SI211 Homework 3

Prof. Boris Houska

Deadline March 29, 2022

1. *Extrapolation.* What is the exact value of the limit

$$\lim_{x \to 0} \frac{x - \tan(x)}{\sin^3(x)} \, ?$$

Next, write a compute program that finds the limit by first constructing the interpolating polynomial $p(x)$ of $f(x) = \frac{x-\tan(x)}{\sin^3(x)}$ at

(a) $x = $ `[1e-6, 5e-6, 9e-6]`,

(b) $x = $ `[1e-7, 5e-7, 9e-7]`,

(c) $x = $ `[1e-6, 3e-6, 5e-6, 7e-6, 9e-6]`,

and then extrapolate f by evaluating $p(0)$. Print the extrapolation error for each choice of x and discuss your results.

2. *Natural cubic splines.*

(a) Write a computer code to implement a function that returns a natural cubic spline $s(x)$ interpolating any continuously differentiable function f.

(b) Consider the function

$$f(x) = \frac{1}{1 + x^2} \, ,$$

which is also known as Runge function (named after German mathematician Carl Runge), and $x = $ `range(-5, stop=5, length=11)`.

i. Plot a polynomial of degree 10 that interpolates $f(x)$ by reusing the code you wrote for Problem 2 in Homework 2,

ii. Plot the natural cubic spline $s(x)$ that interpolates $f(x)$ by using the code you wrote for the previous problem.

iii. Compute $\int_{-5}^{5} |f''(x)|^2 \mathrm{d}x$, $\int_{-5}^{5} |p''(x)|^2 \mathrm{d}x$ and $\int_{-5}^{5} |s''(x)|^2 \mathrm{d}x$. Interpret your result.