EE101 Homework 5

Submit: Blackboard/Paper   Due: Dec. 22th

Please write down Your Name & Student ID

1.

At room temperature (298K), if 1ml water is placed in a magnetic field whose strength is 4.7 Tesla, then what is the protons' magnetization M_0 of that 1ml water?

Solution:

The net magnetization is defined as the sum of magnetic moment of nuclei involved. It is determined by the population difference of nuclei occupying the two energy levels, multiplied by the z component of the magnetic moment for each nucleus:

$$M_0 = \mu_z\left(N_{\text{parallel}} - N_{\text{anti-parallel}}\right) = \frac{\gamma^2 h^2 B_0 N_s}{16\pi^2 kT}$$

1 water molecule (H_2O) has 2 protons, so Ns should be:

$$\mathrm{Ns} = 1\mathrm{ml} \times 1\mathrm{g\ ml}^{-1} \times \frac{1}{18} mol\ g^{-1} \times 2 \times 6.02 \times 10^{23} = 6.69 * 10^{22}$$

At room temperature (298K):

$$\mathrm{M}_0 = \frac{(42.58 * 10^6 * 2\pi)^2 (6.63 * 10^{-34})^2 * 4.7 * 6.69 * 10^{22}}{4 * 4\pi^2 * 1.38 * 10^{-23} * 298} = 1.52 * 10^{-8} \mathrm{J/T}$$

2.

Calculate the effects of the following pulse sequences on thermal equilibrium magnetization. The final answer should include x-, y-, and z-components of magnetization.

a) 90°x (a pulse with tip angle 90°, applied about the x-axis).

b) 30°y

c) 90°x 90°y (the second 90° pulse is applied immediately after the first).

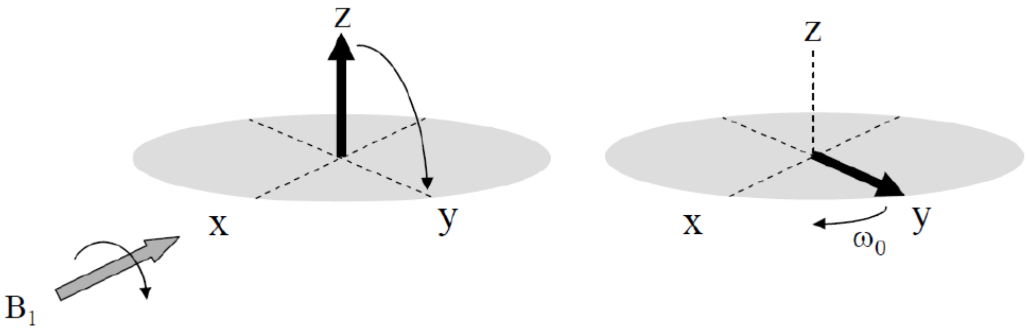


Fig2: Example of the magnetization after a 90°x pulse sequence

Solution:

(a) $\mathrm{M_z} = 0, \mathrm{M_y} = \mathrm{M_0}, \mathrm{M_x} = 0$

(b) $\mathrm{M_z} = \mathrm{M_0} \cos 30 = 0.866\mathrm{M_0}, \mathrm{M_y} = 0, \mathrm{M_x} = \mathrm{M_0} \sin 30 = 0.5\mathrm{M_0}$

(c) $M_z = 0, M_y = M_0, M_x = 0$

3.

The operator wishes to acquire an oblique slice shown by the orientation of the white bar in Figure 3. Draw the gradient echo imaging sequence that would be run to acquire such an image.

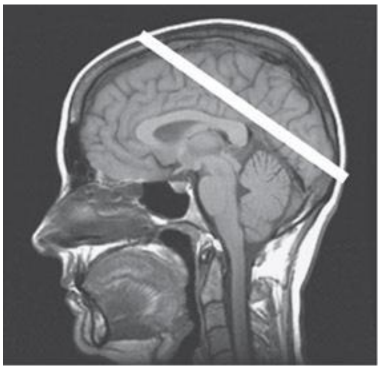
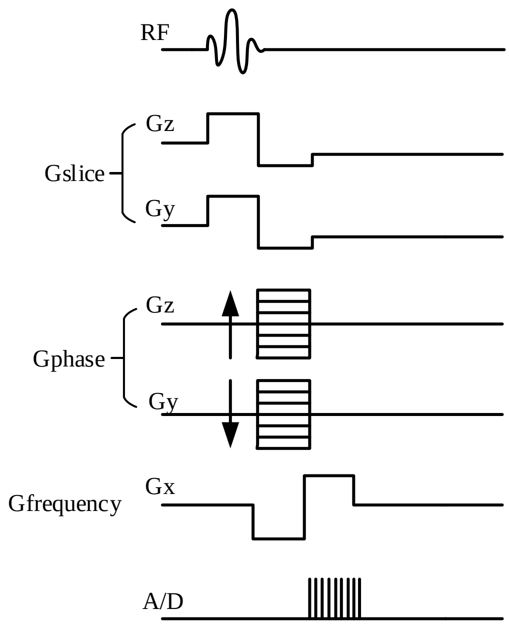
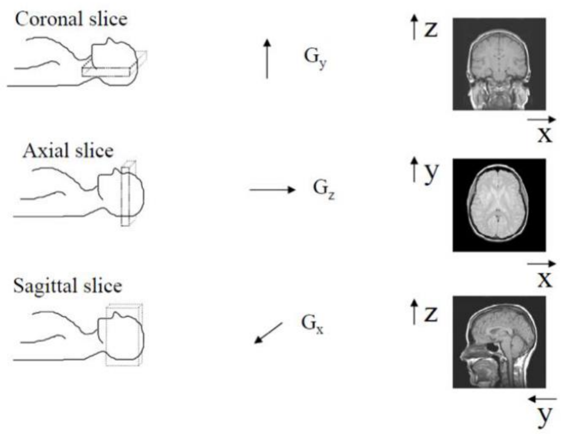


Figure 3: The required oblique slice is shown by the white bar. The brain image is in sagittal orientation.

Solution:

The oblique slice is in the head/foot (z) and anterior/posterior (y) planes, and therefore both gradients must be applied in the slice select direction, with approximately equal strengths since the slice is at an angle of ~45°. Since the phase and frequency encoding gradients are both applied orthogonal to the slice selection gradient, both phase and frequency encoding must also have contributions from two gradients. So the sequence is (note that the phase and frequency axes can be interchanged):


RF
Gslice
Gz
Gy
Gphase
Gz
Gy
Gfrequency
Gx
A/D


Coronal slice
G_y
z
x
Axial slice
G_z
y
x
Sagittal slice
G_x
z
y

4.

Write an expression for the M_z magnetization as a function of time after a 180° pulse. After what time is the M_z component zero? Plot the magnetization M_z after instead applying a 120° pulse.

Solution:

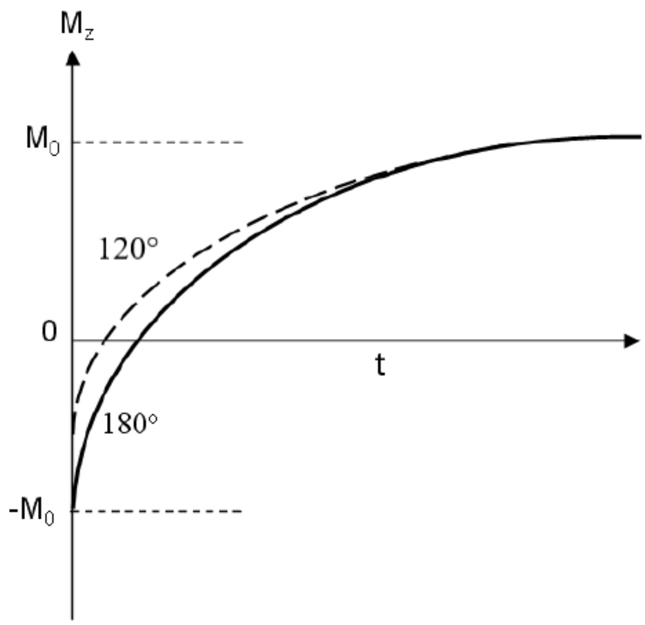
The z-component of magnetization as a function of τ after the RF pulse is given by:

$$M_z(\tau) = M_0\left(1 - 2e^{-\frac{\tau}{T_1}}\right)$$

By setting the left-hand side equal to zero, and solving for τ we obtain:

$$\tau = T_1 \ln 2$$

If, instead of a $180x°$ pulse, a $120x°$ pulse is applied, then the initial component of z magnetization is given by $M_0 \cos(120) = -\frac{1}{2}M_0$



5.

Choose the correct option from (a)-(e) and *explain* why this is your choice.

The maximum MR signal is obtained by using:

(a) $45°$ RF pulse, long TE, and short TR;

(b) $45°$ RF pulse, short TE, and long TR;

(c) $90°$ RF pulse, short TE, and short TR;

(d) $90°$ RF pulse, long TE, and short TR;

(e) $90°$ RF pulse, short TE, and long TR.

Solution:

The correct solution is (e). In order to maximize the signal, the tip angle should be $90°$ which requires a long TR for the magnetization to recover to M0 for full T1 relaxation. T2 relaxation decreases the signal, and so should be minimized by choosing a short value of TE.

6.

Three images are shown in Figure 6: the scaling in each image is different and is normalized to the same maximum value. The imaging parameters are TR = 2000 ms, TE = 20 ms for one image, TR = 750 ms, TE = 80 ms for another, and TR = 2000 ms, TE = 80 ms for the final one.

(i) Assign each image to the appropriate TR and TE values.

(ii) Based on your answer, do the ventricles have a higher or lower T1 value than brain tissue? What is the corresponding answer for T2?

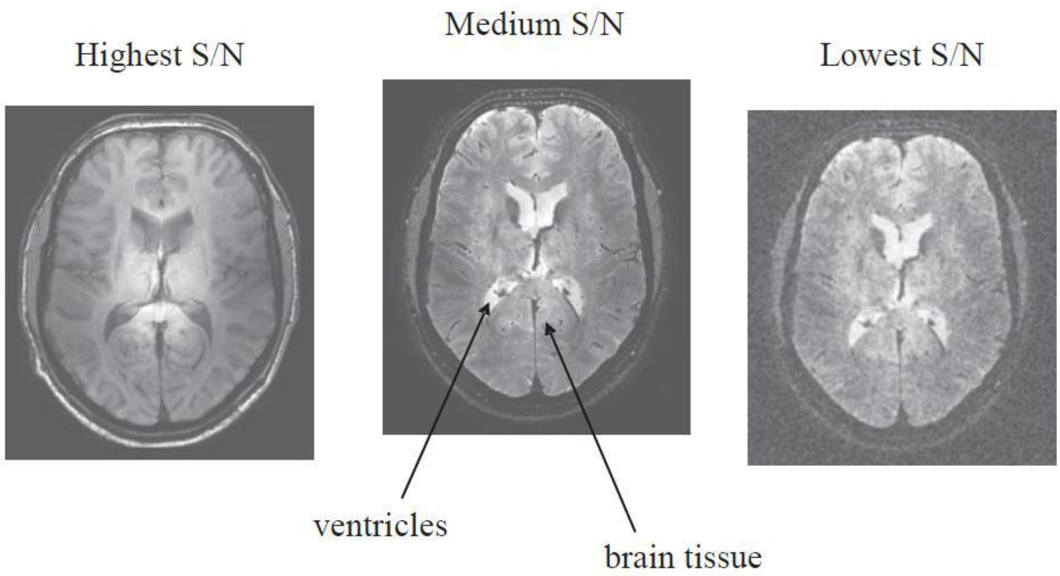


Figure 6: Three brain images with different signal-to-noise ratios

Solution:

(i) The highest S/N corresponds to a long TR and short TE, and so the image on the left has TR 2000 ms, TE 20 ms. The lowest S/N has the shortest TR and longest TE, and so the image on the right has TR 750 ms, TE 80 ms.

(ii) For relative T1 values we compare images with different TR values, the middle and right images. Increasing the TR takes the ventricles from being isointense with brain tissue to much higher value, meaning that the T1 of the ventricles is higher than that of the brain. For relative T2 values we compare images with different TE values, the left and middle images. Increasing the TE takes the ventricles from being darker than brain tissue to brighter, and therefore the ventricles have a longer T2 than brain tissue.

7. There is an MRI machine whose magnetic field has strength of 1.5T. Provided that the gyromagnetic ratio is $\frac{\gamma}{2\pi} = 42.6\text{MHz} * \text{T}^{-1}$ and $1\text{T} = 10^4$ Gauss.

(1) Assume the gradient along z direction is 1 Gauss·cm^{-1}. To get a slice image of thickness 10mm, what should the bandwidth of the RF pulse be?

(2) If the gradient becomes 2 Gauss·cm^{-1}, and the bandwidth of the RF pulse remains unchanged, then what will the slice thickness become?

Solution:

(1) Overlapping linear gradient magnetic field B_z on the static magnetic field B_0, spinning nucleus with different z will have different resonance frequency:

$$\omega = \gamma(\text{B}_0 + z\,G_z)$$

Assume the bandwidth of the RF pulse is $\Delta\omega$, the slice thickness is Δz, then they have the following relationship.

$$\Delta\omega = \gamma\text{G}_\text{z}\Delta z = 2\pi\Delta f$$

Then:

$$\Delta\text{f} = \frac{\gamma\text{G}_\text{z}\Delta z}{2\pi} = 42.6 \times 10^6 \times 1 \times 10^{-4} \times 1 = 4.26 \times 10^3 Hz$$

(2) When the gradient is switched to 2 Gauss·cm^{-1}, and the bandwidth of the RF pulse remains unchanged, the slice thickness becomes:

$$\Delta\text{z} = \frac{2\pi\Delta\text{f}}{\gamma\text{G}_\text{z}} = \frac{4.26 \times 10^3}{42.6 \times 10^6 \times 2 \times 10^{-4}} = 5$$

8. Consider the pulse sequence in Figure 8 (surface 2 equals two times surface 1). Draw the trajectory of k in the k-space.

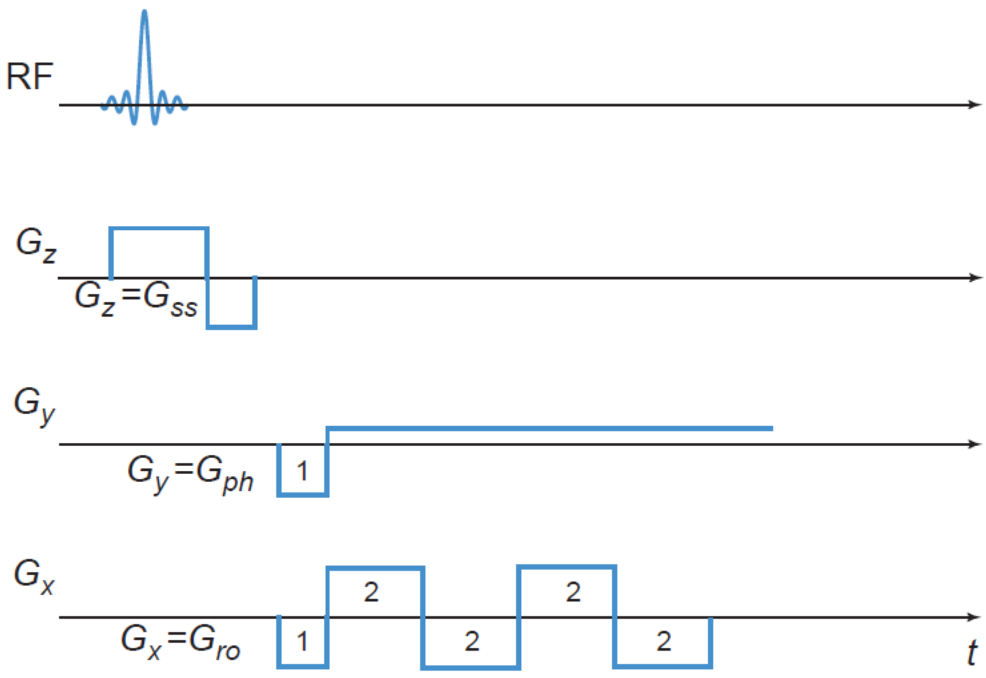
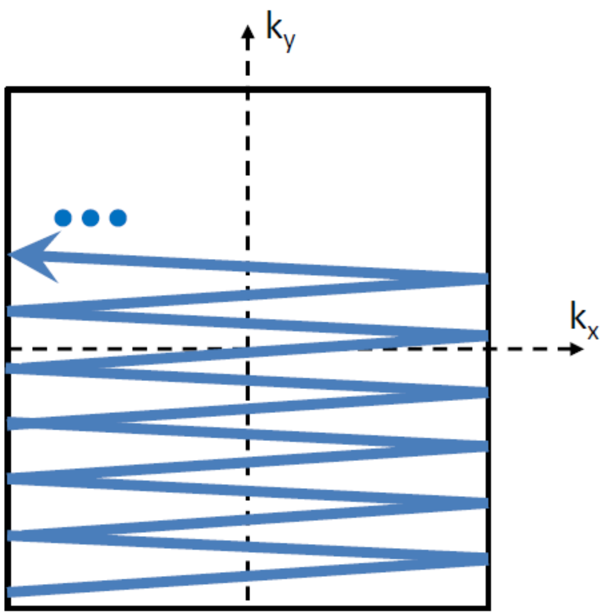


Figure 8

Solution:



After two negative gradient of Gx and Gy, the initial point is on the left-down corner.

The continuous Gy phase encoding gradient leads to continuous increase in ky direction, and the double length frequency encoding gradient shifts the point from the left to the right when the value is positive and from right to left when the value is negative. After this process, the k-space mapping is illustrated by the figure above.

9. Design an EPI pulse sequence that gives the square spiral k-space trajectory shown in Figure 9.

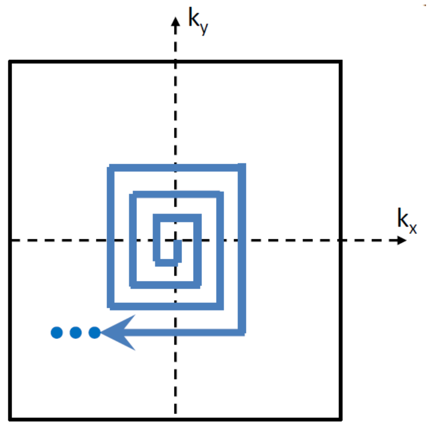


Figure 9

Solution:

In addition to the direction, two characteristics must be shown in your figure:

- The amplitude is gradually increasing.
- A/D is always on.