2021Fall Probability & Mathematical Statistics 2021/09/25

Part II of Homework 2

Professor: Ziyu Shao *Due: 2021/10/08 11:59am*

1. (**Bertrand's Box Paradox**) There are three boxes: (a) a box containing two gold coins (b) a box containing two silver coins (c) a box containing one gold coin and a silver coin. After choosing a box at random and withdrawing one coin at random, if that happens to be a gold coin, find the probability of the next coin drawn from the same box also being a gold coin.

2. (**Probability Function is Continuous**) A sequence of events $\{E_n, n \geq 1\}$ is said to be an increasing sequence if $E_n \subset E_{n+1},\ n \geq 1$ and is said to be decreasing if $E_n \supset E_{n+1},\ n \geq 1$. We define the limits of such event sequences respectively as follows:

$$\lim_{n\to\infty} E_n = \bigcup_{i=1}^{\infty} E_i, \text{ if } E_n \subset E_{n+1}, n \geq 1;$$
$$\lim_{n\to\infty} E_n = \bigcap_{i=1}^{\infty} E_i, \text{ if } E_n \supset E_{n+1}, n \geq 1.$$

Please show the follow results: If $\{E_n, n \geq 1\}$ is either an increasing or decreasing sequence of events, then
$$\lim_{n\to\infty} P(E_n) = P(\lim_{n\to\infty} E_n).$$

3. (**The Borel-Cantelli Lemma**) Given a sequence of events $\{E_i, i \geq 1\}$, we define a new event
$$A = \bigcap_{n=1}^{\infty} \bigcup_{i=n}^{\infty} E_i.$$
Then if $\sum_{i=1}^{\infty} P(E_i) < \infty$, show that $P(A) = 0$.