EE150: Signals and Systems, Spring 2022
Homework 2

(Due Thursday, Mar. 17 at 11:59pm (CST))

1. [12 points] Determine the continuous-time convolution of x(t) and h(t) for the following three cases:

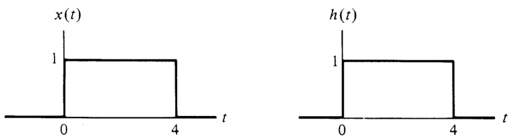

Figure 1.1: (a)

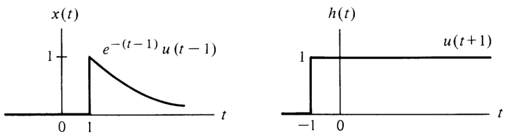

Figure 1.2: (b)

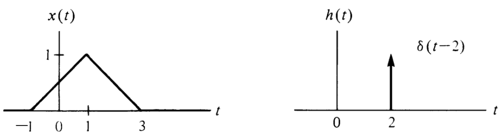

Figure 1.3: (c)

2. [12 points] Consider a discrete-time, linear, shift-invariant system that has unit sample response $h[n]$ and input $x[n]$.

   (a) Sketch the response of this system if $x[n] = \delta[n - n_0]$ for some $n_0 > 0$, and $h[n] = (\frac{1}{2})^n u[n]$.

   (b) Evaluate and sketch the output of the system if $h[n] = (\frac{1}{2})^n u[n]$ and $x[n] = u[n]$.

   (c) Consider reversing the role of the input and system response in part (b). That is,

$$h[n] = u[n],$$
$$x[n] = (\frac{1}{2})^n u[n]$$

   Evaluate the system output $y[n]$ and sketch.

3. [5 points] Compute the convolution $y[n] = x[n] * h[n]$ when

$$x[n] = \alpha^n u[n], 0 < \alpha < 1,$$
$$h[n] = \beta^n u[n], 0 < \beta < 1,$$

Assume that α and β are not equal.

4. [16 points] Determine if each of the following statements is true in general. Provide proofs for those that you think are true and counterexamples for those that you think are false.

   (a) $x[n] * \{h[n]g[n]\} = \{x[n] * h[n]\}g[n]$

   (b) If $y(t) = x(t) * h(t)$, then $y(2t) = 2x(2t) * h(2t)$.

   (c) If $x(t)$ and $h(t)$ are odd signals, then $y(t) = x(t) * h(t)$ is an even signal.

   (d) If $y(t) = x(t) * h(t)$, then $Ev\{y(t)\} = x(t) * Ev\{h(t)\} + Ev\{x(t)\} * h(t)$.

   (Hint: It's taught that for an arbitrary signal $x(t)$, we can have $x(t) = g(t) + h(t)$ where $g(t)$ is an odd signal and $h(t)$ is an even signal, then $Ev\{x(t)\} = h(t)$.)

5. [9 points] Let $x(t) = u(t-3) - u(t-5) \quad \text{and} \quad h(t) = e^{-3t}u(t)$

(a) Compute $y(t) = x(t) * h(t)$.

(b) Compute $g(t) = \frac{dx(t)}{dt} * h(t)$.

(c) How is $g(t)$ related to $y(t)$?

6. [10 points] Consider the cascade interconnection of three causal LTI systems, illustrated in Figure 6.1. The impulse response $h_2[n]$ is

$$h_2[n] = u[n] - u[n-2],$$

and the overall impulse response is as shown in Figure 6.2.

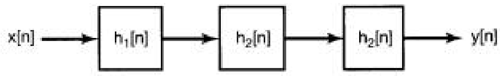

Figure 6.1: LTI systems

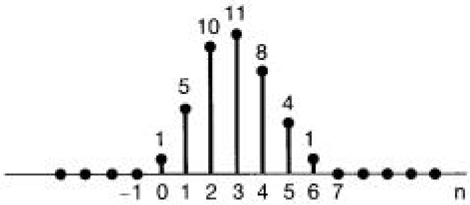

Figure 6.2: overall impulse response

(a) Find the impulse response $h_1[n]$.

(b) Find the response of the overall system to the input

$$x[n] = \delta[n] - \delta[n-1].$$

7. [16 points] Consider the evaluation of

$$y[n] = x_1[n] * x_2[n] * x_3[n],$$

where $x_1[n] = (0.5)^n u[n]$, $x_2[n] = u[n+3]$, and $x_3[n] = \delta[n] - \delta[n-1]$.

(a) Evaluate the convolution $x_1[n] * x_2[n]$.

(b) Convolve the result of part (a) with $x_3[n]$ in order to evaluate $y[n]$.

(c) Evaluate the convolution $x_2[n] * x_3[n]$.

(d) Convolve the result of part (c) with $x_1[n]$ in order to evaluate $y[n]$.

8. [10 points]

(a) Consider an LTI system with input and output related through the equation

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2) d\tau.$$

What is the impulse response $h(t)$ for this system?

(b) Determine the response of the system when the input $x(t)$ is as shown in Figure 8.

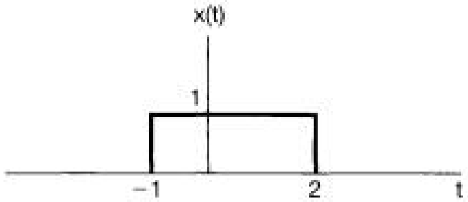

Figure 8: The Figure of $x(t)$

9. [10 points] Suppose that the signal

$$x(t) = u(t + 0.5) - u(t - 0.5)$$

is convolved with the signal

$$h(t) = e^{i\omega_0 t}.$$

(a) Determine a value of w_0 which ensures that

$$y(0) = 0,$$

where $y(t) = x(t) * h(t)$.

(b) Is your answer to the previous part unique?