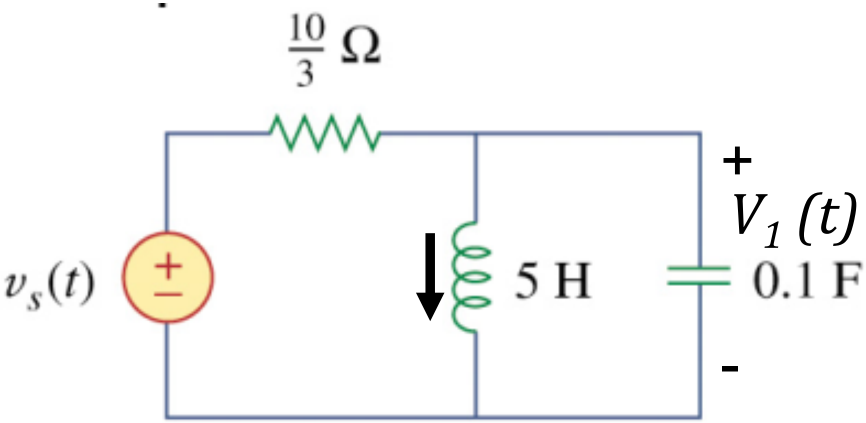
Lecture 14
-- Laplace Transform in Circuit Analysis

Example 2

Find (1) the voltage across the capacitor

(2) current through the inductor

assuming that $v_s(t)$ = 10$u(t)$ V, and assume that at t = 0, *-1 A* flows through the inductor and +5 V is across the capacitor.



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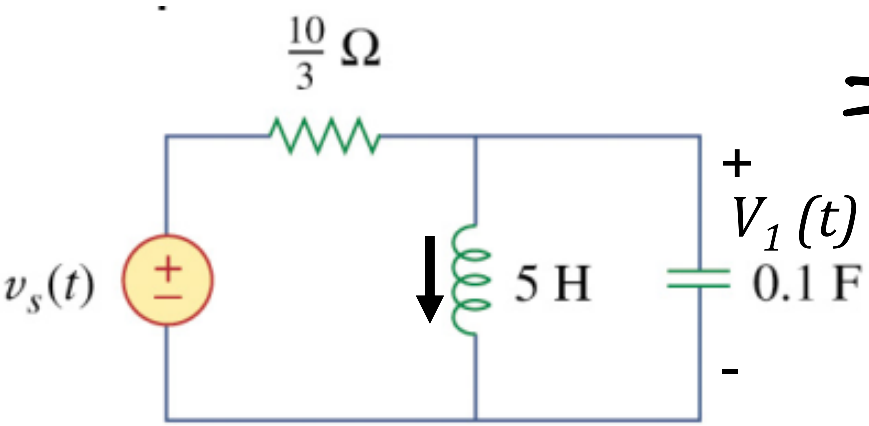
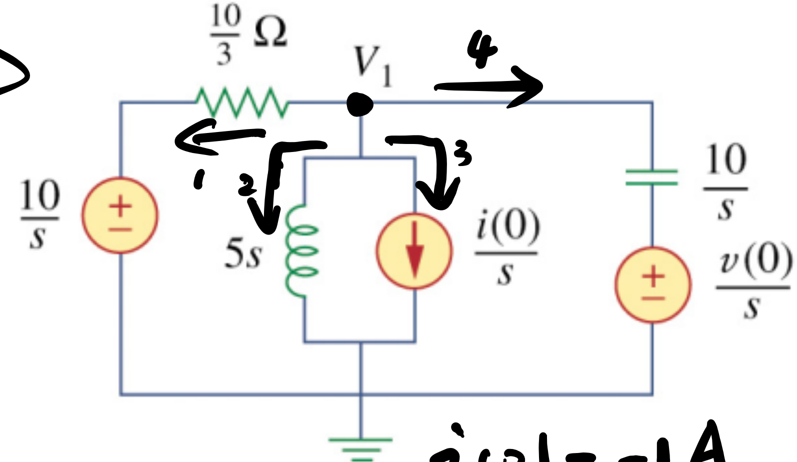




$i(0) = -1\,A$

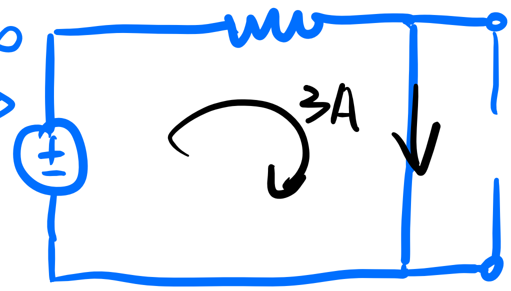
Nodal V_1 : $$\frac{V_1-\frac{10}{s}}{\frac{10}{3}}+\frac{V_1-0}{5s}+\frac{-1}{s}+\frac{V_1-\frac{5}{s}}{\frac{10}{s}}=0$$

$$\Rightarrow V_1(s) = \frac{40+5s}{(s+1)(s+2)} = \frac{\overset{35}{k_1}}{s+1} + \frac{\overset{-30}{k_2}}{s+2}$$

$$V_1(s) = \frac{35}{s+1} - \frac{30}{s+2}$$

$\mathcal{L}^{-1}$: $V_1(t) = [35 \cdot e^{-t} - 30 e^{-2t}] u(t)$ $\quad t>0$

$t \to \infty$



For $I(s)$

$$I(s) = \frac{V_1 - 0}{5s} + \frac{-1}{s}$$

& $V_1(s) = \dfrac{40+5s}{(s+1)(s+2)}$

$$\Rightarrow I(s) = \frac{40+5s}{5s(s+1)(s+2)} - \frac{1}{s}$$

$$= \frac{-s^2-2s+6}{s(s+1)(s+2)}$$

$$= \frac{\overset{3}{k_1'}}{s} + \frac{\overset{-7}{k_2'}}{s+1} + \frac{\overset{3}{k_3'}}{s+2}$$

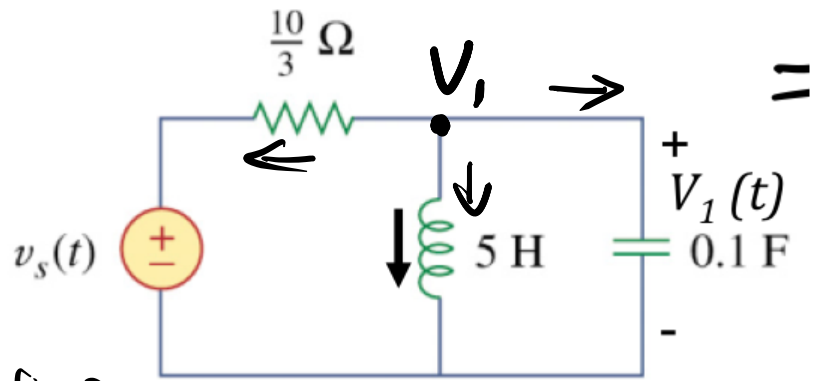
$$\Rightarrow \mathcal{L}^{-1} \quad i(t) = (3 - 7e^{-t} + 3e^{-2t})\, u(t)\ \text{A},\ t>0$$

In T.D.

$$i_L(t) = i_0 + \frac{1}{L}\int_0^t V_L(t)\, dt$$

$$V_L(t) = V_1(t) = [35 \cdot e^{-t} - 30e^{-2t}]\, u(t) \quad t>0$$

$$i_L(t) = (3 - 7e^{-t} + 3e^{-2t})\, u(t)\ \text{A},\ t>0$$



$t>0$:

$$\frac{V_1-10}{\frac{10}{3}} + i_L + C\cdot\frac{dV_1}{dt} = 0 \qquad ①$$

$$i_L = i_0 + \frac{1}{L}\int V_1\,dt \qquad ②$$

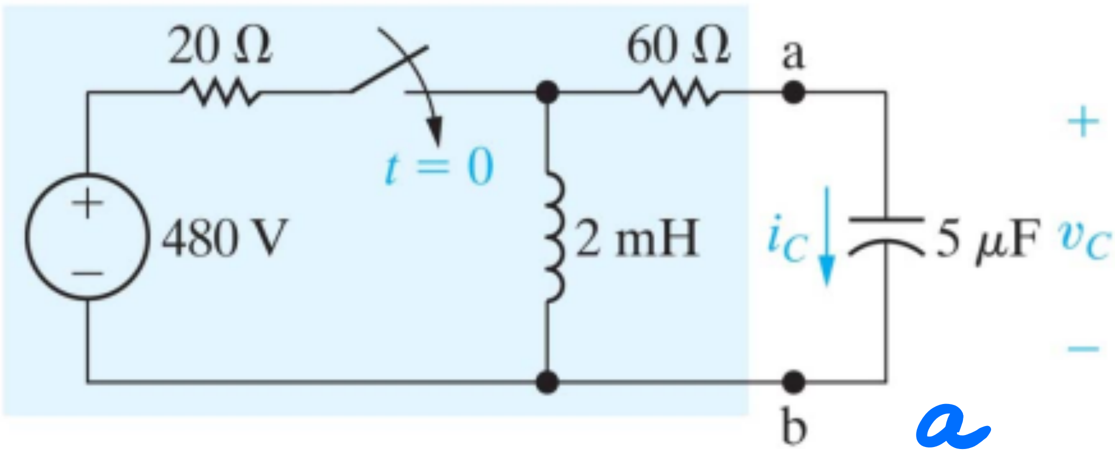
$$V_1'' + 3V_1' + 2V_1 = 0$$

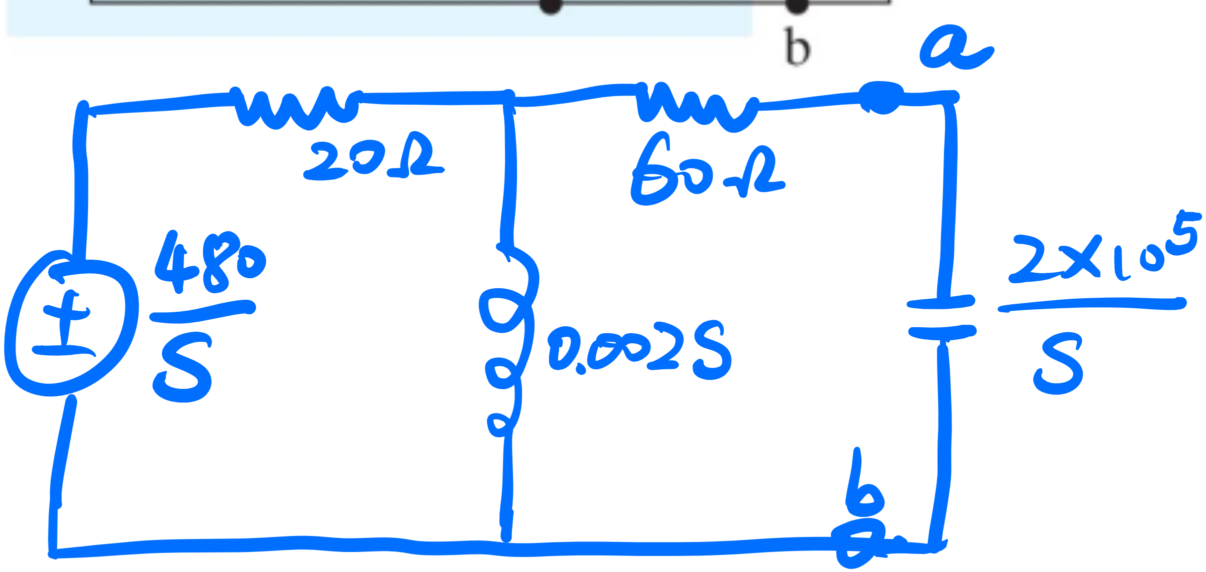
$$s^2+3s+2=0$$

$$(s+1)(s+2)=0$$

Example 3

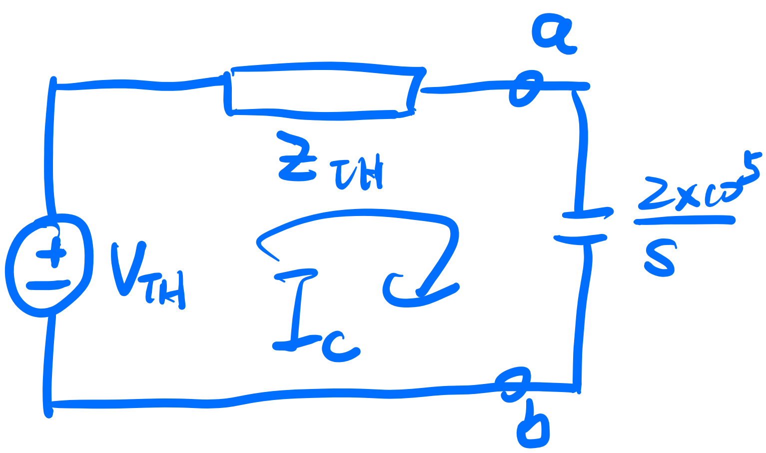
- Use Thevenin's equivalent circuit w.r.t. terminals *a*-*b* to find current $i_C(t)$ for t>0.





$$V_{TH} = V_{oc} = \frac{0.002S}{20+0.002S} \cdot \frac{480}{S} = \frac{480}{S+10^4}$$

$$Z_{TH} = 60 + (0.002S \parallel 20) = \frac{80(S+7500)}{S+10^4}$$



$$I_c(s) = \frac{V_{TH}}{Z_{TH} + Z_c}$$

$$= \frac{6S}{(S+5000)^2}$$

$$= \frac{A}{S+5000} + \frac{B}{(S+5000)^2}$$

$$B = 6S\Big|_{S=-5000} = -30000$$

$$A = (6s)' = 6$$

$$I_c(s) = \frac{6}{s+5000} + \frac{-30000}{(s+5000)^2}$$

$$\xrightarrow{\mathcal{L}^{-1}}$$

$$i_c(t) = \left(6 \cdot e^{-5000t} - 30000\, t \cdot e^{-5000t}\right) u(t)\ A$$

$$t > 0$$

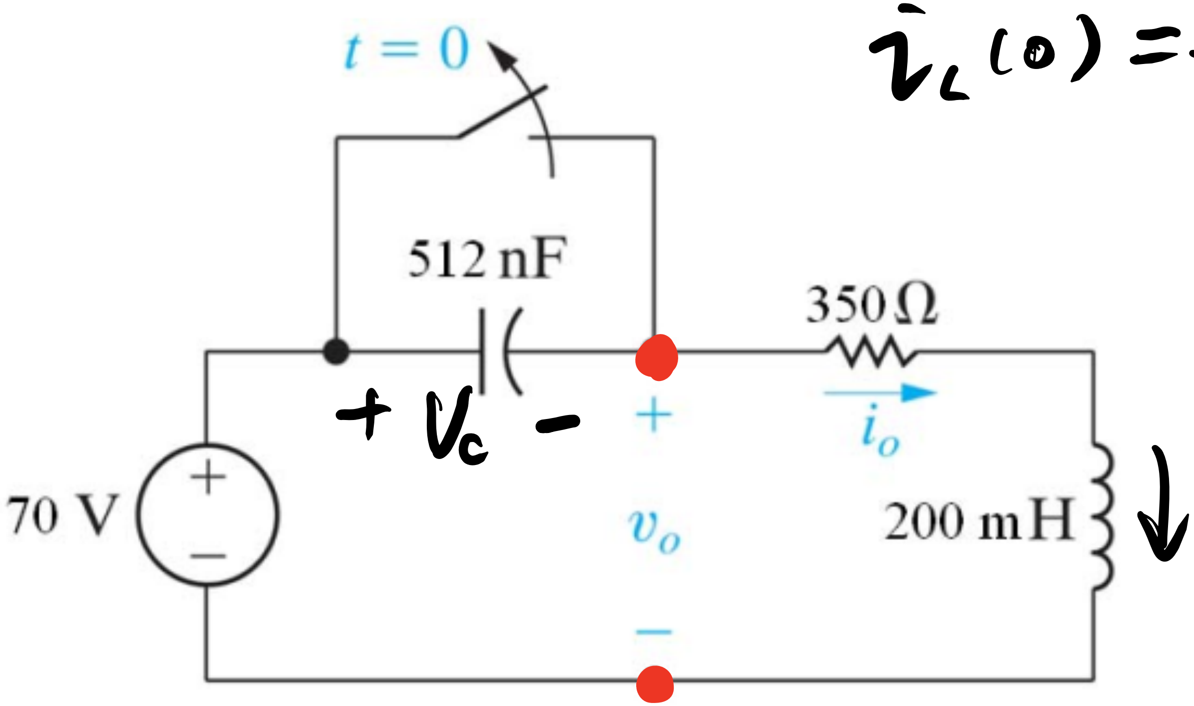
Example 4

- Find $V_o(t)$ for $t>0$

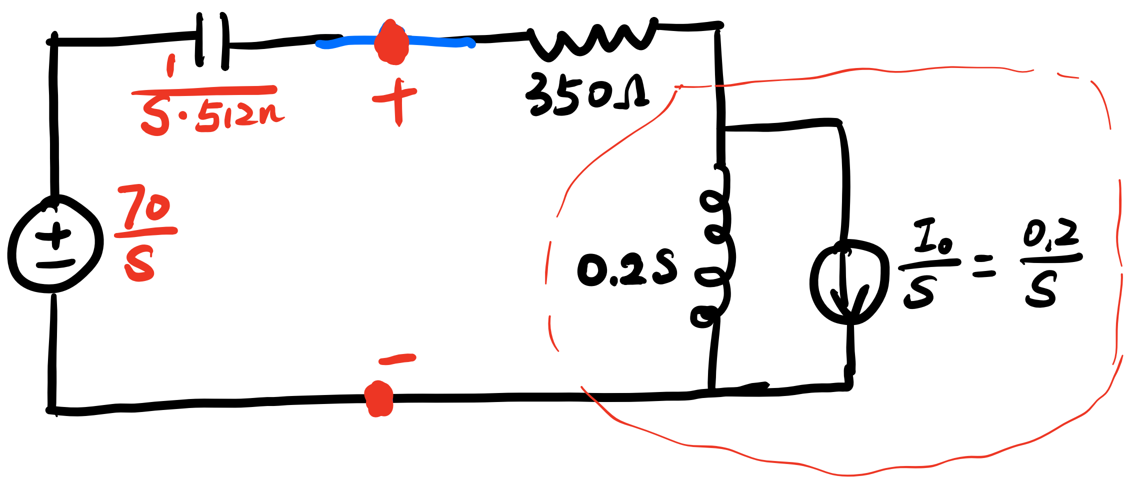
initial condition

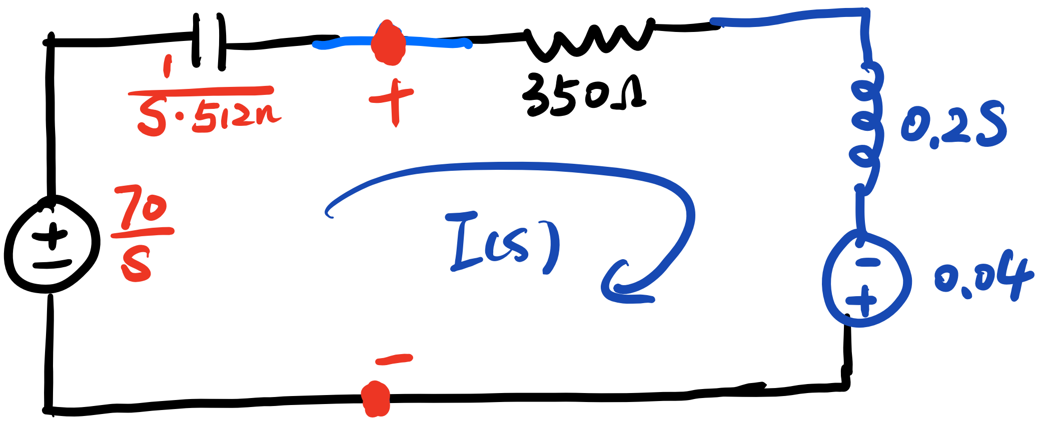
$$V_c(0) = 0$$

$$i_L(0) = \frac{70}{350} = 0.2A$$



t>0. S.D.





$$I(s) = \frac{\frac{70}{s} + 0.04}{\frac{1}{s \cdot 512n} + 350 + 0.2s}$$

$$V(s) = I(s) \cdot (350 + 0.2s) - 0.04$$

$$S_{1,2} = -875 \pm 3000j$$

$$= \frac{K_1}{S-(-875+3000j)} + \frac{K_2}{S-(-875-3000j)}$$

$\alpha = -875$
$\omega = 3000$

$$V_0(s) = \frac{70s - 268{,}125}{s^2 + 1750s + 9{,}765{,}625} = \frac{K_1}{(s+875-j3000)} + \frac{K_2}{(s+875+j3000)}$$

$$K_1 = \left.\frac{70s - 268{,}125}{(s+875+j3000)}\right|_{s=-875+j3000} = \frac{70(-875+j3000) - 268{,}125}{[(-875+j3000)+875+j3000]} = \underbrace{65.1}_{|K_1|}\angle\underbrace{57.48^\circ}_{\varphi_K}$$

$$K_2 = \left.\frac{70s - 268{,}125}{(s+875-j3000)}\right|_{s=-875-j3000} = \frac{70(-875-j3000) - 268{,}125}{[(-875-j3000)+875+-j3000]} = 65.1\angle-57.48^\circ$$

$$V_0(s) = \frac{65.1\angle 57.48^\circ}{(s+875-j3000)} + \frac{65.1\angle-57.48^\circ}{(s+875+j3000)}$$

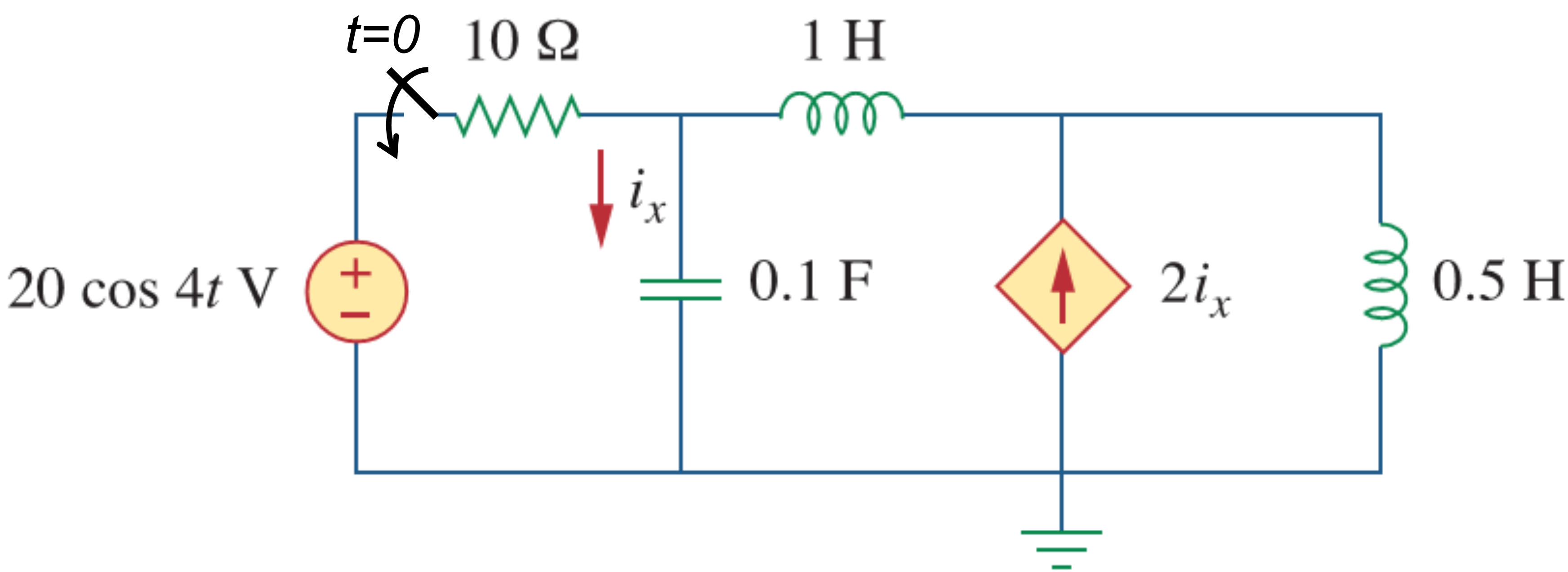
$$v_0(t) = 2(65.1)e^{-875t}\cos(3000t + 57.48^\circ) = \underline{130.2e^{-875t}\cos(3000t + 57.48^\circ)u(t)\ \text{V}}$$

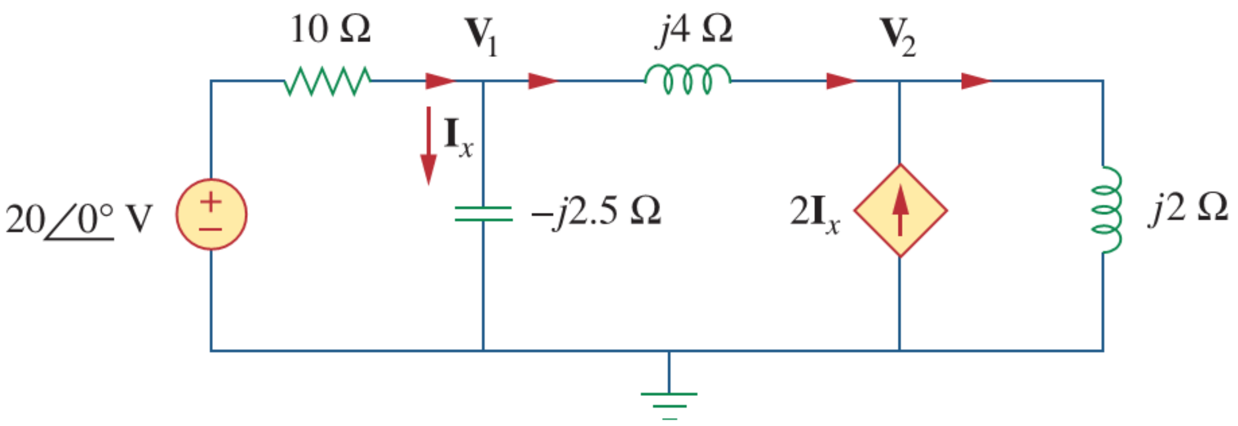
Example 5

- Example---Find i_x(s.s.) assuming no initial energy stored

Using (1)phasor method

(2)Laplace transform method

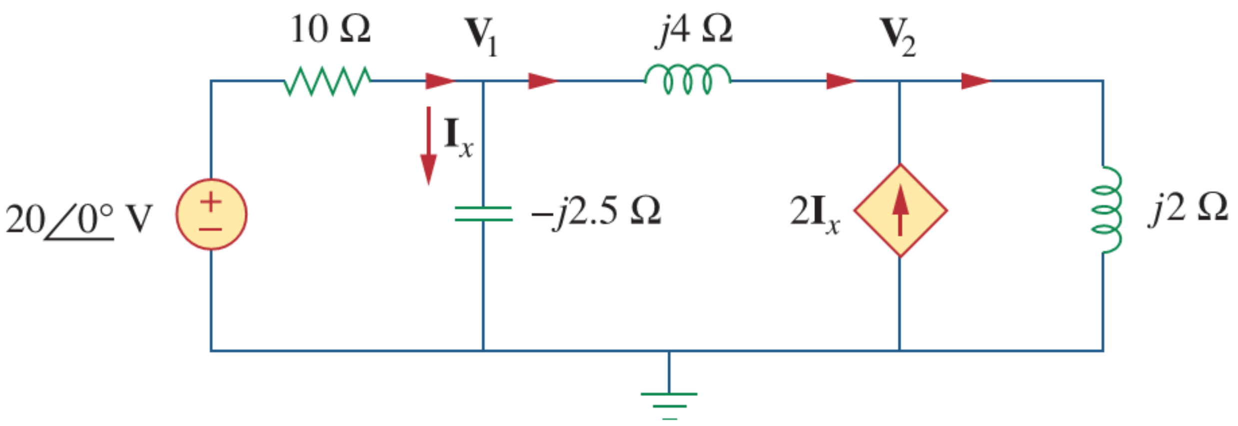




$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

$$I_x = \frac{V_1}{-2.5j}$$



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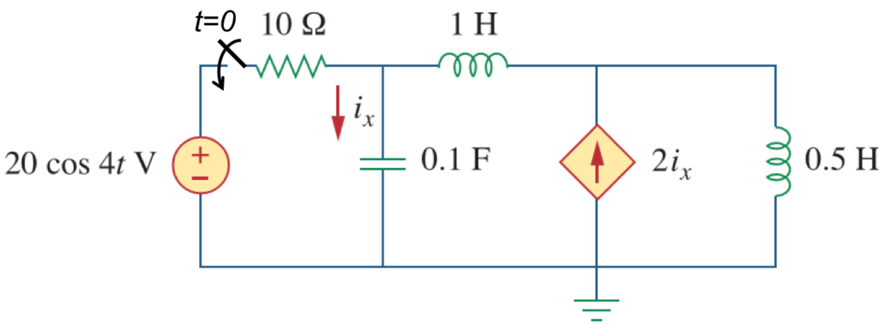
$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

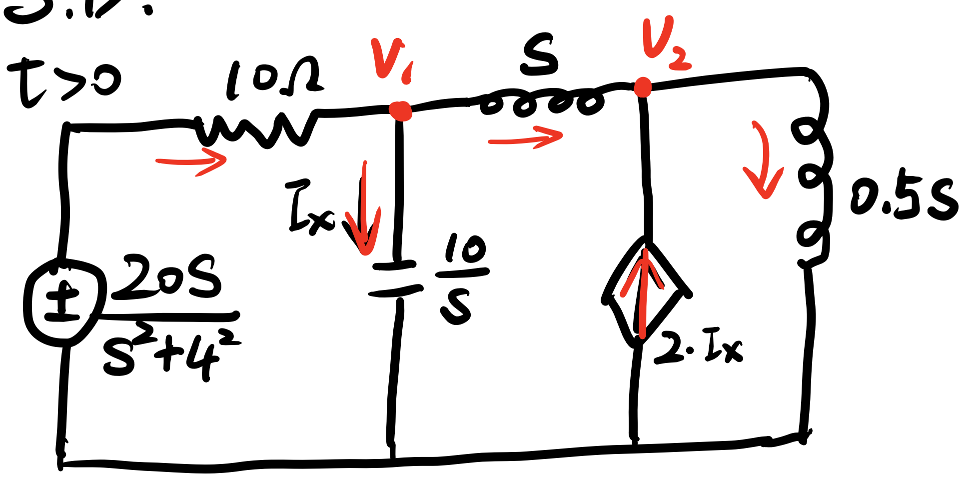
$$I_x = \frac{V_1}{-2.5j}$$

$$i_x = 7.59\cos(4t + 108.4^\circ)\ \text{A}$$



S.D.

t>0



$$\left.\begin{aligned}
&\frac{\frac{20s}{s^2+4^2}-V_1}{10} = \frac{V_1}{\frac{10}{s}} + \frac{V_1-V_2}{s}\\
&\frac{V_1-V_2}{s} + 2I_x = \frac{V_2}{0.5s}\\
&I_x = \frac{V_1}{\frac{10}{s}}
\end{aligned}\right\}$$

$$I_x(s) = \frac{6s^3}{\underbrace{(s^2+4^2)}\underbrace{(s^2+3s+20)}}$$

$S_{1,2} = 0 \pm 4j$ $S_{3,4} = -1.5 \pm 4.2j$

$$I_x(s) = \frac{k_1}{s-4j} + \frac{k_1^*}{s-(-4j)} + \frac{k_2}{s-(-1.5+4.2j)} + \frac{k_2^*}{s-(-1.5-4.2j)}$$

$k_1 = 3.79\angle 108.43^\circ$, $k_2 = 5.0\angle -146.2^\circ$

$$\mathcal{L}^{-1} \quad i_x(t) = 2|k_1|e^{\alpha_1 t}\cos(\omega_1 t + \varphi_{k_1}) + 2|k_2|e^{\alpha_2 t}\cos(\omega_2 t + \varphi_{k_2})$$

$$\alpha_1 \pm j\omega_1 = 0 \pm 4j$$

$$\alpha_1 = 0, \quad \omega_1 = 4$$

$$\alpha_2 \pm j\omega_2 = -1.5 \pm 4.2j$$

$$\alpha_2 = -1.5 \quad \omega_2 = 4.2j$$

$$\mathcal{L}^{-1} \hat{i}_x(t) = 2|k_1|e^{\alpha_1 t}\cos(\omega_1 t + \varphi_{k_1}) + 2|k_2|e^{\alpha_2 t}\cos(\omega_2 t + \varphi_{k_2})$$

S.S.

$$= [\underbrace{7.58e^{0t}\cos(4t + 108.43^\circ)}_{S.S.} + \underbrace{10e^{-1.5t}\cos(4.2t - 146.2^\circ)}_{T.S.}]u(t) \text{ A.}$$

T.S.

$$i_x(S.S.) = 7.58\cos(4t + 108.43^\circ) \text{ A}$$

$t > 0$

Example 6

- There is no initial energy stored in this circuit. Find $i(t)$ if
- $v(t) = e^{-0.6t}\sin 0.8t$ V.