EE150: Signals and Systems, Spring 2022
Homework 3

(Due Friday, Apr. 8 at 11:59pm (CST))

1. [15 points] Find the Fourier series coefficients for each of the following signals:

$$\textbf{(a)} x(t) = sin(10\pi t + \frac{\pi}{6})$$
$$\textbf{(b)} x(t) = 1 + cos(2\pi t)$$
$$\textbf{(c)} x(t) = [1 + cos(2\pi t)][sin(10\pi t + \frac{\pi}{6})]$$

2. [10 points] Derive the Fourier series for the following signals using Fourier series analysis equation.

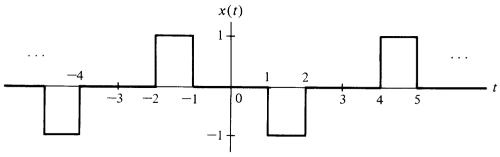

Figure 1: (a)

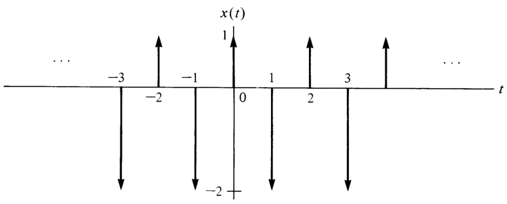

Figure 2: (b)

3. [15 points] Let

$$x(t) = \begin{cases} 1-t, & 0 \le t \le 1 \\ t-1, & 1 \le t \le 2 \end{cases}$$

be a periodic signal with fundamental period $T = 2$ and Fourier coefficients a_k.

(a) Determine the value of a_0.

(b) Determine the Fourier series representation of $\frac{dx(t)}{dt}$.

(c) Use the result of part (b) and the differentiation property of the continuous-time Fourier series to help determine the Fourier series coefficients of $x(t)$.

4. [10 points] Consider a discrete-time LTI system with impulse response

$$h[n] = (\frac{1}{2})^{|n|}$$

Find the Fourier series representation of the output y[n] for each of the following inputs:

(a) $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$

(b) $x[n]$ is periodic with period 6 and

$$x[n] = \begin{cases} 0, & n = 0, \pm 3 \\ 1, & n = \pm 1, \pm 2 \end{cases}$$

5. [10 points] Consider a discrete-time LTI system with impulse response

$$h[n] = \begin{cases} 2, & 0 \le n \le 2 \\ -1, & -2 < n \le -1 \\ 0, & \text{otherwise} \end{cases}$$

Given that the input to this system is

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n - 6k]$$

Determine the Fourier series coefficients of the output $y[n]$.

6. [10 points] Suppose we are given the following information about a signal $x(t)$:

    1. $x(t)$ is a real signal.
    2. $x(t)$ is periodic with period $T = 6$ and has Fourier coefficients a_k.
    3. $a_k = 0$ for $k = 0$ and $k > 2$.
    4. $x(t) = -x(t-3)$
    5. $\frac{1}{6}\int_{-3}^{3} |x(t)|^2 = \frac{1}{2}$
    6. a_1 is a positive real number.

    Show that $x(t) = Acos(Bt + C)$, and determine the values of the constants A, B, and C.

7. [30 points] Let $x(t)$ be a real periodic signal with Fourier series representation given in the sine-cosine form; i.e.,

$$x(t) = a_0 + 2\sum_{k=1}^{\infty}[B_k cos k\omega_0 t - C_k sin k\omega_0 t]$$

(a) Find the exponential Fourier series representation of the even and odd parts of $x(t)$; that is, find the coefficients α_k and β_k in terms of the coefficients in the equation above so that

$$Ev\{x(t)\} = \sum_{k=-\infty}^{+\infty} \alpha_k e^{jk\omega_0 t},$$

$$Od\{x(t)\} = \sum_{k=-\infty}^{+\infty} \beta_k e^{jk\omega_0 t},$$

(b) What is the relationship between α_k and α_{-k} in part (a)? What is the relationship between β_k and β_{-k}?

(c) Suppose that the signals $x(t)$ and $z(t)$ shown in the following figure 7.c have the sine-cosine series representations

$$x(t) = a_0 + 2\sum_{k=1}^{\infty}[B_k cos(\frac{2\pi kt}{3}) - C_k sin(\frac{2\pi kt}{3})],$$

$$z(t) = d_0 + 2\sum_{k=1}^{\infty}[E_k cos(\frac{2\pi kt}{3}) - F_k sin(\frac{2\pi kt}{3})],$$

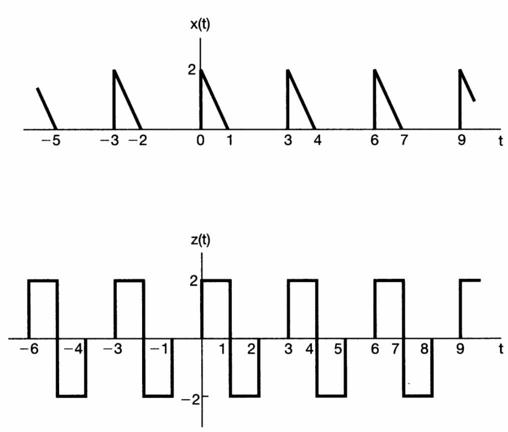

Figure 3: 7.c

Sketch the signal

$$y(t) = 4(a_0 + d_0) + 2\sum_{k=1}^{\infty}\{[B_k + \frac{1}{2}E_k]cos(\frac{2\pi kt}{3}) + F_k sin(\frac{2\pi kt}{3})\}$$