CS244 Theory of Computation
Homework 2

Due: October 23, 2022 at 11:59pm

Name - ID

You may discuss this assignment with other students and work on the problems together. However, your write-up should be your own individual work and you should indicate in your submission who you worked with, if applicable. You should use the LaTeX template provided by us to write your solution and submit the generated PDF file into Gradescope.

I worked with: (Name, ID), (Name, ID), ...

Let $\Sigma = \{0, 1\}$ if not otherwise specified.

Problem 1

(10 points)
Let $C_1 = \{zuz |\ z \in 0^* \text{ and } u \in 0^*10^* \text{ where } |u| = |z|\}$. Show that C_1 is a CFL in two ways:

(a) (5 points) by giving a CFG that generates C_1, and

(b) (5 points) giving a PDA that recognizes C_1.

Problem 2

(15 points)
Let $C_2 = \{ztz |\ z \in 0^* \text{ and } t \in 0^*10^*10^*, \text{ where } |t| = |z|\}$.

(a) (5 points) Show that C_2 is not a CFL.

(b) (5 points) Is $C_2 \cup (\Sigma\Sigma\Sigma)^*$ a CFL? Why or why not?

(c) (5 points) Is $C_2 \cup \Sigma(\Sigma\Sigma\Sigma)^*$ a CFL? Why or why not?

Problem 3

(15 points)
Let $G = (V, \Sigma, R, \langle \text{STMT} \rangle)$ be the following grammar. $\Sigma = \{\texttt{if}, \texttt{condition}, \texttt{then}, \texttt{else}, \texttt{a:=1}\}$, $V = \{\langle \text{STMT} \rangle, \langle \text{IF-THEN} \rangle, \langle \text{IF-THEN-ELSE} \rangle, \langle \text{ASSIGN} \rangle\}$ and the rules are:

$\langle \text{STMT} \rangle \rightarrow \langle \text{ASSIGN} \rangle \mid \langle \text{IF-THEN} \rangle \mid \langle \text{IF-THEN-ELSE} \rangle$

$\langle \text{IF-THEN} \rangle \rightarrow \texttt{if condition then} \langle \text{STMT} \rangle$

$\langle \text{IF-THEN-ELSE} \rangle \rightarrow \texttt{if condition then} \langle \text{STMT} \rangle \texttt{else} \langle \text{STMT} \rangle$

$\langle \text{ASSIGN} \rangle \rightarrow \texttt{a:=1}$

Problem 3 continued on next page...

(a) (10 points) Show that G is ambiguous.

(b) (5 points) Give a new unambiguous grammar that generates $L(G)$.
(You do not need to prove that your grammar works or that it is unambiguous, but please add a few comments about why it does work to help the grader.)

Problem 4

(20 points)
Show that if G is a CFG in Chomsky normal form, then for any string $w \in L(G)$ of length $n \geq 1$, exactly $2n - 1$ steps are required for any derivation of w.
(This is the benefit of Chomsky normal form)

Problem 5

(20 points)
Let the ***rotational closure*** of language A be $\mathrm{RC}(A) = \{yx \mid xy \in A \text{ where } x, y \in \Sigma^*\}$. Show that the class of CFLs is closed under rotational closure.

Problem 6

(20 points)
Let $C = \left\{ww^{\mathcal{R}} \mid w \in \{0,1\}^*\right\}$. We proved that C is a CFL in the Quiz by giving a PDA that recognizes it. Is C a DCFL? Why or why not?
(Hint: Suppose that when some DPDA P is started in state q with symbol x on the top of its stack, P never pops its stack below x, no matter what input string P reads from that point on. In that case, the contents of P's stack at that point cannot affect its subsequent behavior, so P's subsequent behavior can depend only on q and x.)