Lecture 19 – NMR Physics

This lecture will cover: *(CH5.1-5.7, 5.13)*

– Introduction

– Effects of a strong magnetic field

– Magnetic Resonance

(*Supplementary reading: The Essential Physics of Medical Imaging CH12.1-12.3*)

Introduction to MRI

- Provide a spatial map of hydrogen nuclei in different tissues;
- Image intensity depends on the number of proton and physical properties of the tissue;

Fig. The MR coronal, sagittal and axial images, respectively, of the brain.

Introduction to MRI

- Advantages
  - No ionizing radiation;
  - Acquire images in any 2D and 3D plane;
  - Excellent soft-tissue contrast
  - A spatial resolution of 1mm or less
  - Negligible penetration effect
- Disadvantages
  - Slow acquisition time
  - Not able to scan patients with metallic implants;
  - Very expensive

MRI system

- Superconducting magnet
- A set of three magnetic field gradient coils
- Radio frequency transmitter and receiver

(a)

(b)

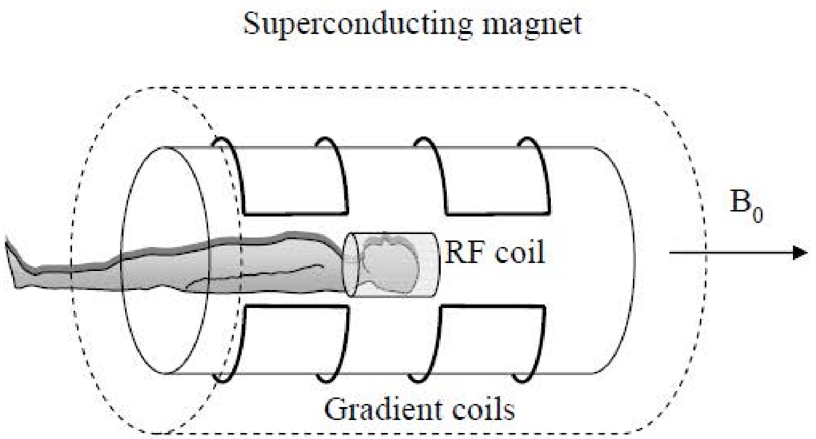

Fig. (a) A high-field clinical magnet with patient bed. (b) The three major components of an MRI system, including the superconducting magnet which produces a strong magnetic field (B0) in the horizontal direction. Only one of the three gradient coils is shown for clarity.

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- **– Effects of a strong magnetic field**
- – Magnetic Resonance

Nuclear Spin

➢ Spin (自旋): a charged particle rotate around an internal axis with a given value of angular momentum;

$$L_I = \sqrt{I(I+1)} \cdot \hbar$$

where $\hbar = h/2\pi$: h is Planck constant

I : spin quantum number (自旋量子数)

- $I = 0$, even number of proton and neutron
- I is integer: odd number of proton and neutron
- I is half: the addition of proton and neutron is odd

L_I in a magnetic field with direction of z

$$L_z = m_I \cdot \hbar, \quad m_I = I, I-1, I-2, \cdots - I$$

m_I: spin magnetic quantum number (自旋磁量子数)

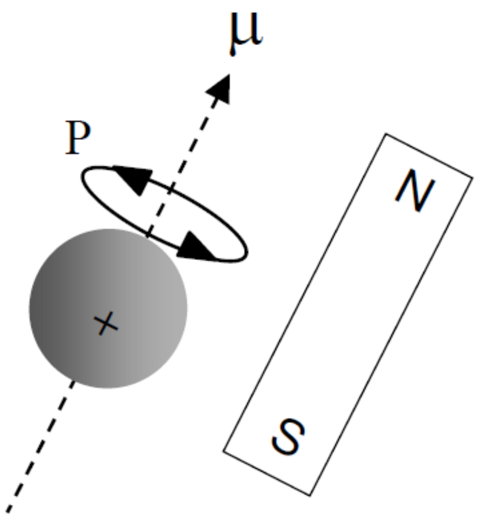

Fig. The internal rotation of a proton creates a magnetic moment, and so the proton acts as a magnet with north and south pole.

Magnetic moment

- Magnetic moment (磁矩): spinning nucleus can be thought of as a very small bar magnet with north and south pole.
- The nuclear magnetic moment:

$$\overrightarrow{\mu_I} = \gamma \cdot \overrightarrow{L_I}$$

Where $\gamma = g_I e/2m_p$ is gyromagnetic ratio (磁旋比)

g_I : g factor, a dimensionless quantity that characterizes the magnetic moment and angular momentum of an atom, a particle or nucleus.

- μ_I in a magnetic field with direction of z

$$\mu_z = \gamma L_z = m_I g_I \mu_N, \quad m_I = I, I-1, I-2, \cdots - I$$

where $\mu_N = e\hbar/2m_p$ is nuclear magneton (核磁子)

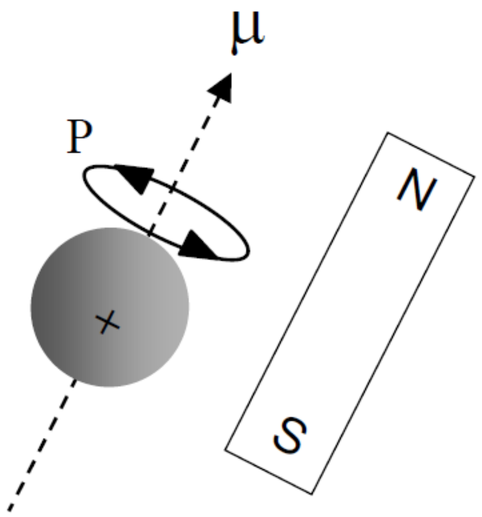

Fig. The internal rotation of a proton creates a magnetic moment, and so the proton acts as a magnet with north and south pole.

Medically useful nuclei

Nucleus	Isotopic abundance	Relative sensitivity	spin quantum number	g factor	Magnetic moment (μ_N)	gyromagnetic ratio (10^8Hz/Tesla)
${}_{1}^{1}H$	99.98%	1	1/2	5.5855	2.7927	2.6753
${}_{6}^{13}C$	1.10%	0.016	1/2	1.4046	0.70216	0.6728
${}_{7}^{14}N$	0.36%	0.001	1	0.7023	0.40357	0.1934
${}_{9}^{19}F$	100%	0.830	1/2	5.256	2.6273	2.5179
${}_{11}^{23}Na$	100%	0.093	3/2	1.478	2.2161	0.7031
${}_{15}^{31}P$	100%	0.066	1/2	2.262	1.1305	1.084

Energy in the magnetic field

When magnetic filed B_0 is applied:

- Two magnetic moments for proton: parallel and anti-parallel to the direction of $\mathbf{B_0}$
- The energy of the protons is related to the magnetic moments
- The energy difference between two states:

$$\Delta E = g_I \mu_N B_0 = \frac{\gamma h B_0}{2\pi}$$

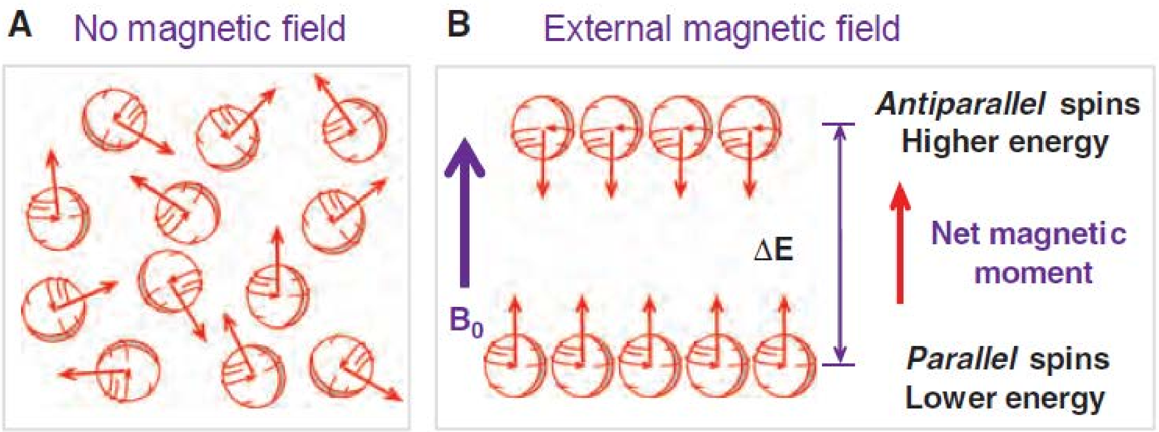

Fig. Simplified distributions of “free” protons without and with an external magnetic field are shown. (**A**) Without an external magnetic field, a group of protons assumes a random orientation of magnetic moments, producing an overall magnetic moment of zero. (**B)** Under the influence of an applied external magnetic field, B_0, the protons assume a nonrandom alignment in two possible orientations: parallel and antiparallel to the applied magnetic field. A slightly greater number of protons exist in the parallel direction, resulting in a measurable net magnetic moment in the direction of B_0.

Energy in the magnetic field

- The relative number of protons in the two states:

$$\frac{N_{\text{anti-parallel}}}{N_{\text{parallel}}} = e^{-\frac{\Delta E}{kT}} = e^{-\frac{\gamma h B_0}{2\pi kT}} \approx 1 - \frac{\gamma h B_0}{2\pi kT}$$

Where B_0: static magnetic field

T: temperature in Kelvin

k: Boltzmann's constant ($1.38 * 10^{-23}$J/K)

- The difference in population between two energy levels:

$$N_{\text{parallel}} - N_{\text{anti-parallel}} = N_{\text{total}} \frac{\gamma h B_0}{4\pi kT}$$

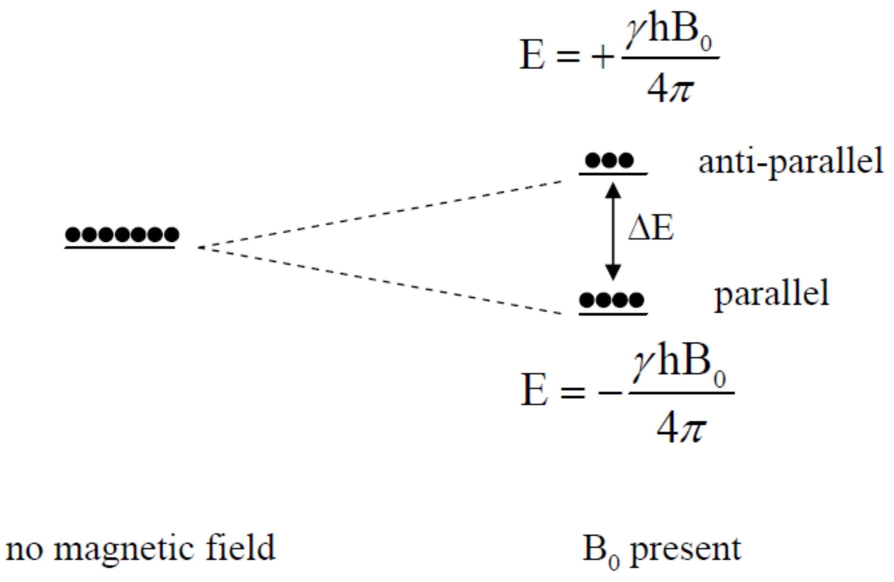

Fig. Proton configurations. (left) In the absence of a strong magnetic field, the energies of all the random orientations of the magnetic moments are the same. (right) When a strong magnetic field is applied, the single energy level splits into two levels, one corresponding to the magnetic moments being in the parallel state, and the other the anti-parallel state. The energy difference between the two levels depends upon the value of B_0.

Classical Precession

- There is an angle between the axis of spin and the direction of magnetic field;
- A torque is created by the combination of the magnetic field and the spin, which is perpendicular to the spin angular moment and causes proton precess (进动、旋进) around the axis of the magnetic field;
- Larmor precession (拉摩尔旋进):

$$\omega_0 = 2\pi f_0 = \gamma B_0$$

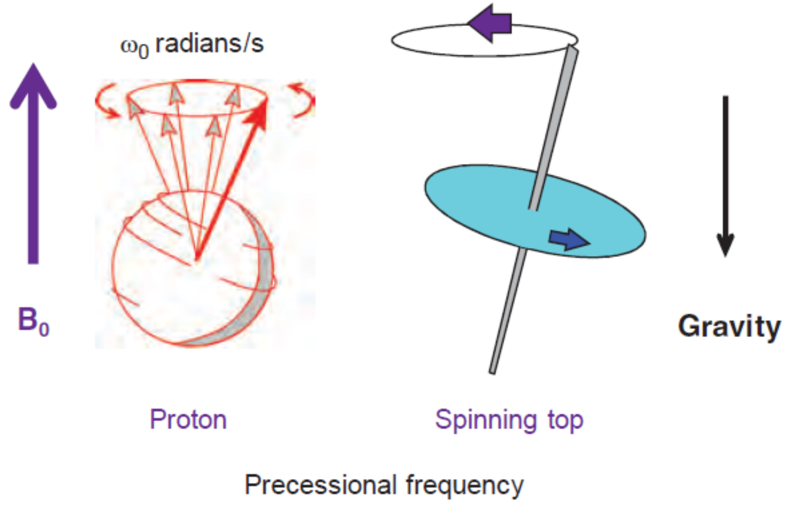

Fig. A single proton *precesses* about its axis with an angular frequency, ω, proportional to the externally applied magnetic field strength, according to the *Larmor* equation. A well-known example of precession is the motion a spinning top makes as it interacts with the force of gravity as it slows.

Net magnetization

➢ The net magnetization (磁化强度) with direction of B_0:

$$M_0 = \sum_{n=1}^{N_{\text{total}}} \mu_{z,n}$$

$$= \frac{\gamma h}{4\pi}\left(N_{\text{parallel}} - N_{\text{anti-parallel}}\right)$$

$$= \frac{\gamma^2 h^2 B_0 N_{\text{total}}}{16\pi^2 kT}$$

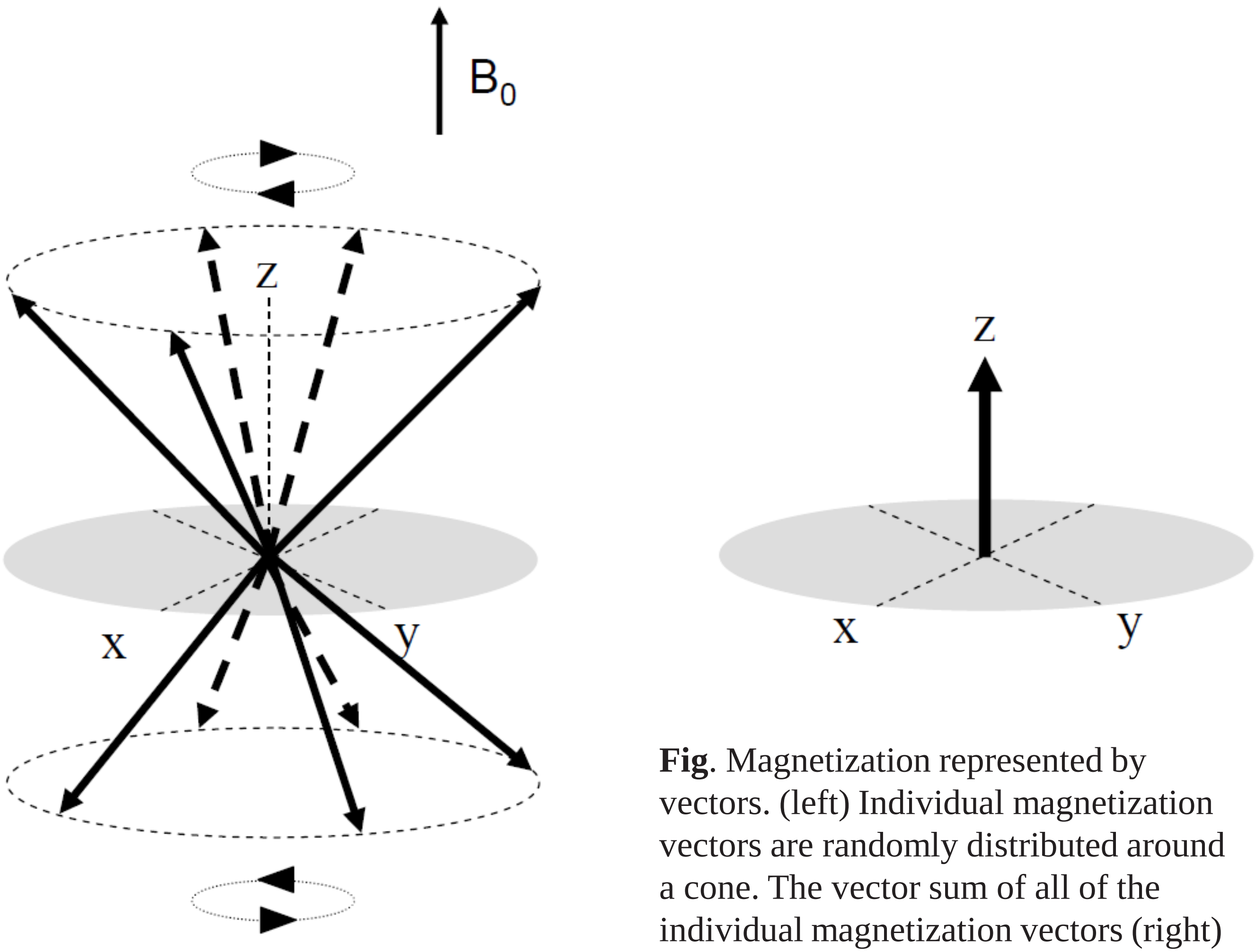

Fig. Magnetization represented by vectors. (left) Individual magnetization vectors are randomly distributed around a cone. The vector sum of all of the individual magnetization vectors (right) is simply a static component in the direction of B_0.

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- **Magnetic Resonance**

Magnetic Resonance

- Radiofrequency (RF, 射频) pulse: $hf = \Delta E = \frac{\gamma h B_0}{2\pi} \quad \Rightarrow f = \frac{\gamma B_0}{2\pi}$
  - Stimulate transition between the energy levels;
  - The magnetic component of the RF pulse as B_1
  - Perpendicular to the direction of static magnetic field or net magnetization z
  - The frequency of RF is identical to the procession frequency
  - Creation of transverse magnetization, and the “tip angle” defined as the angle through which the net magnetization is rotated: $\alpha = \gamma B_1 \tau_{B_1}$, where τ_{B_1} is the time of RF pulse

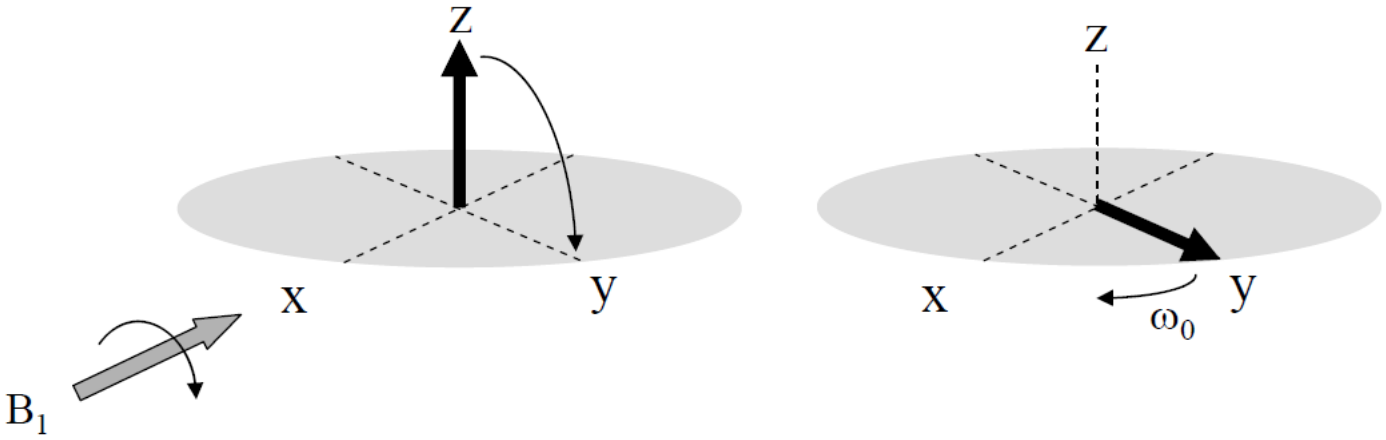

Fig. (left) Application of an RF pulse about the x-axis rotates the magnetization from the z-direction towards the y-axis. If the RF pulse strength and duration are chosen to produce a 90° pulse, then the magnetization lies directly along the y-axis. When the RF pulse is switched off (right), the magnetization precesses around the z-axis at the Larmor frequency ω_0.

MR signal detection

- A pair of conductive loops perpendicular to each other and static magnetic field;
- The induced voltage is proportional to the time rate of change of the magnetic flux $\boldsymbol{\varphi}$

$$V \propto -\frac{d\varphi}{dt}$$

- For a 90° pulse

$$V_y \propto M_0\omega_0 \sin\omega_0 t$$

$$V_x \propto -M_0\omega_0 \cos\omega_0 t$$

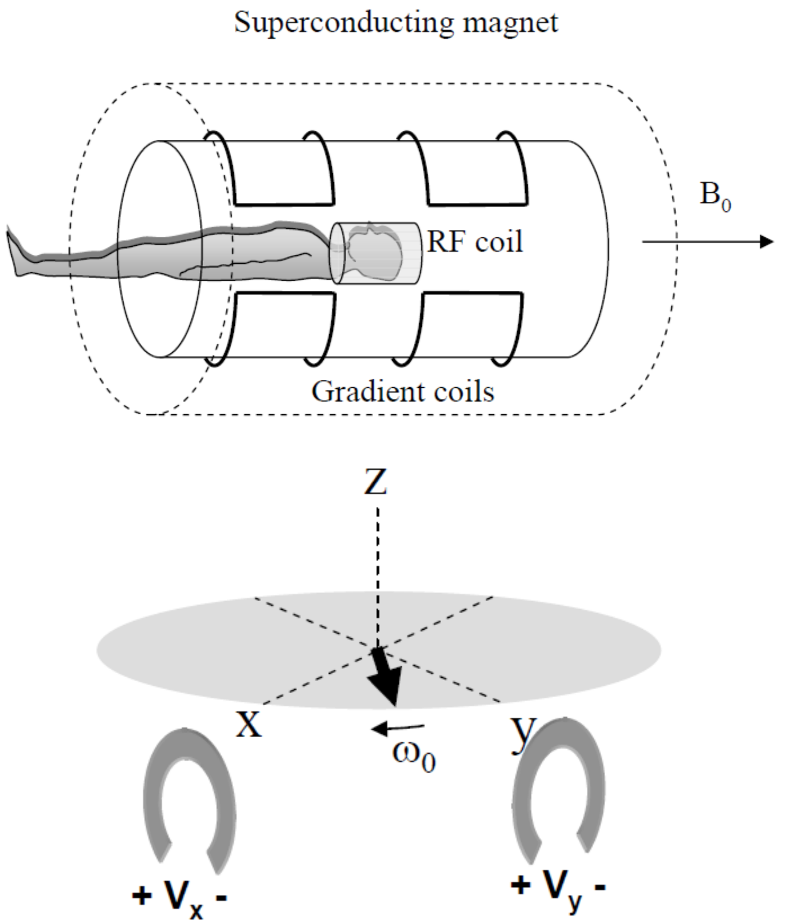

Fig. The MR signal is measured via Faraday induction. Either one or two RF coils can be used, with a voltage being induced across the ends of the conductor loops by the precessing magnetization

MR signal intensity

$$V_y \propto M_0 \omega_0 \sin\omega_0 t \qquad V_x \propto -M_0 \omega_0 \cos\omega_0 t$$

where $M_0 = \frac{\gamma^2 h^2 B_0 N_{\text{total}}}{16\pi^2 kT}$, $\omega_0 = 2\pi f_0 = \gamma B_0$

The intensity of MR signal is determined by

- Proportional to the number of protons in the object (N_{total})
- The value of B_0 (proportional to $(B_0)^2$ from M_0 and ω_0)