EE160 Homework 3 Solution

1. *(2 points) Set points.*

   Substitute u with the affine feedback law then a closed-loop system

   $$\dot{x} = 3x + 4\left(k(x - x_s) + u_s\right) = (3 + 4k)\left(x - x_s\right) + 3x_s + 4u_s$$

   is obtained. To make $x(t)$ converge to $x_s = 5$, we need to choose k and u_s satisfying

   $$3 + 4k < 0 \quad \text{and} \quad 3x_s + 4u_s = 0,$$

   one option is let $k = -1$ and $u_s = -\frac{15}{4}$ then the derivative

   $$\dot{x}(t) = -(x(t) - x_s).$$

2. *(4 points) Uncertain control system with bounded noise.*

   With the feedback law $\mu(x) = k(x - x_s) + u_s$, the differential equation is given by

   $$\dot{x} = ax + b\mu(x) + cw(t) = (a + bk)(x - x_s) + cw(t),$$

   let $y(t) = x(t) - x_s$, we derive an equation and an explicit expression of y,

   $$\dot{y}(t) = (a + bk)\, y(t) + cw(t) \quad \text{and} \quad y(t) = e^{(a+bk)t} y_0 + \int_0^t e^{(a+bk)\tau} cw(\tau)\, \mathrm{d}\tau.$$

   We'll show that when choosing k such that $a + bk < 0$ $y(t)$ is bounded for arbitrary t, that is

   $$|y(t)| < |y_0| + \left| \int_0^t e^{(a+bk)\tau} c\bar{w}\, \mathrm{d}\tau \right| < |y_0| + \left| \frac{c\bar{w}}{a + bk} \left(e^{(a+bk)t} - 1 \right) \right| < |y_0| + \left| \frac{c\bar{w}}{a + bk} \right| < \infty\ .$$

   Recall that $|x(t)| = |y(t) + x_s| < |y(t)| + |x_s|$, so trajectory $x(t)$ is bounded as well.

3. *(4 points) Proportional control of an RC-circuit.*

   To let $V_C(t)$ converge to 10V, steady-state condition need to be satisfied

   $$u_s - V_C(\infty) = 0 \quad \Longrightarrow \quad u_s = 10\text{V}\ .$$

   Now we design the feedback control law in form of $V(t) = k\left(V_C(t) - 10\right) + u_s$, the differential equation of V_C is given by

   $$\dot{V}_C(t) = \frac{k\left(V_C(t) - 10\right) + u_s - V_C(t)}{RC} = \frac{k - 1}{RC} \cdot \left(V_C(t) - 10\right).$$

   To achieve the fastest convergence rate, we shall make $k - 1$ as small as possible. With the constraint of input function

   $$-220\text{V} \le V(t) \le 220\text{V} \quad \Rightarrow \quad -220 \le k(V_C(0) - 10) + u_s \le 220 \quad \Rightarrow \quad -21 \le k \le 23$$

   we shall choose $k = -21$ then the feedback law is $V(t) = -21(V_C(t) - 10) + 10$.

   Here, we assume that $V_C(0) = 0$ but it's definitely okay if you discuss more situations for different initial state $V_C(0)$.