Lecture 11: Course Review

Ziyu Shao

School of Information Science and Technology
ShanghaiTech University

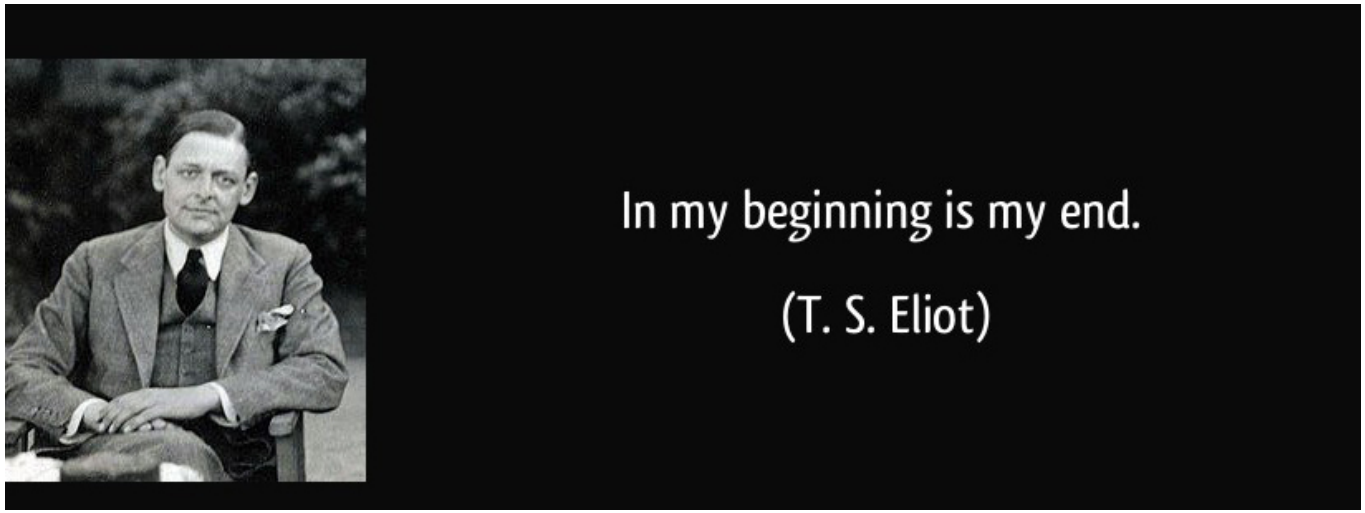
December 30, 2021

Final Exam

Outline

Outline

In My Beginning is My End



Basic Contents: Part I

- Probability and Counting: Definition of Probability, Counting, Bose-Einstein Model, Birthday Match Problem, Hash Table.
- Conditional Probability: Bayes'rule, LOTP, Conditioning, Gambler's ruin, Simpson's Paradox, Monty Hall.
- Random Variables and Distributions: Bernoulli, Binomial, Story for distributions, entropy.
- Expectation: Indicator r.v., Geometric, Coupon Collector, Poisson, Probability Generating Function.
- Continuous Random Variables: Universality of the Uniform, Normal, Exponential, Memoryless, Moment Generating Function.

Basic Contents: Part II

- Joint Distributions: Joint/Marginal distribution, chicken-egg model, Poisson, meaning of conditioning on zero-probability event, Four Forms of Bayes' Rule, Four Forms of LOTP, Poisson clock, Covariance, Correlation, Multivariate Normal Distribution.

- Transformations: Change of Variables, Jacobian Matrix, Bivariate Normal Joint PDF, Convolution, Order Statistics, Beta-Binomial Conjugacy, Dirichlet-Multinomial Conjugacy, Bayesian Ranking, Gamma, Gamma-Poisson Conjugacy, Bank–post Office model.

Basic Contents: Part III

- Conditional Expectation: Conditional Expectation Given An Event , Conditional Expectation Given An R.V., LOTE, Adam's Law, Adam's Law with Extra Conditioning, Eve's law, Linear Regression, Projection Interpretation, Geometric Perspective, Prediction Perspective (MMSE).
- Inequalities and Limit Theorems: Cauchy-Schwarz Inequality, Jensen's Inequality, Kullback-Leibler Divergence (Entropy), Markov's Inequality, Chebyshev's Inequality, Chernoff's Inequality (related to MGF), Chernoff's Technique, Hoeffding Lemma, Hoeffding Bound, Parameter Estimation (confidence interval), Law of Large Numbers, Monte Carlo Method for Estimation π, Central Limit Theorem.

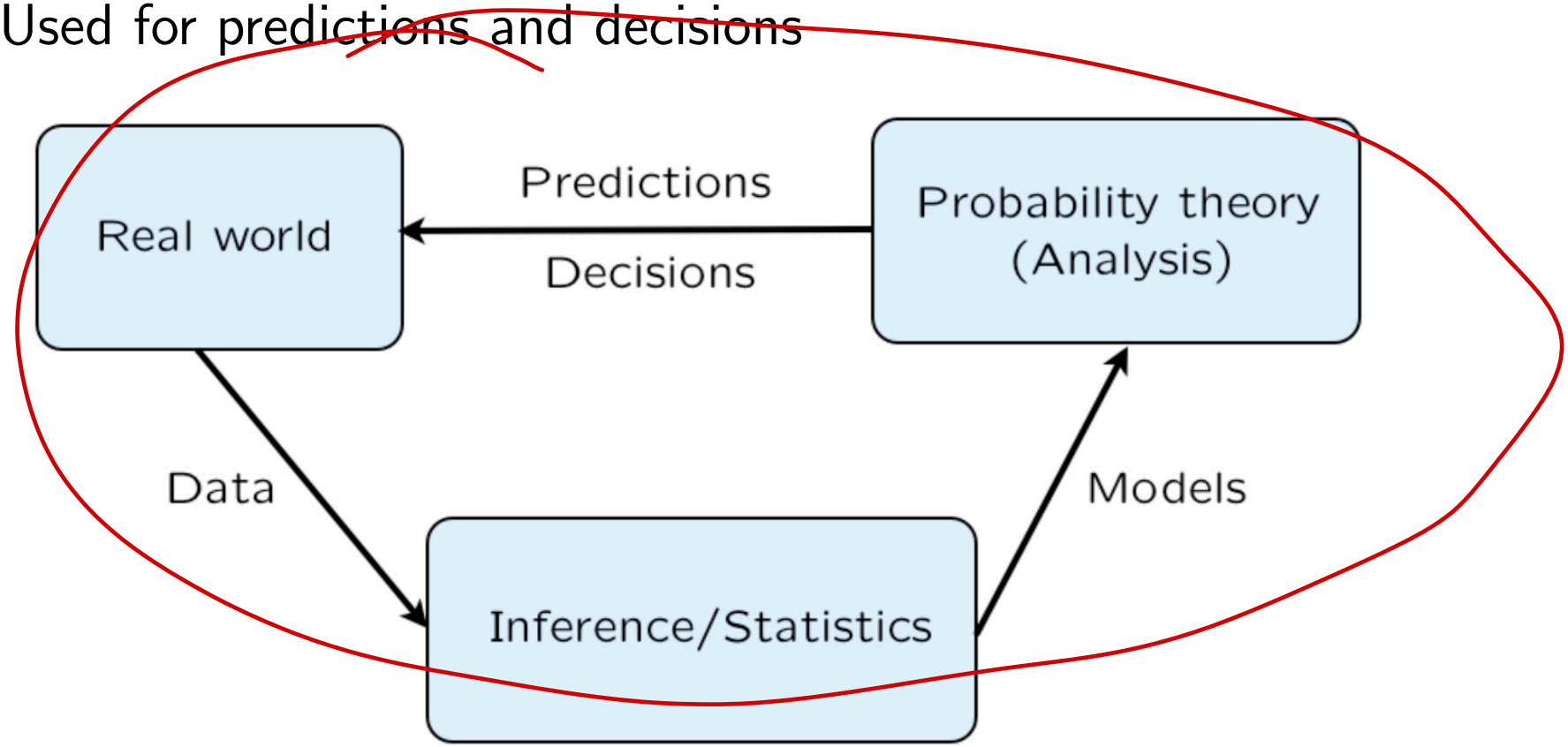
Basic Contents: Part IV

- Classical Statistical Inference: Maximum Likelihood Estimation, MLE & Information Theory Perspective for Normal Distribution, Family of Normal Distribution, Confidence Interval.
- Bayesian Statistical Inference: Statistical Inference, Bayesian Inference and the Posterior Distribution, The Maximum A Posteriori Probability (MAP) Rule, Bayesian Estimation, Linear Least Square Estimate (LLSE), Minimum Mean Square Error Estimator (MMSE), Orthogonality Property of MMSE, MMSE and LLSE, MMSE for Jointly Gaussian Random Variables, Kalman Filter.

The Role of Probability & Statistics

A framework for analyzing phenomena with uncertain outcomes:

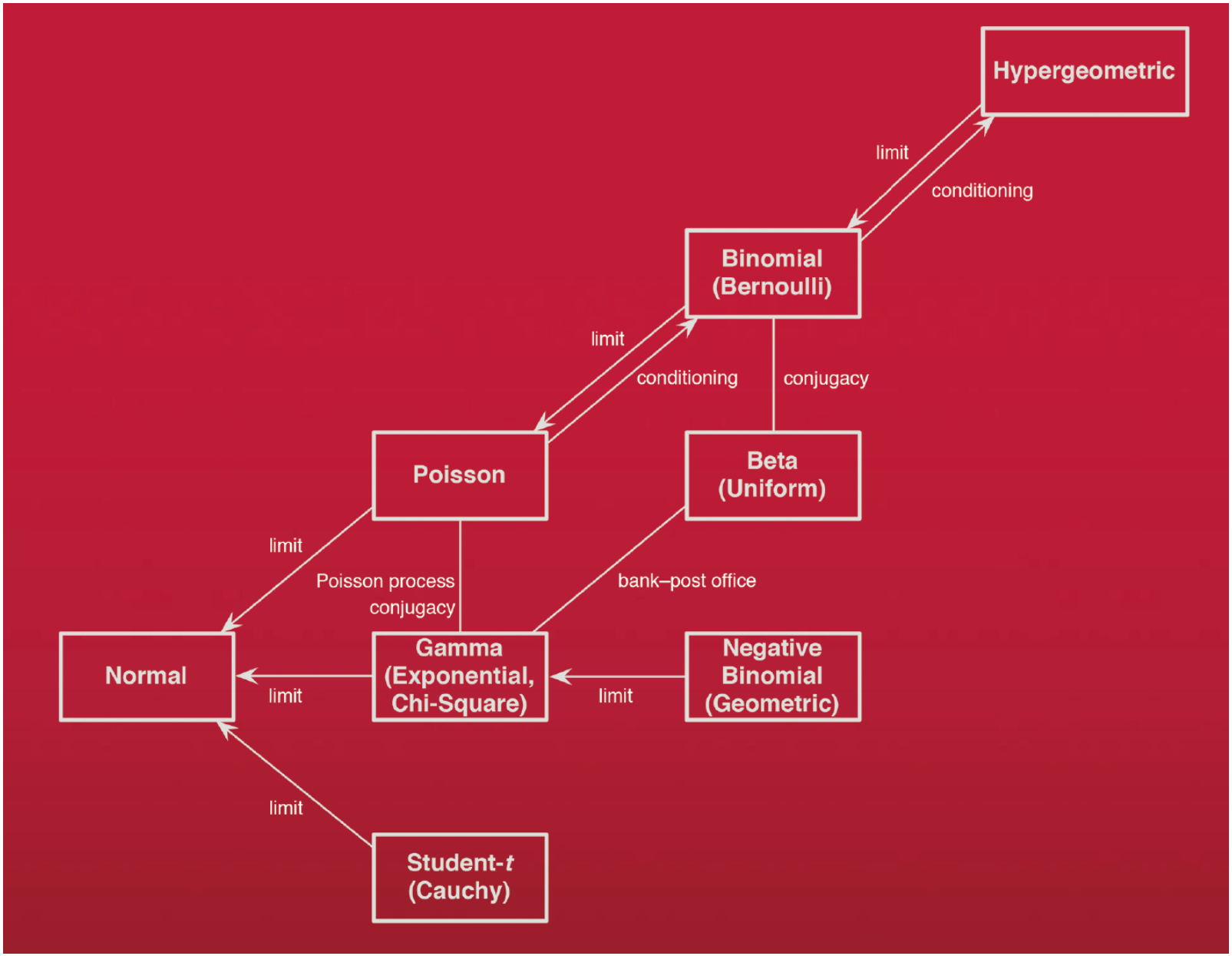
- Rules for consistent reasoning
- Used for predictions and decisions



Typical Distributions

Name	Param.	PMF or PDF	Mean	Variance
Bernoulli	p	$P(X=1)=p, P(X=0)=q$	p	pq
Binomial	n,p	$\binom{n}{k}p^k q^{n-k}$, for $k \in \{0,1,\dots,n\}$	np	npq
FS	p	pq^{k-1}, for $k \in \{1,2,\dots\}$	$1/p$	q/p^2
Geom	p	pq^k, for $k \in \{0,1,2,\dots\}$	q/p	q/p^2
NBinom	r,p	$\binom{r+n-1}{r-1}p^r q^n, n \in \{0,1,2,\dots\}$	rq/p	rq/p^2
HGeom	w,b,n	$\frac{\binom{w}{k}\binom{b}{n-k}}{\binom{w+b}{n}}$, for $k \in \{0,1,\dots,n\}$	$\mu = \frac{nw}{w+b}$	$(\frac{w+b-n}{w+b-1})n\frac{\mu}{n}(1-\frac{\mu}{n})$
Poisson	λ	$\frac{e^{-\lambda}\lambda^k}{k!}$, for $k \in \{0,1,2,\dots\}$	λ	λ
Uniform	$a<b$	$\frac{1}{b-a}$, for $x \in (a,b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Normal	μ,σ^2	$\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/(2\sigma^2)}$	μ	σ^2
Log-Normal	μ,σ^2	$\frac{1}{x\sigma\sqrt{2\pi}}e^{-(\log x-\mu)^2/(2\sigma^2)}, x>0$	$\theta = e^{\mu+\sigma^2/2}$	$\theta^2(e^{\sigma^2}-1)$
Expo	λ	$\lambda e^{-\lambda x}$, for $x>0$	$1/\lambda$	$1/\lambda^2$
Gamma	a,λ	$\Gamma(a)^{-1}(\lambda x)^a e^{-\lambda x}x^{-1}$, for $x>0$	a/λ	a/λ^2
Beta	a,b	$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1}$, for $0<x<1$	$\mu = \frac{a}{a+b}$	$\frac{\mu(1-\mu)}{a+b+1}$
Chi-Square	n	$\frac{1}{2^{n/2}\Gamma(n/2)}x^{n/2-1}e^{-x/2}$, for $x>0$	n	$2n$
Student-t	n	$\frac{\Gamma((n+1)/2)}{\sqrt{n\pi}\Gamma(n/2)}(1+x^2/n)^{-(n+1)/2}$	0 if $n>1$	$\frac{n}{n-2}$ if $n>2$

Relationship Among Distributions

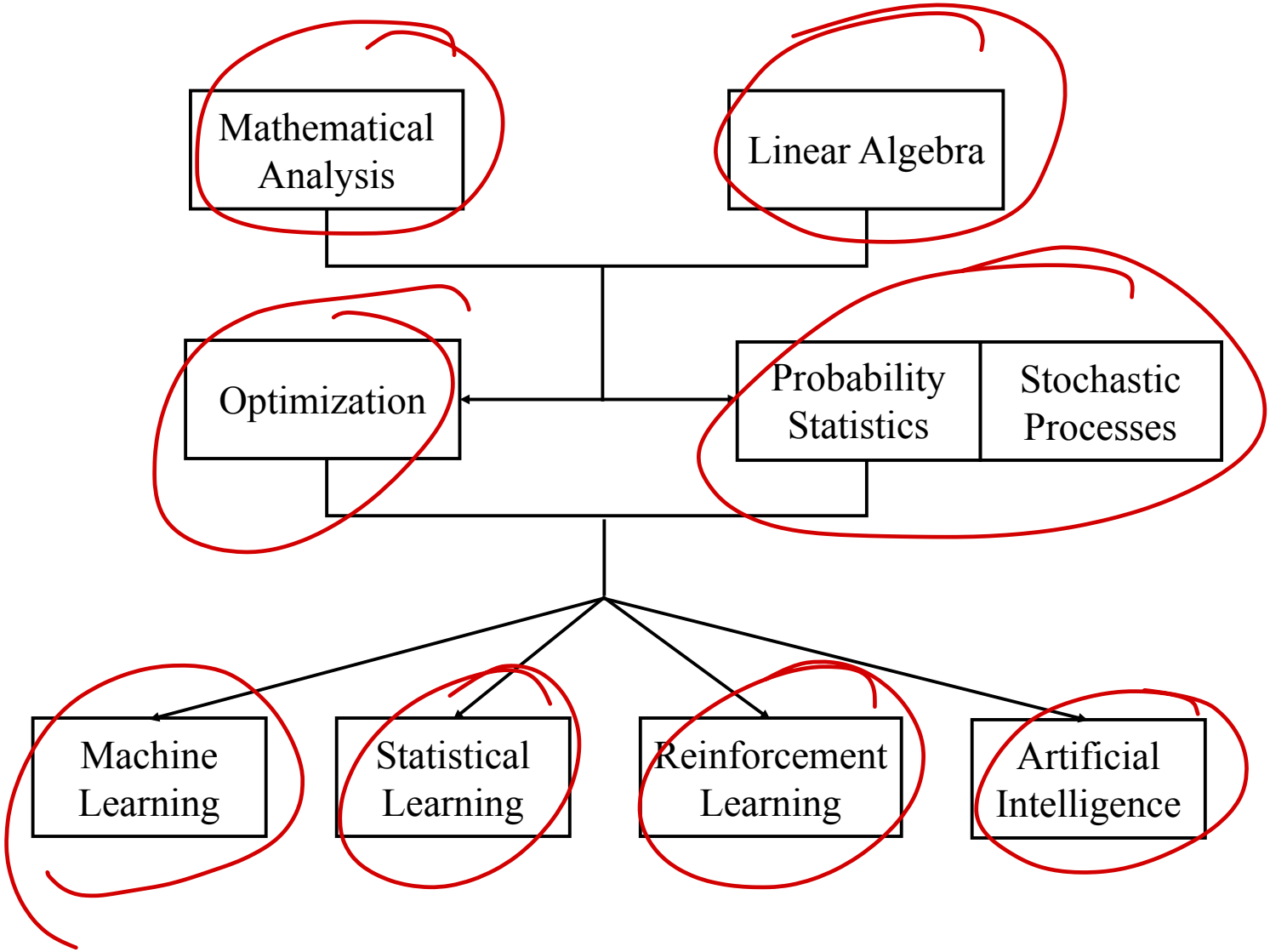


In My End is My Beginning

- Now this is not the end. It is not even the beginning of the end.
- But it is, perhaps, the end of the beginning.

Outline

The Role of This Course



Course Reward

Good Luck!

- Nine problems.
- Closed book without cheating sheets.
- 9:00am-11:30am in the morning of Jan. 13.

In My End is My Beginning