1. (a) $X(z) = \dfrac{z^2}{(z-\frac{1}{2})(z-1)}$

(b) $X(z) = z^2\left(\dfrac{A}{z-\frac{1}{2}} + \dfrac{B}{z-1}\right) = z^2\left(\dfrac{(A+B)z-(A+\frac{1}{2}B)}{(z-1)(z-\frac{1}{2})}\right)$

$\begin{cases} A+B=0 \\ A+\frac{1}{2}B=-1 \end{cases}$ $\begin{cases} A=-2 \\ B=2 \end{cases}$

$X(z) = z^2\left(\dfrac{-2}{z-\frac{1}{2}} + \dfrac{2}{z-1}\right)$

(c) $X(z) = \dfrac{A}{1-\frac{1}{2}z^{-1}} + \dfrac{B}{1-z^{-1}} = \dfrac{(A+B)-(A+\frac{1}{2}B)z^{-1}}{(1-\frac{1}{2}z^{-1})(1-z^{-1})}$

$\begin{cases} A+B=1 \\ -A-\frac{1}{2}B=0 \end{cases}$ $\begin{cases} A=-1 \\ B=2 \end{cases}$

$X(z) = \dfrac{-1}{1-\frac{1}{2}z^{-1}} + \dfrac{2}{1-z^{-1}}$

(left side) $= (\frac{1}{2})^n u[-n-1] - 2u[-n-1]$

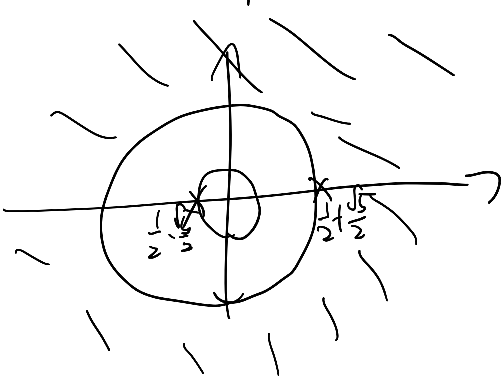
2. $Y(z) = z^{-1}Y(z) + z^{-2}Y(z) + z^{-1}X(z)$

(a) $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{z^{-1}}{1-z^{-1}-z^{-2}}$

pole: $1-z^{-1}-z^{-2}=0$

$z = (\frac{1}{2}) \pm \frac{\sqrt{5}}{2}$

no zero (or $z=0$ is zero)



(b) $H(z)=\dfrac{A}{1-\frac{-1+\sqrt{5}}{2}z^{-1}}+\dfrac{B}{1-\frac{-1-\sqrt{5}}{2}z^{-1}}$

$$\begin{cases} A+B=0 \\ \frac{-1+\sqrt{5}}{2}A+\frac{-1-\sqrt{5}}{2}B=-1 \end{cases} \qquad \begin{cases} A=-\frac{\sqrt{5}}{5} \\ B=\frac{\sqrt{5}}{5} \end{cases}$$

$$H(z)=-\frac{\sqrt{5}}{5}\cdot\frac{1}{1-\frac{-1+\sqrt{5}}{2}z^{-1}}+\frac{\sqrt{5}}{5}\frac{1}{1-\frac{-1-\sqrt{5}}{2}z^{-1}}$$

$$h[n]=\left[-\frac{\sqrt{5}}{5}\left(\frac{-1+\sqrt{5}}{2}\right)^n+\frac{\sqrt{5}}{5}\cdot\left(\frac{-1-\sqrt{5}}{2}\right)^n\right]u[n]$$

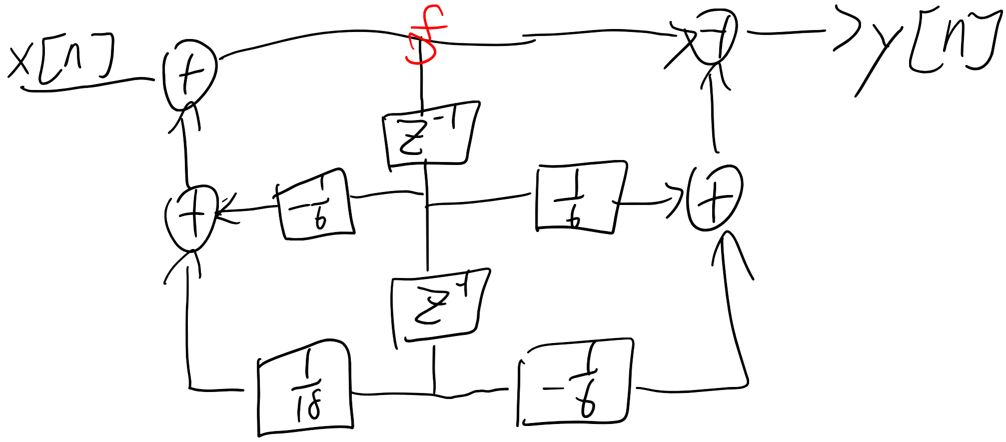
(casual)

(c) $\left|\frac{1-\sqrt{5}}{2}\right|<1<\left|\frac{1+\sqrt{5}}{2}\right|$, so ROC: $\left|\frac{1-\sqrt{5}}{2}\right|<|z|<\left|\frac{1+\sqrt{5}}{2}\right|$

$$h[n]=\frac{\sqrt{5}}{5}\left(\frac{-1+\sqrt{5}}{2}\right)^n u[-n-1]+\frac{\sqrt{5}}{5}\left(\frac{-1-\sqrt{5}}{2}\right)^n u[n]$$

3.

(a)



$$\begin{cases} X(z) - \frac{1}{6}z^{-1}F(z) + \frac{1}{18}z^{-2}F(z) = F(z) & ① \\ F(z) + \frac{1}{6}z^{-1}F(z) - \frac{1}{6}z^{-2}F(z) = Y(z) & ② \end{cases}$$

①: $X(z) = (1 + \frac{1}{6}z^{-1} - \frac{1}{18}z^{-2})F(z)$, $F(z) = \frac{X(z)}{1+\frac{1}{6}z^{-1} - \frac{1}{18}z^{-2}}$

②: $(1 + \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2})F(z) = Y(z)$, $X(z) \cdot \frac{1+\frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}}{1+\frac{1}{6}z^{-1} - \frac{1}{18}z^{-2}} = Y(z)$

So $H(z) = \frac{1+\frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}}{1+\frac{1}{6}z^{-1} - \frac{1}{18}z^{-2}} = \frac{(1+\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})}{(1+\frac{1}{3}z^{-1})(1-\frac{1}{6}z^{-1})}$

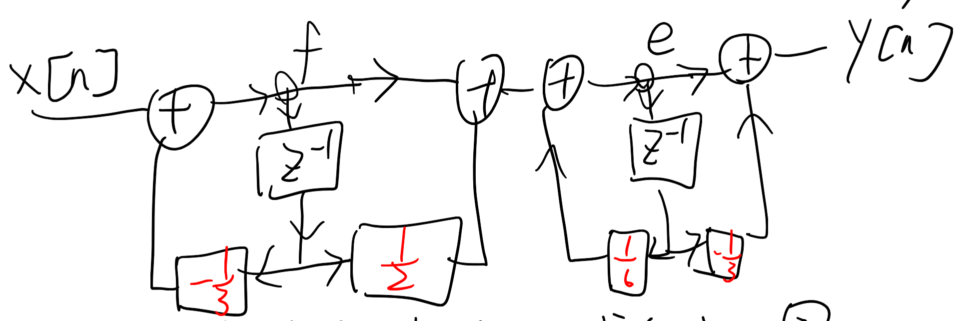
equation: $x[n] + \frac{1}{6}x[n-1] - \frac{1}{18}x[n-2] = y[n] + \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2]$

(b) $(1+\frac{1}{6}z^{-1} - \frac{1}{18}z^{-2}) = (1+\frac{1}{3}z^{-1})(1-\frac{1}{6}z^{-1})$

pole: $|z| = \frac{1}{3}$ or $\frac{1}{6}$ as casual, so $|z| > \frac{1}{3}$

contain $|z|=1$, so stable

(c)



$$\begin{cases} X(z) + a_1 z^{-1}F(z) = F(z) & ③ \\ F(z) + a_2 z^{-1}F(z) + a_3 z^{-1}E(z) = E(z) & ④ \\ E(z) + a_4 z^{-1}E(z) = Y[n] & ⑤ \end{cases}$$

③: $F(z) = \frac{1}{1-a_1 z^{-1}} X(z)$

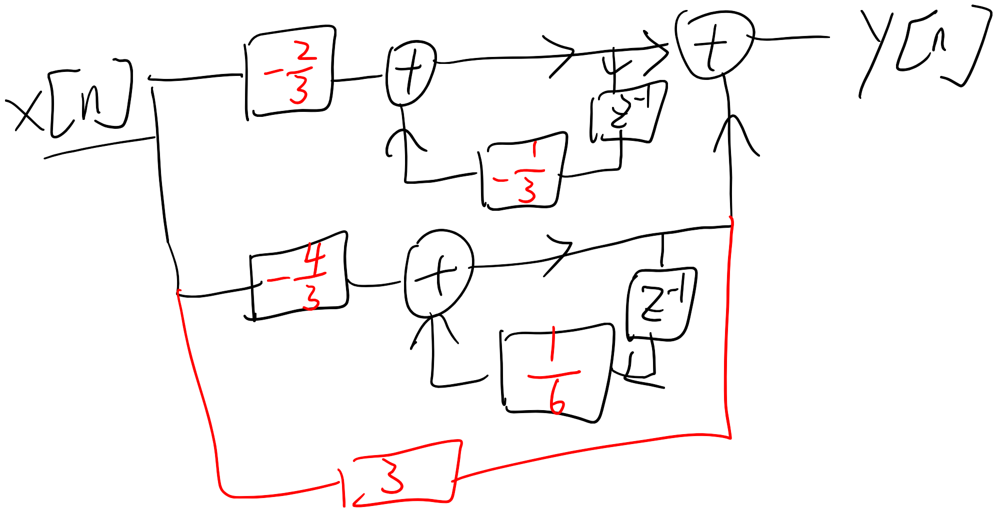
④: $(1+a_2 z^{-1})\, F(z) = (1-a_3 z^{-1}) E(z)$

$$E(z) = \frac{c(1+a_2 z^{-1}) \quad X(z)}{(1-a_3 z^{-1})(1-a_1 z^{-1})}$$

⑤: $$\frac{c(1+a_4 z^{-1})(1+a_2 z^{-1})}{(1-a_3 z^{-1})(1-a_1 z^{-1})} X(z) = Y(z)$$

So $(a_1, a_2, a_3, a_4) = (-\frac{1}{3}, \frac{1}{2}, \frac{1}{6}, -\frac{1}{3})$

($a_1 a_3$, $a_2 a_4$ can be swapped)



$$H(z) = \frac{1+\frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}}{1+\frac{1}{6}z^{-1} - \frac{1}{18}z^{-2}} = 3 + \frac{-2-\frac{1}{3}z^{-1}}{1+\frac{1}{6}z^{-1} - \frac{1}{18}z^{-2}}$$

$$= 3 + \frac{a}{1+\frac{1}{3}z^{-1}} + \frac{b}{1-\frac{1}{6}z^{-1}} = 3 + \frac{(a+b)+(-\frac{1}{6}a+\frac{1}{3}b)}{(1+\frac{1}{3}z^{-1})(1-\frac{1}{6}z^{-1})}$$

$\begin{cases} a = -\frac{2}{3} \\ b = -\frac{4}{3} \end{cases}$, $H(z) = 3 + \frac{-\frac{2}{3}}{1+\frac{1}{3}z^{-1}} + \frac{-\frac{4}{3}}{1-\frac{1}{6}z^{-1}}$

$(b_1, b_2, b_3, b_4) = (-\frac{2}{3}, -\frac{1}{3}, -\frac{4}{3}, \frac{1}{6})$ (or $(-\frac{4}{3}, \frac{1}{6}, -\frac{2}{3}, -\frac{1}{3})$)

and a line of $\boxed{3}$ is added.