SI251 Convex Optimization, Fall 2022
Quiz 1

Monday, Sep. 26

1. **Convex Set:** Describe the dual cone for each of the following cones:
   (a) $K = \mathbb{R}^2$. (10 points)
   (b) $K = \{(x_1, x_2) | x_1 + x_2 = 0\}$. (10 points)
2. **Convex Function:** Determine the convexity (i.e., convex, concave, or neither) of the following functions.
   (a) $f(x_1, x_2) = 1/(x_1 x_2)$. (10 points)
   (b) $f(x_1, x_2) = x_1^2/x_2$ on $\mathbb{R} \times \mathbb{R}_{++}$. (10 points)
3. **Convex Optimization:** Find all of the stationary points of the following functions. For each stationary point, determine if it is a local minimum, local maximum, or neither. Justify your answer.
   (a) $f_1(x, y) = \frac{x^2}{y^4 - 4y^2 + 5}$ on $\mathbb{R}^2$. (15 points)
   (b) $f_2(x, y) = 100(y - x^2)^2 - x^2$ on $\mathbb{R}^2$. (15 points)
4. **Duality:**
   (a) Derive the dual problems of the following primal problem:

$$\begin{array}{ll} \text{minimize} & \mathrm{Tr}(\mathbf{X}) \\ \text{subject to} & \mathbf{X} \succeq \mathbf{A} \\ & \mathbf{X} \succeq \mathbf{B} \end{array} \tag{1}$$

   where $\mathbf{A}, \mathbf{B} \in \mathbb{S}^n$. (15 points)

   (b) Consider the following compressive sensing problem via ℓ_1-minimization:

$$\begin{array}{ll} \underset{\boldsymbol{x}}{\text{minimize}} & \|\boldsymbol{x}\|_1 \\ \text{subject to} & \boldsymbol{A}\boldsymbol{x} = \boldsymbol{z}, \end{array} \tag{2}$$

   where $\boldsymbol{A} \in \mathbb{R}^{m \times d}, \boldsymbol{x} \in \mathbb{R}^d$ and $\boldsymbol{z} \in \mathbb{R}^m$. Please write down the equivalent linear programming reformulation of preblem (2), and then write down the dual problem of the reformulated linear program. (15 points)