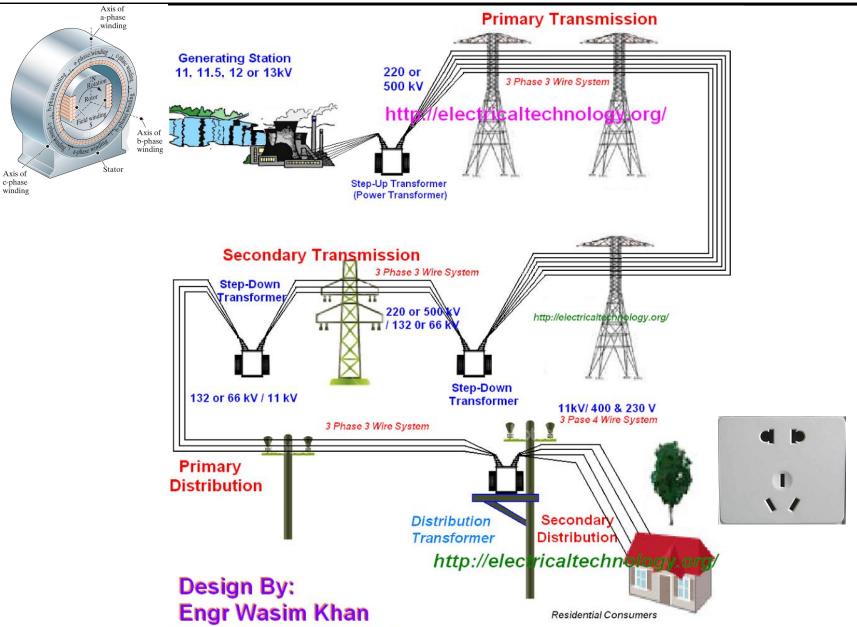
Lecture 12

- Transformers/Three-Phase Circuits

Outline

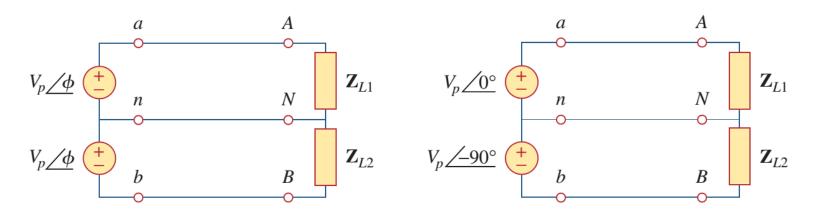
- Balanced Three-Phase System (Wye Wye)
 - Balanced sources
 - Balanced loads
- Circuit analysis (Wye Wye)
 - Phase voltage/current
 - Line voltage/current



Single Phase vs. Polyphase

- Households have single-phase power supply
 - This typically in a three wire form

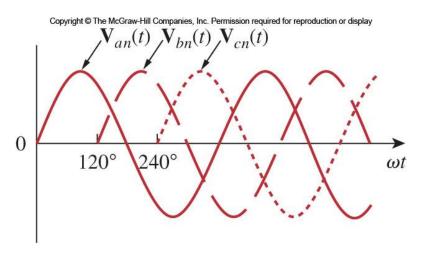
• Circuits that operate at the same frequency but with multiple sources at different phases are called <u>polyphase</u>.

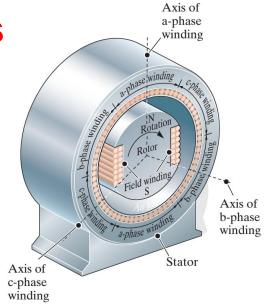


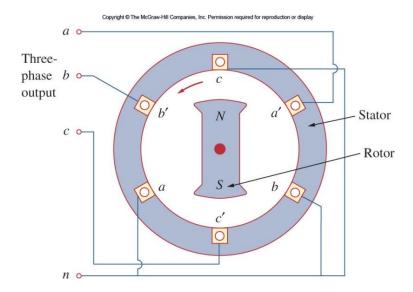
Balanced Three-Phase Sources

 Three phase voltages are typically produced by a three-phase AC generator.

The output voltages look like below.

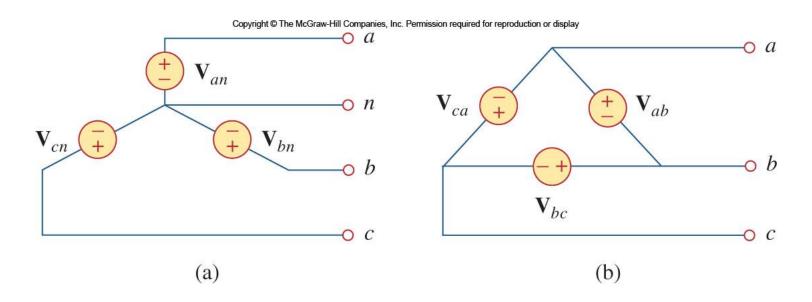






Connecting the Sources

- Three phase voltage sources can be connected the loads by either three or four wire configurations.
 - Three-wire configuration accomplished by Delta connected source.
 - Four-wire system accomplished using a Y connected source.

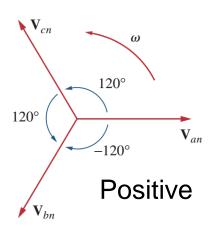


Balanced Sources

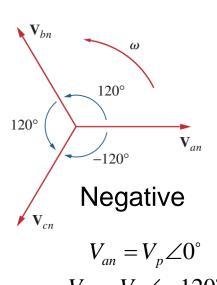
 A wye connected source is said to be balanced when

$$V_{an} + V_{bn} + V_{cn} = 0$$

Two sequences for the phases:



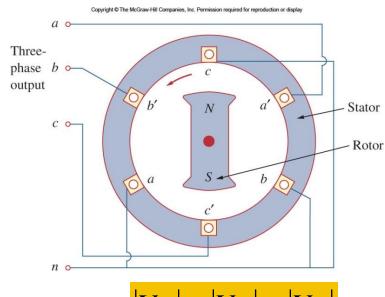
$$\begin{split} V_{an} &= V_p \angle 0^{\circ} & V_{an} &= V_p \angle 0^{\circ} \\ V_{bn} &= V_p \angle -120^{\circ} & V_{cn} &= V_p \angle -120^{\circ} \\ V_{cn} &= V_p \angle -240^{\circ} &= V_p \angle +120^{\circ} & V_{bn} &= V_p \angle -240^{\circ} &= V_p \angle +120^{\circ} \\ \end{split}$$



$$V_{an} = V_p \angle 0^{\circ}$$

$$V_{cn} = V_p \angle -120^{\circ}$$

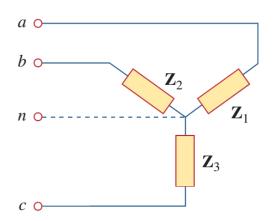
$$= V_p \angle -240^{\circ} = V_p \angle +120^{\circ}$$

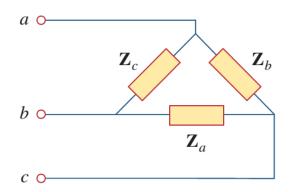


$$\left|V_{an}\right| = \left|V_{bn}\right| = \left|V_{cn}\right|$$

Balanced Loads

- Similar to the source, a <u>balanced</u> load is one that has the same impedance presented to all three voltage sources.
- They may also be connected in either Delta or wye
 - For a balanced wye connected load: $Z_1 = Z_2 = Z_3 = Z_Y$
 - For a balanced delta connected load: $Z_a = Z_b = Z_c = Z_\Lambda$



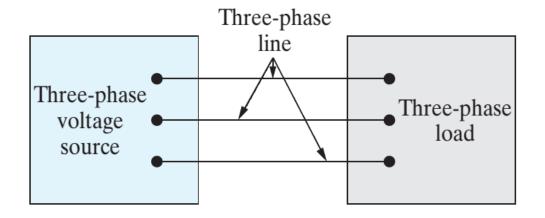


 The load impedance per phase for the two load configurations can be interchanged.

Outline

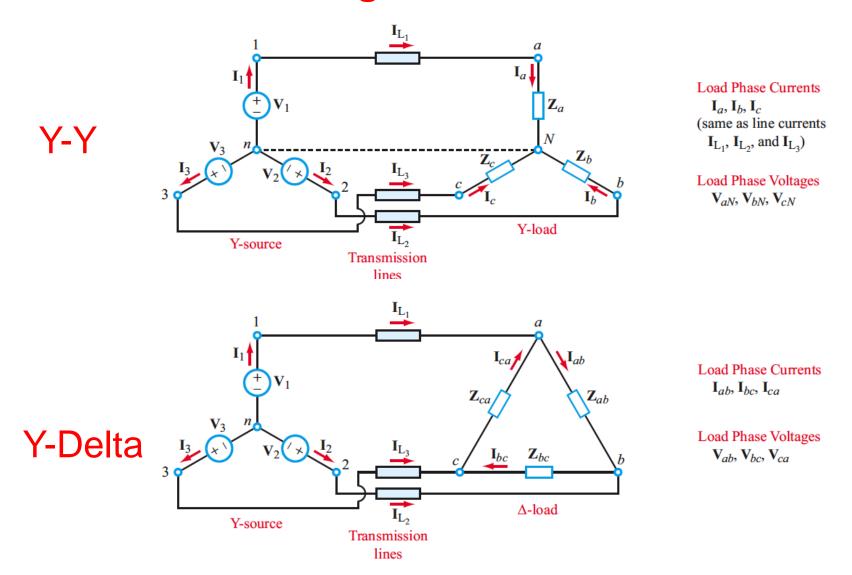
- Balanced Three-Phase System (Wye Wye)
 - Balanced sources
 - Balanced loads
- Circuit analysis (Wye Wye)
 - Phase voltage/current
 - Line voltage/current

Source-Load configurations

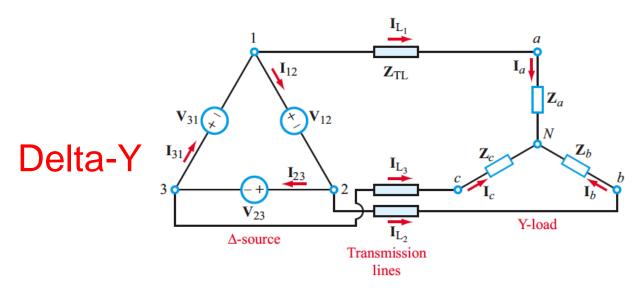


Source	Load
Y	Y
Y	Δ
Δ	Y
Δ	Δ

Source-Load Configurations



Source-Load Configurations

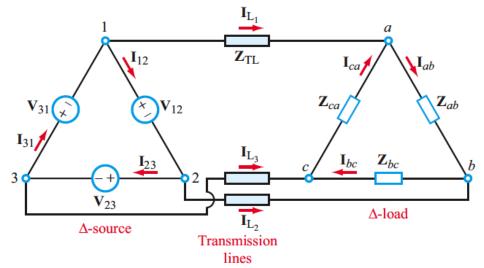


Load Phase Currents

 \mathbf{I}_a , \mathbf{I}_b , \mathbf{I}_c (same as line currents \mathbf{I}_{L_1} , \mathbf{I}_{L_2} , and \mathbf{I}_{L_3})

Load Phase Voltages V_{aN} , V_{bN} , V_{cN}

Delta-Delta



Load Phase Currents

 I_{ab}, I_{bc}, I_{ca}

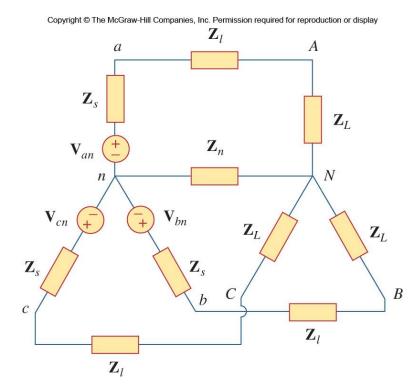
Load Phase Voltages

 V_{ab}, V_{bc}, V_{ca} (same as source voltages if Z_{TL} is negligible)

Balanced Y-Y connection

- Any three-phase system can be reduced to an equivalent Y-Y system.
- The load impedances Z_y will be assumed to be balanced.
 - This can be the source Z_s , line Z_l and load Z_L together.

$$\mathbf{Z}_Y = \mathbf{Z}_s + \mathbf{Z}_\ell + \mathbf{Z}_L$$



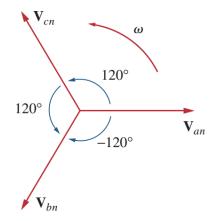
Line-to-Line Voltage

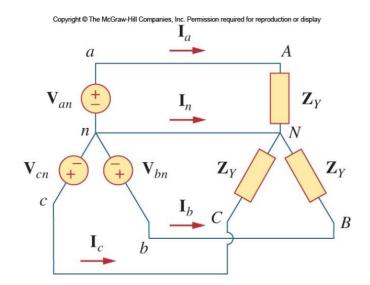
Use the positive sequence:

$$V_{an} = V_p \angle 0^{\circ}$$

$$V_{bn} = V_p \angle -120^{\circ} \quad V_{cn} = V_p \angle +120^{\circ}$$

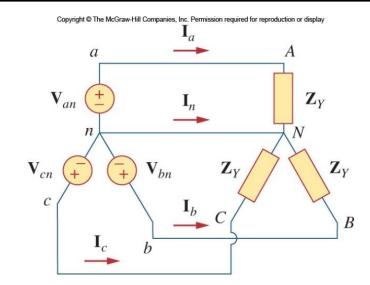
The line to line (or line in short) voltages:





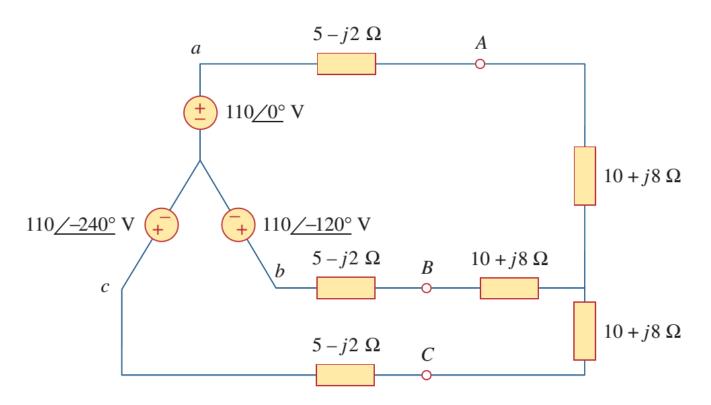


Line Currents



Example

Calculate the line currents.



Y-Y

$$\mathbf{V}_{an} = V_p / 0^{\circ}$$

$$\mathbf{V}_{bn} = V_p / -12$$

$$V_{ab} =$$

$$\mathbf{V}_{ab} = \sqrt{3}V_p/30^{\circ}$$

$$\mathbf{V}_{bn} = V_p / -120^{\circ}$$

$$\mathbf{V}_{bc} = \mathbf{V}_{ab} / -120^{\circ}$$

$$\mathbf{V}_{cn} = V_p / +120^{\circ}$$

$$\mathbf{V}_{ca} = \mathbf{V}_{ab} / +120^{\circ}$$

$$\mathbf{I}_a = \mathbf{V}_{an}/\overline{\mathbf{Z}_Y}$$

$$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$$

$$\mathbf{I}_b = \mathbf{I}_a/-120^\circ$$

$$\mathbf{I}_b = \mathbf{I}_a / -120$$

$$\mathbf{I}_c = \mathbf{I}_a / +120^{\circ}$$

$$Y$$
- Δ

$$V_{an} = V_p/0^{\circ}$$

$$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p/30^{\circ}$$

$$\mathbf{V}_{bn}$$

$$\mathbf{V}_{an} = V_p / 0^{\circ}$$

$$\mathbf{V}_{bn} = V_p / -120^{\circ}$$

$$\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab} / -120^{\circ}$$
$$\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab} / +120^{\circ}$$

Same as phase voltages

$$\mathbf{I}_{AB}$$

$$\mathbf{V}_{cn} = V_p / +120^{\circ}$$

$$\mathbf{I}_{AB} = \mathbf{V}_{AB} / \mathbf{Z}_{\Delta}$$

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}/-30^{\circ}$$

$$\mathbf{I}_{BC}$$
 =

$$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_{\Delta}$$

$$I_b = I_a / -120^\circ$$

$$\mathbf{I}_{CA}$$

$$\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_{\Delta} \qquad \qquad \mathbf{I}_{c} = \mathbf{I}_{a}/+120^{\circ}$$

 Δ - Δ

 Δ -Y

$$\mathbf{V}_{ab} = V_p / 0^{\circ}$$

$$\mathbf{V}_{ab} = V_p / -120$$

$$\mathbf{V}_{bc} = V_p / -120^{\circ}$$

$$\mathbf{V}_{ca} = V_p / +120^{\circ}$$

$$V_{ca} - V_{p} / + 120$$

$$\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_{\Delta}$$

$$I_{BC} = V_{ba}/Z_{\Delta}$$

$$\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_{\Delta}$$

$$\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_{\Delta}$$

$$\mathbf{I}_a = \mathbf{I}_{AB} \sqrt{3} / -30^{\circ}$$

$$\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_{\Delta}$$
 $\mathbf{I}_{b} = \mathbf{I}_{a}/-120^{\circ}$ $\mathbf{I}_{c} = \mathbf{I}_{a}/+120^{\circ}$

$$\mathbf{V}_{ab} = V_p/0^{\circ}$$

$$V_p = V_p / 0^{\circ}$$

$$V_{bc} = V_p / -120^{\circ}$$

$$\mathbf{I}_c = \mathbf{I}_a / + 120$$

Same as phase voltages

$$\mathbf{V}_{bc} = V_p / -120$$

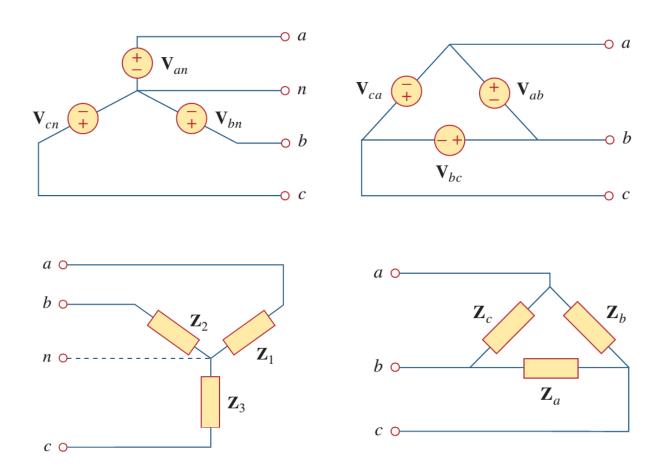
$$\mathbf{V}_{ca} = V_p / +120^{\circ}$$

Same as line currents
$$\mathbf{I}_a = \frac{V_p / -30^\circ}{\sqrt{3} \mathbf{Z}_Y}$$

$$\mathbf{I}_b = \mathbf{I}_a / -120^\circ$$

$$\mathbf{I}_c = \mathbf{I}_a / +120^{\circ}$$

Y and ∆, Which One Better?



http://www.allaboutcircuits.com/textbook/alternating-current/chpt-10/three-phase-y-delta-configurations/

Lecture 10