Lecture 10: Course Review (Revised Version)

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Final Exam

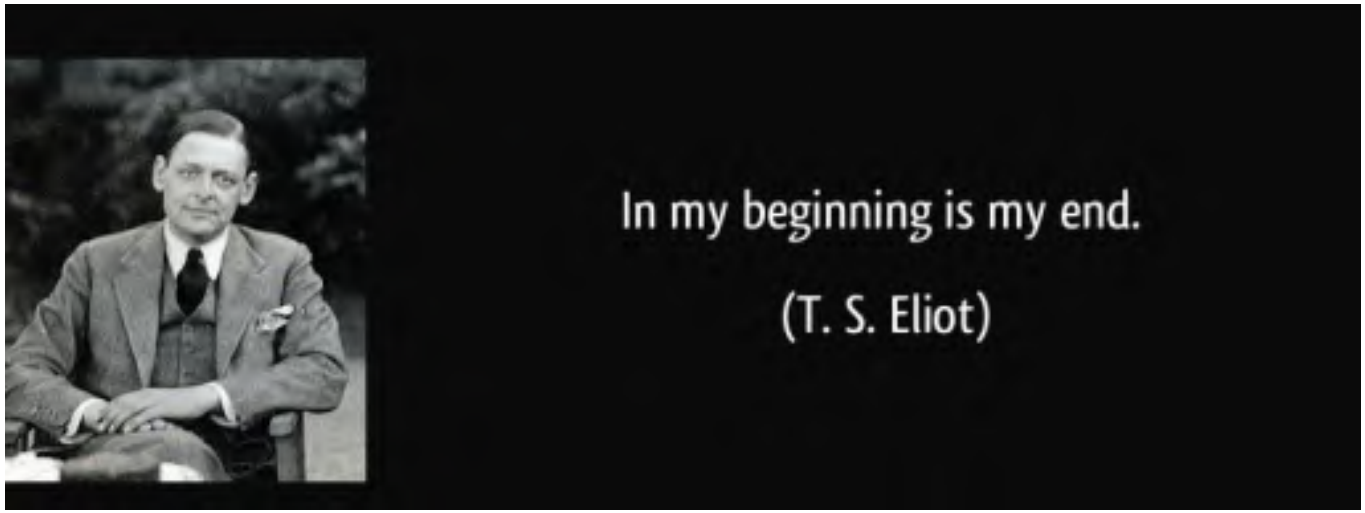
- Nine problems
- Closed book without cheating sheets
- 9:00am-12:00pm in the morning of Dec. 29

Course Reward

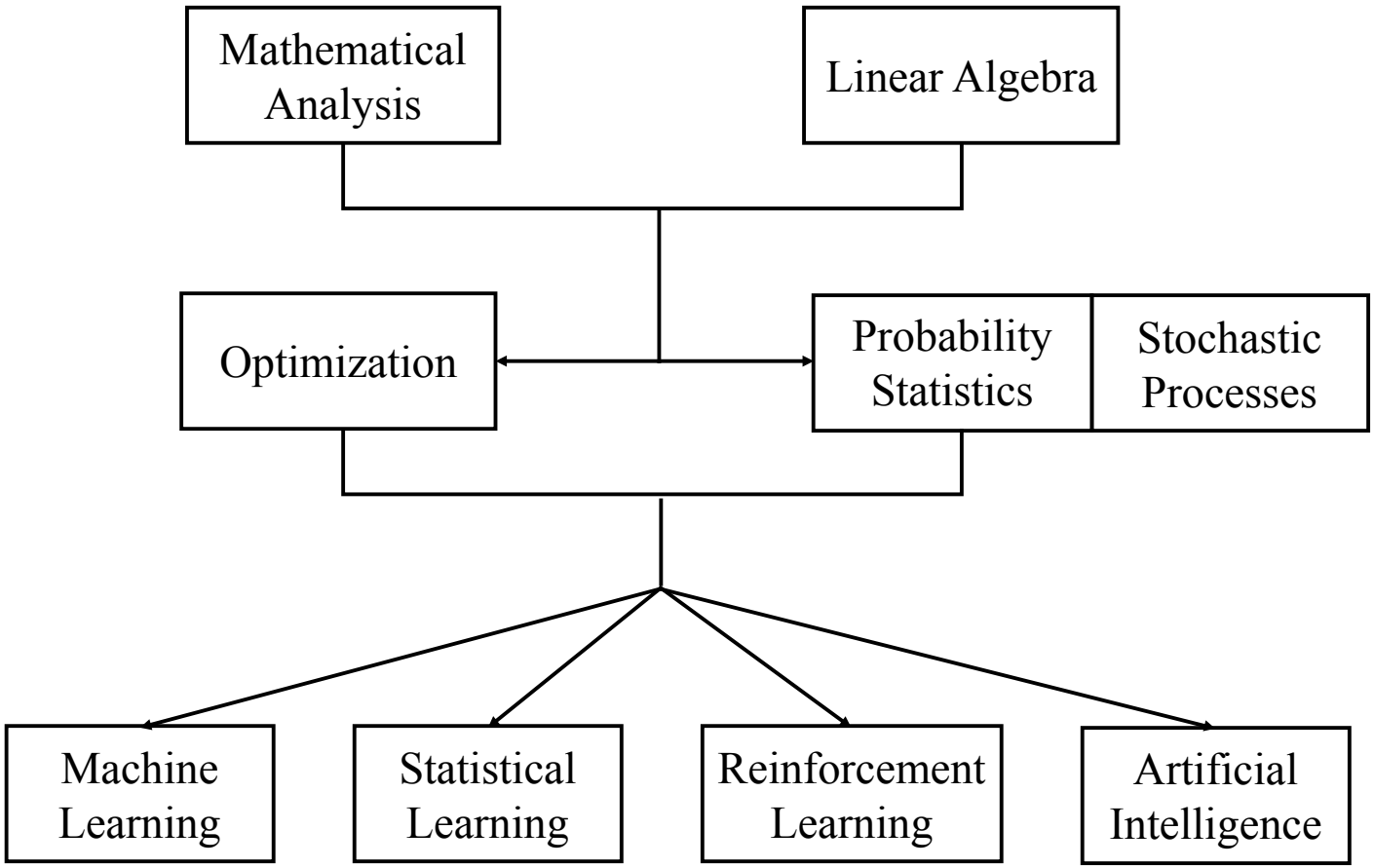
- Gauss Award
- Markov Award
- Bernoulli Award
- Poisson Award
- Fisher Award

Final Exam

In My Beginning is My End



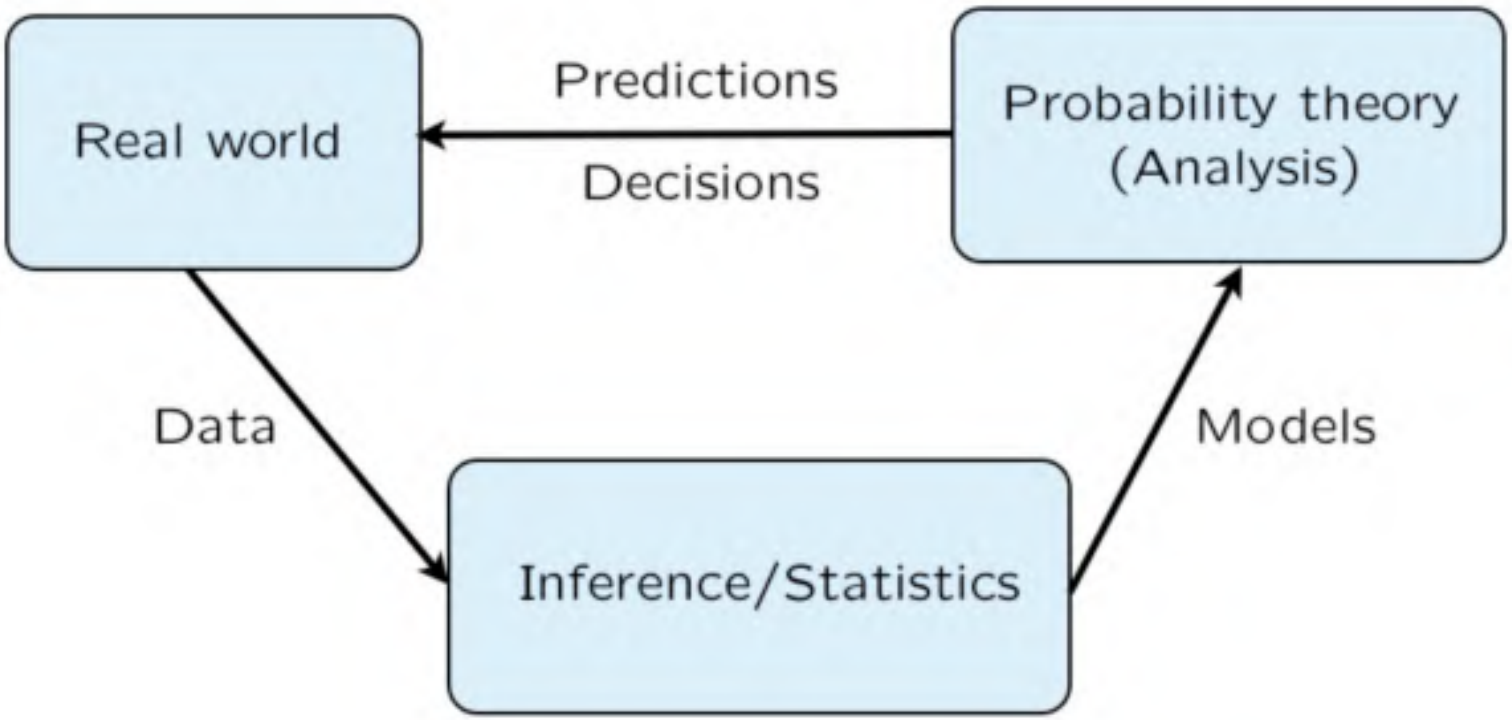
The Role of This Course



The Role of Probability & Statistics

A framework for analyzing phenomena with uncertain outcomes:

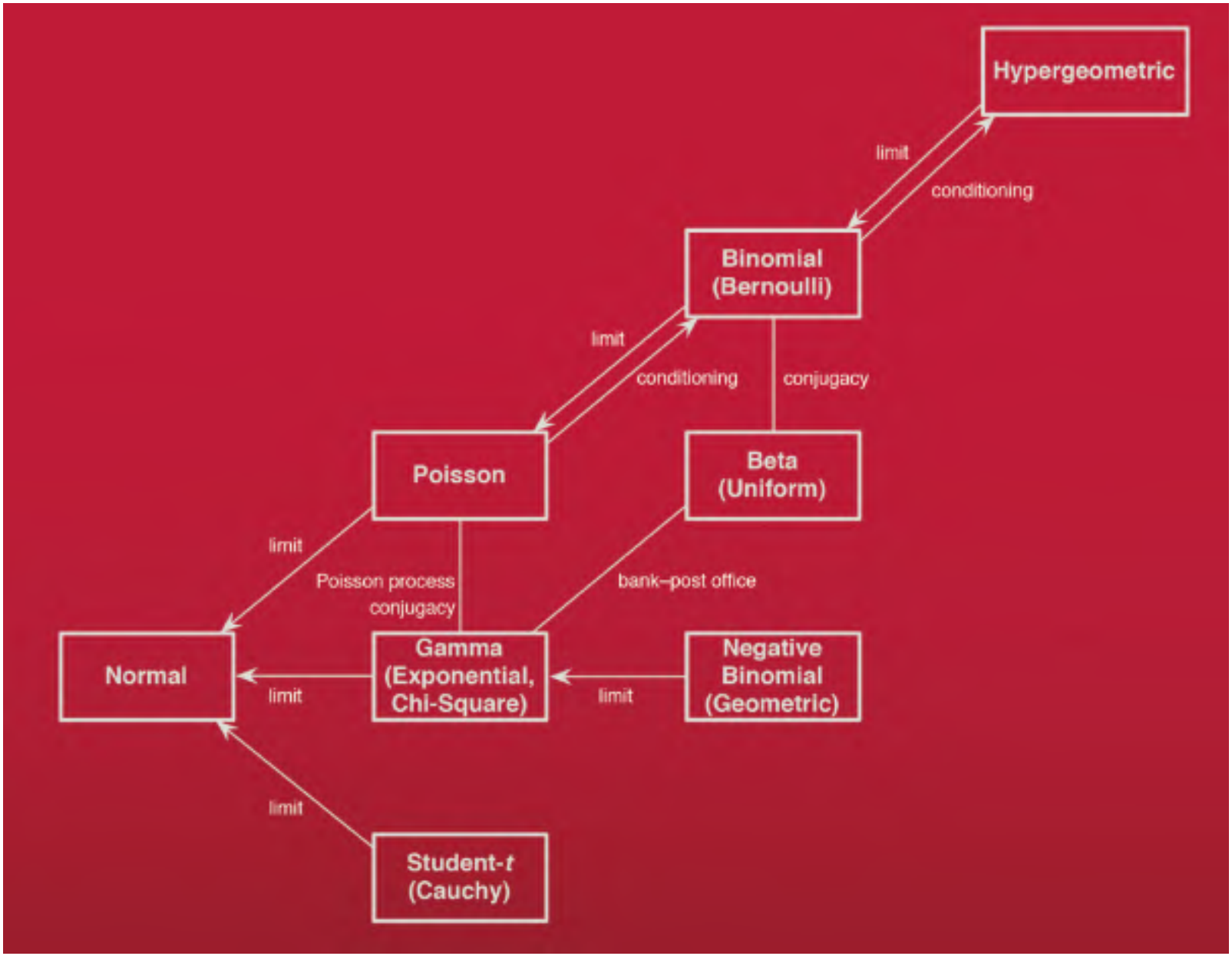
- Rules for consistent reasoning
- Used for predictions and decisions



Typical Distributions

Name	Param.	PMF or PDF	Mean	Variance
Bernoulli	p	$P(X=1)=p, P(X=0)=q$	p	pq
Binomial	n,p	$\binom{n}{k}p^k q^{n-k}$, for $k \in \{0,1,\dots,n\}$	np	npq
FS	p	pq^{k-1}, for $k \in \{1,2,\dots\}$	$1/p$	q/p^2
Geom	p	pq^k, for $k \in \{0,1,2,\dots\}$	q/p	q/p^2
NBinom	r,p	$\binom{r+n-1}{r-1}p^r q^n, n \in \{0,1,2,\dots\}$	rq/p	rq/p^2
HGeom	w,b,n	$\frac{\binom{w}{k}\binom{b}{n-k}}{\binom{w+b}{n}}$, for $k \in \{0,1,\dots,n\}$	$\mu = \frac{nw}{w+b}$	$(\frac{w+b-n}{w+b-1})n\frac{\mu}{n}(1-\frac{\mu}{n})$
Poisson	λ	$\frac{e^{-\lambda}\lambda^k}{k!}$, for $k \in \{0,1,2,\dots\}$	λ	λ
Uniform	$a<b$	$\frac{1}{b-a}$, for $x \in (a,b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Normal	μ,σ^2	$\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/(2\sigma^2)}$	μ	σ^2
Log-Normal	μ,σ^2	$\frac{1}{x\sigma\sqrt{2\pi}}e^{-(\log x-\mu)^2/(2\sigma^2)}, x>0$	$\theta = e^{\mu+\sigma^2/2}$	$\theta^2(e^{\sigma^2}-1)$
Expo	λ	$\lambda e^{-\lambda x}$, for $x>0$	$1/\lambda$	$1/\lambda^2$
Gamma	a,λ	$\Gamma(a)^{-1}(\lambda x)^a e^{-\lambda x}x^{-1}$, for $x>0$	a/λ	a/λ^2
Beta	a,b	$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1}$, for $0<x<1$	$\mu = \frac{a}{a+b}$	$\frac{\mu(1-\mu)}{a+b+1}$
Chi-Square	n	$\frac{1}{2^{n/2}\Gamma(n/2)}x^{n/2-1}e^{-x/2}$, for $x>0$	n	$2n$
Student-t	n	$\frac{\Gamma((n+1)/2)}{\sqrt{n\pi}\Gamma(n/2)}(1+x^2/n)^{-(n+1)/2}$	0 if $n>1$	$\frac{n}{n-2}$ if $n>2$

Relationship Among Distributions



Basic Contents: Part I

- Probability and Counting: Definition of Probability, Counting, Bose-Einstein Model, Birthday Match Problem, Hash Table.
- Conditional Probability: Bayes'rule, LOTP, Conditioning, Gambler's ruin, Simpson's Paradox, Monty Hall.
- Random Variables and Distributions: Bernoulli, Binomial, Story for distributions, entropy.
- Expectation: Indicator r.v., Geometric, Coupon Collector, Poisson, Probability Generating Function.
- Continuous Random Variables: Universality of the Uniform, Normal, Exponential, Memoryless, Moment Generating Function. Central Limit Theorem.

Basic Contents: Part II

- Joint Distributions: Joint/Marginal distribution, chicken-egg model, Poisson, meaning of conditioning on zero-probability event, Four Forms of Bayes' Rule, Four Forms of LOTP, Poisson clock, Covariance, Correlation, Multivariate Normal Distribution.
- Transformations: Change of Variables, Jacobian Matrix, Bivariate Normal Joint PDF, Convolution, Order Statistics, Beta-Binomial Conjugacy, Dirichlet-Multinomial Conjugacy, Bayesian Ranking, Gamma, Gamma-Poisson Duality, Bank–post Office model.

Basic Contents: Part III

- Classical Statistical Inference: Maximum Likelihood Estimation, Confidence Interval.
- Bayesian Statistical Inference: Statistical Inference, Bayesian Inference and the Posterior Distribution, The Maximum A Posteriori Probability (MAP) Rule, Bayesian Estimation, Linear Least Square Estimate (LLSE), Minimum Mean Square Error Estimator (MMSE), Orthogonality Property of MMSE, MMSE and LLSE, MMSE for Jointly Gaussian Random Variables, Kalman Filter.
- Conditional Expectation: Conditional Expectation Given An Event , Conditional Expectation Given An R.V., LOTE, Adam' s Law, Eve' s law, Linear Regression, Projection Interpretation, Geometric Perspective, Prediction Perspective (MMSE).

Basic Contents: Part IV

- Monte Carlo Statistical Methods: Random Variable Generation, Acceptance-Rejection Method, Monte Carlo Integration, Importance Sampling, Law of Large Numbers, Cauchy-Schwarz Inequality, Jensen's Inequality, Kullback-Leibler Divergence (Entropy), Markov's Inequality, Chebyshev's Inequality, Chernoff's Inequality (related to MGF), Chernoff's Technique, Hoeffding Lemma, Hoeffding Bound, Parameter Estimation (confidence interval), Monte Carlo Method for Estimation π,

In My End is My Beginning

- Now this is not the end. It is not even the beginning of the end.
- But it is, perhaps, the end of the beginning.

In My End is My Beginning

昔年曾见此湖图，

不信人间有此湖。

今日打从湖上过，

画工还欠费工夫。

In My End is My Beginning