SI231b - Matrix Computations, 2020-21 Spring

Homework Set #4

Prof. Ziping Zhao

Notice:

1) Deadline: **2021-04-28 23:59:59**
2) Submit your homework in pdf format to Email: zhangzp1@shanghaitech.edu.cn.
3) You can write your homework using LaTeX/Word, or you can write in handwriting and submit the scanned pdf.
4) You need to submit the code for Problem 5 as well.

Problem 1. (20 points) Compute the QR factorization of $\mathbf{A} = \begin{bmatrix} 0 & \sqrt{3} & 4 \\ 0 & 1 & 5 \\ 1 & 2 & -3 \end{bmatrix}$ through Householder reflection. (Hint: You need to show the procedure step by step.)

Problem 2. (10 points) Prove that for a matrix $\mathbf{A} \in \mathbb{C}^{n \times n}$, $\mathsf{rank}(\mathbf{A}) < n$ (i.e., rank-deficient) if and only if 0 is an eigenvalue of $\mathbf{A}$.

Problem 3. (20 points) Prove that for a matrix $\mathbf{A} \in \mathbb{C}^{n \times n}$, $\mathsf{rank}(\mathbf{A}) \geq$ number of nonzero eigenvalues of $\mathbf{A}$ and the equality holds if $\mathbf{A}$ admits an eigendecomposition.

Problem 4. (10 points) Prove that for orthogonal iteration (see pages 5-52 ~ 5-55 of the slides) we have $\mathcal{R}(\mathbf{V}^{(k)}) = \mathcal{R}(\mathbf{A}^k \mathbf{v}^{(0)})$.

Problem 5. (40 points) Given the matrix $\mathbf{A} = \begin{bmatrix} 7 & 2 \\ 2 & 4 \end{bmatrix}$,

1) find the eigenvalues and eigenvectors of $\mathbf{A}$ by hand; (10 points)
2) program the power iteration method (see pages 5-45 ~ 5-47 of the slide) and compute all the eigen-pairs of $\mathbf{A}$; (15 points)
3) program the QR iteration method (see pages 5-56 ~ 5-66 of the slide) and compute all the eigen-pairs of $\mathbf{A}$. (15 points)