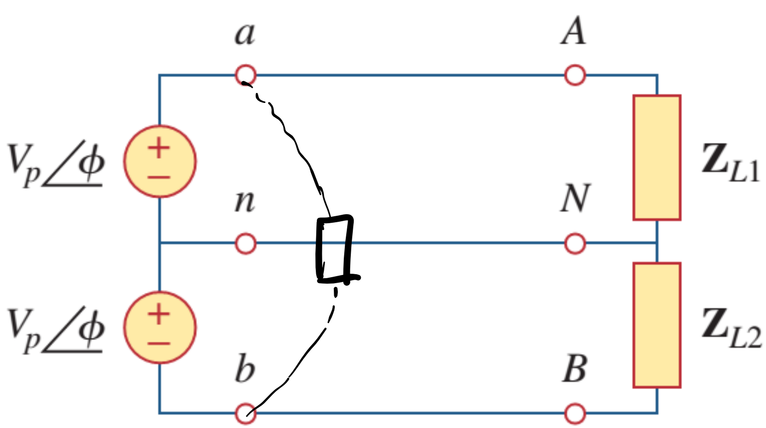
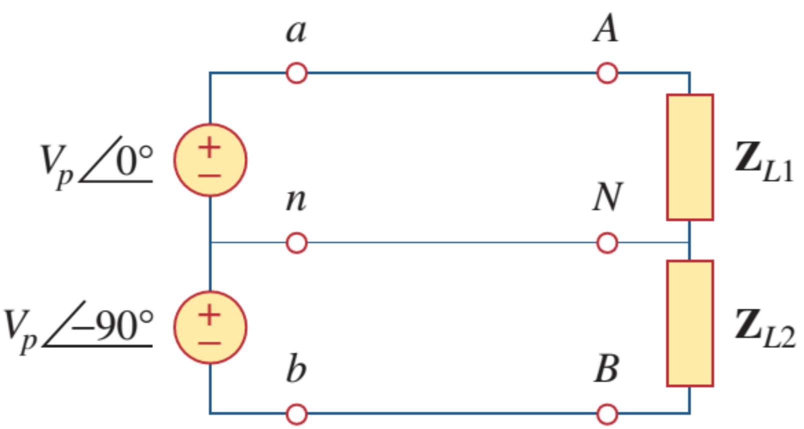
Lecture 10

- Three-Phase Circuits

Single phase vs. Polyphase

- Single-phase power supply
  - For example, two 120V sources with the same phase are connected in series.
  - This allows for appliances to use either 120 or 240V
- Circuits that operate with multiple sources, at the same frequency but *at different phases* are called polyphase.





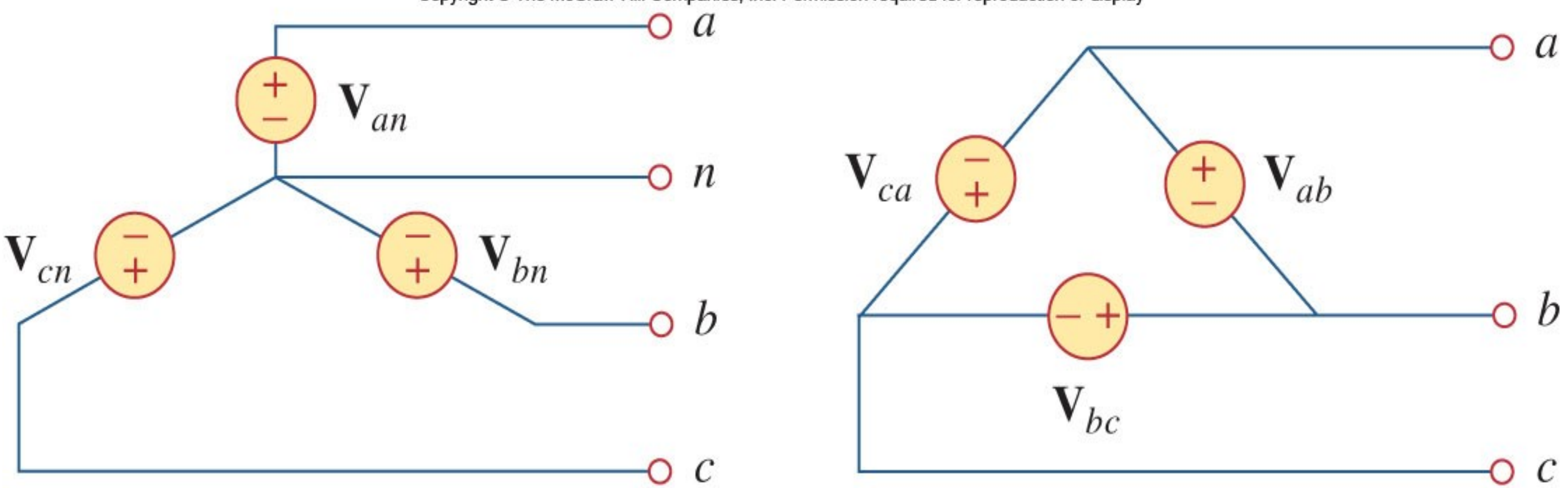
Outline--Three-Phase Circuits

- Balanced Three-Phase System
  - Balanced sources
  - Balanced loads
- Circuit analysis
  - Phase voltage/current
  - Line voltage/current

Balanced Three-Phase Sources

Connecting the Sources

- Three phase voltage sources can be connected by either four or three wire configurations.
  - Four-wire system accomplished using a Y(Wye) connected source.
  - Three-wire configuration accomplished by Delta connected source.



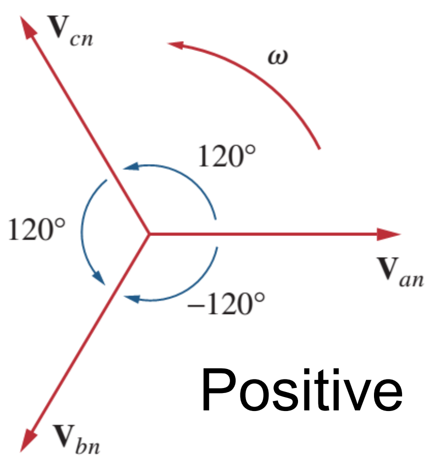
(a) (b)

$$\dot{V}_{an} = V_p\angle 0^\circ$$
$$\dot{V}_{bn} = V_p\angle -120^\circ$$
$$\dot{V}_{cn} = V_p\angle -240^\circ = V_p\angle +120^\circ$$

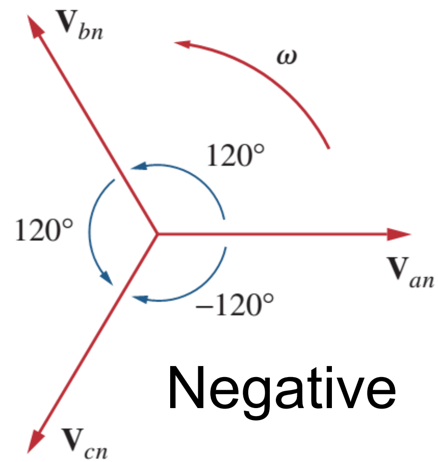
$$\dot{V}_{ab} = V_p\angle 0^\circ$$
$$\dot{V}_{bc} = V_p\angle -120^\circ$$
$$\dot{V}_{ca} = V_p\angle -240^\circ = V_p\angle +120^\circ$$

Balanced Three-Phase Sources

- Balanced phase voltage are equal in magnitude and are out of phase with each other by 120deg
- It's easy to know $\boxed{\dot{V}_{an} + \dot{V}_{bn} + \dot{V}_{cn} = 0}$
- Two sequences for the phases:



Positive



Negative

Positive:

$$\dot{V}_{an} = V_p \angle 0^\circ \qquad V_p' \angle 10^\circ$$

$$\dot{V}_{bn} = V_p \angle -120^\circ \qquad V_p' \angle -110^\circ$$

$$\dot{V}_{cn} = V_p \angle -240^\circ = V_p \angle +120^\circ \qquad V_p' \angle 130^\circ$$

$a \to b \to c \to a \to b \to c$

Negative:

$$\dot{V}_{an} = V_p \angle 0^\circ$$

$$\dot{V}_{cn} = V_p \angle -120^\circ$$

$$\dot{V}_{bn} = V_p \angle -240^\circ = V_p \angle +120^\circ$$

$a \to c \to b \to a \to c \to b$

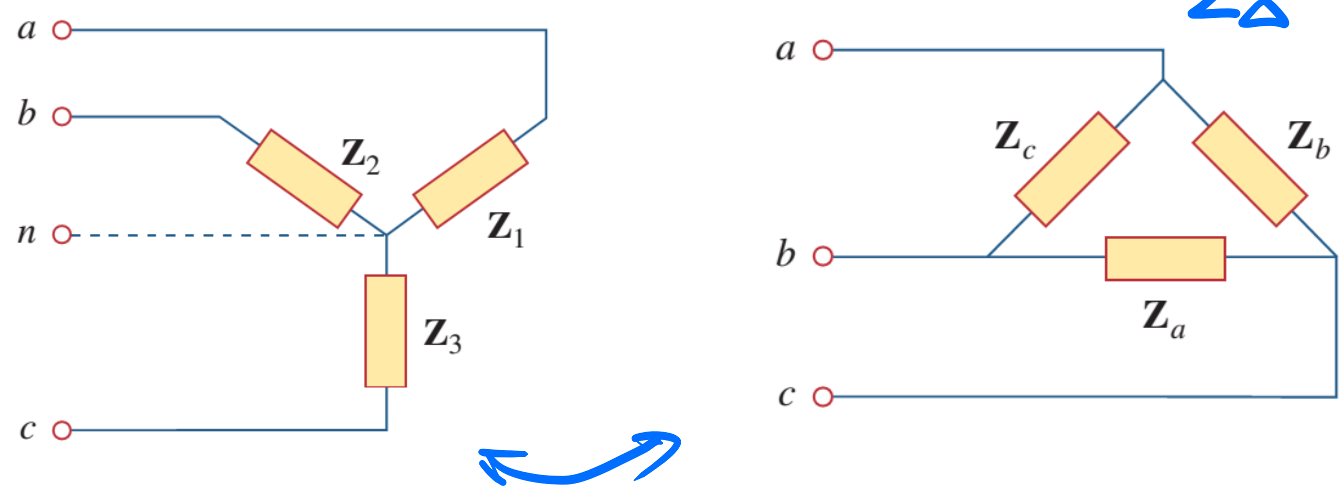
Balanced Loads

- A balanced load means the same impedance for each load.

-- Impedance are equal in magnitude and in phase

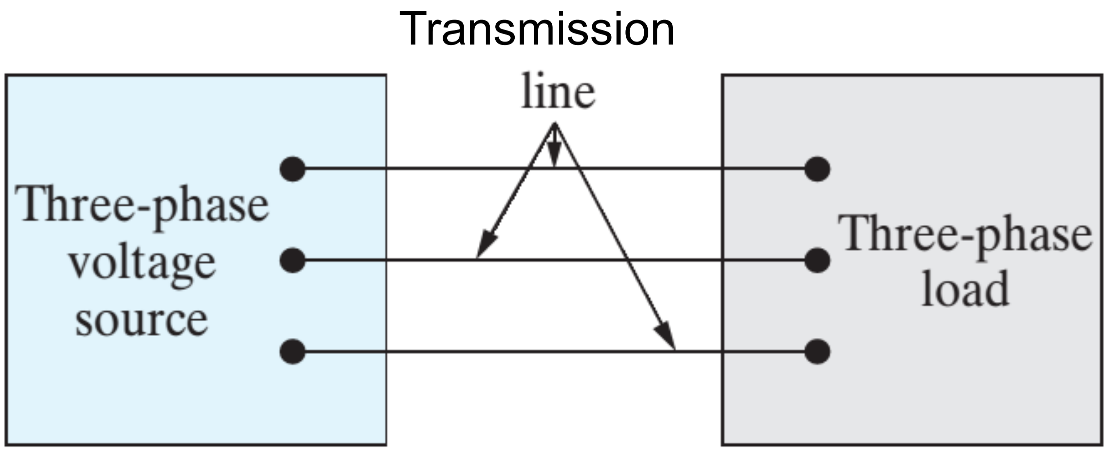
- They may also be connected in either Delta or wye
  - For a balanced wye connected load: $Z_1 = Z_2 = Z_3 = Z_Y$
  - For a balanced delta connected load: $Z_a = Z_b = Z_c = Z_\Delta$

$Z_\Delta = 3 \cdot Z_Y$



- The load impedance per phase for the above configurations can be interchanged.

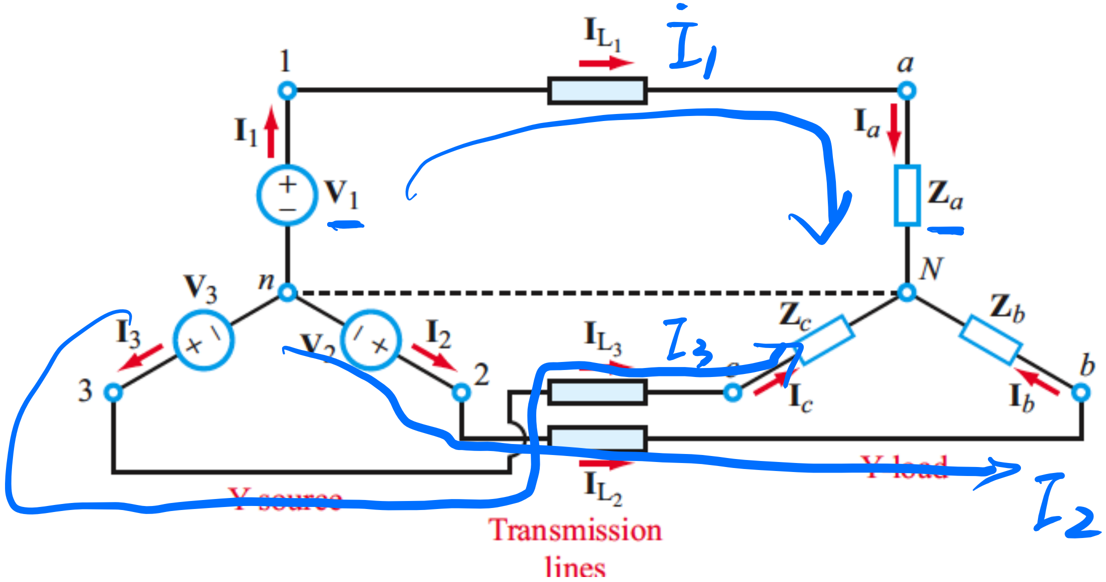
Source-Load configurations



Source	Load
Y	Y
Y	Δ
Δ	Y
Δ	Δ

Source-Load Configurations

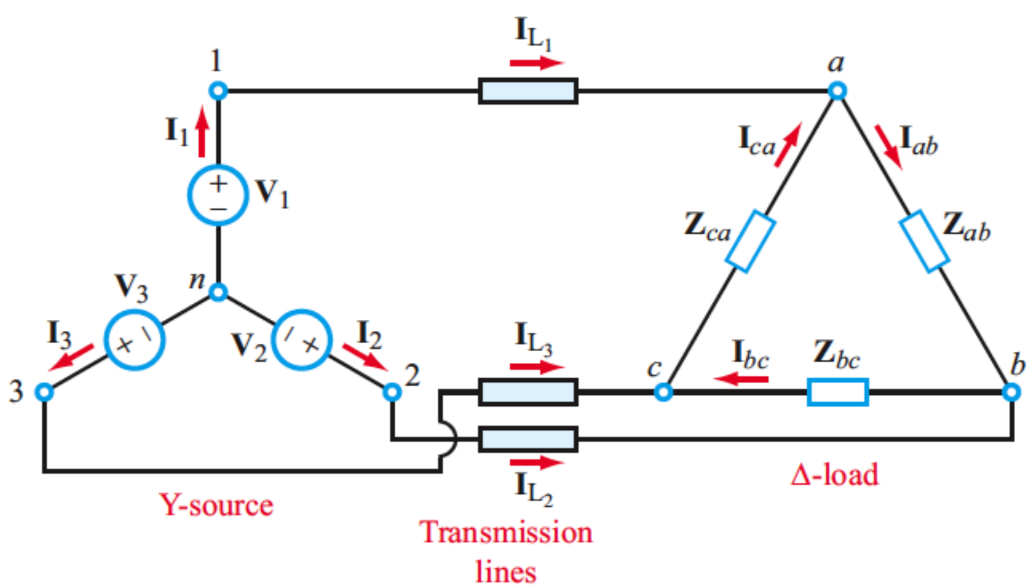
Y-Y



Load Phase Currents
I_a, I_b, I_c
(same as line currents I_{L_1}, I_{L_2}, and I_{L_3})

Load Phase Voltages
V_{aN}, V_{bN}, V_{cN}

Y-Delta

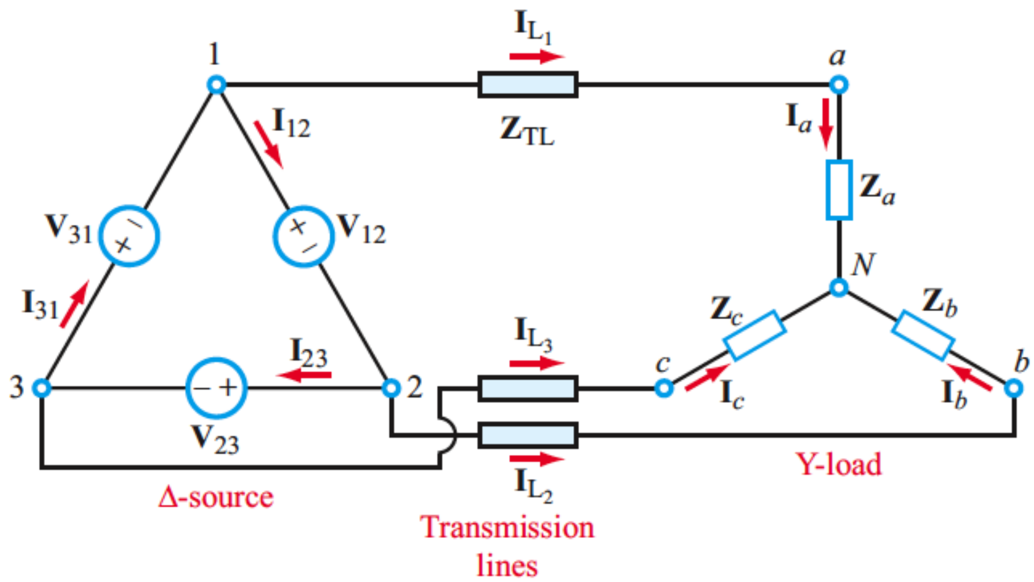


Load Phase Currents
I_{ab}, I_{bc}, I_{ca}

Load Phase Voltages
V_{ab}, V_{bc}, V_{ca}

Source-Load Configurations (optional)

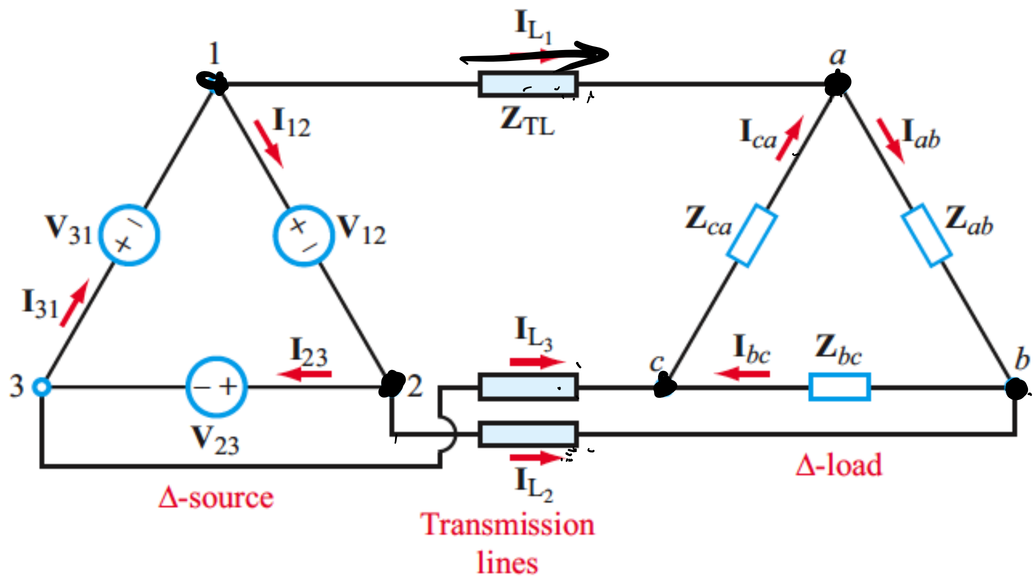
Delta-Y



Load Phase Currents
I_a, I_b, I_c
(same as line currents
I_{L_1}, I_{L_2}, and I_{L_3})

Load Phase Voltages
V_{aN}, V_{bN}, V_{cN}

Delta-Delta



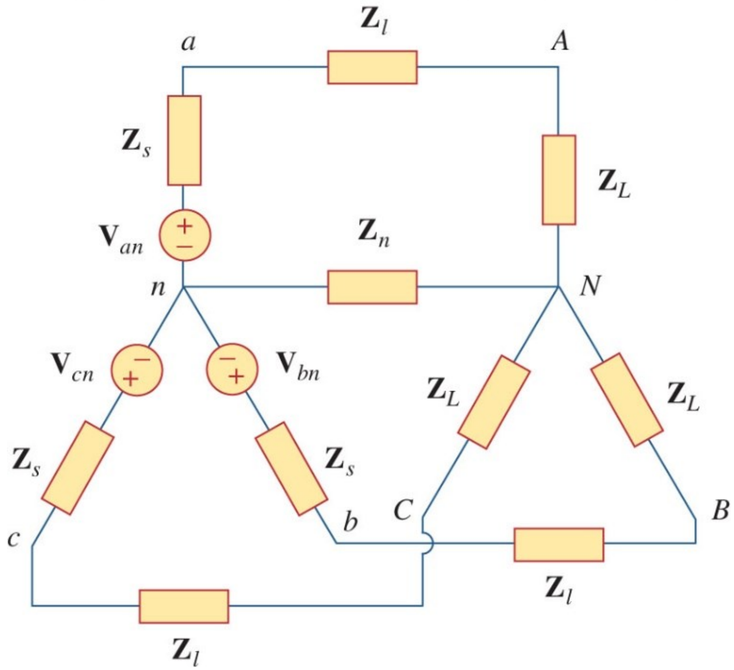
Load Phase Currents
I_{ab}, I_{bc}, I_{ca}

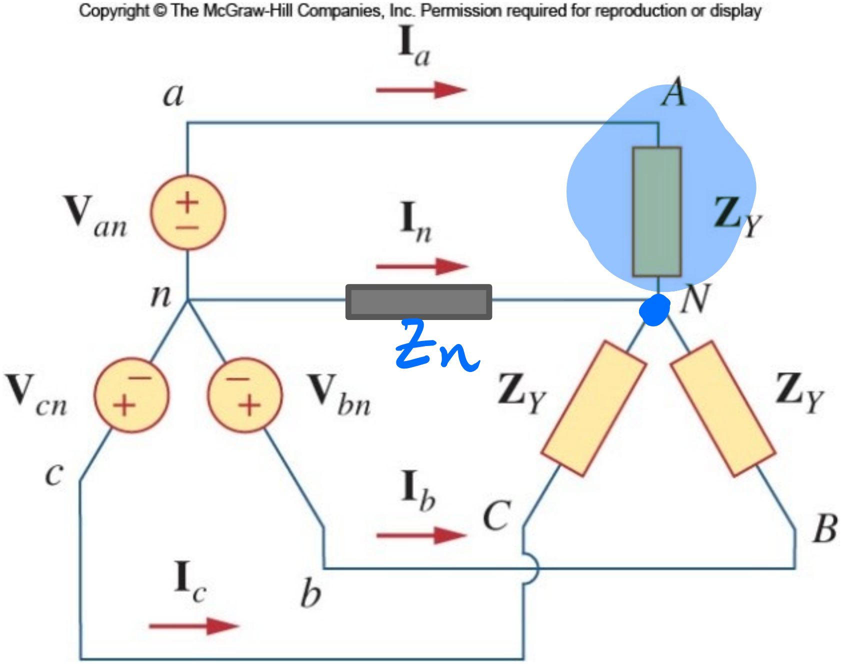
Load Phase Voltages
V_{ab}, V_{bc}, V_{ca}
(same as source voltages
if Z_{TL} is negligible)

Balanced Y-Y connection

- The load impedances Z_Y will be assumed to be balanced.
  - This can be the source Z_s, line Z_l and load Z_L together.

$$\mathbf{Z}_Y = \mathbf{Z}_s + \mathbf{Z}_\ell + \mathbf{Z}_L$$





Assume Z_n 为 minor value.

$$\left.\begin{aligned} \dot{I}_a &= \frac{\dot{V}_{an}}{Z_Y} \\ \dot{I}_b &= \frac{\dot{V}_{bn}}{Z_Y} \\ \dot{I}_c &= \frac{\dot{V}_{cn}}{Z_Y} \end{aligned}\right\} \qquad \dot{I}_a + \dot{I}_b + \dot{I}_c + \dot{I}_n = 0$$

$$0 + \dot{I}_n = 0$$

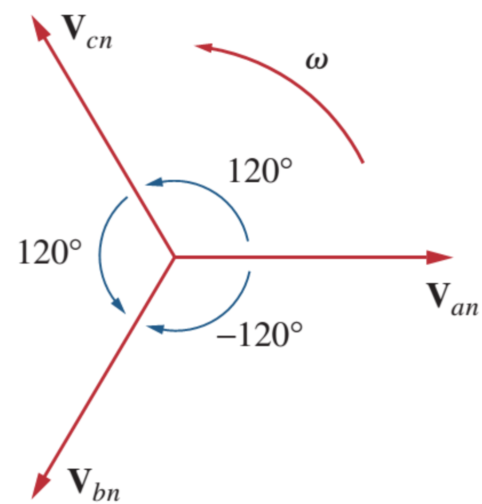
Phase Voltage & Line-to-Line Voltage

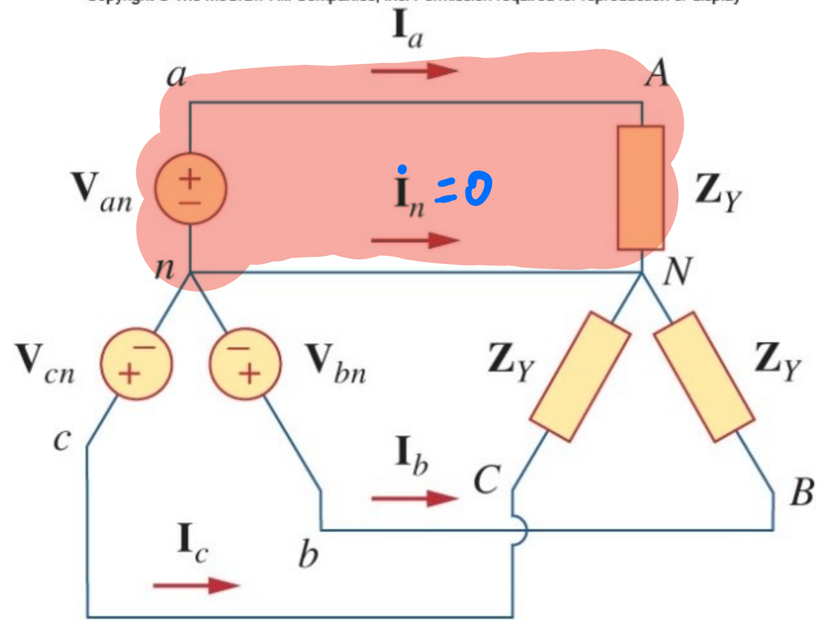
- Use the positive sequence:

$\dot{I}_a = \frac{\dot{V}_{an}}{Z_Y}$

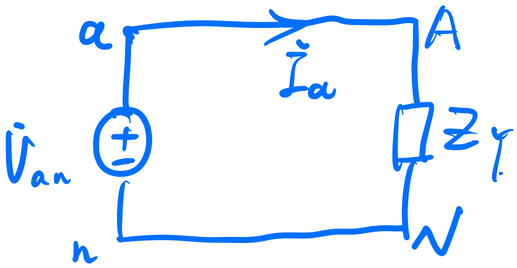
Phase Voltage $\quad V_{an} = V_p \angle 0^\circ$

$V_{bn} = V_p \angle -120^\circ \quad V_{cn} = V_p \angle +120^\circ$





- ***The line to line voltages (or just line voltages in short):***

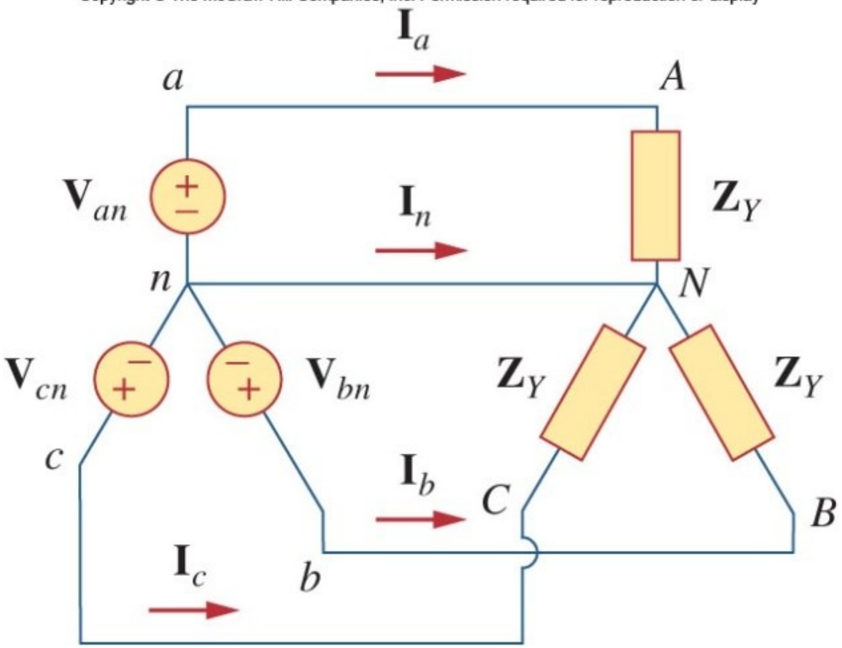


$$\dot{I}_a = \frac{\dot{V}_{an}}{Z_Y}$$

$$\dot{I}_b = \dot{I}_a \cdot 1\angle -120^\circ$$

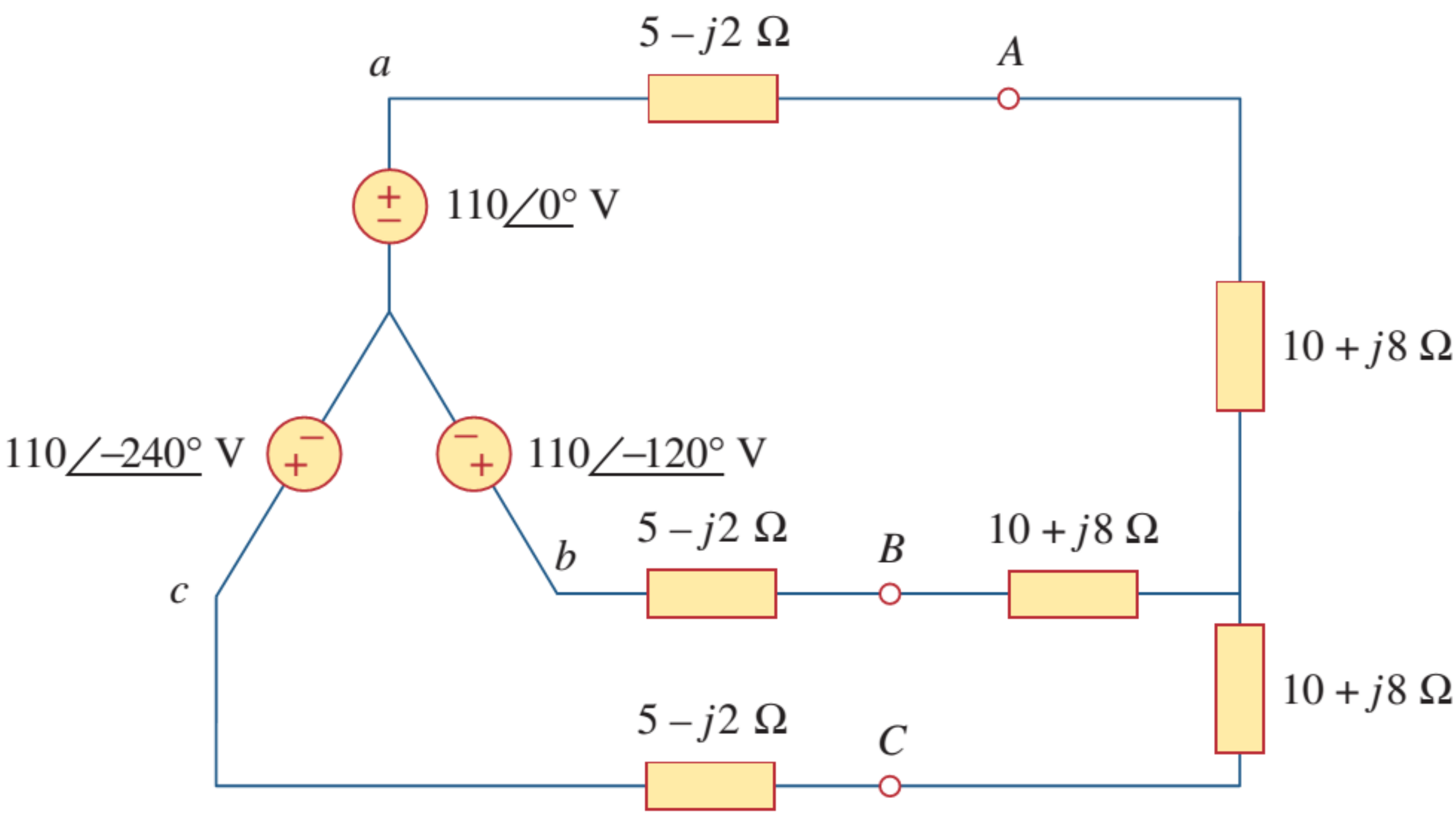
$$\dot{I}_c = \dot{I}_b \cdot 1\angle -120^\circ$$

Line Currents



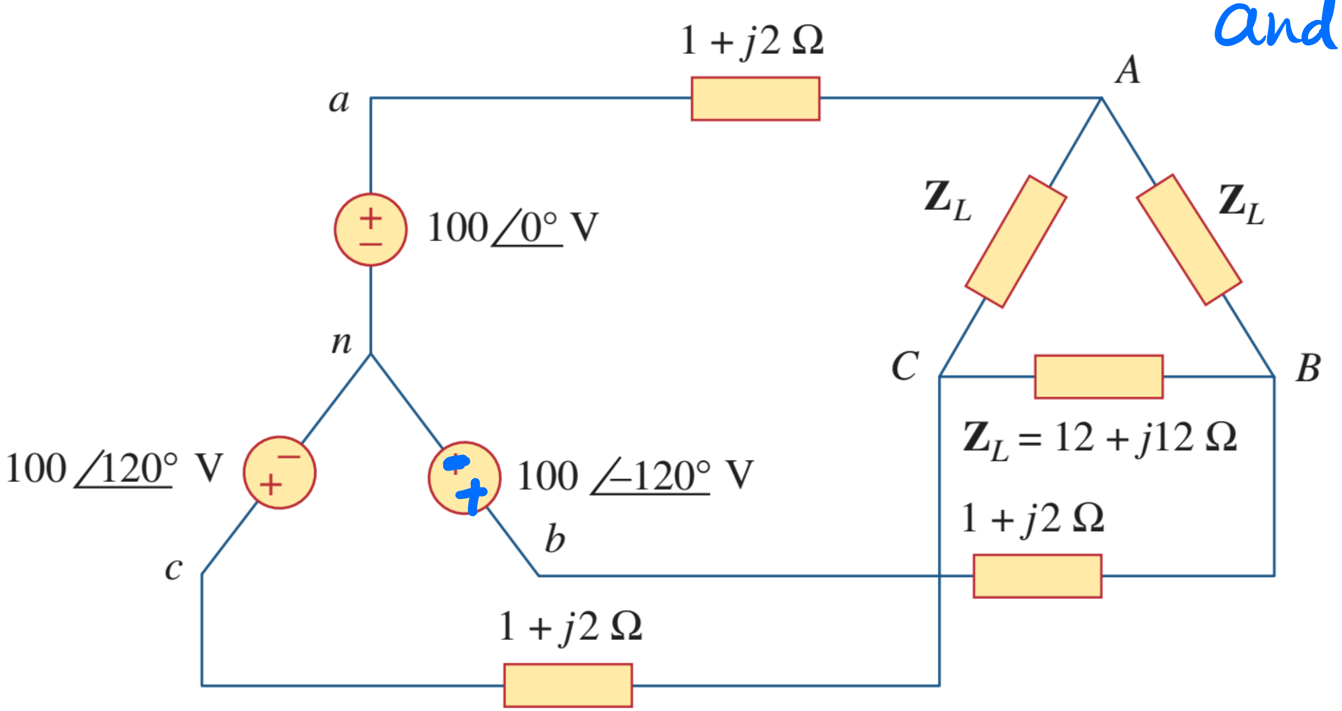
Example

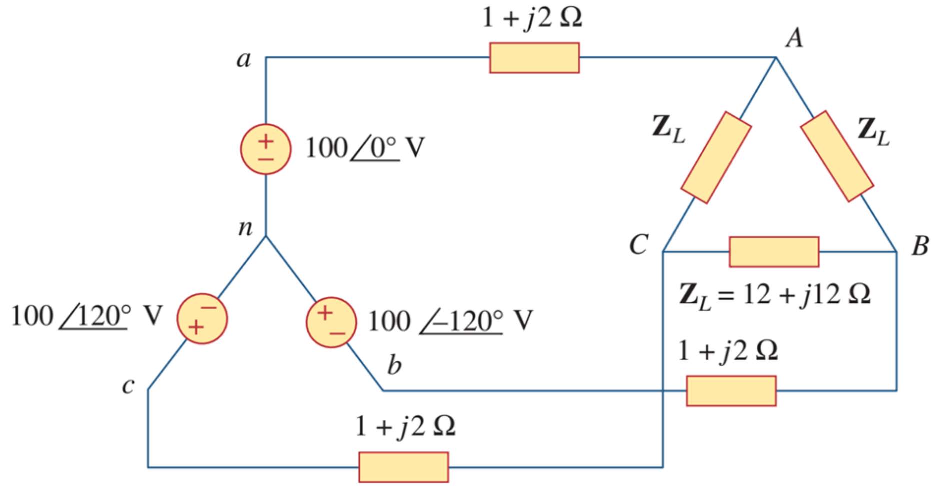
- Calculate the line currents.

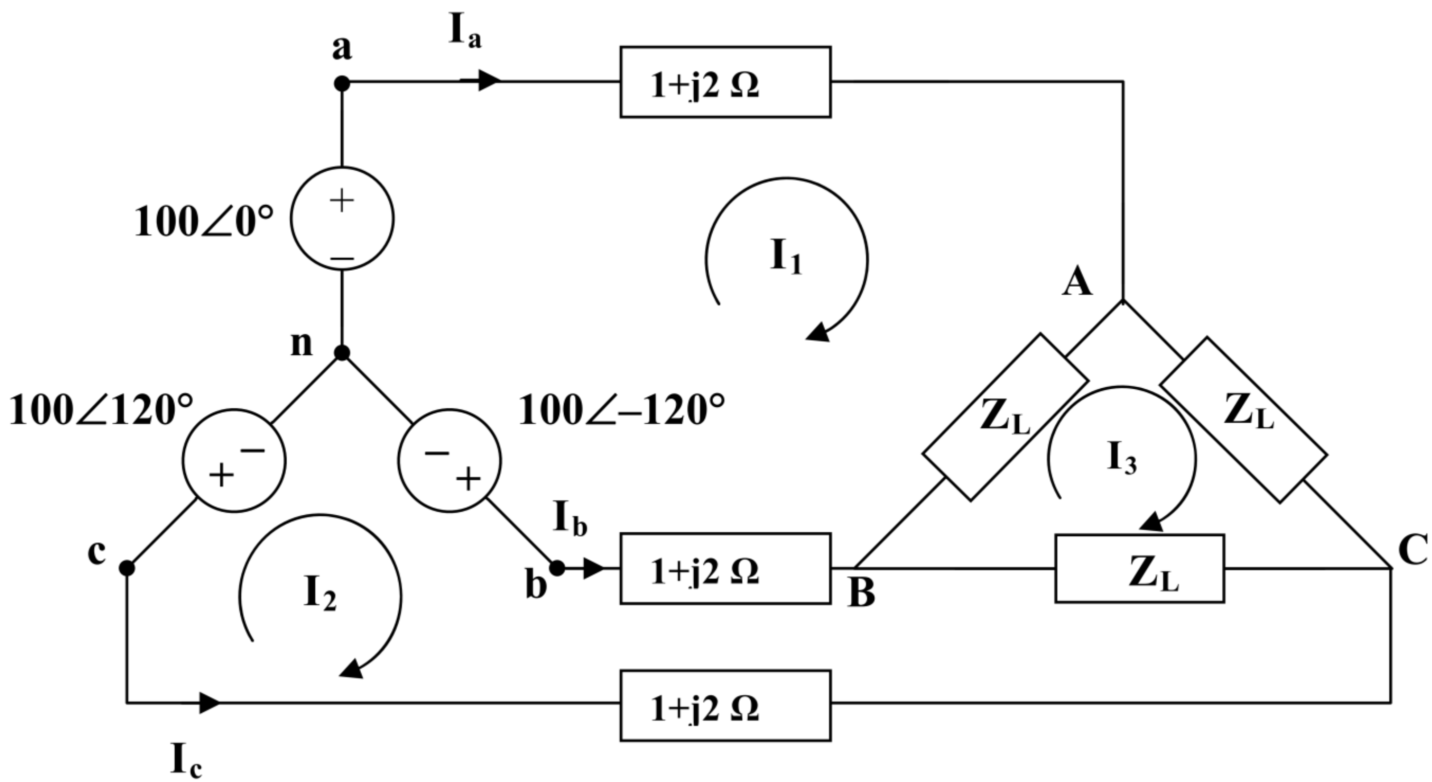


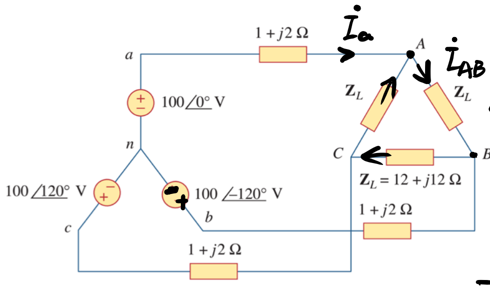
Wye-Δ

find complex power for each Z_L?
and complete Load.
?

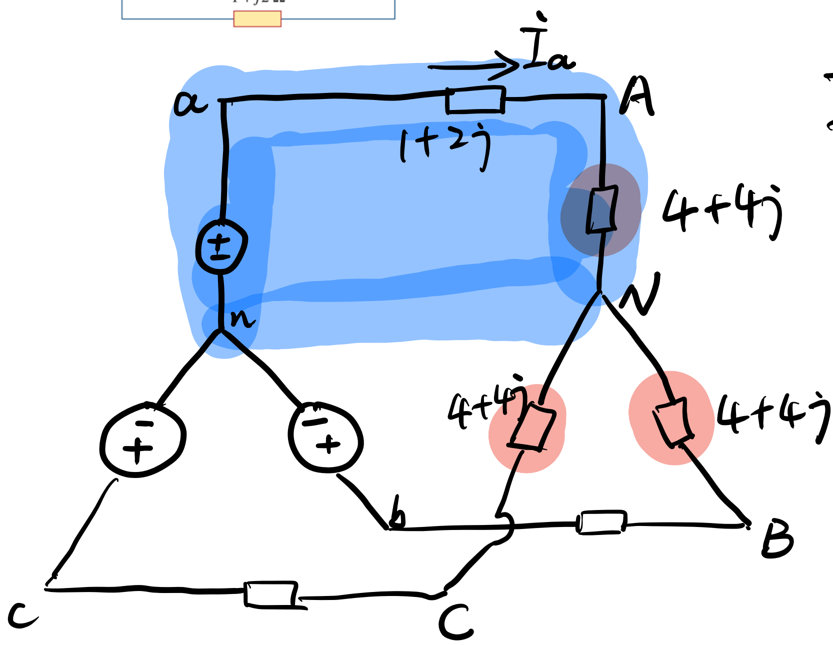



1 + j2 Ω
a
A
Z_L
Z_L
100 /0° V
n
C
B
Z_L = 12 + j12 Ω
100 /120° V
100 /−120° V
1 + j2 Ω
b
c
1 + j2 Ω


I_a
a
1+j2 Ω
100∠0°
I_1
A
n
100∠120°
100∠–120°
Z_L
Z_L
I_3
I_b
c
I_2
b
1+j2 Ω
B
Z_L
C
1+j2 Ω
I_c



complex



$$\dot{I}_a = \frac{\dot{V}_{an}}{Z^{1P}_{total}} = \frac{\dot{V}_{an}}{1+2j+4+4j}$$

$$= \frac{100\angle 0^\circ}{5+6j} = 12.8\angle -50.19^\circ$$

$$\dot{I}_b = \dot{I}_a \cdot 1\angle -120^\circ$$
$$= 12.8\angle -170.19^\circ$$

$$\dot{I}_c = \dot{I}_b \cdot 1\angle -120^\circ$$
$$= \dot{I}_a \cdot \angle +120^\circ$$
$$= 12.8\angle 69.81^\circ$$

A: KCL:

$$\dot{I}_a + \dot{I}_{CA} = \dot{I}_{AB} \quad (1)$$

$$\dot{I}_{AB} = \dot{I}_{BC} \cdot 1\angle +120^\circ = \dot{I}_{CA} \cdot 1\angle -120^\circ \quad (2)$$

$$\Rightarrow \dot{I}_{CA} = \frac{\dot{I}_a}{(1\angle -120^\circ - 1)}$$

$$= 7.39\angle 99.81^\circ$$

$$\tilde{S}_{1p} = \tilde{S}_{1p}^{CA} = |\dot{I}_{CA}|^2 \cdot Z_{CA}$$

$$= |\dot{I}_{CA}|^2 \cdot Z_L$$

$$= 7.39^2 \times (12 + 12j)$$

$$\tilde{S}_{3p} = 3 \times \tilde{S}_{1p} = \cdots$$