#### Homework 4

Submit: Blackboard/Paper Due: Nov. 16<sup>th</sup> Please write down Your Name & Student ID

1. Within tissue lies a strongly reflecting boundary, which backscatters 70% of the intensity of the ultrasound beam. Given a 100dB receiver dynamic range, and an operating frequency of 3 MHz, what is the maximum depth within tissue at which this boundary can be detected? (the frequency dependence of the attenuation coefficient for soft tissue is 1 dB cm<sup>-1</sup>MHz<sup>-1</sup>. (10)

### Solution:

Since the frequency dependence of the attenuation coefficient for soft tissue is 1 dB/cm/MHz., the attenuation coefficient at 3 MHz is 3 dB/cm. 70% of the energy is backscattered, which means that the loss is  $10 \log(0.7) = -1.6 \, \text{dB}$ . Given the limit of  $100 \, \text{dB}$  attenuation, the maximum travel path for the ultrasound beam corresponds to the distance that results in 98.4 dB of loss. The total path is therefore given by  $98.4/3 = 32.8 \, \text{cm}$ . Knowing that the beam traveling length is twice as long as the reflection depth, the maximum detectable depth of the reflector is  $16.4 \, \text{cm}$ .

2. Calculate the distance at which the intensity of a 1 MHz and 5 MHz ultrasound beam will be reduced by half traveling through (a) bone, (b) air, and (c)muscle. (The attenuation coefficient for muscle, bone and air are 1, 8.7 and 45 dB cm<sup>-1</sup> MHz<sup>-1</sup>, respectively dB =  $10 \log_{10} \left(\frac{l_x}{l_0}\right)$ ) (24)(4 for each)

## Solution:

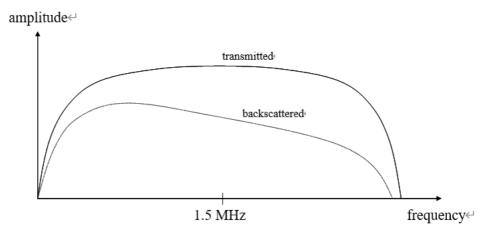
As stated in the text the values of  $\mu$  for muscle, bone and air are 1,8.7 and 45 dB/cm/MHz , respectively. For the intensity to be reduced by half, the value of  $(\mu x)$  must be ~3 dB. Therefore at 1 MHz, the half value distance is 0.34 cm for bone, 0.067 cm for air and 3 cm for muscle. At 5 MHz, the distances are one-fifth those at 1 MHz, i.e. 0.068 cm for bone, 0.013 cm for air and 0.6 cm for muscle.

Tips: If you confuse the intensity attenuation coefficient and frequecy dependence, half of the points will be deducted. Note:  $\mu(db/cm) = 4.343\mu(/cm)$ 

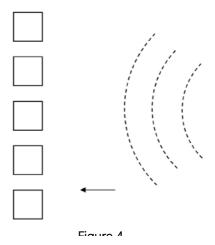
3. Plot the transmitted frequency spectrum of an ultrasound beam from a transducer operating at a central frequency of 1.5 MHz Assume that the transducer is damped. Repeat the plot for the beam returning to the transducer after having passed through tissue and been backscattered. (10) (4+6)

### Solution:

In the plot shown below, the central frequency is 1.5 MHz, and the bandwidth of the transducer is very large corresponding to a low Q (you can draw the bandwidth to be whatever you want of course). For the backscattered energy, the intensity is lower, and the higher frequencies are preferentially attenuated, meaning that the frequency spectrum becomes skewed towards the lower frequencies. (3)

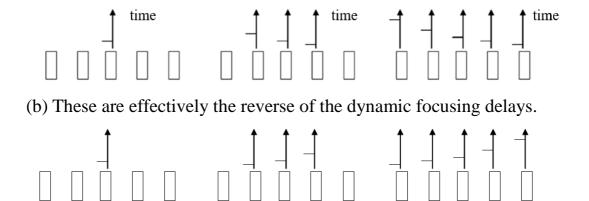


- 4. (a) Show the required timing for simultaneous steering and dynamic focusing of a phased array. For simplicity, sketch the general scheme using a small number (for example five) of elements. (9)
  - (b) Sketch the corresponding delays required for dynamic beamforming during signal reception. (9)



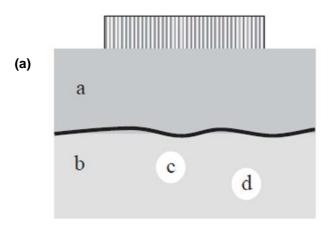
### Solution:

(a) Three successive time points are shown, the first fires a single element, the second three elements and the third five elements. The timings steer the beam towards the left and also focus the beam.



Tips: Not sketch the corresponding delays (-3pts)

5. A B-mode scan is taken of the object in Figure 5 with a linear array. There are four tissue components, a and b with a boundary in-between and two spherical tumors c and d. Given the corresponding ultrasound image shown on the 5.b what can you deduce about the acoustic characteristics of components a, b, c and d? (20) (5 for each)



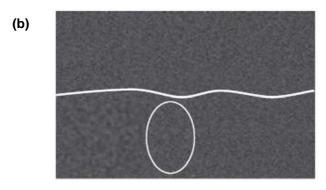


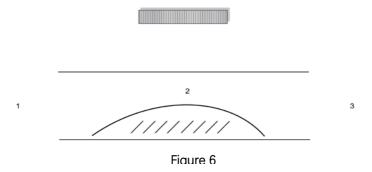
Figure 5

# Solution:

There are five different things that can be determined.

- (i) Since the boundary between a and b is visible,  $Z_a \neq Z_b$ .
- (ii) Since tumor d is not visible,  $Z_b = Z_d$ .
- (iii) Since tumor c is visible,  $Z_c \neq Z_b$ .
- (iv) Since tumor c is elongated,  $c_c < c_b$ .

6. Doppler ultrasound is currently the preferred non-invasive carotid artery examination method. It can not only display the location and size of the plaque, the location and severity of the lumen stenosis, but also perform hemodynamic measurement and morphological evaluation of the plaque. Sketch the Doppler spectral patterns at points 1, 2, and 3 in a carotid artery shown in Figure 6. (All of the plots need be made over one cardiac cycle) (18)



### Solution:

All of the plots are made over one cardiac cycle. At position 1 the flow is characterized by a range of relatively low velocities predominantly flowing towards the transducer. At 2, since the vessel narrows, the velocities become much higher. However, there are equal contributions from flow towards and away from the transducer, and so equal positive and negative frequencies. At 3, there will be a broad range of velocities, probably including turbulent flow. The Doppler spectral patterns will have the general appearance shown below.

