Homework 5

Submit: Blackboard/Paper Due: Dec. 13th 13：00

Please write down Your Name & Student ID

1. Assuming that there are $5 * 10^{22}$ protons in a cubic centimeter of water, what is the magnetization contained within this volume at a magnetic field strength of 2 Tesla at room temperature (300K)? (10)

   Solution:
   The net magnetization is defined as the sum of magnetic moment of nuclei involved. It is determined by the population difference of nuclei occupying the two energy levels, multiplied by the z component of the magnetic moment for each nucleus:

$$\mathrm{M}_0 = \mu_z\left(N_{parallel} - N_{anti-parallel}\right) = \frac{\gamma^2 h^2 B_0 N_s}{16\pi^2 kT}$$

   At room temperature,

$$\mathrm{M}_0 = \frac{(42.58 * 10^6 * 2\pi)^2 (6.63 * 10^{-34})^2 * 2 * 5 * 10^{22}}{4 * 4\pi^2 * 1.38 * 10^{-23} * 300} = 4.81 * 10^{-9} J/T$$

2. Calculate the effects of the following pulse sequences on thermal equilibrium magnetization. The final answer should include x-, y-, and z-components of magnetization.) (9)
   a) 90°x (a pulse with tip angle 90°, applied about the x-axis).
   b) 60°x.
   c) 90°x 90°y (the second 90° pulse is applied immediately after the first).

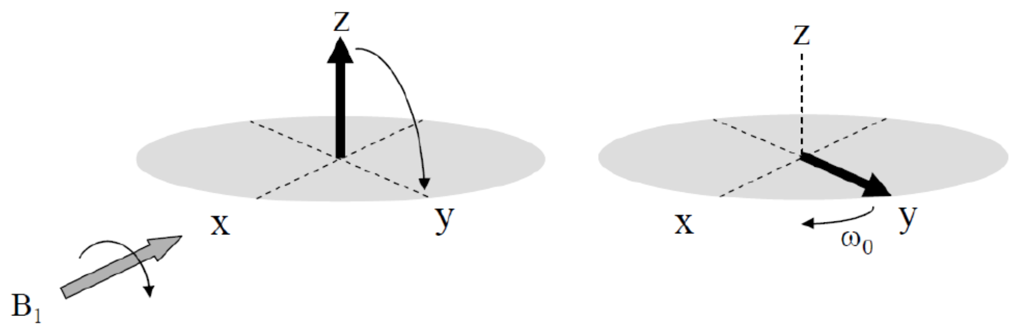


Figure 2

Solution:

(a) $M_z = 0, M_y = M_0, M_x = 0$

(b) $M_z = M_0 \cos 60 = 0.5\, M_0, M_y = M_0 \sin 60 = 0.866 M_0, M_x = 0$

(c) $M_z = 0, M_y = M_0, Mx = 0$

3. The operator wishes to acquire an oblique slice shown by the orientation of the white bar in Figure 3 (coronal plane). Draw the gradient echo imaging sequence that would be run to acquire such an image. (10)

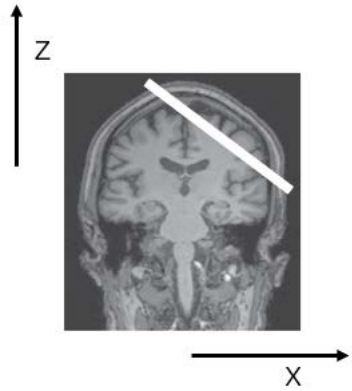
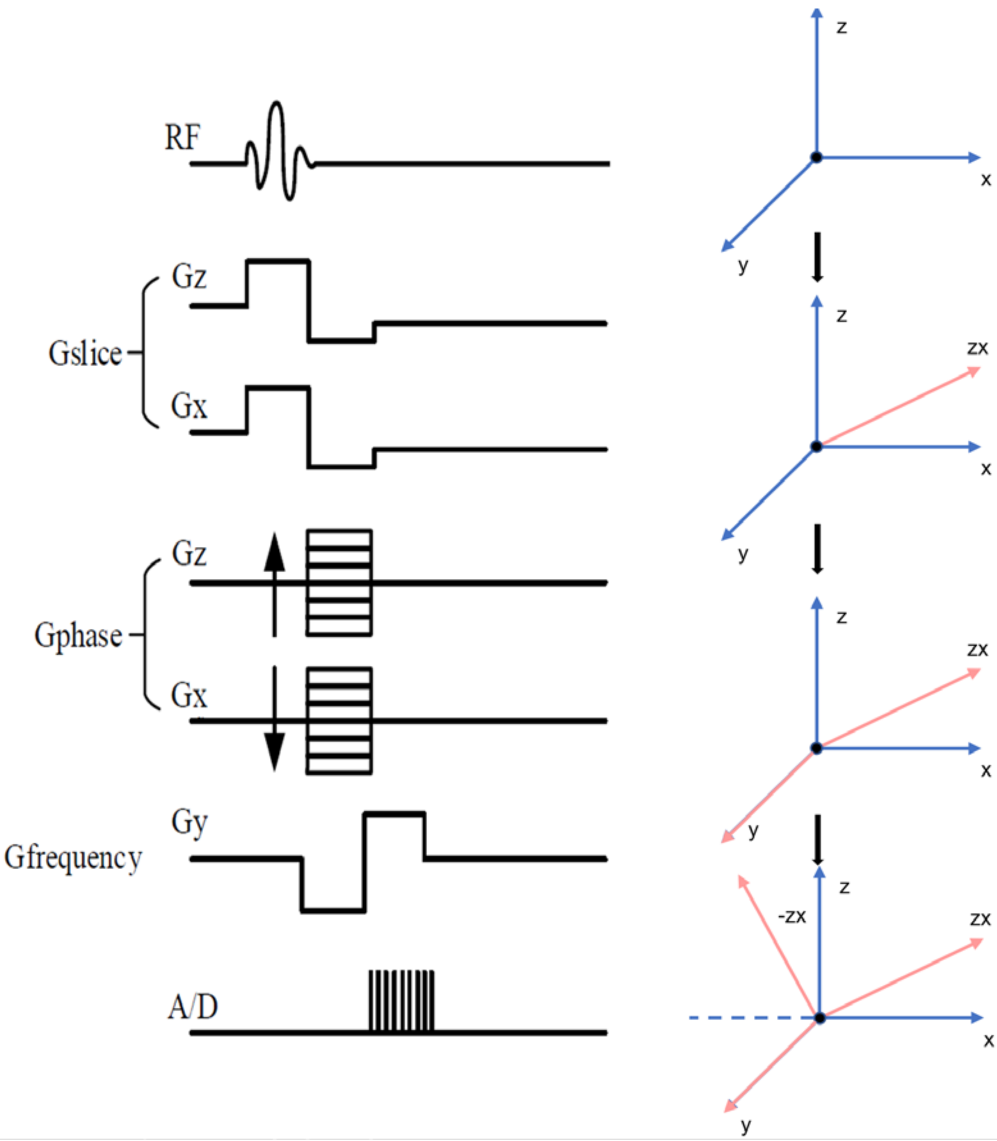


Figure 3

Solution:

The oblique slice is in the coronal plane(z-x), and therefore both gradients must be applied in the slice select direction, with approximately equal strengths since the slice is at an angle of $\sim 45°$. Since the phase and frequency encoding gradients are both applied orthogonal to the slice selection gradient, both phase and frequency encoding must also have contributions from two gradients. So, the sequence is (note that the phase and frequency axes can be interchanged).


RF
Gslice
Gz
Gx
Gphase
Gz
Gx
Gfrequency
Gy
A/D
z
x
y
zx
-zx

4. Derive the value of the Ernst angle given in Equation (10)

$$\alpha_{\mathrm{Ernst}} = \cos^{-1} e^{-\frac{TR}{T1}}$$

Solution:
In the gradient-echo sequence, the $\mathrm{M_z}$ magnetization reaches a steady state. This means that the reduction in $\mathrm{M_z}$ from a tip angle of α degrees, is e balanced by the gain in $\mathrm{M_z}$ due to $\mathrm{T_1}$ relaxation during the TR interval, i.e.

$$\mathrm{M_z} = M_z \cos\alpha + (M_0 - M_z \cos\alpha)\left(1 - e^{-\frac{TR}{T_1}}\right)$$

Re-arranging and solving for the steady-state value of $\mathrm{M_z}$ gives:

$$\mathrm{M_z} = M_0 \frac{1 - e^{-\frac{TR}{T_1}}}{1 - \cos\alpha\, e^{-\frac{TR}{T_1}}}$$

Therefore, the steady state signal intensity is given by:

$$\mathrm{M_y} = \frac{M_0 \sin\alpha \left(1 - e^{-\frac{TR}{T_1}}\right)}{1 - \cos\alpha\, e^{-\frac{TR}{T_1}}}$$

In order to determine the optimum value of α for a given value of TR, set:

$$\frac{\partial \mathrm{M_y}}{\partial \mathrm{\alpha}} = 0$$

Which leads directly to:

$$\alpha = \cos^{-1} e^{-\frac{TR}{T1}}$$

5. Choose the correct option from (a)-(e) and explain why this is your choice. The maximum MR signal is obtained by using (10)
   (a) $90°$ RF pulse, short TE, and long TR;
   (b) $45°$ RF pulse, short TE, and short TR;
   (c) $90°$ RF pulse, long TE, and short TR;
   (d) $90°$ RF pulse, short TE, and short TR;
   (e) $45°$ RF pulse, long TE, and short TR.

   Solution:
   The correct solution is (a). In order to maximize the signal, the tip angle should be $90°$ which requires a long TR for the magnetization to recover to M_0 for full T_1 relaxation. T_2 relaxation decreases the signal, and so should be minimized by choosing a short value of TE.

6. Choose the correct option from (a)-(e) and explain why this is your choice. Water in tendons is bound very strongly and cannot diffuse freely. It produces very low MR signal intensity because (10)
   (a) T1 is too short;
   (b) T2 is too short;
   (c) T2* is very long;
   (d) T2 is longer than T1;
   (e) T2* is longer than T2.

   Solution:
   Very tightly bound tissue has a very short T_2 and T_2^* which produces a low signal intensity. Therefore, (b) is the only possibility that is correct. T_2 can never be longer than T1 and so (d) is incorrect, and T_2^* can never be longer than T_2 so (e) is incorrect. A short T_1 gives a high MR signal, as does a long T_2^*, so both (a) and (c) are wrong.

7. Three images are shown in Figure 7: the scaling in each image is different and is normalized to the same maximum value. for three images are (1) TR = 3000 ms, TE = 15 ms, (2) TR = 900 ms, TE = 60 ms (3) TR = 3000 ms, TE = 60 ms respectivly.
   (i) Assign each image to the appropriate TR and TE values. (6)
   (ii) Given that the ventricles in the middle image is much brighter than in the right image, while the brain tissue is slightly brighter. Based on your answer, do the ventricles have a higher or lower T1 value than brain tissue? What is the corresponding answer for T2 (5)

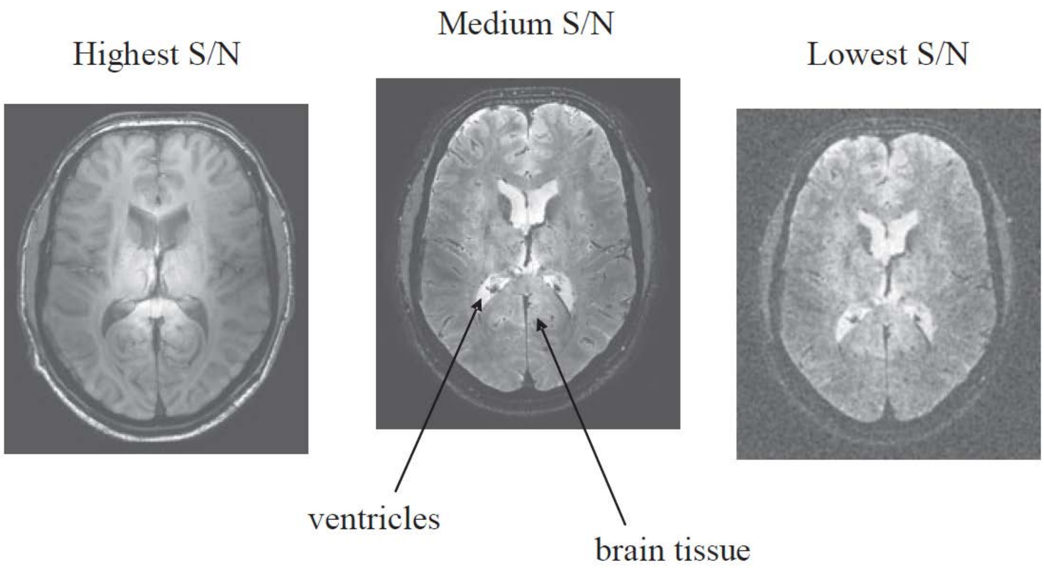


Figure 7

Solution.

(i) The highest S/N corresponds to a long TR and short TE, and so the image on the left has TR 3000 ms, TE 15 ms. The lowest S/N has the shortest TR and longest TE, and so the image on the right has TR 900 ms, TE 60 ms.

(ii) For relative T1 values we compare images with different TR values, the middle and right images. Increasing the TR takes the ventricles from being isointense with brain tissue to much higher value, meaning that the T1 of the ventricles is higher than that of the brain. For relative T2 values we compare images with different TE values, the left and middle images. Increasing the TE takes the ventricles from being darker than brain tissue to brighter, and therefore the ventricles have a longer T2 than brain tissue.

8. Consider the pulse sequence in Figure 8 (surface 2 equals two times surface 1). Draw the trajectory of k in the k-space. shown in Figure 9. (15)

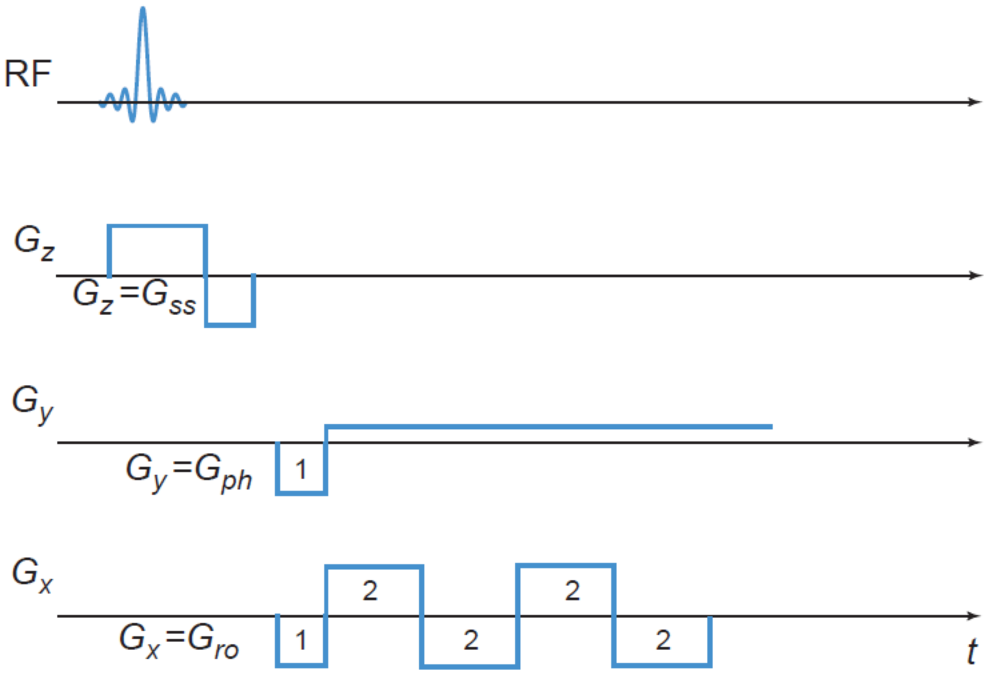
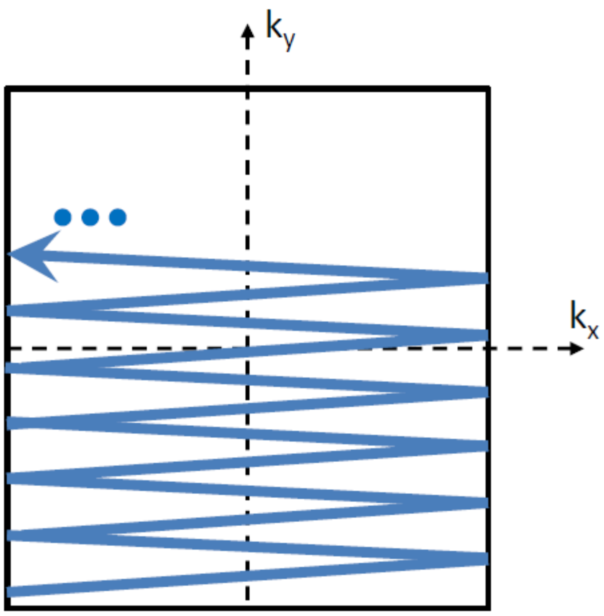


Figure 8

Solution:



9. Sketch an EPI pulse sequence that gives the square spiral k-space trajectory shown in Figure 9. Assume that the height of the right triangle is always greater than the base. (15)

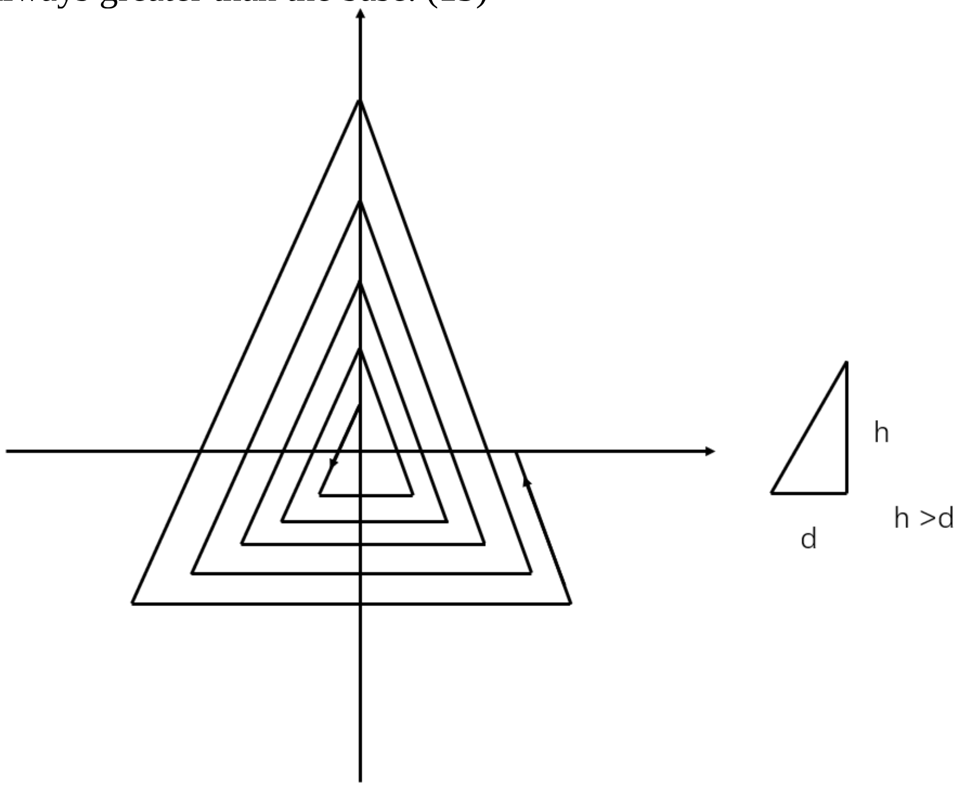
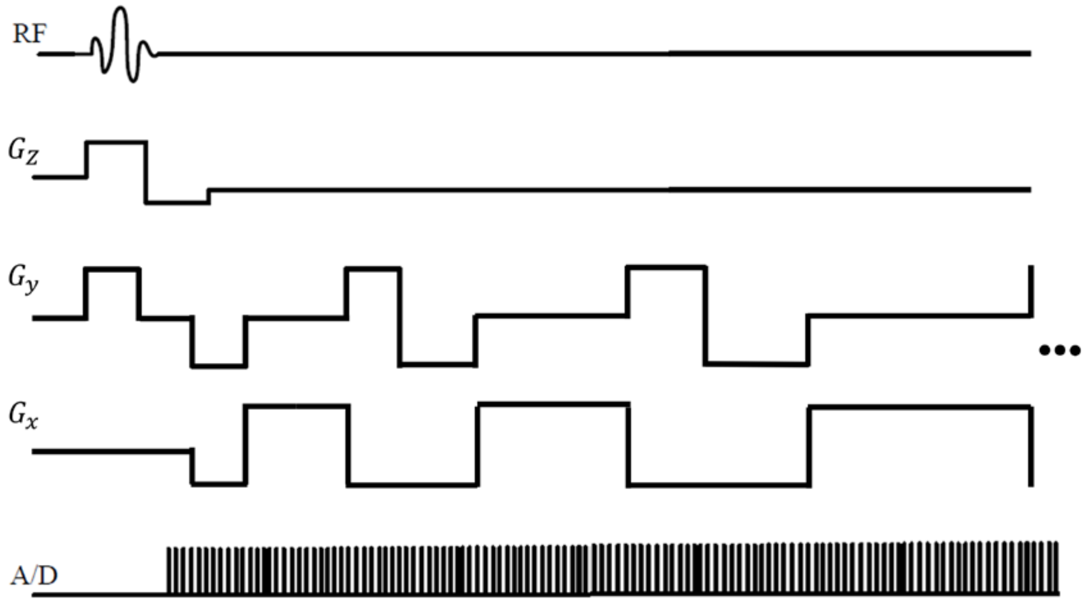


Figure 9

Solution:



Tips:

1. You should show the length of the encoding time or the gradient magnitude is gradually increasing.
2. You should show “the height of the right triangle is always greater than the base” by length of the encoding time of gradient magnitude.