ShanghaiTech University

EE 115B: Digital Circuits

Fall 2022

Solution

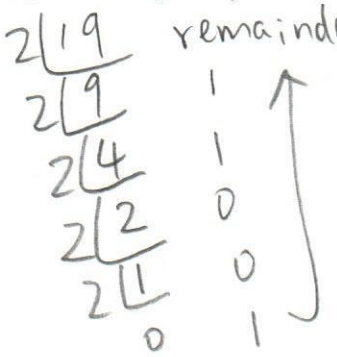
Midterm Exam 1, October 20, 2022

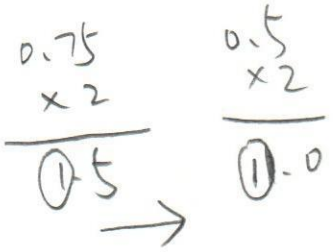
My signature below indicates that I understand and have complied with the Academic Integrity Policy of ShanghaiTech University. I have neither received nor given any unauthorized aid.

Student ID: _____ Name in Chinese: _____

1. Short questions. (27 points, 3 points each.)

(1) Convert $(19.75)_{10}$ to binary.





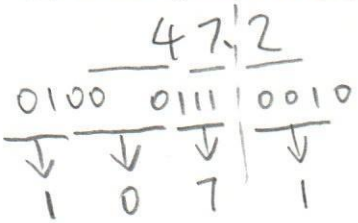
$(19.75)_{10} = (10011.11)_2$

(2) Convert $(11010.01)_2$ to decimal.

$(11010.01)_2 = 1\times2^4 + 1\times2^3 + 0 + 1\times2^1 + 0 + 0 + 1\times2^{-2}$

$= (26.25)_{10}$

(3) Convert $(47.2)_{16}$ to octal.



$(47.2)_{16} = (107.1)_8$

(4) Convert $(35)_8$ to BCD.

$(35)_8 = 3\times8^1 + 5\times8^0 = (29)_{10}$

$= (0010\ 1001)_{BCD}$

(5) Convert $(000110010110)_{BCD}$ to hexadecimal.

$(0001\ 1001\ 0110)_{BCD} = (196)_{10}$

1 9 6

16 | 196 remainder 4

16 | 12 remainder 12 → C

0

$(196)_{10} = (C4)_{16}$

(6) Determine the even parity bit for 11010101.

5 1's → even parity bit is "1".

(7) (True or False) The XOR gate is also called the equivalence gate.

False

(8) (True or False) For any combination of inputs, the sum of any two minterms is 0.

False

(9) (True or False) For any combination of inputs, the product of all maxterms is 1.

False

2. Consider the XNOR gate. (9 points.)

(1) Draw its circuit symbol. (3 points.)

(2) Assuming that its two inputs are labeled as A and B and its output is labeled as Y, build and complete its truth table. (4 points.)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(3) Write its operation in two ways: (a) using its own Boolean operator and (b) using the three basic Boolean operators: AND, OR, and NOT. The input and output notations are specified in (2). (2 points.)

$Y = A \odot B = \bar{A}\bar{B} + AB$

3. Design a 4-bit "Gray" code. Note that in a "Gray" code system, any two adjacent codes have one and only one different bit. The first code corresponding to the decimal number of 0 is "1111". Denote the code as $G_3G_2G_1G_0$. The bit pattern for G_0 is "1001". You need to: (a) determine the bit patterns for the remaining three bits and (b) build and complete the 4-bit "Gray" code table. (28 points.)

(a) Bit patterns. (12 points, 4 points each.)

G_1: 11 00 00 11

G_2: 11 11 00 00 00 00 11 11

G_3: 11 11 11 11 00 00 00 00 00 00 00 00 11 11 11 11

(b) Build and complete the 4-bit "Gray" code table. (16 points.)

Decimal	Gray Code
0	1 1 1 1
1	1 1 1 0
2	1 1 0 0
3	1 1 0 1
4	1 0 0 1
5	1 0 0 0
6	1 0 1 0
7	1 0 1 1

Decimal	Gray Code
8	0 0 1 1
9	0 0 1 0
10	0 0 0 0
11	0 0 0 1
12	0 1 0 1
13	0 1 0 0
14	0 1 1 0
15	0 1 1 1

4. Derive the standard SOP and POS expressions for the following function. Write your results in two forms: (a) explicit expressions with minterms/maxterms (e.g., Y=ABC) and (b) compact expressions with indexes of minterms/maxterms (e.g., Y=∑m(7) for Y=ABC). (20 points, 5 points each.)

$$\boldsymbol{Y(A,B,C,D) = (A + B + \overline{C})(\overline{B} + D)}$$

$$Y(A,B,C,D) = (A+B+\overline{C}+D\overline{D})(A\overline{A}+\overline{B}+D)$$

$$= (A+B+\overline{C}+D)(A+B+\overline{C}+\overline{D})(A+\overline{B}+D)(\overline{A}+\overline{B}+D)$$

$$= (A+B+\overline{C}+D)(A+B+\overline{C}+\overline{D})(A+\overline{B}+C\overline{C}+D)(\overline{A}+\overline{B}+C\overline{C}+D)$$

$= (A + B + \bar{C} + D)(A + B + \bar{C} + \bar{D})(A + \bar{B} + C + D)(A + \bar{B} + \bar{C} + D) \cdot$
$(\bar{A} + \bar{B} + C + D)(\bar{A} + \bar{B} + \bar{C} + D)$

$= \Pi M(2, 3, 4, 6, 12, 14)$

$= \Sigma m(0, 1, 5, 7, 8, 9, 10, 11, 13, 15)$

$= \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}D + \bar{A}BCD + A\bar{B}\bar{C}\bar{D} +$
$A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + A\bar{B}CD + AB\bar{C}D + ABCD$

5. Determine the standard SOP expression and the equivalent standard POS expression for the truth table below. Write your results in both the explicit and compact forms illustrated in Problem 4. (16 points, 4 points each.)

Inputs			Output
A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

SoP: $Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$

$= \Sigma m(1, 2, 4, 7)$

PoS: $Y = (A + B + C)(A + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + C)$

$= \Pi M(0, 3, 5, 6)$