Numerical Optimization, 2023 Fall

Homework 3

Due 23:59 (CST), Nov. 16, 2023

Problem 1. Prove the dual of the dual of a linear programming (standard form) is itself.[25pts]

Problem 2. Prove the dual objective increases after a pivot of the dual simplex method.[25pts]

Problem 3. Let $L(\boldsymbol{x}, \boldsymbol{\lambda})$ be the Lagrangian of a linear programming problem, and $(\boldsymbol{x}^*, \boldsymbol{\lambda}^*)$ be the optimal primal-dual solution. Prove that

$$L(\boldsymbol{x}, \boldsymbol{\lambda}^*) \geq L(\boldsymbol{x}^*, \boldsymbol{\lambda}^*) \geq L(\boldsymbol{x}^*, \boldsymbol{\lambda}),$$

for any primal feasible $\boldsymbol{x}$ and dual feasible $\boldsymbol{\lambda}$.[25pts]

Problem 4. Construct a linear programming problem for which both the primal and the dual problem has no feasible solution.[25pts]