Lecture 14
-- Laplace Transform in Circuit Analysis

V-I relations of R,L,C

- R

$$U_R(s) = RI_R(s)$$

- C

$$\boxed{V(s) = \frac{1}{sC} I(s) + \frac{V_0}{s}}$$

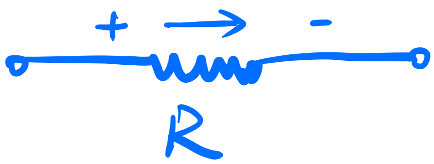
$$\boxed{I(s) = sCV(s) - CV_0}$$

- L

$$\boxed{I(s) = \frac{1}{sL} V(s) + \frac{I_0}{s}}$$

$$\boxed{V(s) = sLI(s) - LI_0}$$

- R $\qquad U_R(s) = RI_R(s)$



T.D. : $v(t) = R \cdot i(t)$

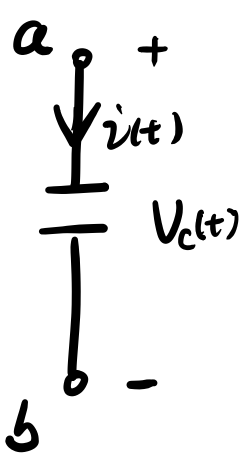
$\downarrow \qquad \downarrow$

S.D. : $V(s) = R \cdot I(s)$

- C

$$\boxed{V(s) = \frac{1}{sC} I(s) + \frac{V_0}{s}}$$

$$\boxed{I(s) = sCV(s) - CV_0}$$



$$i(t) = C \cdot \frac{dV_c(t)}{dt}$$

$\downarrow \mathcal{L}$

$$I(s) = C \cdot [sV_c(s) - V_c(0_-)]$$
$$= C \cdot [sV(s) - V_0] \quad ①$$

$$\Rightarrow V(s) = \frac{1}{sC} \cdot I(s) + \frac{V_0}{s} \quad ②$$

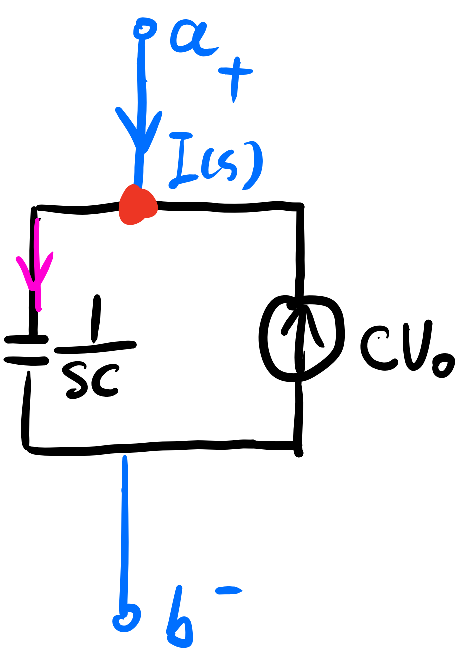
$$V_c(t) = V_0 + \frac{1}{C}\int i(t)\,dt$$

$\downarrow \mathcal{L}$

$$V(s) = \frac{V_0}{s} + \frac{1}{Cs} \cdot I(s)$$

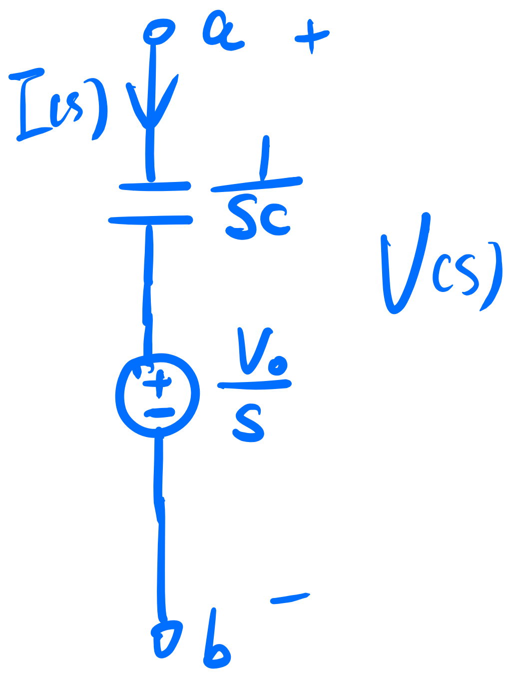
From

$$I(s) = C \cdot [s V(s) - V_0]$$

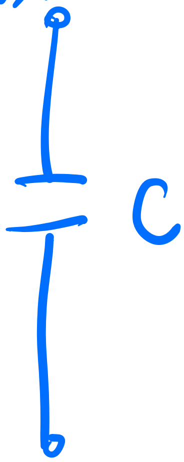


$\Longrightarrow$

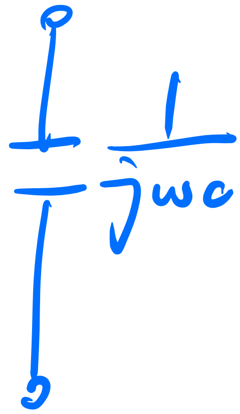
①



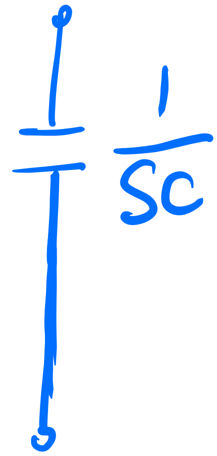
T.D.



Phasor. D.



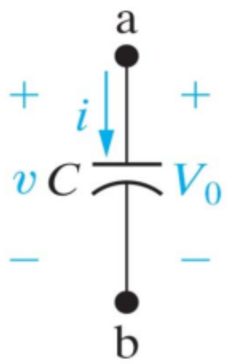
S.D. with $V_0=0$



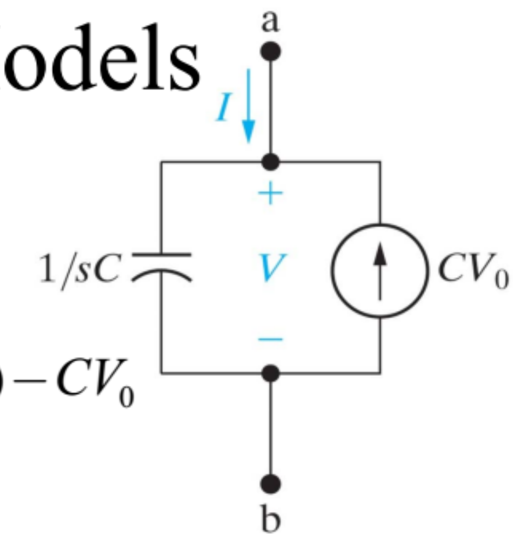
S-domain circuit models for a capacitor

s-Domain Circuit Models

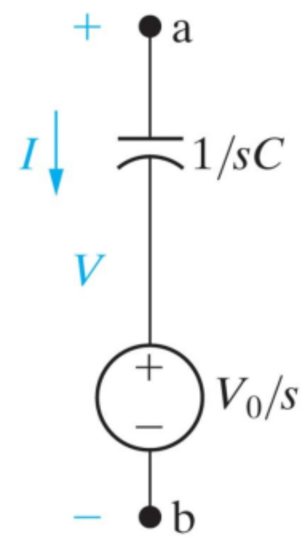
For a capacitor
(with initial conditions)



$$i(t)=C\frac{dv(t)}{dt} \Rightarrow I(s)=sCV(s)-CV_0$$



$$V(s)=\frac{1}{sC}I(s)+\frac{V_0}{s}$$



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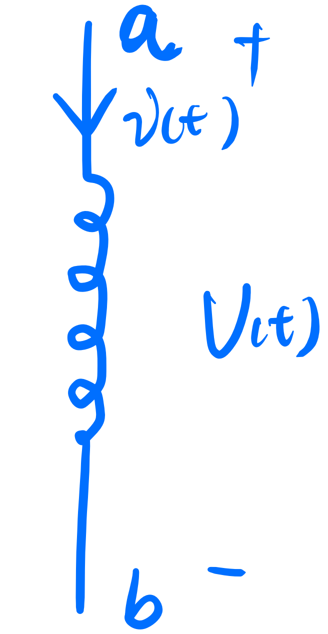
T.D.



$$V(t) = L \cdot \frac{d\, i(t)}{dt}$$

$$\xrightarrow{\mathcal{L}}$$

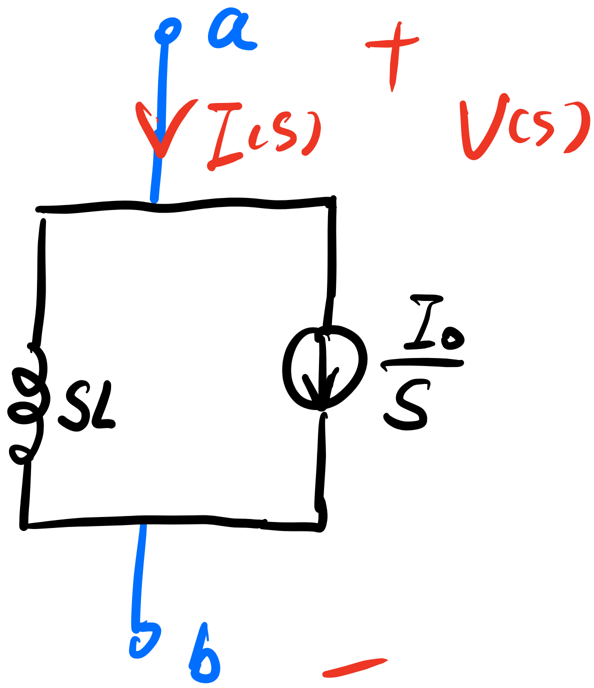
$$V(s) = L[s \cdot I(s) - i(0_-)]$$

$$= L \cdot s \cdot I(s) - L I_0 \quad ①$$

$$I(s) = \frac{1}{sL} \cdot V(s) + \frac{I_0}{s} \quad ②$$

$$V(s) = L \cdot s \cdot I(s) - L I_0 \quad ①$$

$$\xRightarrow{S.T.}$$



T.D.

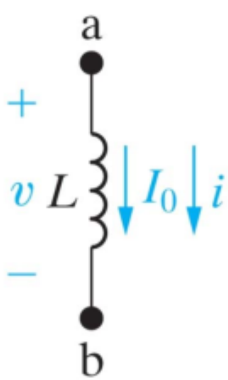
Phasor D.

S.D. with $I_0 = 0$

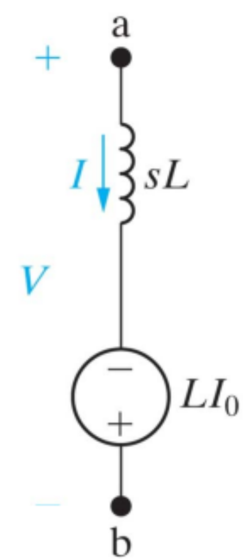
S-domain circuit models for an inductor

s-Domain Circuit Models

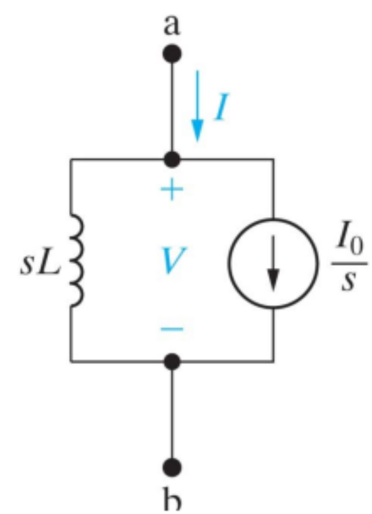
For an inductor
(with initial conditions)

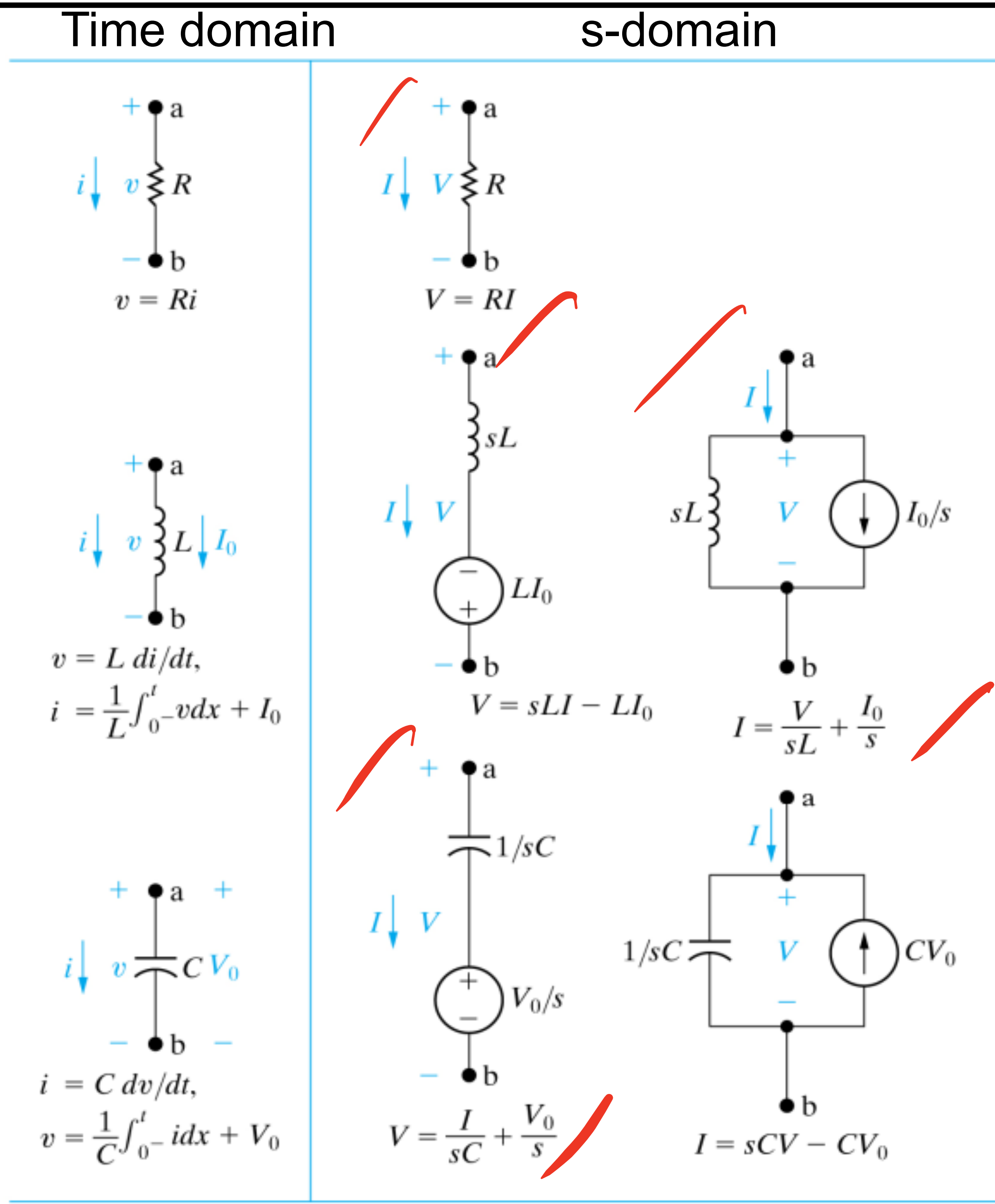


$$v(t) = L\frac{di(t)}{dt} \Rightarrow V(s) = sLI(s) - LI_0$$

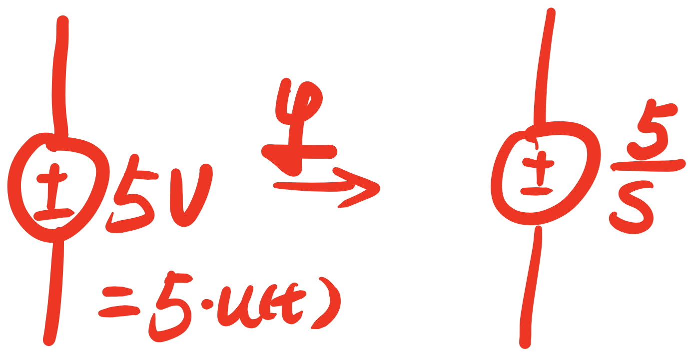


$$I(s) = \frac{1}{sL}V(s) + \frac{I_0}{s}$$




Time domain
s-domain
$v = Ri$
$V = RI$
$v = L\,di/dt,$
$i = \frac{1}{L}\int_{0^-}^{t} v\,dx + I_0$
$V = sLI - LI_0$
$I = \frac{V}{sL} + \frac{I_0}{s}$
$i = C\,dv/dt,$
$v = \frac{1}{C}\int_{0^-}^{t} i\,dx + V_0$
$V = \frac{I}{sC} + \frac{V_0}{s}$
$I = sCV - CV_0$

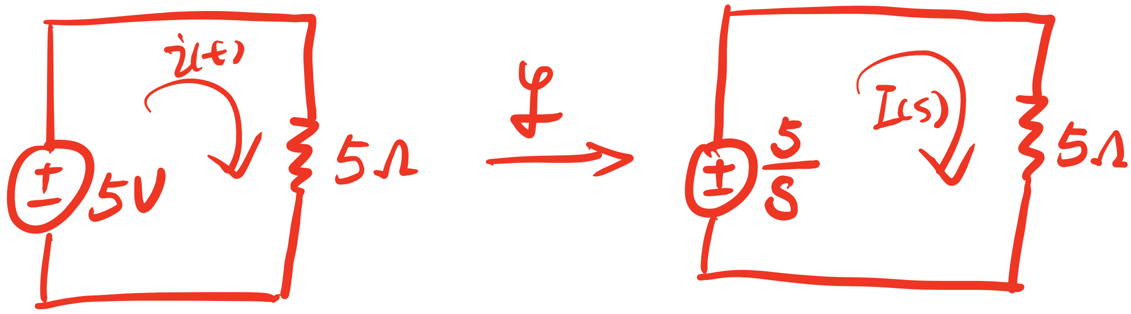
D.C. sources and Dependent Sources



- The models for dependent sources are easy to develop, drawing from the simple fact that if the Laplace transform of *f(t)* is *F(s)*, then the Laplace transform of *af*(*t*) is *aF*(*s*) — the linearity property.

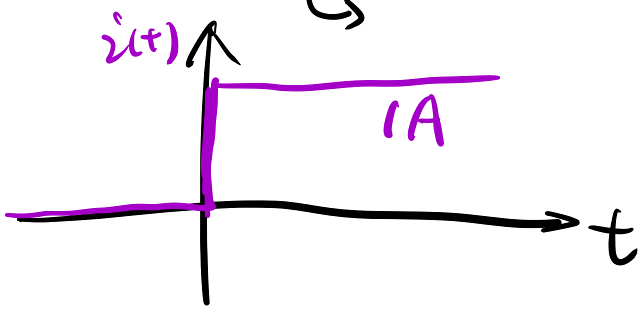
$$\mathcal{L}[av(t)] = aV(s)$$

$$\mathcal{L}[ai(t)] = aI(s)$$



$$I(s) = \frac{5/s}{5} = \frac{1}{s}$$

$$\mathcal{L}^{-1}: \quad \underline{i(t) = u(t)} = 1 \cdot u(t) \quad \underbrace{\qquad}_{} \; t>0$$



Steps in Applying the Laplace transform

- **Transform the circuit from the time domain to the Laplace (*s*) domain, including initial conditions.**

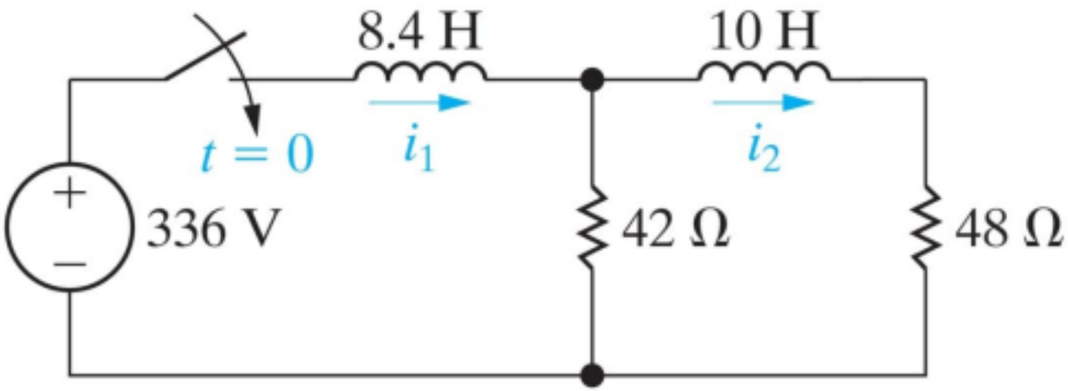
--*The elegance of using the Laplace transform in circuit analysis lies* in (1) transforming the differential equation into an *algebraic* equation; and (2) automatic inclusion of initial conditions in the transformation process, thus providing a complete (**transient and steady-state**) solution.

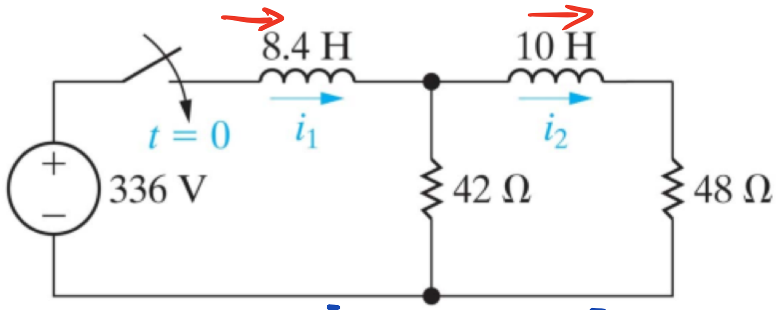
- **Solve the circuit using nodal analysis, mesh analysis, source transformation, superposition, or any other analysis technique with which we are familiar.**

- Take the inverse transform of the solution and thus obtain the solution in the time domain.

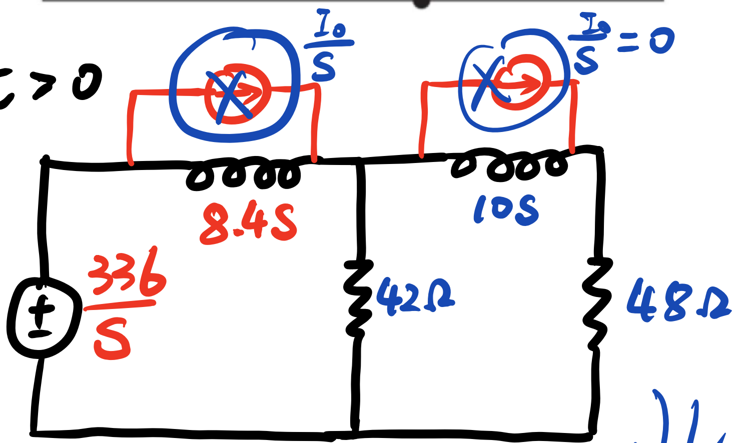
Example 1

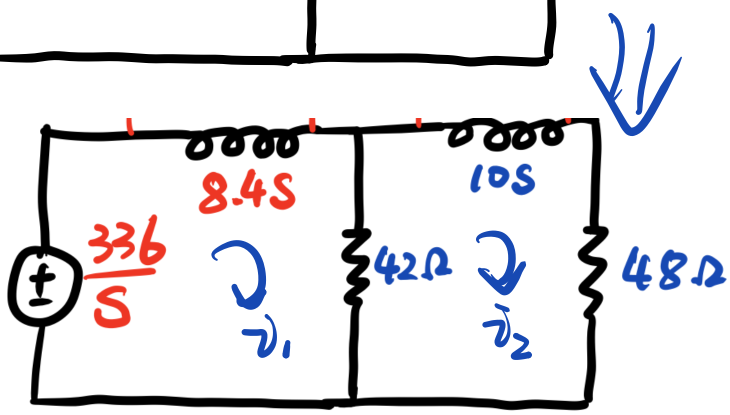
Assuming no initial energy storage, find $i_1(t)$ and $i_2(t)$ for $t > 0$.





t > 0





$$\begin{cases} \dfrac{336}{S} = 8.4 i_1 + 42(i_1 - i_2) & ① \\ 42(i_1 - i_2) = 10S \cdot i_2 + 48 i_2 & ② \end{cases}$$

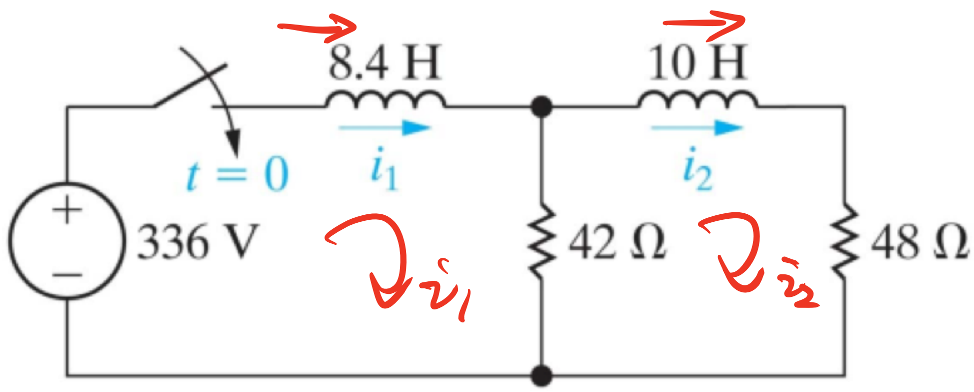
$$i_2 = \frac{168}{S^3 + 14S^2 + 24S} = \frac{168}{S(S+2)(S+12)}$$

$$= \frac{K_1}{S} + \frac{K_2}{S+2} + \frac{K_3}{S+12}$$

7 -8.4 1.4

$$\Rightarrow i_2(t) = \left[7 - 8.4e^{-2t} + 1.4e^{-12t}\right]u(t)$$

$$i_1(s) = \frac{90+10s}{42} \cdot \frac{168}{s^3+14s^2+24s}$$



$$336 = 8.4\frac{di_1}{dt} + 42(i_1 - i_2) \quad ①$$

$$42(i_1 - i_2) = 10\cdot\frac{di_2}{dt} + 48i_2 \quad ②$$

由②

$$42i_1 = 10\frac{di_2}{dt} + 90i_2$$

$$\text{"}\ i_2'' + 14i_2' + 24i_2 = 168\ \text{"}$$

$$s^2 + 14s + 24 = 0$$