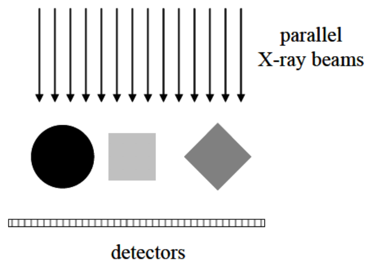
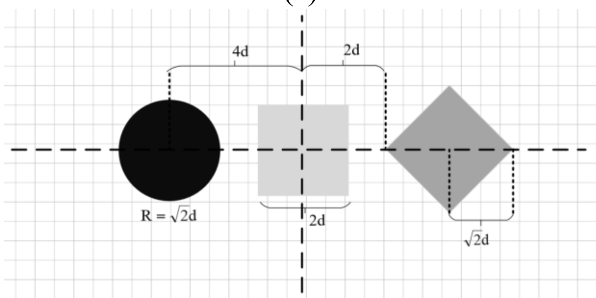
Homework 3 Solution

1. For the object shown in Figure 1(a), a circle and two squares. Draw the projections that would be acquired at angles φ=0, 45, 90, 135 and 180º (ignore beam hardening). Sketch the sinogram for values of φ from 0 to 360º. Assume that a dark area corresponds to an area of high signal. The detail geometry relationship is shown as Figure 1(b). (20point)



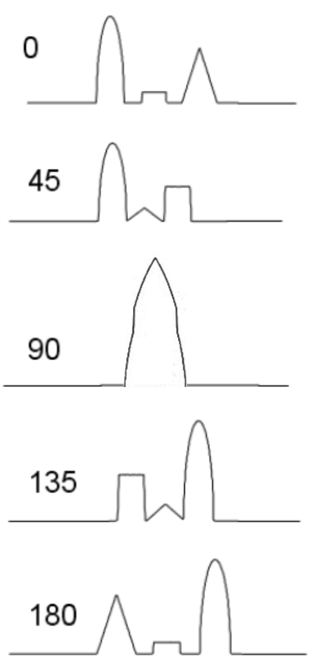
(a)

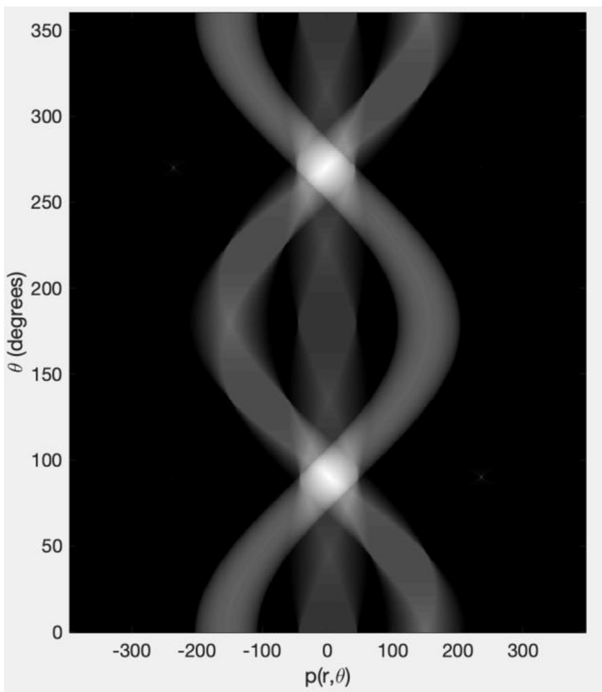


(b)

Figure 1

Solution





2. For Figure 2, suggest one possible shape that could have produced the sinogram and **explain why**. (10point)

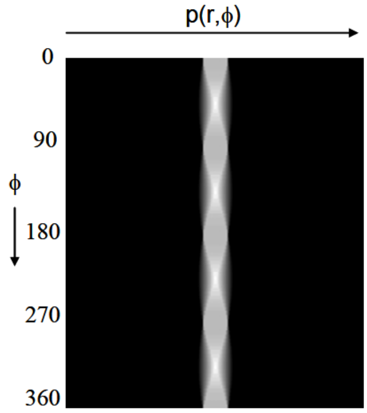
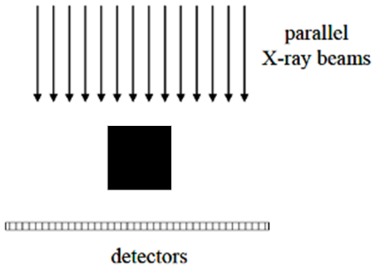


Figure 2

Solution. There are four repeating structures in the sinogram as the angle goes through 360 degrees, suggesting two-fold symmetry. One possible solution, therefore, is a square.



3. Cardiac CT. The following conditions are given: (30point)
   - A CT scanner with 128 detector rows.
   - The detector width in the center of the FOV is 0.5 mm.
   - A full rotation (360^o) of the X-ray tube takes 0.33 s.
   - A full data set for reconstruction requires projection values for a range of 210^o.
   - Maximum 1/4 of the heart cycle can be used for acquiring projection data.
   - The heart rhythm is 72 bpm.
   - The scan length is 20 cm.

   (a) Calculate the duration of 1/4 of heart cycle (in seconds). (6)

   (b) Calculate (in seconds) the time needed to obtain projection values for a range of 210^o. (6)

   (c) What can you conclude from (a) and (b)? (6)

   (d) Assume that the table shift per heart beat is equal to the total width of the detector rows (i.e. the total z-collimation). Calculate the acquisition time. (6)

   (e) The assumption under (d) is approximate. Explain why? How does this approximation influence the acquisition time? (6)

Solution

(a) 72 heart cycles per minute = 60 seconds

$\Rightarrow$ Δb = duration of 1 heart cycle = 60/72 seconds = 5/6 seconds

$\Rightarrow$ duration of 1/4 of a heart cycle = 5/24 seconds $\approx$ 5/25 = 0.2 s (somewhat more)

(b) 360^o rotation in 0.33s = 1/3 s

$\Rightarrow$ 210^o degrees (= 180^o + fan angle) in Δt seconds

$\Rightarrow$ Δt = 210/360 x 1/3 = 7/36 $\approx$ 7/35 = 0.2 s (somewhat less)

(c) The time needed to collect a full set of projection data fits within the 1/4 of the heart cycle that can be used for imaging (when cardiac motion is relatively small).

(d) Total width of the detector = 128 x 0.5 mm = 64 mm

$\Rightarrow$ table feed = 64 mm / heart beat

$\Rightarrow$ to cover 20cm, 4 heart beats in total are needed (= 200 / 64)

$\Rightarrow$ acquisition time = 4 x Δb = 4 x 5/6 s = 20/6 = 3.33 s

(e) During the measurement time Δt, the detector moves relative to the table over a distance $\alpha\Delta$t, with α the table feed (in mm/s). The total length covered by the detector during collection of the projections is thus W + α.Δt, with W the detector width. However, only the central part of length W − $\alpha\Delta$t is covered by the detector during the full duration of the measurement, such that only from this part complete projection data are obtained during the time Δt. Within one heart beat of duration Δb, a slab of length W − $\alpha\Delta$t can thus be imaged. Over a time period Δb of one heart beat, the table should thus move a distance W − $\alpha\Delta$t. This means that the table feed α should be equal to:

$$\alpha = \frac{W - \alpha\Delta t}{\Delta b} \quad => \alpha = \frac{W}{\Delta b + \Delta t}$$

Comparing this to the previous assumption which stated that α = W/Δb, accounting for the detector motion during the measurement itself makes the table feed smaller, and hence the total acquisition time longer (in principle). It was argued above that Δt $\leq$ 1/4 $\times$ Δb. Hence:

$$\alpha \approx \frac{W}{\Delta b + \frac{\Delta b}{4}} = \frac{4}{5}\left(\frac{W}{\Delta b}\right)$$

i.e. 20% smaller than the assumption above would yield. The total acquisition time is thus also likely higher, if an additional heart beat is needed to cover the entire volume. For the example given above:

Table feed = 4/5 $\times$64 mm per heart beat = 52 mm / heart beat

$\Rightarrow$ to cover 20cm, 4 heart beats in total are needed (= 200 / 52)

$\Rightarrow$ acquisition time = 4 $\times$$\Delta$b = 4 $\times$5/6 s = 20/6 = 3.33 s

In this case, there is no difference in acquisition time. However, if a volume of 22cm had to be scanned, an additional heart beat was needed and the actual scan time would be 5 $\times$ Δb = 4.16s, while the assumption under (d) above would underestimate the scan time by 1 heart beat.

4. Window operation. Here is a 10*10 matrix described an image with bone and lung. The values in the matrix are indicated CT number. The original window is set as

W=1000 and L=1000, Then, (20point)

(a) Design a window for Bone and calculate the corresponding result after window operation.

(b) Design a window for lung and calculate the corresponding result after window operation.

	1970	1850	1800	1900	
1940	1800	1830	1850	1910	1880
1880	0	40	60	60	1910
1950	20	50	20	40	1800
1900	30	0	30	10	1950
1910	1800	1920	1900	1840	1890
	1860	1800	1900	1920	
		1940	1920		

Solution

(a) (An example) Set W=200, L = 1900.

	3400	1000	0	2000	
2800	0	600	1000	2200	1600
1600	0	0	0	0	2200
3000	0	0	0	0	0
2000	0	0	0	0	3000
2200	0	2400	2000	800	1800
	1200	0	2000	2400	
		2800	2400		

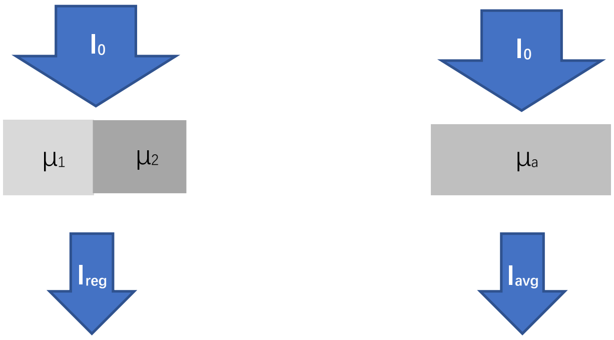
(b) (An example) Set W = 100, L = 50.

	4000	4000	4000	4000	
4000	4000	4000	4000	4000	4000
4000	0	1600	2400	2400	4000
4000	800	2000	800	1600	4000
4000	1200	0	1200	400	4000
4000	4000	4000	4000	4000	4000
	4000	4000	4000	4000	
		4000	4000		

5. Nonlinear partial volume effect(20point)

(1) When there is the finite width for the X-ray beam, the averaged attenuation coefficient is counted over the beam width to measure the intensity. The simple example model is shown as below. Please compare the X-ray intensity Ireg and Iavg where $\mu a=(\mu 1+\mu 2)/2$.

(2) Based on the results from (1), please explain the nonlinear partial volume effect, i.e. considering the finite width of X-ray beam, there will be an underestimation of the integrated averaged attenuation.



Solution:

(1) Method 1:

$$I_{\text{reg}} = \frac{I_0}{2} e^{-\mu_1 d} + \frac{I_0}{2} e^{-\mu_2 d}$$

$$I_{\text{avg}} = I_0 e^{-\mu_a d} = I_0 e^{-(\mu_1+\mu_2)d/2}$$

$$I_{\text{reg}}{}^2 - I_{\text{reg}}{}^2 = \left(\frac{I_0}{2} e^{-\mu_1 d} - \frac{I_0}{2} e^{-\mu_2 d}\right)^2 \geq 0$$

Therefore $I_{\text{reg}} \geq I_{\text{avg}}$

(It can also be solved using Taylor series of e^{-x})

(2) From the result of (1), when using the average attenuation along the beam width to calculate the beam intensity after passing through the matter, there is a lower intensity received from detectors. It means that in the case of $I_{\text{reg}} = I_{\text{avg}}$, the actual attenuation value is greater than the measured attenuation value with assumption of average. Therefore, it will be an underestimation of the actual integrated averaged attenuation.