一、掌握热力学第零定律，理解温度的概念，温度和能量、热量的关系。

温度：
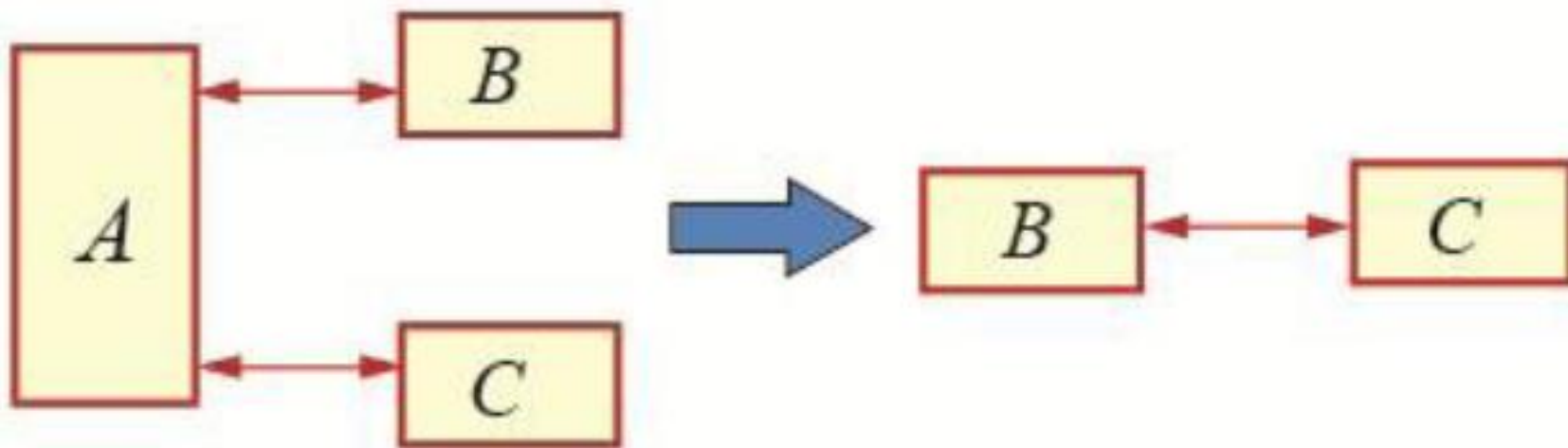
温度是对微观运动的统计描述

温度是个态函数，定义这个态函数为温度T，即标志着互为热平衡的系统具有同一个温度。

思考：给定一个体系，只跟温度相关，或者只由温度决定的的物理量有哪些？

如，理想气体分子平均动能/内能，卡诺循环的效率，。。。

热平衡: 两热力学系统相互接触, 而与外界没有热量交换, 当经过了足够长的时间后, 它们的冷热程度不再发生变化, 则称这两系统达到了热平衡。

热力学第零定律（热平衡的传递性）

如果系统A、B同时和系统C达到热平衡，则系统A和B也处于热平衡—**热力学第零定律**。

$$T_A(x_A, y_A) = T_B(x_B, y_B) = T_C(x_C, y_C)$$

可以定义这个态函数为温度 T，即标志着互为热平衡的系统 具有同一个温度。

•热力学第零定律是热力学三大定律的基础: 它定义了温度——达到 热平衡的系统具有共同的内部属性.

•热力学第零定律的物理意义：1）互为热平衡的物体之间必存在一个 相同的特征，即它们的温度是相同的。2）第零定律不仅给出了温度的概念, 而且指出了判别温度是否相同的方法。

二、掌握热量的计算

热容量：使系统温度升高1K所需的吸收的热量

$$C = \lim_{\mathrm{D}T \to 0} \frac{\mathrm{D}Q}{\mathrm{D}T} \qquad \text{单位：} \quad \mathbf{J \times K^{-1}}$$

比热容：单位质量的热容, specific heat

等容热容量：等容过程中系统温度升高1K所需的吸收的热量。

$$C_V = \lim_{\mathrm{D}T \to 0} \left(\frac{\mathrm{D}Q}{\mathrm{D}T} \right)_V$$

等压热容量：等压过程中系统温度升高1K所需的吸收的热量。

$$C_p = \lim_{\mathrm{D}T \to 0} \left(\frac{\mathrm{D}Q}{\mathrm{D}T} \right)_p$$

摩尔热容量： $C_m = C / n$

n 是系统的物质量(mol) $m = nM, C_m = MC, M$ 为摩尔质量

同相内的热量计算：

$$Q = mC\Delta T \qquad Q = nC_m\Delta T$$

有相变的热量计算还要考虑相变热：

Heat transfer in a phase change ·····→ $Q = \pm mL$

Mass of material that changes phase

Latent heat for this phase change

$+$ if heat enters material, $-$ if heat leaves

典型例题：

EXAMPLE 17.7 A temperature change with no phase change

A camper pours 0.300 kg of coffee, initially in a pot at 70.0°C, into a 0.120 kg aluminum cup initially at 20.0°C. What is the equilibrium temperature? Assume that coffee has the same specific heat as water and that no heat is exchanged with the surroundings.

IDENTIFY and SET UP The target variable is the common final temperature T of the cup and coffee. No phase changes occur, so we need only Eq. (17.13). With subscripts C for coffee, W for water, and Al for aluminum, we have $T_{0C} = 70.0°C$ and $T_{0Al} = 20.0°C$; Table 17.3 gives $c_W = 4190 \text{ J/kg} \cdot \text{K}$ and $c_{Al} = 910 \text{ J/kg} \cdot \text{K}$.

EXECUTE The (negative) heat gained by the coffee is $Q_C = m_C c_W \Delta T_C$. The (positive) heat gained by the cup is $Q_{Al} = m_{Al} c_{Al} \Delta T_{Al}$. We set $Q_C + Q_{Al} = 0$ (see Problem-Solving Strategy 17.2) and substitute $\Delta T_C = T - T_{0C}$ and $\Delta T_{Al} = T - T_{0Al}$:

$$Q_C + Q_{Al} = m_C c_W \Delta T_C + m_{Al} c_{Al} \Delta T_{Al} = 0$$

$$m_C c_W (T - T_{0C}) + m_{Al} c_{Al} (T - T_{0Al}) = 0$$

Then we solve this expression for the final temperature T. A little algebra gives

$$T = \frac{m_C c_W T_{0C} + m_{Al} c_{Al} T_{0Al}}{m_C c_W + m_{Al} c_{Al}} = 66.0°C$$

EVALUATE The final temperature is much closer to the initial temperature of the coffee than to that of the cup; water has a much higher specific heat than aluminum, and we have more than twice as much mass of water. We can also find the quantities of heat by substituting the value $T = 66.0°C$ back into the original equations. We find $Q_C = -5.0 \times 10^3$ J and $Q_{Al} = +5.0 \times 10^3$ J. As expected, Q_C is negative: The coffee loses heat to the cup.

KEYCONCEPT In a calorimetry problem in which two objects at different temperatures interact by exchanging heat, energy is conserved: The sum of the heat flows (one positive, one negative) into the two objects is zero. The heat flow stops when the two objects reach the same temperature.

三、掌握理想气体的定义、方程，分子平均动能，能量均分定理，C_p、C_V及其关系的内在机制

理想气体方程:

Ideal-gas equation:

Gas pressure ⋯⋯⋯ $pV = nRT$ ⋯⋯⋯ Number of moles of gas

Gas volume ⋯⋯⋯ ⋯⋯⋯ Absolute temperature of gas

⋯⋯⋯ Gas constant

Gas constant R=8.314 J·K⁻¹·mol⁻¹

理想气体为假想的气体。假设气体分子本身不占有体积，气体分子持续以直线运动，并且与容器器壁间发生弹性碰撞，因而对器壁施加压强，气体分子间无作用力，气体分子的平均能量与开尔文温度成正比。适用理想气体状态方程。

- 普通气体的内能是由分子热运动的动能和分子间相互作用势能之和构成的。气体分子热运动的动能只是温度的函数，但是气体体积变化，分子间的平均距离会跟着变化，因而分子间的相互作用势能也会变化，因而气体的内能不仅与温度有关，而且还与体积（或压强）有关。

- 而对于忽略分子间相互作用的理想气体，其内能 U 与 V 和 P 都无关，或者说理想气体的内能只是温度T的函数。

- 理想气体的严格定义：严格遵守理想气体状态方程和焦耳定律的气体。

$$pV = RT, \quad U = U(T)$$

理想气体分子动力学

$$(v^2)_{av} = (v_x^2)_{av} + (v_y^2)_{av} + (v_z^2)_{av}$$

$$(v_x^2)_{av} = \tfrac{1}{3}(v^2)_{av}$$
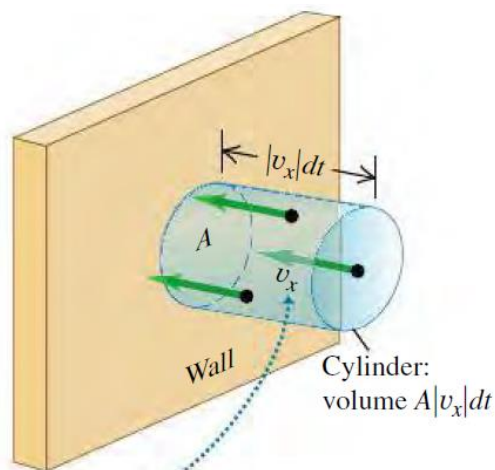
$$\tfrac{1}{2}m(v^2)_{av} = \frac{3nRT}{2N}$$

All molecules are assumed to have the same magnitude $|v_x|$ of x-velocity.

$|v_x|dt$

A

v_x

Wall

Cylinder: volume $A|v_x|dt$

Mass of a molecule

Average translational kinetic energy of a gas molecule

$$\tfrac{1}{2}m(v^2)_{av} = \tfrac{3}{2}kT$$

Absolute temperature of gas

Average value of the square of molecular speeds

Boltzmann constant

理想气体中分子的平均动能，只和温度相关。

思考：3从哪里来？

单原子理想气体等容热容（3平动自由度）

Molar heat capacity at constant volume, ideal gas of point particles $\longrightarrow C_V = \frac{3}{2}R \longleftarrow$ Gas constant

能量按自由度均分定理

双原子理想气体等容热容（3平动+2转动）

Molar heat capacity at constant volume, ideal diatomic gas $\longrightarrow C_V = \frac{5}{2}R \longleftarrow$ Gas constant

很高温下为 $\frac{7}{2}R$

很低温下为 $\frac{3}{2}R$

一般不考虑这两种情形

熟练掌握理想气体C_P和C_V的关系 $C_{p,m} = C_{V,m} + R$
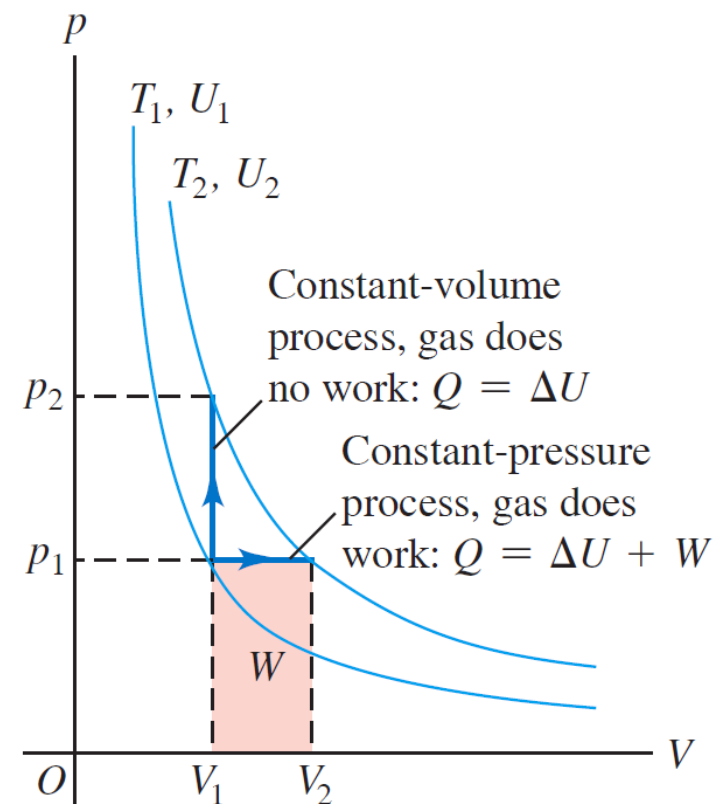
C_p: 等压热容; C_V: 等容热容

等容过程： $W=0$ ➡ $dU=dQ = nC_V dT$

等压过程 $dQ = nC_p dT$

$dW = pdV= nRdT$ ➡ $dU=nC_p dT-nRdT$

理想气体的内能只和温度有关，因此:

$nC_V dT=nC_p dT-nRdT$ ➡ $C_V=C_p-R$

（英文教参p. 626-627）

For an ideal gas:

Molar heat capacity at constant *pressure*

$$C_p = C_V + R$$

Gas constant

Molar heat capacity at constant *volume*

**Ratio of
heat capacities** $\cdots\cdots\cdots\blacktriangleright \gamma = \dfrac{C_p}{C_V}$

Molar heat capacity at
constant *pressure*

Molar heat capacity at
constant *volume*

绝大部分气体在室温常压下可近似理想气体

单原子气体:

$$C_V = \tfrac{3}{2}R \qquad\qquad \gamma = \frac{C_p}{C_V} = \frac{\tfrac{5}{2}R}{\tfrac{3}{2}R} = \tfrac{5}{3} = 1.67$$

双原子气体:

$$C_V = \tfrac{5}{2}R \qquad\qquad \gamma = \frac{C_p}{C_V} = \frac{\tfrac{7}{2}R}{\tfrac{5}{2}R} = \tfrac{7}{5} = 1.40$$

麦克斯韦速率分布

Maxwell–Boltzmann distribution function

$$f(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}$$

Mass of a gas molecule — Molecular speed — Mass of a gas molecule — Molecular speed — Boltzmann constant — Boltzmann constant — Absolute temperature of gas

速率分布公式不要求记忆，但要理解速率分布的几个重要物理量的意义和它们的大小

$$v_{\text{mp}} = \sqrt{\frac{2kT}{m}}$$

- 最概然速率

$$v_{\text{av}} = \sqrt{\frac{8kT}{\pi m}}$$

- 平均速率

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

- 均方根速率

四、掌握重要的热力学过程

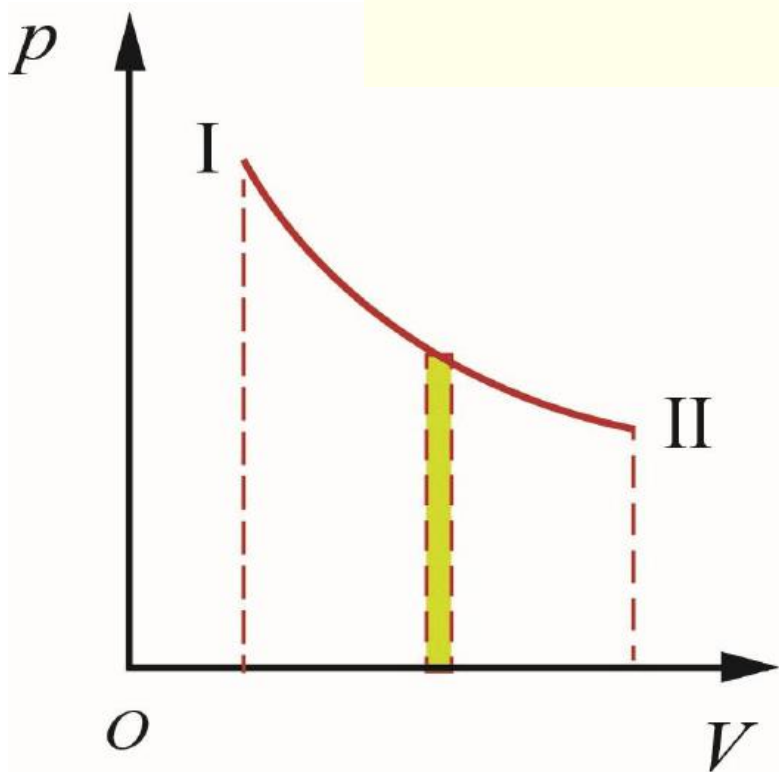# 熟练应用热力学第一定律

熟练判断 PV图上的过程：

哪些做功？哪些不做功？哪些吸热？哪些放热？哪些绝热？

(注意箭头方向！)

掌握气体做功过程的概念和计算：

对于一个有限的准静态压缩过程，体积由 V_1 变为 V_2，系统对外界做的总功为：
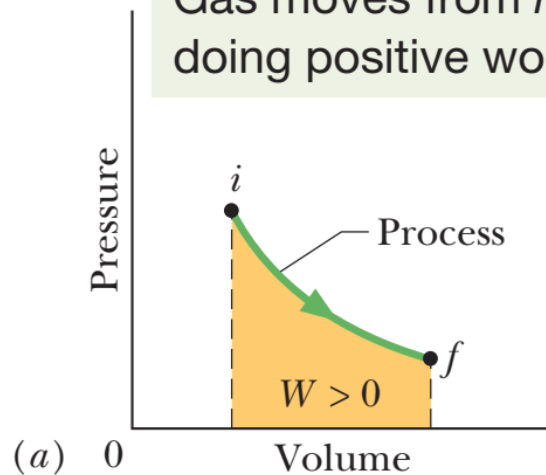
Work done in a volume change $\longrightarrow$ $W = \displaystyle\int_{V_1}^{V_2} p\, dV$ $\begin{cases} \text{Upper limit} = \text{final volume} \\ \text{Integral of the pressure with respect to volume} \\ \text{Lower limit} = \text{initial volume} \end{cases}$

当气体被压缩，系统对外界做负功，
当气体膨胀时，系统对外界做正功。

由积分的意义可知：
曲线下的总面积等于系统对外界做的总功。

(a) Gas moves from *i* to *f*, doing positive work.

Pressure — Volume. Process. $W > 0$.

(b) It still goes from *i* to *f*, but now it does *more* work.

Pressure — Volume. $W > 0$.
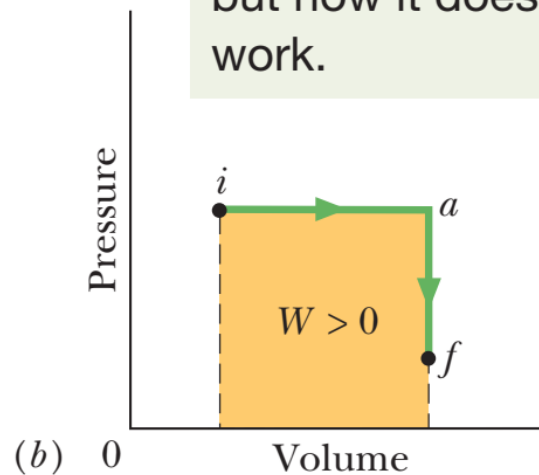
(c) It still goes from *i* to *f*, but now it does *less* work.

Pressure — Volume. $W > 0$.
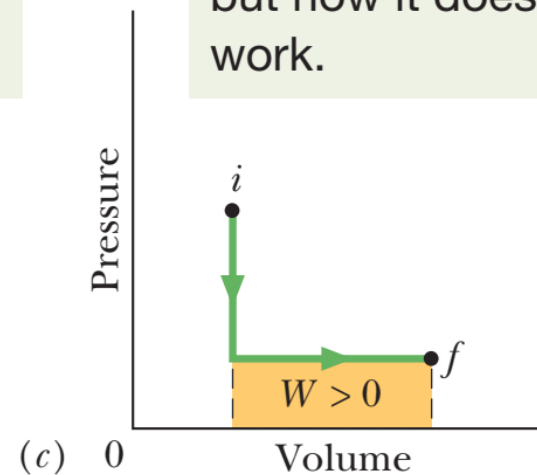
(d) We can control how much work it does.

Pressure — Volume.
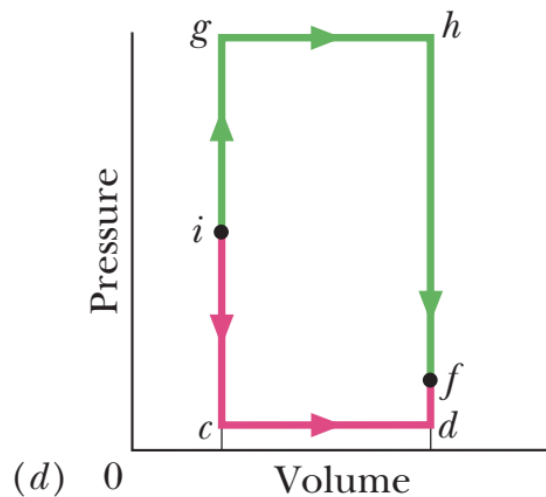
(e) Moving from *f* to *i*, it does negative work.

系统对外界做负功

Pressure — Volume. $W < 0$.
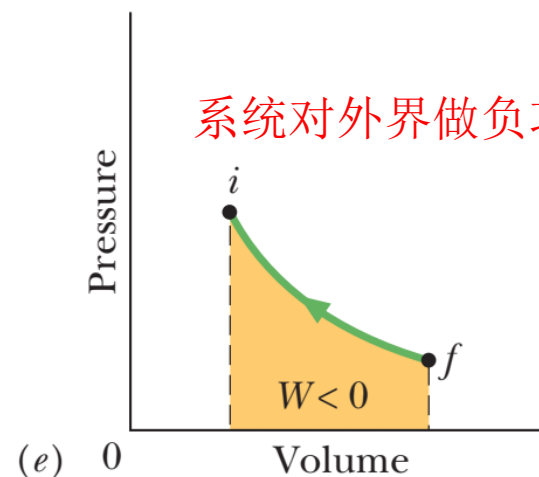
(f) Cycling clockwise yields a positive net work.

Pressure — Volume. $W_{net} > 0$.
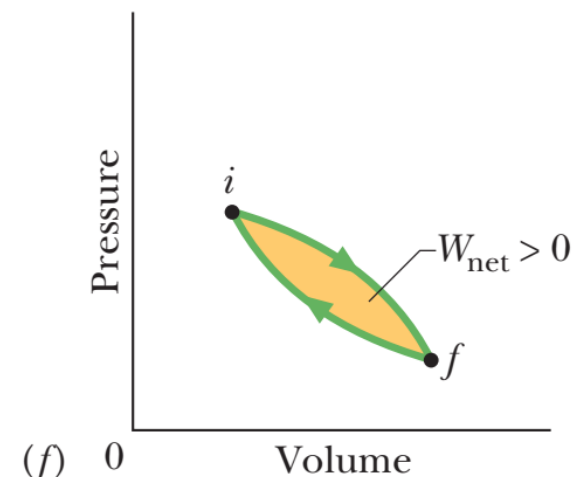
思考：原路返回，是否有净功？

例：如图，理想气体, $I(2p_0, V_0) \rightarrow II(p_0, 2V_0)$，求：
计算沿路径a, b, c时外界对气体所作的功。

解：理想气体

   (a)等体过程不作功，故

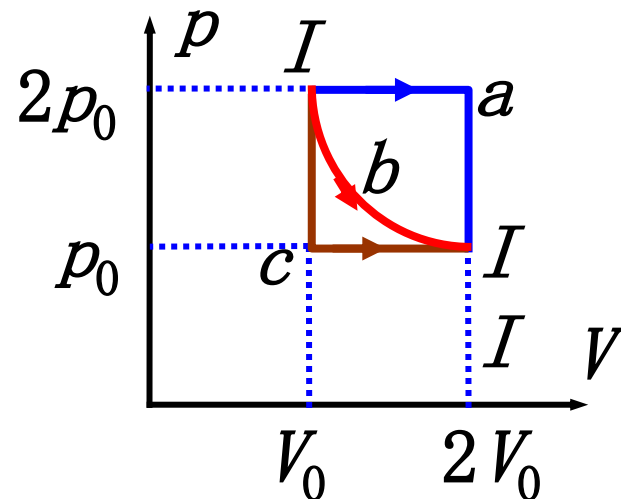$$W_a = -\int_{V_0}^{2V_0} p\,dV = -2p_0 \int_{V_0}^{2V_0} dV = -2p_0V_0$$

   (b)等温膨胀作功，其一般公式：

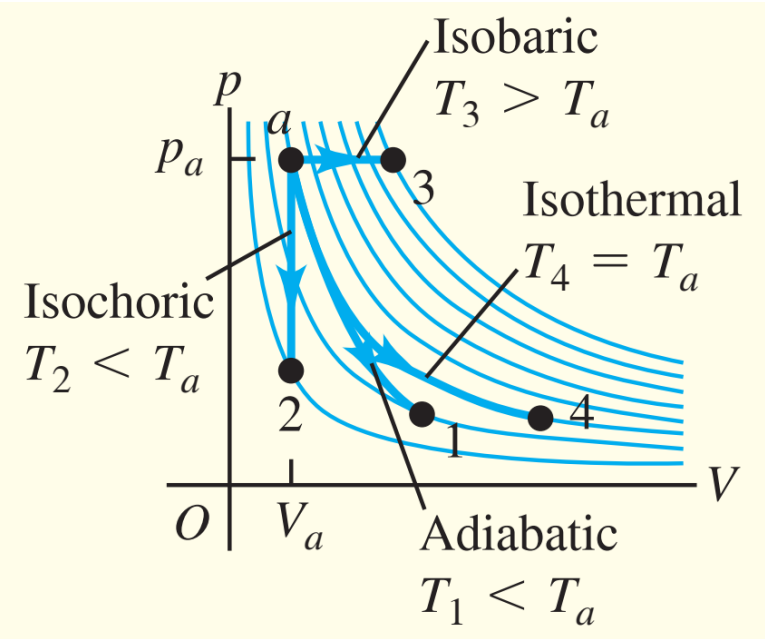$$W_b = -\int_{V_I}^{V_{II}} p\,dV = -\int_{V_I}^{V_{II}} \frac{\nu RT}{V}\,dV = -\nu RT\,ln\frac{V_{II}}{V_I}$$

本例中 $p_0 \cdot 2V_0 = \nu RT$，$V_{II}/V_I = 2$

$\therefore\ W_b = -2p_0V_0\,ln\,2$

(c) 与(a)类似，也可用面积法求：$p_0 \cdot (2V_0 - V_0) = p_0V_0, \Rightarrow\ W_c = -p_0V_0$

熟练掌握等压、等容、等温、绝热过程

Isobaric Process 等压过程 $pV^0 = p =$ 常数

Isochoric Process 等容过程 $pV^\infty =$ 常数
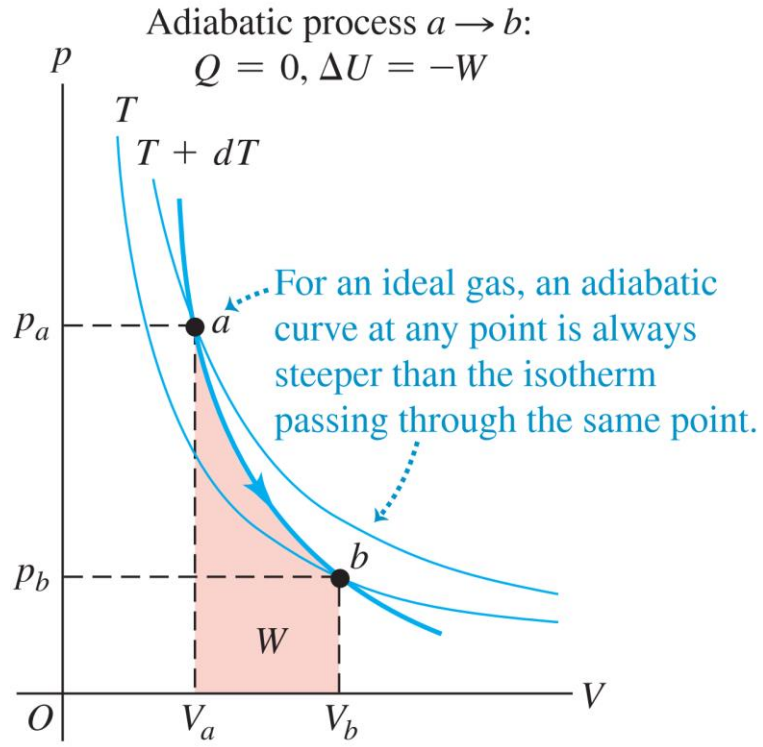
Isothermal Process 等温过程 $pV = nRT =$ 常数

Adiabatic Process 绝热过程 $pV^\gamma =$ 常数; $TV^{\gamma-1} =$ 常数

上述过程均满足 pV^λ=常数，思考：任意两种过程函数是否有且只有一个交点？

Constant Quantity	Process Type	Some Special Results ($\Delta U = Q - W$ and $\Delta U = nC_V \Delta T$ for all paths)
p	Isobaric	$Q = nC_p \Delta T; W = p\,\Delta V$
T	Isothermal	$Q = W = nRT \ln(V_f/V_i);$ $\Delta U = 0$ 等温过程，内能不变
$pV^\gamma, TV^{\gamma-1}$	Adiabatic	$Q = 0;$ $W = -\Delta U$ 绝热过程，热量不变
V	Isochoric	$Q = \Delta U = nC_V \Delta T;$ $W = 0$ 等容过程，不做功

熟练掌握绝热过程功的推导、$\gamma - 1$ 怎么出来的?

Adiabatic process $a \rightarrow b$:
$$Q = 0, \quad \Delta U = -W$$

For an ideal gas, an adiabatic curve at any point is always steeper than the isotherm passing through the same point.

Work done by an ideal gas, adiabatic process

- Number of moles
- Initial temperature

$$W = nC_V(T_1 - T_2)$$

- Final temperature
- Molar heat capacity at constant volume

Work done by an ideal gas, adiabatic process

- Molar heat capacity at constant volume
- Initial pressure, volume

$$W = \frac{C_V}{R}(p_1 V_1 - p_2 V_2) = \frac{1}{\gamma - 1}(p_1 V_1 - p_2 V_2)$$

- Gas constant
- Ratio of heat capacities
- Final pressure, volume

$$nC_V\, dT = -\frac{nRT}{V}dV$$

$$\frac{dT}{T} + \frac{R}{C_V}\frac{dV}{V} = 0$$

$$\frac{R}{C_V} = \frac{C_p - C_V}{C_V} = \frac{C_p}{C_V} - 1 = \gamma - 1$$

$$\frac{dT}{T} + (\gamma - 1)\frac{dV}{V} = 0$$
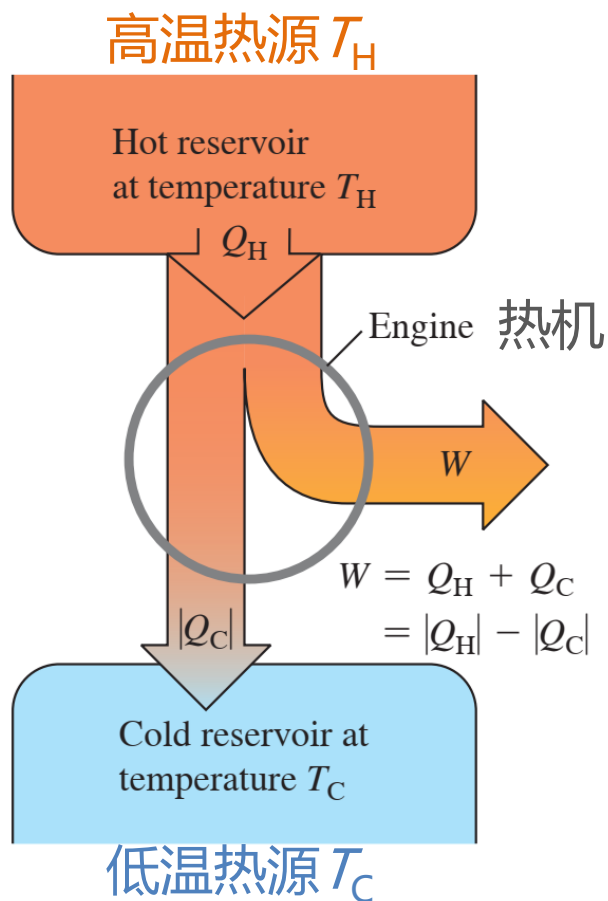
$$\ln T + (\gamma - 1)\ln V = \text{constant}$$

$$\ln T + \ln V^{\gamma - 1} = \text{constant}$$

$$\ln(TV^{\gamma - 1}) = \text{constant}$$

$$TV^{\gamma - 1} = \text{constant}$$

五、掌握重要的循环

掌握卡诺循环、奥托循环的特点，能够对照PV图对它们进行效率推导

高温热源 T_H

Hot reservoir
at temperature T_H

Q_H

Engine 热机

W

$W = Q_H + Q_C$
$= |Q_H| - |Q_C|$

$|Q_C|$

Cold reservoir at
temperature T_C

低温热源 T_C

$$Q = Q_H + Q_C = |Q_H| - |Q_C|$$

$$W = Q = Q_H + Q_C = |Q_H| - |Q_C|$$

Work done by engine   Heat rejected by engine

Thermal efficiency of an engine

$$e = \frac{W}{Q_H} = 1 + \frac{Q_C}{Q_H} = 1 - \left|\frac{Q_C}{Q_H}\right|$$

Heat absorbed by engine

提高热机效率的办法：

- 尽可能接近卡诺循环； $e = 1 - \dfrac{T_C}{T_H}$

- 尽可能提高高温热源温度T_H和低温热源温度T_C的比值

Otto cycle

定容加热

Q_H

c ② Heating at constant volume (fuel combustion)

b

③ Adiabatic expansion (power stroke)

W

d $|Q_\text{C}|$

a

① Adiabatic compression (compression stroke)

④ Cooling at constant volume (cooling of exhaust gases)
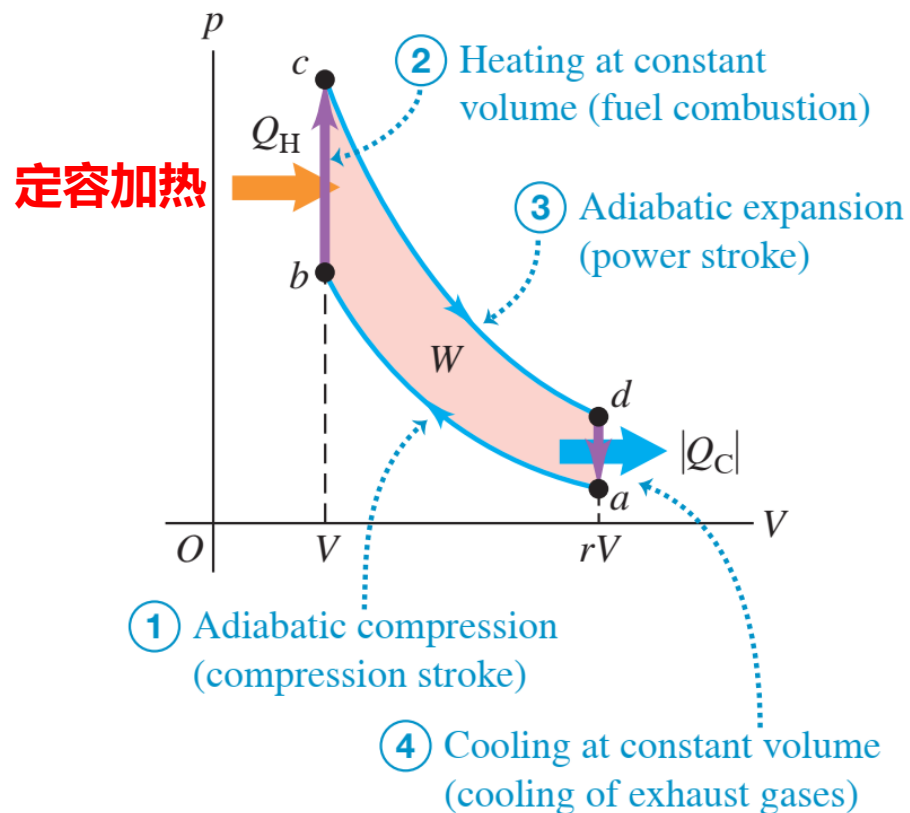
$a \rightarrow b$: 绝热压缩
$b \rightarrow c$: 定容加压(吸热)
$c \rightarrow d$: 绝热膨胀
$d \rightarrow a$: 定容降压(放热)

$$a \to b \qquad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \qquad\qquad V^{\gamma-1}T = 恒量$$

$$c \to d \qquad \frac{T_3}{T_4} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \qquad\qquad Q_{\mathrm{H}} = nC_{V,m}(T_3 - T_2)$$

$$Q_{\mathrm{C}} = nC_{V,m}(T_4 - T_1)$$

$$e = 1 - \frac{Q_{\mathrm{C}}}{Q_{\mathrm{H}}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{1}{\left(\frac{V_1}{V_2}\right)^{\gamma-1}} = 1 - \frac{1}{r^{\gamma-1}}$$

- 注意热量的正负和绝对值的关系

Thermal efficiency in Otto cycle $\cdots\cdots e = 1 - \dfrac{1}{r^{\gamma-1}}$

Compression ratio $\cdots$ Ratio of heat capacities

a→b 等温膨胀：与温度为T_H的高温热源接触，T_H不变，体积由V_a膨胀到V_b，W>0，从热源吸收热量为：
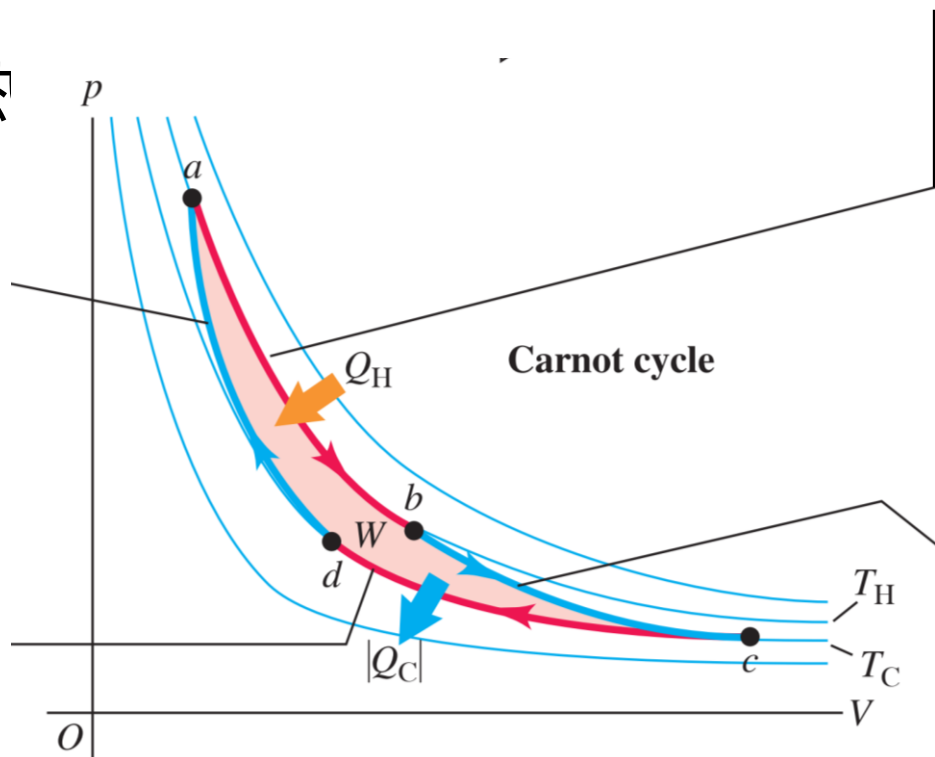
$$Q = Q_H = W = \int_{V_a}^{V_b} pdV$$

$$= nRT_H \int_{V_a}^{V_b} \frac{dV}{V} = nRT_H \ln\frac{V_b}{V_a}$$

d→a 绝热压缩：绝热压缩，体积由V_d变到V_a，W<0，吸热Q为零。

Carnot cycle

Q_H

W

$|Q_C|$

T_H

T_C

c→d 等温压缩:

b→c 绝热膨胀：绝热膨胀，体积由V_b变到V_c，吸热为零。

与温度为T_C的低温热源接触,T_C不变，体积由V_c压缩到V_d，从热源放热为：

$$Q = Q_C = nRT_C \ln\frac{V_c}{V_d}$$

注意热量的正负和绝对值的关系

Because V_d is less than V_c, Q_C is negative ($Q_C = -|Q_C|$); heat flows out of the gas during the isothermal compression at temperature T_C.

The ratio of the two quantities of heat is thus

$$\frac{Q_C}{Q_H} = -\left(\frac{T_C}{T_H}\right)\frac{\ln\left(V_c/V_d\right)}{\ln\left(V_b/V_a\right)} \qquad (20.12)$$

This can be simplified further by use of the temperature–volume relationship for an adiabatic process, Eq. (19.22). We find for the two adiabatic processes:
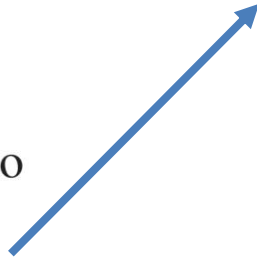
$$T_H V_b^{\gamma-1} = T_C V_c^{\gamma-1} \qquad \text{and} \qquad T_H V_a^{\gamma-1} = T_C V_d^{\gamma-1}$$

Dividing the first of these by the second, we find

$$\frac{V_b^{\gamma-1}}{V_a^{\gamma-1}} = \frac{V_c^{\gamma-1}}{V_d^{\gamma-1}} \qquad \text{and} \qquad \frac{V_b}{V_a} = \frac{V_c}{V_d}$$

$$\boxed{e = 1 - \frac{T_C}{T_H}}$$

Thus the two logarithms in Eq. (20.12) are equal, and that equation reduces to

$$\frac{Q_C}{Q_H} = -\frac{T_C}{T_H} \quad \text{or} \quad \frac{|Q_C|}{|Q_H|} = \frac{T_C}{T_H} \qquad \text{(heat transfer in a Carnot engine)} \quad (20.13)$$

六、掌握热力学第二定律

掌握热二的概念、两种表述和数学表达，理解两种表述的等价性

理解熵、熵增的概念

了解热二的统计学意义

热力学第二定律的第一种表述

热力学第二定律的开尔文表述：
不可能从单一热源吸取热量，使之完全变为有用功而不引起其它变化。

注释：

- "单一热源"指温度处处相同恒定不变的热源。

- "其它影响"指除了"由单一热源吸收热量全部转化为功"以外的任何其他变化。

- 开尔文表述指出，"热变功"过程一定会引起系统或外界变化。

开尔文表述可简化为第二类永动机（效率为100%）不可能造成。

热力学第二定律的第二种表述

SECOND LAW OF THERMODYNAMICS ("refrigerator" statement)：It is impossible for any process to have as its sole result the transfer of heat from a cooler to a hotter object.

以上为热力学第二定律的克劳修斯表述：**不可能把热量从低温物体传到高温物体而不引起其它变化**。

Notes (注释):

(1) 克氏表述说明使热量从低温物体传到高温物体，一定会使系统或外界引起变化，即"热传导"是不可逆过程。

(2) **克氏表述和开尔文表述是等价的**: 否定克劳修斯表述即否定了开尔文表述，否定开尔文表述即否定了克劳修斯表述。

热力学第二定律的意义

第二定律实质： 一切与热相联系的自然现象中它们自发地实现的过程都是不可逆的。

与第一定律的比较：

第一定律主要从数量上说明功和热量的等价性；

第二定律却从能量转换的角度说明功与热的本质区别，从而揭示了自然界中普遍存在的一类不可逆过程，并指出吸收的热量不可能全部用来做有用功．

与第零定律的比较：

第零定律并不能比较尚未达热平衡的两物体间温度的高低；

第二定律却能从热量的自发流动方向判别出物体温度的高低，所以第零定律与第二定律是两个相互独立的基本定律。

热力学第二定律的熵表述

$$S_b - S_a \geq \int_a^b \frac{dQ}{T} \quad \text{或} \quad dS \geq \frac{dQ}{T}$$

其中等号对应着任一可逆过程。 – 此式更为普遍，即为热力学第二定律的数学表达式。<u>孤立系统的任何过程都是绝热过程，所以有</u>：

SECOND LAW OF THERMODYNAMICS ("entropy" statement)： No process is possible in which the total entropy decreases, when all systems that take part in the process are included.

<u>孤立系统的熵不可能变小</u>。

思考：绝热过程熵如何变化？可逆过程的熵如何变化？为什么？
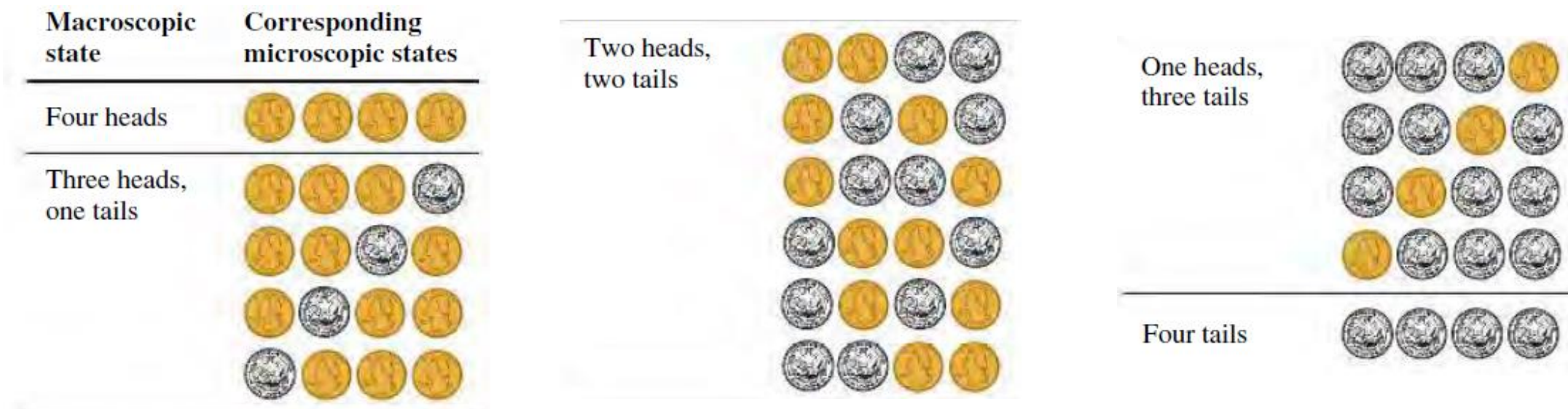
绝热+可逆同时满足，熵才不变；

不绝热，熵增；

不可逆，熵增。

摩擦、热传导、热对流、热辐射都是不可逆现象。自然界中一切与热相关的自发过程都是不可逆过程！

熵的微观理解

统计力学中定义熵: $S = klnw$

k: 波尔兹曼常数。
w : 系统处于稳定状态下，对应的微观状态数目。

四个硬币分上下:
宏观状态：{4, 0}, {3, 1}, {2, 2}, {1, 3},{0, 4}
微观状态：2X2X2X2 = 16
宏观-微观状态：{4, 0}-1, {3, 1}-4, {2, 2}-6, {1, 3}-4,{0, 4}-1

每个微观态出现的机率一样，具有微观态数目多的宏观态分布机率大。

# 七、狭义相对论部分

理解光速在所有惯性参照系中相同给伽利略变换带来的局限性

熟练掌握狭义相对论的两个基本假设

理解和熟练掌握狭义相对论的主要的几个结论

- 同时性的相对性→时间延缓效应
- 长度的相对性→尺寸收缩效应

能熟练解决类似隧道悖论的习题

Invariance of physical laws, simultaneity: All of the fundamental laws of physics have the same form in all inertial frames of reference. The speed of light in vacuum is the same in all inertial frames and is independent of the motion of the source. Simultaneity is not an absolute concept; events that are simultaneous in one frame are not necessarily simultaneous in a second frame moving relative to the first.
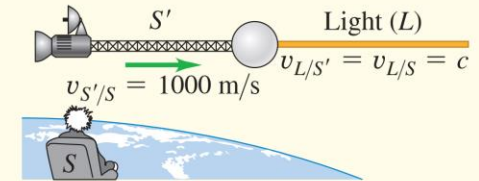
Time dilation: If two events occur at the same space point in a particular frame of reference, the time interval Δt_0 between the events as measured in that frame is called a proper time interval. If this frame moves with constant velocity u relative to a second frame, the time interval Δt between the events as observed in the second frame is longer than Δt_0. (See Examples 37.1–37.3.)

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}} = \gamma \, \Delta t_0 \qquad (37.6), (37.8)$$

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} \qquad (37.7)$$
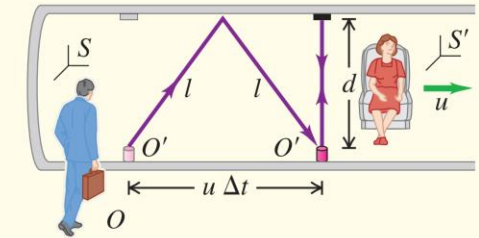
Length contraction: If two points are at rest in a particular frame of reference, the distance l_0 between the points as measured in that frame is called a proper length. If this frame moves with constant velocity u relative to a second frame and the distances are measured parallel to the motion, the distance l between the points as measured in the second frame is shorter than l_0. (See Examples 37.4 and 37.5.)

$$l = l_0 \sqrt{1 - \frac{u^2}{c^2}} = \frac{l_0}{\gamma} \qquad (37.16)$$
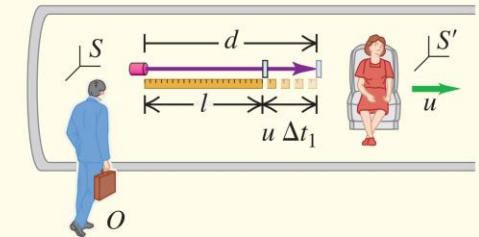
从上述效应理解洛伦兹因子的导出

熟练掌握洛伦兹变换

能应用洛伦兹变换进行逆变换

理解和熟练掌握质能关系方程，能运用质能关系、动量p-动能K-总能E
关系求解速率v接近光速c运动的物体的质量、能量、动量

The Lorentz transformations: The Lorentz coordinate transformations relate the coordinates and time of an event in an inertial frame S to the coordinates and time of the same event as observed in a second inertial frame S' moving at velocity u relative to the first. For one-dimensional motion, a particle's velocities v_x in S and v'_x in S' are related by the Lorentz velocity transformation. (See Examples 37.6 and 37.7.)
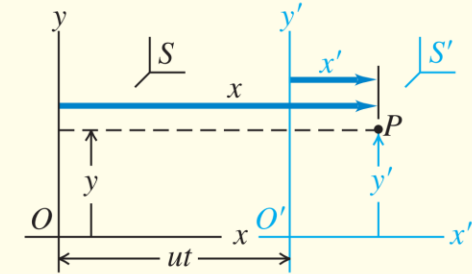
$$x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}} = \gamma(x - ut)$$

$$y' = y \qquad z' = z \tag{37.21}$$

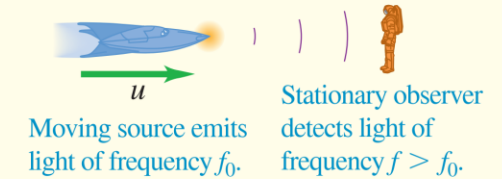$$t' = \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}} = \gamma(t - ux/c^2)$$

$$v'_x = \frac{v_x - u}{1 - uv_x/c^2} \tag{37.22}$$

$$v_x = \frac{v'_x + u}{1 + uv'_x/c^2} \tag{37.23}$$

The Doppler effect for electromagnetic waves: The Doppler effect is the frequency shift in light from a source due to the relative motion of source and observer. For a source moving toward the observer with speed u, Eq. (37.25) gives the received frequency f in terms of the emitted frequency f_0. (See Example 37.8.)

$$f = \sqrt{\frac{c + u}{c - u}}\, f_0 \tag{37.25}$$

Moving source emits light of frequency f_0. Stationary observer detects light of frequency $f > f_0$.

Relativistic momentum and energy: For a particle of rest mass m moving with velocity $\vec{v}$, the relativistic momentum $\vec{p}$ is given by Eq. (37.27) or (37.31) and the relativistic kinetic energy K is given by Eq. (37.36). The total energy E is the sum of the kinetic energy and the rest energy mc^2. The total energy can also be expressed in terms of the magnitude of momentum p and rest mass m. (See Examples 37.9–37.11.)

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}} = \gamma m\vec{v} \qquad (37.27),\ (37.31)$$

$$K = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2 = (\gamma - 1)mc^2 \tag{37.36}$$

$$E = K + mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \tag{37.38}$$

$$= \gamma mc^2$$

$$E^2 = (mc^2)^2 + (pc)^2 \tag{37.39}$$