SI152 - Numerical Optimization homework 2

Deadline: 2022-04-01 23:59:00

1. You can use Word, Latex or handwriting to complete this assignment. If you want to submit a handwritten version, scan it clearly.
2. The **report** has to be submitted as a PDF file to Blackboard, other formats are not accepted.
3. The submitted file name is **student_id+your_student_name.pdf**.
4. Late policy: You have 4 free late days for the quarter and may use up to 2 late days per assignment with no penalty. Once you have exhausted your free late days, we will deduct a late penalty of 25% per additional late day. Note: The timeout period is recorded in days, even if you delay for 1 minute, it will still be counted as a 1 late day.
5. You are required to follow ShanghaiTech's academic honesty policies. You are not allowed to copy materials from other students or from online or published resources. Violating academic honesty can result in serious sanctions.

Any plagiarism will get Zero point.

1. Using Simplex to solve the following problems. (30 pt)

$$\begin{aligned} \min \quad & 3x_1 - x_2 - 3x_3 + x_4 \\ \text{s.t.} \quad & x_1 + 2x_2 - x_3 + x_4 = 0 \\ & x_1 - x_2 + 2x_3 - x_4 = 6 \\ & 2x_1 - 2x_2 + 3x_3 + 3x_4 = 9 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned} \tag{1}$$

2. As shown in figure 1, please explain: If the primal simplex tableau contains a unit matrix, why is the bottom position corresponding to the optimal multiplier after pivot completed? (Note: The answer needs to be directed to the general linear programming problem revolving axis, and cannot just explain the given example.) (20 pt)

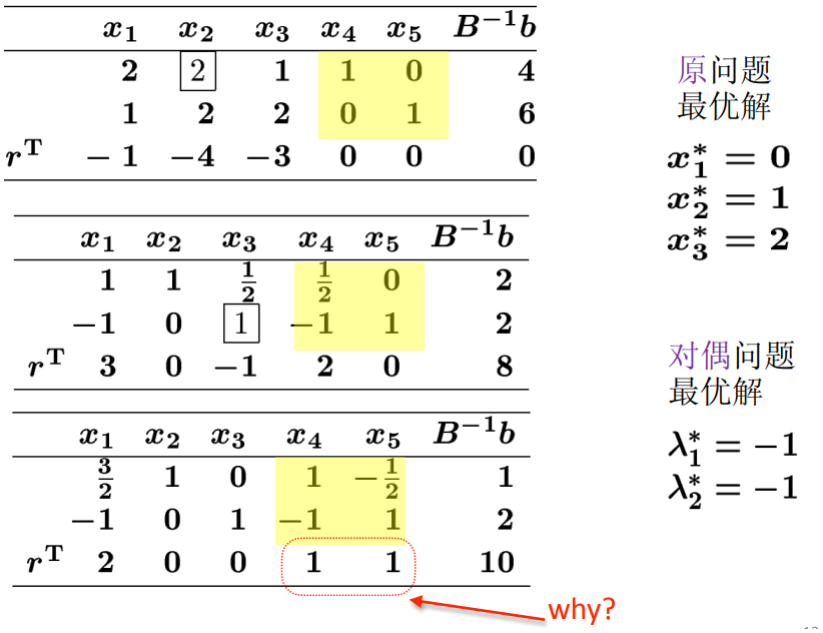

Figure 1: Slides in lecture 5 page 13

3. Prove complementary slackness.
Suppose $\hat{X}$ and $\hat{Y}$ is a feasible solution of primal and dual problem respectively. Then $\hat{Y}X_s = 0$ and $Y_s\hat{X} = 0$ holds iff $\hat{X}$ and $\hat{Y}$ are optimal values, where X_s and Y_s being the vector of slack variables. (20 pt)

4. Given a linear programming problem:

$$\begin{aligned} \max \quad & 10x_1 + 7x_2 + 30x_3 + 2x_4 \\ \text{s.t.} \quad & x_1 - 6x_3 + x_4 \leq -2 \\ & x_1 + x_2 + 5x_3 - x_4 \leq -7 \\ & x_2, x_3, x_4 \geq 0 \end{aligned} \tag{2}$$

a) Write the dual problem (15 pt)
b) Use graphical methods to solve the optimal solution of the dual problem. (15 pt)