

# CS244 Theory of Computation

## Homework 4

Due: November 12, 2020 at 11:59 p.m.

Name - ID

You may discuss this assignment with other students and work on the problems together. However, your write-up should be your own individual work and you should indicate in your submission who you worked with, if applicable. You should use the L<sup>A</sup>T<sub>E</sub>X template provided by us to write your solution and submit the generated PDF file into Gradescope.

I worked with: (Name, ID), (Name, ID), ...

Let  $\Sigma = \{0, 1\}$  if not otherwise specified.

### Problem 1

Say that a variable  $A$  in CFG  $G$  is **redundant** if removing it and its associated rules leaves  $L(G)$  unchanged. Let  $REDUNDANT_{CFG} = \{\langle G, A \rangle \mid A \text{ is a redundant variable in } G\}$ . Show that  $REDUNDANT_{CFG}$  is **undecidable**.

### Problem 2

Consider the problem of testing whether a TM accepts the empty string  $\epsilon$ . Formally, let  $EPSILON_{TM} = \{\langle T \rangle \mid T \text{ is a TM that accept } \epsilon\}$ .

- (a) Use a **reduction** or the **recursion theorem** to prove that  $EPSILON_{TM}$  is **undecidable**.
- (b) Answer YES or NO and give a brief reason for your answer.  
Is  $A_{TM}$  **mapping reducible** to  $\overline{EPSILON_{TM}}$ ?

### Problem 3

Show that  $EQ_{TM} \not\leq_m \overline{EQ_{TM}}$ .

### Problem 4

Let  $SET-SPLITTING = \{\langle S, C \rangle \mid S \text{ is a finite set and } C = \{C_1, \dots, C_k\} \text{ is a collection of subsets of } S, \text{ where the elements of } S \text{ can be colored red or blue so every } C_i \text{ has at least one red element and at least one blue element}\}$ . Show that  $SET-SPLITTING$  is **NP-complete**.