

Homework 6

Due date: 29th Nov.

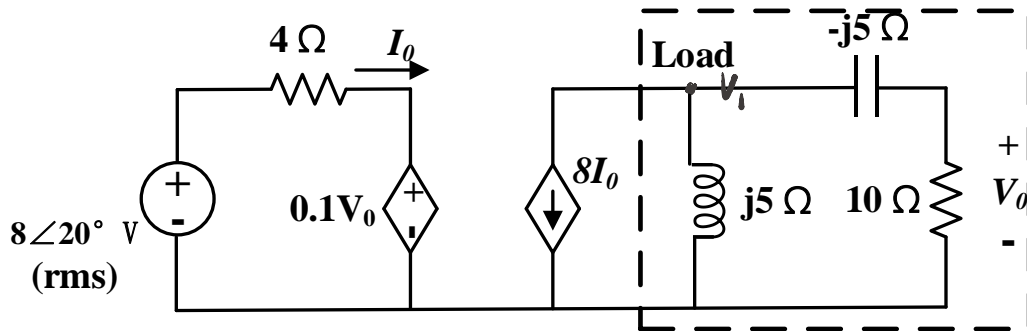
Turn in your homework in class

Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

1. For the circuit below, please find:

- (1) The average power absorbed by the load;
- (2) The reactive power absorbed by the load;
- (3) The complex power S absorbed by the load;
- (4) The power factor pf of the load.



$$1) \quad 8\angle 20^\circ = 4I_o + 0.1V_o \quad \Rightarrow \quad 4I_o = 8\angle 20^\circ - \frac{1}{10-j5} V_o$$

$$6' \quad 8I_o + \frac{V_o}{j5} + \frac{V_o}{-j5+10} = 0 \quad \Rightarrow \quad V_o = 20\sqrt{10} \angle -51.57^\circ$$

$$V_o = \frac{10}{10-j5} V_o$$

$$I_o = \sqrt{2} \angle 65^\circ$$

$$I_L = -8I_o = 8\sqrt{2} \angle -115^\circ$$

$$\Rightarrow P = 20\sqrt{10} \cdot 8\sqrt{2} \cos(-51.57^\circ + 115^\circ) = 320.06 \text{ W}$$

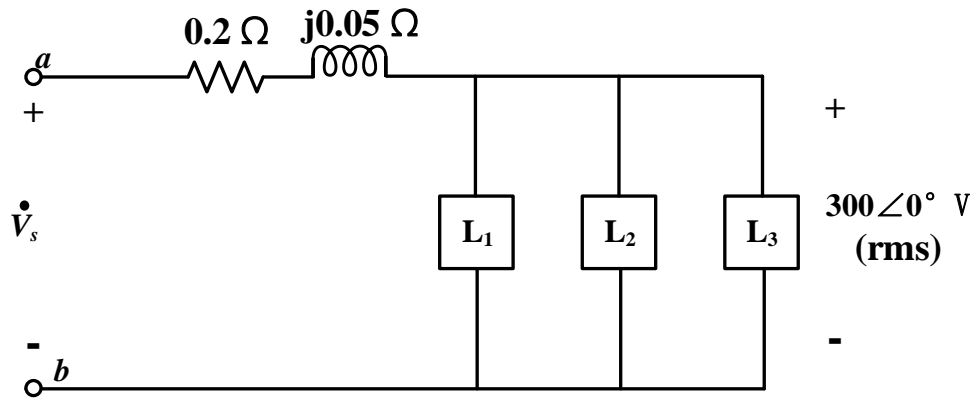
$$2) \quad Q = 20\sqrt{10} \cdot 8\sqrt{2} \sin(-51.57^\circ + 115^\circ) = 639.97 \text{ Var}$$

$$3) \quad S = P + jQ = 320.06 + j639.97 \text{ VA}$$

$$4) \quad \text{p.f.} = \cos(\theta_v - \theta_i) = \cos(-51.57^\circ + 115^\circ) = 0.45$$

2. For the three-load circuit shown below, L_1 absorbs 3 kW at unity power factor; L_2 absorbs 5 kVA at 0.8 leading; L_3 absorbs 5 kW and delivers 6 kvars.

- (1) Calculate the voltage \dot{V}_s .
 (2) Calculate the average power and reactive power associated with the line impedance ($0.2\Omega + j0.05\Omega$).
 (3) Calculate the average power and reactive power between port **a** and port **b**.



1) $S_1 = 3 + j0 \text{ kVA}$

3' $S_2 = 4 - j3 \text{ kVA} \Rightarrow S_T = S_1 + S_2 + S_3 = 12 - j9 \text{ kVA}$

$S_3 = 5 - j6 \text{ kVA}$

$300 \hat{I}_L^* = (12 - j9) \times 10^3$

$\hat{I}_L = 40 + j30 \text{ A}$ 2'

$\therefore V_s = 300 \angle 0^\circ + (40 + j30)(0.2 + j0.05) = 306.5 + j8$ 2'
 $= 306.6 \angle 1.5^\circ \text{ V}$

2) $|\hat{I}_L| = 50$

$P_L = 2500 \times 0.2 = 500 \text{ W}$ 3'

$Q_L = 2500 \times 0.05 = 125 \text{ VAR}$ 3'

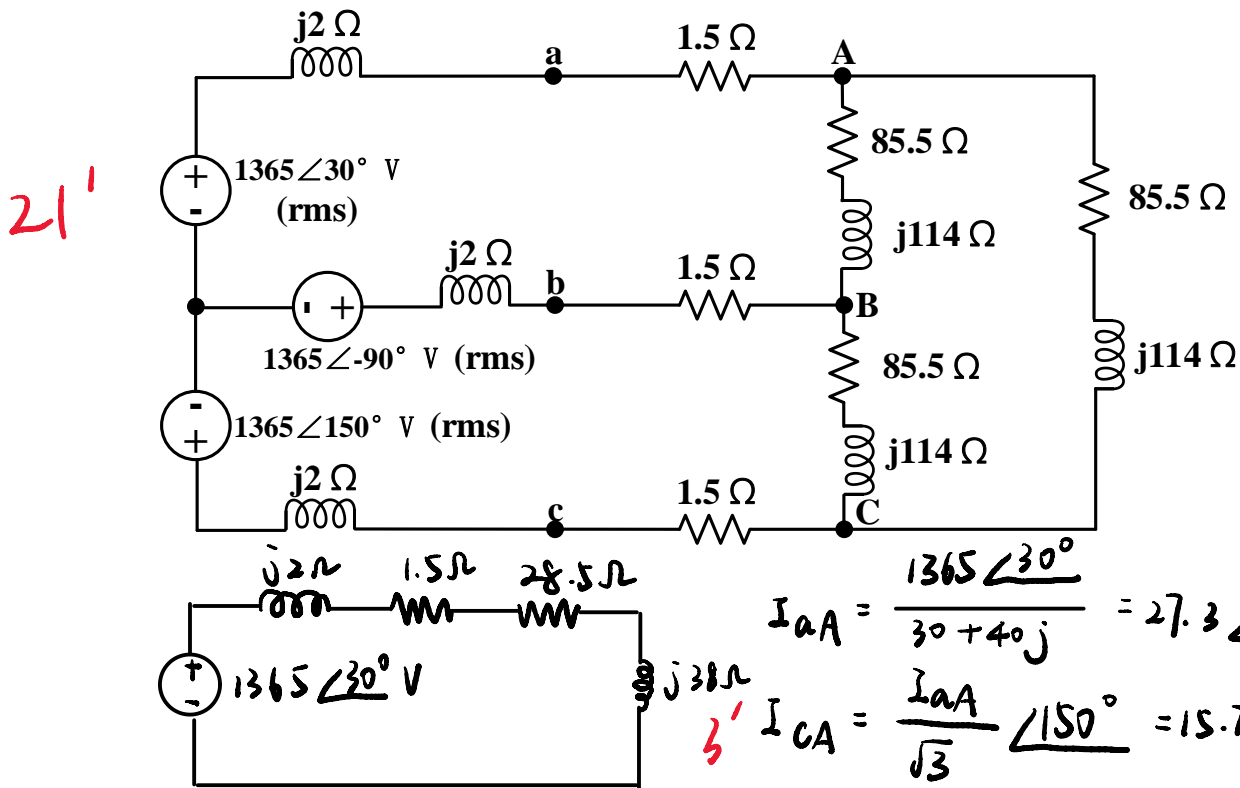
3) $P_{ab} = 12000 + 500 = 12.5 \text{ kW}$ 3'

$Q_{ab} = -9000 + 125 = -8.875 \text{ kVAR}$ 3'

3. For the circuit below,

(1) Find \dot{I}_{AB} , \dot{I}_{BC} , \dot{I}_{CA} , and \dot{V}_{CA} in the circuit.

(2) What percent of the average power delivered by the three-phase source is dissipated in the three-phase load?



$$I_{aA} = \frac{1365 \angle 30^\circ}{30 + 40j} = 27.3 \angle -23.13^\circ \text{ A}$$

3'

$$I_{CA} = \frac{I_{aA}}{\sqrt{3}} \angle 150^\circ = 15.76 \angle 126.87^\circ \text{ A}$$

3'

$$V_{CA} = I_{CA} \cdot (85.5 + j114) = -2245.8 \text{ V}$$

3'

$$I_{AB} = I_{CA} \angle -120^\circ = 15.76 \angle 6.87^\circ \text{ A}$$

3'

$$I_{BC} = I_{CA} \angle -240^\circ = 15.76 \angle -113.13^\circ \text{ A}$$

2)

$$S_{g/p} = -1365 \angle 30^\circ I_{aA}^* = -22358.75 - j29811.56 \text{ VA}$$

2'

$$\therefore P_{\text{deliver}} = 22.359 \text{ kW}$$

2'

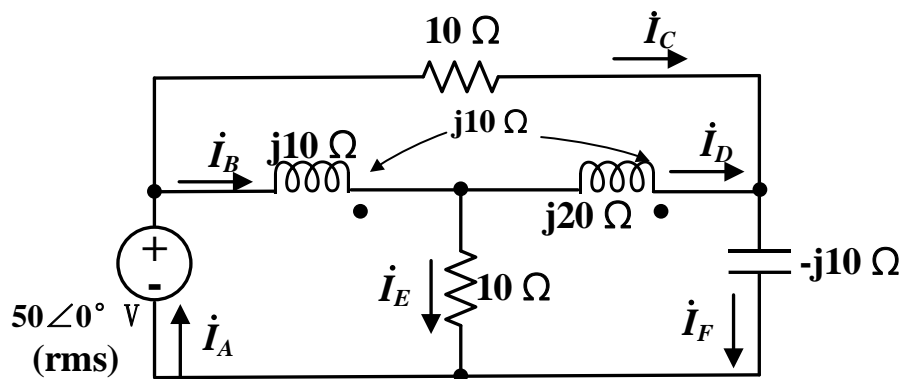
$$P_{\text{dissipate}} = |I_{aA}|^2 \cdot 28.5 = 21.241 \text{ kW}$$

3'

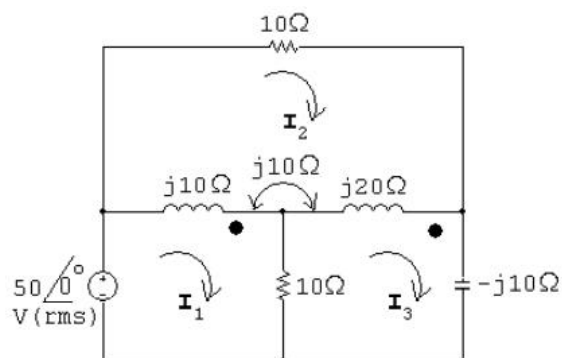
$$\therefore \eta = \frac{21.241}{22.359} \times 100\% = 95\%$$

4. For the circuit below, please find:

- (1) Current \mathbf{i}_D , \mathbf{i}_E , and \mathbf{i}_F
 (2) The complex power on the capacitor, and the power factor pf of the capacitor.



2)



$$50 = j10(I_1 - I_2) + j10(I_3 - I_2) + 10(I_1 - I_3)$$

$$0 = 10I_2 + j20(I_2 - I_3) + j10(I_2 - I_1) + j10(I_2 - I_1) + j10(I_2 - I_3)$$

$$0 = -j10I_3 + 10(I_3 - I_1) + j20(I_3 - I_2) + j10(I_1 - I_2)$$

Solving,

$$\mathbf{I}_1 = 5.5 + j0.5 \text{ A(rms)}; \quad \mathbf{I}_2 = 3 + j2 \text{ A(rms)}; \quad \mathbf{I}_3 = 2 + j2 \text{ A(rms)}$$

$$\mathbf{I}_a = \mathbf{I}_1 = 5.5 + j0.5 \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_1 - \mathbf{I}_2 = 2.5 - j1.5 \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_2 = 3 + j2 \text{ A}$$

$$\mathbf{I}_d = \mathbf{I}_3 - \mathbf{I}_2 = -1 \text{ A}$$

$$\mathbf{I}_e = \mathbf{I}_1 - \mathbf{I}_3 = 3.5 - j1.5 \text{ A}$$

$$\mathbf{I}_f = \mathbf{I}_3 = 2 + j2 \text{ A}$$

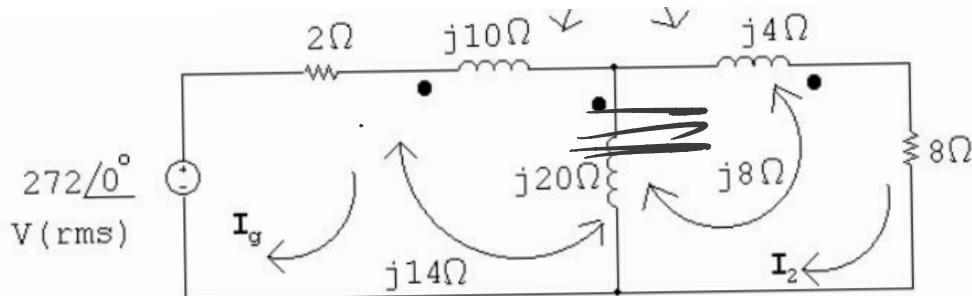
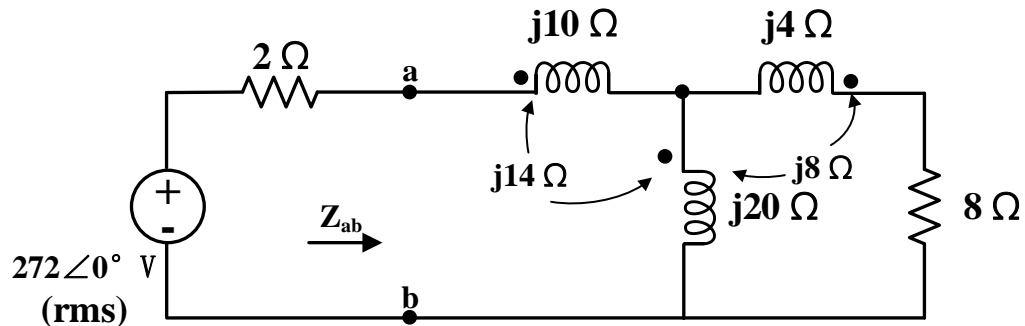
$$2) \quad \mathbf{V}_f = -j10 \mathbf{I}_f = 20 - j20 \text{ V}$$

$$\mathbf{S}_f = \mathbf{V}_f \mathbf{I}_f^* = 0 - j80 \text{ VA}$$

$$\text{pf} = \cos(135^\circ - 45^\circ) = 0.$$

18' 5. For the circuit below:

- Find the average power delivered to the 8Ω resistor.
- Find the average power generated by the ideal sinusoidal source
- Find the impedance of Z_{ab}



$$\begin{aligned}
 272\angle 0^\circ &= 2\mathbf{I}_g + j10\mathbf{I}_g + j14(\mathbf{I}_g - \mathbf{I}_2) \\
 &\quad + j14\mathbf{I}_g - j8\mathbf{I}_2 + j20(\mathbf{I}_g - \mathbf{I}_2) \\
 0 &= j20(\mathbf{I}_2 - \mathbf{I}_g) - j14\mathbf{I}_g + j8\mathbf{I}_2 + j4\mathbf{I}_2 \\
 &\quad + j8(\mathbf{I}_2 - \mathbf{I}_g) + 8\mathbf{I}_2
 \end{aligned}$$

Solving,

$$\mathbf{I}_g = 14.48\angle -56.1^\circ \text{ A} \quad \mathbf{I}_2 = 14.9^\circ \angle -44.79^\circ \text{ A}$$

$$P_{8\Omega} = 14.9^2 (8) = 1776 \text{ W}$$

$$\text{b) } P_g (\text{developed}) = 272 \times 14.48 \cos(-56.1^\circ) = 2196.7 \text{ W}$$

$$\text{c) } Z_{ab} = \frac{V_g}{I_g} - 2 = \frac{272}{14.48\angle -56.1^\circ} - 2 = 17.75\angle 61.5^\circ \Omega$$