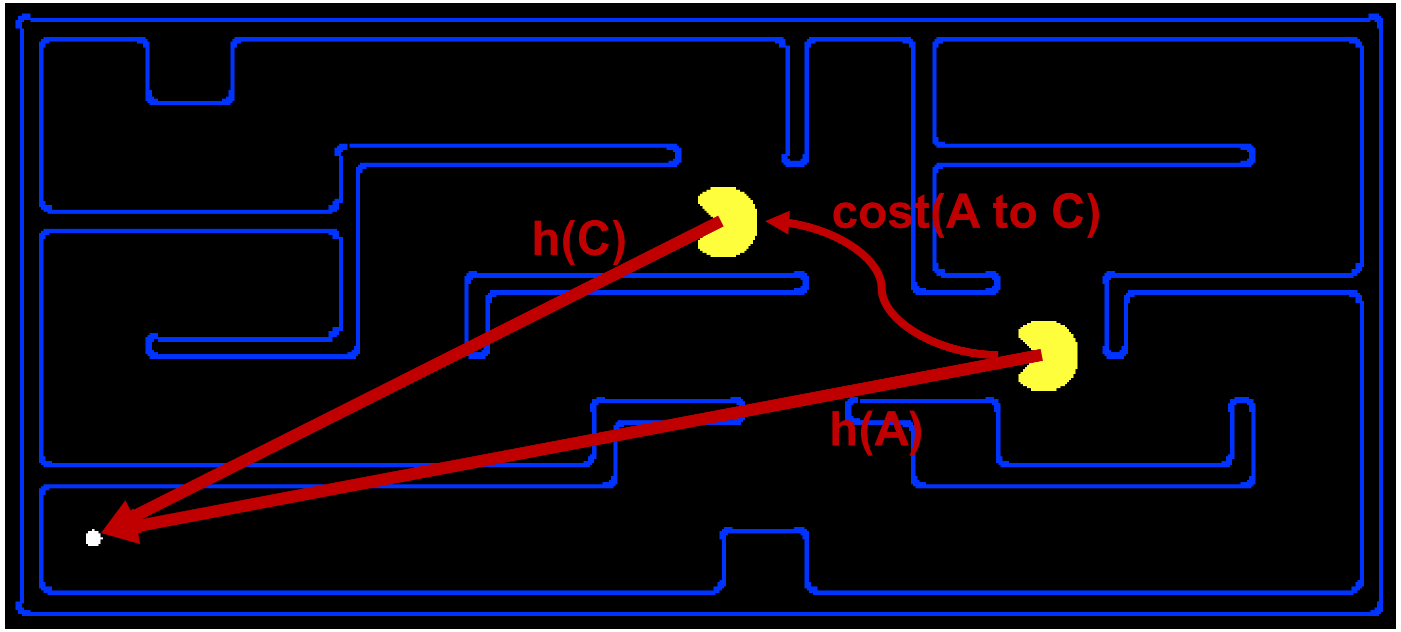
    h(A) – h(C) ≤ cost(A to C)

  - Consistency implies admissibility
- Consequences of consistency:
  - The f value along a path never decreases, because:

    h(A) ≤ cost(A to C) + h(C)

Consistency of Heuristics

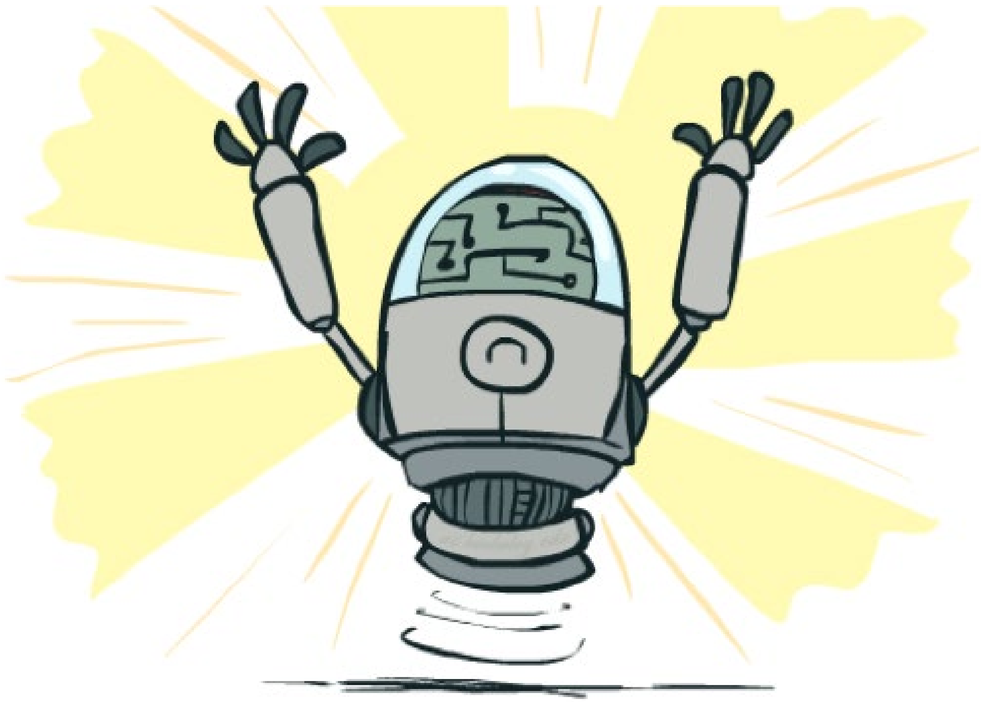
- In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems
- Example: Manhattan distance or Euclidean distance for pathing



Optimality

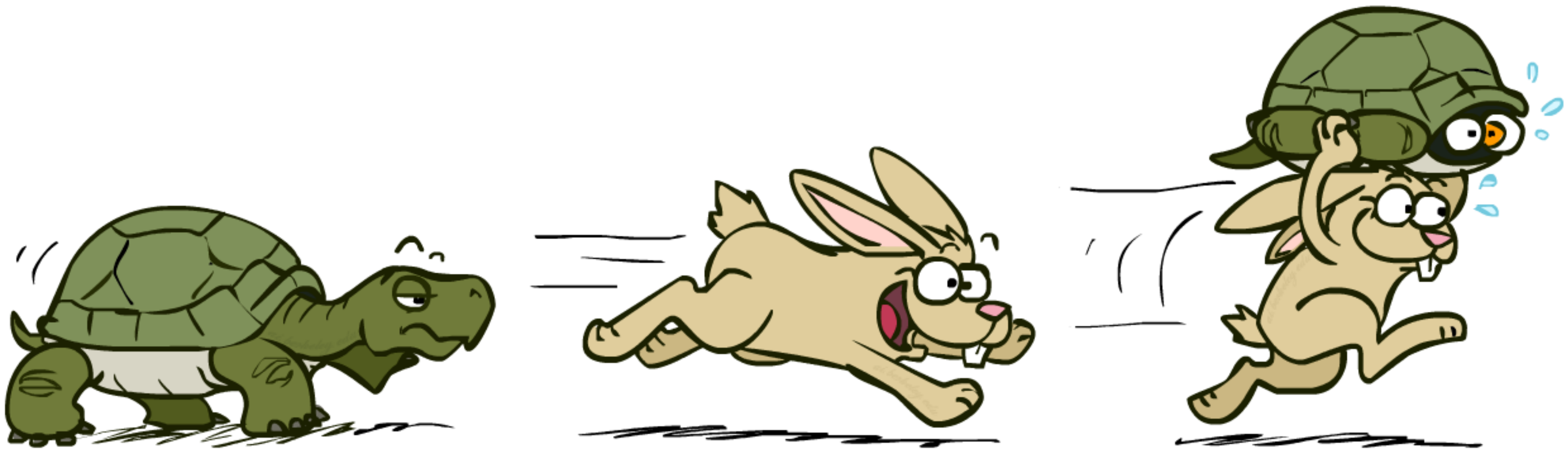
- Tree search:
  - A* is optimal if heuristic is admissible
  - UCS is a special case (h = 0)

- Graph search:
  - A* optimal if heuristic is consistent
    - See textbook for a proof
  - UCS optimal (h = 0 is consistent)



A*: Summary

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible / consistent heuristics
- Heuristic design is key: often use relaxed problems



Summary

- Why search
  - Agents that Plan Ahead
- Search Problems
  - state space, successor function
  - start state and goal test
- Uninformed Search Methods
  - DFS, BFS, UCS
- Informed Search
  - Greedy, A*
- Graph Search