Lecture 4 – X ray Physics

This lecture will cover:

- Structure of the Atom (*Ref: Essential Physics 2.2*)
- Radioactivity (*CH3.2*)
- Generation of X ray (*CH2.1-2.3*)
    - The X-ray tube
    - The X-ray energy spectrum
- Interactions of X-ray with body (*CH2.4*)
- Linear and mass attenuation coefficients (*CH2.5*)

(*Supplementary reading: The Essential Physics of Medical Imaging CH3.1-3.3, CH6*)

Atom

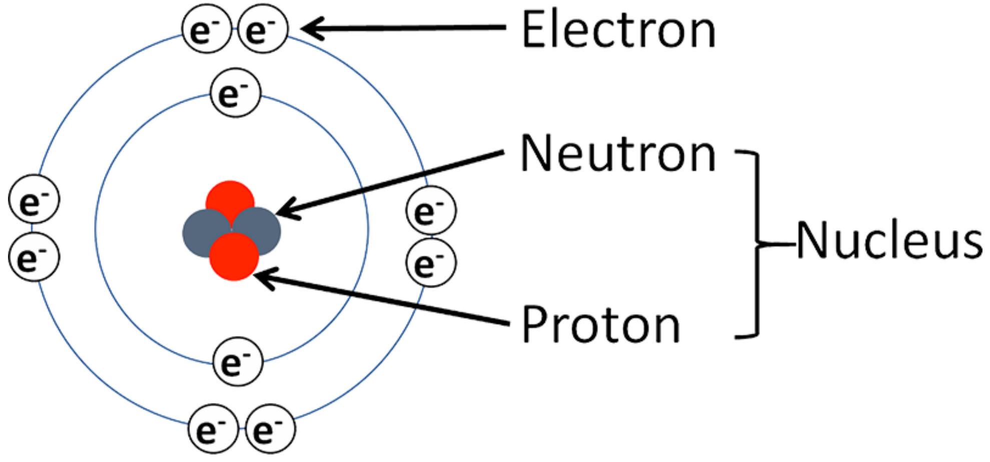

Figure. Structure of the atom.

Atom (原子) is the smallest division of an element.

- Nucleus (原子核) and electron (电子)
- Diameter: Angstrom (埃) ~10^{-10}m

  nm (nanometer)
- Mass: 1u=1.66 *10^{-27}kg (1/12 of C^{12})
- Charge: 1e =1.6*10^{-19} C (单位电子电荷)
- Energy: 1eV=1.6*10^{-19}J (电子伏)
- Expression: ${}^{A}_{Z}X$, where ***A*** is mass number (质量数), ***Z*** is atomic number (原子序数)

Electron

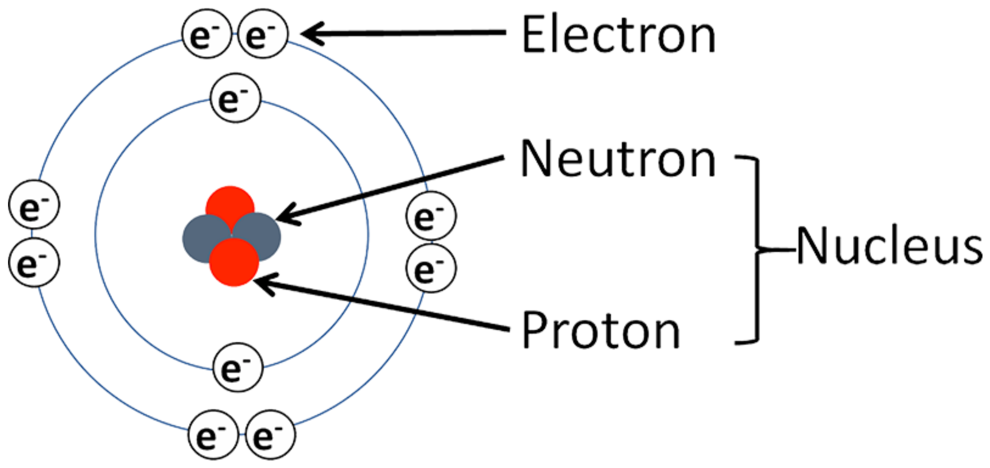

Figure. Structure of the atom.

- Mass: $9.108*10^{-31}$kg, 0.00055u
- Charge: $-1.6*10^{-19}$C
- Orbit around Nucleus
- Shell (壳层)：$2n^2$, (K,L,M…)
- Binding Energy (结合能): the energy required to remove an electron completely from the atom
- Electron transition (电子跃迁)
  - electron moved from outer shell to the inner vacancy shell;
  - Resulting in characteristic X-rays

Nucleus

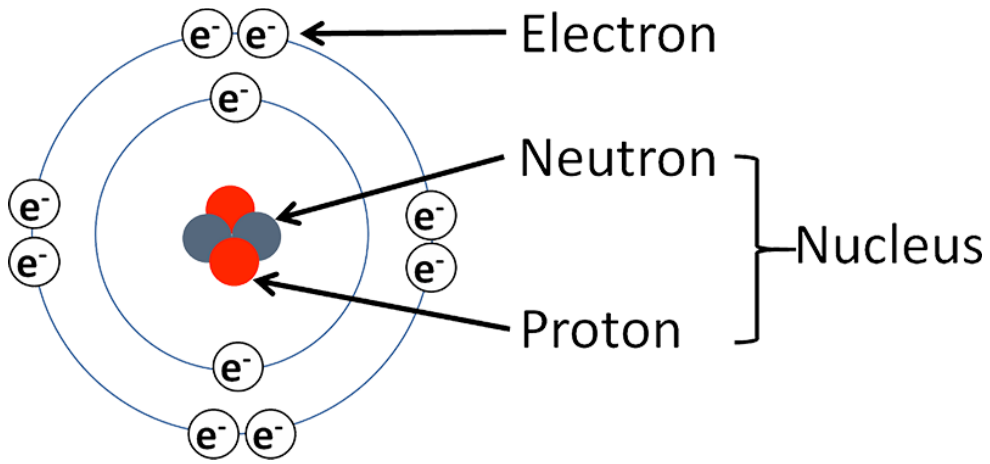

Figure. Structure of the atom.

- ~10^{-14}m
- Proton (质子): 1.6734 $*10^{-27}$kg (1.00727u), $+1.6*10^{-19}$C

  Neutron (中子): $1.6747*10^{-27}$kg (1.00866u)
- Nuclear families
  - Isotopes (同位素): same atomic number **Z**
  - Isobar (同量异位素): same mass number **A**
  - Isotones (同中子异位素): same number of neutrons **A-Z**
  - Isomers (同质异能素): same **A** and **Z** but different energy states in the nucleus

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Radioactivity (放射性衰变)

- Nuclear stability (核稳定性)
- Radioactivity (放射性活度): $Q = -dN/dt$, unit: Bq 贝克勒尔，1Ci(居里)=3.7*10^{10}Bq
- Decay constant (衰变常数): $\lambda = Q/N$, $N^* = Ne^{-\lambda t}$
- Half life (半衰期): the time required for radioactivity to drop to one half of its value.

$$\tau_{1/2} = \frac{\ln 2}{\lambda} = 0.693/\lambda$$

- Types of radioactivity
  - α decay: alpha particle (helium nucleus, 2 protons and 2 neutrons)
  - β decay: emitting electron, neutron convert to proton
  - γ (gamma) ray: excited state to stable energy state.

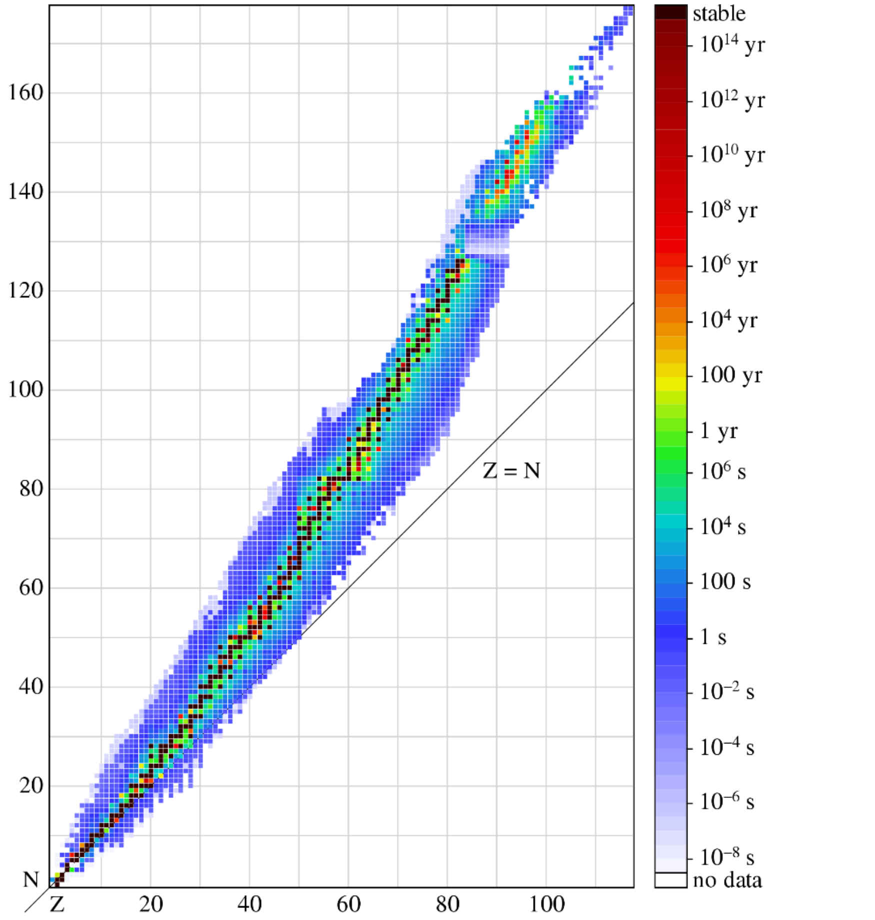

Figure. A plot of the nuclides where the number of protons (i.e., atomic number or Z) and neutrons of each nuclide is shown on the x- and y-axes, respectively. The stable nuclides are indicated by small black squares, whereas the colored squares represent radioactive (i.e., unstable) nuclides or radionuclides.

Energy

- **Particle Characteristics (photon-光子)**

$$E = h\nu = \frac{hc}{\lambda}$$

where E : energy of the photon (x or gamma ray)

h : Planck's constant (普朗克常数), $6.626 * 10^{-34}$ J*s

ν : frequency

c : speed of electromagnetic waves

λ : wavelength

- **Mass energy equivalence (质能方程)**

$$E = mc^2$$

an electron: 511 keV 1u: 931.5 MeV

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The X-ray tube (X射线管)

- Source to produce electrons (电子源)
- High speed electron flow (高速电子流): strong electric field and vacuum
- Target (靶): metal anode (Tungsten-钨, Molybdenum-钼, Rhodium-铑)

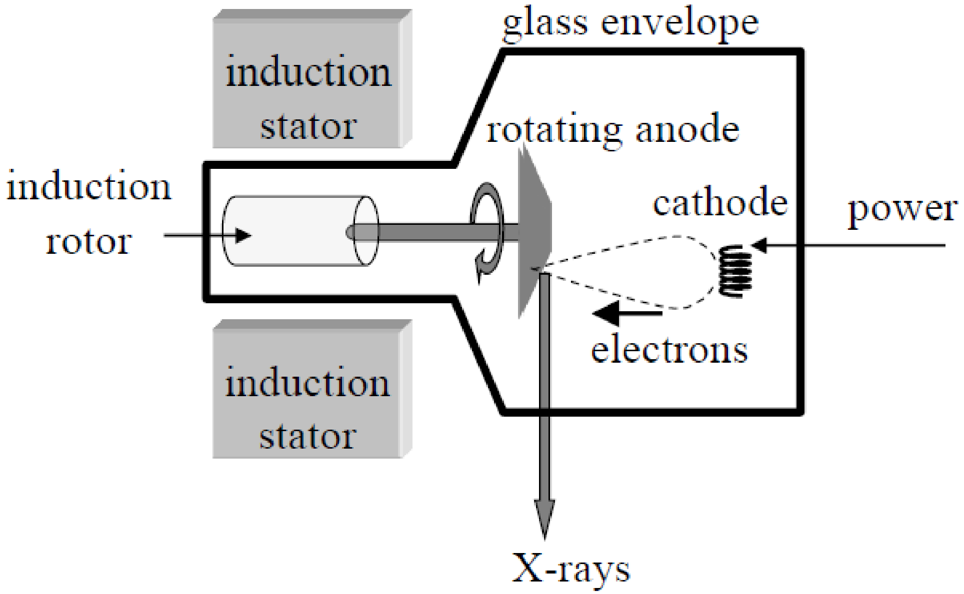

Figure. (left) An X-ray tube enclosed in an evacuated glass enclosure. (right) The individual components of an X-ray tube.

Focusing Cup（聚焦杯）

- To produce a narrow beam of electrons
- Constructed around the cathode
- Negatively-charged
- Multiple cathodes for the X-ray beams with different width

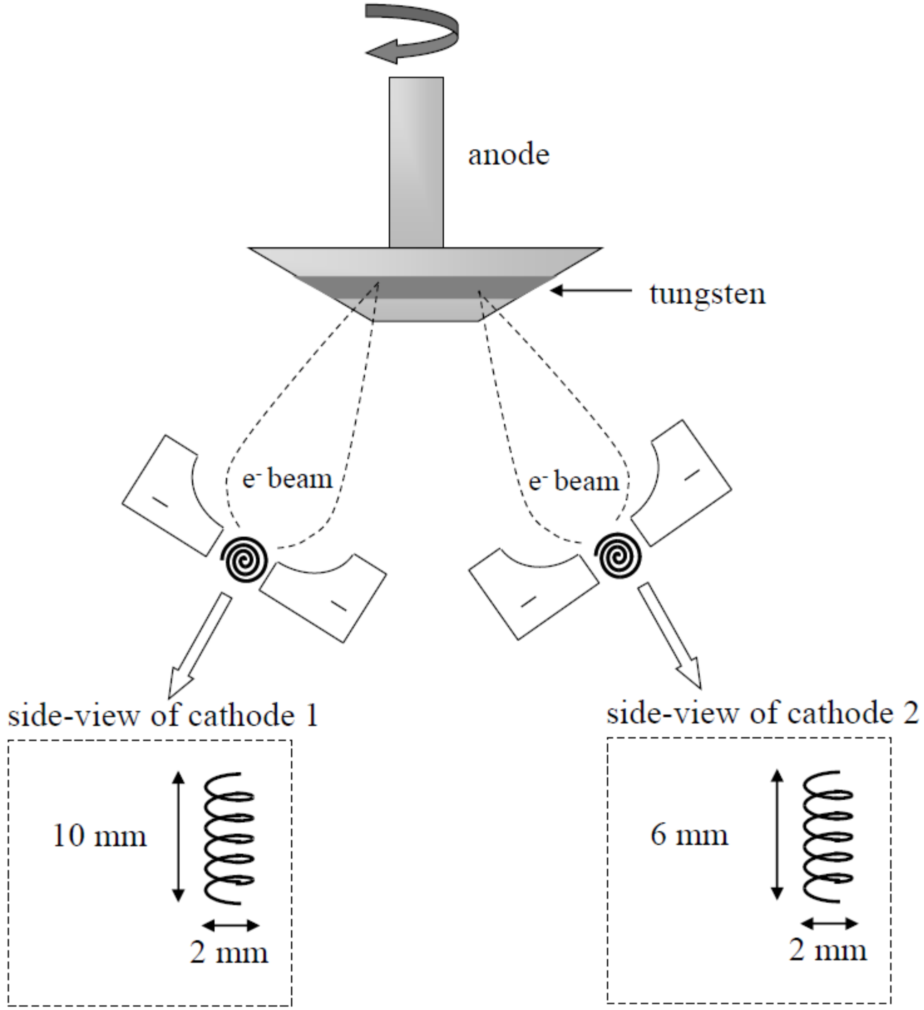

Figure. Top view showing the effect of the focusing cup on the shape of the electron beam striking the rotating anode. There are two cathodes (expanded in the side-views) which produce a wider or narrower beam depending upon the particular application. The direction of the X-ray beam produced is out-of-the page towards the reader.

Focal Spot and Coverage

- **Focal Spot (焦点) size:** $f = F \sin \theta$

  where θ: bevel angle
  F: the width of the electron beam

- **Coverage** = 2(source-patient distance) tanθ

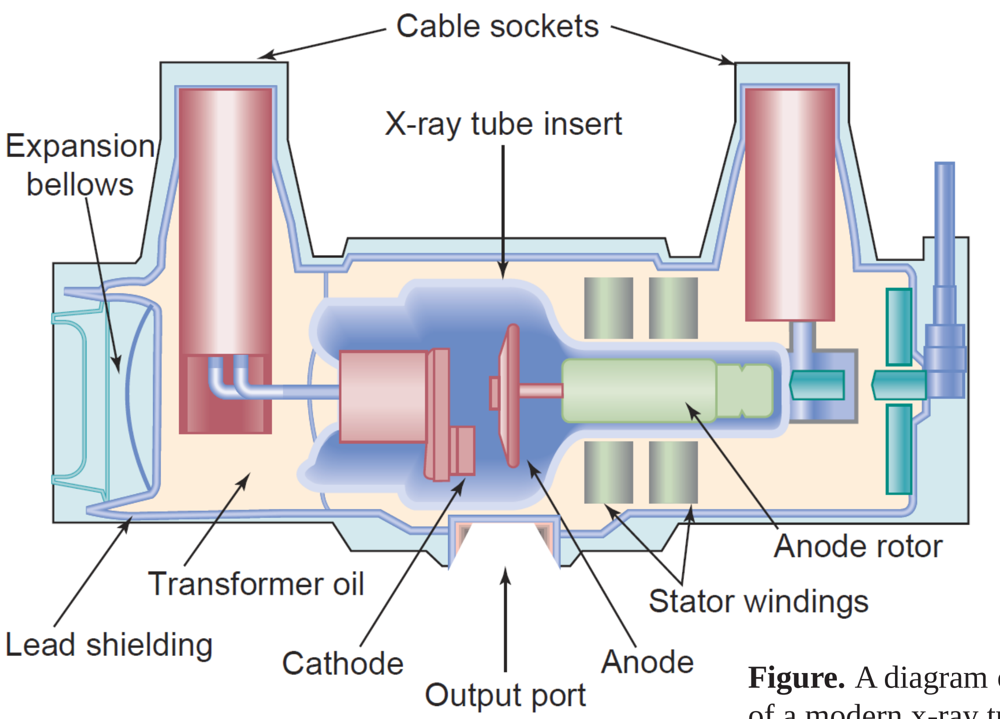

Figure. A diagram of the Cable sockets major components of a modern x-ray tube and housing assembly is shown.

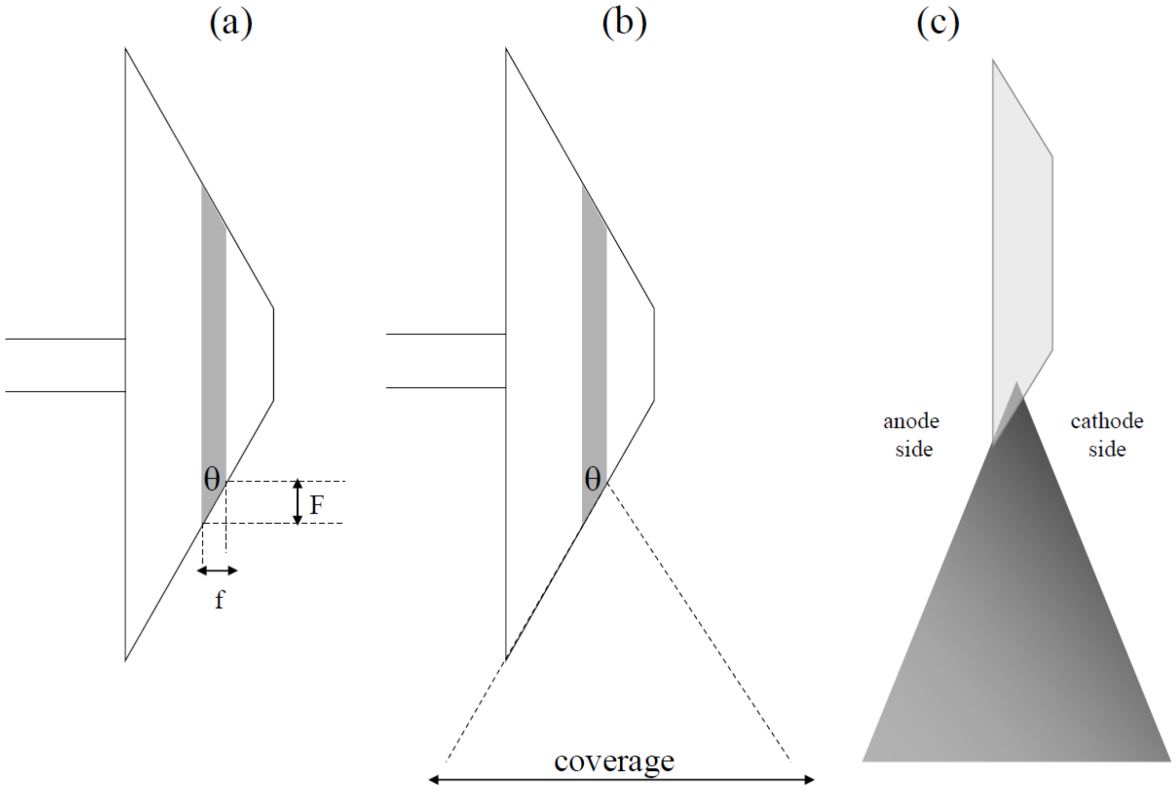

Figure. (a) The effect of the bevel angle θ on the effective focal spot size (f) as a function of the width of the electron beam (F). (b) Corresponding diagram for the effect of θ on the coverage of the X-ray beam. (c) Illustration of the Heel effect, in which the X-ray intensity is higher on the 'cathode side' of the beam than on the 'anode side', since the X-rays on the anode side have to travel further through the anode itself before leaving the tube, and are therefore more highly attenuated.

Parameters

The parameters chosen by operator for X-ray imaging:

- Accelerating voltage (25-140 kVp) – 管电压
- Tube current (50-1000 mA) – 管电流
- Exposure time – 曝光时间
- Limitation of kVp and tube current are defined by the maximum power dissipated in an exposure time of 0.1s

Interaction of incident electron

- Collision Loss (碰撞损失) : electron in outer shell － heat (热能)
- Radiation Loss (辐射损失)
  - Nucleus: bremsstrahlung (韧致辐射)

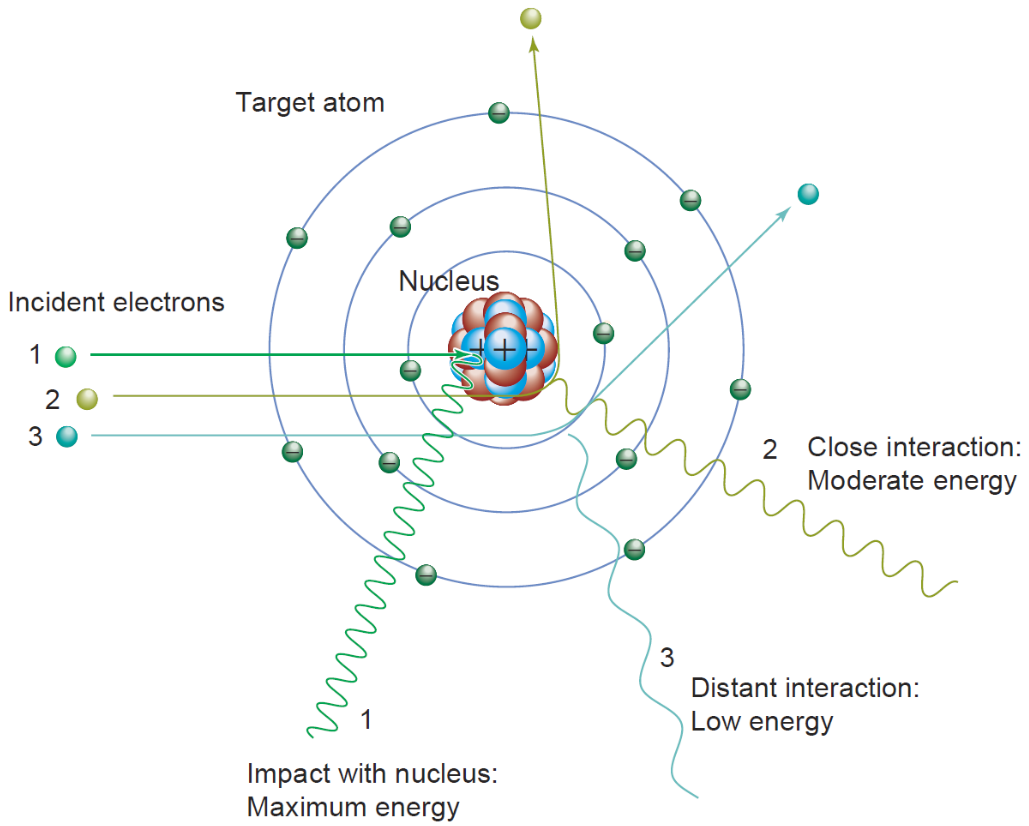

Figure. Bremsstrahlung radiation arises from energetic electron interactions with an atomic nucleus of the target material. In a "close" approach, the positive nucleus attracts the negative electron, causing deceleration and redirection, resulting in a loss of kinetic energy that is converted to an x-ray. The x-ray energy depends on the interaction distance between the electron and the nucleus; it decreases as the distance increases.

Interaction of incident electron

- Radiation Loss (辐射损失)
  - Electron: characteristic X-ray (特征X射线)

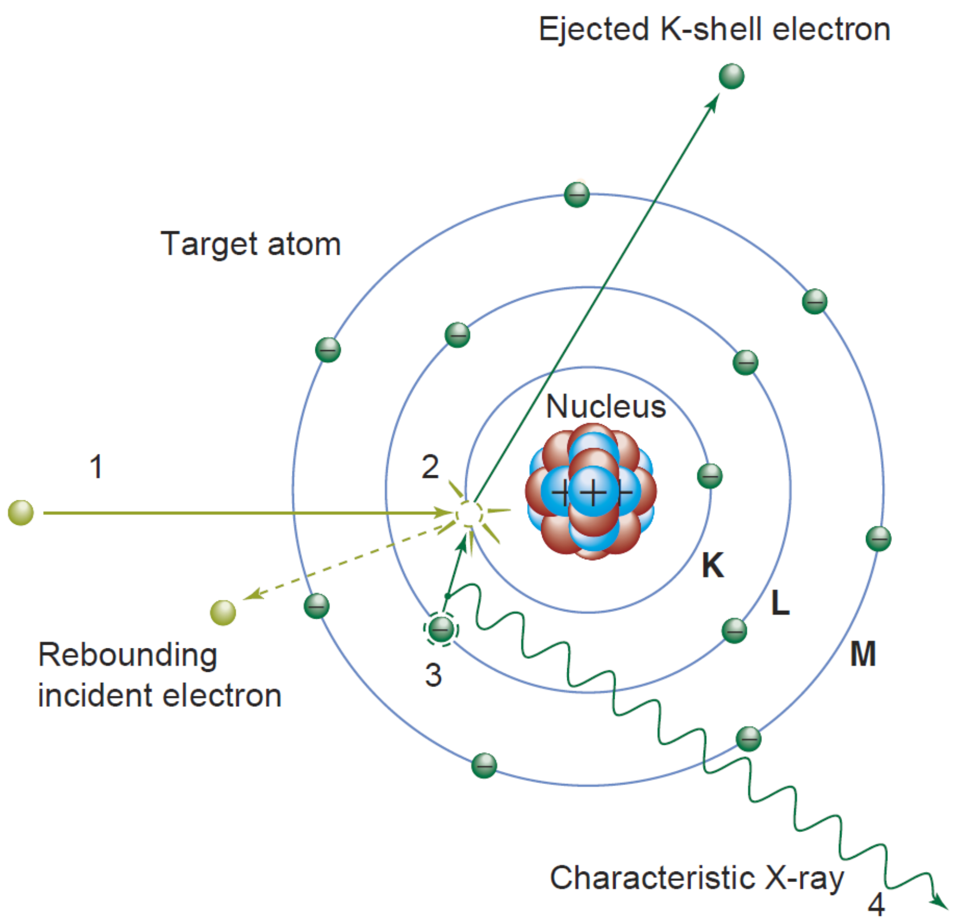

Figure. Generation of a characteristic x-ray in a target atom occurs in the following sequence: (1) The incident electron interacts with the K-shell electron via a repulsive electrical force. (2) The K-shell electron is removed (only if the energy of the incident electron is greater than the K-shell binding energy), leaving a vacancy in the K-shell. (3) An electron from the adjacent L-shell (or possibly a different shell) fills the vacancy. (4) A Ka characteristic x-ray photon is emitted with energy equal to the difference between the binding energies of the two shells. In this case, a 59.3-keV photon is emitted.

X-ray Energy Spectrum (X射线谱)

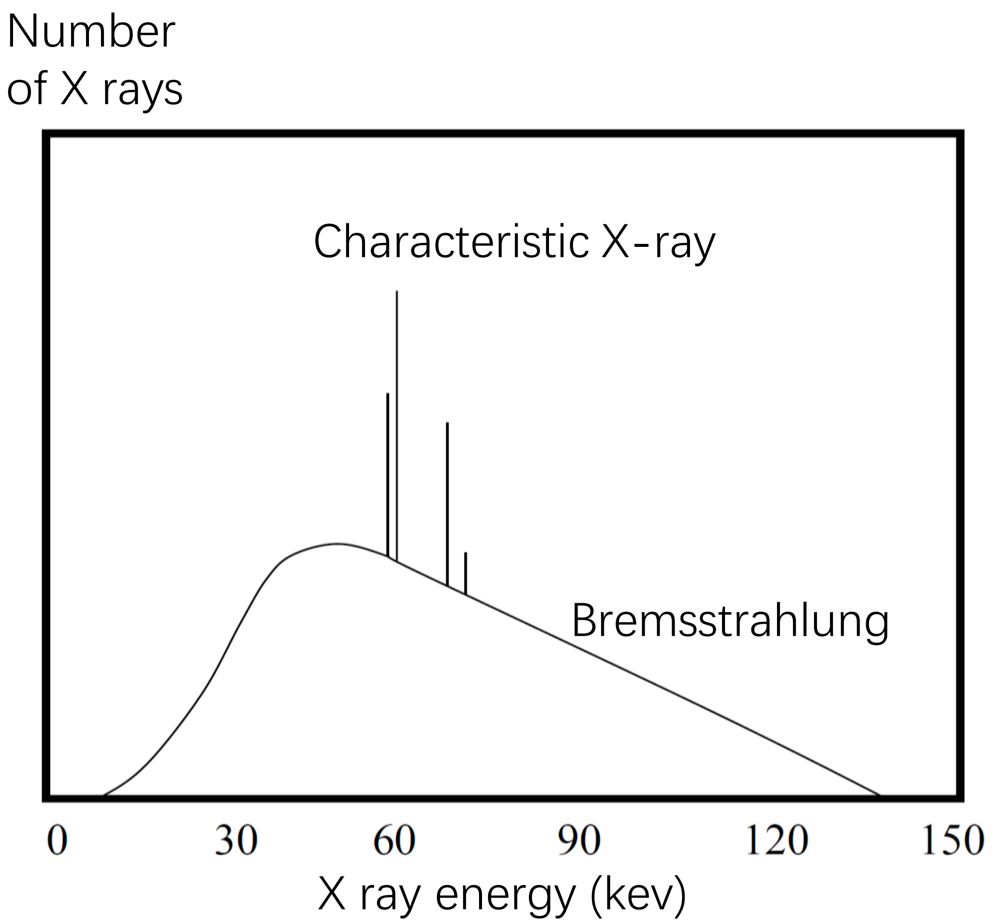

Figure. The energy spectrum of a beam emitted from an X-ray tube with a tungsten anode operating at 140 kVp. The very low energies are absorbed by the tube itself. Characteristic lines are seen as sharp lines, superimposed upon a broad energy distribution from general radiation.

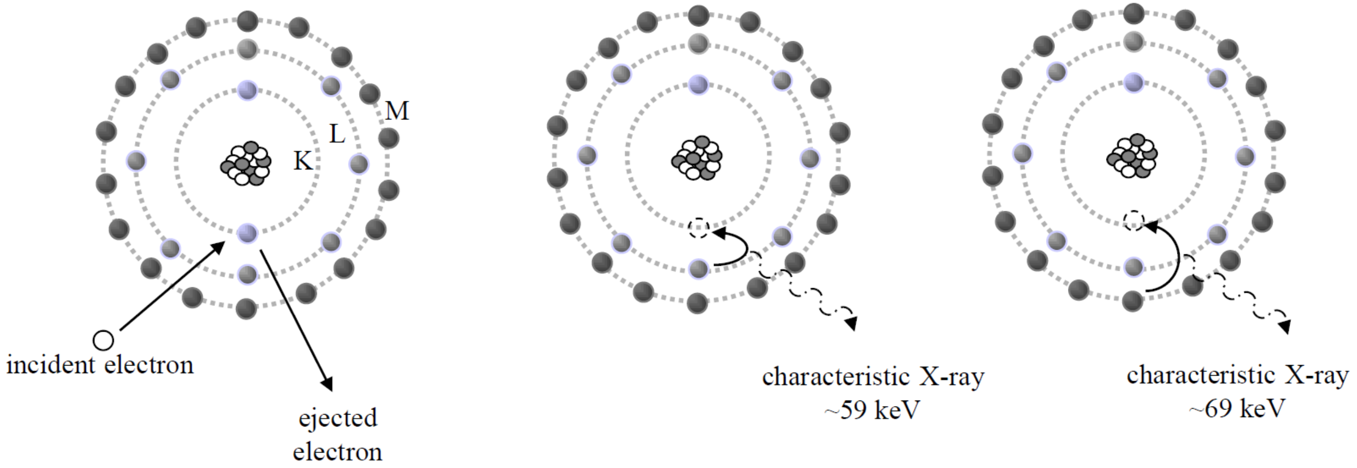

Figure. The chain-of-events involved in production of characteristic radiation from the metal anode in an X-ray tube. (left) A high energy electron from the cathode ejects an inner electron from the metal target in the anode. An outer electron fills the hole in the inner shell and the difference in binding energies of the inner and outer shell electrons is converted into a characteristic X-ray which is emitted. The outer electron can come from the L-shell (centre) or M-shell (right), resulting in two different characteristic X ray energies.

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Interaction of X-ray with matter

- Photoelectric effect (光电效应)
- Compton scattering (康普顿散射)
- Pair production (电子对效应)
- Coherent scattering (相干散射)

Photoelectric effect

- Energy of electron: $E = h\nu - E_B$
- Characteristic X-ray: Auger electron (俄歇电子)
- The probability of a photoelectric effect : $P_{\mathrm{pe}} \propto \rho \frac{(Z_{\mathrm{eff}})^3}{(h\nu)^3}$

Where $\boldsymbol{\rho}$: tissue density; $\boldsymbol{Z_{eff}}$: the effective atomic number; $\boldsymbol{h\nu}$: energy of incident X-ray

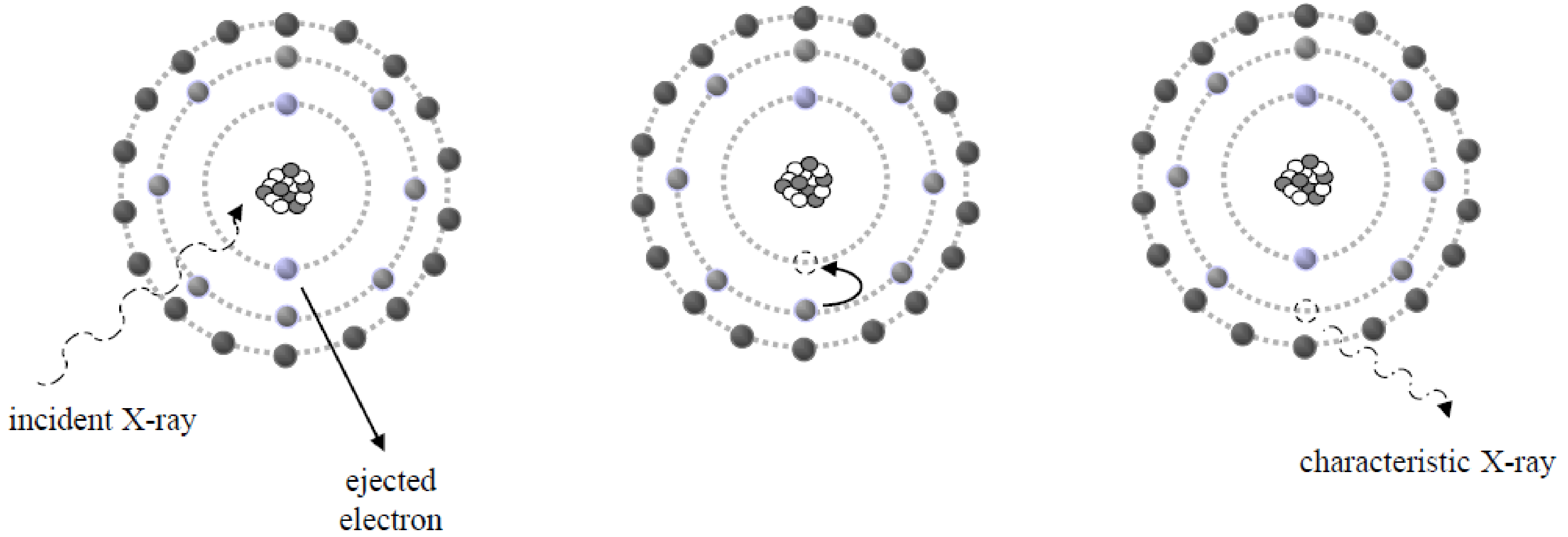
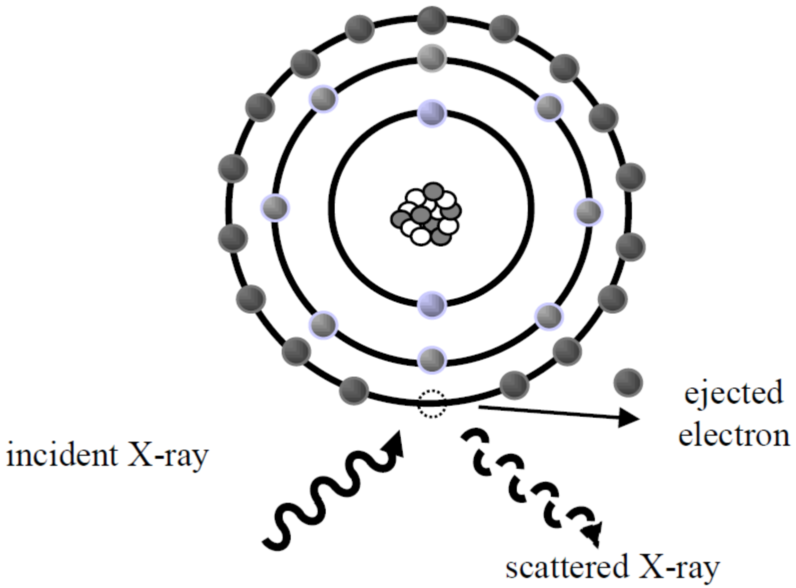


Figure. A photoelectric interaction between an incident X-ray and tissue involves an inner electron being emitted (left), an electron from an outer shell filling the hole in the inner shell (middle), and the difference in the binding energies being transferred to a characteristic X-ray. This X-ray has very low energy and is absorbed after travelling ~1 mm in tissue..

Compton scattering

- Wavelength change: $\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$

- Energy of scattered X-ray: $E_{\text{scat}} = \frac{E_{\text{inc}}}{1+\left(\frac{E_{\text{inc}}}{m_e c^2}\right)(1-\cos\theta)}$



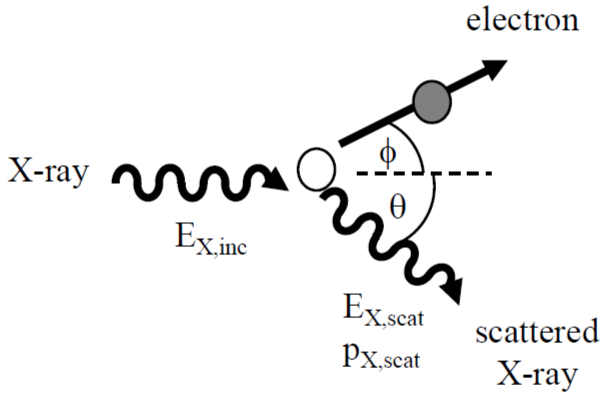


Figure. Compton scattering of an incident X-ray involves an outer electron being ejected from a tissue molecule (left), with the X-ray being scattered at an angle h with respect to its initial trajectory (right).

Compton scattering

The probability of Compton scattering

- Independent of atomic number
- Proportional to the tissue electron density
- Weakly dependent on the energy of incident X-ray

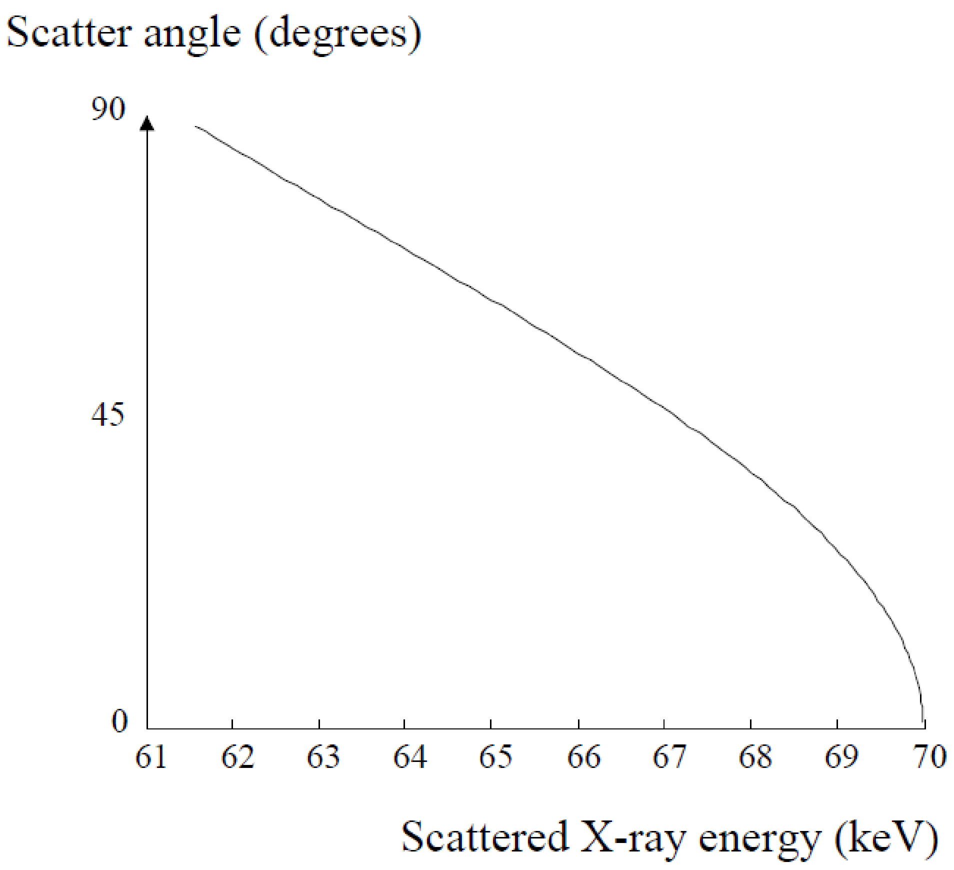


Figure. The energy of a Compton-scattered X-ray as a function of the scatter angle for a 70 keV incident energy.

Other Interactions

- Pair production(电子对效应) : with nucleus & E⩾1.02MeV
- Coherent scattering (相干散射) : photon only changes direction.

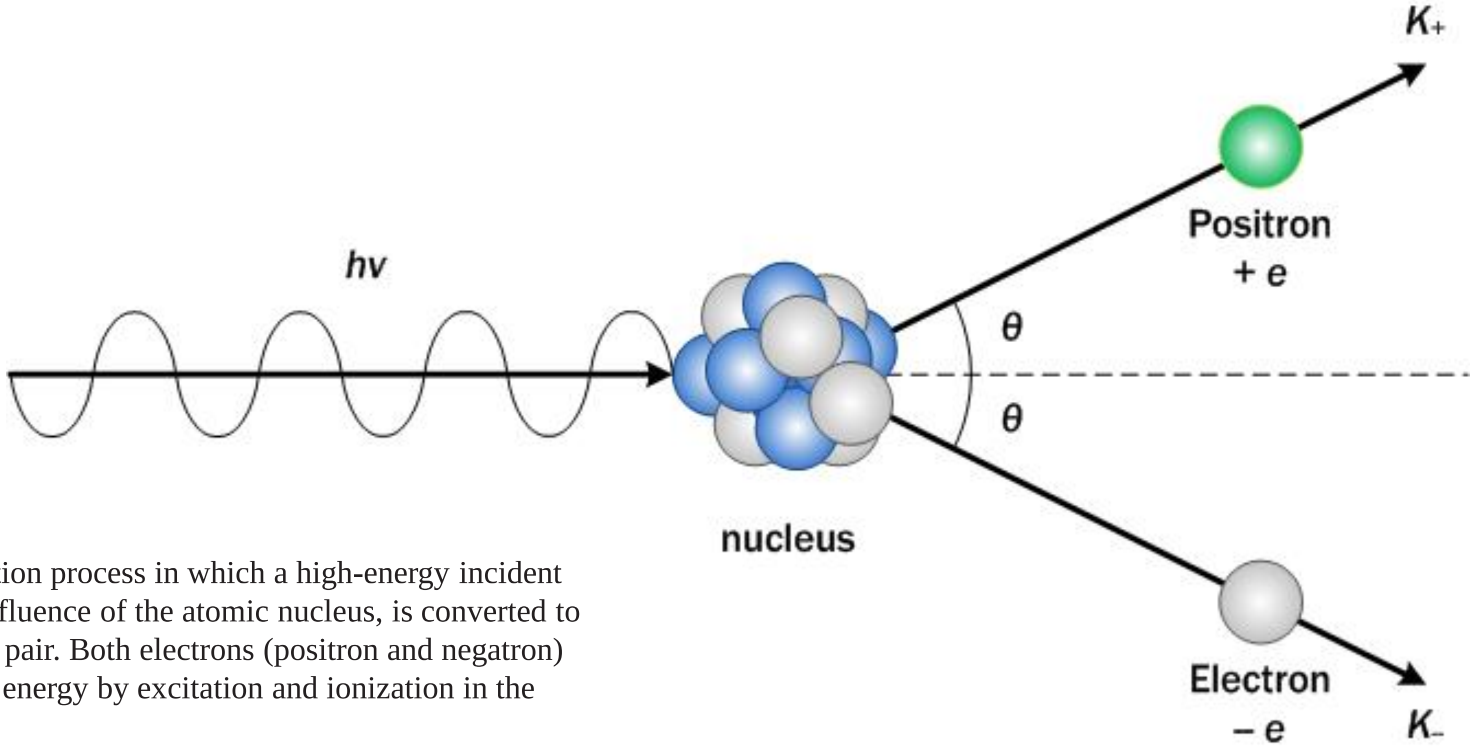
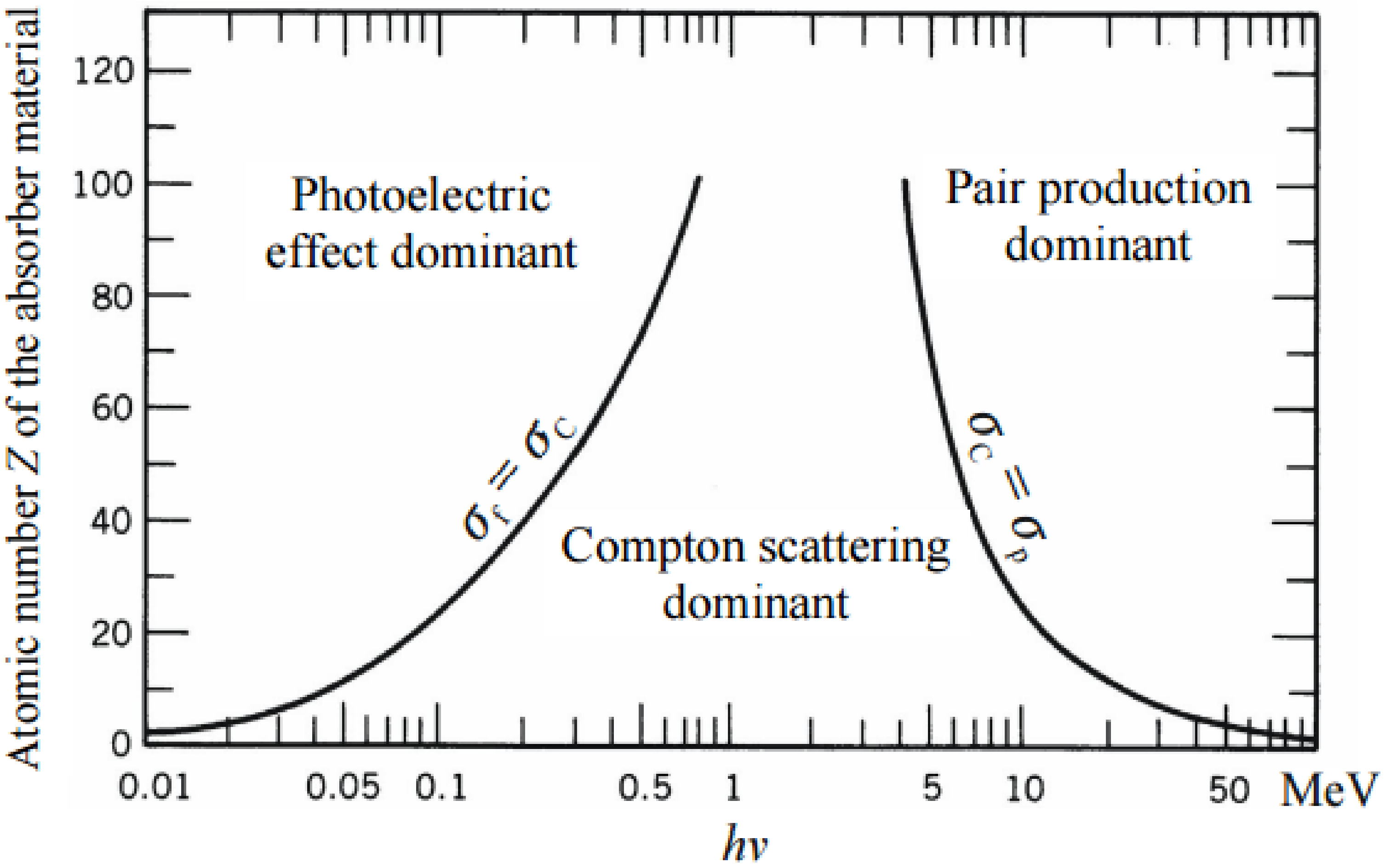


Figure. pair production process in which a high-energy incident photon, under the influence of the atomic nucleus, is converted to an electron-positron pair. Both electrons (positron and negatron) expend their kinetic energy by excitation and ionization in the matter they traverse.

Interaction regions of X-rays with matter

- The interaction modes of photons with matter depending on elemental atomic number Z and the photon energy (MeV)



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Attenuation coefficient

- Number of X-ray transmitted:

$$N = N_0 e^{-\mu(E)x}$$

Where

N_0: the number of incident X-ray

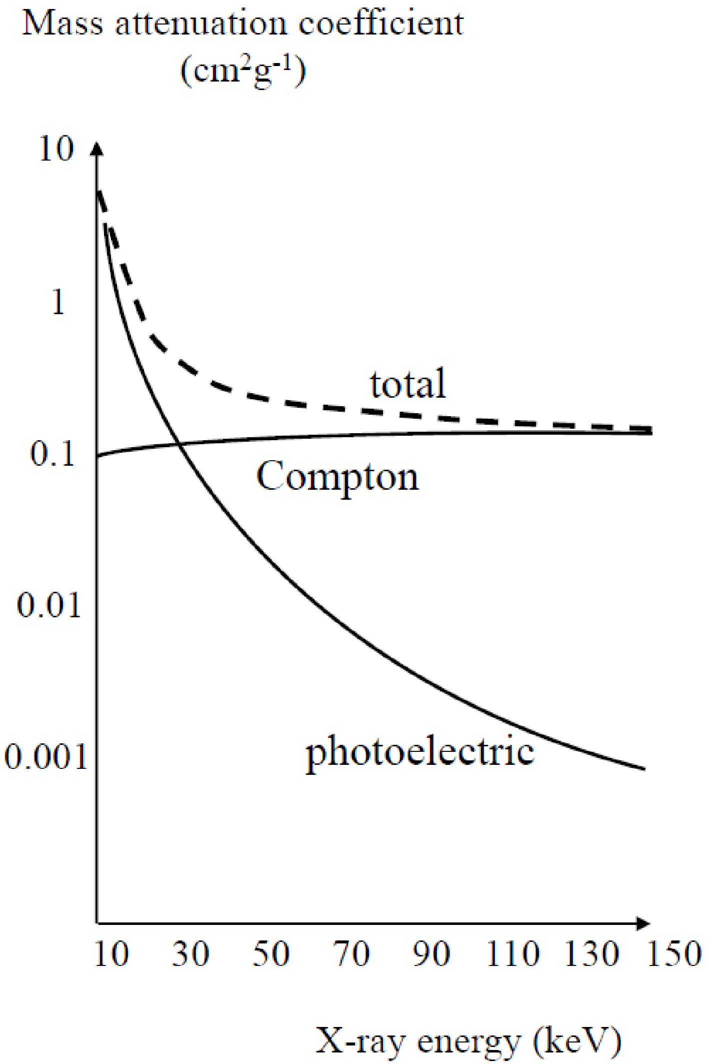
x : thickness of tissue

$\mu(E)$: linear attenuation coefficient and $\mu(E) = \mu(E)_{\mathrm{pe}} + \mu(E)_{\mathrm{compton}}$

- Mass attenuation coefficient（质量衰减系数）： $\mu_m(E) = \frac{\mu(E)}{\rho}$

- Half value layer (半价层) : HVL = $\frac{\ln 2}{\mu}$

Attenuation coefficient



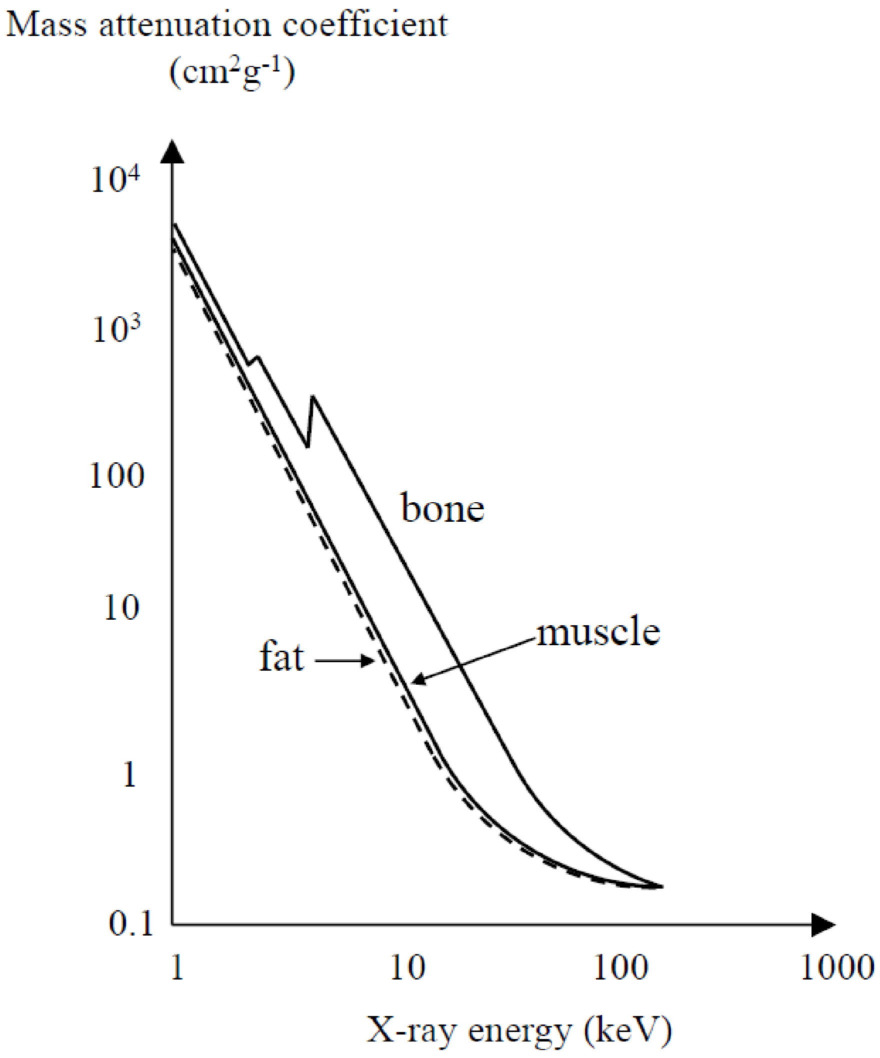


Figure. (left) The individual contributions from photoelectric attenuation and Compton scatter add together to give a net tissue linear attenuation coefficient (the specific data are shown for water). The contribution from the photoelectric effect dominates at low X-ray energies, but Compton scatter is the more important term at high energies. (right) The mass attenuation coefficient of lipid, muscle and bone as a function of X-ray energy.

Attenuation coefficient

TABLE3-1 MATERIAL DENSITY, ELECTRONS PER MASS, ELECTRON DENSITY, AND THE LINEAR ATTENUATION COEFFIENT (AT 50 keV) FOR SEVERAL MATERIALS

MATERIAL	DENSITY(g/cm^3)	ELECTRONS PER MASS(e/g) $\times 10^{23}$	ELECTRON DENSITY (e/cm^3) $\times 10^{23}$	μ @ 50keV(cm^{-1})
Hydrogen gas	0.000084	5.97	0.0005	0.000028
Water vapor	0.000598	3.34	0.002	0.000128
Air	0.00129	3.006	0.0038	0.00029
Fat	0.91	3.34	3.04	0.193
Ice	0.917	3.34	3.06	0.196
Water	1	3.34	3.34	0.214
Muscle	1	3.36	3.36	0.214
Compact bone	1.85	3.192	5.91	0.573

X-ray Imaging Physics

- Differential absorption of X-rays by various tissues.
- Not considering scattering (散射)
- 2D projection(投影) of the tissues between the X-ray source and detector

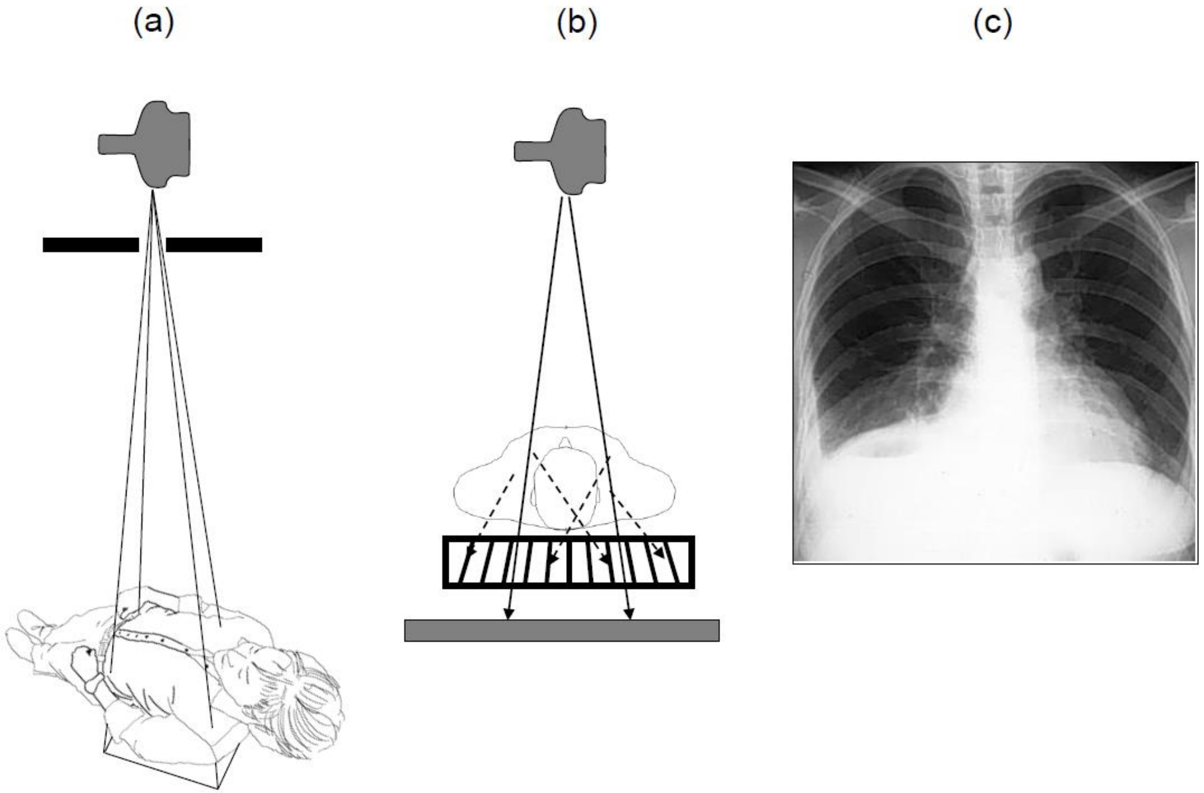
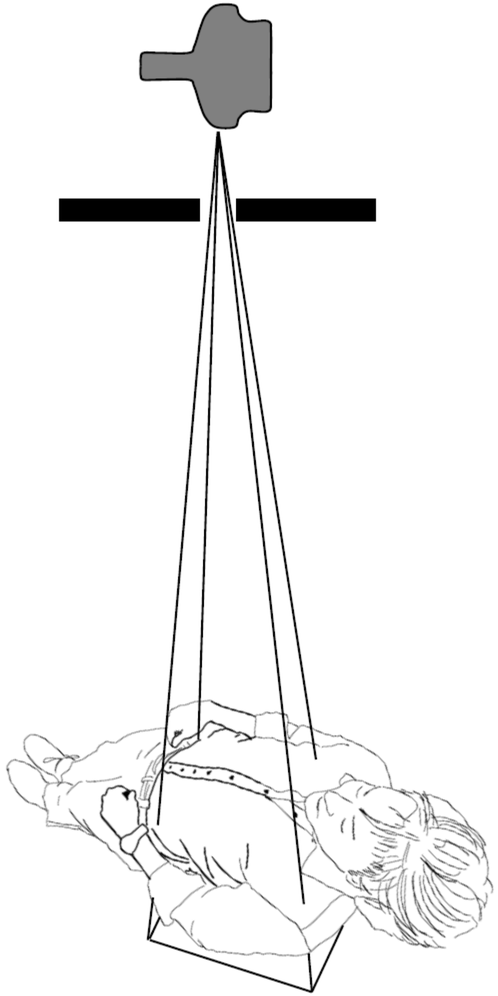


Figure. (a) The set-up for planar radiography. The X-ray beam from the tube is collimated, passes through the patient, and forms an image on the digital detector placed below the patient. (b) An anti-scatter grid is placed directly in front of the detector to reduce the contribution from scattered X-rays in order to increase the image contrast. (c) An example of a planar radiograph through the chest. The bones attenuate X-rays to a much greater degree than the soft tissue of the lungs, and appear bright on the image.

Basic criteria for imaging

For high SNR and CNR of images

- Sufficient X-rays transmitting through the body
- Sufficiently different absorption between different tissue-types
- Removal of X-rays scattered through unknown angles



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