Electromagnetics Spring 2020
Homework 7

说明：全用英文作答；
每道题要对所有小问作答，要给出全部必要的推导过程，计算题要算出最终的数值结果， 比如开根号之类的；
所有计算出来的结果如果是有单位的物理量，一定要写明单位；
每题的分数在括号中给出；
可以互相讨论，也可以上网查，但是不能抄袭，也不能找别人代做；
可以在电脑敲字解答，也可以手写解答，最后统一转换为 PDF 格式，按分组信息邮件 或 BB 上提交；
邮件主题&附件命名规范:姓名_章节，不按规范发送扣除一半分数；
请在作业 PDF 的第一行写上姓名和学号；
有问题请给老师或助教发邮件；

Textbook: Fundamentals of Applied Electromagnetics, 7th edition

Part I. Problems in textbook.
8.4 (50 points)
8.16 (20 points)
8.9 (100 points) You need to first do it using the infinite reflection method. Then do it again by assuming all the waves in the medium 2 can be classified to Ae^{-jk_2z} and Be^{jk_2z}.
8.22 (20 points)
8.30 (20 points)
8.36 (50 points)
8.40 (40 points)
8.42 (20 points) You need to specify the four walls by $x = 0, a$ or $y = 0, b$.
8.44 (20 points)

PART II. Problems in quiz
1. (32 points)
(a) (points) Consider a $+z$-direction propagating plane wave normally incident on the interface between two media with the interface at $z = 0$. Write out the expressions of the electric field and magnetic field of the incident wave, reflected wave and transmitted wave in their phasor forms. The quantities you can use are: magnitude of the incident electric field intensity E_0, reflection coefficient Γ, transmission coefficient τ, intrinsic impedance of the two media η_1 and η_2, and wave number in the two media k_1, and k_2.

(b) (2 points) Express the reflection coefficient and transmission coefficient by η_1 and η_2.

(c) (2 points) Give the possible value ranges of the reflection coefficient and transmission coefficient.

(d) (2 points) Is it possible that the magnitude of the reflected electric field is larger than that of the incident electric field? Is it possible that the magnitude of the transmitted electric field is larger than that of the incident electric field?

(e) (2 points) Is it possible that the direction of the reflected electric field at $z = 0$ is different from that of the incident electric field at $z = 0$? Is it possible that the direction of the transmitted electric field at $z = 0$ is different from that of the incident electric field at $z = 0$?

(f) (2 points) Is it possible that the magnitude of the reflected magnetic field is larger than that of the incident magnetic field? Is it possible that the magnitude of the transmitted magnetic field is larger than that of the incident magnetic field?

(g) (2 points) Is it possible that the direction of the reflected magnetic field at $z = 0$ is different from that of the incident magnetic field at $z = 0$? Is it possible that the direction of the transmitted magnetic field at $z = 0$ is different from that of the incident magnetic field at $z = 0$?

(h) (4 points) Calculate the total time-average power density (vector) in medium 1.

(i) (5 points) Assume the medium 2 is an infinitely large PEC, what is the reflection coefficient? What kind of wave is formed in this case in the medium 1? Is the total electric field at $z = 0$ zero? Is the total magnetic field at $z = 0$ zero? What is the maximum magnitude the total electric field in medium I can get?

(j) (2 points) For the case in (i), what can be considered as the source of the reflected field?

2. (14 points) Consider a +z-direction propagating plane wave normally incident on the interface between two media with the interface at z = 0. Intrinsic impedance of the two media are $\eta_1 = \eta_0$ for $z < 0$ and $\eta_2 = 2\eta_0$ for $z > 0$. Magnitude of the electric field of the incident wave is E_0.

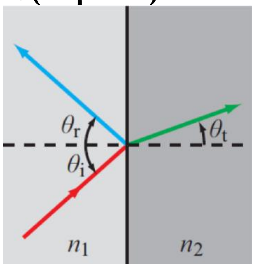
(a) (4 points) Calculate the reflection coefficient and standing wave ratio.

(b) (2 points) What is the theoretical value range of the standing wave ratio?

(c) (4 points) At what locations in the $z < 0$ region does the total electric field reaches maximum? Calculate the maximum electric field.

(d) (4 points) At what locations in the $z < 0$ region does the total magnetic field reaches maximum? Calculate the maximum magnetic field.

3. (12 points) Consider the oblique incidence case shown below.



(a) (2 points) Write out the expression of the Snell's law.

(b) (2 points) Which of n_1 and n_2 is bigger in the case shown in the figure?

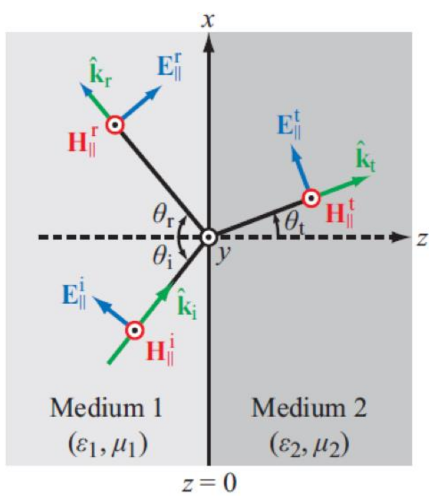
(c) (2 points) To enable total reflection, what is the relationship between n_1 and n_2?

(d) (2 points) Express the critical angle in terms of n_1 and n_2

(e) (2 points) Which polarization (parallel or perpendicular) can enable total transmission? Assume nonmagnetic materials.

(f) (2 points) Express the Brewster angle in terms of n_1 and n_2.

4. (15 points) Write out the phasor-form expressions of the electric field intensity and magnetic field intensity of the incident, reflected and transmitted wave using the magnitude of the incident electric field intensity E_0. reflection coefficient Γ, transmission coefficient τ, wave number k_1 and k_2, intrinsic impedance η_1 and η_2, and angles shown in the figure.



5. (46 points)

(a) (2 points) If a TE wave is propagating along $+y$, which one of these six terms (E_x, E_y, E_z, H_x, H_y, H_z) must be 0?

(b) (2 points) If a TM wave is propagating along $-x$, which one of these six terms (E_x, E_y, E_z, H_x, H_y, H_z) must be 0?

(c) (4 points) Can a rectangular waveguide carry a TEM wave? Can a coaxial waveguide (coaxial cable) carry a TEM wave?

(d) (5 points) What is the dominant mode of a rectangular waveguide? Assume the wave is propagating in $+z$, what field components does this mode have?

(e) (4 points) Write out the expression of the cutoff wavenumber of a rectangular waveguide in terms of waveguide dimension. Is the cutoff wavenumber dependent on the material property?

(f) (2 points) Write out the expression of the cutoff frequency of a rectangular waveguide in terms of waveguide dimension.

(g) (2 points) For a rectangular waveguide filled with a dielectric material having relative permittivity of 4, how does the cutoff frequency of a specific mode change compared with an-air filled waveguide of the same size?

(h) (4 points) If the longer dimension of the rectangular waveguide is 3 times of its shorter dimension, what is the second TE mode? What is the second TM mode?

(i) (4 points) If the longer dimension of the rectangular waveguide is 1.5 times of its shorter dimension, what is the second TE mode? What is the second TM mode?

(j) (2 points) If a mode in a rectangular waveguide has a cutoff frequency of 1 GHz, what does this physically mean?

(k) (8 points) Write out the expressions of the guide wavelength and phase velocity of a rectangular waveguide filled with vacuum. Is the guide wavelength greater than, smaller than or equal to the wavelength of a plane wave in vacuum? Is the phase velocity greater than, smaller than or equal to the phase velocity of a plane wave in vacuum?

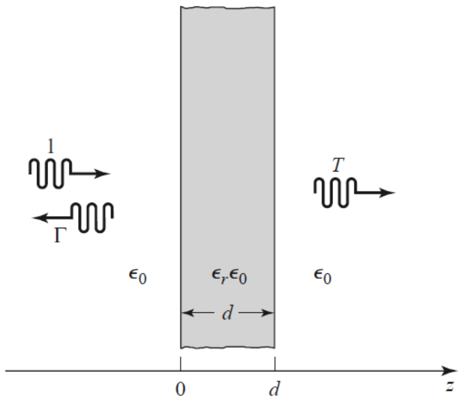
(l) (**2 points**) For solving TE modes in a rectangular waveguide, write out the wave equation for $h_z(x, y)$.

(m) (2 points) Write out the expressions of the coefficients k_x and k_y.

(n) (3 points) Write out the relationship between λ, λ_x, λ_y and λ_g.

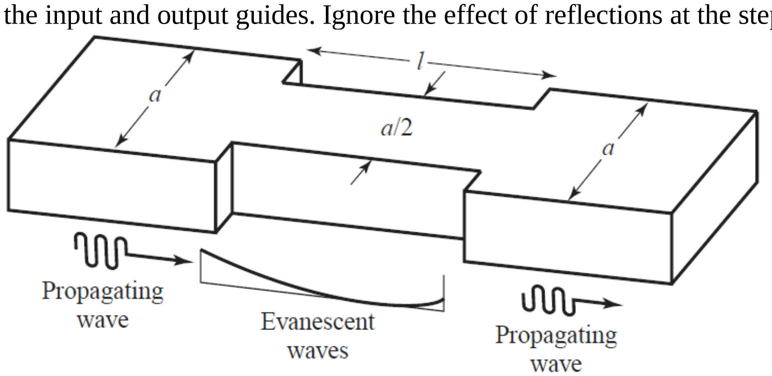
PART III. Homemade

1. (80 points) A plane wave is normally incident on a dielectric slab of permittivity ε_r and thickness d, where $d = \lambda_0/(4\sqrt{\varepsilon_r})$ and λ_0 is the free-space wavelength of the incident wave, as shown in the accompanying figure. If free-space exists on both sides of the slab, find the reflection coefficient of the wave reflected from the front of the slab. Then try it again using $d = \lambda_0/(2\sqrt{\varepsilon_r})$.



2. (30 points) An empty X-band (8.2–12.4 GHz) rectangular waveguide, with dimensions of 2.286 cm by 1.016 cm, is to be connected to an X-band waveguide of the same dimensions but filled with lossless polystyrene (ε_r = 2.56). To avoid reflections, an X-band waveguide (of the same dimensions) quarter-wavelength long section is inserted between the two. Assume dominant-mode propagation and that matching is to be made at 10 GHz. Hint: use the conclusion obtained in problem 8.9. Determine: (a) Wave impedance of the quarter-wavelength section waveguide. (b) Dielectric constant of the lossless medium that must be used to fill the quarter-wavelength section waveguide. (c) Length (in cm) of the quarter-wavelength section waveguide.

3. (20 points) An attenuator can be made using a section of waveguide operating below cutoff, as shown in the accompanying figure. If a = 2.286 cm and the operating frequency is 12 GHz, determine the required length of the below-cutoff section of waveguide to achieve an attenuation of 100 dB between the input and output guides. Ignore the effect of reflections at the step discontinuities.



4. (50 points) Assume only TE_{11} and TM_{11} modes are propagating in an air-filled X-band (8.2–12.4 GHz) rectangular waveguide. Calculate the cutoff frequency of these two modes. At 20 GHz, calculate the total time-average power passing a transverse cross section of this waveguide. Assume the maximum amplitude of E_y of the two modes are both 1 V/m. Hint: you need to use the TOTAL E and H fields to do this. Then individually calculate the time-average power of the TE_{11} and TM_{11} modes. Now you need to use the separate E and H fields of these two modes.