Electromagnetics Spring 2020

Homework 6

Deadline:4.21 23:59pm

说明：全用英文作答；

每道题要对所有小问作答，要给出全部必要的推导过程，计算题要算出最终的数值结果，比如开根号之类的；

所有计算出来的结果如果是有单位的物理量，一定要写明单位；

每题的分数在括号中给出；

可以互相讨论，也可以上网查，但是不能抄袭，也不能找别人代做；

可以在电脑敲字解答，也可以手写解答，最后统一转换为 PDF 格式，按分组信息邮件 或 BB 上提交；

邮件主题&附件命名规范:姓名_章节，不按规范发送扣除一半分数；

请在作业 PDF 的第一行写上姓名和学号；

有问题请给老师或助教发邮件；

Textbook: Fundamentals of Applied Electromagnetics, 7th edition

Part I. Problems in textbook.

7.7 (20 points)
7.8 (20 points)
7.10 (30 points)
7.11 (10 points)
7.13 (10 points)
7.15 (10 points)
7.16 (10 points)
7.18 (20 points)
7.22 (10 points)
7.24 (20 points)
7.27 (20 points)

PART II. Problems in quiz

1. (**7 points**)

(a) (**2 point**) Write out the wave equation that $\vec{E}$ satisfies using the wave number.

(b) (**2 points**) If the fields of a plane wave do not vary with x and z, write out the wave equation (in the simplest form) that E_z satisfies using the wave number.

(c) (**3 point**) What is the general form of the solution to the wave equation you obtained in (b)? Is it possible to solve the amplitude of E_z from this wave equation.

2. (**12 points**)
(a) (**2 point**) Write out the complex permittivity by its real part ε' and imaginary part ε''

(b) (**2 point**) Write out the relationship between ε'' and conductivity σ

(c) (**2 points**) Write out the expression of loss tangent

(d) (**2 points**) For a lossless medium, write out the expression of wave number in terms of angular frequency, permittivity and permeability.

(e) (**4 points**) For a lossless medium, write out the expression of speed of sound in terms of permittivity and permeability. Demonstrate that the unit of this expression equals to m/s.

3. (**15 points**) In a nonconducting medium with $\varepsilon - 4\varepsilon_0$ and $\mu = 4\mu_0$, the electric field intensity of a plane wave is $E(y,t) = \hat{x}\sin(10^{10}t + ky)\,\mathrm{V/m}$
(a) (**2 points**) What is the direction of propagation of this wave?

(b) (**2 points**) Calculate the intrinsic impedance of this medium.

(c) (**2 points**) Calculate the phase velocity of any plane wave in this medium.

(d) (**2 points**) Calculate the wavelength of this wave.

(e) (**3 points**) Obtain the expression of the corresponding magnetic field in time-domain form.

(f) (**2 points**) Is it possible for this plane wave to have magnetic field pointing to the $-\hat{x}$ direction at some locations.

(g) (**2 points**) If another uniform plane wave is propagating in the $+\hat{z}$ direction and its magnetic field is in the $-\hat{y}$ direction, what is the direction of its electric field?

4. (**14 points**) For an infinite large sheet of surface current having a current density of $\overrightarrow{J_s} = J_0\hat{z}$ and existing on the $y = 0$ plane in free space, derive expressions of the fields of the fields of the plane waves generated by this current source. First, judge the directions of the plane waves. Second, judge the directions of the electric and magnetic fields of the plane waves. Third, write out the complete expressions of the electric and magnetic fields by some unknown quantities representing amplitudes of the fields. Last, solve for the unknown amplitudes.

5. (**5 points**)

(a) (**3 points**) For a plane wave propagating in the +z direction, its electric field can have both x and y components, which can be expressed as $\vec{E} = \hat{x}E_x + \hat{y}E_y$ Write out the corresponding total magnetic field in terms of E_x and E_y.

(b) (**2 points**) If both E_x and E_y can be complex numbers, which one (s) of the following polarization states are possible for this plane wave?

(1)Linear (2) left-hand circular (3) right-hand circular (4) Elliptical

6. (**10 points**)

(a) (**2 points**) For a $\hat{y}$-polarized plane wave traveling in the -$\hat{z}$ direction in a lossy medium, write out its complex propagation constant by the attenuation constant α and phase constant β.

(b) (**2 points**) Write out the phasor form of the electric field intensity of this plane wave given the magnitude of the electric field is E_0.

(c) (**2 points**) Write out the expression and unit of the skin depth δ_s of this medium.

(d) (**2 points**) Write out the expression by which you can calculate the wavelength and phase velocity of this plane wave in this medium.

(e) (**2 points**) Write out the range of possible value of the phase difference between the electric field and magnetic field of this plane wave.

7. (12 points) For a general plane wave propagating along -$\hat{y}$ direction, its electric field intensity can be written as $\vec{E}(y) = (\hat{x}a_x + \hat{z}a_z e^{j\delta})e^{jky}$ with $a_x \geq 0, a_z \geq 0$.

(a) (2 points) For linear polarization, what requirements do a_x , a_z, δ need to satisfy?

(b) (2 points) For linear polarization, is it possible that a_x is zero?

(c) (2 points) For LH circular polarization, what requirements do a_x , a_z, δ need to satisfy?

(d) (2 points) For RH circular polarization, what requirements do a_x , a_z, δ need to satisfy?

(e) (2 points) For circular polarization, does the magnitude of magnetic field change with time?

(f) (2 points) For circular polarization, does the direction of electric field change with time?

8. (8 points)

(a) (2 points) Write out the general expression and unit of the Poynting vector for a plane wave.

(b) (2 points) Write out the general expression of time-average power density for a plane wave.

(c) (2 points) For a plane wave propagating in $\hat{x}$ direction in free space with electric field intensity of $\vec{E}$, write out the time-average power density (vector) of this wave.

(d) (2 points) For a plane wave propagating in the direction of $\hat{k} = \frac{\sqrt{2}}{2}(\hat{x} - \hat{z})$in a medium with $|\vec{H}|$=2 A/m and$\eta = 100\Omega$, calculate the total time-average power flowing through a square region with a size of 2 m x 2 m in the xz plane.

PART III. Homemade

1. (10 points) Why did Maxwell notice that a displacement current is necessary? 2. (20 points) The time-domain magnetic flux density in free space is given by $B^{\rightharpoonup} = x\hat{}Bx\cos(2y)\sin(\omega t - \pi z) + y\hat{}By\cos(2x)\sin(\omega t - \pi z)$, where Bx and By are constants. Assuming no conduction current exists, determine the electric displacement current density.

2. (40 points) For a steady surface current density $x\hat{}Js$ on the xoy plane, obtain the magnetic fields generated by this source in both the $z > 0$ and $z < 0$ regions. Then verify that your results agree with the magnetic boundary conditions.

3. (50 points) If a material has conductivity of 0.4 S/m at 5 GHz and wavelength in it is 2 cm, express its permittivity in a complex number and determine its loss tangent. Assuming the material is non-magnetic ($\mu = \mu 0$), calculate the phase velocity and wave impedance. How big is the phase

difference between the electric field and magnetic field of a plane wave propagating in this material? Is this material a good conductor at 5 GHz? Calculate its complex propagation constant. How many percentage of the electric field intensity is lost after a plane wave propagates 5 cm in this material?

4. (20 points) Calculate the Poynting vector of a left-hand circular polarized plane wave in air using time-domain expressions. Assume the electric field intensity magnitude is a.