ShanghaiTech University

EE 115B: Digital Circuits

Fall 2022

Lecture 7

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Boolean Algebra and Logic Simplification

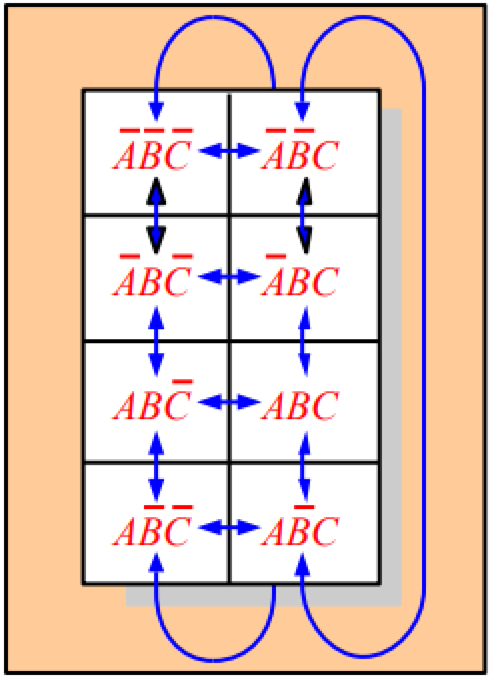
- Boolean Algebra
- Logic Simplification
  - Karnaugh Map
  - Quine-McCluskey Method

Karnaugh Maps

The Karnaugh map (K-map) is a tool for simplifying combinational logic with 3 or 4 variables. For 3 variables, 8 cells are required (2^3).

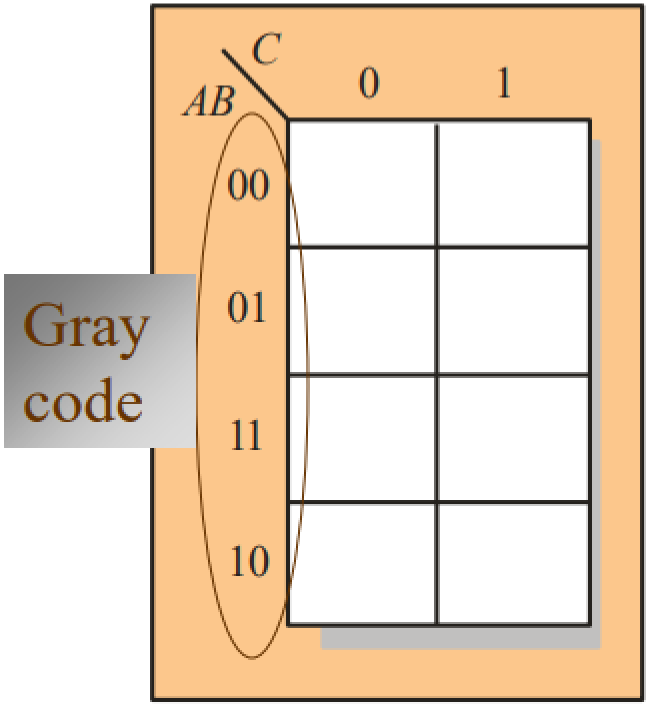
The map shown is for three variables labeled *A*, *B*, and *C*. Each cell represents one possible product term.

Each cell differs from an adjacent cell by only one variable.



Karnaugh Maps

Cells are usually labeled using 0's and 1's to represent the variable and its complement.



The numbers are entered in gray code, to force adjacent cells to be different by only one variable.

Ones are read as the true variable and zeros are read as the complemented variable.

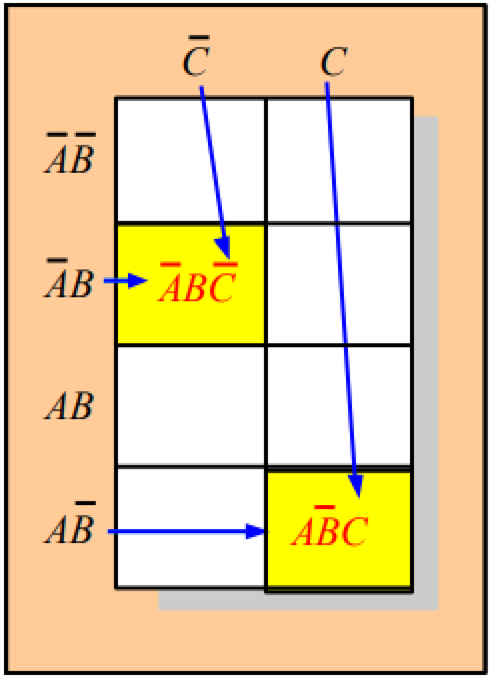
Karnaugh Maps

Alternatively, cells can be labeled with the variable letters. This makes it simple to read, but it takes more time preparing the map.

Example Read the terms for the yellow cells.

Solution

The cells are $\bar{A}B\bar{C}$ and $A\bar{B}C$.



表示最小项的卡诺图

- ## 二变量卡诺图

$A \backslash B$	0	1
0	$A'B'$ m_0	$A'B$ m_1
1	AB' m_2	AB m_3

三变量的卡诺图

$A \backslash BC$	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

- 4变量的卡诺图

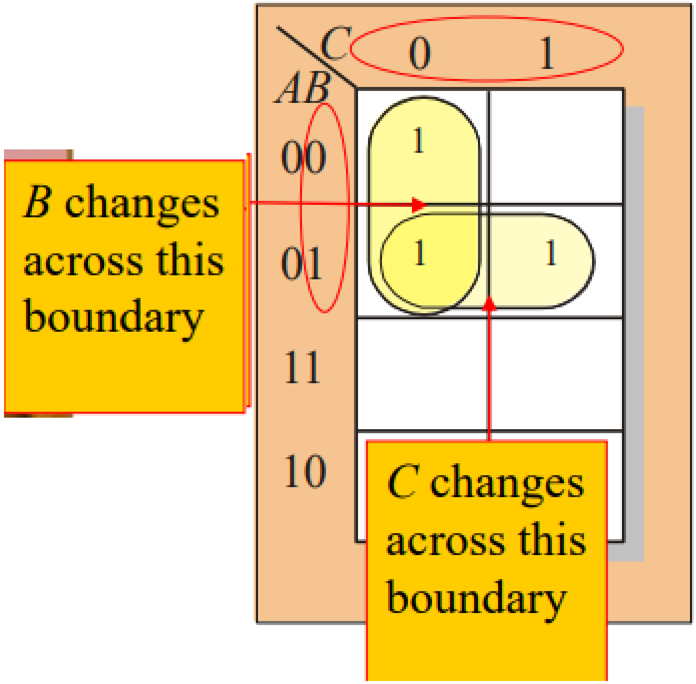
$AB \backslash CD$	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

- **五变量的卡诺图**

AB \ CDE	000	001	011	010	110	111	101	100
00	m_0	m_1	m_3	m_2	m_6	m_7	m_5	m_4
01	m_8	m_9	m_{11}	m_{10}	m_{14}	m_{15}	m_{13}	m_{12}
11	m_{24}	m_{25}	m_{27}	m_{26}	m_{30}	m_{31}	m_{29}	m_{28}
10	m_{16}	m_{17}	m_{19}	m_{18}	m_{22}	m_{23}	m_{21}	m_{20}

K-maps can simplify combinational logic by grouping cells and eliminating variables that change.

Example Group the 1's on the map and read the minimum logic.



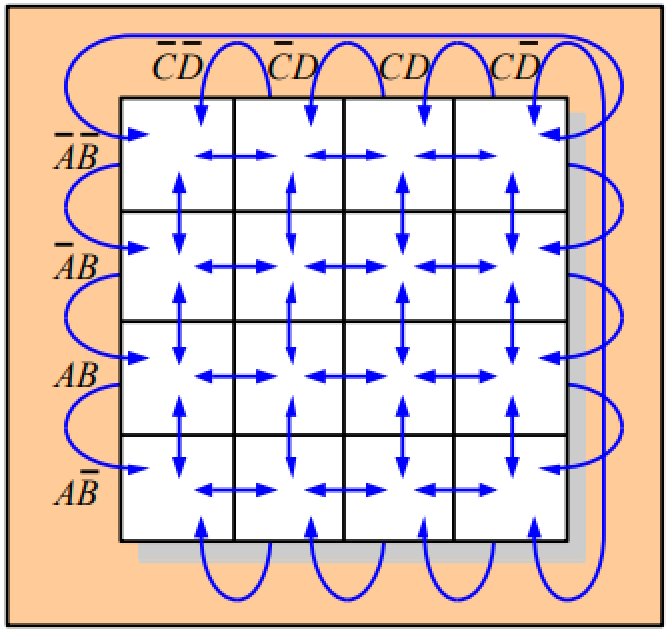
Solution

1. Group the 1's into two overlapping groups as indicated.
2. Read each group by eliminating any variable that changes across a boundary.
3. The vertical group is read $\overline{A}\overline{C}$.
4. The horizontal group is read $\overline{A}B$.

$$X = \overline{A}\overline{C} + \overline{A}B$$

Karnaugh Maps

A 4-variable map has an adjacent cell on each of its four boundaries as shown.

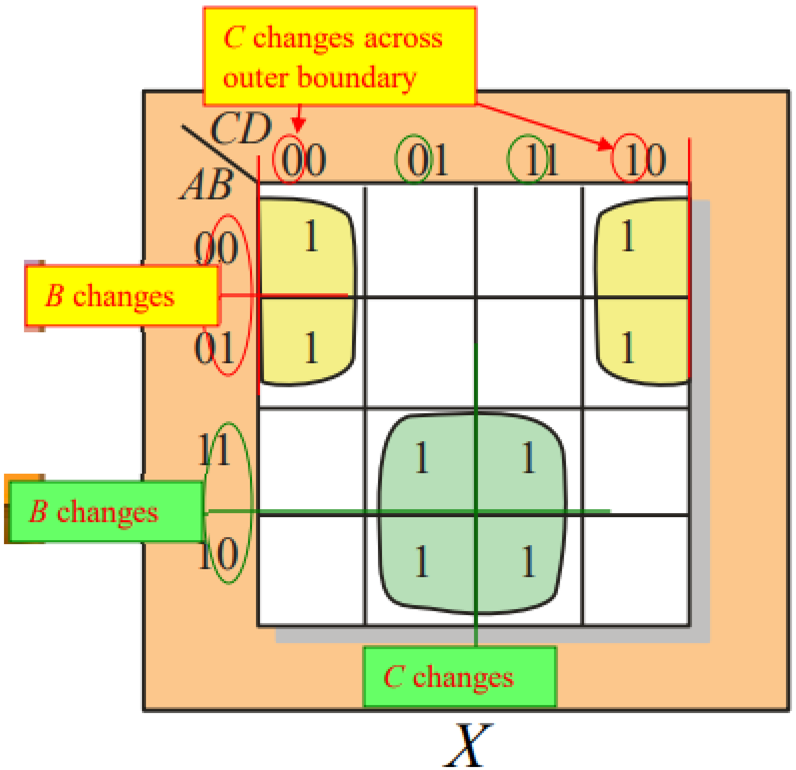


Each cell is different only by one variable from an adjacent cell.

Grouping follows the rules given in the text.

The following slide shows an example of reading a four variable map using binary numbers for the variables…

Example Group the 1's on the map and read the minimum logic.



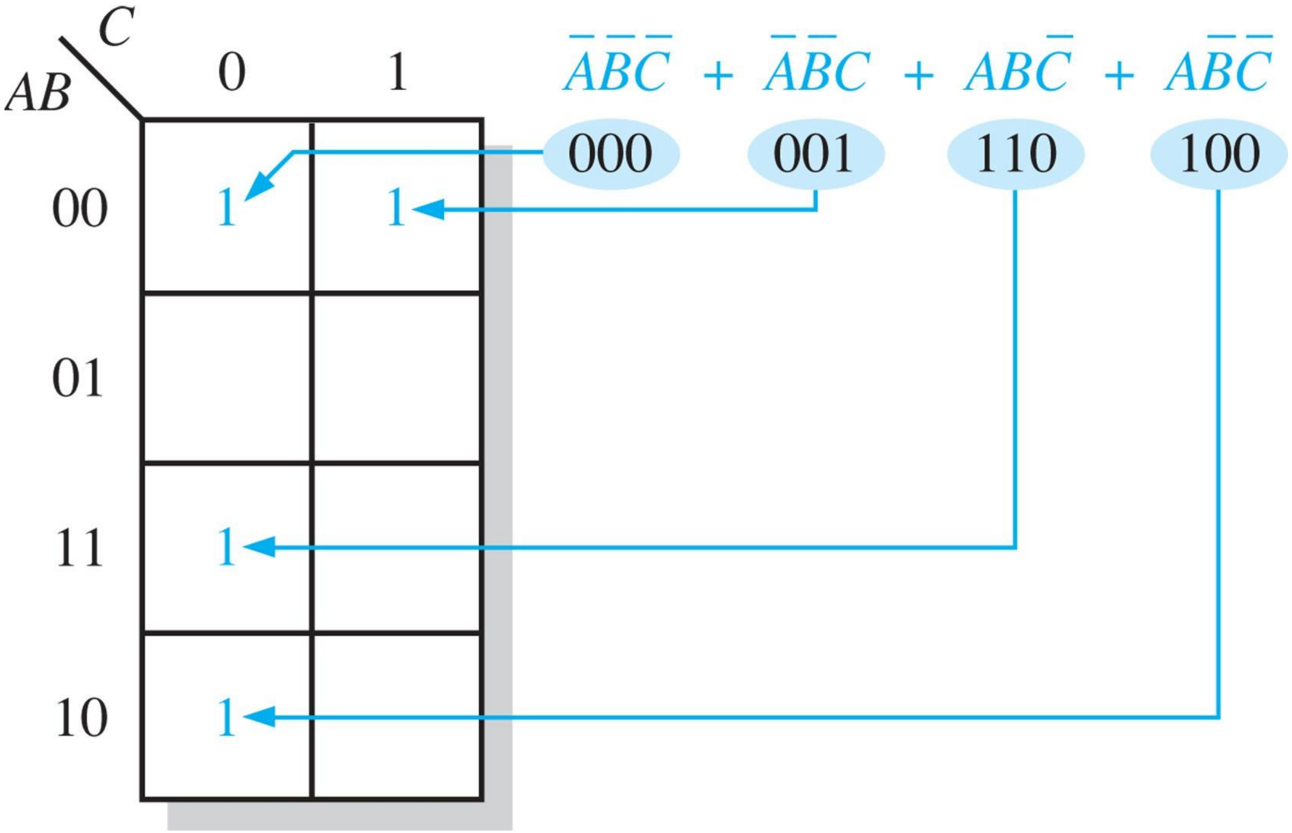
Solution

1. Group the 1's into two separate groups as indicated.
2. Read each group by eliminating any variable that changes across a boundary.
3. The upper (yellow) group is read as $\overline{A}\overline{D}$.
4. The lower (green) group is read as AD.

$$X = \overline{A}\overline{D} + AD$$

Mapping a standard SOP expression

Place a "1" for each minterm.



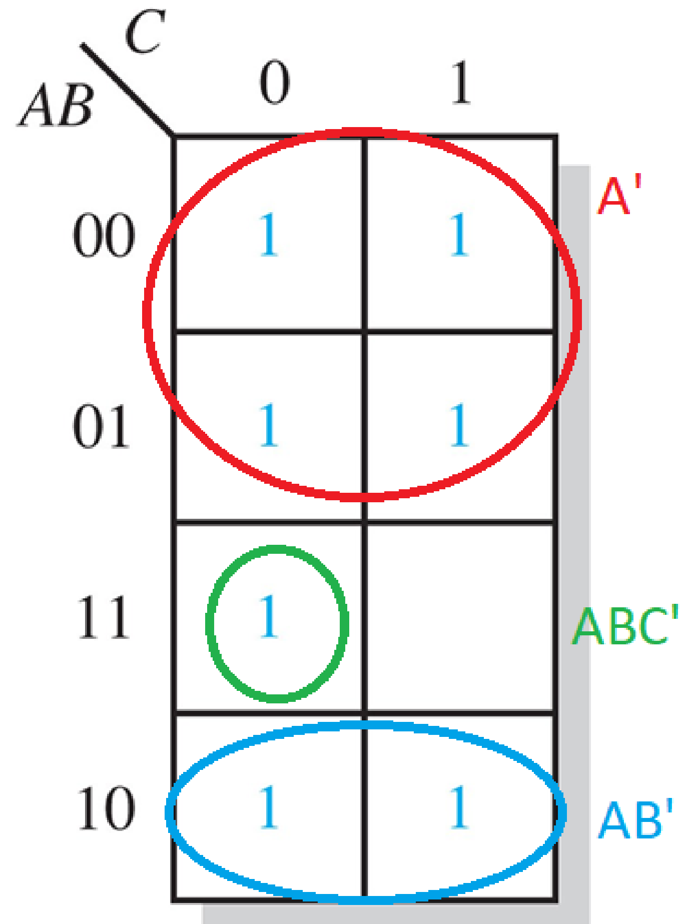
Mapping a nonstandard SOP expression

Expand product terms to minterms and place a "1" for each minterm.

Example: Map $\overline{A} + A\overline{B} + AB\overline{C}$.

Solution:

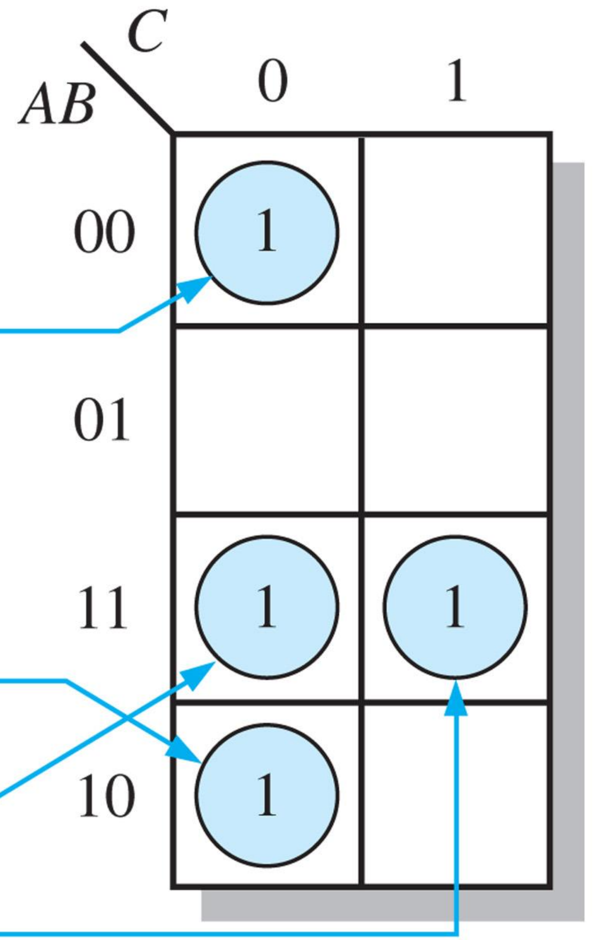
$\overline{A}$	$+ A\overline{B}$	$+ AB\overline{C}$
000	100	110
001	101	
010		
011		



Mapping directly from a truth table

$$X = \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C} + AB\overline{C} + ABC$$

Inputs			Output
A	*B*	*C*	*X*
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



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“Don’t care” conditions

“Don’t care” term can be “1” or “0”. Denoted by “X”.

Example: Output is “1” when input BCD code is 7, 8, or 9.

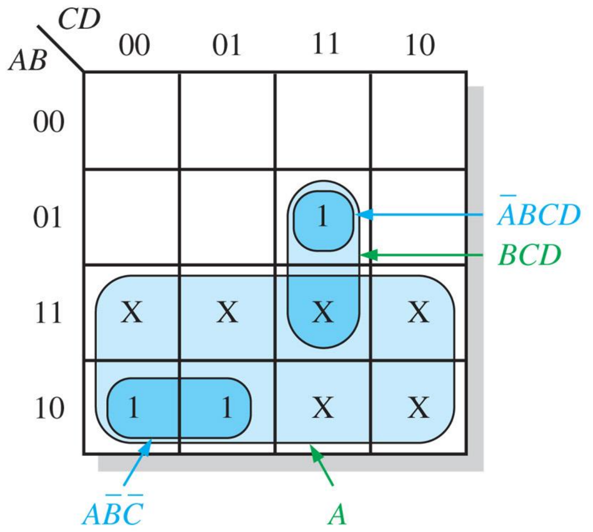
$Y=\sum m(7,8,9)+D(10,11,12,13,14,15)$

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

(Inputs: A B C D; Output: Y)

Don’t cares

(a) Truth table



(b) Without “don’t cares” $Y = A\overline{B}\overline{C} + \overline{A}BCD$
With “don’t cares” $Y = A + BCD$

例：Determine the minimum SOP expression.

$$Y(A,B,C) = AC' + A'C + B'C + BC'$$

A \ BC	00	01	11	10
0	0	1	1	1
1	1	1	0	1

$$AB' + A'C + BC'$$

A \ BC	00	01	11	10
0	0	1	1	1
1	1	1	0	1

$$AC' + A'B + B'C$$

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例：Determine the minimum SOP expression.

$$Y = ABC + ABD + AC'D + C' \cdot D' + AB'C + A'CD'$$

1. Work on 1’s
2. Work on 0’s
   (a) Take 0’s as 1’s (i.e., invert truth table or K-map)
   (b) Write SOP for these 1’s to get Y’
   (c) Invert Y’ to get Y

Y’=A’D

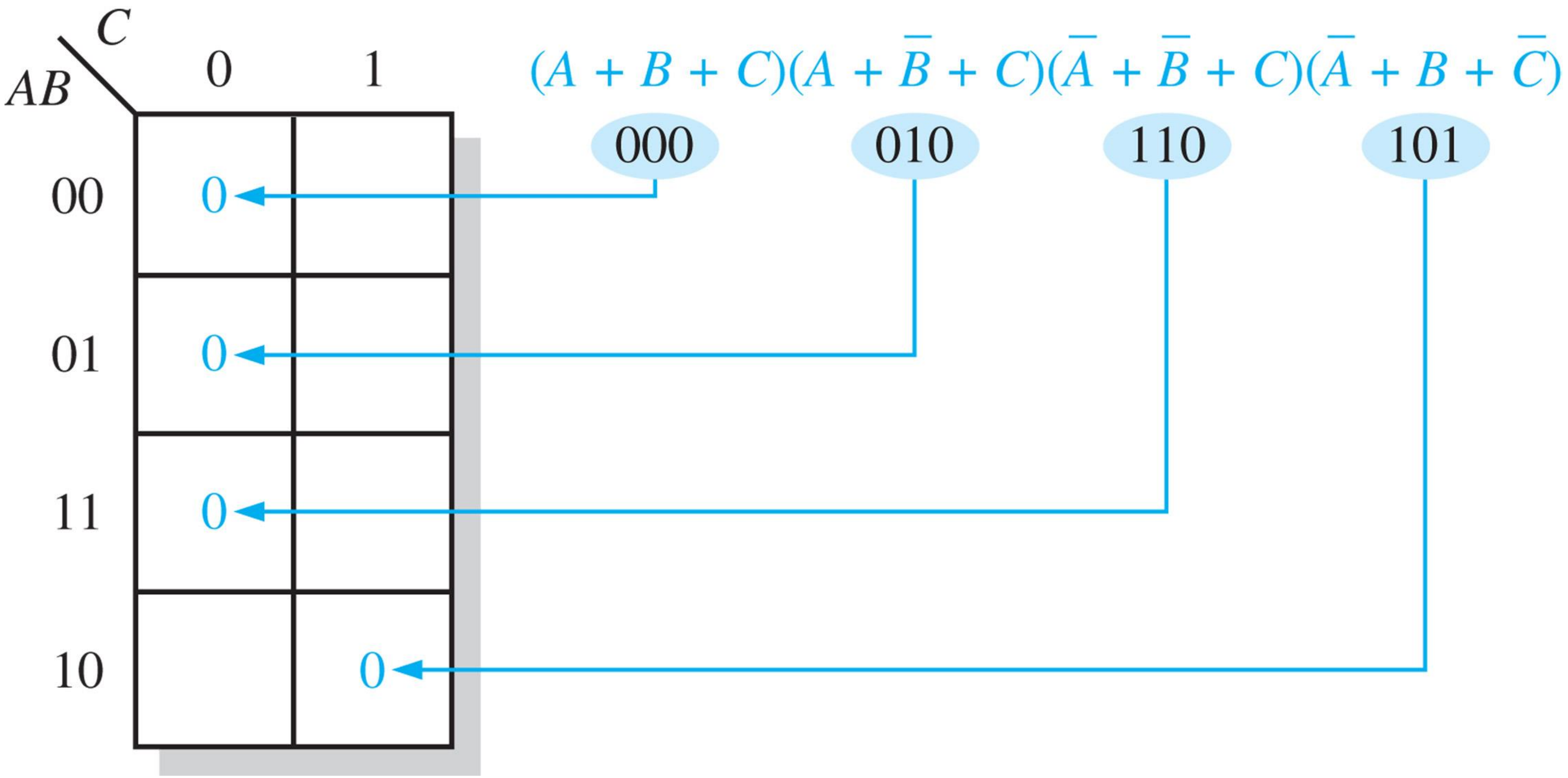
Y=(Y’)’=A+D’

AB \ CD	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	1	1	1
10	1	1	1	1

$$A + D'$$

Mapping a standard POS expression

Place a “0” for each maxterm.



Karnaugh map simplification of POS expressions

Group 0's to determine the minimum POS expression.

Use a Karnaugh map to minimize the following standard POS expression:

$$(A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + \overline{B} + C)$$

Also, derive the equivalent SOP expression.

Solution: (1) Minimum POS: group 0's

$$A(\overline{B} + C)$$

(2) Minimum SOP: group 1's

$$AC + A\overline{B}$$