Lecture 3 – Digital Image Processing

This lecture will cover:

– Digital Image

– Basic Image Operation

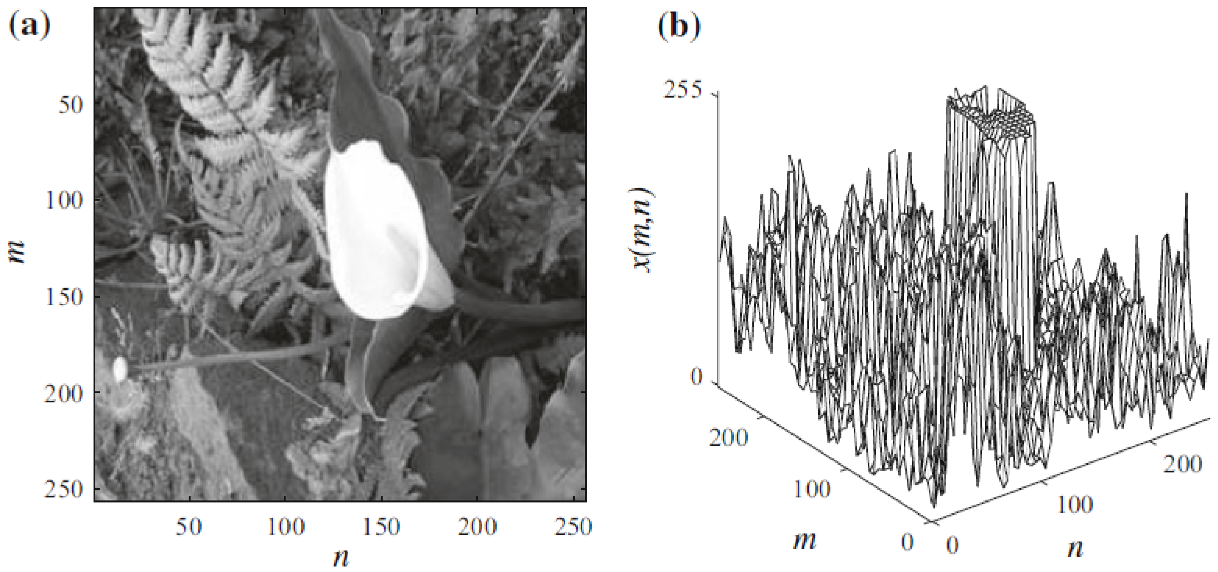
- Array and Matrix Operation
- Vector and Matrix Operation
- Linear and Nonlinear Operation
- Set and Logical Operation
- Arithmetic Operation

– Spatial Operation

– Image filtering

Digital image

- **A visual representation in form of a function *f*(*x*,*y*), where**
  - *f* is related to the intensity or brightness (color) at point
  - (*x*, *y*) are spatial coordinates
  - *x*, *y*, and the amplitude of *f* are finite and discrete quantities

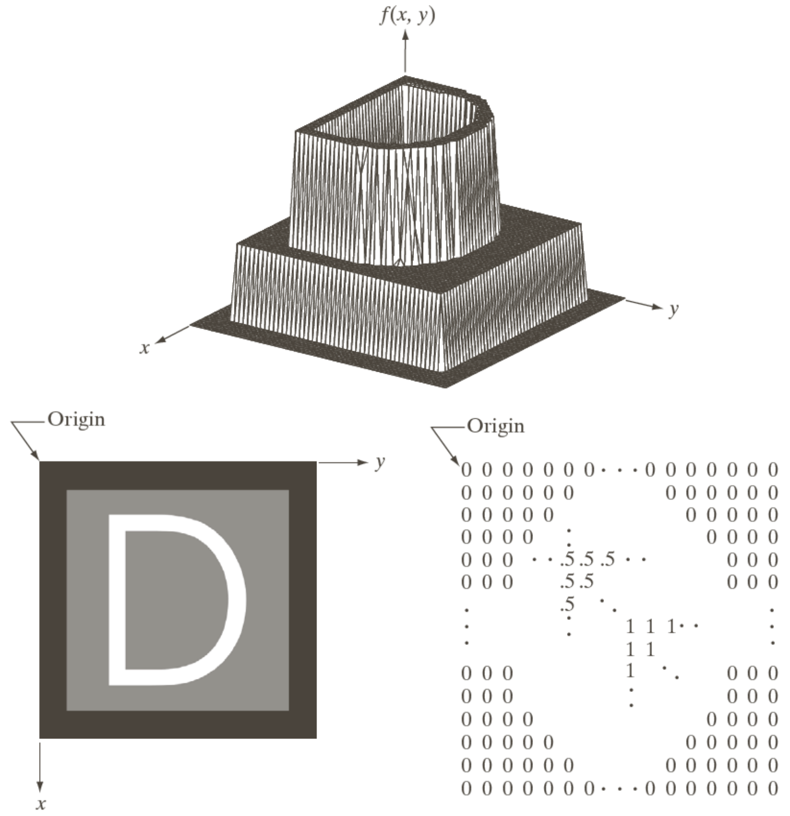


(a) A 256X256 image with 256 gray levels; (b) its amplitude profile

Matrix Representation

Three basic ways to represent *f*(*x, y*)

- Plot of function: *difficult to view and interpret*
- Visual intensity array: *for view*
- numerical array: *for processing and algorithm development*



$$[f(x,y)] = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \ddots & \cdots & \vdots \\ f(M-1,0) & f(M-1,1)\cdots & \cdots & f(M-1,N-1) \end{bmatrix}$$

$$A = \begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \vdots & \ddots & \cdots & \vdots \\ a_{M-1,0} & a_{M-1,1}\cdots & \cdots & a_{M-1,N-1} \end{bmatrix}$$

Intensity level $L = 2^k$, then $b = M \times N \times k$

Array and Matrix Operation

Consider two 2 x 2 image

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

- **Array product**

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix}$$

- **Matrix product**

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Vector and Matrix Operation

- **Multispectral image processing**

A pixel in a n-dimensional space can be expressed as a column vector $Z = [z_1, z_2 \dots z_n]^T$, then a vector norm between two pixels Z and A

$$\|Z - A\| = [(Z - A)^T (Z - A)]^{\frac{1}{2}}$$
$$= [(z_1 - a_1)^2 + (z_2 - a_2)^2 + \cdots + (z_n - a_n)^2]^{\frac{1}{2}}$$

- **Linear transformations**

$$g = Hf + n$$

Linear and Nonlinear Operation

An operator

$$H[f(x,y)] = g(x,y)$$

is linear if

$$\begin{aligned} H[a_i f_i(x,y) + a_j f_j(x,y)] &= a_i H[f_i(x,y)] + a_j H[f_j(x,y)] \\ &= a_i g_i(x,y) + a_j g_j(x,y) \end{aligned}$$

- Additivity (相加性)
- Homogeneity (同质性)

Set Operation (Coordinates)

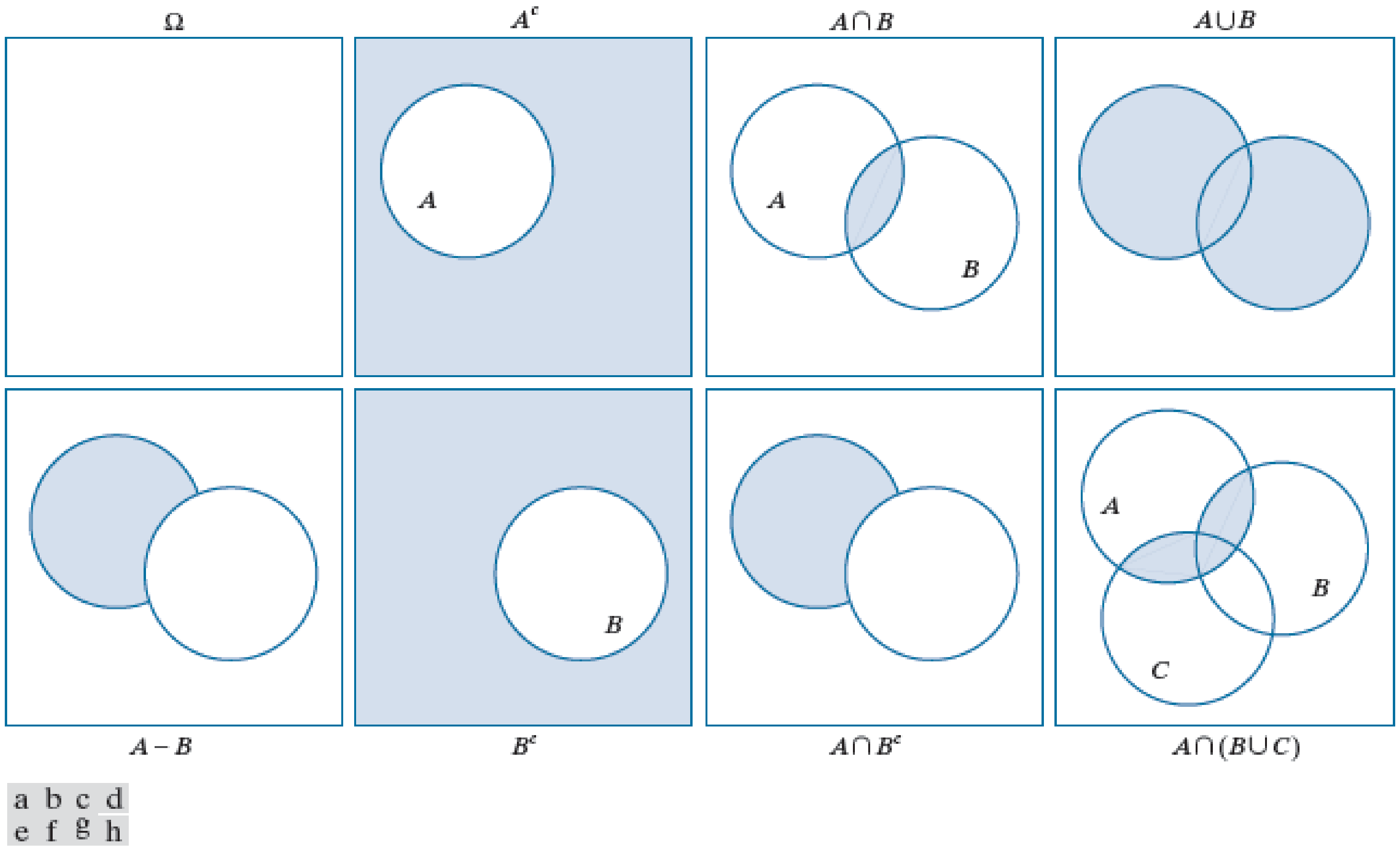


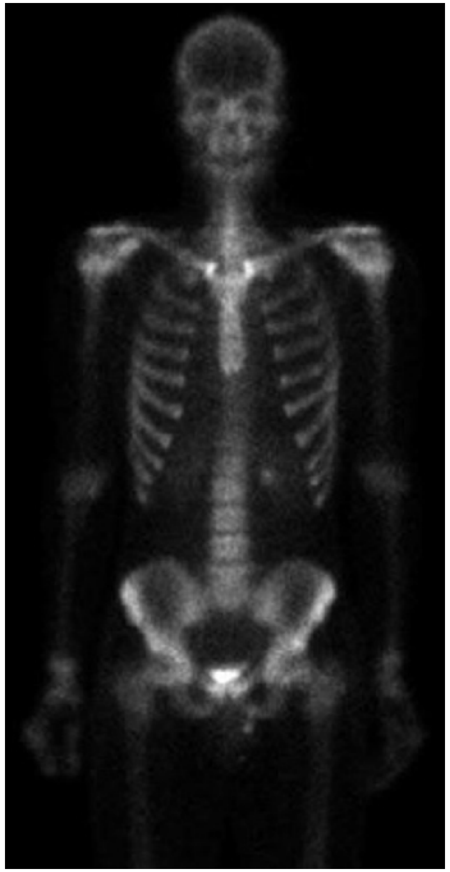
FIGURE 2.35 Venn diagrams corresponding to some of the set operations in Table 2.1. The results of the operations, such as A^c, are shown shaded. Figures (e) and (g) are the same, proving via Venn diagrams that $A - B = A \cap B^c$ [see Eq. (2-40)].

Set Operation (Intensity)

a b c

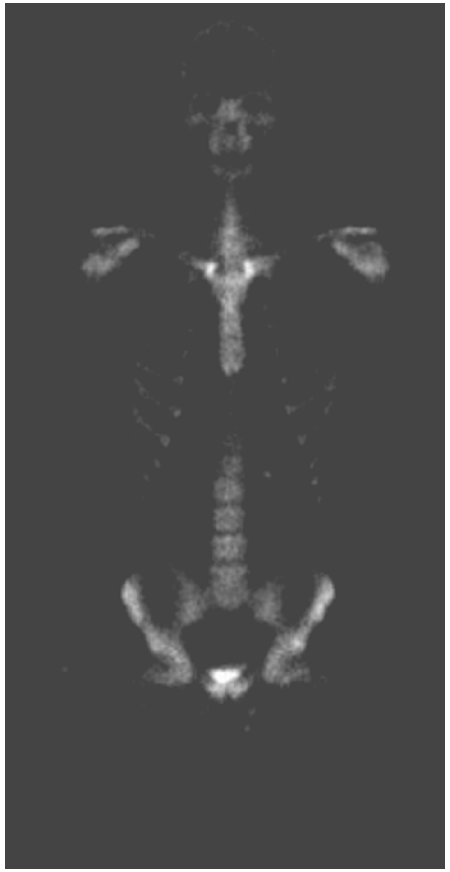
FIGURE 2.36 Set operations involving grayscale images. (a) Original image. (b) Image negative obtained using grayscale set complementation. (c) The union of image (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)

(A) Original image



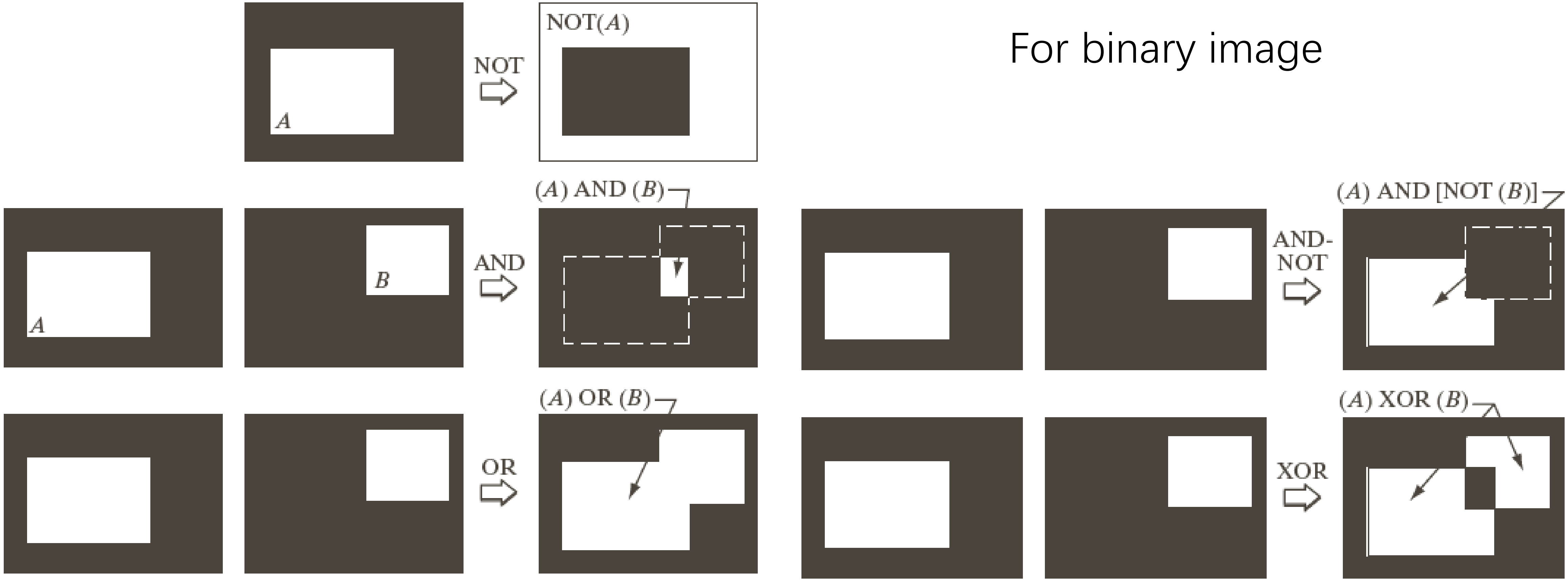
(B) Complement image $Z = 255 - A$

(C) Union: $A \cup 3\bar{Z} = \{\max(a, 3\bar{Z}) \,|a \in A\,\}$



Logical Operation

For binary image



Arithmetic Operation

- **Addition**

$$s(x,y) = f(x,y) + g(x,y)$$

- **Subtraction**

$$d(x,y) = f(x,y) - g(x,y)$$

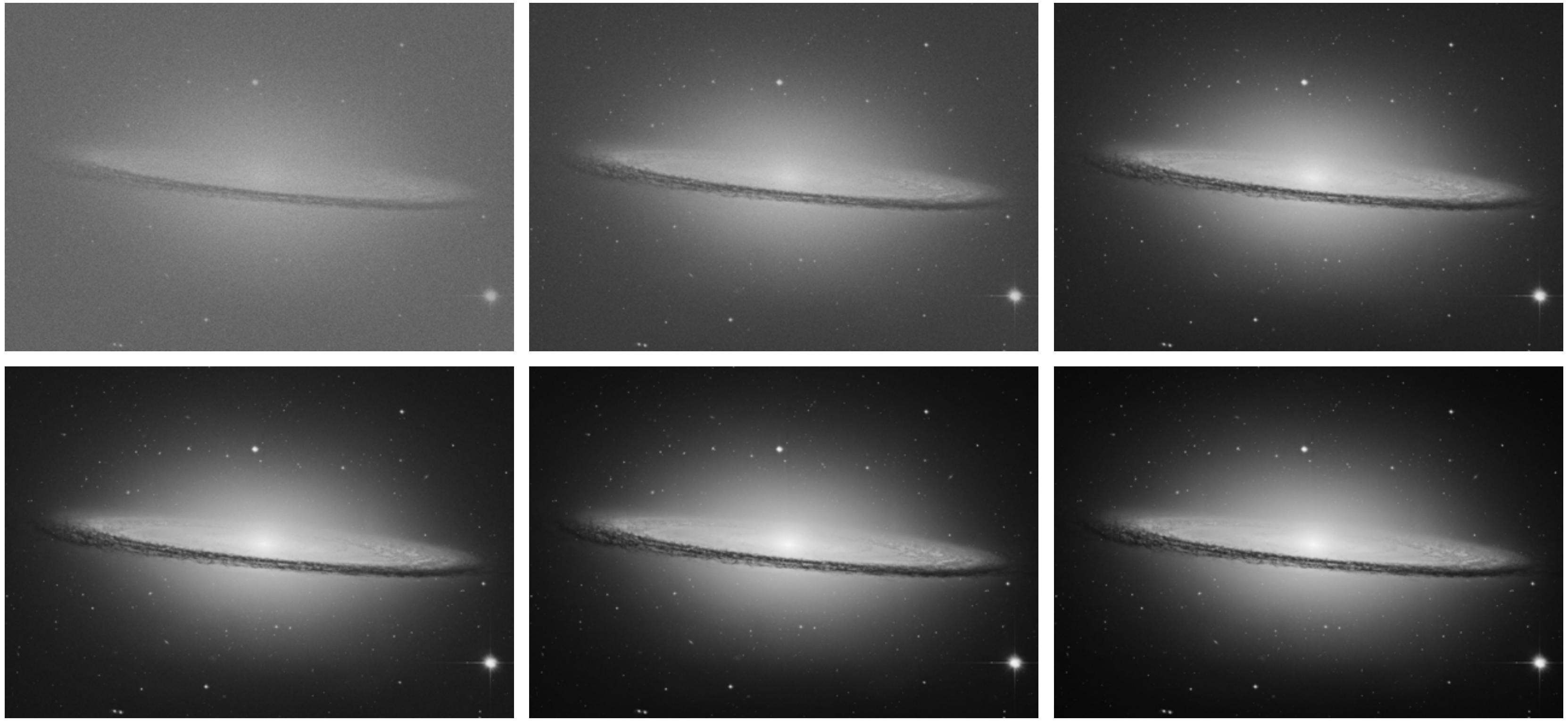
- **Multiplication**

$$p(x,y) = f(x,y) \times g(x,y)$$

- **Division**

$$v(x,y) = f(x,y) \div g(x,y)$$

Image Addition



a b c
d e f

FIGURE 2.29 (a) Sample noisy image of the Sombrero Galaxy. (b)-(f) Result of averaging 10, 50, 100, 500, and 1,000 noisy images, respectively. All images are of size 1548×2238 pixels, and all were scaled so that their intensities would span the full [0, 255] intensity scale. (Discovered in 1767, the Sombrero Galaxy is 28 light years from Earth. Original image courtesy of NASA.)

Image Subtraction

a b
c d

FIGURE 2.32 Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of the Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)

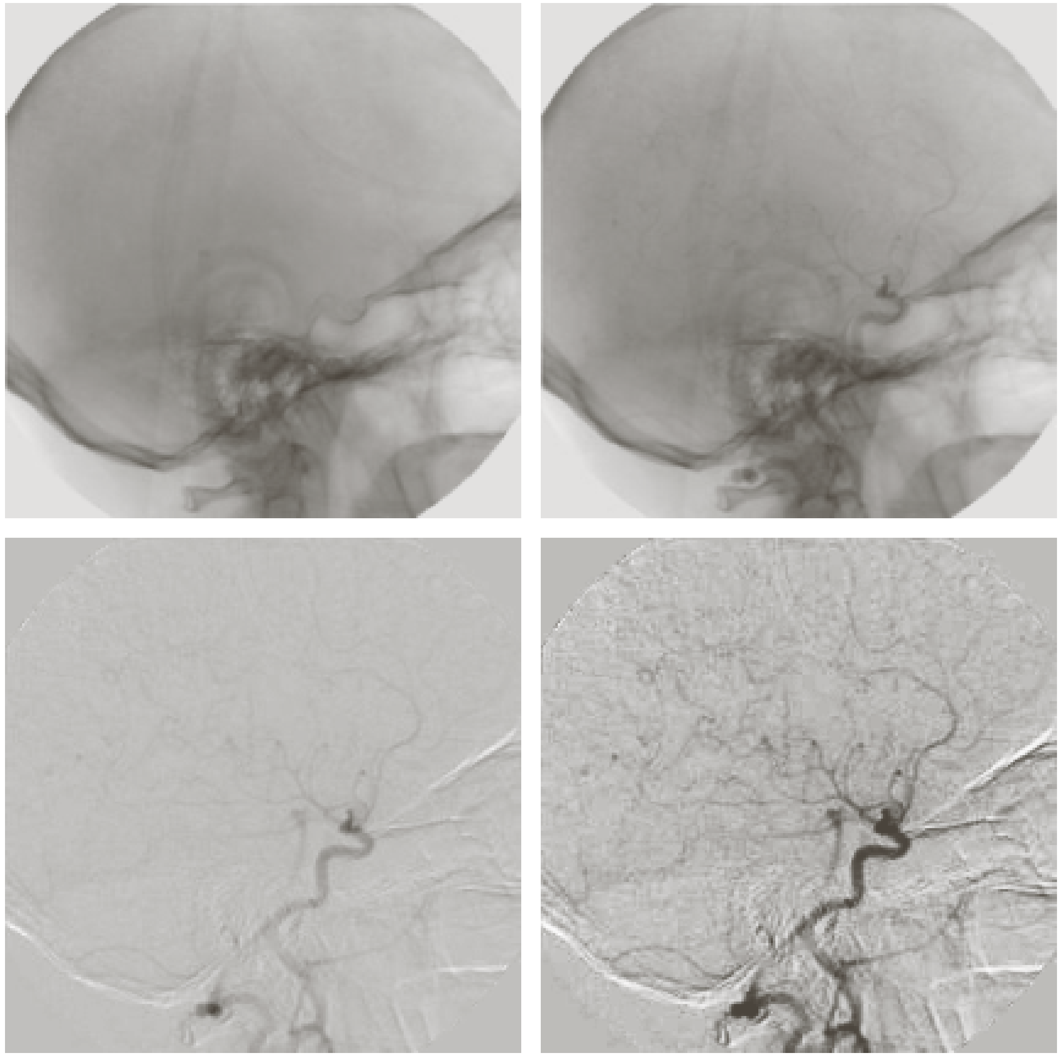
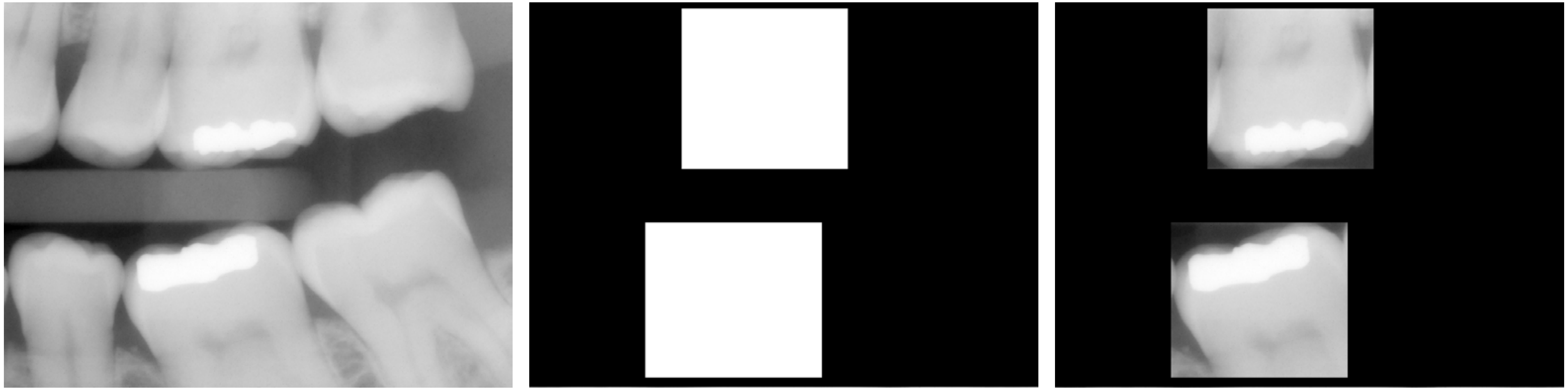


Image Multiplication



a b c

FIGURE 2.34 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

Image Division

$g(x, y) = f(x, y)\,h(x, y)$ $h(x, y)$ $f(x, y)$

$$f(x, y) = g(x, y) / h(x, y)$$

Spatial Operation

Performed directly on the pixels of the image

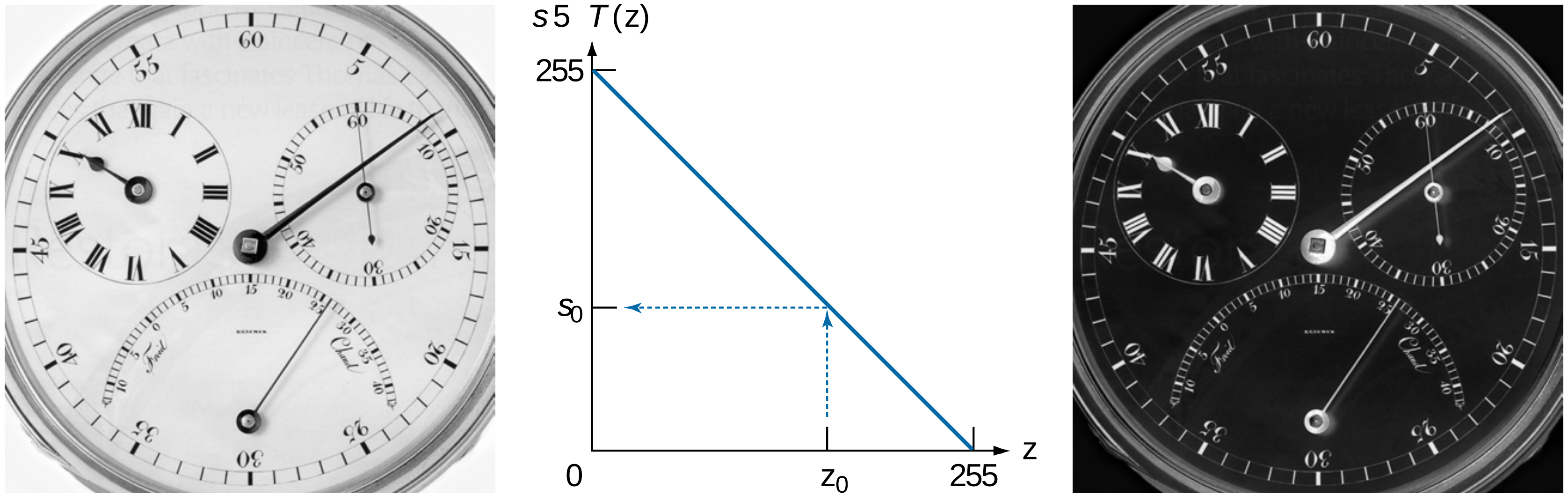
- Single-pixel operations
- Neighborhood operations
- Image geometry

  Scale, Rotate, Translate, Mirror, Transpose, Shear, etc.

- Interpolation

Single-pixel Operation

$$S = T(z)$$

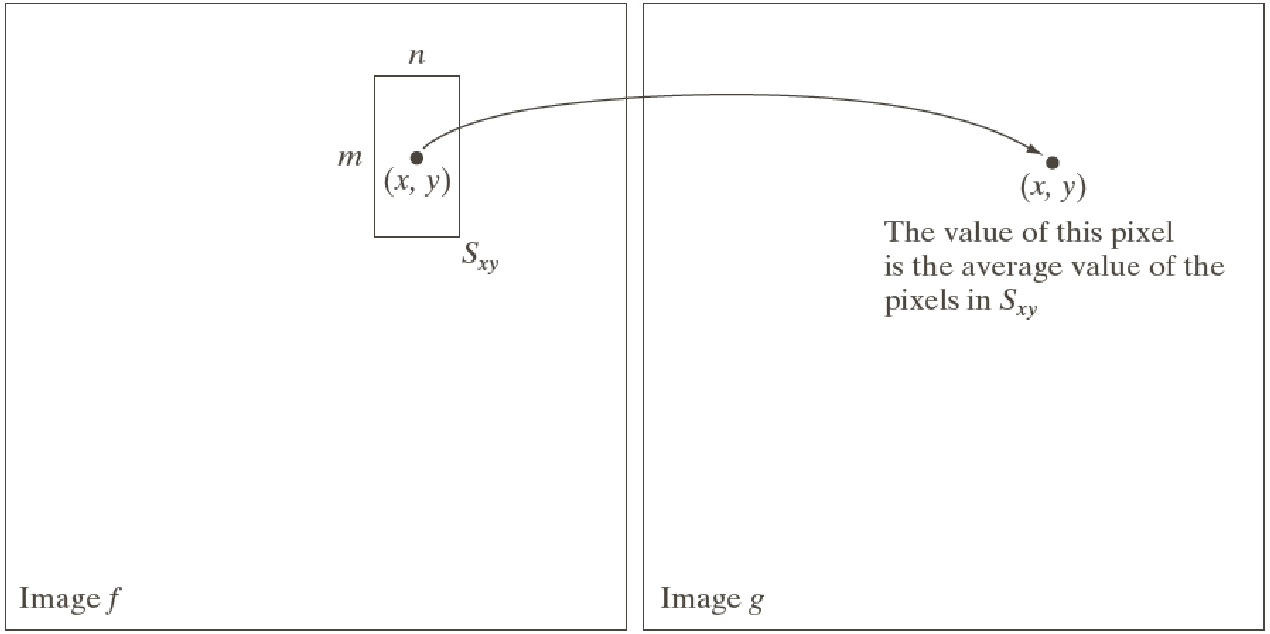


a b c

Figure 2.38 (a) An 8-bit image. (b) Intensity transformation function used to obtain the digital equivalent of a "photographic" negative of an 8-bit image. The arrows show transformation of an arbitrary input intensity value z_0 into its corresponding output value s_0. (c) Negative of (a), obtained using the transformation function in (b).

Neighborhood operation

S_{xy} is a region with center (x, y), $g(x,y) = \frac{1}{mn}\sum_{(r,c)\in S_{xy}} f(r,c)$



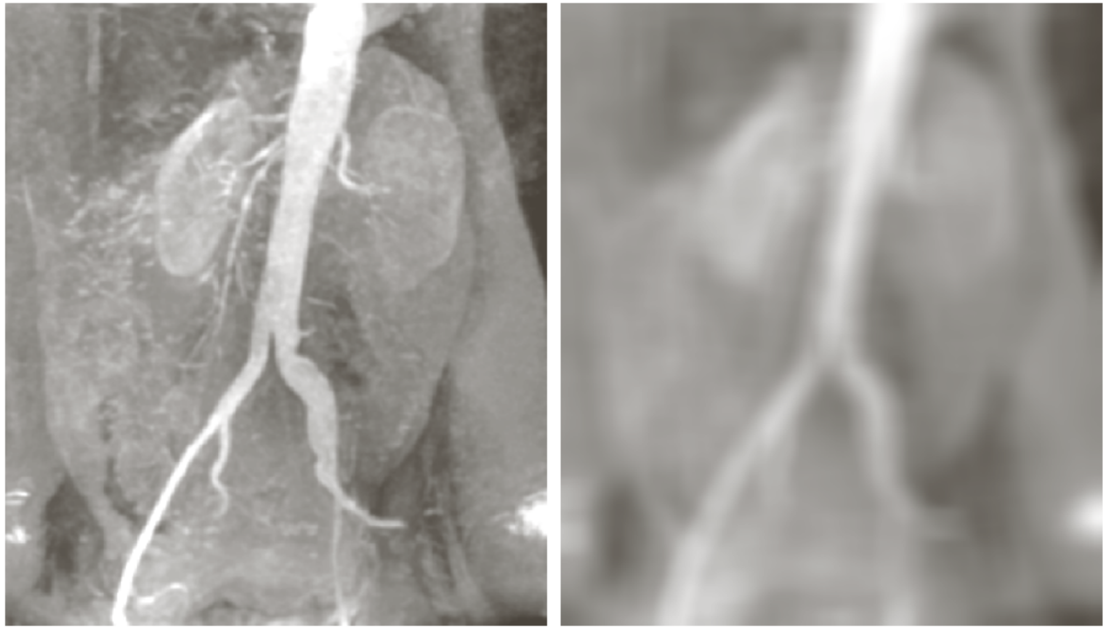


Image geometry

- **Modify spatial relationship between pixels –** *rubber-sheet*
  - Forward mapping (前向映射): $(x\ y) = T(v\ w)$
  - Inverse mapping (反向映射): $(v\ w) = T^{-1}(x\ y)$
- **Affine transform (仿射变换)**

$$[x\ y\ 1] = [v\ w\ 1]T = [v\ w\ 1]\begin{bmatrix} t_1 & t_4 & 0 \\ t_2 & t_5 & 0 \\ t_3 & t_6 & 1 \end{bmatrix}$$

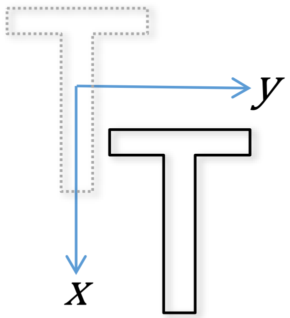
or

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = T\begin{bmatrix} v \\ w \\ 1 \end{bmatrix} = \begin{bmatrix} t_1 & t_2 & t_3 \\ t_4 & t_5 & t_6 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} v \\ w \\ 1 \end{bmatrix}$$

Affine Transform

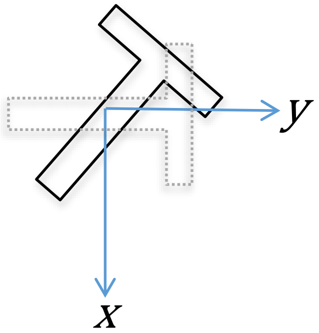
➢ **Translation**

$$\begin{cases} x = v + \Delta v \\ y = w + \Delta w \end{cases} \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \Delta v \\ 0 & 1 & \Delta w \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \\ 1 \end{bmatrix}$$



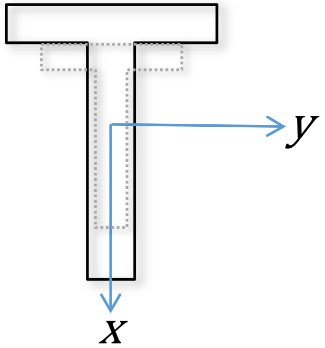
➢ **Rotation**

$$\begin{cases} x = v cos\beta - w sin\beta \\ y = v sin\beta + w cos\beta \end{cases} \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} cos\beta & -sin\beta & 0 \\ sin\beta & cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \\ 1 \end{bmatrix}$$



➢ **Scaling**

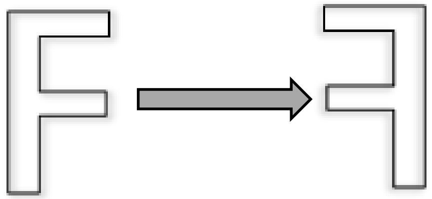
$$\begin{cases} x = c_x v \\ y = c_y w \end{cases} \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \\ 1 \end{bmatrix}$$



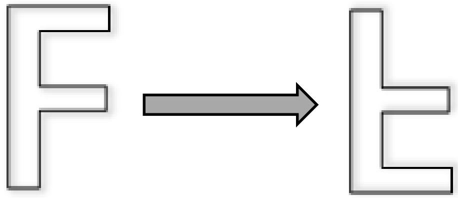
Affine Transform

- **Mirror**

Horizontal: $\begin{cases} x = W - v \\ y = w \end{cases} \Longrightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & W \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \\ 1 \end{bmatrix}$

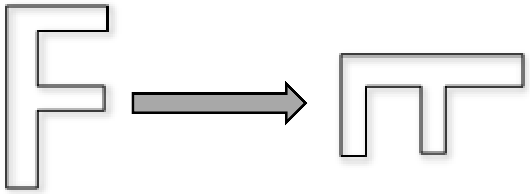


Vertical: $\begin{cases} x = v \\ y = H - w \end{cases} \Longrightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & H \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \\ 1 \end{bmatrix}$



- **Transpose**

$$\begin{cases} x = w \\ y = v \end{cases} \Longrightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \\ 1 \end{bmatrix}$$



Affine Transform

- **Shear**

Horizontal: $\begin{cases} x = v + c_y w \\ y = w \end{cases} \Longrightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & c_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \\ 1 \end{bmatrix}$

Vertical: $\begin{cases} x = v \\ y = c_x v + w \end{cases} \Longrightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ c_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \\ 1 \end{bmatrix}$

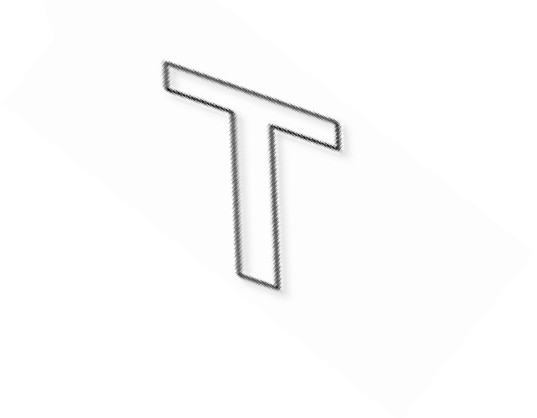
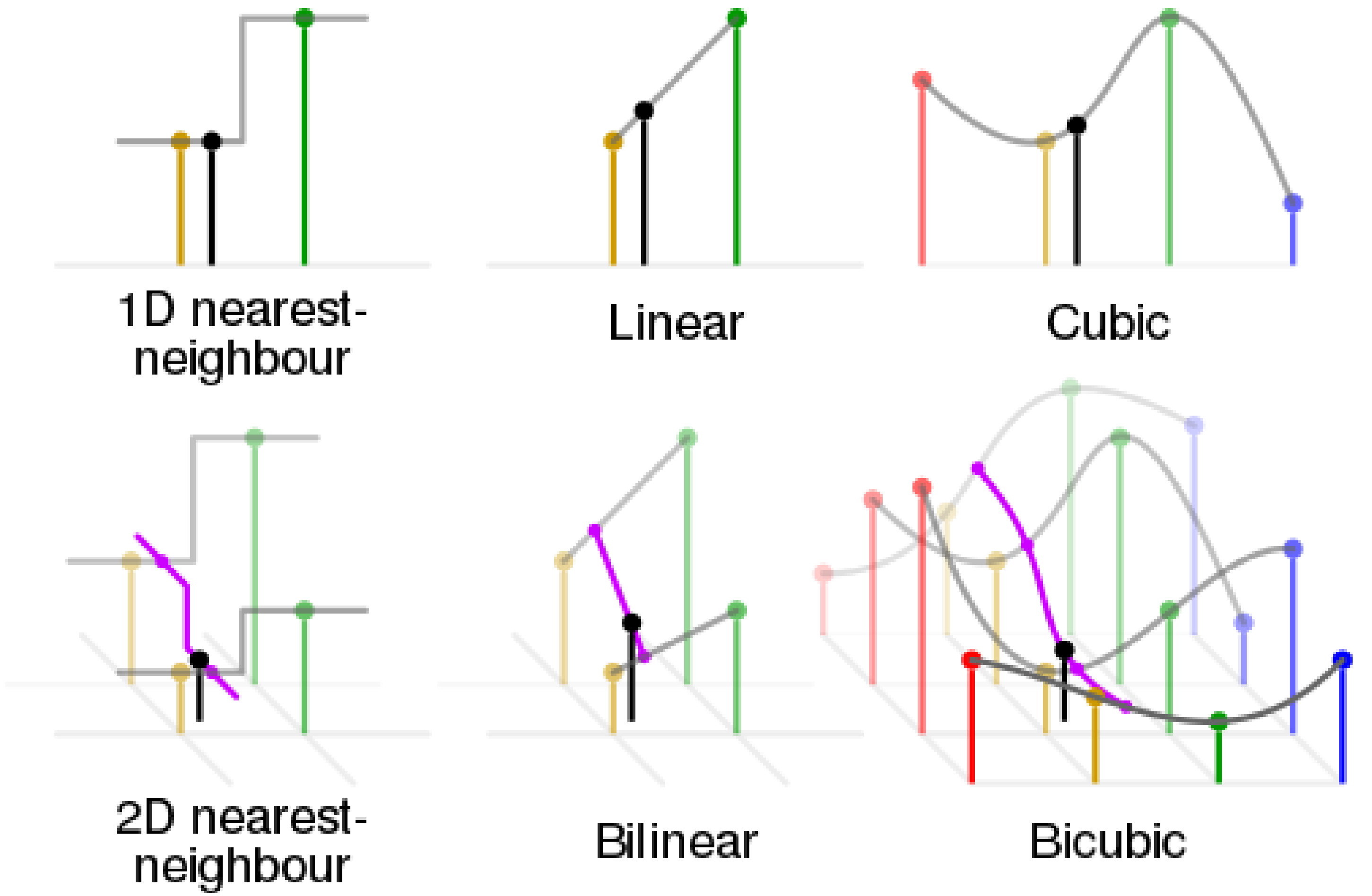
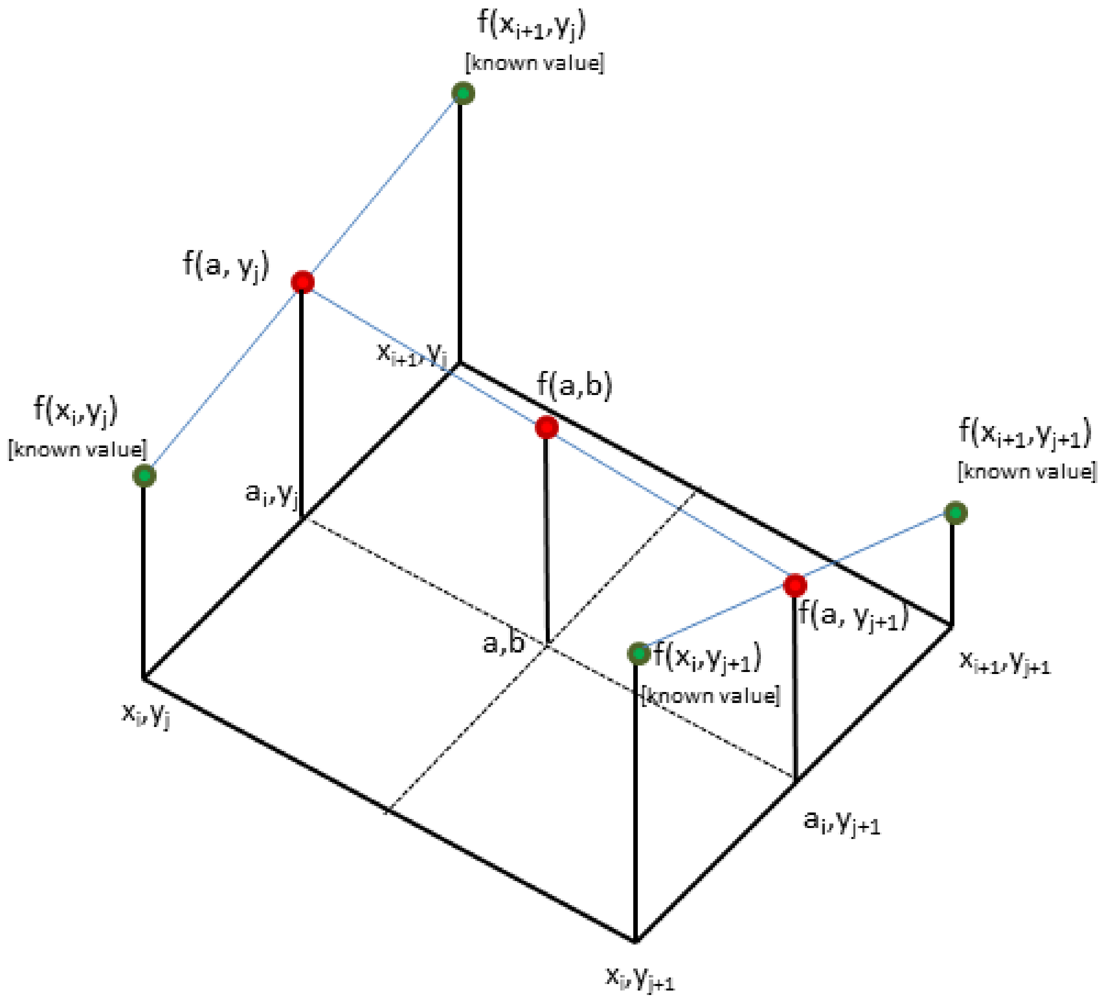


Image Interpolation (插值)

- ➢ Use known data to estimate values at unknown locations
- ➢ A resampling method
- ➢ Intensity interpolation



Bilinear interpolation



Interpolation

a b c
d e f

Image interpolation:

interpolate the image from 72dpi to 1250 dpi using (a)nearest neighbor method, (b) Bilinear method, (c) Bicubic method;

interpolate the image from 150dpi to 1250 dpi using (a)nearest neighbor method, (b) Bilinear method, (c) Bicubic method;

Image Filtering

- **High-pass**
  - Small objects and edges
  - Sharpening
  - Improving spatial resolution
  - More noise
  - SNR decrease
- **Low-pass**
  - Smoothing
  - Little effect on spatial resolution
  - Attenuating noise
  - SNR increase

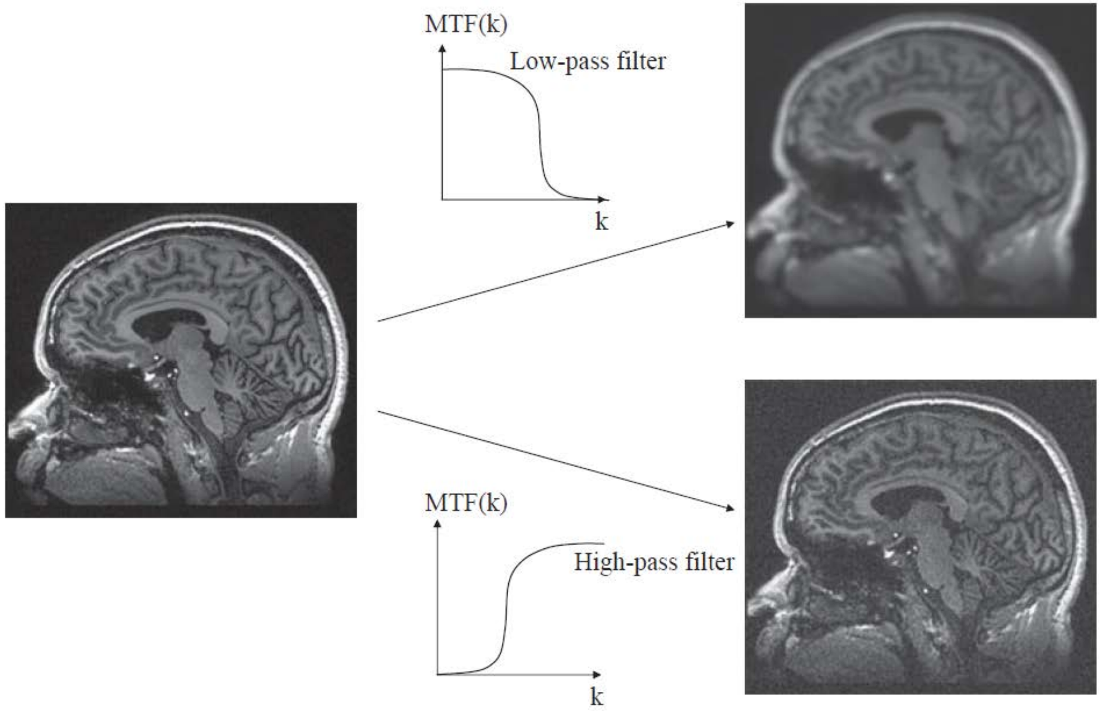


Image Filtering

original image

1	5	3	5	4	6
4	3	32	5	6	9
6	10	4	8	8	7

*

filter

1/12	1/12	1/12
1/12	4/12	1/12
1/12	1/12	1/12

=

filtered image

1	5	3	5	4	6
4	a	b	c	d	9
6	10	4	8	8	7

$$\begin{cases} a=(1)(1/12)+(5)(1/12)+(3)(1/12)+(4)(1/12)+(3)(4/12)+(32)(1/12)+(6)(1/12)+(10)(1/12)+(4)(1/12)=6.4 \\ b=(5)(1/12)+(3)(1/12)+(5)(1/12)+(3)(1/12)+(32)(4/12)+(5)(1/12)+(10)(1/12)+(4)(1/12)+(8)(1/12)=14.3 \\ c=(3)(1/12)+(5)(1/12)+(4)(1/12)+(32)(1/12)+(5)(4/12)+(6)(1/12)+(4)(1/12)+(8)(1/12)+(8)(1/12)=7.5 \\ d=(5)(1/12)+(4)(1/12)+(6)(1/12)+(5)(1/12)+(6)(4/12)+(9)(1/12)+(8)(1/12)+(8)(1/12)+(7)(1/12)=6.3 \end{cases}$$

1	5	3	5	4	6
4	6.4	14.3	7.5	6	9
6	10	4	8	8	7

filtered image