Lecture 15&16 – NMR Physics

This lecture will cover: *(CH5.1-5.7, 5.13)*

- Introduction
- Effects of a strong magnetic field
- Magnetic Resonance
- Relaxation time
- Free induction decay

(*Supplementary reading: The Essential Physics of Medical Imaging CH12.1-12.3)*

Introduction to MRI

- Provide a spatial map of hydrogen nuclei in different tissues;
- Image intensity depends on the number of proton and physical properties of the tissue;

Fig. The MR coronal, sagittal and axial images, respectively, of the brain.

Introduction to MRI

- Advantages
  - No ionizing radiation;
  - Acquire images in any 2D and 3D plane;
  - Excellent soft-tissue contrast
  - A spatial resolution of 1mm or less
  - Negligible penetration effect
- Disadvantages
  - Slow acquisition time
  - Not able to scan patients with metallic implants;
  - Very expensive

MRI system

- Superconducting magnet
- A set of three magnetic field gradient coils
- Radio frequency transmitter and receiver

(a)

(b)

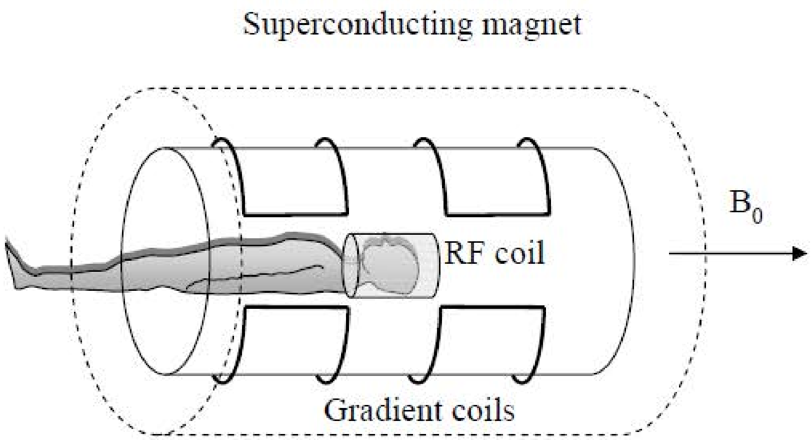

Fig. (a) A high-field clinical magnet with patient bed. (b) The three major components of an MRI system, including the superconducting magnet which produces a strong magnetic field (B0) in the horizontal direction. Only one of the three gradient coils is shown for clarity.

Lecture 15 – Magnetic Resonance Imaging

This lecture will cover:

- – Introduction
- **– Effects of a strong magnetic field**
- – Magnetic Resonance
- – Relaxation time
- – Free induction decay

Nuclear Spin

- Spin (自旋): a charged particle rotate around an internal axis with a given value of angular momentum;

$$L_I = \sqrt{I(I+1)} \cdot \hbar$$

where $\hbar = h/2\pi$: h is Planck constant

I : spin quantum number (自旋量子数)

- $I = 0$, even number of proton and neutron
- I is integer: odd number of proton and neutron
- I is half: the addition of proton and neutron is odd

L_I in a magnetic field with direction of z

$$L_z = m_I \cdot \hbar, \quad m_I = I, I-1, I-2, \cdots - I$$

m_I: spin magnetic quantum number (自旋磁量子数)

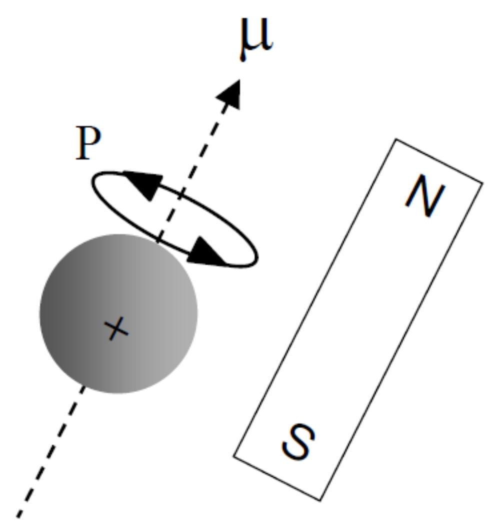

Fig. The internal rotation of a proton creates a magnetic moment, and so the proton acts as a magnet with north and south pole.

Magnetic moment

- Magnetic moment (磁矩): spinning nucleus can be thought of as a very small bar magnet with north and south pole.
- The nuclear magnetic moment:

$$\overrightarrow{\mu_I} = \gamma \cdot \overrightarrow{L_I}$$

Where $\gamma = g_I e/2m_p$ is gyromagnetic ratio (磁旋比)

g_I : g factor, a dimensionless quantity that characterizes the magnetic moment and angular momentum of an atom, a particle or nucleus.

- μ_I in a magnetic field with direction of z

$$\mu_z = \gamma L_z = m_I g_I \mu_N, \quad m_I = I, I-1, I-2, \cdots - I$$

where $\mu_N = e\hbar/2m_p$ is nuclear magneton (核磁子)

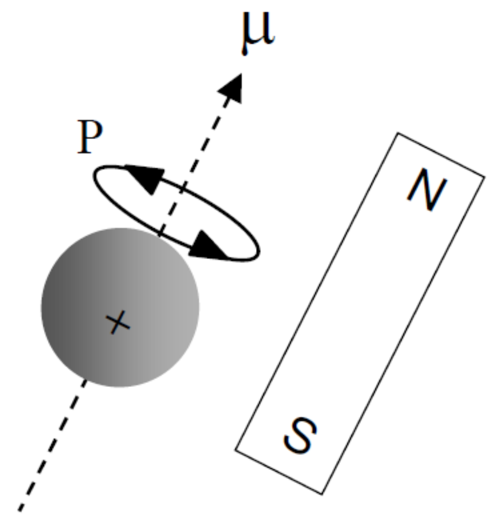

Fig. The internal rotation of a proton creates a magnetic moment, and so the proton acts as a magnet with north and south pole.

Medically useful nuclei

Nucleus	Isotopic abundance	Relative sensitivity	spin quantum number	g factor	Magnetic moment (μ_N)	gyromagnetic ratio (10^8Hz/Tesla)
${}_{1}^{1}H$	99.98%	1	1/2	5.5855	2.7927	2.6753
${}_{6}^{13}C$	1.10%	0.016	1/2	1.4046	0.70216	0.6728
${}_{7}^{14}N$	0.36%	0.001	1	0.7023	0.40357	0.1934
${}_{9}^{19}F$	100%	0.830	1/2	5.256	2.6273	2.5179
${}_{11}^{23}Na$	100%	0.093	3/2	1.478	2.2161	0.7031
${}_{15}^{31}P$	100%	0.066	1/2	2.262	1.1305	1.084

Energy in the magnetic field

When magnetic filed B_0 is applied:

- Two magnetic moments for proton: parallel and anti-parallel to the direction of $\mathbf{B_0}$
- The energy of the protons is related to the magnetic moments
- The energy difference between two states:

$$\Delta E = g_I \mu_N B_0 = \frac{\gamma h B_0}{2\pi}$$

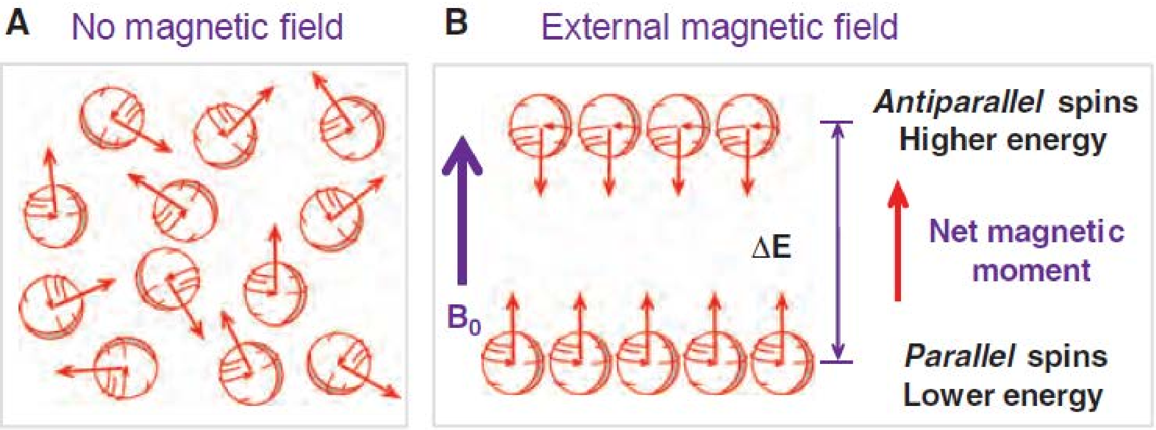

Fig. Simplified distributions of "free" protons without and with an external magnetic field are shown. (**A**) Without an external magnetic field, a group of protons assumes a random orientation of magnetic moments, producing an overall magnetic moment of zero. (**B)** Under the influence of an applied external magnetic field, B_0, the protons assume a nonrandom alignment in two possible orientations: parallel and antiparallel to the applied magnetic field. A slightly greater number of protons exist in the parallel direction, resulting in a measurable net magnetic moment in the direction of B_0.

Energy in the magnetic field

- The relative number of protons in the two states:

$$\frac{N_{\text{anti-parallel}}}{N_{\text{parallel}}} = e^{-\frac{\Delta E}{kT}} = e^{-\frac{\gamma h B_0}{2\pi kT}} \approx 1 - \frac{\gamma h B_0}{2\pi kT}$$

Where B_0: static magnetic field

T: temperature in Kelvin

k: Boltzmann's constant ($1.38*10^{-23}$J/K)

- The difference in population between two energy levels:

$$N_{\text{parallel}} - N_{\text{anti-parallel}} = N_{\text{total}} \frac{\gamma h B_0}{4\pi kT}$$

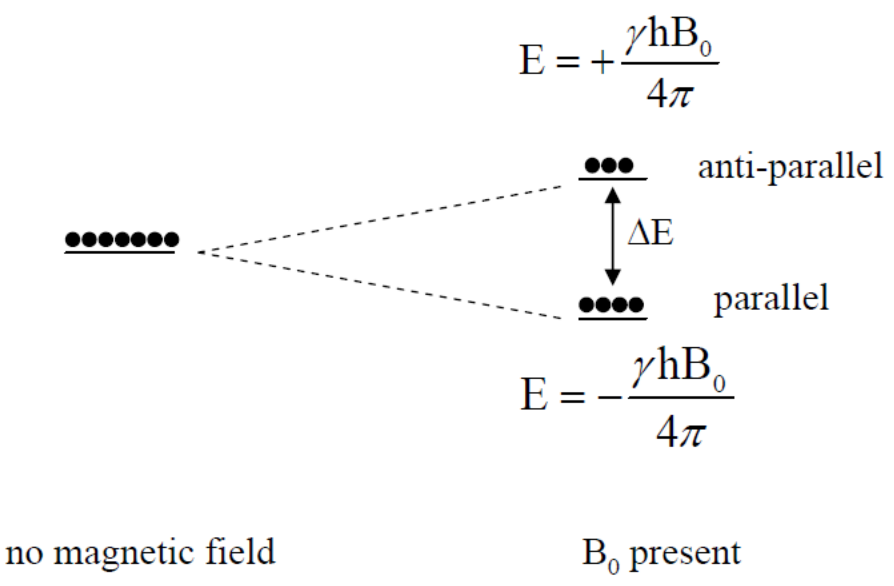

Fig. Proton configurations. (left) In the absence of a strong magnetic field, the energies of all the random orientations of the magnetic moments are the same. (right) When a strong magnetic field is applied, the single energy level splits into two levels, one corresponding to the magnetic moments being in the parallel state, and the other the anti-parallel state. The energy difference between the two levels depends upon the value of B_0.

Classical Precession

- ➢ There is an angle between the axis of spin and the direction of magnetic field;
- ➢ A torque is created by the combination of the magnetic field and the spin, which is perpendicular to the spin angular moment and causes proton precess (进动、旋进) around the axis of the magnetic field;
- ➢ Larmor precession (拉摩尔旋进):

$$\omega_0 = 2\pi f_0 = \gamma B_0$$

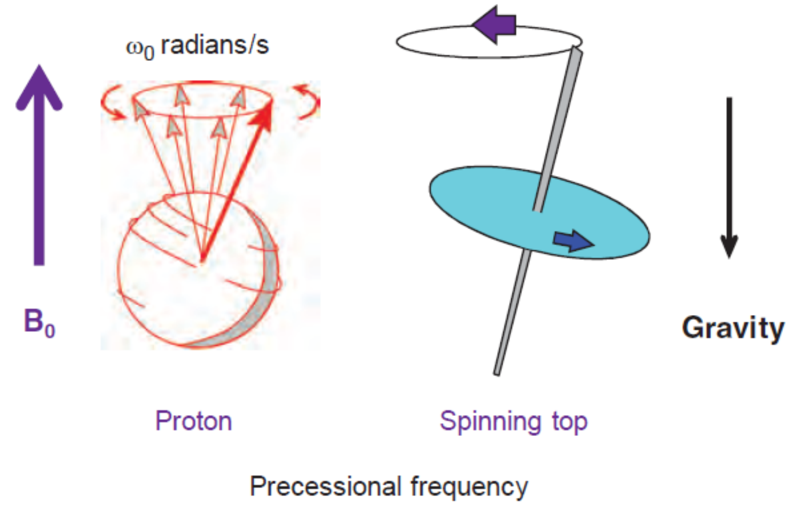

Fig. A single proton *precesses* about its axis with an angular frequency, ω, proportional to the externally applied magnetic field strength, according to the *Larmor* equation. A well-known example of precession is the motion a spinning top makes as it interacts with the force of gravity as it slows.

Net magnetization

- The net magnetization (磁化强度) with direction of B_0:

$$M_0 = \sum_{n=1}^{N_{\mathrm{total}}} \mu_{z,n}$$

$$= \frac{\gamma h}{4\pi}\left(N_{\mathrm{parallel}} - N_{\mathrm{anti-parallel}}\right)$$

$$= \frac{\gamma^2 h^2 B_0 N_{\mathrm{total}}}{16\pi^2 kT}$$

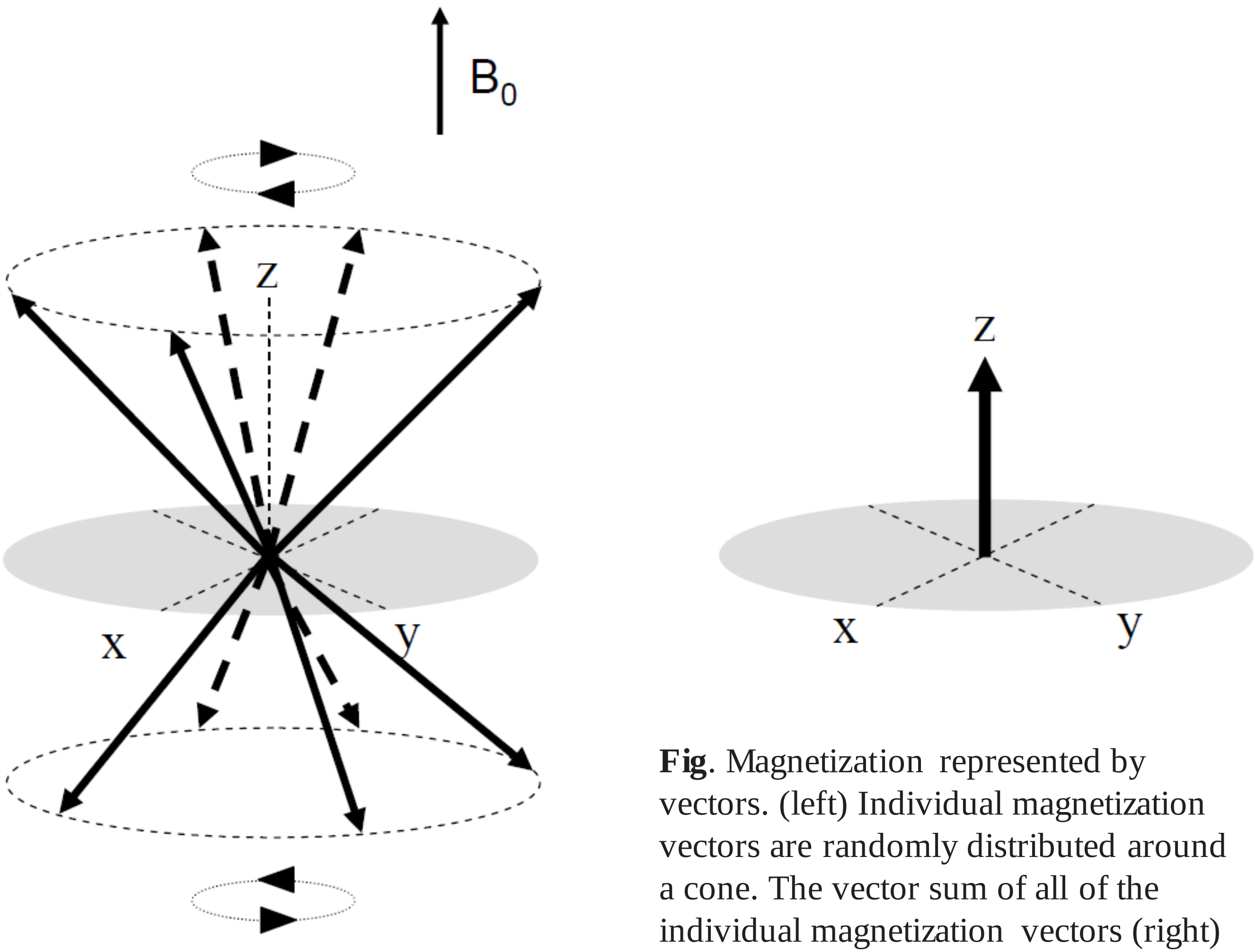

Fig. Magnetization represented by vectors. (left) Individual magnetization vectors are randomly distributed around a cone. The vector sum of all of the individual magnetization vectors (right) is simply a static component in the direction of B_0.

Lecture 15 – Magnetic Resonance Imaging

This lecture will cover:

- – Introduction
- – Effects of a strong magnetic field
- – **Magnetic Resonance**
- – Relaxation time
- – Free induction decay

Magnetic Resonance

- Radiofrequency (RF, 射频) pulse: $hf = \Delta E = \frac{\gamma h B_0}{2\pi} \Rightarrow f = \frac{\gamma B_0}{2\pi}$
  - Stimulate transition between the energy levels;
  - The magnetic component of the RF pulse as B_1
  - Perpendicular to the direction of static magnetic field or net magnetization z
  - The frequency of RF is identical to the procession frequency
  - Creation of transverse magnetization, and the "tip angle" defined as the angle through which the net magnetization is rotated: $\alpha = \gamma B_1 \tau_{B_1}$, where τ_{B_1} is the time of RF pulse

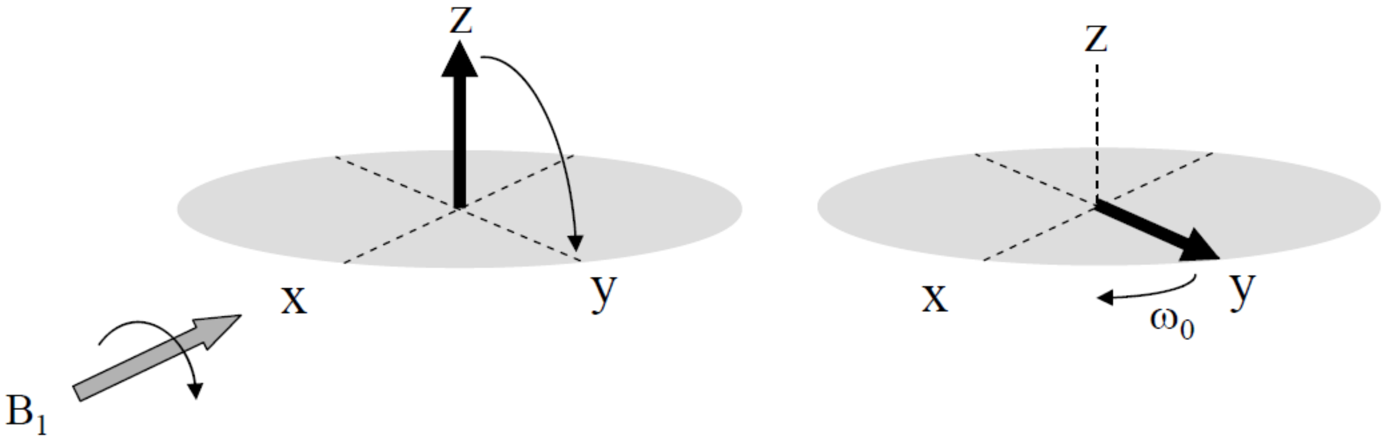

Fig. (left) Application of an RF pulse about the x-axis rotates the magnetization from the z-direction towards the y-axis. If the RF pulse strength and duration are chosen to produce a 90° pulse, then the magnetization lies directly along the y-axis. When the RF pulse is switched off (right), the magnetization precesses around the z-axis at the Larmor frequency ω_0.

MR signal detection

- A pair of conductive loops perpendicular to each other and static magnetic field;
- The induced voltage is proportional to the time rate of change of the magnetic flux $\boldsymbol{\varphi}$

$$V \propto -\frac{d\varphi}{dt}$$

- For a 90° pulse

$$V_y \propto M_0 \omega_0 \sin\omega_0 t$$

$$V_x \propto -M_0 \omega_0 \cos\omega_0 t$$

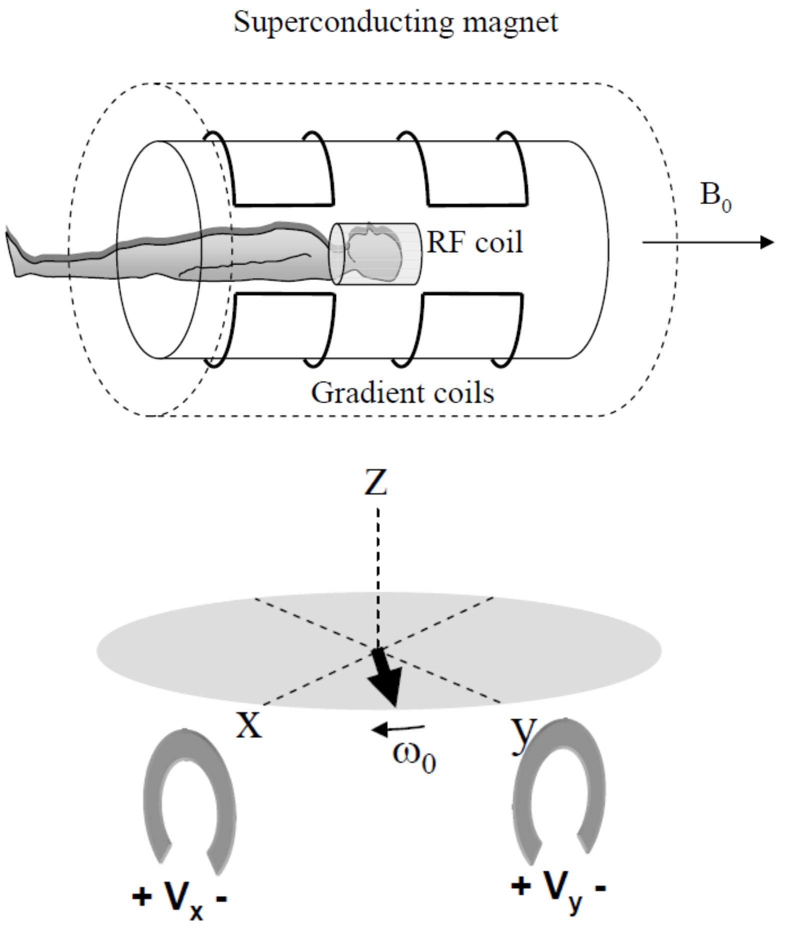

Fig. The MR signal is measured via Faraday induction. Either one or two RF coils can be used, with a voltage being induced across the ends of the conductor loops by the precessing magnetization

MR signal intensity

$$V_y \propto M_0 \omega_0 \sin\omega_0 t \qquad V_x \propto -M_0 \omega_0 \cos\omega_0 t$$

where $M_0 = \frac{\gamma^2 h^2 B_0 N_{\text{total}}}{16\pi^2 kT}$, $\omega_0 = 2\pi f_0 = \gamma B_0$

The intensity of MR signal is determined by

- Proportional to the number of protons in the object (N_{total})
- The value of B_0 (proportional to $(B_0)^2$ from M_0 and ω_0)

Lecture 15 – Magnetic Resonance Imaging

This lecture will cover:

– Introduction

– Effects of a strong magnetic field

– Magnetic Resonance

– Relaxation time (弛豫时间)

– Free induction decay

Relaxation time

- The equilibrium magnetization state
  - The z-component, M_z equal to M_0
  - The transverse components, M_x and M_y , equal to zero
- Two relaxation time (弛豫时间)
  - T_1-relaxation (纵向弛豫): the z-component from M_z to M_0 (spin-lattice relaxation, 自旋-晶格弛豫)
  - T_2-relaxation (横向弛豫): the transverse components from M_x and M_y to 0 (spin-spin relaxation, 自旋-自旋弛豫)

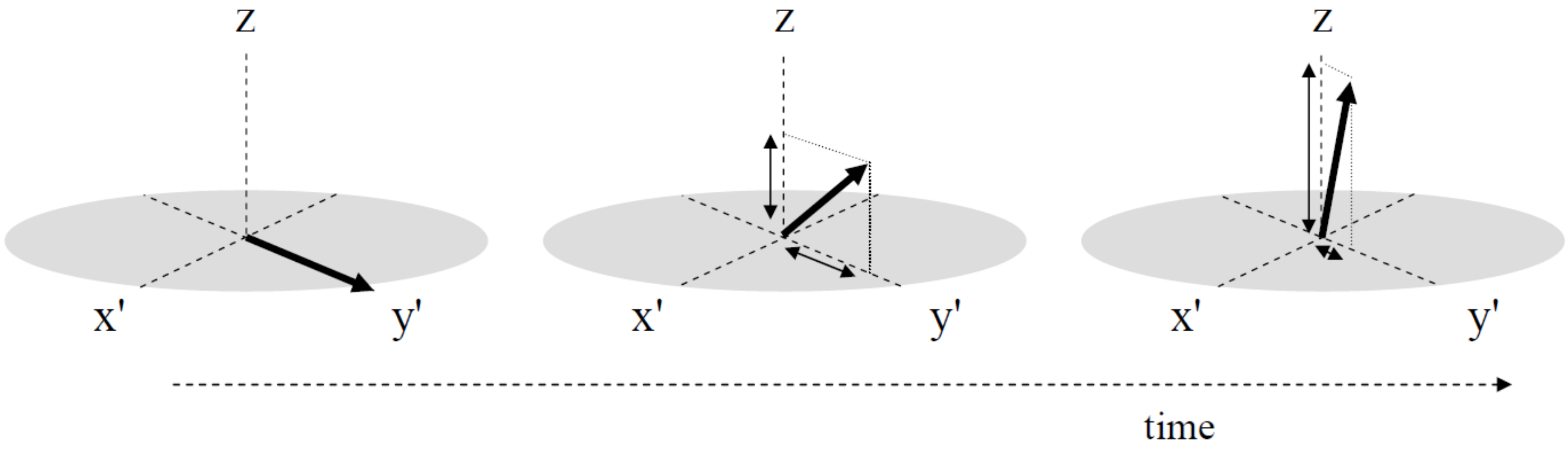

Fig. (left) Magnetization vector after a 90° RF pulse about the x-axis. (centre) T_1 and T_2 relaxation of the magnetization a certain time after the pulse has been applied results in an increased M_z component and reduced My component, respectively. (right) After a further time, the M_z and M_y components have almost returned to their equilibrium values of M_0 and zero, respectively.

Relaxation time

- For an arbitrary tip angle α for M_z component:

$$M_z(t) = M_0\cos\alpha + (M_0 - M_0\cos\alpha)(1 - e^{-\frac{t}{T_1}})$$

- For an arbitrary tip angle α for $M_{x,y}$ component:

$$M_{x,y}(t) = M_0\sin\alpha e^{-\frac{t}{T_2}}$$

Table Tissue relaxation times (ms) at 1.5 and 3 Tesla

Tissue	T_1 (1.5 T)	T_1 (3 T)	T_2 (1.5 T)	T_2 (3 T)
Brain (white matter)	790	1100	90	60
Brain (grey matter)	920	1600	100	80
Liver	500	800	50	40
Skeletal muscle	870	1420	60	30
Lipid (subcutaneous)	290	360	160	130
Cartilage	1060	1240	42	37

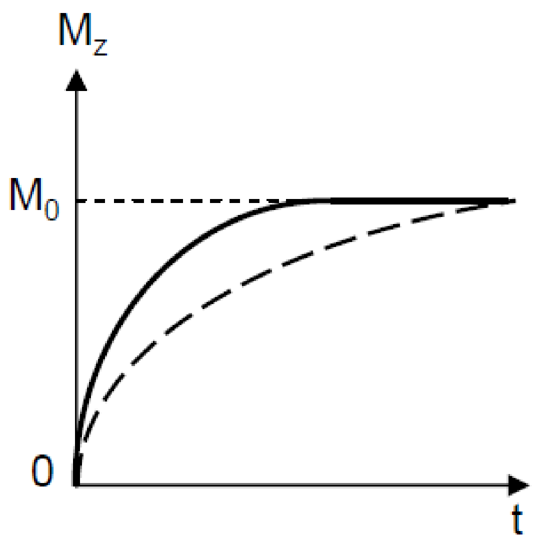

Fig. The recovery of M_z magnetization as a function of time after a 90 pulse for a tissue with short T_1 relaxation time (solid line) and long T_1 relaxation time (dashed line). When $t = 5*T_1$, M_z~99%M_0,which is assumed to be full recovery.

T_2-relaxation time

- T_2-relaxation time is affected by the spatial inhomogeneity in the B_0 field which is caused by
  - Non-uniform B_0 over the entire imaging volume
  - Different magnetic susceptibilities (磁化率) of different parts of the body, i.e. metal implant.

- The combined relaxation time

$$\frac{1}{T_2^*} = \frac{1}{T_2^+} + \frac{1}{T_2}$$

Where T_2^+: a relaxation time characterized by B_0 inhomogeneity

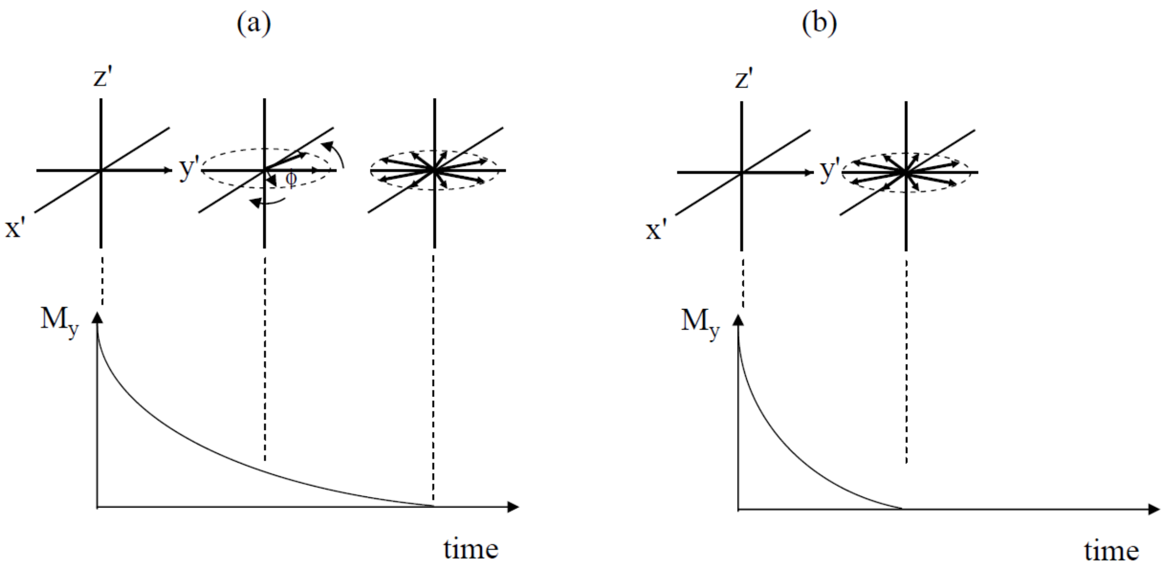

Fig. The time-dependence of the M_y component of magnetization for (a) a tissue with relatively long T_2* and (b) one with a shorter T_2*. The decrease in signal occurs due to the loss of phase coherence of the protons, i.e. protons precess at slightly different frequencies, thus acquiring different phases and reducing the net magnetization along the y-axis. The faster the dephasing process the shorter the T_2* relaxation time.

Chemical shift (化学位移)

- Protons resonate very close to the same frequency for water within tissue, but protons in lipid resonate at a significantly different frequency.
- The effective magnetic field:

$$B_{\text{eff}} = B_0(1 - \sigma)$$

where σ is the shielding constant;

- The resonant frequency of the proton in lipid:

$$\omega = \gamma B_{\text{eff}} = \gamma B_0(1 - \sigma)$$

- Magnetic resonance spectroscopy (MRS): study metabolic changes in organs or tissues based on the resonant frequency and intensity

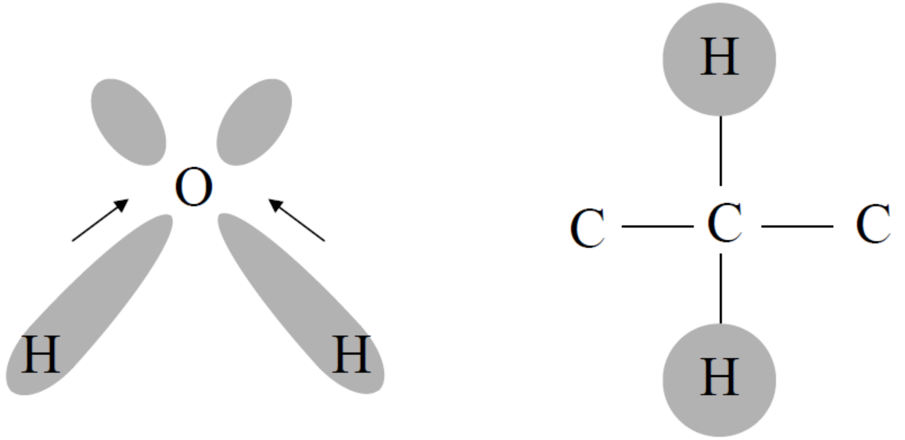

Fig. The electron density distribution (shaded area) surrounding protons in water and lipid. The strong electronegativity of the oxygen atom in water pulls electrons away from the proton, leaving it unshielded compared to the protons in lipid.

Tissue relaxation time

- Free water (自由/游离水) and bound water (束缚/结合水)
  - Free water (~90%): longer T_1 and T_2
  - bound water (~10%): bound with large molecules, shorter T_1 and T_2
- Factors affecting relaxation time
  - Water content (free water)
  - The movement of water molecules
  - The movement of large molecules
  - Lipid content
  - Paramagnetic particles (顺磁粒子)

Lecture 15 – Magnetic Resonance Imaging

This lecture will cover:

– Introduction

– Effects of a strong magnetic field

– Magnetic Resonance

– Relaxation time

– Free induction decay

Free induction decay

The free induction decay (FID, 自由感应衰减)

- The measured MR signal from tissues;
- Caused by the change of magnetization during the relaxation;
- The signal processed freely after the RF pulse has been turned off;
- Decay to a zero equilibrium value;
- Both M_x and M_y components can be detected;
- Electronic signal produced by EM induction with frequency of ω_0 and time constant T_2^*;
- Most convenient to observe in the frequency domain
- The linewidth of each peak give by $1/\pi T_2^*$

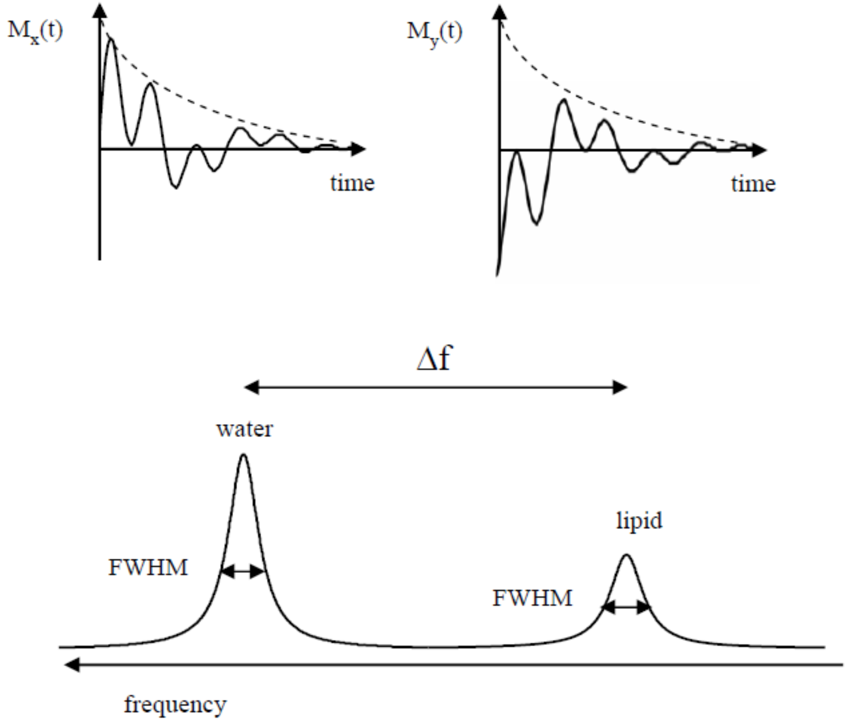

Fig. (top) x- and y-components of magnetization as a function of time, showing 'beat patterns' which come from the two different resonant frequencies of lipid and water. The real part of the frequency spectrum, shown on the bottom, shows the two peaks separated by Δf Hz.

FID signal

Characteristics of FID signals

- Only M_x and M_y can be measured, M_z can be measured if it is rotated to x-y plane;
- Initial amplitude of FID is proportional to the density of protons in tissues;
- Under the circumstance of same density of protons, the longer T_2 , the slower decay, the greater FID signal;
- Under the circumstance of same measurement time, the shorter T_1 , the greater FID signal;
- The intensity of FID signals are affected by density of protons, T_1 and T_2 , therefore MRI is multiple-parameter imaging.