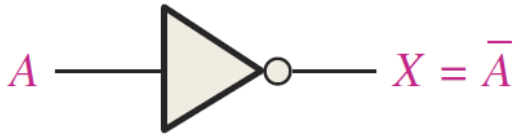
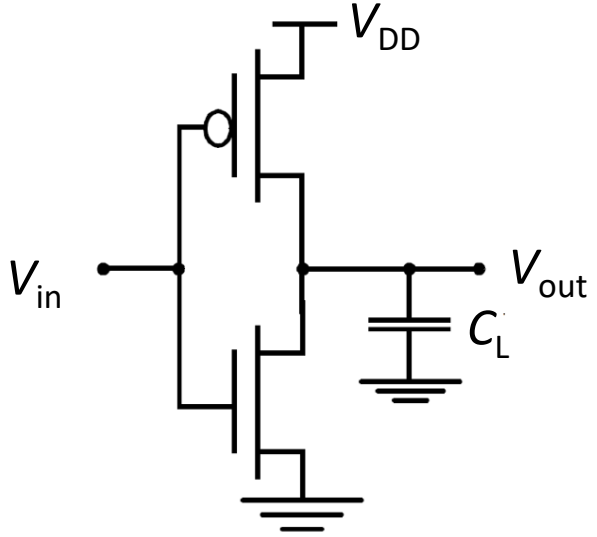
NOT function (Inverter)

- Boolean expression $X = \overline{A}$
- Truth table

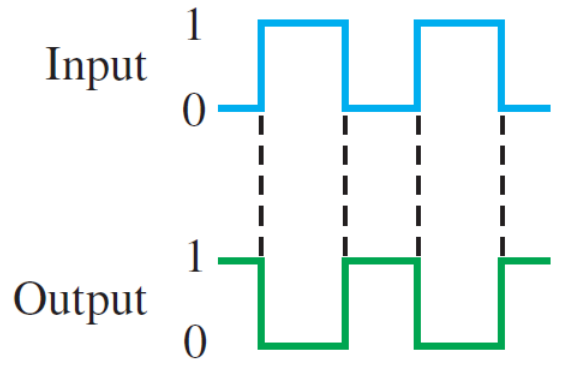
Input	Output
LOW (0)	HIGH (1)
HIGH (1)	LOW (0)

- Logic circuit





Circuit implementation



Waveform

AND operation

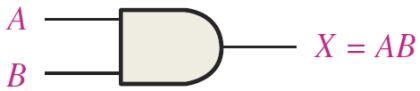
- Boolean expression $X = AB$

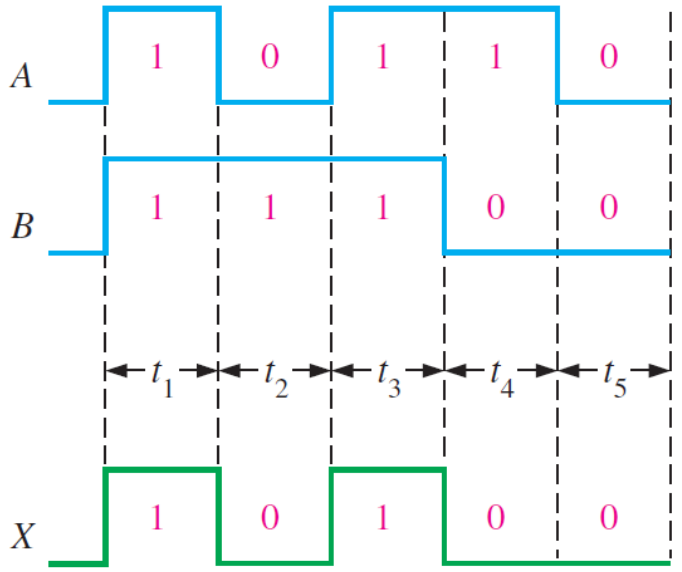
- Truth table

Inputs		Output
A	*B*	*X*
0	0	0
0	1	0
1	0	0
1	1	1

Inputs			Output
A	*B*	*C*	*X*
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

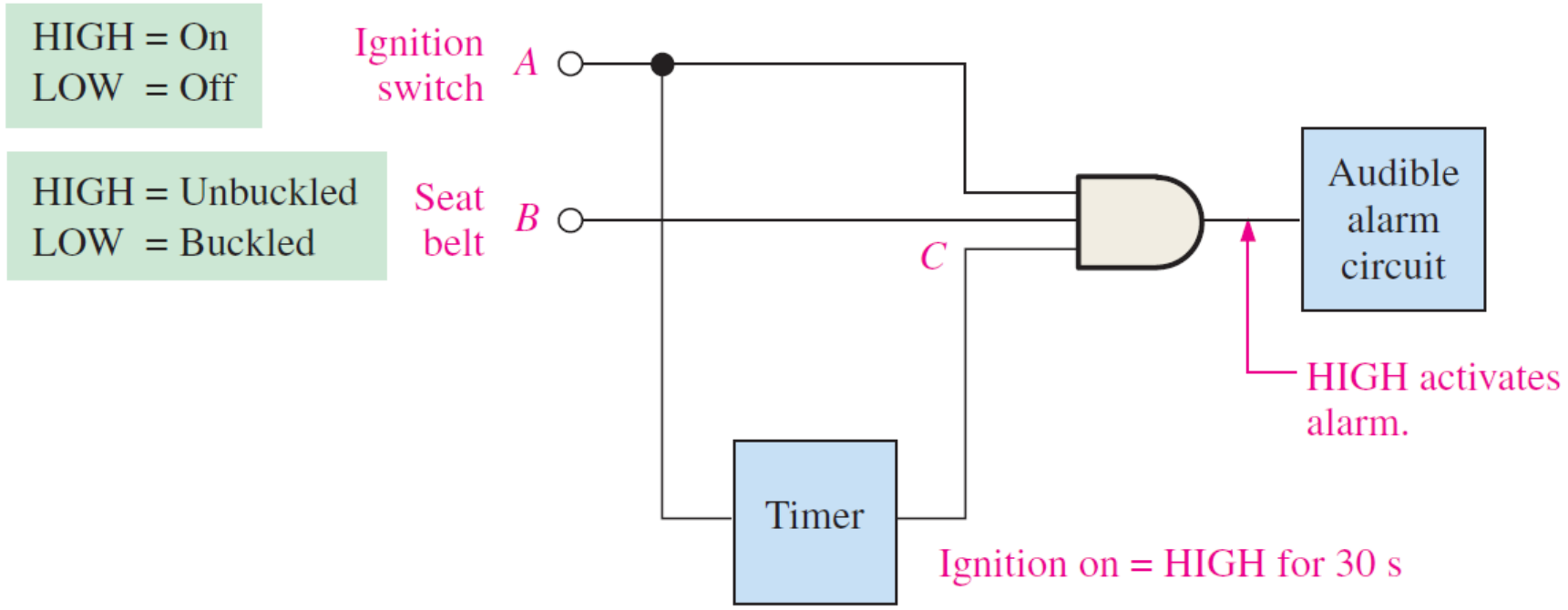
- Logic circuit





Application of AND gate

Seat Belt Alarm System



If the ignition is on and the seat belt is unbuckled and the timer is running, the output is HIGH.

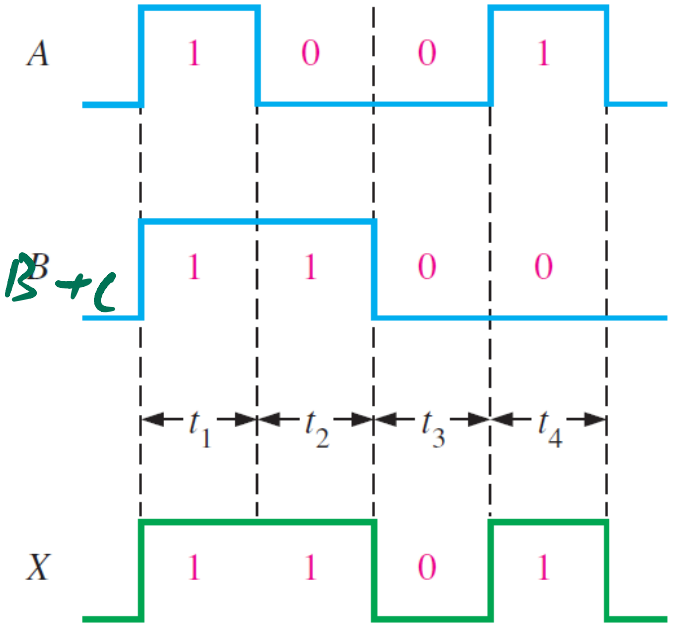
OR operation

- Boolean expression $X = A + B$
- Truth table

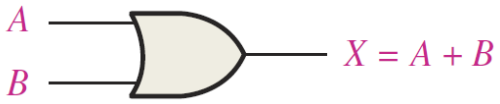
A	B	$A + B = X$
0	0	$0 + 0 = 0$
0	1	$0 + 1 = 1$
1	0	$1 + 0 = 1$
1	1	$1 + 1 = 1$

A B C

$X = A + B + C$

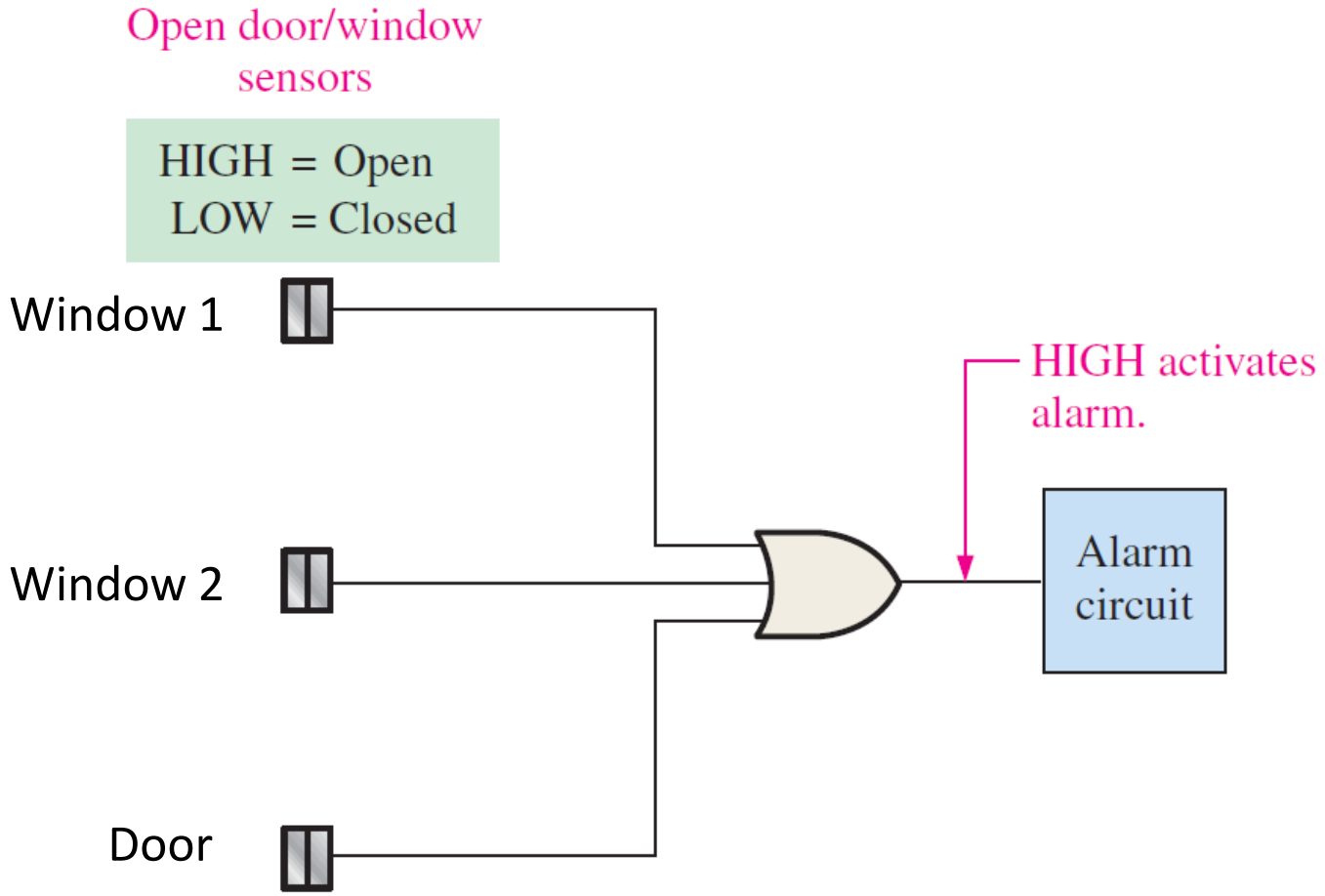


- Logic circuit



Question: draw the truth table for 3-input OR gate

Application of OR gate



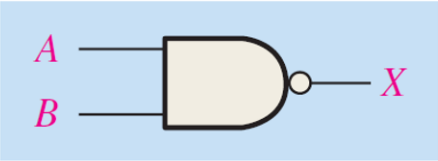
When one of the windows or the door is opened, the gate output goes HIGH.

NAND gate

- Boolean expression

$$X = \overline{AB}$$

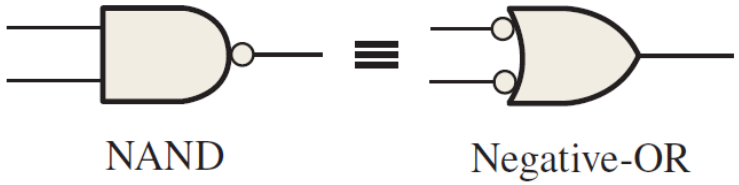
- Logic circuit



- Truth table

Inputs		Output
A	*B*	*X*
0	0	1
0	1	1
1	0	1
1	1	0

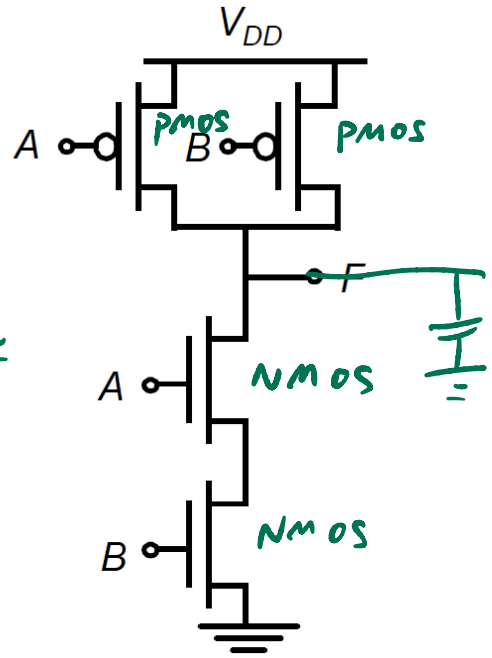
- Inherent in a NAND gate's operation is the fact that one or more LOW inputs produce a HIGH output



$$\overline{AB} = \overline{A} + \overline{B}$$

Verify this using truth table

A	B	F
0	0	1
0	1	1
1	0	1
1	1	0



Circuit

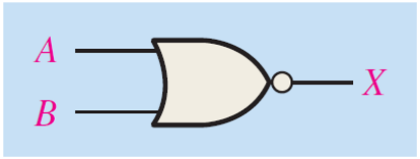
Question: draw the truth table for 3-input NAND gate

NOR gate

- Boolean expression

$$X = \overline{A + B}$$

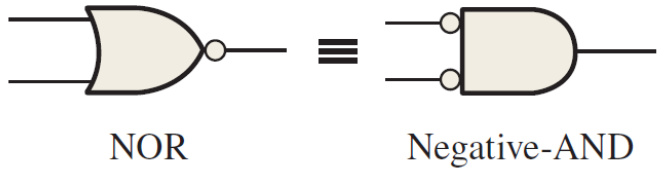
- Logic circuit



- Truth table

Inputs		Output
A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

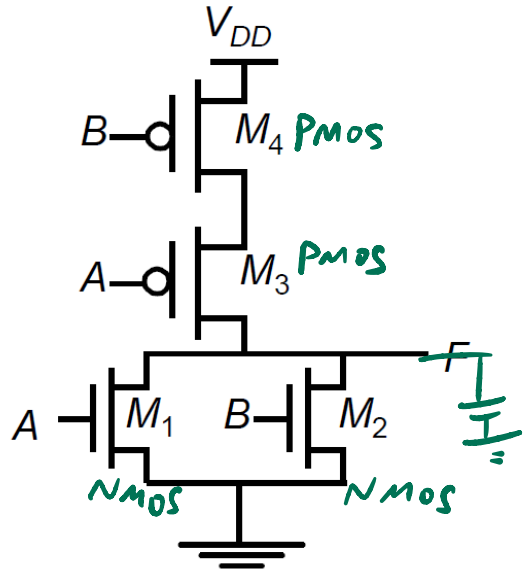
- A HIGH is produced on the gate output only when all of the inputs are LOW



$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

Verify this using truth table

A	B	F
0	0	1
0	1	0
1	0	0
1	1	0

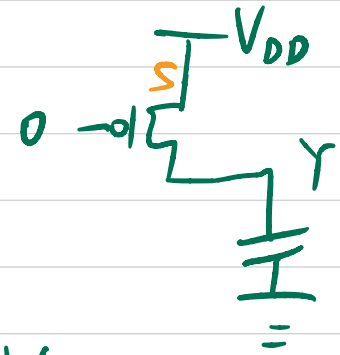


Circuit

Question: draw the truth table for 3-input NOR gate

pull up $\Rightarrow V_Y = V_{DD} \Rightarrow$ pmos

pull down $\Rightarrow V_Y = 0 \Rightarrow$ nmos



$V_{GS} = -V_{DD} < V_{TP}$

$V_{Y,max} = V_{DD}$

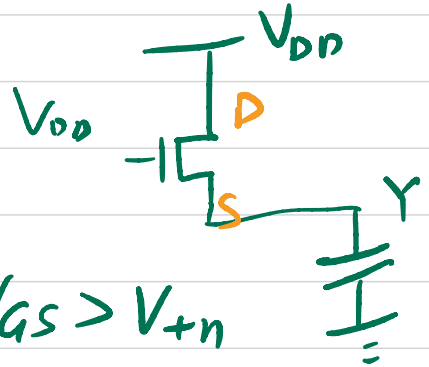
nmos is not good pull up

pmos is not good pull down

$V_Y = V_{DD} - V_{tn}$

$\Rightarrow V_{GS} = V_{DD} - (V_{DD} - V_{tn})$

$= V_{tn}$



$V_{GS} > V_{tn}$

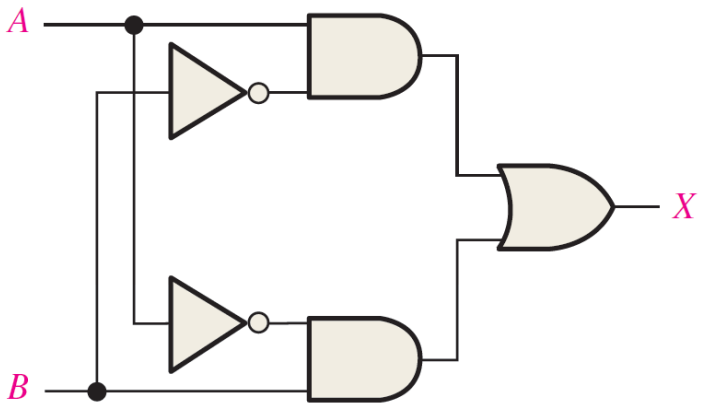
$V_{Y,max} = V_{DD} - V_{Tn}$

Exclusive-OR (XOR) gate

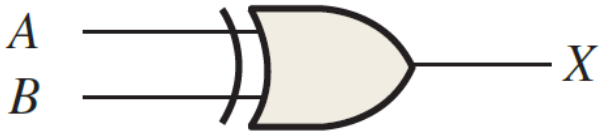
- The output of XOR is HIGH only when the two inputs are at opposite logic levels.
- Truth table

Inputs		Output
A	*B*	*X*
0	0	0
0	1	1
1	0	1
1	1	0

- Boolean expression $X = A\overline{B} + \overline{A}B$ or $X = A \oplus B$



- Logic circuit



Question: draw the truth table for 3-input XOR gate

$$(A+B)' = A'B'$$

$$(AB)' = A'+B'$$

$$A \oplus B = A'B + AB'$$

$$A \oplus B \oplus C = (A'B + AB') \oplus C$$

$$= (A'B + AB')'C + (A'B + AB')C'$$

$$= (A'B)'(AB')'C + \cdots$$

$$= (A+B')(A'+B)C + \cdots$$

$$= \cancel{AA'}C + ABC + A'B'C + \cancel{BB'}C + A'BC' + AB'C'$$

$$= ABC + A'B'C + A'BC' + AB'C'$$

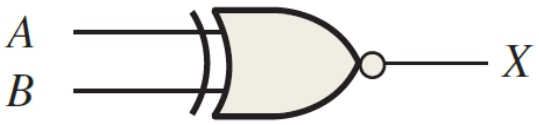
A	B	C	$A \oplus B$	$A \oplus B \oplus C$	X
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	1	1	1
0	1	1	1	0	0
1	0	0	1	1	1
1	0	1	1	0	0
1	1	0	0	0	0
1	1	1	0	1	1

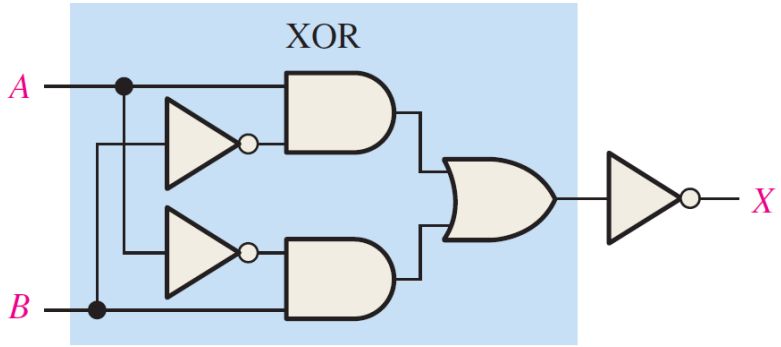
Exclusive-NOR (XNOR) gate

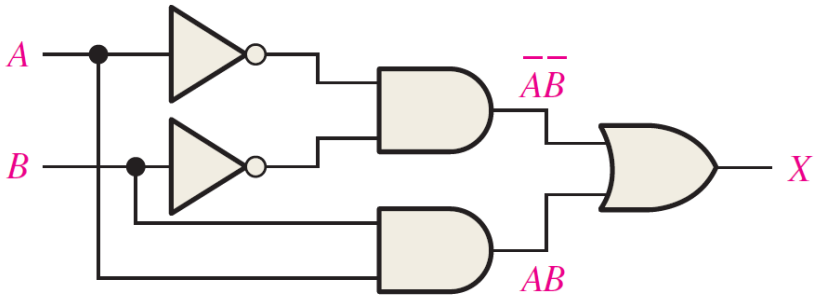
- The output of XNOR is LOW only when the two inputs are at opposite logic levels.
- Truth table

Inputs		Output
A	*B*	*X*
0	0	1
0	1	0
1	0	0
1	1	1

- Boolean expression $X = \overline{A}\overline{B} + AB$
- Logic circuit







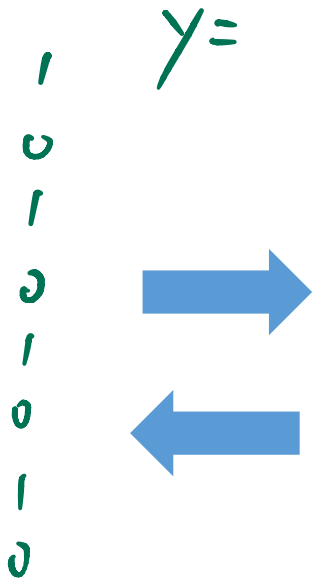
Think about it:

- In the truth table with two inputs, how many lines? How many columns?

Truth Table & Boolean Expression

Even or odd 1s

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

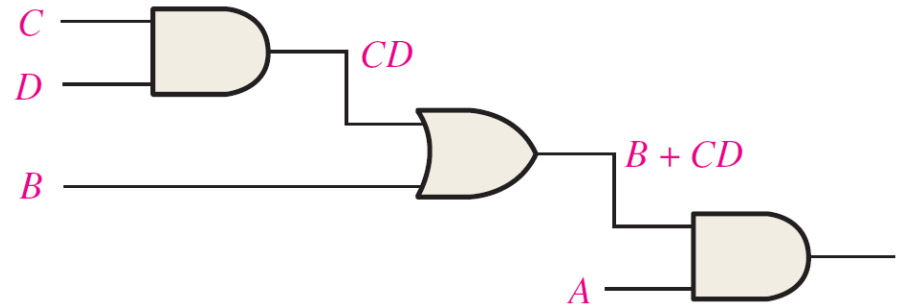


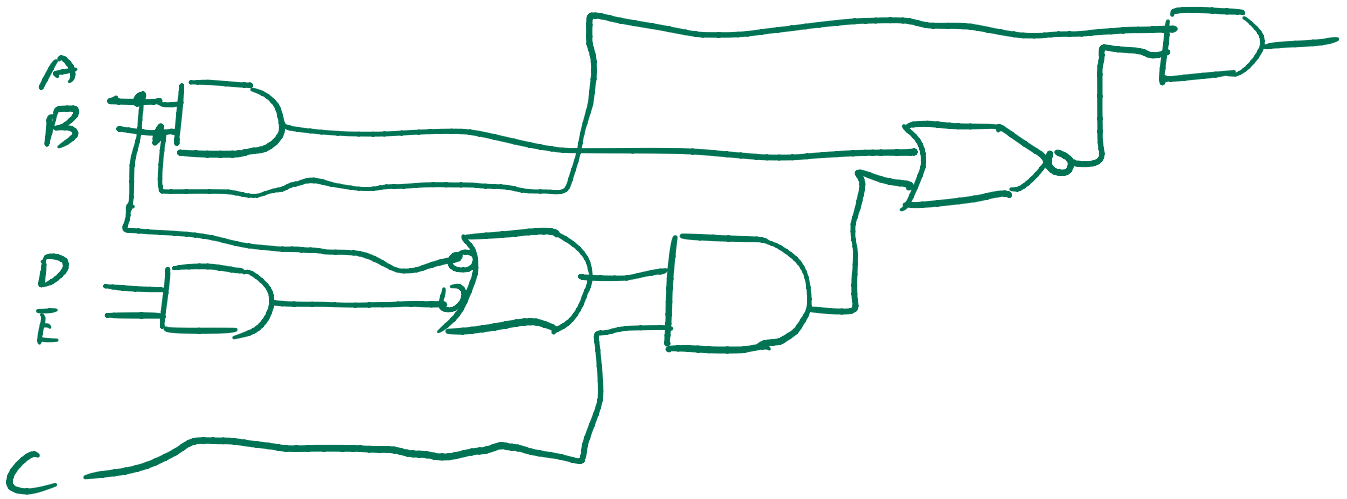
y = A’BC + AB’C + ABC’

Boolean Expression & Logic Circuit

$B[(AB)' + C(DE + A)']$ ⇒ Circuit ?

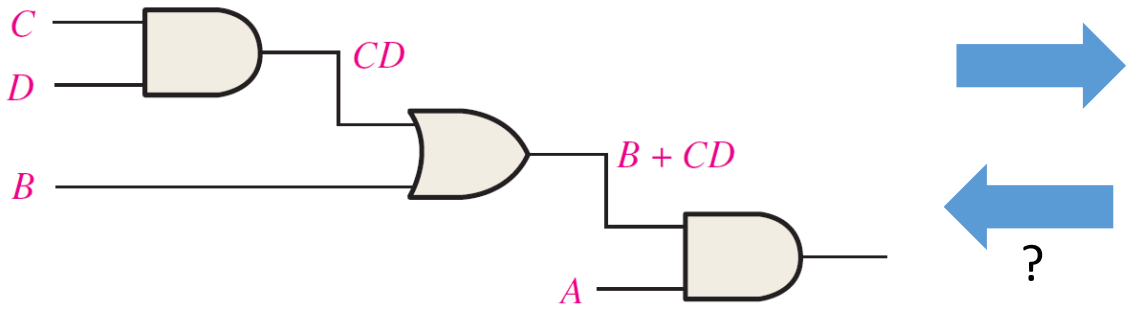
$A(B + CD)$





⇒ Boolean expression ?

Truth Table & Logic Circuit



Inputs				Output
A	*B*	*C*	*D*	*A*(*B* + *CD*)
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Reading materials

- Chapter 3 of Floyd book