Homework 2

Submit: Blackboard/Paper   Due: Oct. 10th

Please write down Your Name & Student ID

1. $5 * 10^{16}$ X-ray photons with energy of $50keV$ were incident into a 4cm thickness phantom, and finally reached the detector. The length of lead strip is 4cm and the separation of lead strip is 1cm. Assuming that Compton scattering will at most happen once during the travelling, and it can happen anywhere. Figure 1 shows several possibilities of X-ray propagation. Provided the linear attenuation of Phantom 5cm^{-1}, calculate the energy range of X-ray which was detected by detector. (If you need any physical constant for the calculation, please refer to the course PPT) (24)

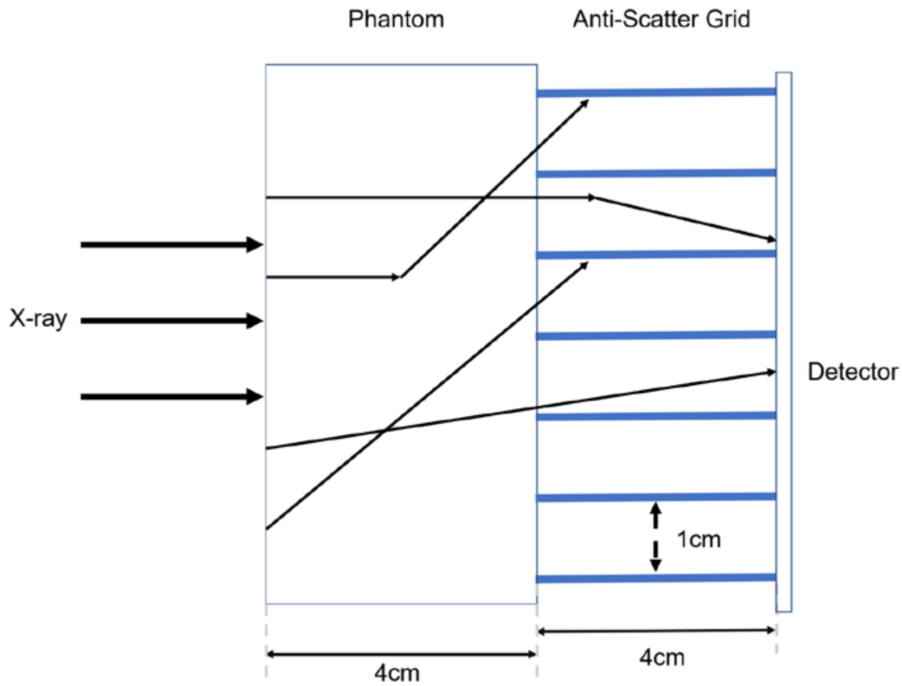
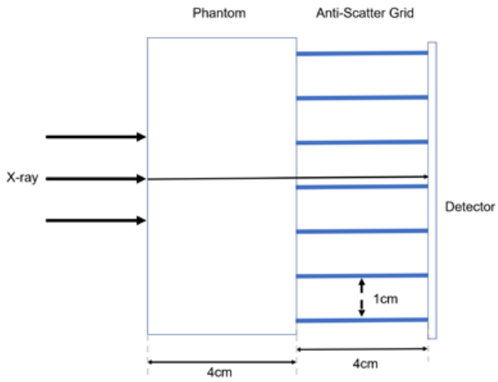


Figure 1

Solution: $[49.86keV, 50keV]$

Extreme case 1: (No Compton Scattering)

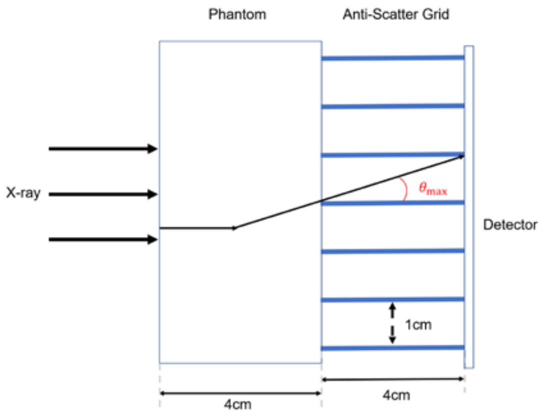


$$\mathrm{E_{max}} = E_{inc} = 50keV$$

Extreme case 2:

Compton Scattering Function: $\mathrm{E_{scat}} = \frac{E_{inc}}{1+\left(\frac{E_{inc}}{m_e c^2}\right)(1-\cos\theta)}$

In order to minimize $\mathrm{E_{scat}}$, we should find the maximum value of θ



$$\cos(\theta_{max}) = \frac{4}{\sqrt{(4^2+1^2)}} = \frac{4}{\sqrt{17}}$$

$$\mathrm{E_{scat\_min}} = \frac{E_{inc}}{1+\left(\frac{E_{inc}}{m_e c^2}\right)(1-\cos\theta_{max})}$$

$$= \frac{E_{inc}}{1+\left(\frac{E_{inc}}{m_e c^2}\right)\left(1-\cos\left(arctan\left(\frac{d}{h}\right)\right)\right)} \approx 49.86keV$$

Tips:

1) If you considered the condition that the Compton Scattering may happen in Anti-Scatter Grid and derived the answer $[45.55keV, 50keV]$, It is also correct.
2) If you confused the concepts of x-ray Numbers and Energy, you will lose 3pts for each.
3) If you considered the minimum energy is 0, this part you will have no pt.

2. For the object shown in Figure 2(a), draw the projections that would be acquired at angles φ=0°, 45°, 90°, 135° and 180° (ignore beam hardening), and sketch the sinogram for values of φ from 0 to 360°. Assume that a dark area corresponds to an area of high signal. The detail geometry relationship is shown as in Figure 2(b). (28)

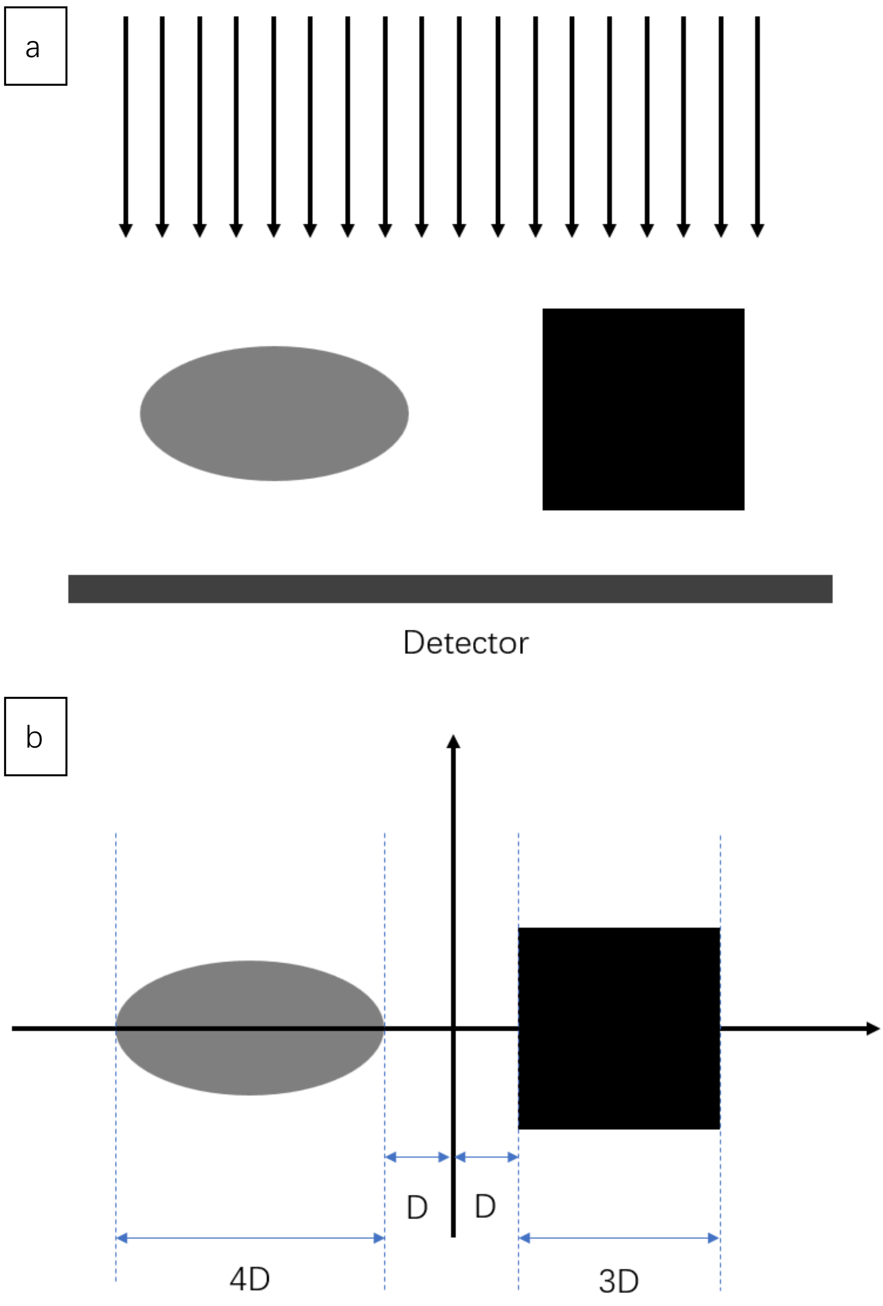
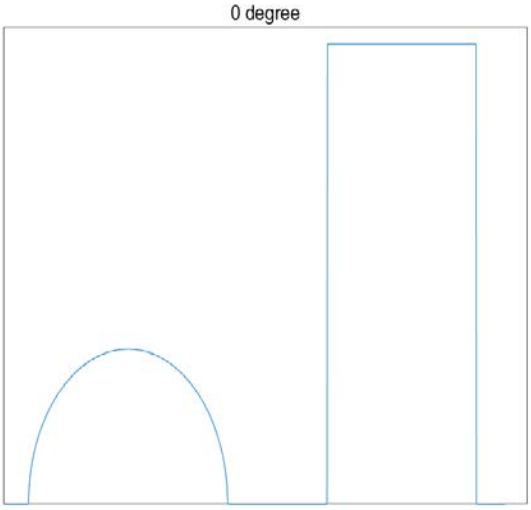


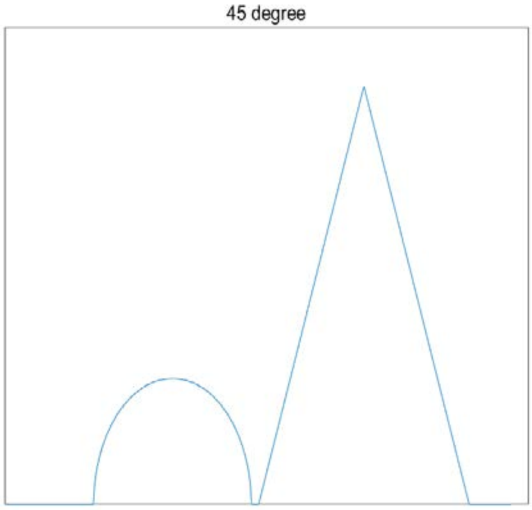
Figure 2

Solution: (4 pts for each projection and 8pts for sinogram)

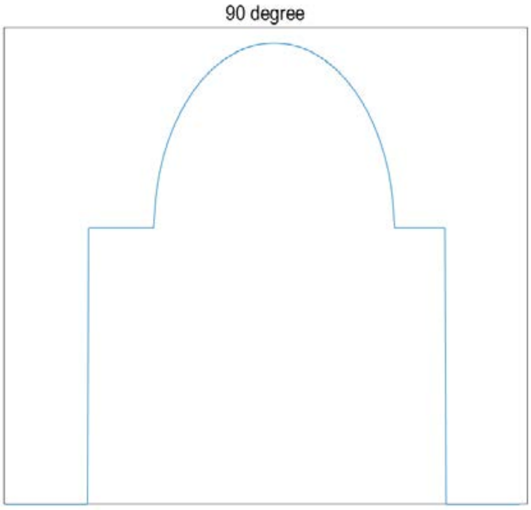
0 degree



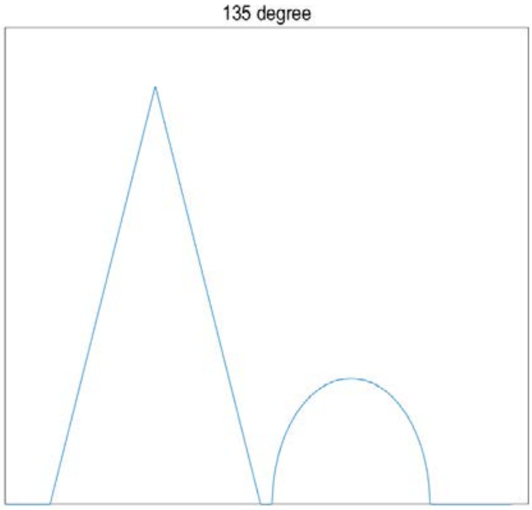
45 degree



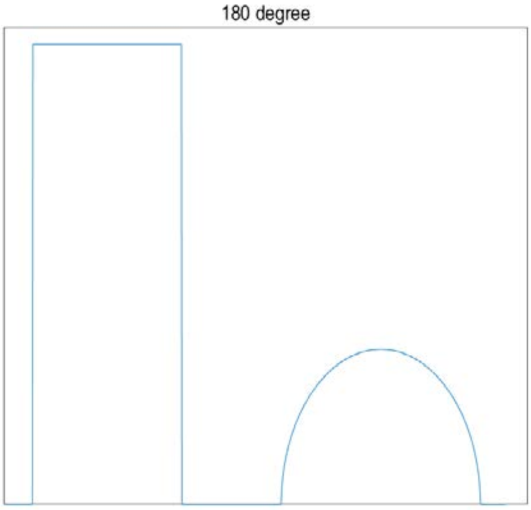
90 degree

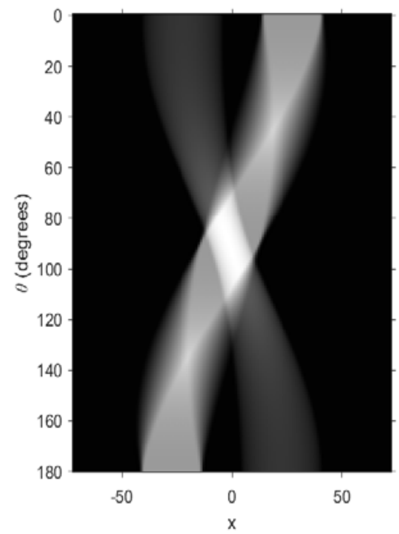


135 degree



180 degree





3. In mammographic examinations, the breast is compressed between two plates. Answer the following with a brief explanation (24)
   (a) Is the geometric unsharpness increased or decreased by compression?
   (b) Why is the image contrast improved by this procedure?
   (c) Is the required X-ray dose for a given image SNR higher or lower with compression?

   Solution: 8pts for each, 3pts(answer) and 5pts(explanation)

   (a) Compression of the breast reduces the object to detector distance (S1- S0), but also the object to source distance (S0). However, the former is reduced to a greater degree. Therefore, the size of the penumbra will decrease, and the geometric unsharpness becomes less.

   (b) The image contrast is degraded by Compton scattered X-rays originating in the breast. Compression makes the object thinner in the direction of X-ray transmission, and therefore the number of Compton scattered X-rays decreases, and image contrast increases.

   (c) Firm compression reduces overlapping anatomy, decreases tissue thickness, it results in fewer scattered X-rays, less geometric blurring of anatomic structures, and lower radiation dose to the breast tissues. So, for a given image SNR, when with compression, the required dose decreases.

4. Given two different tissues *a* and *b*, two different detector sizes are used as indicated in the Figure 4. In the first case the detector is twice as large as in the second case. (24)
   a) Calculate the linear attenuation coefficients μ_a, μ_b and μ_{a+b} from the input intensity I_i and the output intensities I_o, I_{oa} and I_{ob}.
   b) Show that μ_{a+b} is always an underestimate of the mean linear attenuation $\frac{(\mu_a + \mu_b)}{2}$.
   c) What is the influence of this underestimate on a reconstructed CT image? Explain.

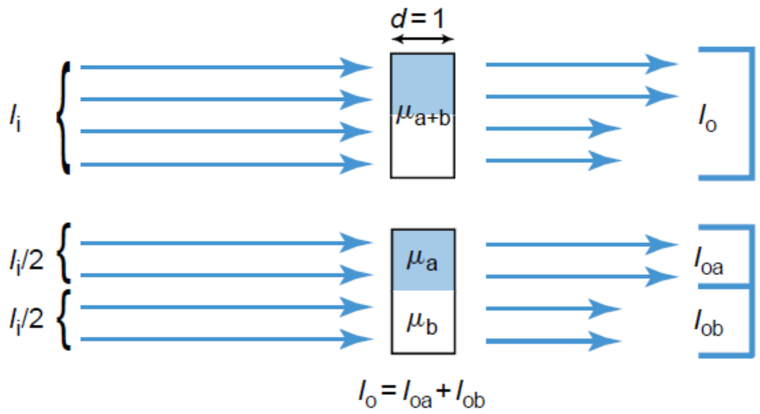


Figure 4

Solution:

a) (8pts)

From the figure, we have:

$$\mathrm{I_{oa}} = \frac{1}{2} I_i * e^{-\mu_a * d}$$

$$\mathrm{I_{ob}} = \frac{1}{2} I_i * e^{-\mu_b * d}$$

$$\mathrm{I_o} = I_{oa} + I_{ob}$$

$$\mathrm{I_o} = I_i * e^{-\mu_{a+b} * d}$$

Such that with d=1

$$\mu_{\mathrm{a}} = \log \frac{I_i}{2 I_{oa}}$$

$$\mu_{\mathrm{b}} = \log \frac{I_i}{2 I_{ob}}$$

$$\mu_{\mathrm{a+b}} = \log \frac{I_i}{I_{oa} + I_{ob}}$$

b) (8pts)

$$\begin{aligned}\frac{1}{2}(\mu_a + \mu_b) &= \frac{1}{2}\left(\log \frac{I_i}{2 I_{oa}} + \log \frac{I_i}{2 I_{ob}}\right) \\ &= \frac{1}{2} * \log \frac{I_i^2}{4 I_{oa} I_{ob}} \\ &= \log \frac{I_i}{2\sqrt{I_{oa} I_{ob}}} \\ &\geq \log \frac{I_i}{I_{oa} + I_{ob}} = \mu_{a+b}\end{aligned}$$

c) (8pts, 3pts(answer) and 5pts(explanation))

Nonlinear partial volume effect.

From the result of (b), when using the average attenuation $\mu_{\mathrm{a+b}}$ along the beam width to calculate the beam intensity after passing through the matter, there is a lower intensity received from detectors. It means that in the case of $\mathrm{I_o} = \mathrm{I_{oa}} + \mathrm{I}_{ob}$, the actual attenuation value is greater than the measured attenuation value with assumption of average. Therefore, it will be an underestimation of the actual integrated averaged attenuation.