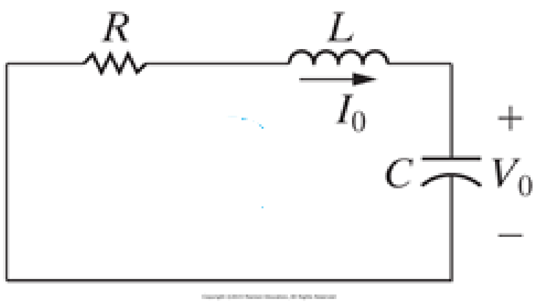
Properties of Series RLC Network - *v(t)*



- Behavior captured by damping
  - Gradual loss of the initial stored energy
  - α determines the rate of damping

$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

- $\alpha > \omega_0$, overdamped

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

- $\alpha = \omega_0$, critically damped

$$v(t) = (A_1 t + A_2) e^{-\alpha t}$$

- $\alpha < \omega_0$, underdamped

$$v(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$

[Source: Berkeley]

Example

- Find $v(t)$ in the circuit below. Assume the circuit has reached steady state at $t = 0^{-}$.

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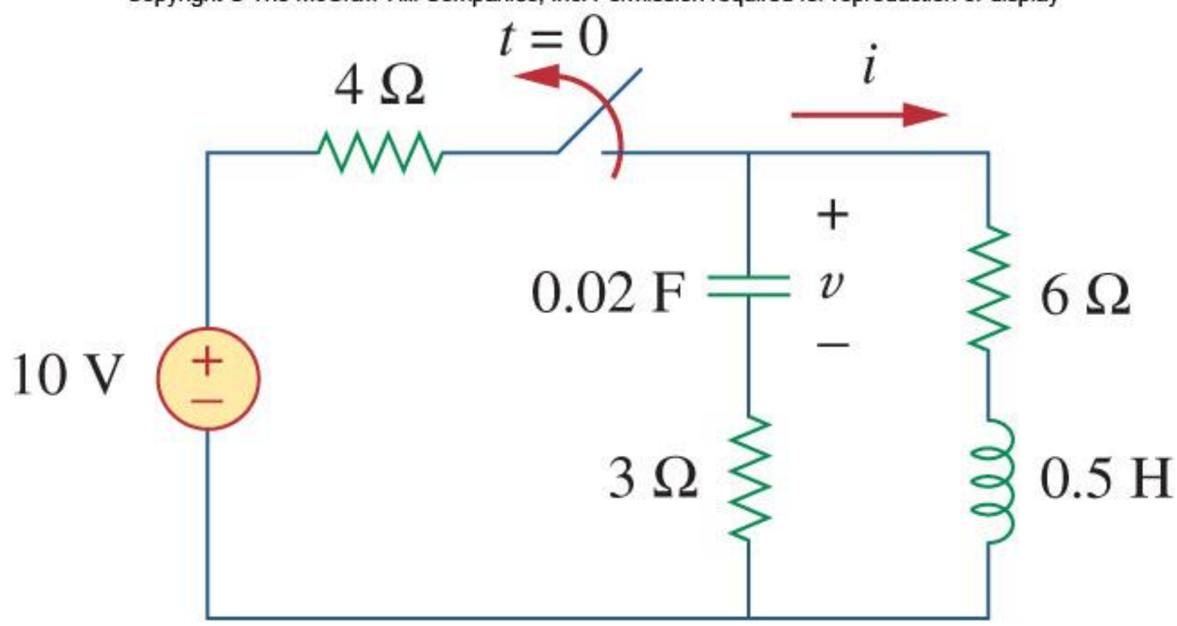
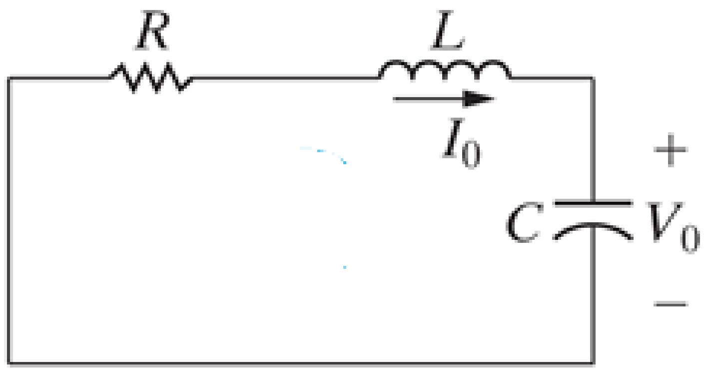
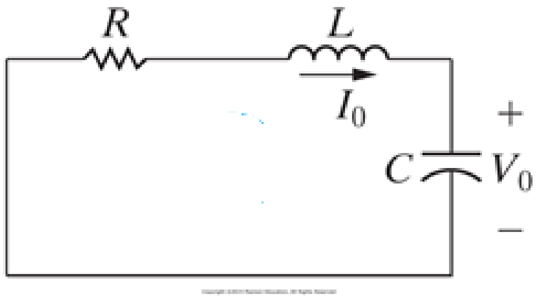


Source-Free Series RLC Circuit



Properties of Series RLC Network - *i(t)*



- Behavior captured by <u>damping</u>
  - Gradual loss of the initial stored energy
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Example

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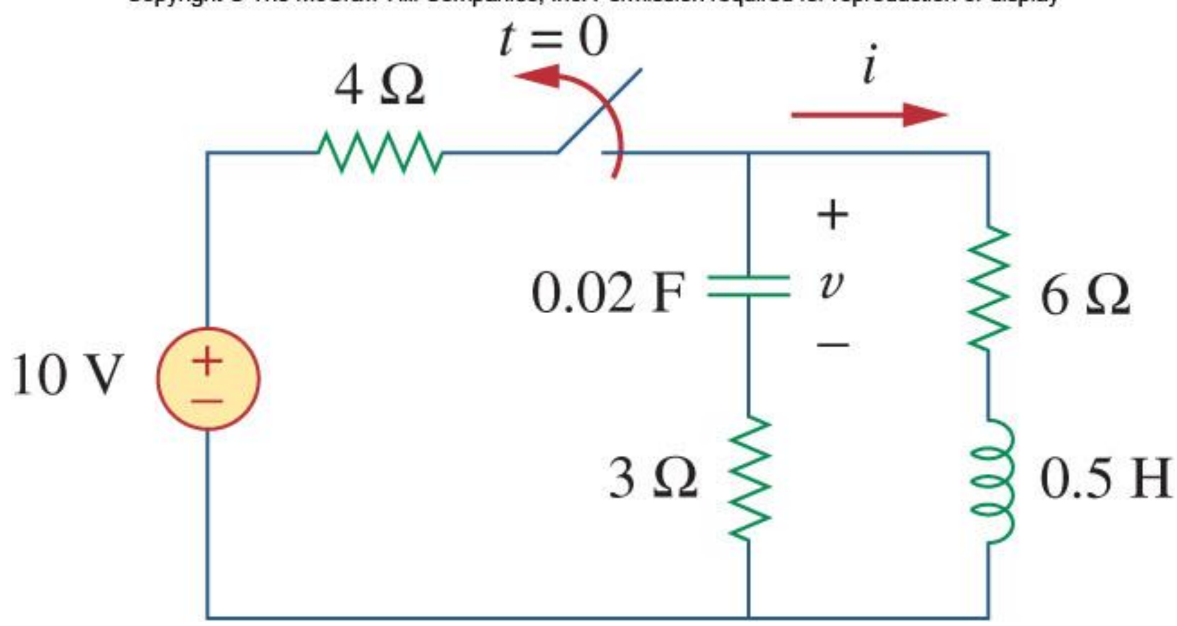

Example

- Find $i(t)$ in the circuit below. Assume the circuit has reached steady state at $t = 0^-$.

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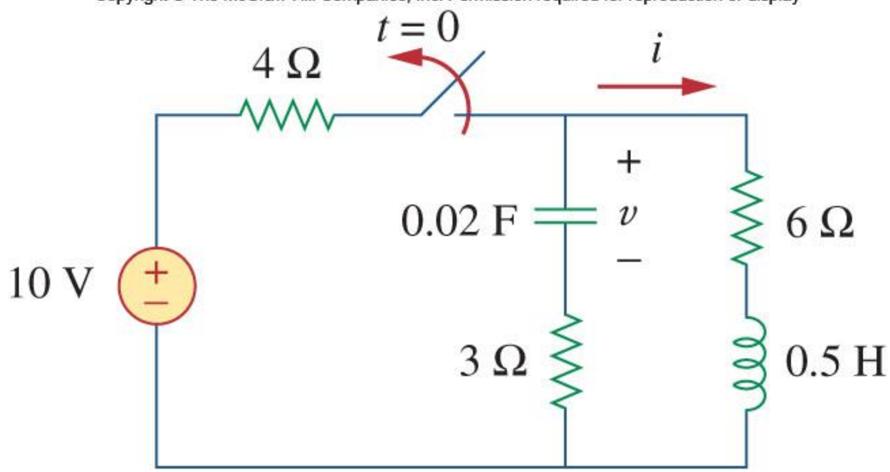

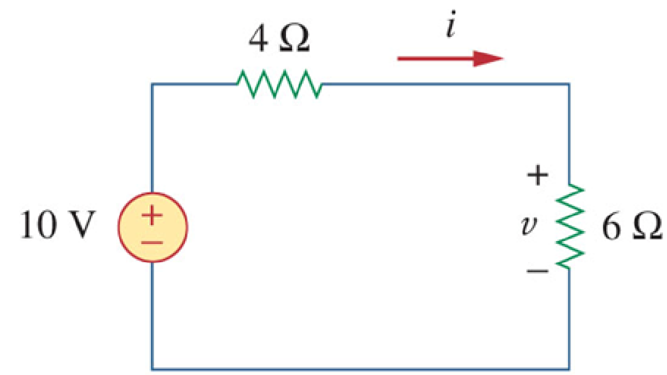

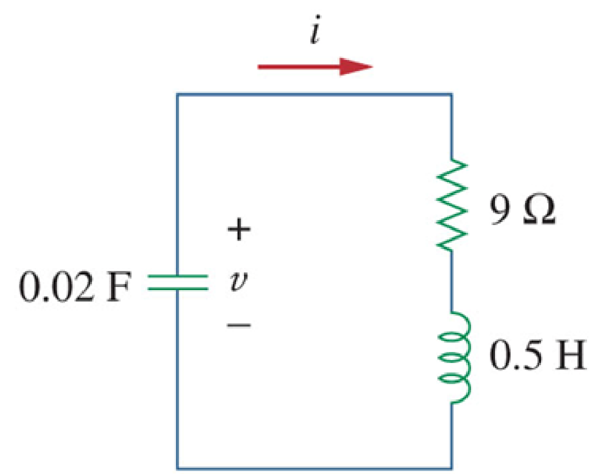

$$\alpha = \frac{R}{2L} = 9 \qquad \omega_0 = \frac{1}{\sqrt{LC}} = 10$$

$$s_{1,2} = -\alpha \mp \sqrt{\alpha^2 - \omega_0^2} = -9 \mp j4.359$$

Source-Free Parallel RLC Network

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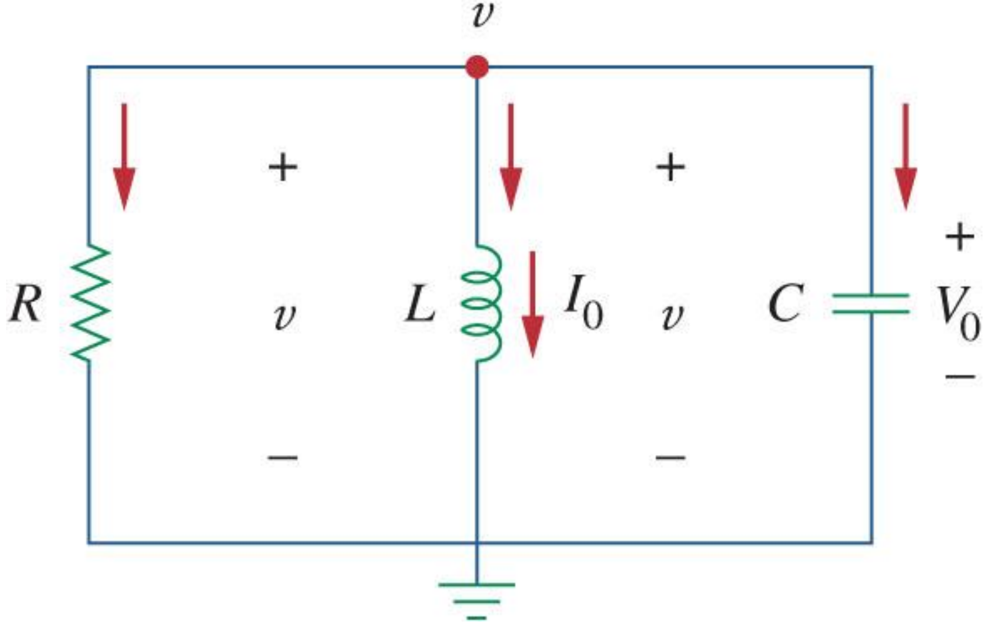
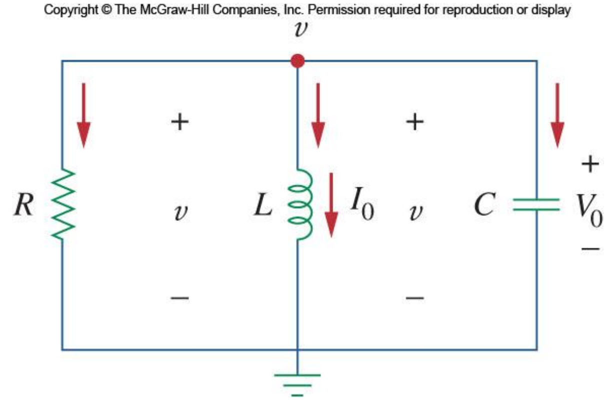


Source-Free Parallel RLC Network - *v(t)*

$$\frac{d^2v}{dt^2}+\frac{1}{RC}\frac{dv}{dt}+\frac{1}{LC}v=0$$



- The characteristic equation is:

$$s^2+\frac{1}{RC}s+\frac{1}{LC}=0$$

From this, we can find the roots of the characteristic equation to be:

$$s_{1,2}=-\alpha\pm\sqrt{\alpha^2-\omega_0^2}$$

$$\alpha=\frac{1}{2RC}\qquad \omega_0=\frac{1}{\sqrt{LC}}$$

As in last time, there are three scenarios to consider.

Three Damping Cases - *v(t)*

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{1}{2RC} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

- For the overdamped case, the roots are real and negative,

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

- For critically damped, the roots are real and equal

$$v(t) = (A_1 t + A_2) e^{-\alpha t}$$

- In the underdamped case, the roots are complex

$$v(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$

Three Damping Cases - *i(t)*

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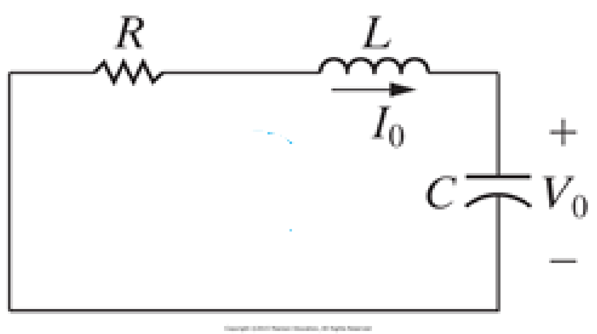
Series vs. Parallel (Source-Free RLC Circuit)

- Series $\alpha = \dfrac{R}{2L} \quad \omega_0 = \dfrac{1}{\sqrt{LC}}$

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$
$$v(t) = (A_1 t + A_2) e^{-\alpha t}$$
$$v(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$



- Parallel $\alpha = \dfrac{1}{2RC} \quad \omega_0 = \dfrac{1}{\sqrt{LC}}$

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$
$$v(t) = (A_1 t + A_2) e^{-\alpha t}$$
$$v(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$

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