CS270-B Advanced Digital Image Processing

Lecture 11 Image Reconstruction

(Problem Definition)

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Outline

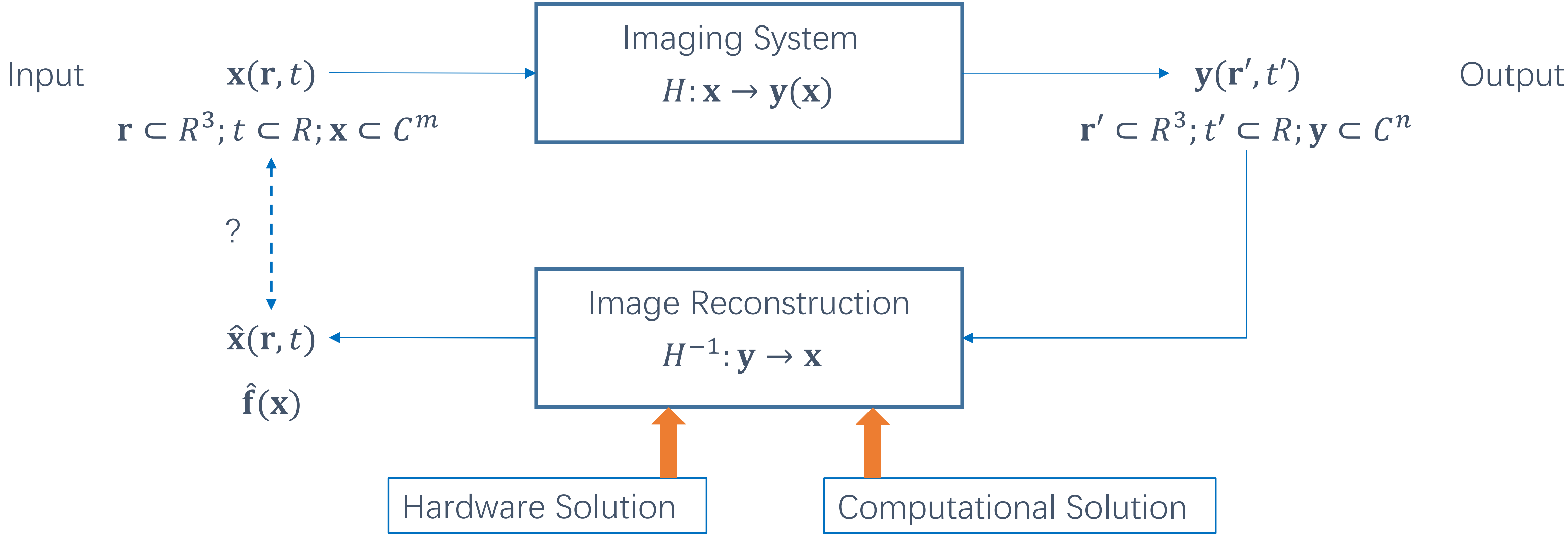
– Projection and back-projection

– Radon transform

– Fourier-Slice Theorem

– Filtered back-projection

Image Reconstruction: A System’ s View



1. Is H even known?

2. What’ s the evaluation criteria for reconstruction?

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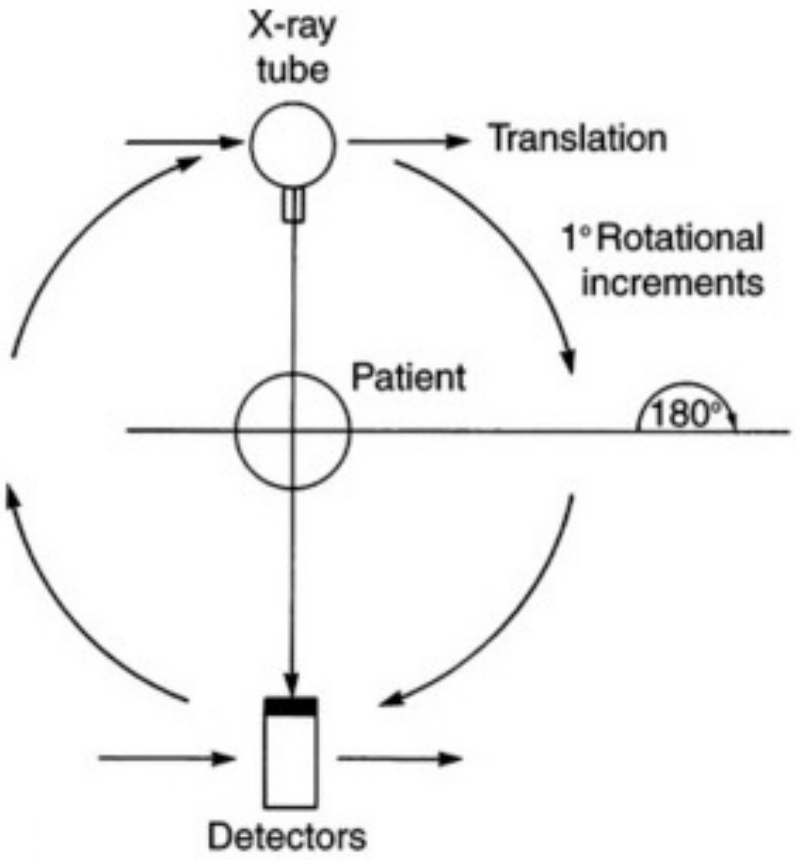
Reconstruction Evaluation Criteria

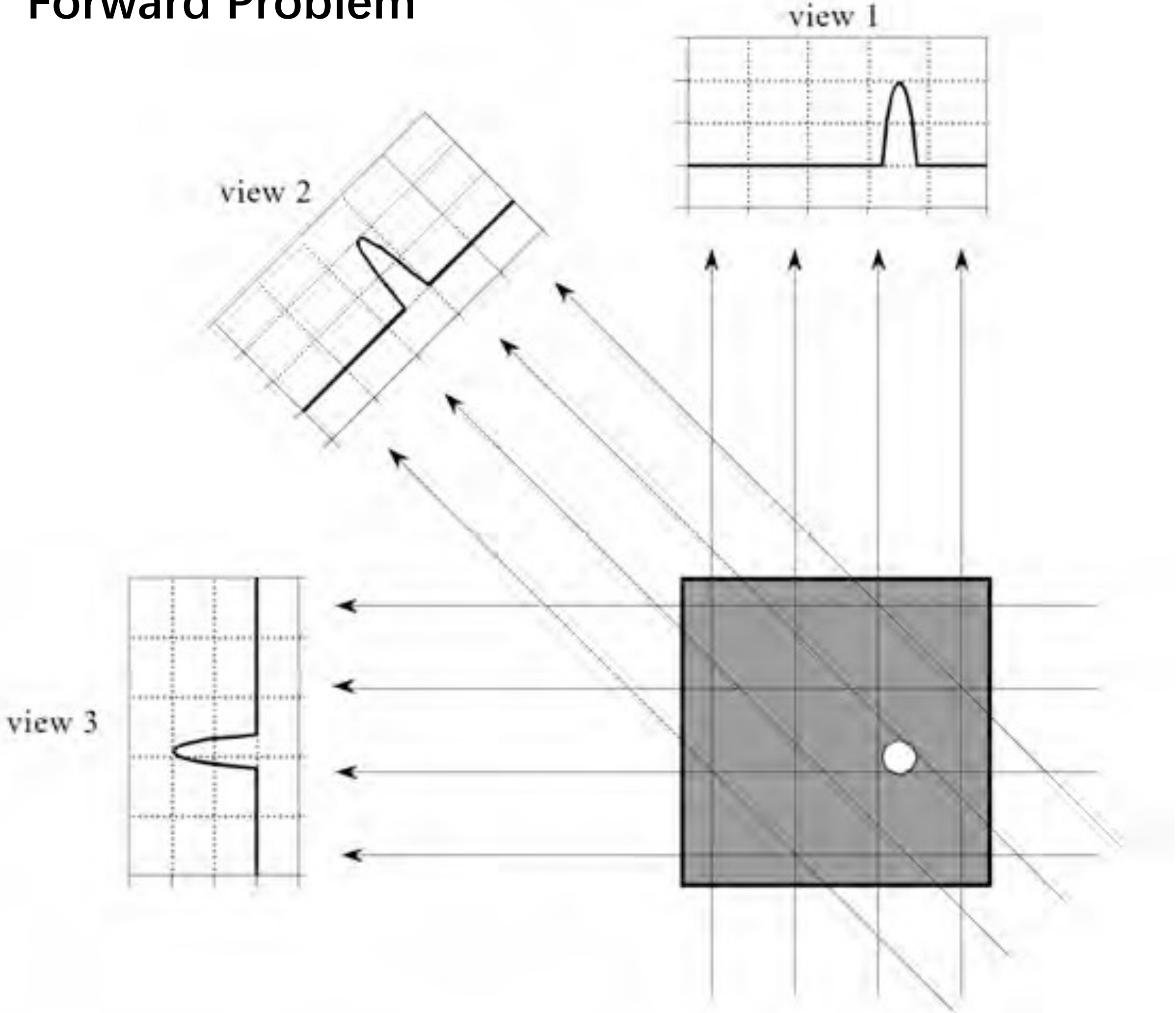
- Least Mean Square Errors: needs to know the ground truth. Typically done in numerical simulation;
- Point Spread Functions: evaluating image blurring;
- Signal-to-Noise (SNR): sensitivity to noise amplification;
- Contrast-to-Noise (CNR) ratio: e.g. sampling density correction in gridding;
- Ultimate test: real applications, e.g. by clinical practice.

Computational Solution

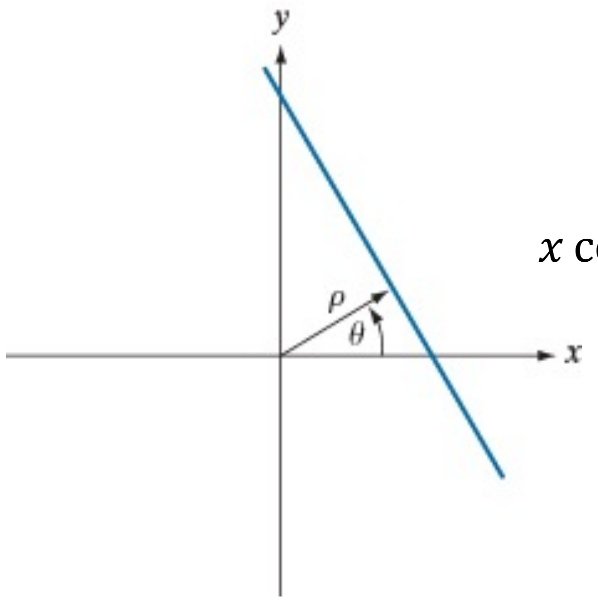
Computed Tomography



Forward Problem



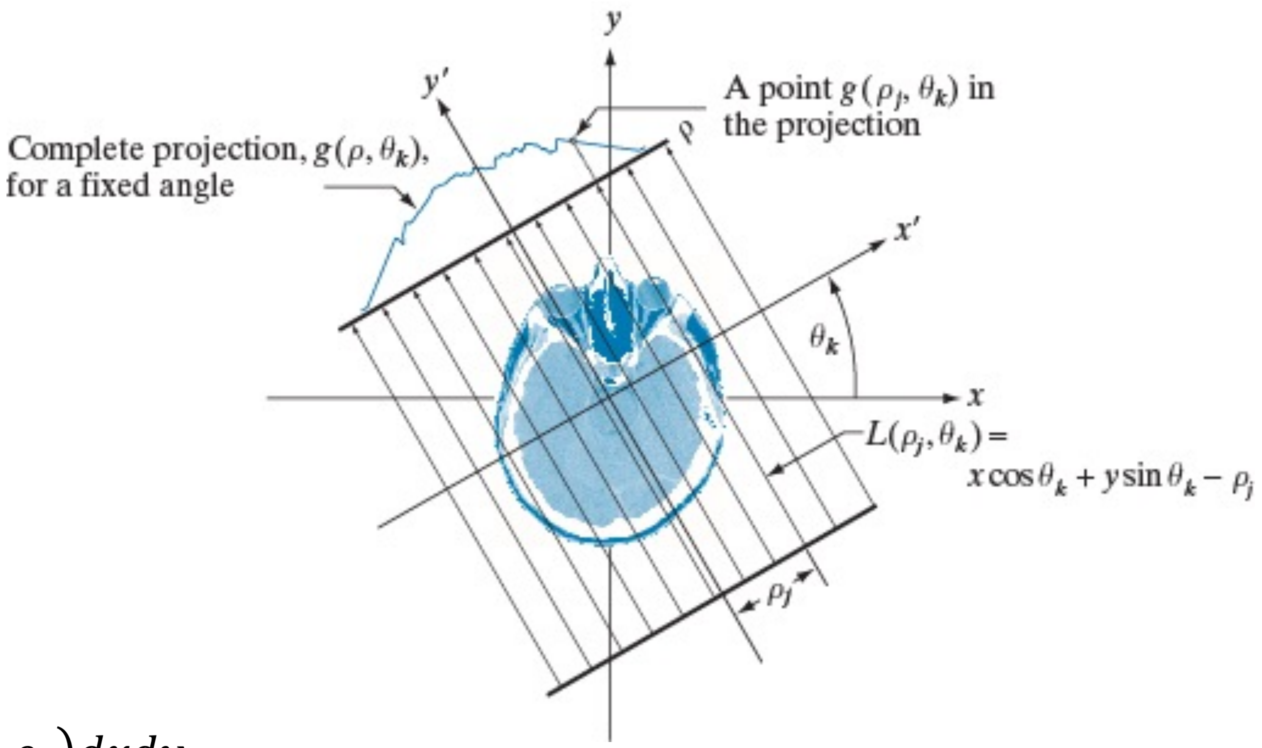
Radon transform



$$y = ax + b$$

$$\downarrow$$

$$x\cos\theta + y\sin\theta = \rho$$



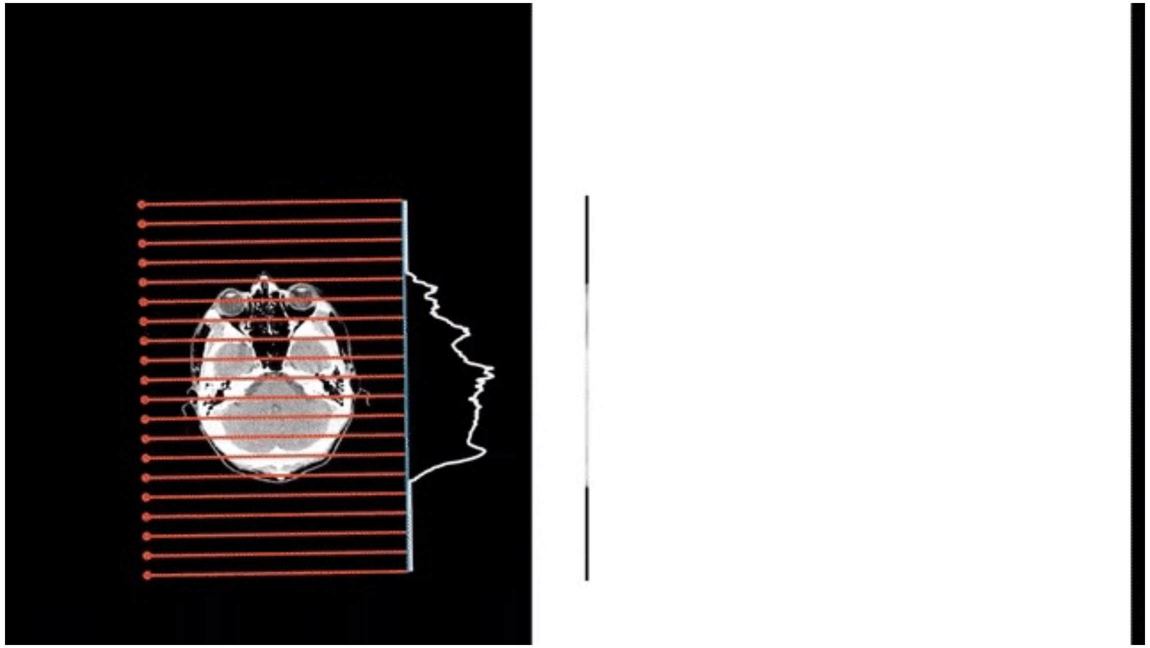
$$g(\rho_j, \theta_k) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\delta(x\cos\theta_k + y\sin\theta_k - \rho_j)dxdy$$

$$g(\rho, \theta) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\delta(x\cos\theta + y\sin\theta)dxdy$$

$$g(\rho, \theta) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\delta(x\cos\theta + y\sin\theta - \rho)$$

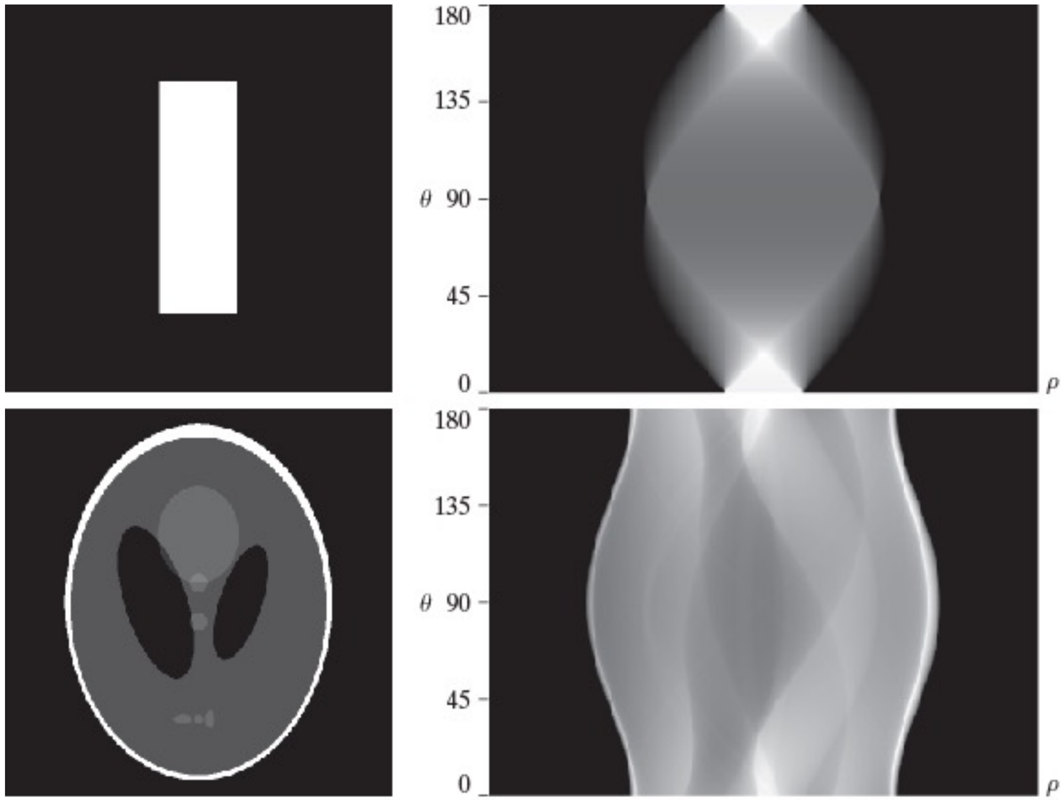
Sinogram

➢ Radon transform $g(\rho, \theta)$ is displayed as an image with ρ and θ as rectilinear coordinates



Radon Transform

$f(x, y) \longrightarrow g(\rho, \theta)$ (Sinogram)



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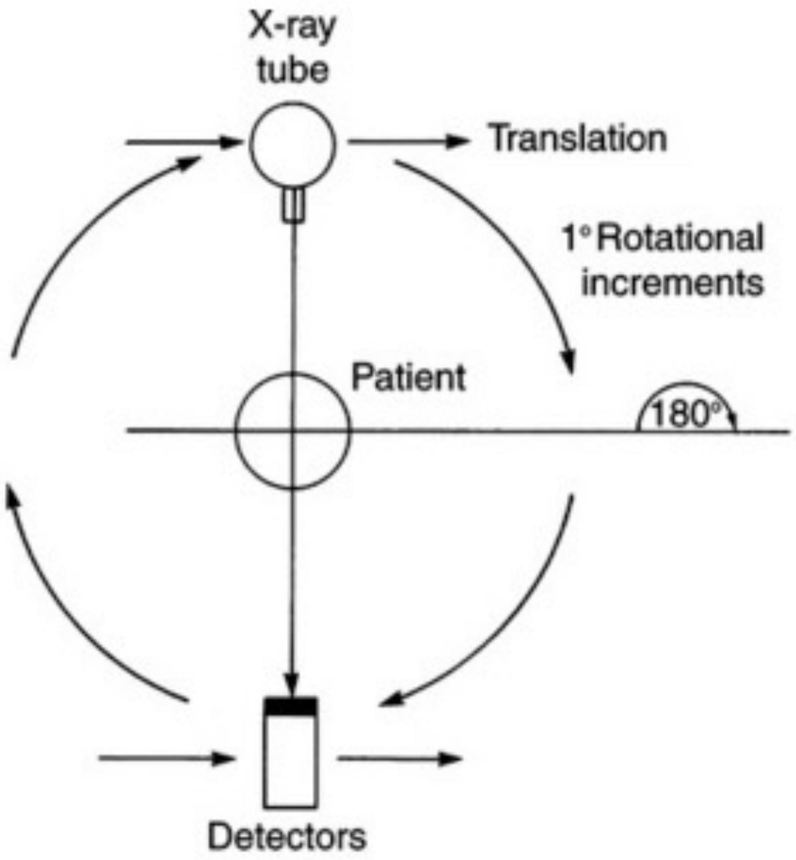
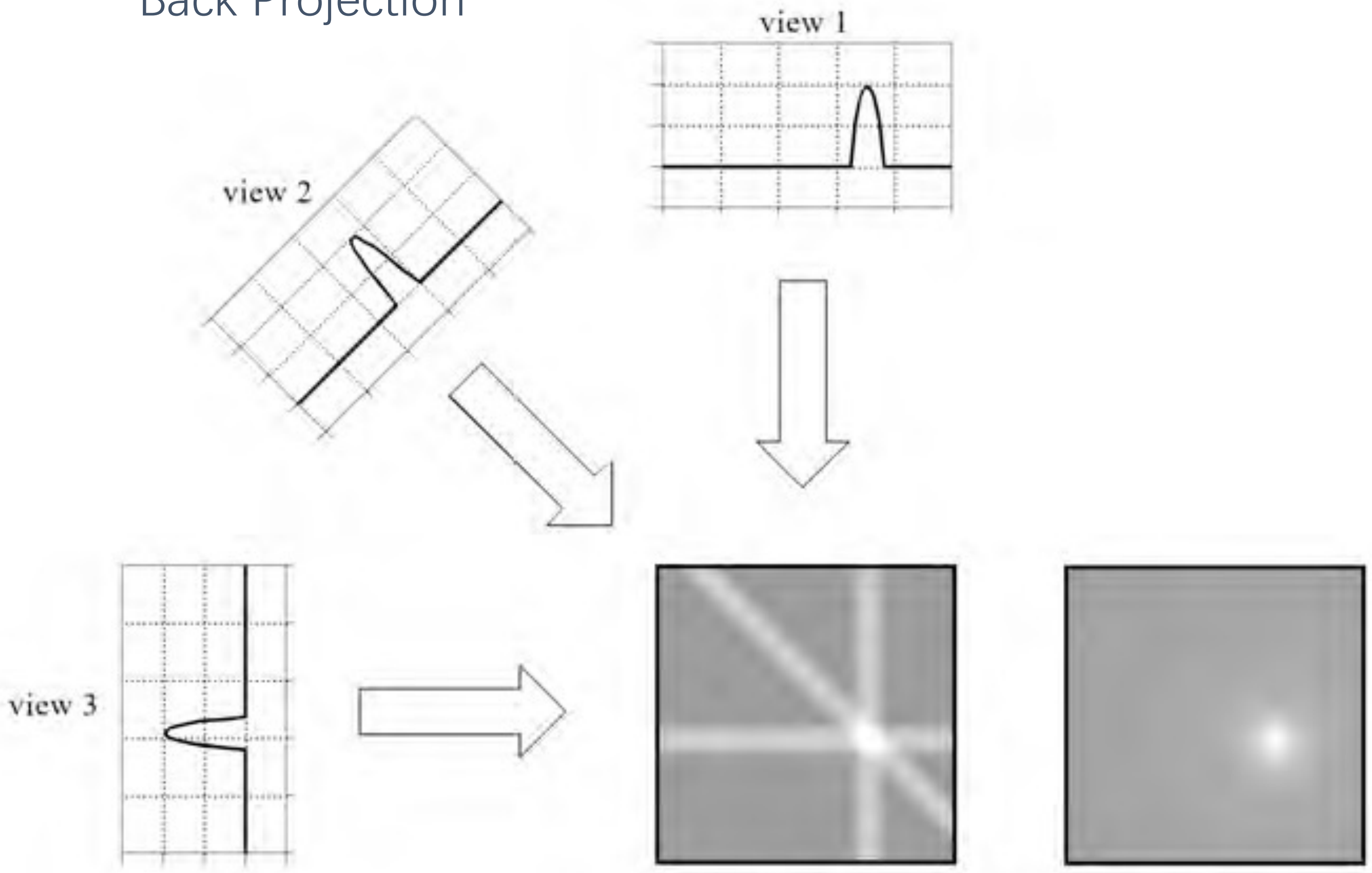
CT: Back Projection in Image Domain



Inverse Problem
Back Projection



a. Using 3 views
b. Using many views

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Back-projection from Sinogram

- For a fixed value of rotation θ_k:

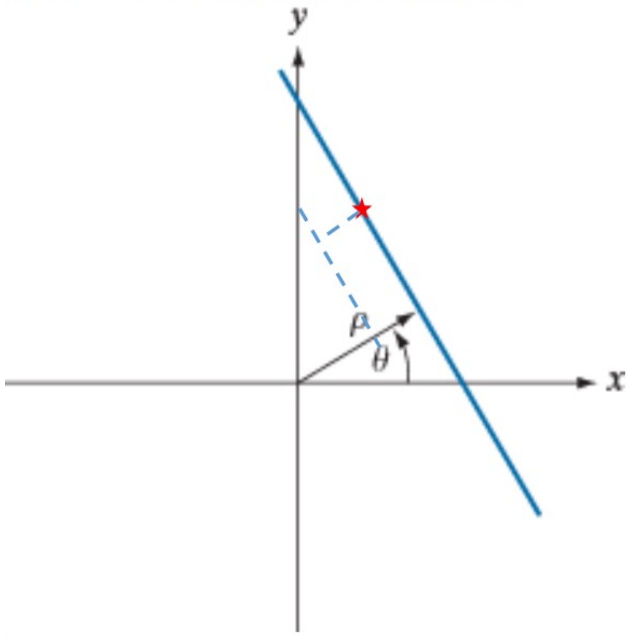
$$f_{\theta_k}(x, y) = g(\rho, \theta_k) = g(x \cos\theta_k + y \sin\theta_k\,, \theta_k)$$

- Then a single back-projection image obtained at an angle θ is :

$$f_{\theta}(x, y) = g(x \cos\theta + y \sin\theta\,, \theta)$$

- The reconstructed image is obtained by summing over all the back-projected images:

$$f(x, y) = \sum_{\theta=0}^{\pi} f_{\theta}(x, y)$$



Fourier-Slice Theorem

- The 1D FT of a projection with respect to ρ is:

$$G(\omega,\theta)=\int_{-\infty}^{+\infty} g(\rho,\theta)\mathrm{e}^{-j2\pi\omega\rho}d\rho$$

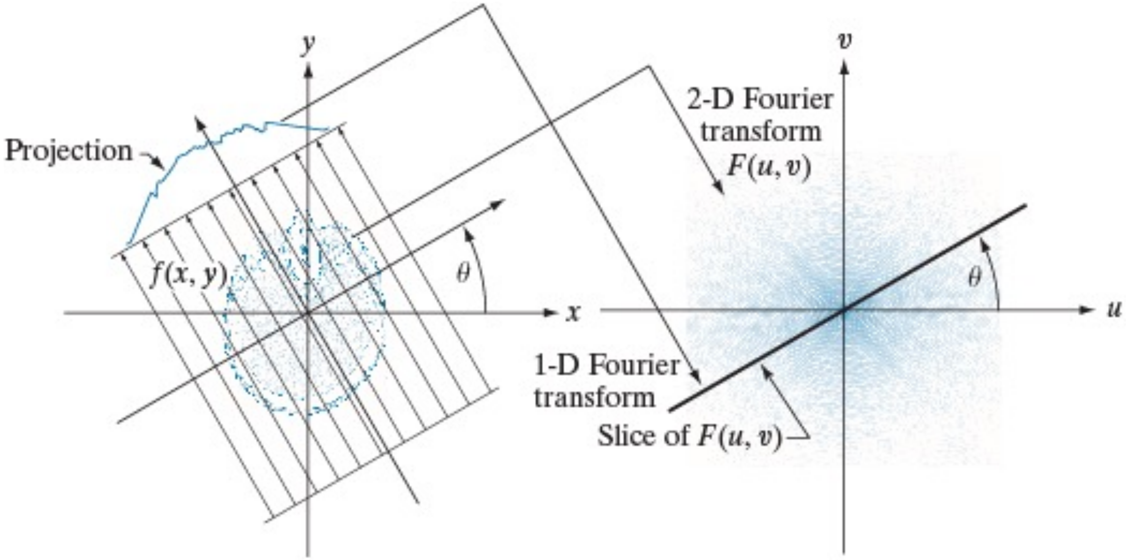
- Then

$$G(\omega,\theta)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\mathrm{e}^{-j2\pi\omega(x\cos\theta+y\sin\theta)}dxdy$$

$$=\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\mathrm{e}^{-j2\pi(ux+vy)}dxdy\right]_{u=\omega\cos\theta;v=\omega\sin\theta}$$

- Therefore

$$G(\omega,\theta)=[F(u,v)]_{u=\omega\cos\theta;v=\omega\sin\theta}=F(\omega\cos\theta,\omega\sin\theta)$$



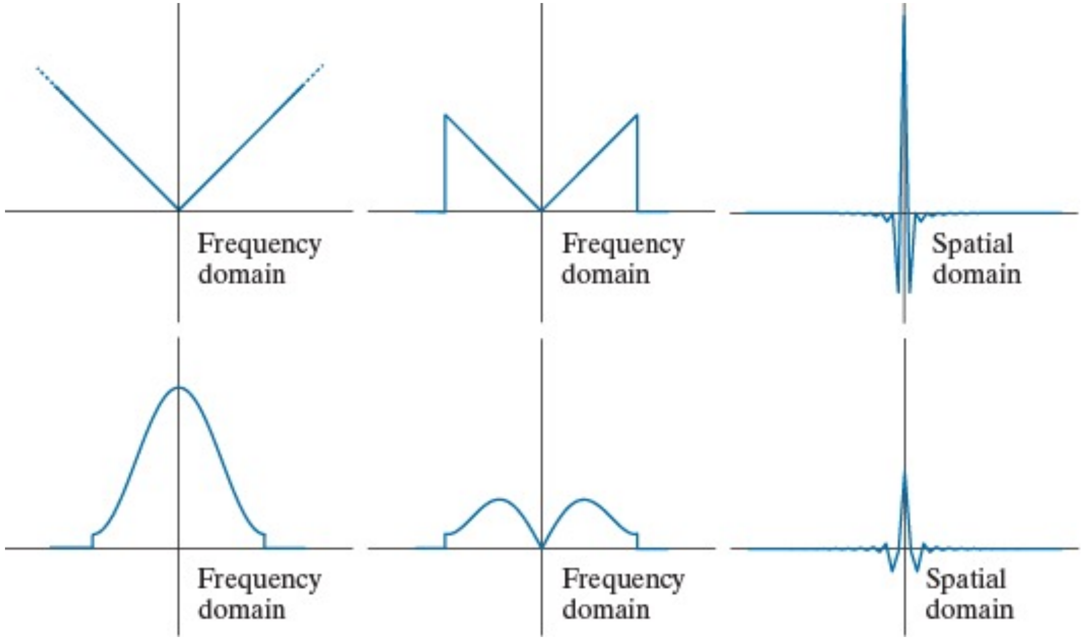
Filtered back-projection

- The 2D IFT of $\mathrm{F(u,v)}$ is:

$$f(x,y)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}F(u,v)\mathrm{e}^{j2\pi(ux+vy)}dudv$$

$$=\int_{0}^{2\pi}\int_{0}^{\infty}G(\omega,\theta)\mathrm{e}^{j2\pi\omega(x\cos\theta+y\sin\theta)}\omega d\omega d\theta$$

$$=\int_{0}^{\pi}\left[\left[\int_{-\infty}^{+\infty}|\omega|G(\omega,\theta)\mathrm{e}^{j2\pi\omega\rho}d\omega\right]_{\rho=x\cos\theta+y\sin\theta}\right.d\theta$$

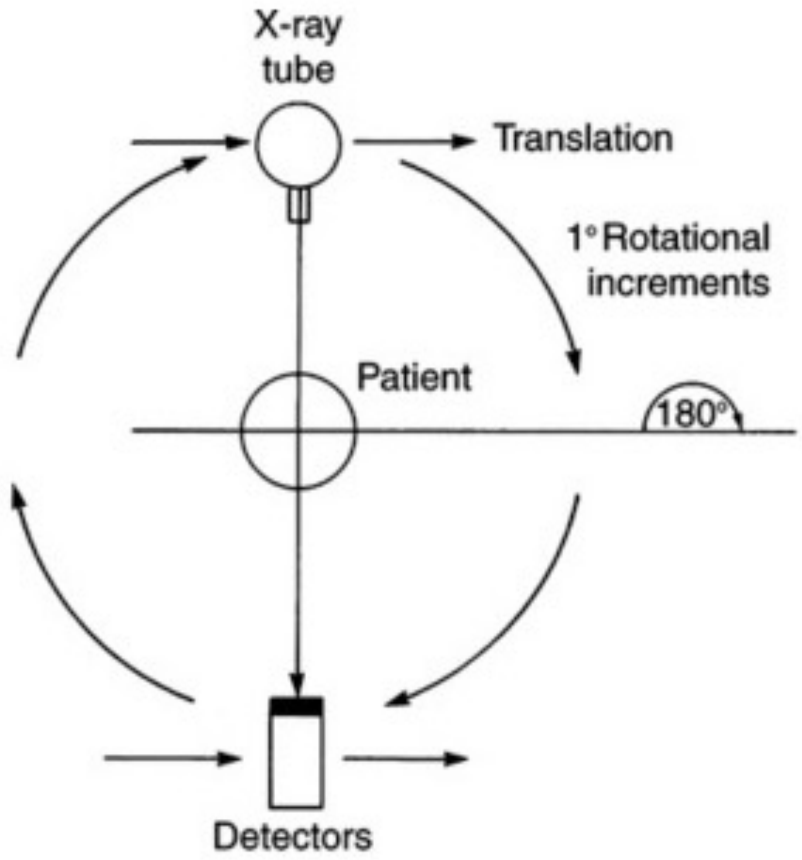


- Back-projection with convolution

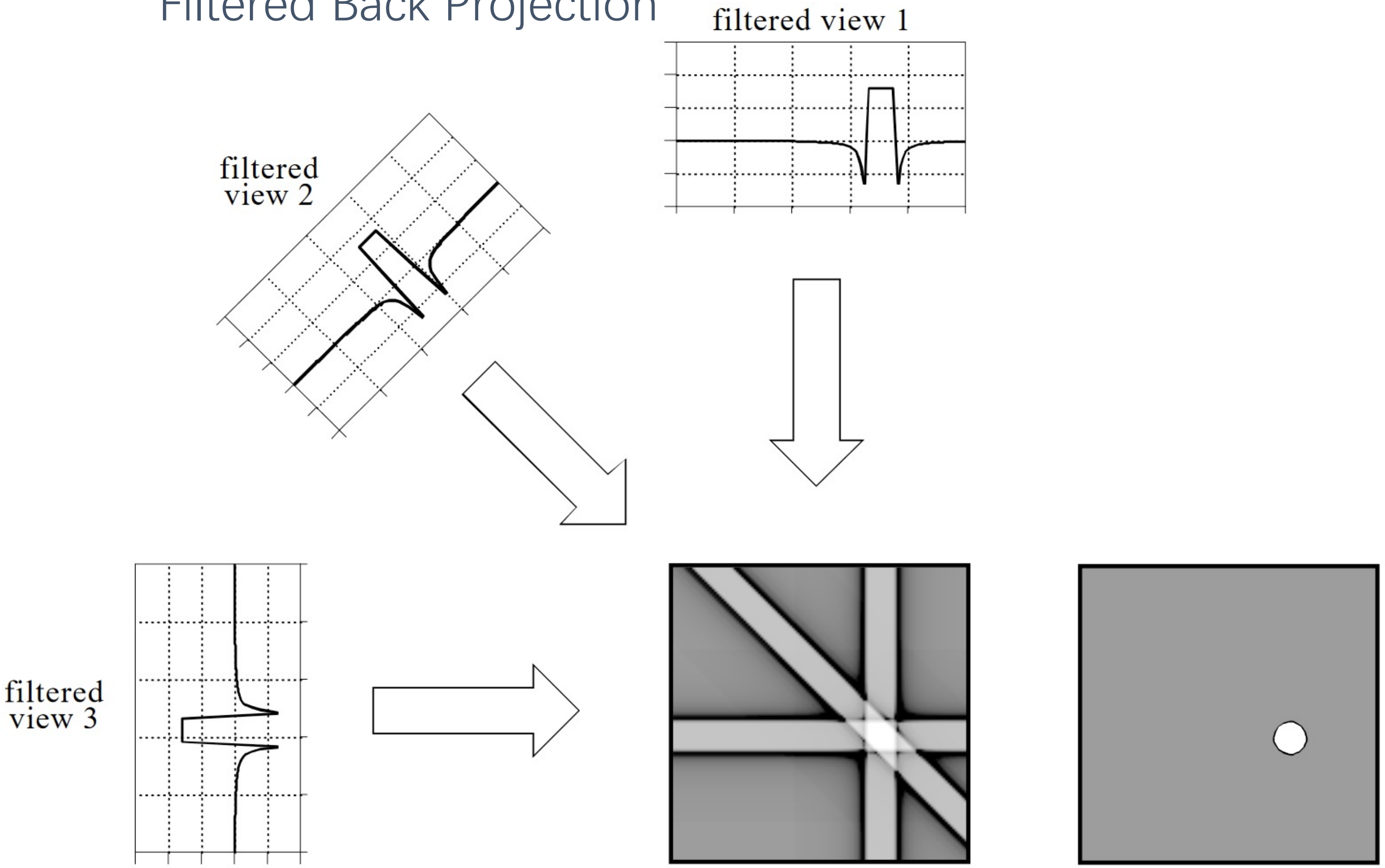
$$f(x,y)=\int_{0}^{\pi}[s(\rho)\otimes g(\rho,\theta)]_{\rho=x\cos\theta+y\sin\theta}d\theta$$

Where $s(\rho)=IFT(|\omega|)$, $g(\rho,\theta)=IFT(G(\omega,\theta))$

CT: Back Projection



Inverse Problem
Filtered Back Projection

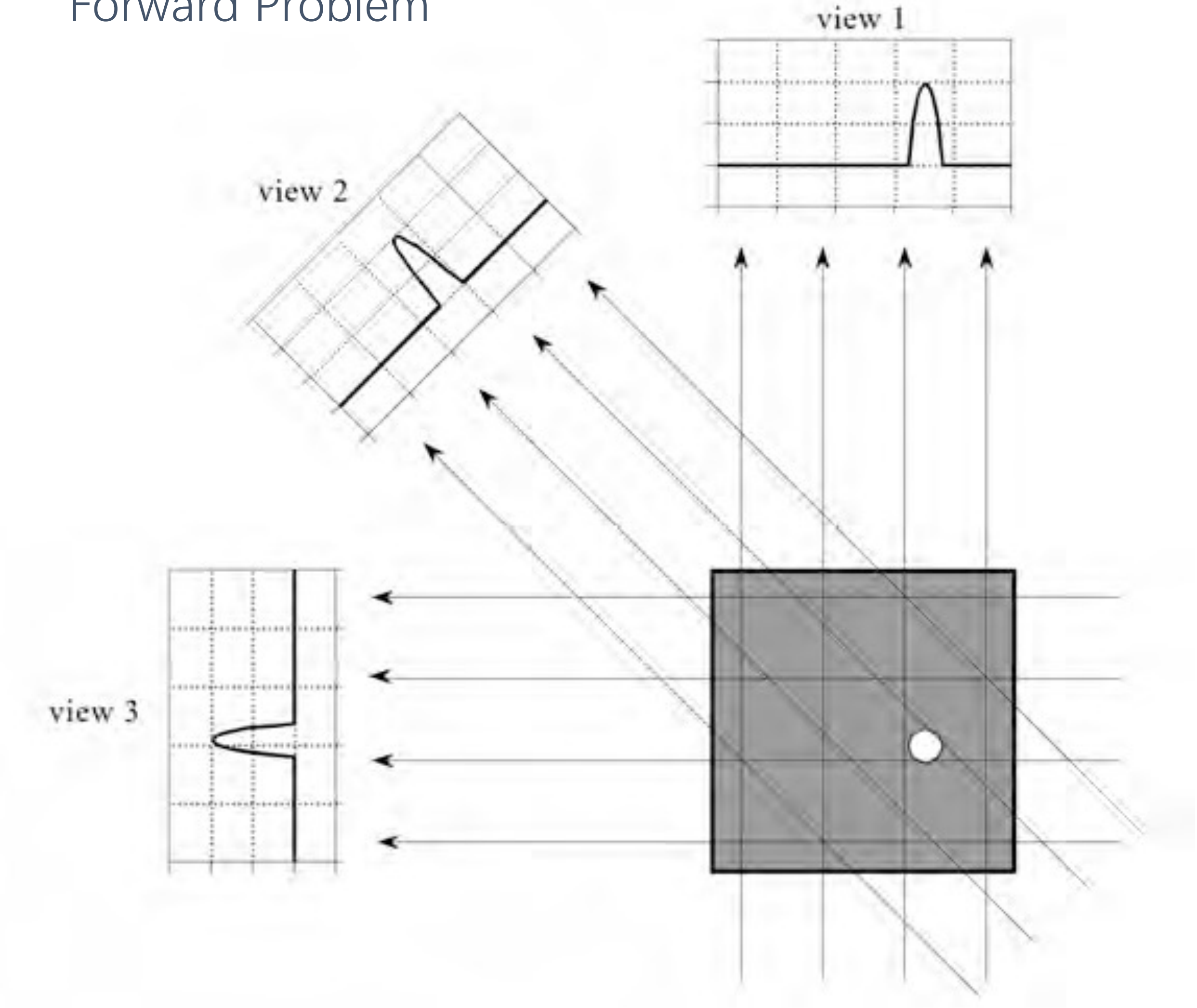


a. Using 3 views

b. Using many views

CT: Discrete Optimization Formulation

Forward Problem



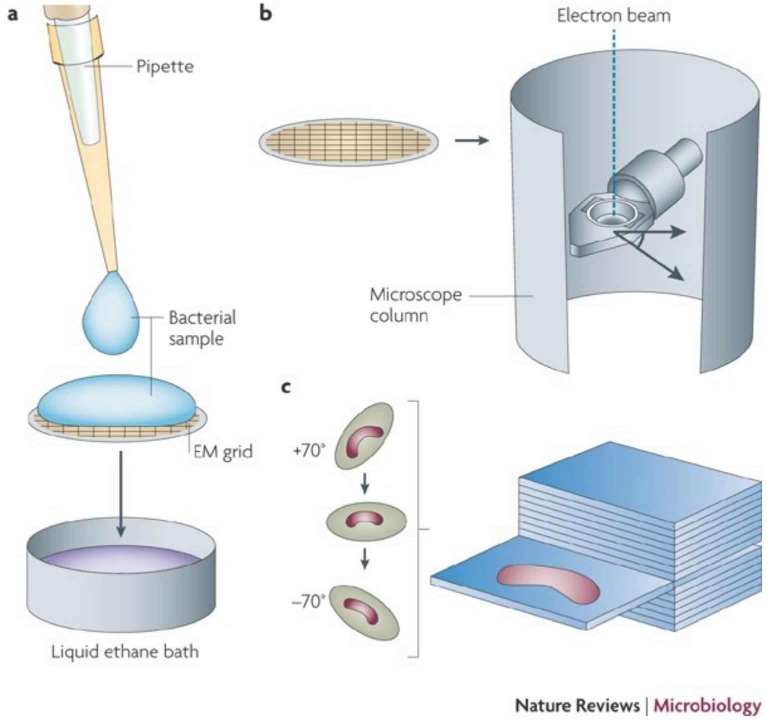
$$\mathbf{x}(\mathbf{r}, t) \rightarrow \boxed{\begin{array}{c}\text{X-ray Projection} \\ H: \mathbf{x} \rightarrow \mathbf{y}(\mathbf{x})\end{array}} \rightarrow \mathbf{y}(\mathbf{r}', t')$$

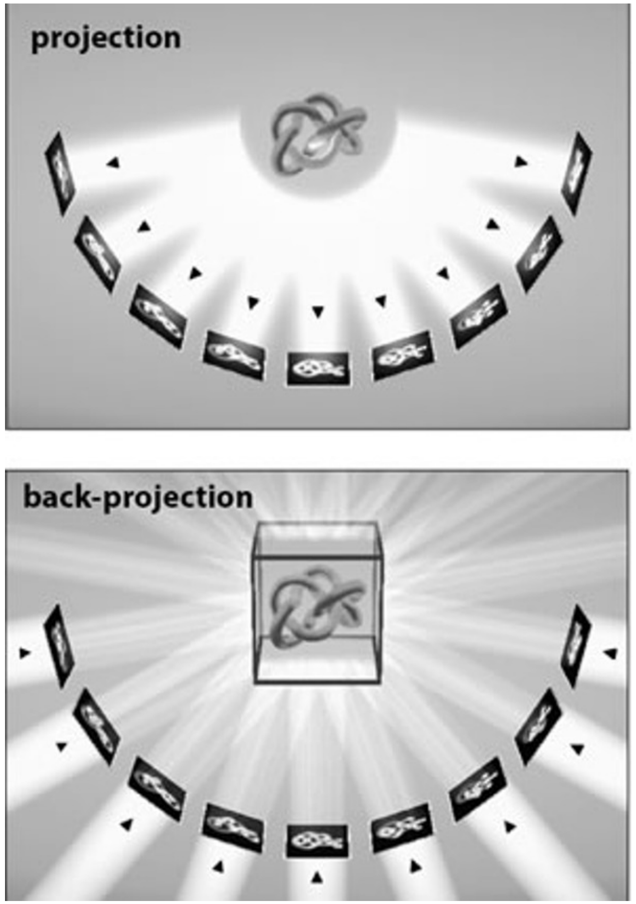
$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

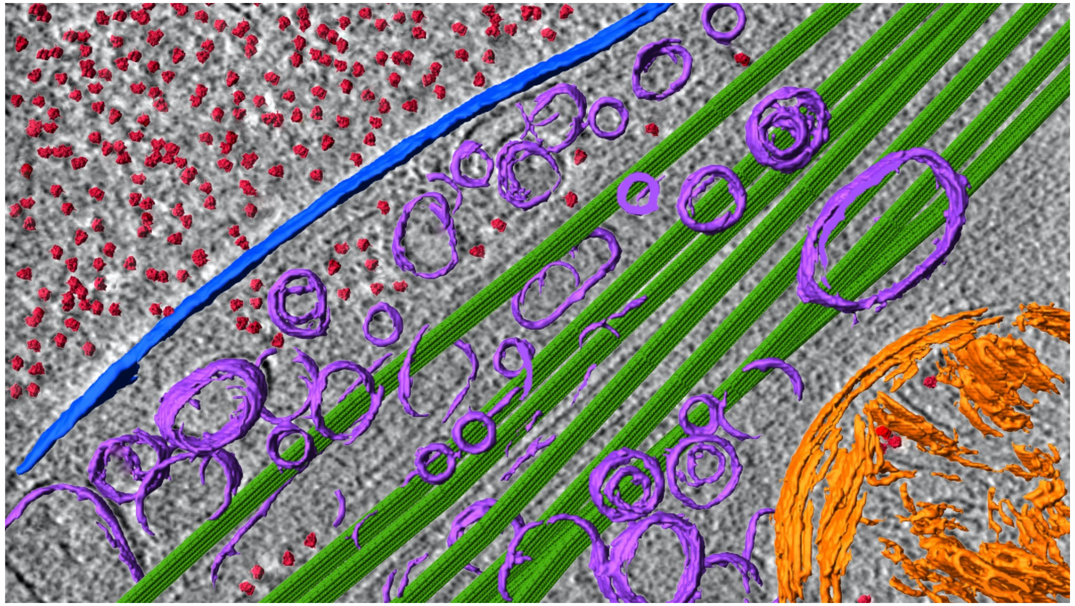
$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2$$

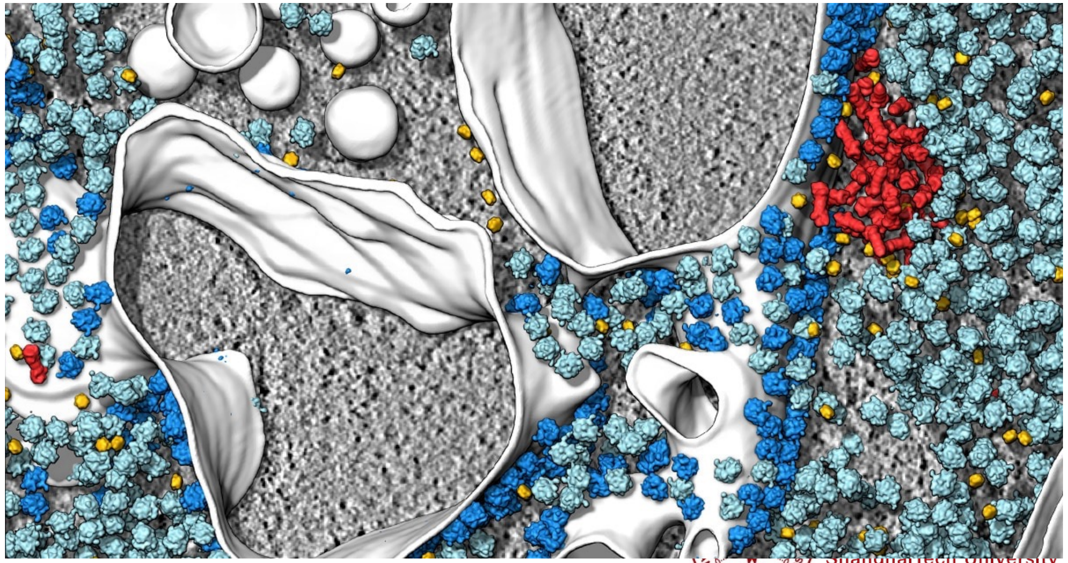
$$s.t. \quad some\ constraints$$

Cryo Electron Tomography （冷冻电子断层扫描）

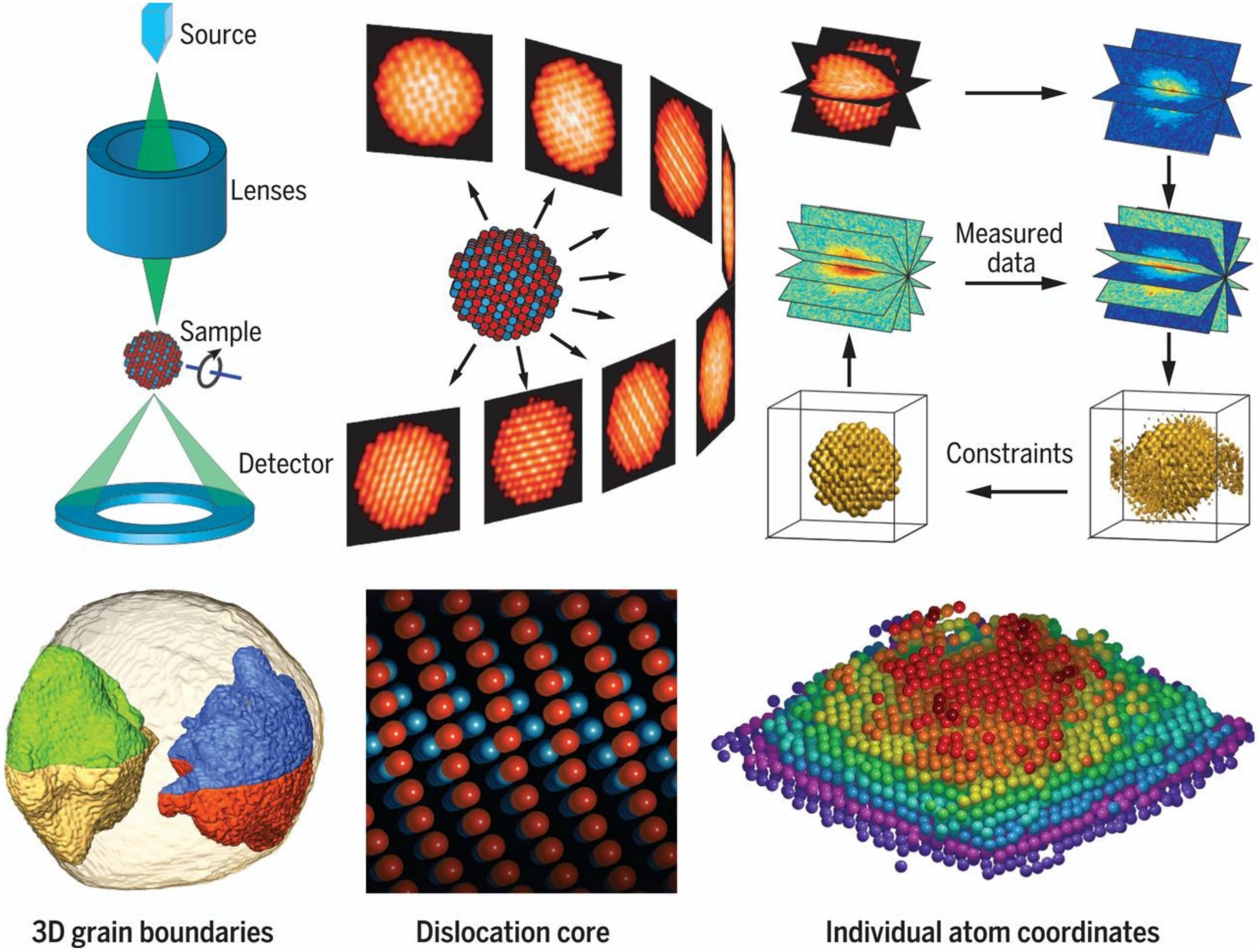








Atomic electron tomography（原子电子断层扫描）



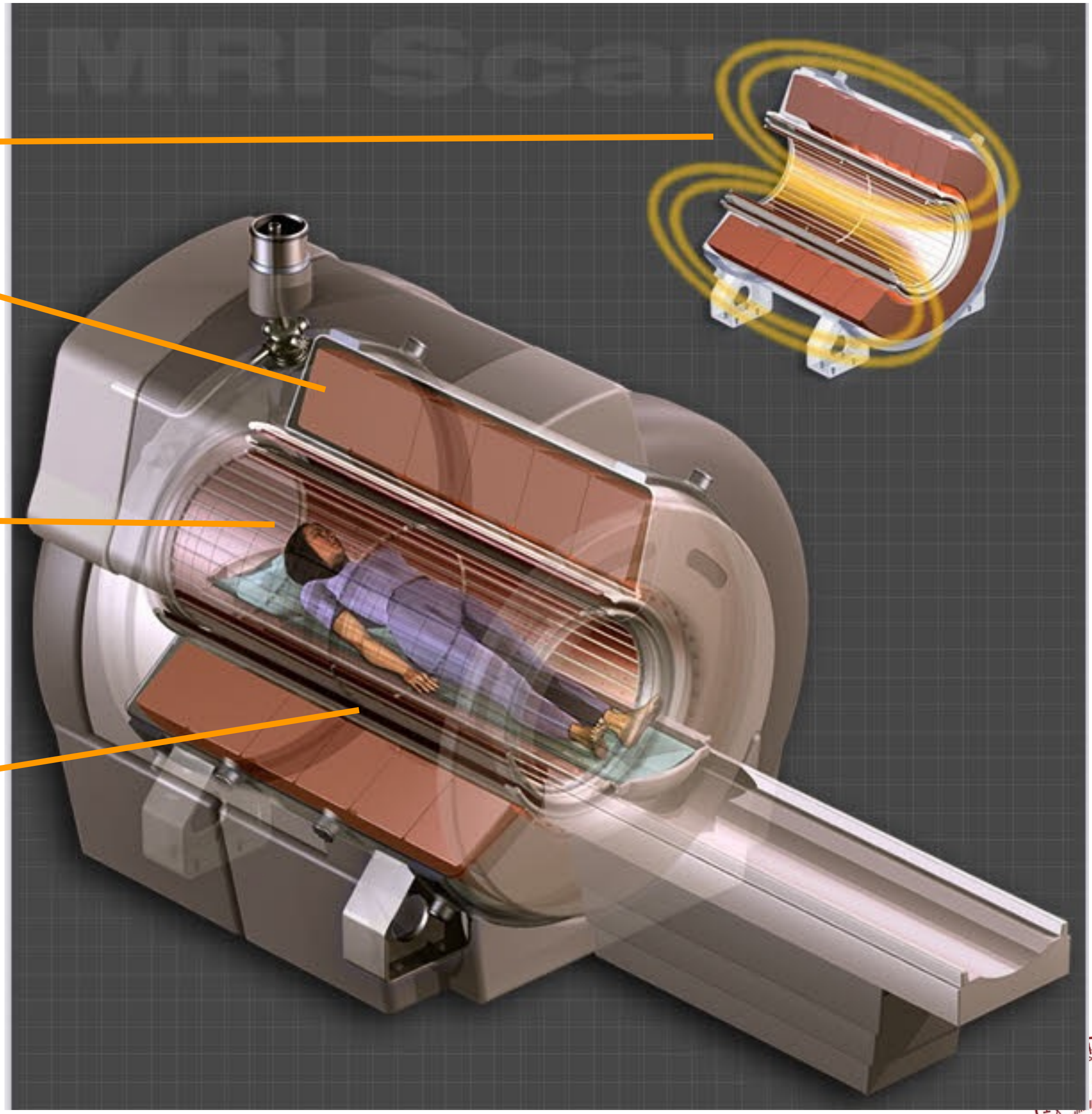
Magnetic Resonance Imaging (MRI): Fourier Encoding

Basic Components of MRI System

B_0 Field

RF Coil: B_1

Gradient Coil: G



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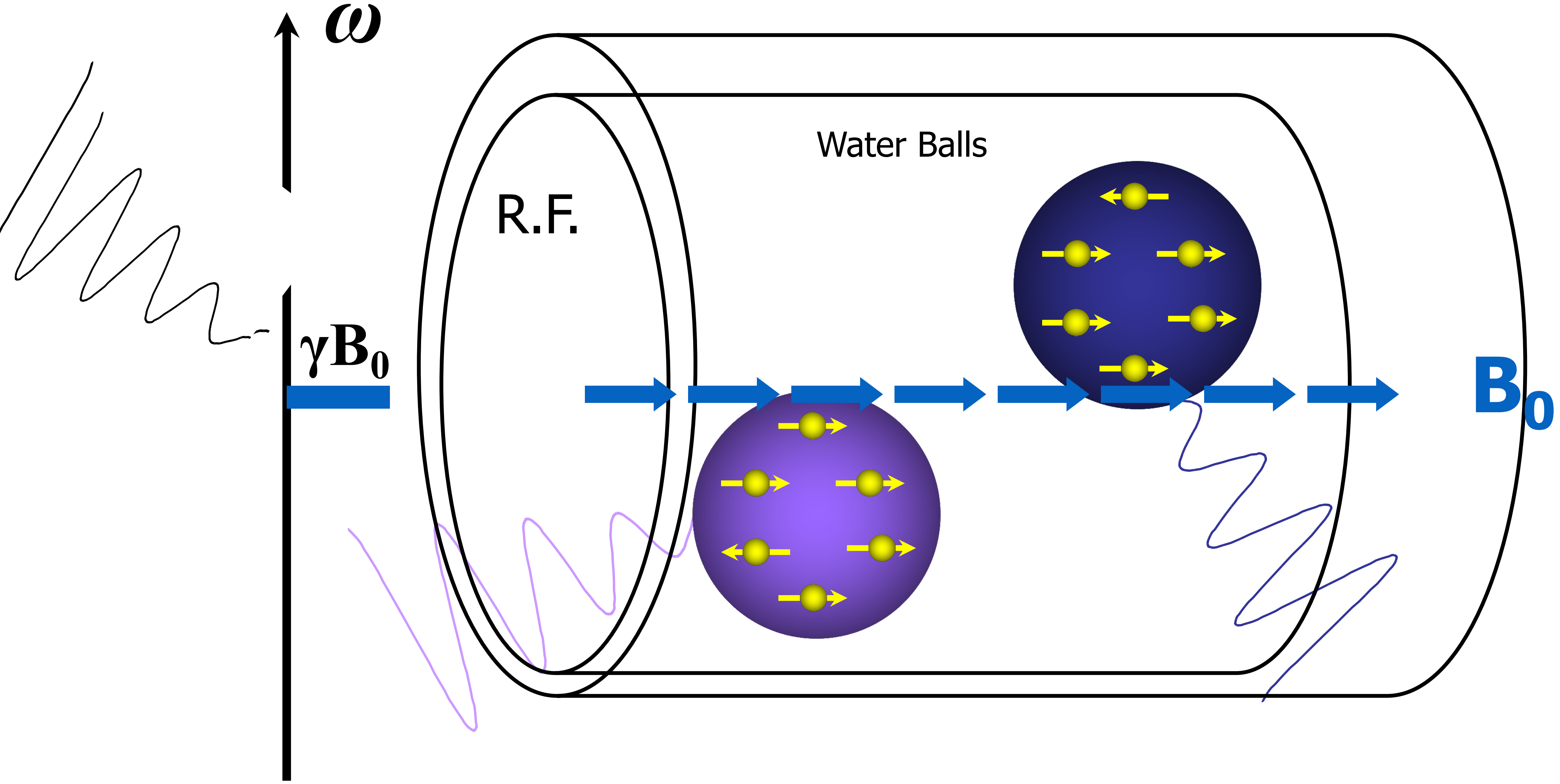
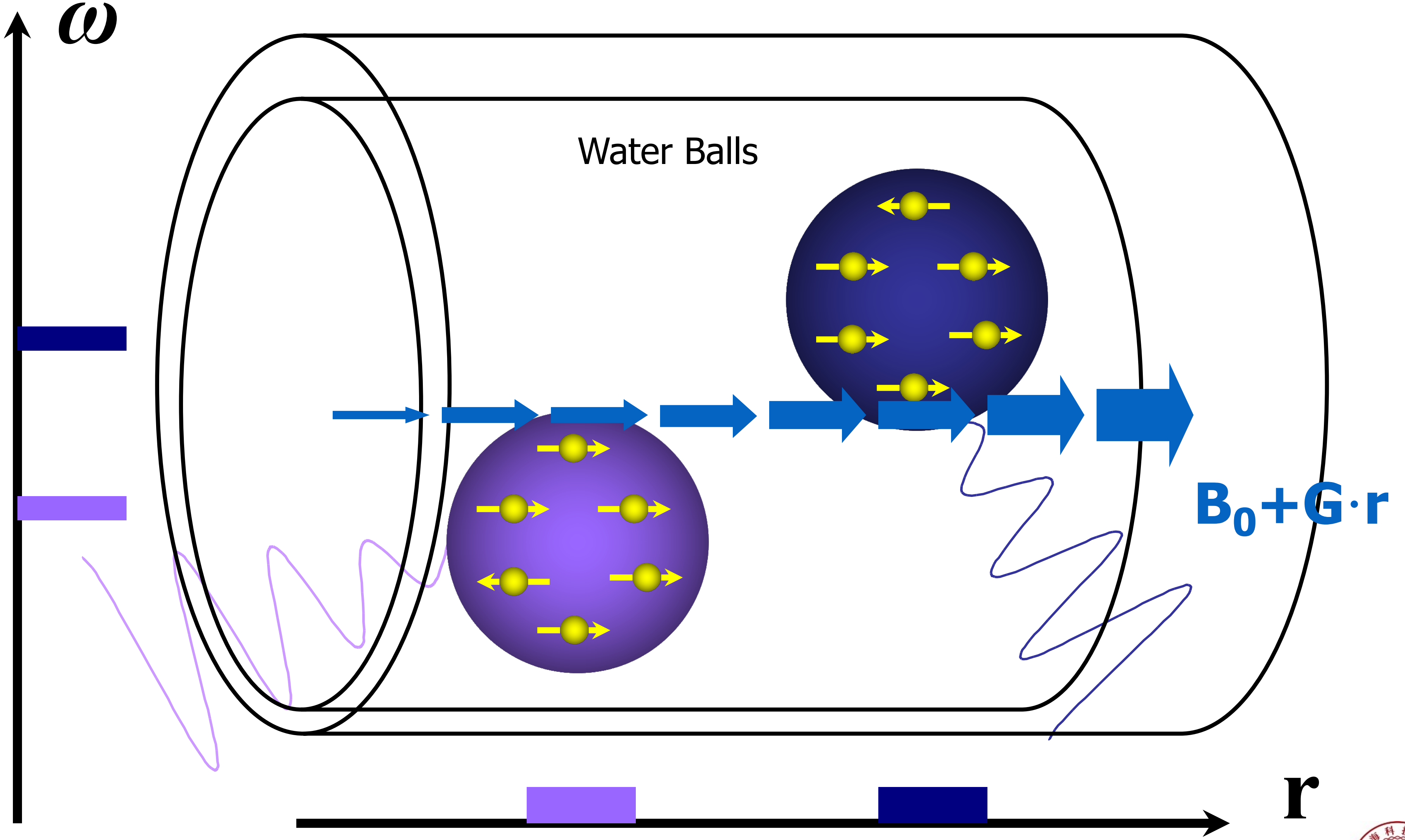
Magnetic Resonance Phenomenon



"Big" Idea: Magnetic Field Gradient



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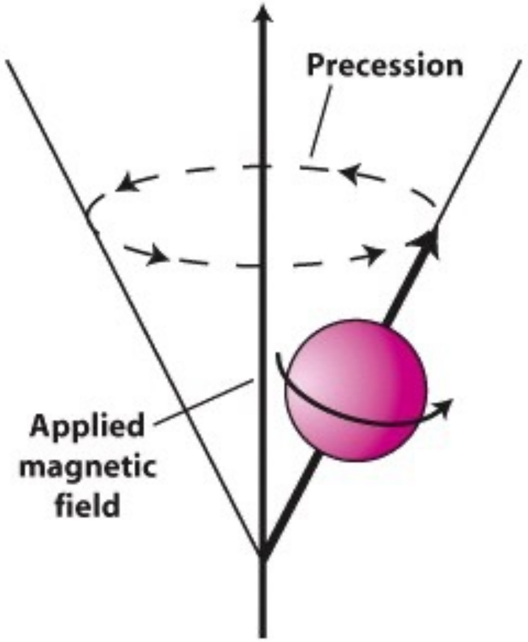
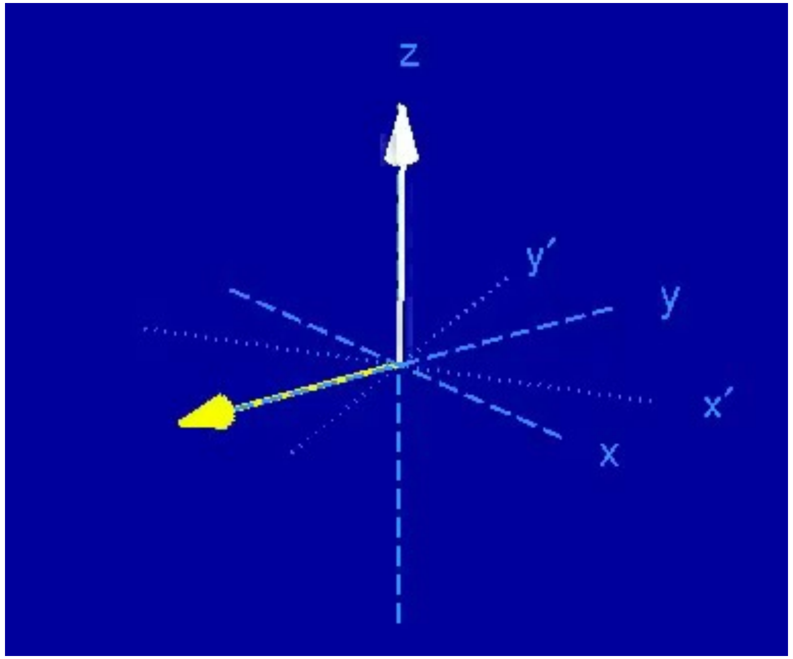
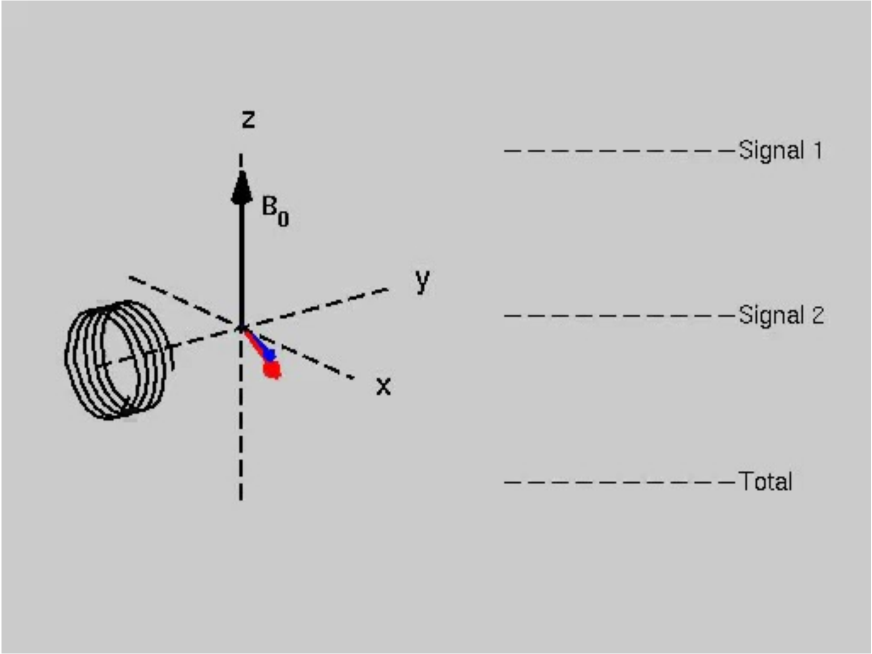
MR Signal Detection



B1 excitation

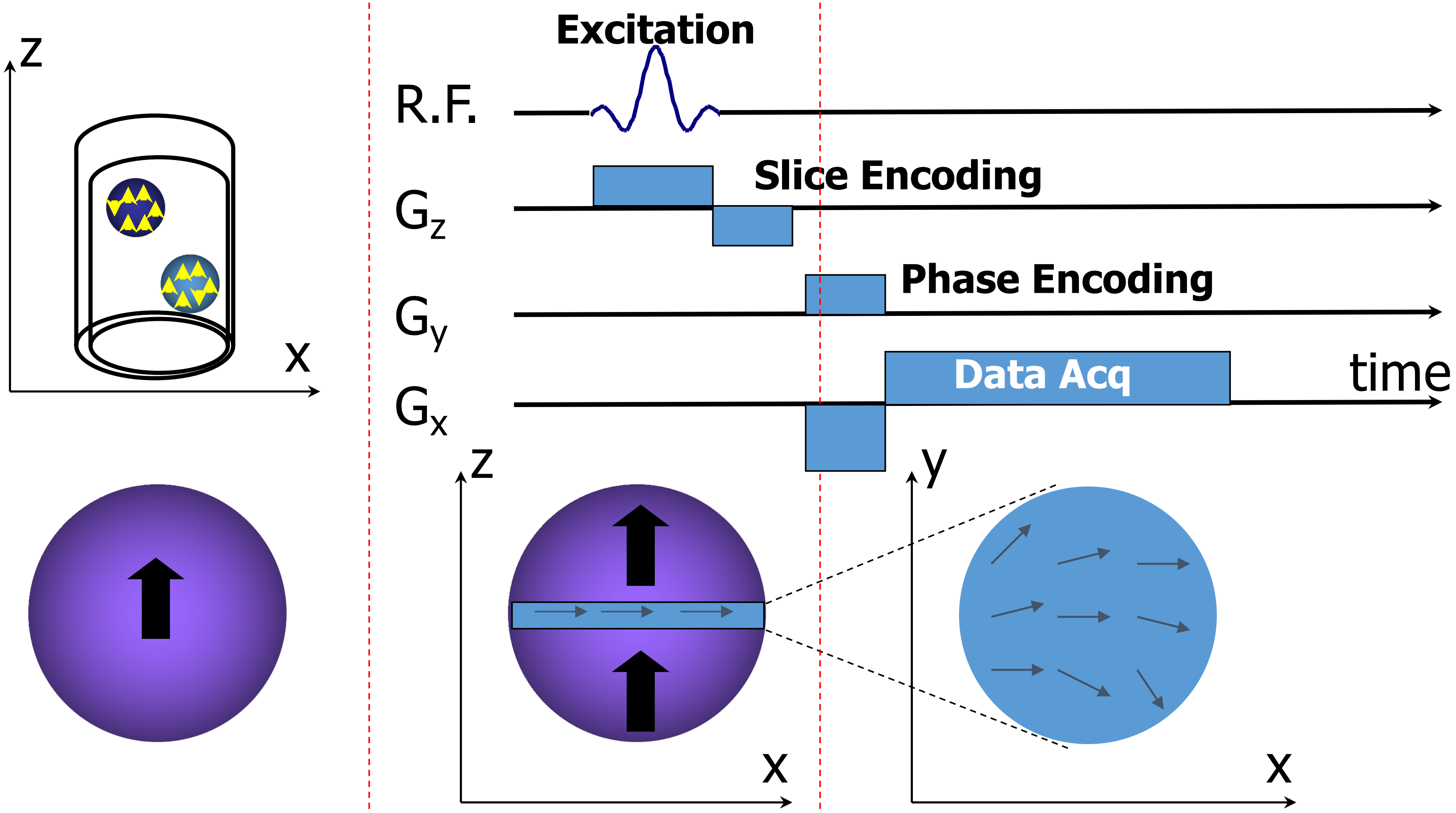


Spin dephasing



Q: 什么是FID信号,有何特点？是否包含spin的位置信息？

Pulse Sequence: Excitation, Encoding & Acquisition

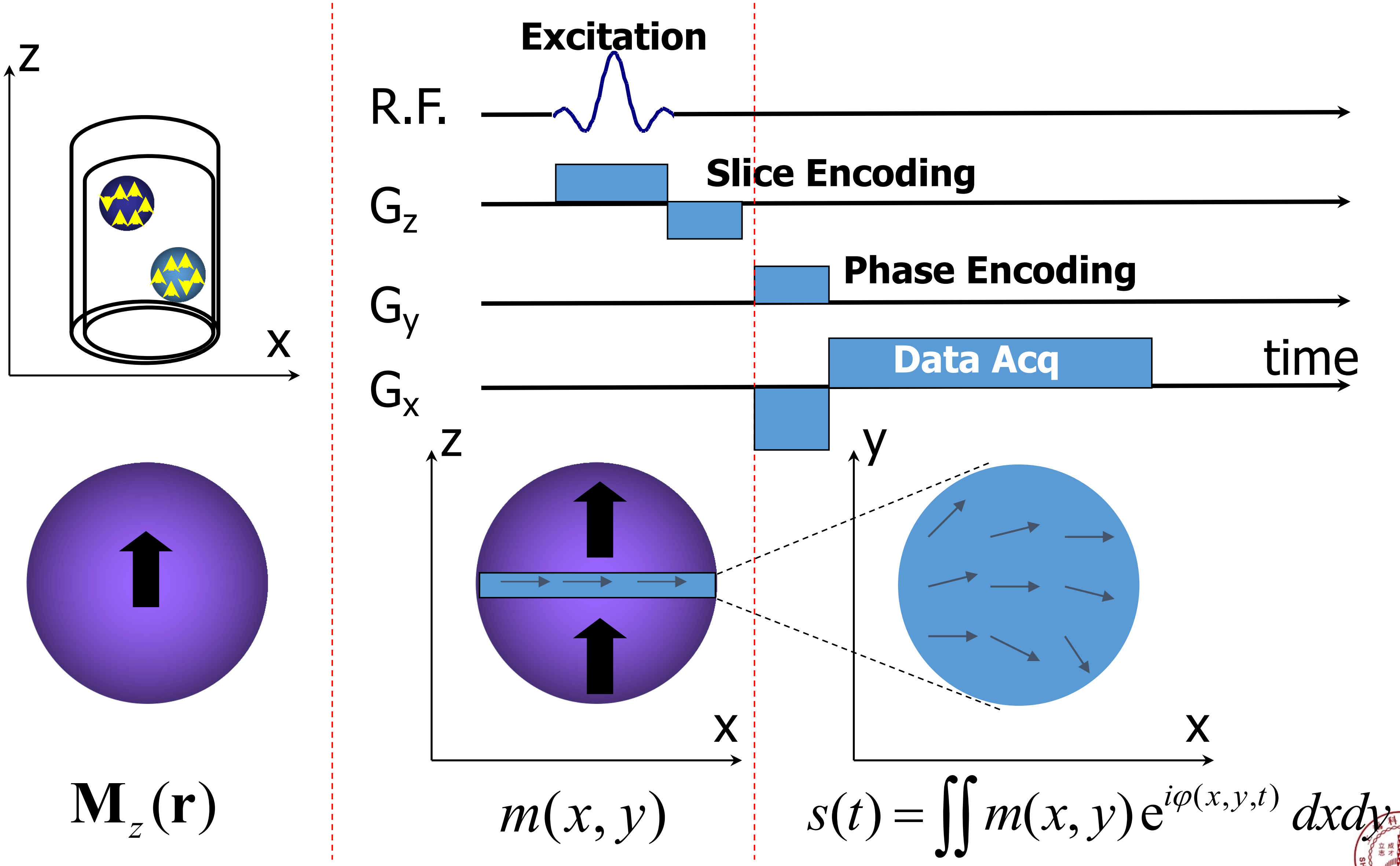


$$\mathbf{M}_z(\mathbf{r})$$

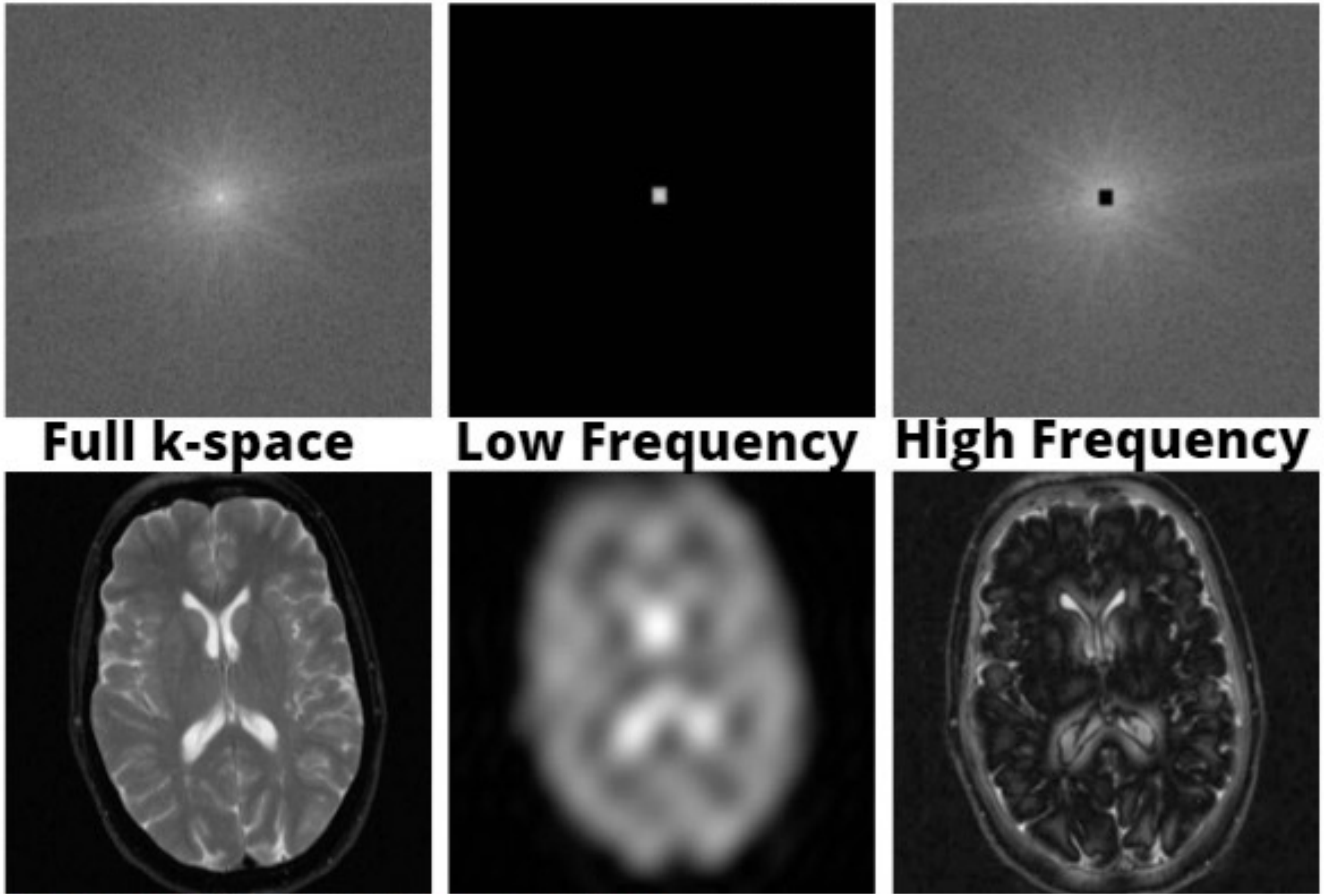
$$m(x, y)$$

$$s(t) = \iint m(x,y)\, e^{i\varphi(x,y,t)}\, dxdy$$

Pulse Sequence: Excitation, Encoding & Acquisition

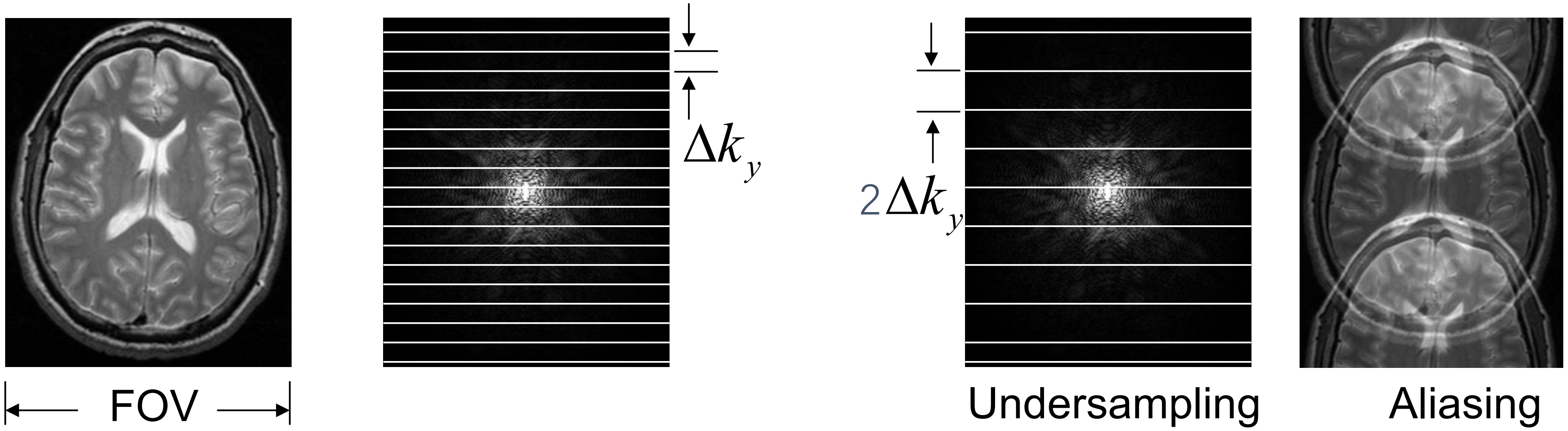


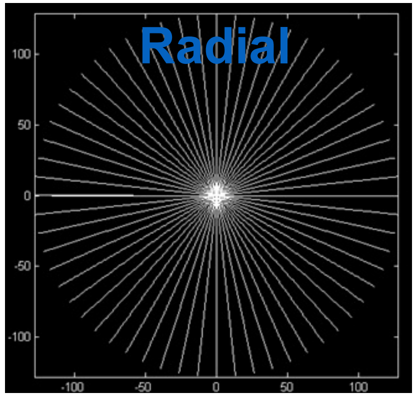
k-Space

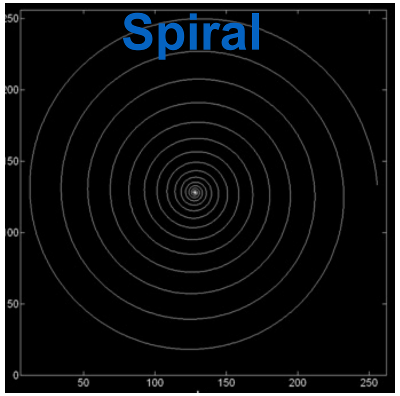


k-Space Sampling

Nyquist rate: f = 2*Bw → $\Delta k_x = 1/FOV_x, \Delta k_y = 1/FOV_y$



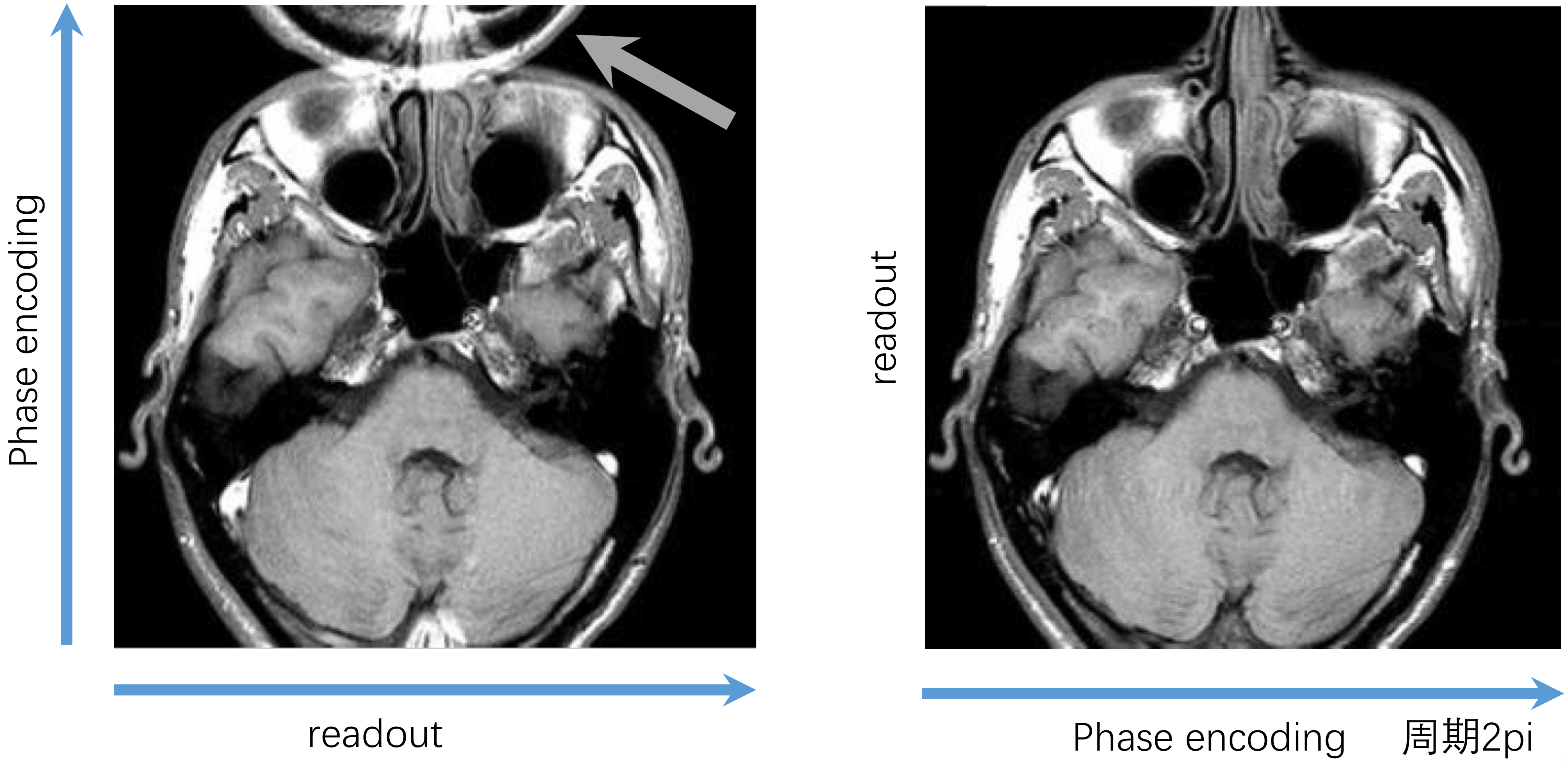




- Non-Cartesian Sampling
  - Design of gradient waveform
  - Image recon needs gridding

Aliasing

What is the difference in acquisition between the two images?



Take home message

- Tomography imaging reconstruction is based on accumulating back-projections data directive, while the reconstructed images are blurred.
- The 2D Fourier transform of an image for reconstruction can be obtained by accumulating the Fourier transform of the projections at different angles (Fourier-Slice Theorem).
- Key problem: how to use less projections to reconstruct clear images?