Lecture 11

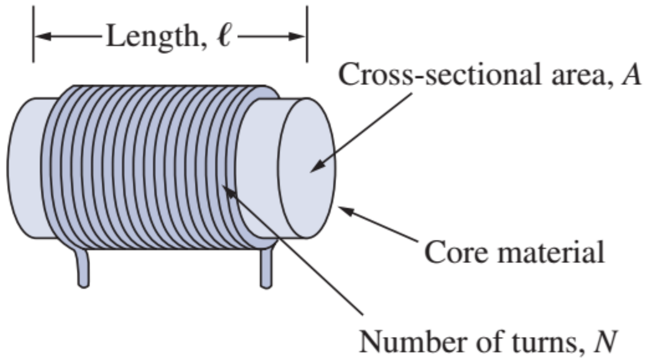
- Transformers/Three-Phase Circuits

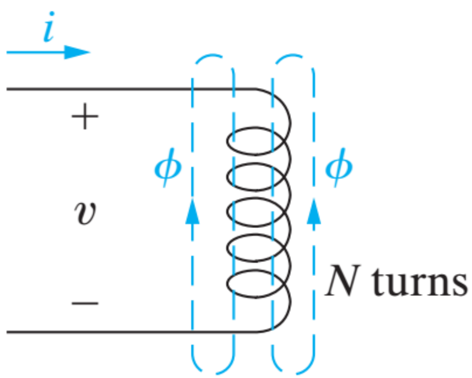
Outline

- Mutual Inductance
- Transformers

Review: Self Inductance

- Self inductance: reaction of the inductor to the change in current through itself.

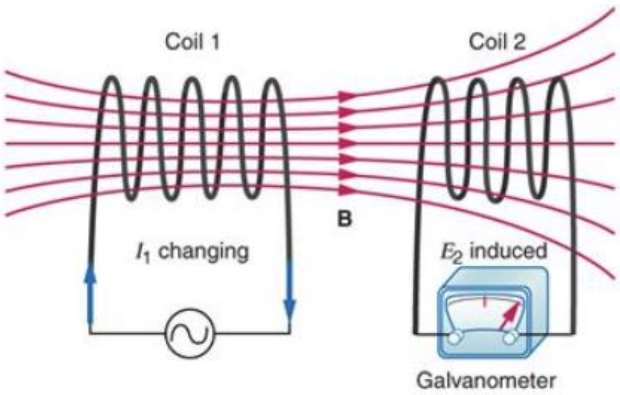


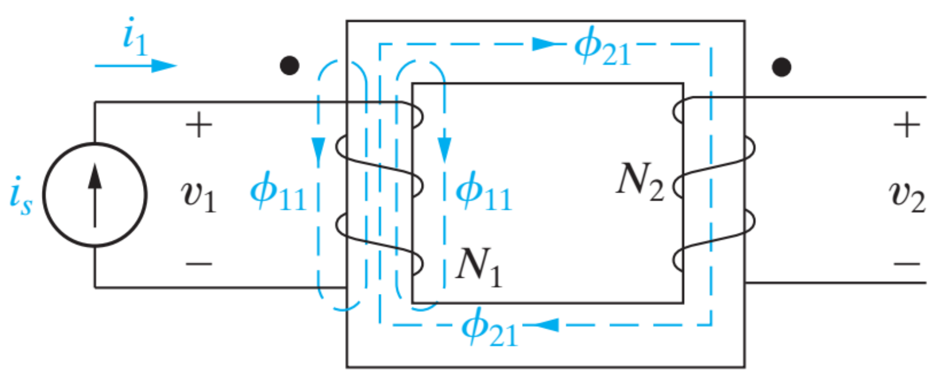


$$v = L\frac{di}{dt}$$

Mutual Inductance

- Mutual inductance: reaction of the inductor to change in current through another inductor.





$$\emptyset_{11}=L_1 i_1$$

$$\emptyset_{21}=M i_2$$

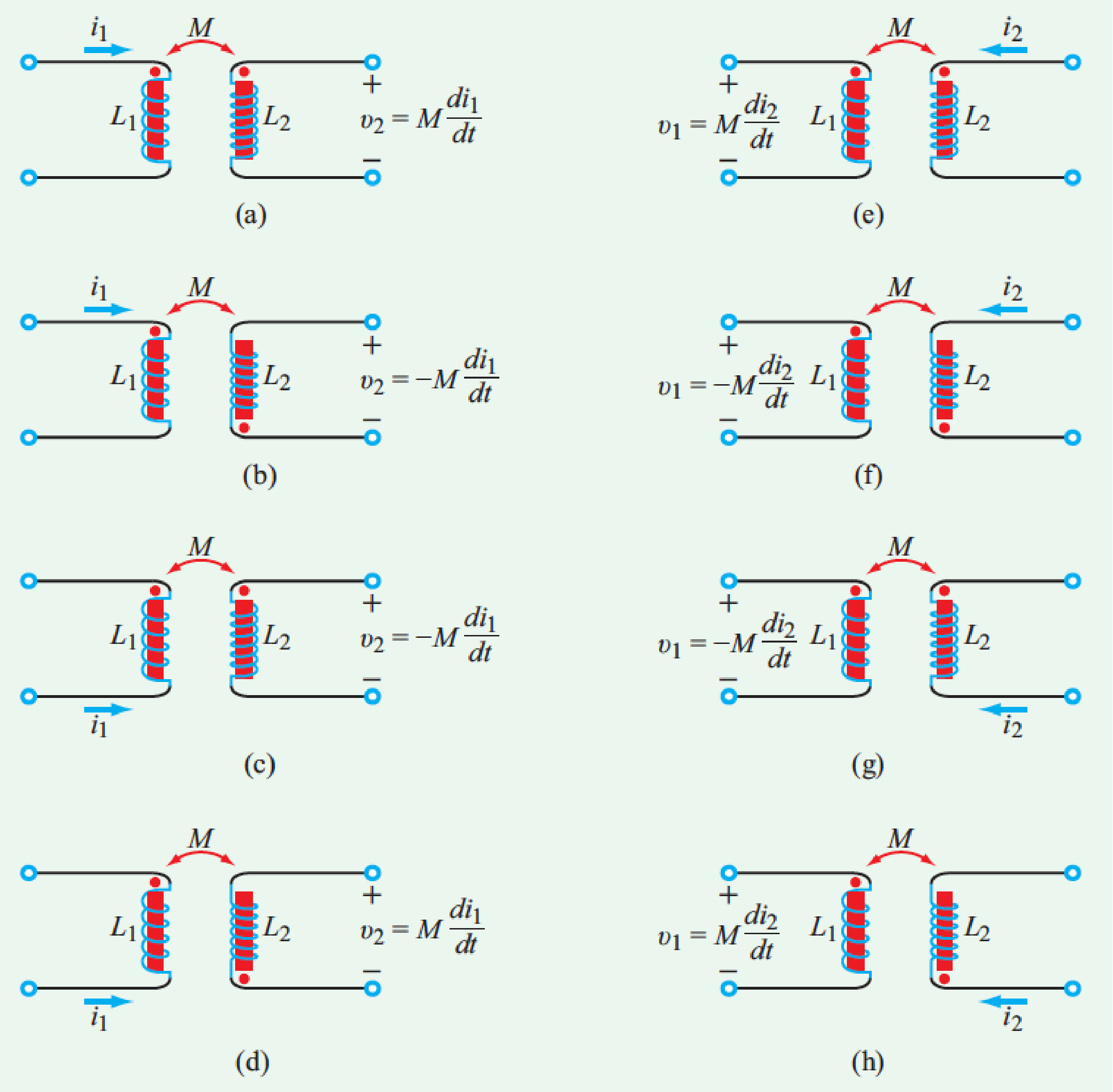
$$v_1 = \frac{d\emptyset_1}{dt}$$

$$= \frac{d\emptyset_{11}}{dt} + \frac{d\emptyset_{21}}{dt}$$

$$= L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

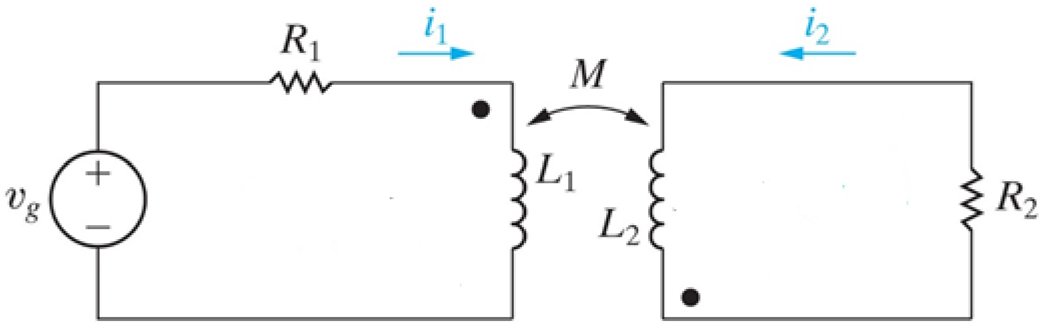
Dot Convention: Defines Directions of Windings



[Source: Berkeley]

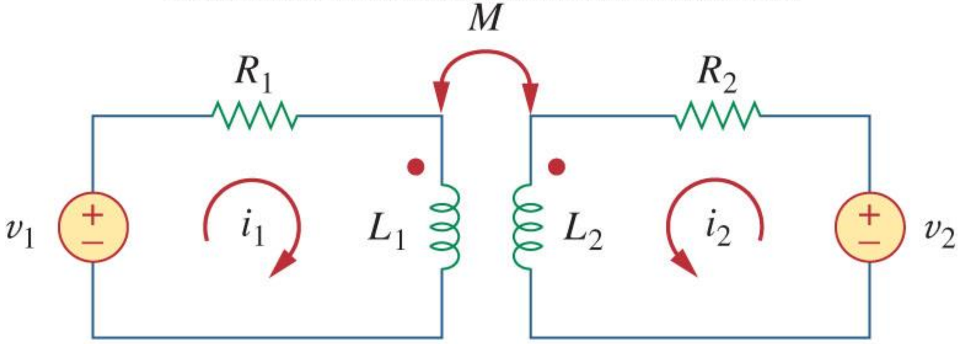
Mutual Inductance: General Case

- Two circuits lined by a magnetic field
  - L_1, L_2: self-inductances
  - M: mutual inductance
  - Dots: indicating polarity of mutually induced voltages.



Exercise

- Relate v_1, v_2 with $i_1$and i_2.
  - In time domain
  - In phasor domain



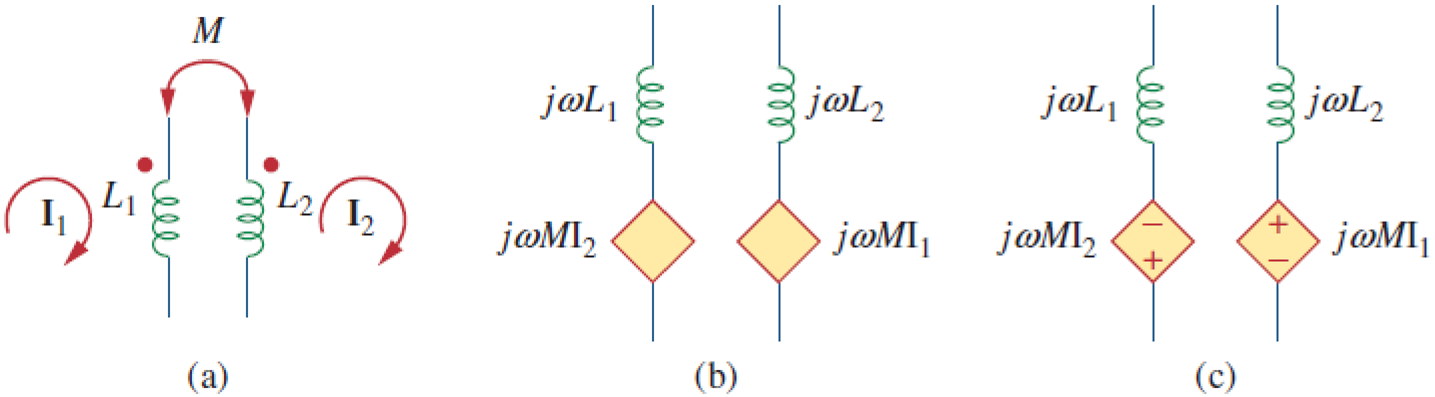
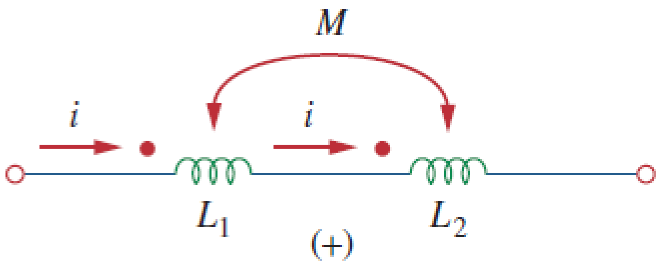
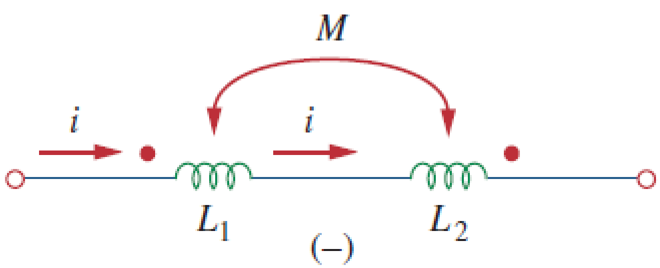


Figure 13.8
Model that makes analysis of mutually coupled easier to solve.



(a)



(b)

Calculate the mesh currents in the circuit of Fig. 13.11.

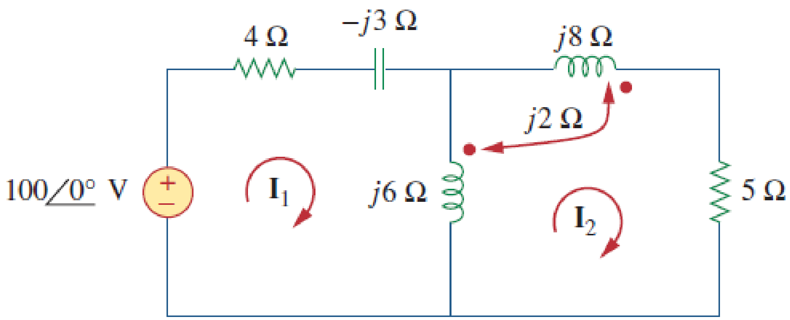
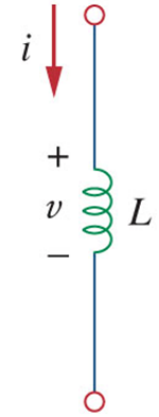


Figure 13.11
For Example 13.2.

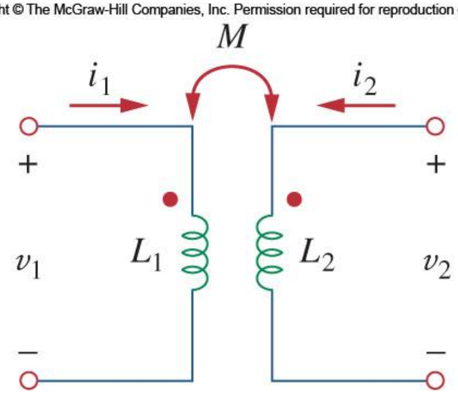
Energy in a Coupled Circuit

- The energy stored in an inductor is



- For coupled inductors, the total energy stored is

$$w = \frac{1}{2} L_1 i_1^{\,2} + \frac{1}{2} L_2 i_2^{\,2} \pm M i_1 i_2$$



  - The positive sign is selected when the currents both enter or leave the dotted terminals.

[Text, Ch. 6.5]

Coupling Coefficient *k*

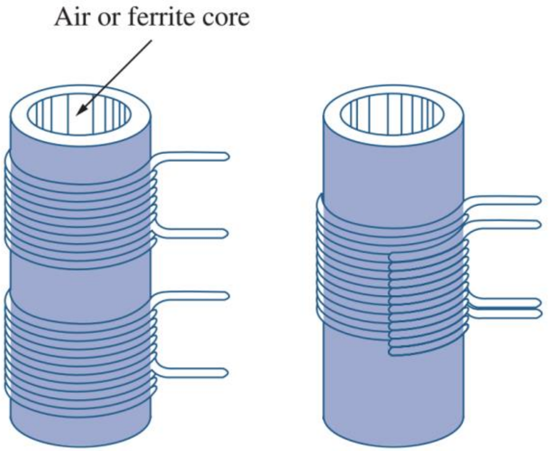
- The system cannot have negative energy

$$\frac{1}{2}L_1 i_1^{\,2} + \frac{1}{2}L_2 i_2^{\,2} - M i_1 i_2 \geq 0 \quad \Rightarrow \quad \boxed{M \leq \sqrt{L_1 L_2}}$$

- Define a parameter describes how closely M approaches upper limit.

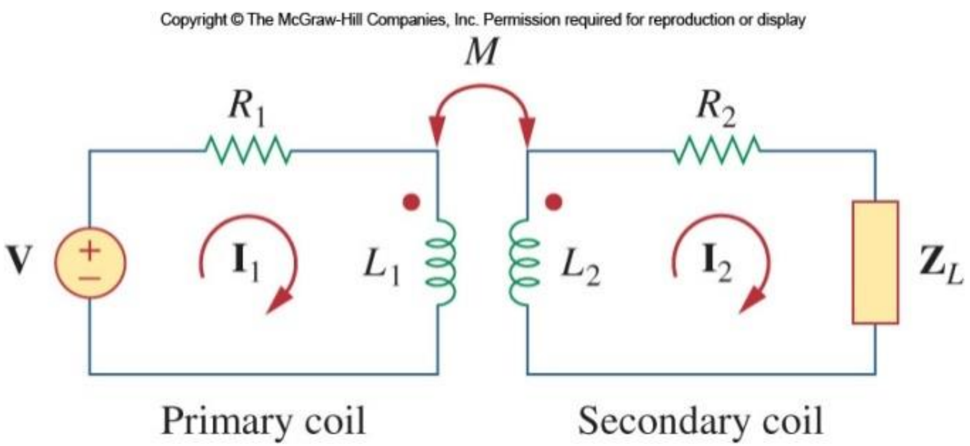
$$\boxed{k = \frac{M}{\sqrt{L_1 L_2}}}$$

  - Coupling coefficient, $0 \leq k \leq 1$.
  - determined by the physical configuration of the coils.



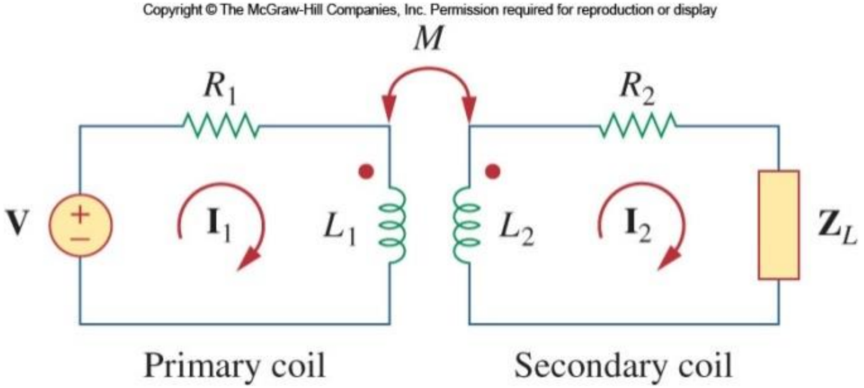
Linear Transformers

- A transformer is a magnetic device that takes advantage of mutual inductance.
  - Called linear if the coils are wound on a magnetically linear material, i.e. permeability μ is constant.



Transformer Impedance

- An important parameter to know for a transformer is how the input impedance Z_{in} is seen from the source.
  - Z_{in} is important because it governs the behavior of the primary circuit.



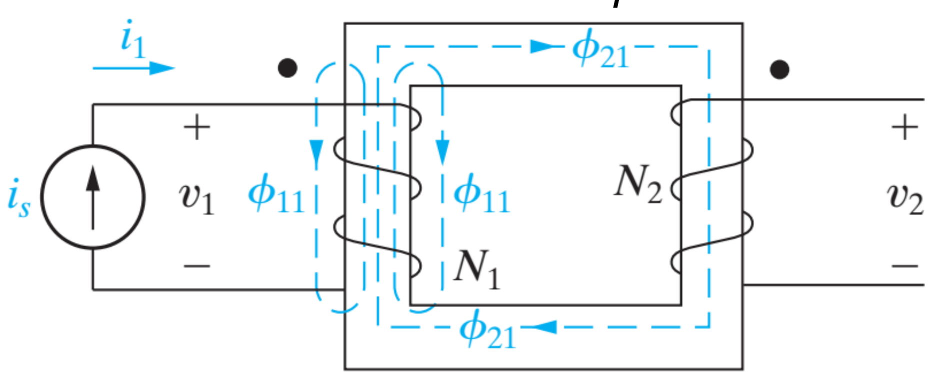
$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}}{\mathbf{I}_1} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}$$

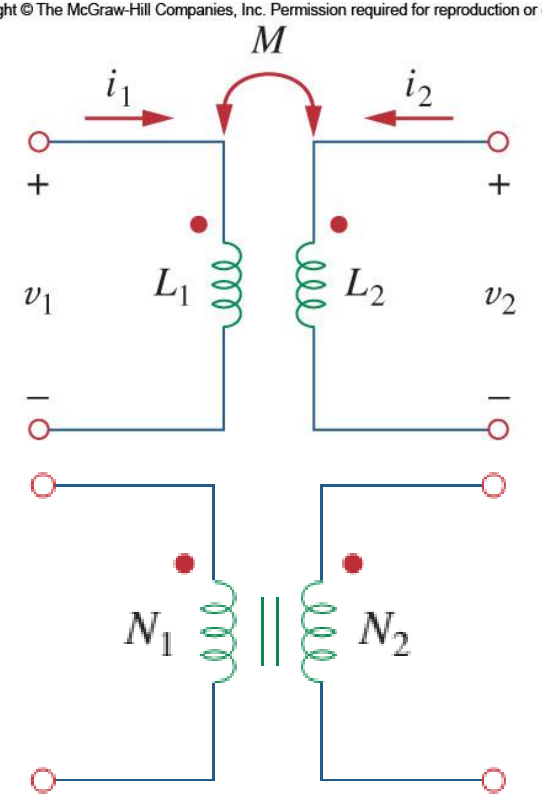
Reflected impedance from secondary to primary

Ideal Transformers

- The ideal transformer has:
  - Coils with very large reactance
    (L_1, L_2, $M \rightarrow \infty$)
  - Coupling coefficient k=1.
  - Primary and secondary coils are lossless, $R_1 = R_2 = 0$.

$$\phi = \mathcal{P}Ni$$

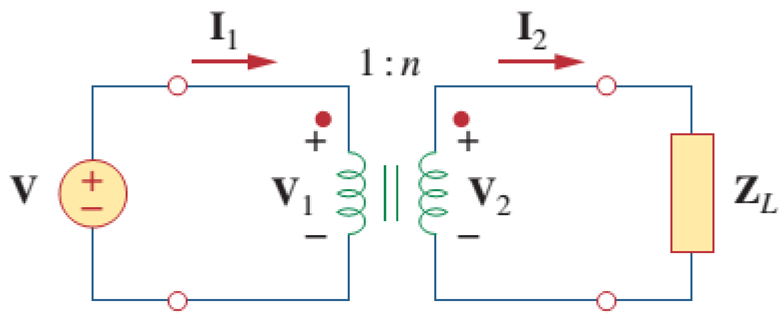




$$\frac{\mathbf{V_2}}{\mathbf{V_1}} = \frac{N_2}{N_1} = n$$

Ideal Transformers II

$$\frac{\mathbf{V_2}}{\mathbf{V_1}} = \frac{N_2}{N_1} = n$$



- The current is related as:

- Reflected impedance

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} =$$

Example

Find $\mathbf{V}_o$ in the circuit of Fig. 13.40.

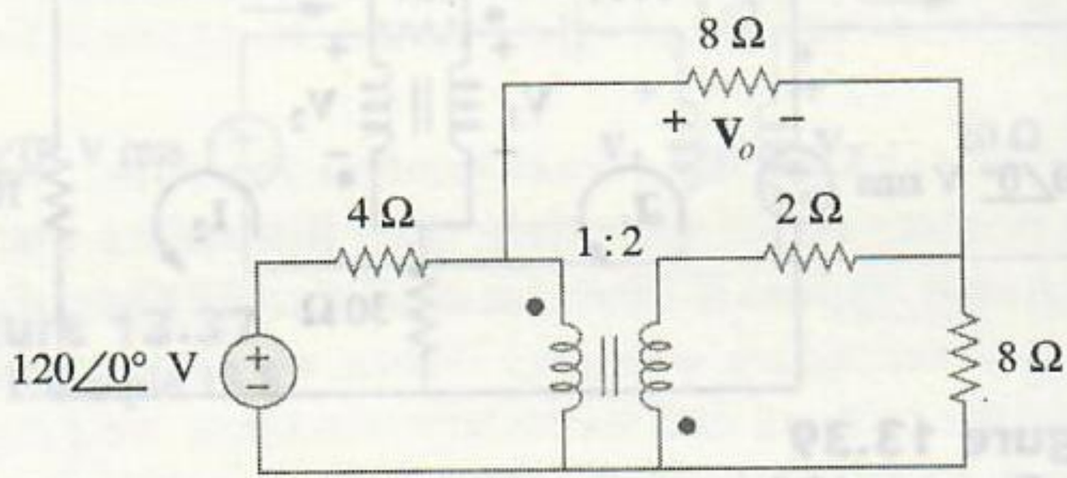
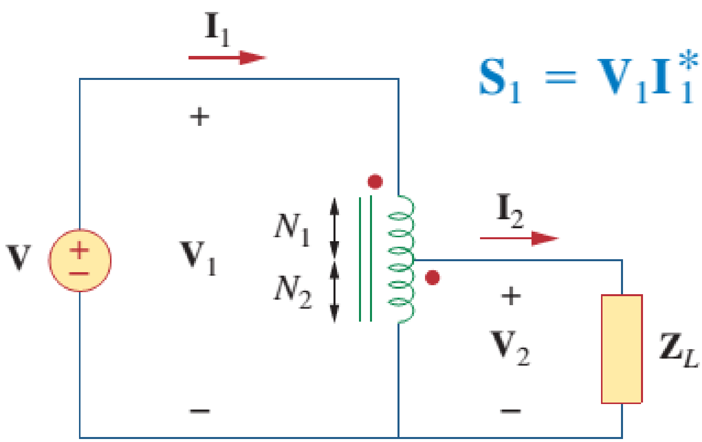


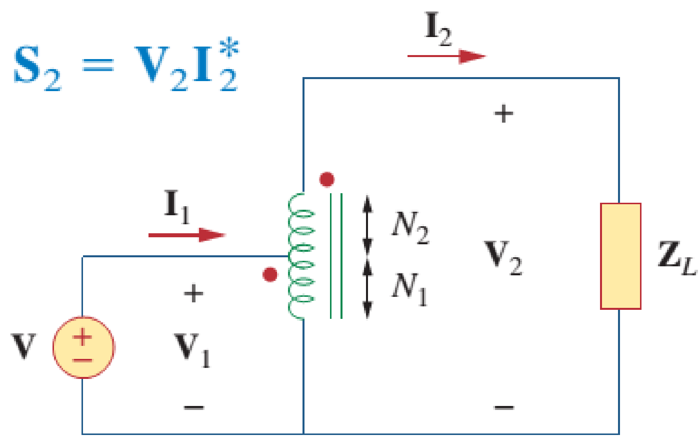
Figure 13.40
For Practice Prob. 13.9.

Ideal Autotransformer

- Autotransformer uses one winding for primary & secondary
  - It does not offer isolation!



$$\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1^* = \mathbf{S}_2 = \mathbf{V}_2\mathbf{I}_2^*$$



$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1 + N_2}{N_2} = 1 + \frac{N_1}{N_2}$$

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_1 + N_2}$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_2}{N_1 + N_2}$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_1 + N_2}{N_1} = 1 + \frac{N_2}{N_1}$$