1. $X(s) = \frac{1}{s+1} - \frac{1}{s+2}$

(a) ROC: $Re(s) > -1$ $\quad x(t) = e^{-t}u(t) - e^{-2t}u(t)$

(b) ROC: $Re(s) < -2$ $\quad x(t) = -e^{-t}u(-t) + e^{-2t}u(-t)$

(c) ROC: $-2 < Re(s) < -1$ $\quad x(t) = -e^{-t}u(-t) - e^{-2t}u(t)$

($Re(s) > -1$ and $Re(s) < -2$ is impossible)

4. $X(s) = \frac{\beta}{s+1}$

$G(s) = X(s) + \alpha X(-s) = \frac{\beta}{s+1} + \frac{\alpha\beta}{-s+1}$

$$= \frac{(\beta - \alpha\beta)s - (\alpha\beta + \beta)}{s^2 - 1}$$

$$\begin{cases} \beta - \alpha\beta = 1 \\ \alpha\beta + \beta = 0 \end{cases} \rightarrow \begin{cases} \alpha = -1 \\ \beta = \frac{1}{2} \end{cases}$$

3 (a) $X(s) = \frac{s}{s^2+4}$, $Re\{s\} > 0$.

$x(t) = \cos(2t)\, u(t)$

(b) $X(s) = \frac{s^2 - s + 1}{(s+1)^2} = 1 - \frac{3s}{(s+1)^2} = 1 - \frac{A}{s+1} - \frac{B}{(s+1)^2}$, $Re\{s\} > -1$

$A = \lim_{s \to -1} \frac{d}{ds} (s+1)^2 \cdot \frac{3s}{(s+1)^2} = 3$

$B = \lim_{s \to -1} \frac{d}{ds} (s+1)^2 \cdot \frac{3s}{(s+1)^2} = -3$

$X(s) = 1 - \frac{3}{s+1} + \frac{3}{(s+1)^2}$

$x(t) = \delta(t) - 3te^{-t}u(t) - 3e^{-t}u(t)$

(c) $X(s) = \frac{s+1}{(s+1)^2+4}$, $Re\{s\} > -1$. $\quad \frac{s+a}{(s+a)^2 + w_0^2} \Longleftrightarrow \cos(w_0 t)\, e^{-at}u(t)$

$x(t) = \cos(2t)\, e^{-t}u(t)$

2. [20 points] An LTI system has an impulse response $h(t)$ for which the Laplace transform $H(s)$ is

$$H(s) = \int_{-\infty}^{\infty} h(t)e^{-st}dt = \frac{1}{s+1}, \quad Re\{s\} > -1$$

Determine the system output $y(t)$ for all t if the input $x(t)$ is given by

$$x(t) = e^{\frac{-t}{2}} + 2e^{\frac{-t}{3}} \quad \forall t$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

suppose $x(t) = e^{at}$

$$\Rightarrow y(t) = \int_{-\infty}^{\infty} h(\tau)\, e^{a(t-\tau)}\, d\tau$$

$$= e^{at} \int_{-\infty}^{\infty} h(\tau)\, e^{-a\tau}\, d\tau$$

by $H(s)$

$$= e^{at} \cdot \frac{1}{a+1}$$

Now $x(t) = e^{-\frac{1}{2}t} + 2e^{-\frac{1}{3}t}$

$$\Rightarrow y(t) = e^{-\frac{1}{2}t} \cdot \frac{1}{-\frac{1}{2}+1} + 2 \cdot e^{-\frac{1}{3}t} \cdot \frac{1}{1-\frac{1}{3}}$$

$$= 2e^{-\frac{1}{2}t} + 3e^{-\frac{1}{3}t}$$

5. [20 points] Consider a signal $y(t)$ which is related to two signals $x_1(t)$ and $x_2(t)$ by

$$y(t) = x_1(t-2) * x_2(-t+3)$$

where

$$x_1(t) = e^{-2t}u(t) \quad and \quad x_2(t) = e^{-3t}u(t)$$

Given that

$$e^{-at}u(t) \xrightarrow{\mathcal{L}} \frac{1}{s+a}, \quad Re\{s\} > -a$$

use properties of the Laplace transform to determine the Laplace transform $Y(s)$ of $y(t)$.

Solution:

From Table 9.2 we have

$$x_1(t) = e^{-2t}u(t) \stackrel{\mathcal{L}}{\longleftrightarrow} X_1(s) = \frac{1}{s+2}, \quad \mathcal{R}e\{s\} > -2$$

and

$$x_1(t) = e^{-3t}u(t) \stackrel{\mathcal{L}}{\longleftrightarrow} X_1(s) = \frac{1}{s+3}, \quad \mathcal{R}e\{s\} > -3.$$

Using the time-shifting time-scaling properties from Table 9.1, we obtain

$$x_1(t-2) \stackrel{\mathcal{L}}{\longleftrightarrow} e^{-2s}X_1(s) = \frac{e^{-2s}}{s+2}, \quad \mathcal{R}e\{s\} > -2$$

and

$$x_2(-t+3) \stackrel{\mathcal{L}}{\longleftrightarrow} e^{-3s}X_2(-s) = \frac{e^{-3s}}{3-s}, \quad \mathcal{R}e\{s\} > -3.$$

Therefore, using the convolution property we obtain

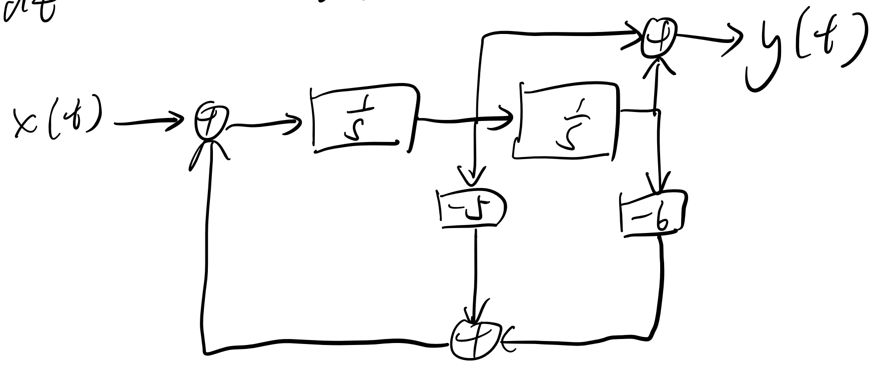
$$y(t) = x_1(t-2) * x_2(-t+3) \stackrel{\mathcal{L}}{\longleftrightarrow} Y(s) = \left[\frac{e^{-2s}}{s+2}\right]\left[\frac{e^{-3s}}{3-s}\right].$$

Re{s}>-2

6.

(a) $Y(s)(s^2+5s+6) = X(s)(s+1)$

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + x(t)$$



(b)

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 4y(t) = \frac{dx(t)}{dt}$$