Homework 4 Solution

1. In a nuclear medicine scan using ^{99m}Tc, the image SNR for a 30 minute scan was 25:1 for an injected radioactive dose of 1 mCi. Imaging began immediately after injection.

(a) If the injected dose were tripled to 3 mCi, what would be the image SNR for a 30 minute scan? (5 Point)

(b) If the scan time were doubled to 60 minutes with an initial dose of 1 mCi, what would be the image SNR? (10 Point)

Solution

(a) Since the SNR is proportional to the square root of the number of counts, tripling the injected dose increases the SNR by the square root of 3 to give a value of 43.3:1.

(b) After 30 minutes the number of nuclei is reduced to (exp–3.22×10^{-5}*30*60)= 94.4% of the original number. After 60 minutes, the number is reduced to 89.1%. The total number of disintegrations during the second 30 minutes is approximately 94% (89.1/94.4) that of the first 30 minutes. Therefore the S/N is 25*sqrt((1-0.891)/(1-0.944)) = 34.84:1

2. Do the tops of the curves in Figure 1 lie at the same values that would have been obtained if the technetium cow were not milked at all? (14 Point)

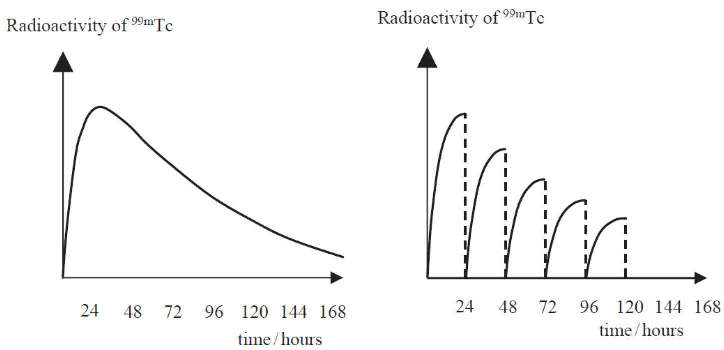
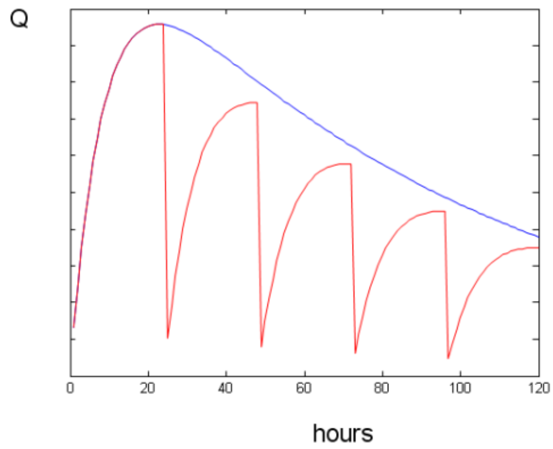


Figure 1. (Left) Theoretical plot of the radioactivity of ^{99m}Tc vs time for a generator that is not milked.
(Right) Practical radioactivity curve in which the generator is milked every 24 hours, as shown by the dashed lines.

Solution

Intuitively one would say not, since after milking it will take time for the radioactivity to build up again. Plotting this mathematically gives the graph below with the blue line representing with no milking, and the red with milking.



For blue line,

$$Q = \frac{\lambda_1 \lambda_2 N_0}{\lambda_1 - \lambda_2}(e^{-\lambda_1 t} - e^{\lambda_2 t})$$

For red line,

$$Q = \frac{\lambda_1 \lambda_2 N_0}{\lambda_1 - \lambda_2}\left(e^{-\lambda_1 t} - e^{\lambda_2 t}\right), t = 24\text{h}$$

$$Q = \frac{\lambda_1 \lambda_2 N_1}{\lambda_1 - \lambda_2}\left(e^{-\lambda_1 t} - e^{\lambda_2 t}\right), N_1 = N_0 e^{-\lambda_1 24}, t = 48\text{h}$$

$$Q = \frac{\lambda_1 \lambda_2 N_2}{\lambda_1 - \lambda_2}\left(e^{-\lambda_1 t} - e^{\lambda_2 t}\right), N_2 = N_1 e^{-\lambda_1 24}, t = 72\text{h}$$

$$\ldots$$

3. (i) The thickness of the lead septa is chosen to ensure that only 5% of the γ-rays penetrate from one collimator hole to the adjacent one. Using Figure 2 show that the thickness is given by [6d/μ]/[L-3/μ]. (12 Point)

   (ii) Calculate the septal thickness required for γ-rays of 140 keV for lead collimators with a hole diameter of 0.1 cm and a length of 2.5 cm. The attenuation coefficient for lead is 30 cm^{-1} at 140 keV. (5 Point)

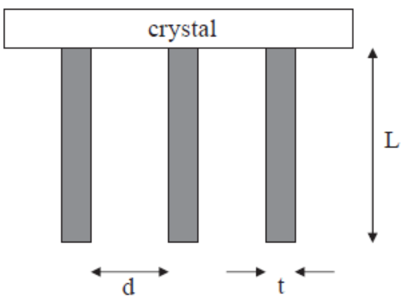


Figure 2

Solution

(i) From the figure below, the minimum path distance (x) for a γ-ray to pass through the collimator and be detected is related to l, t and d by:

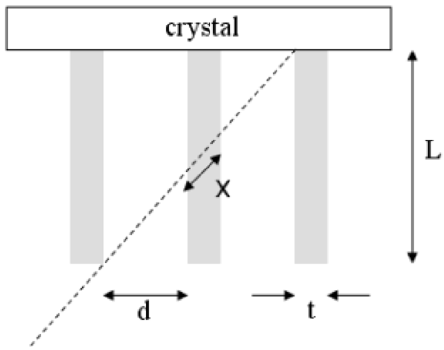
$$x = \frac{tL}{2d + t}$$

Since only 5% of the γ-rays can penetrate:

$$e^{-\mu x} = 0.05 => x = \frac{3}{\mu}$$

Rearranging the above two equations gives:

$$t = \frac{\frac{6d}{\mu}}{L - \frac{3}{\mu}}$$



(ii)

$$t = \frac{6\left(\frac{0.1}{30}\right)}{2.5 - \frac{3}{30}} = 0.008cm$$

4. Three parameters which affect the image SNR in nuclear medicine are the thickness of the detector crystal, the length of the lead septa in the anti-scatter grid, and the FWHM of the energy window centred around 140 keV. For each parameter, does an increase in the value of the particular parameter increase or decrease the image SNR? In each case, name one other image characteristic (e.g. CNR, spatial resolution) that is affected, and explain whether this image characteristic is improved or degraded. (18 Point)

Solution

Increases in the detector crystal thickness increases the signal-to-noise and CNR since a larger proporation of the γ-rays are detected, but decreases the spatial resolution due to the increased light spread function.

Increasing the length of the lead septa decreases the signal-to-noise since more γ-rays strike the collimator and are absorbed before reaching the detector, but increases the contrast since more Compton scattered γ-rays are absorbed and also the spatial resolution since the solid angle of acceptance is lower.

Increasing the width of the energy window increases the signal-to-noise since more detected γ-rays are accepted as having not been scattered, but decreases the contrast since some of these have, in fact, been scattered. The spatial resolution is not affected in this part.

5. Calculate the maximum angle and corresponding energy of Compton scattered γ-rays accepted for energy resolutions of 5, 15 and 25%. (18 Point)

Solution.
For a 5% energy resolution, the limits are 136.5 to 143.5 keV. The lower energy corresponds to a scatter angle determined by:

$$136.5 = \frac{140}{1 + \left(\frac{140}{511}\right)(1 - cos(\theta))}$$

Solving gives θ=25°
For a 15% energy resolution, the limits are 129.5 to 150.5 keV. The value of θ is 45°.
For a 25% energy resolution, the limits are 122.5 to 157.5 keV. The value of θ is 61°.

6. What timing resolution would be necessary to obtain a position resolution of 5 mm in TOF PET based only upon time-of-flight considerations? (8 Point)

Solution.
Applying equation (3.21):

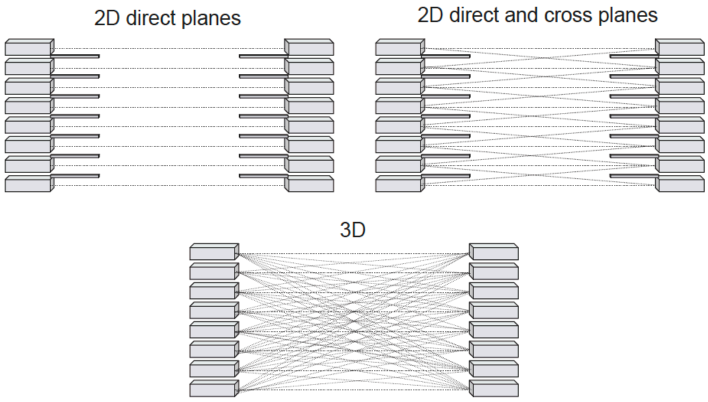
$$\Delta x = \frac{c\Delta t}{2}$$

$$0.005 = \frac{3 \times 10^8 \, \Delta t}{2} \Rightarrow \Delta t = 33ps$$

This is well below the resolving power of any current PET detectors.

7. Suggest why a PET/CT scanner operating in 2D mode has a relatively uniform axial sensitivity profile, whereas in 3D mode the sensitivity is much higher at the centre of the scanner. (10 Point)

Solution.
In two-dimensional mode (with the septa in place) the PET signal at each crystal ring comes only from a very well-defined region in the z-direction which is adjacent to that ring. In three-dimensional mode, on the other hand, as shown below, the PET signal detected by each crystal ring can originate from anywhere within the body. Since the central ring in the z-direction detects γ-rays that have been attenuated in the body to a lower degree than for the outer rings, due to a shorter pathway through tissue, the sensitivity will be higher. The central ring also receives signal from tissue lying either side of it, whereas the outer rings only receive signal from one side.


2D direct planes
2D direct and cross planes
3D