# Homework 1

**Submit: Blackboard/Paper Due: Sept. 21th**

**Please write down Your Name & Student ID**

1. The sinc function, denoted by $\mathrm{Sinc}(t) = \frac{\sin(\pi t)}{\pi t}$ occurs often in Fourier analysis. Fig.1a,1b shows the function graph and spectrogram of Sinc(40t), its cut-off frequency is 20Hz. Fig1c shows an example of sampling sinc(40t) at 50Hz. Given the sampling frequency 30Hz\ 40Hz\80Hz\200Hz, please plot each spectrogram after sampling. (24pts)

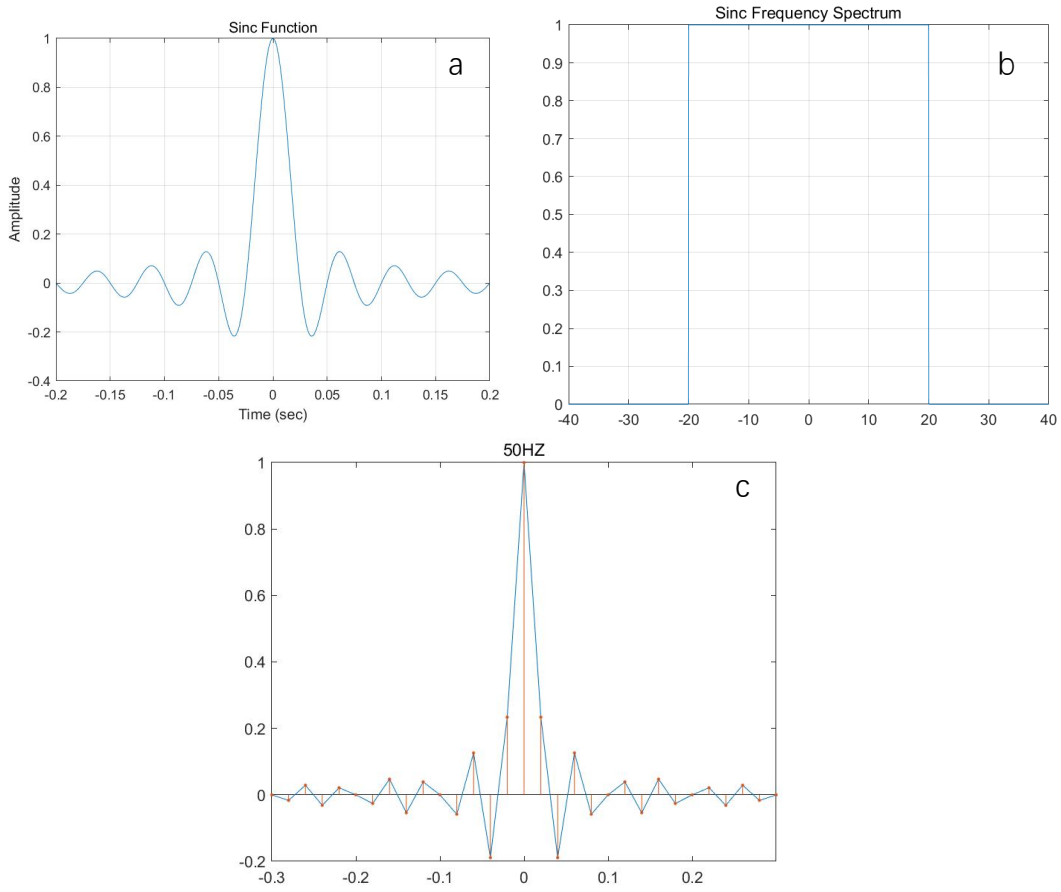
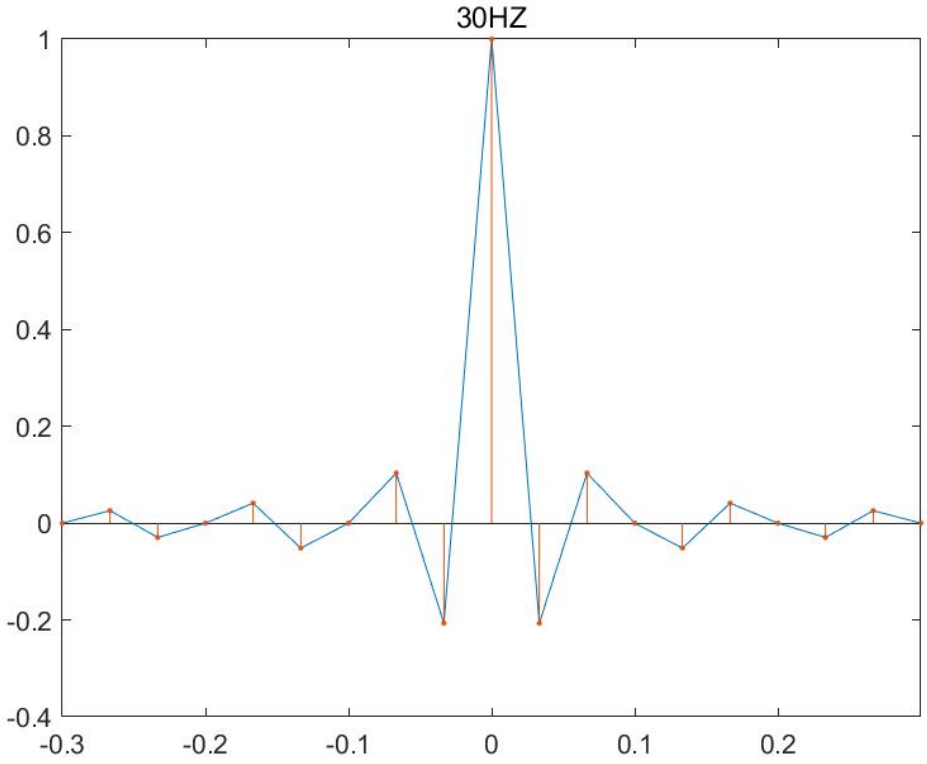
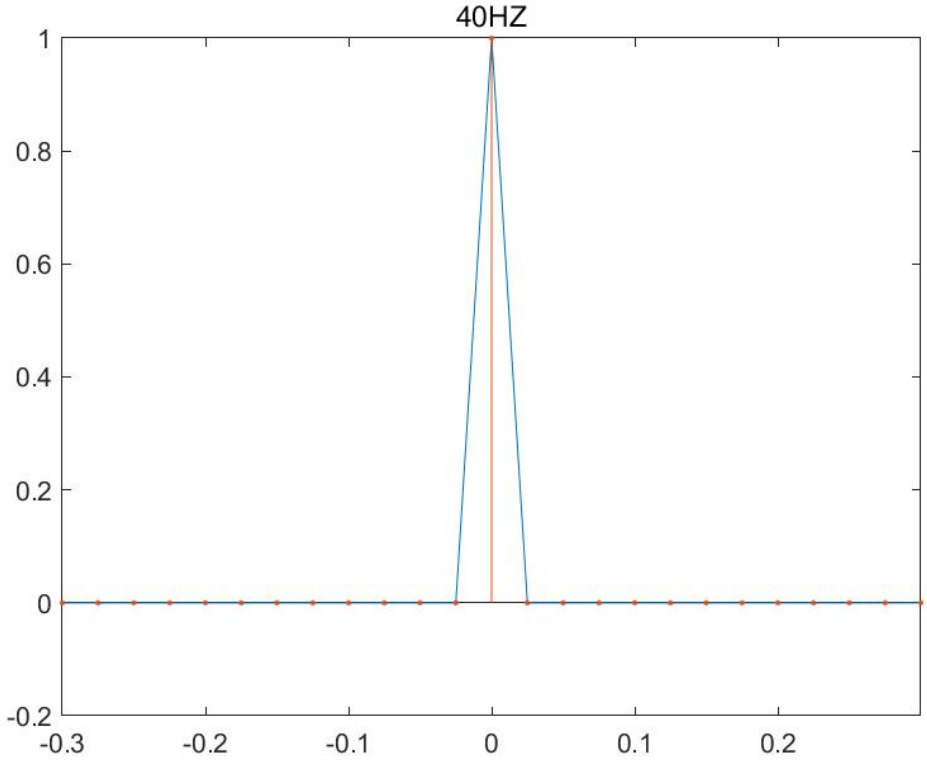
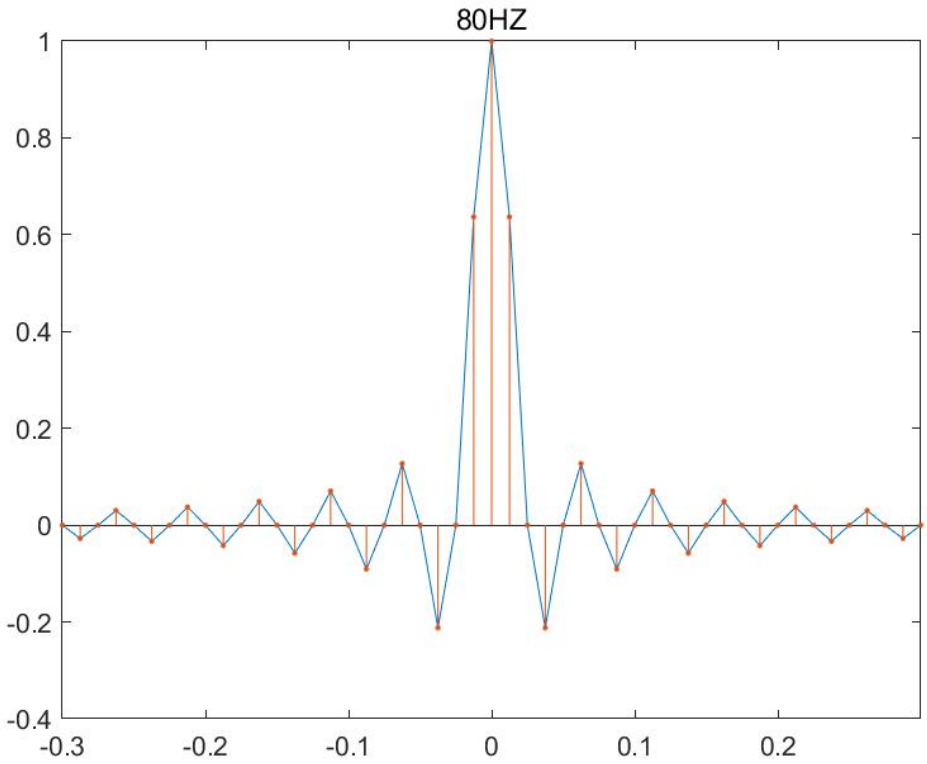


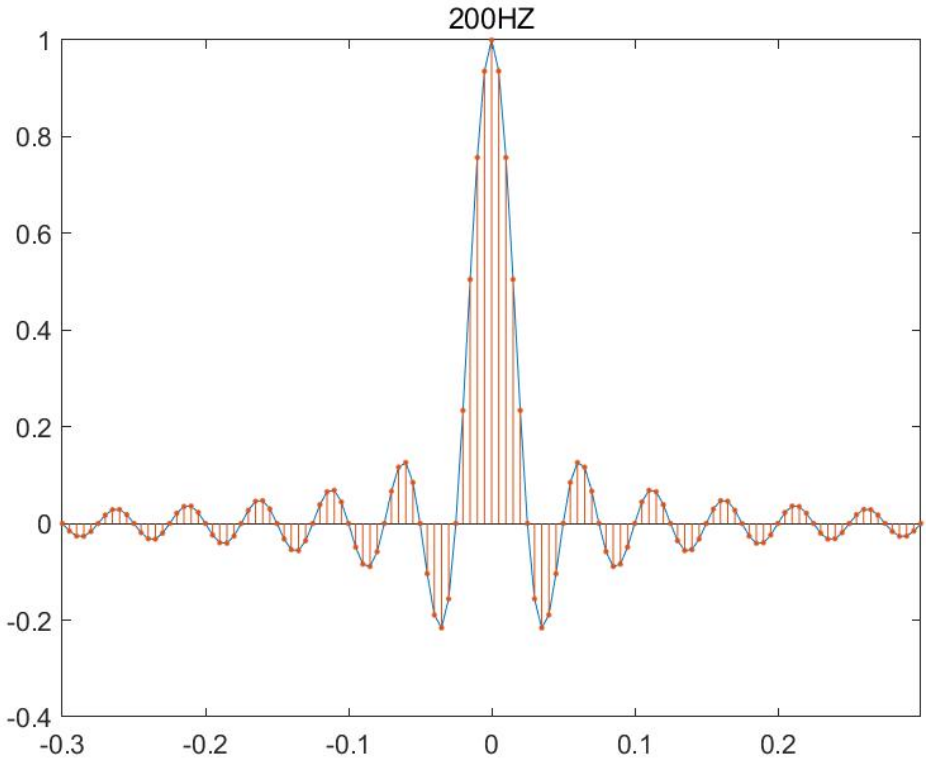

Figure 1 **a)** Function $\mathrm{sinc}(40t)$ **b)** Frequency Spectrum of $\mathrm{sinc}(40t)$ **c)** Sampling at 50Hz

Solution:


30HZ
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2


40HZ
1
0.8
0.6
0.4
0.2
0
-0.2
-0.3
-0.2
-0.1
0
0.1
0.2


80HZ
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2


200HZ
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2

2. For an X-ray imaging system, the tube accelerating voltage is 50kVp. Provided the following condition: a) the X-ray tube operated at 80 mA for 0.1 seconds; 2) the efficiency of transferring electron to output X-ray of the tube is 30%; 3) The linear attenuation coefficients of the bone and tissue at the average received energy of X-ray is 10 $cm^{-1}$and 2 $cm^{-1}$, and the thickness of the bone(dark) and tissue(light) is shown in the Fig.2. Calculate how many photons are received by the detector for each X-ray. Where Planck's constant is 6.63∗10-34J*s, Speed of light is $3 * 10^8$m/s, electron charge is $1.6 * 10^{-19}$C. (24pts)

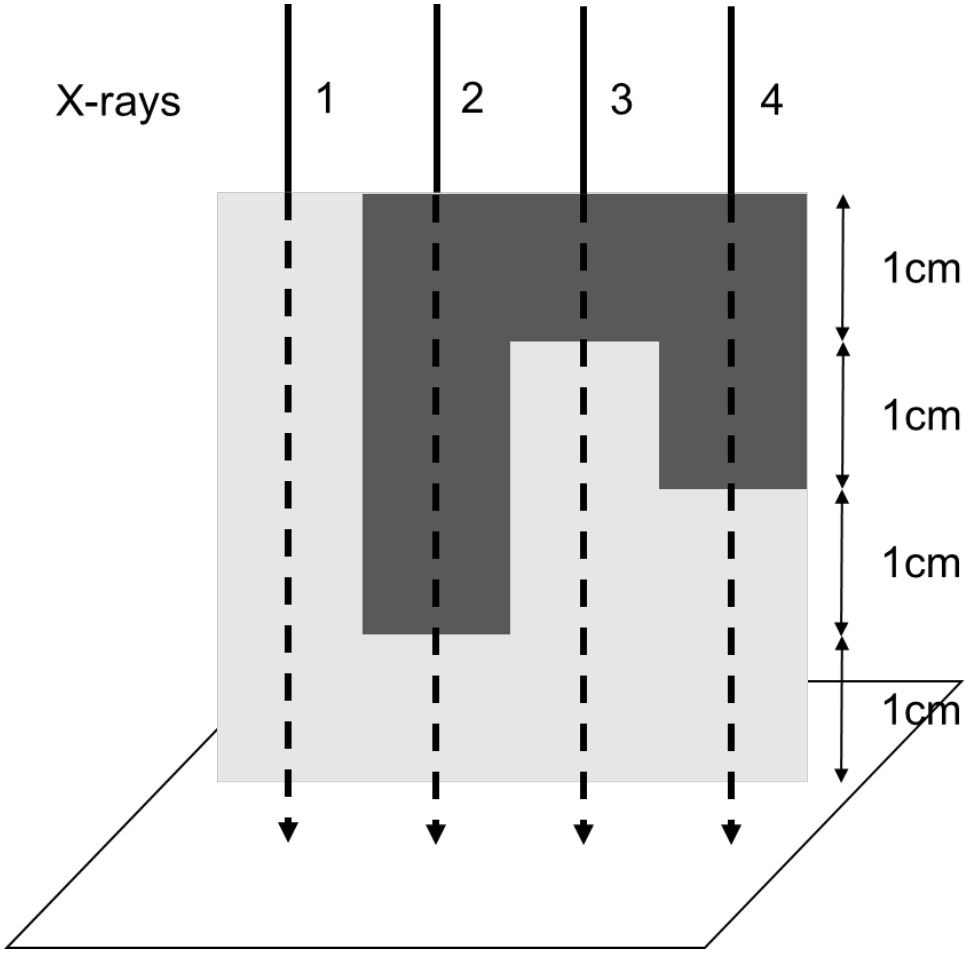



Figure 2 the thickness of the bone(dark) and tissue(light)

Solution:

Beam1:

$$\mathrm{N1} = \frac{\mathrm{tI}}{\mathrm{q}} = \left(\frac{80\mathrm{mA}\ * 0.1\mathrm{s}}{1.6 * 10^{-19}C}\right) * 30\% * \mathrm{e}^{-2\mathrm{cm}^{-1} * 4\mathrm{cm}} = 5.03 * 10^{12}$$

Beam2:

$$\mathrm{N2} = \frac{\mathrm{tI}}{\mathrm{q}} = \left(\frac{80\mathrm{mA}\ * 0.1\mathrm{s}}{1.6 * 10^{-19}C}\right) * 30\% * \mathrm{e}^{-2\mathrm{cm}^{-1} * 1\mathrm{cm} - 10\mathrm{cm}^{-1} * 3\mathrm{cm}} \approx 189$$

Beam3:

$$\mathrm{N3} = \frac{\mathrm{tI}}{\mathrm{q}} = \left(\frac{80\mathrm{mA}\ * 0.1\mathrm{s}}{1.6 * 10^{-19}C}\right) * 30\% * \mathrm{e}^{-2\mathrm{cm}^{-1} * 3\mathrm{cm} - 10\mathrm{cm}^{-1} * 1\mathrm{cm}}$$
$$= 1.69 * 10^{9}$$

Beam4:

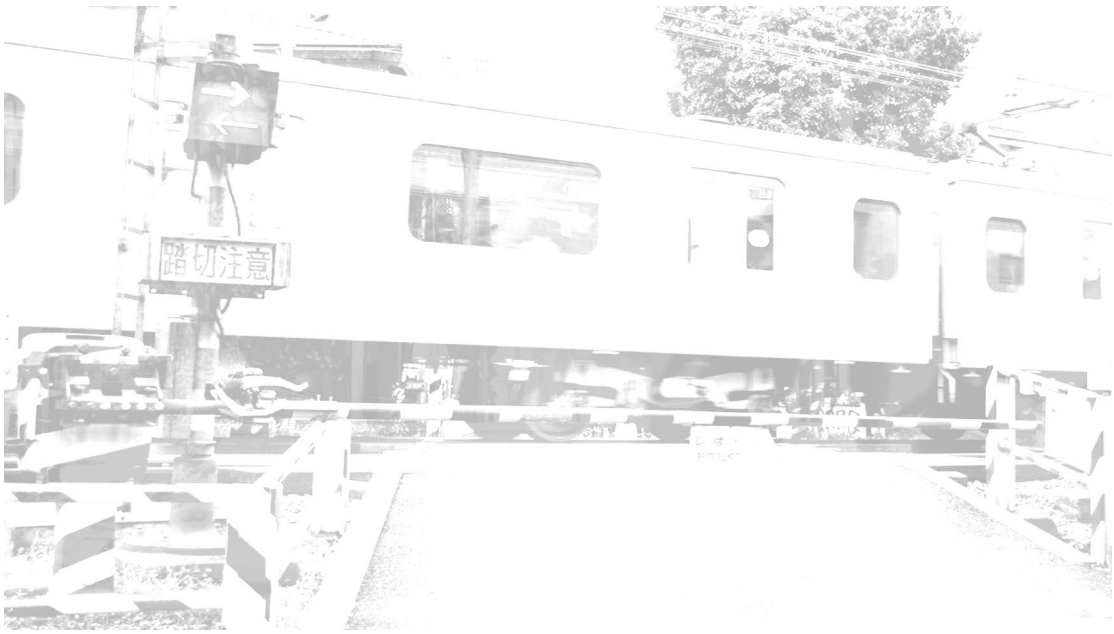
$$\mathrm{N4} = \frac{\mathrm{tI}}{\mathrm{q}} = \left(\frac{80\mathrm{mA}\ * 0.1\mathrm{s}}{1.6 * 10^{-19}C}\right) * 30\% * \mathrm{e}^{-2\mathrm{cm}^{-1} * 2\mathrm{cm} - 10\mathrm{cm}^{-1} * 2\mathrm{cm}}$$
$$= 5.66 * 10^{5}$$

3. *(MATLAB)* Given 2 grayscale images “img1.jpg” and “img2.jpg” in file “Problem 3”, implement *Image Addition* algorithms “Addition.m” using the following three different methods and show your processed images. （24pts）
   1) Mean Value
   2) Maximum Value
   3) Scale

   Solution：

   1）


   2）



3)


Code:

```
%% Problem 1 - Mean Value

p1=uint8((img1+img2)/2);
figure;title('Mean Value');imshow(p1);imwrite(p1,'reslut1.jpg');
```

```
%% Problem 2 - Max Intensity
p2=img1+img2;
p2(p2>255)=255;
p2=uint8(p2);
figure;title('Maximum Value');imshow(p2);imwrite(p2,'reslut2.jpg');
```

```
%% Problem 3
p3_temp=img1+img2;
p3_temp=p3_temp-min(min(p3_temp));
p3=p3_temp/(max(max(p3_temp))-min(min(p3_temp)));
p3=uint8(p3*255);
figure;title('Scale');imshow(p3);imwrite(p3,'reslut3.jpg');
```

4. In a study for testing a new form of cancer among 1000 suspected patients, multiple medical methods had been considered for auxiliary diagnosis. For each method, the number of positive and negative people corresponding to the different threshold values was given in the Table.1 (28pts)

Table1 Diagnosis Results

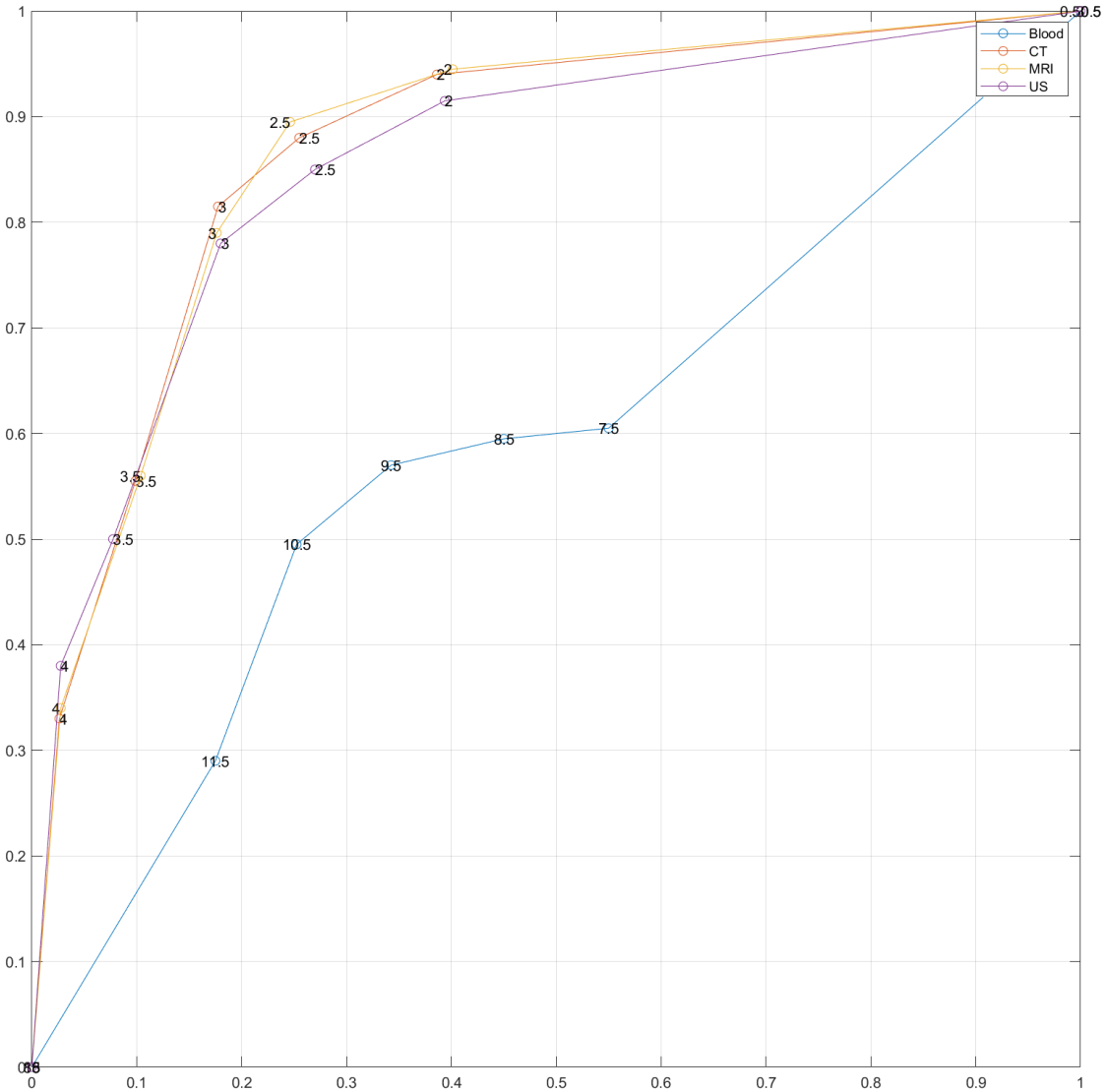
| Biopsy | | | | | | | |
|---|---|---|---|---|---|---|---|
| Positive | | | | 200 | | | |
| Negative | | | | 800 | | | |
| Blood(Immune cell concentration 10^9/L) | | | | | | | |
| Threshold | 3 | 7.5 | 8.5 | 9.5 | 10.5 | 11.5 | 15 |
| Positive(>Thr) | 1000 | 561 | 479 | 388 | 301 | 198 | 0 |
| TP | 200 | 121 | 119 | 114 | 99 | 58 | 0 |
| CT(length(cm)) | | | | | | | |
| Threshold | 0.5 | 2 | 2.5 | 3 | 3.5 | 4 | 8 |
| Positive(>Thr) | 1000 | 497 | 380 | 309 | 190 | 87 | 0 |
| TP | 200 | 188 | 176 | 163 | 111 | 66 | 0 |
| MRI(length(cm)) | | | | | | | |
| Threshold | 0.5 | 2 | 2.5 | 3 | 3.5 | 4 | 8 |
| Positive(>Thr) | 1000 | 510 | 376 | 299 | 195 | 90 | 0 |
| TP | 200 | 189 | 179 | 158 | 112 | 68 | 0 |
| Ultrasound(length(cm)) | | | | | | | |
| Threshold | 0.5 | 2 | 2.5 | 3 | 3.5 | 4 | 8 |
| Positive(>Thr) | 1000 | 498 | 386 | 300 | 162 | 98 | 0 |
| TP | 200 | 183 | 170 | 156 | 100 | 76 | 0 |

(1) Plot the ROC curve. (8pts) (If ROC curves are not on one Figure, -2pts)
(2) Determine a standard Threshold for each method(8pts) (2 for each) ( If only give answers, not even mention the distance to (0,1), -2pts)
(3) Consider which method is best for auxiliary diagnosis and explain. (12pts) (4pts for correct method and 8pts for correct explanation);

Solution:

| Blood(Immune cell concentration 10^9/L) | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Threshold | | 3 | | 7.5 | | 8.5 | | 9.5 | | 10.5 | | 11.5 | | 15 | |
| Positive(>Thr) | Negative(<Thr) | 1000 | 0 | 561 | 439 | 479 | 521 | 388 | 612 | 301 | 699 | 198 | 802 | 0 | 1000 |
| TP | FP | 200 | 800 | 121 | 440 | 119 | 360 | 114 | 274 | 99 | 202 | 58 | 140 | 0 | 0 |
| FN | TN | 0 | 0 | 79 | 360 | 81 | 440 | 86 | 526 | 101 | 598 | 142 | 660 | 200 | 800 |
| Sensitivity | FPR | 1 | 1 | 0.605 | 0.55 | 0.595 | 0.45 | 0.57 | 0.3425 | 0.495 | 0.2525 | 0.29 | 0.175 | 0 | 0 |
| CT(length(cm)) | | | | | | | | | | | | | | | |
| Threshold | | 0.5 | | 2 | | 2.5 | | 3 | | 3.5 | | 4 | | 8 | |
| Positive(>Thr) | Negative(<Thr) | 1000 | 0 | 497 | 503 | 380 | 620 | 309 | 691 | 190 | 810 | 87 | 913 | 0 | 1000 |
| TP | FP | 200 | 800 | 188 | 309 | 176 | 204 | 163 | 146 | 111 | 79 | 66 | 21 | 0 | 0 |
| FN | TN | 0 | 0 | 12 | 491 | 24 | 596 | 37 | 654 | 89 | 721 | 134 | 779 | 200 | 800 |
| Sensitivity | FPR | 1 | 1 | 0.94 | 0.38625 | 0.88 | 0.255 | 0.815 | 0.1825 | 0.555 | 0.09875 | 0.33 | 0.02625 | 0 | 0 |
| MRI(length(cm)) | | | | | | | | | | | | | | | |
| Threshold | | 0.5 | | 2 | | 2.5 | | 3 | | 3.5 | | 4 | | 8 | |
| Positive(>Thr) | Negative(<Thr) | 1000 | 0 | 510 | 490 | 376 | 624 | 299 | 701 | 195 | 805 | 90 | 910 | 0 | 1000 |
| TP | FP | 200 | 800 | 189 | 321 | 179 | 197 | 158 | 141 | 112 | 83 | 68 | 22 | 0 | 0 |
| FN | TN | 0 | 0 | 11 | 479 | 21 | 603 | 42 | 659 | 88 | 717 | 132 | 778 | 200 | 800 |
| Sensitivity | FPR | 1 | 1 | 0.945 | 0.40125 | 0.895 | 0.24625 | 0.79 | 0.17625 | 0.56 | 0.10375 | 0.34 | 0.0275 | 0 | 0 |
| US(length(cm)) | | | | | | | | | | | | | | | |
| Threshold | | 0.5 | | 2 | | 2.5 | | 3 | | 3.5 | | 4 | | 8 | |
| Positive(>Thr) | Negative(<Thr) | 1000 | 0 | 498 | 502 | 386 | 614 | 300 | 700 | 162 | 838 | 98 | 902 | 0 | 1000 |
| TP | FP | 200 | 800 | 183 | 315 | 170 | 216 | 156 | 144 | 100 | 62 | 76 | 22 | 0 | 0 |
| FN | TN | 0 | 0 | 17 | 485 | 30 | 584 | 44 | 656 | 100 | 738 | 124 | 778 | 200 | 800 |
| Sensitivity | FPR | 1 | 1 | 0.915 | 0.39375 | 0.85 | 0.27 | 0.78 | 0.18 | 0.5 | 0.0775 | 0.38 | 0.0275 | 0 | 0 |

1)




2) Calculate the minimum distance compared to (0,1) for each threshold of each method to determine the standard threshold:

Blood (9.5) CT (3) MRI (2.5) US (3)

3)Area under ROC Curve (AUC) is used to evaluate the performance of each method.

**Area under the curve** [ edit ]

When using normalized units, the area under the curve (often referred to as simply the AUC) is equal to the probability that a classifier will rank a randomly chosen positive instance higher than a randomly chosen negative one (assuming 'positive' ranks higher than 'negative').[15] This can be seen as follows: the area under the curve is given by (the integral boundaries are reversed as large T has a lower value on the x-axis)

$TPR(T) : T \to y(x)$
$FPR(T) : T \to x$

$$A = \int_{x=0}^{1} \mathrm{TPR}(\mathrm{FPR}^{-1}(x))\, dx = \int_{\infty}^{-\infty} \mathrm{TPR}(T)\mathrm{FPR}'(T)\, dT = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(T' > T) f_1(T') f_0(T)\, dT'\, dT = P(X_1 > X_0)$$

where $X_1$ is the score for a positive instance and $X_0$ is the score for a negative instance, and $f_0$ and $f_1$ are probability densities as defined in previous section.

AUC(Blood)=0.5875;
AUC(CT)=0.8708;
AUC(MRI)=0.8718;
AUC(Ultrasound)=0.8559;
So, MRI is the best method