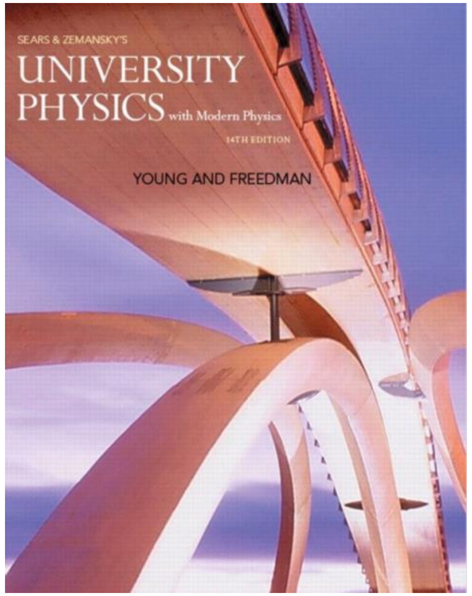
普通物理I PHYS1181.01　　第3章

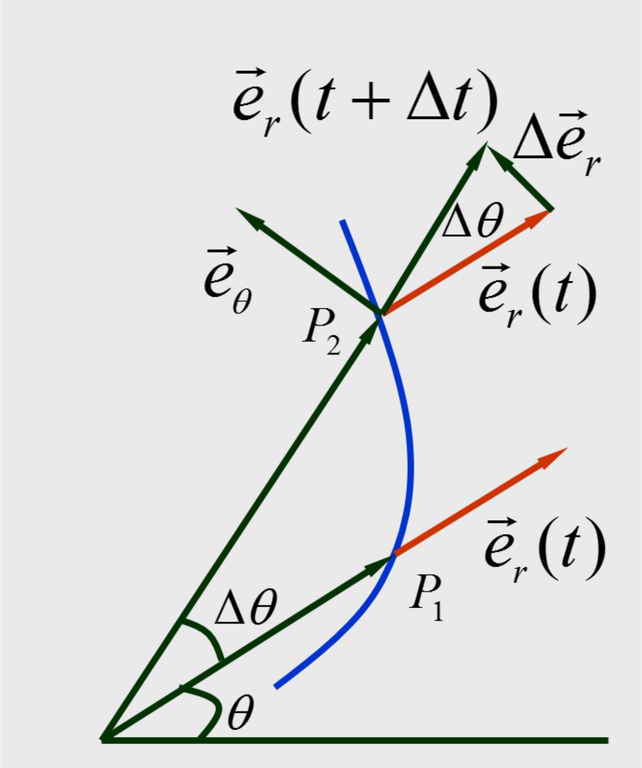
质点动力学 Dynamics



其他重要的坐标系：极坐标系、自然坐标系

速度的分量形式：平面极坐标系

$$\vec{r}(t)=r(t)\vec{e}_r(t)$$



$$\vec{v}=\frac{\mathrm{d}\vec{r}}{\mathrm{d}t}=\frac{\mathrm{d}}{\mathrm{d}t}(r\vec{e}_r)=\frac{\mathrm{d}r}{\mathrm{d}t}\vec{e}_r+r\frac{\mathrm{d}\vec{e}_r}{\mathrm{d}t}$$

$\Delta t \longrightarrow 0$时，方向：$\Delta\vec{e}_r \,//\, \vec{e}_\theta$

大小：$\left|\Delta\vec{e}_r\right|=\left|\vec{e}_r\right|\Delta\theta=\Delta\theta \Longrightarrow \Delta\vec{e}_r=\Delta\theta\vec{e}_\theta$

$$\frac{\mathrm{d}\vec{e}_r}{\mathrm{d}t}=\lim_{\Delta t\to 0}\frac{\Delta\vec{e}_r}{\Delta t}=\lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t}\vec{e}_\theta=\frac{\mathrm{d}\theta}{\mathrm{d}t}\vec{e}_\theta$$

$$\vec{v}=\frac{\mathrm{d}r}{\mathrm{d}t}\vec{e}_r+r\frac{\mathrm{d}\vec{e}_r}{\mathrm{d}t}=\frac{\mathrm{d}r}{\mathrm{d}t}\vec{e}_r+r\frac{\mathrm{d}\theta}{\mathrm{d}t}\vec{e}_\theta=v_r\vec{e}_r+v_\theta\vec{e}_\theta$$

加速度的分量形式

直角坐标系

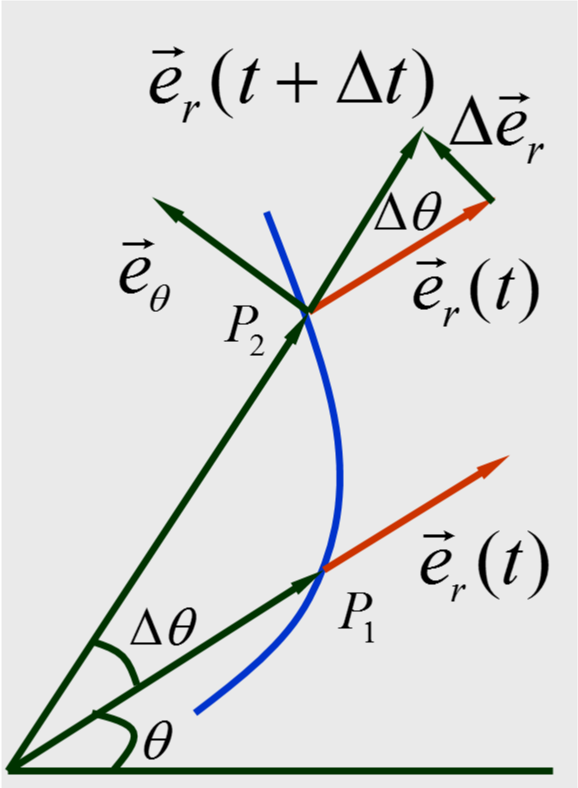
$$\vec{a} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} + \frac{dv_z}{dt}\hat{k} = \frac{d^2x}{dt^2}\hat{\imath} + \frac{d^2y}{dt^2}\hat{\jmath} + \frac{d^2z}{dt^2}\hat{k}$$

平面极坐标系

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d(\frac{\mathrm{d}r}{\mathrm{d}t}\vec{e}_r + r\frac{\mathrm{d}\theta}{\mathrm{d}t}\vec{e}_\theta)}{dt}$$

$$= \frac{\mathrm{d}^2 r}{\mathrm{d}t^2}\vec{e}_r + \frac{\mathrm{d}r}{\mathrm{d}t}\cdot\frac{\mathrm{d}\vec{e}_r}{\mathrm{d}t} + \frac{\mathrm{d}r}{\mathrm{d}t}\cdot\frac{\mathrm{d}\theta}{\mathrm{d}t}\vec{e}_\theta + r\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}\vec{e}_\theta + r\frac{\mathrm{d}\theta}{\mathrm{d}t}\cdot\frac{\mathrm{d}\vec{e}_\theta}{\mathrm{d}t}$$

加速度的分量形式：平面极坐标系



如前所述： $\dfrac{\mathrm{d}\vec{e}_r}{\mathrm{d}t} = \dfrac{\mathrm{d}\theta}{\mathrm{d}t}\vec{e}_\theta$ 同理可得：$\dfrac{\mathrm{d}\vec{e}_\theta}{\mathrm{d}t} = -\dfrac{\mathrm{d}\theta}{\mathrm{d}t}\vec{e}_r$

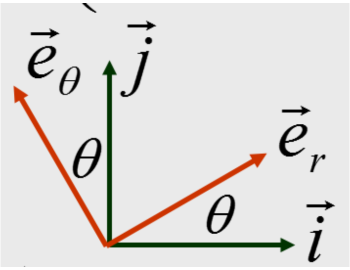
简单的推导：

$$\vec{e}_r = \cos\theta\vec{i} + \sin\theta\vec{j}$$

$$\vec{e}_\theta = -\sin\theta\vec{i} + \cos\theta\vec{j}$$

$$\dot{\vec{e}}_r = \left(-\sin\theta\vec{i} + \cos\theta\vec{j}\right)\dot{\theta} = \dot{\theta}\vec{e}_\theta$$

$$\dot{\vec{e}}_\theta = \left(-\cos\theta\vec{i} - \sin\theta\vec{j}\right)\dot{\theta} = -\dot{\theta}\vec{e}_r$$



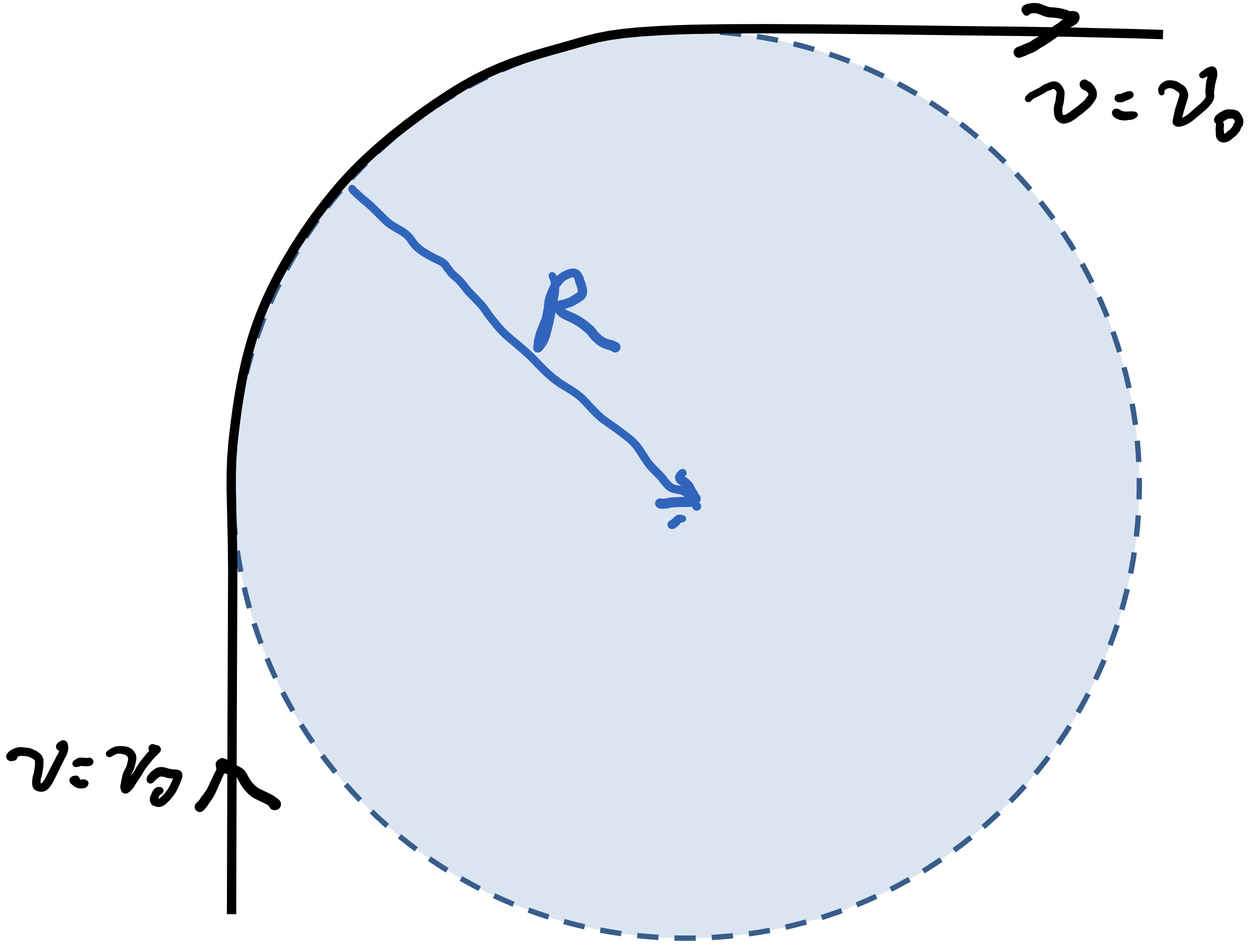
加速度的分量形式：平面极坐标系

$$\vec{a}=\frac{\mathrm{d}^2 r}{\mathrm{d}t^2}\vec{e}_r+\frac{\mathrm{d}r}{\mathrm{d}t}\cdot\frac{\mathrm{d}\theta}{\mathrm{d}t}\vec{e}_\theta+\frac{\mathrm{d}r}{\mathrm{d}t}\cdot\frac{\mathrm{d}\theta}{\mathrm{d}t}\vec{e}_\theta+r\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}\vec{e}_\theta-r\frac{\mathrm{d}\theta}{\mathrm{d}t}\cdot\frac{\mathrm{d}\theta}{\mathrm{d}t}\vec{e}_r$$

$$=\left(\frac{\mathrm{d}^2 r}{\mathrm{d}t^2}-r\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2\right)\vec{e}_r+\left(r\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}+2\frac{\mathrm{d}r}{\mathrm{d}t}\cdot\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)\vec{e}_\theta$$

$$\vec{a}=\left(\ddot{r}-r\dot{\theta}^2\right)\vec{e}_r+\left(r\ddot{\theta}+2\dot{r}\dot{\theta}\right)\vec{e}_\theta$$

例子：匀速率过弯



$$\vec{a}=\left(\ddot{r}-r\dot{\theta}^2\right)\vec{e}_r+\left(r\ddot{\theta}+2\dot{r}\dot{\theta}\right)\vec{e}_\theta$$

$$\vec{a}=-r\dot{\theta}^2\vec{e}_r+r\ddot{\theta}\vec{e}_\theta$$

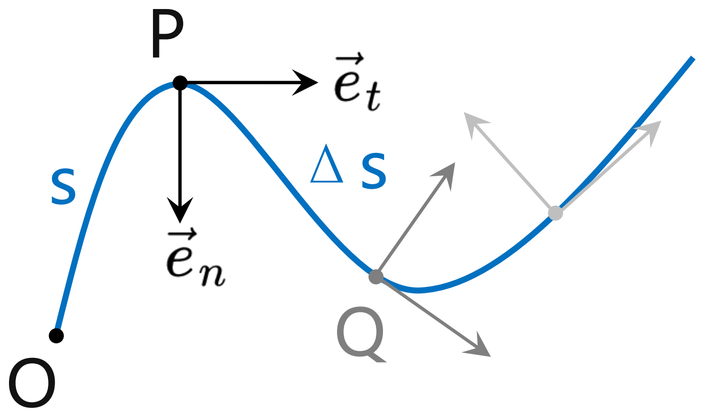
$$r\dot{\theta}^2=v^2/r$$

$$r\ddot{\theta}=0$$

自然坐标系

在某质点运动的轨迹线上任取一点O为自然坐标原点，以质点所在位置P点与O点间轨迹的路程 **s** 来确定质点的位置，则称**s** 为质点的自然坐标，其运动方程：

$$s = s(t)$$



当质点经Δt时间内从P点达Q点，运动的路程为**Δs**：

$$\Delta s = s(t + \Delta t) - s(t)$$

在质点轨迹切向取一单位矢量$\boldsymbol{e}_t$，其称之为**切向单位矢量** $\vec{e}_t$

在$\mathbf{e}_t$垂直且指向质点轨迹凹侧的方向处取单位矢量$\boldsymbol{e}_n$，称作**法向单位矢量** $\vec{e}_n$

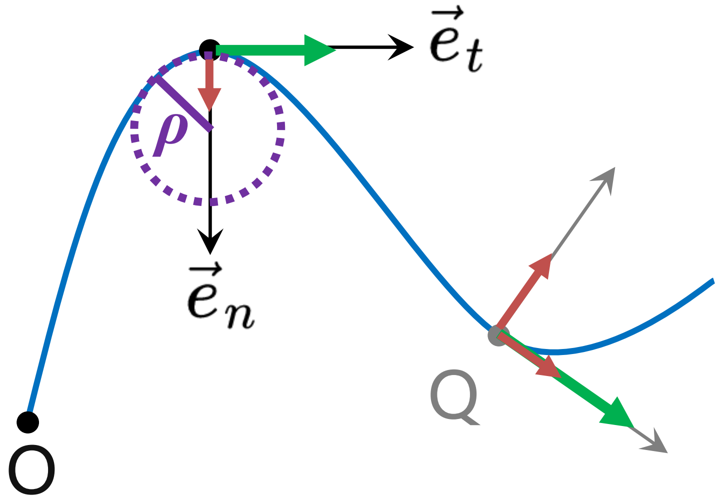
速度矢量的大小（速率）：　$v=\lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t}=\frac{ds}{dt}$

速度矢量的方向总是沿着为轨迹的切向，表示为：$\vec{v}=\frac{ds}{dt}\vec{e}_t$

加速度矢量：　$\vec{a}=\underline{a_1\vec{e}_t}+\underline{a_2\vec{e}_n}$

$\boldsymbol{a}_1$:切向加速度，改变速度的大小

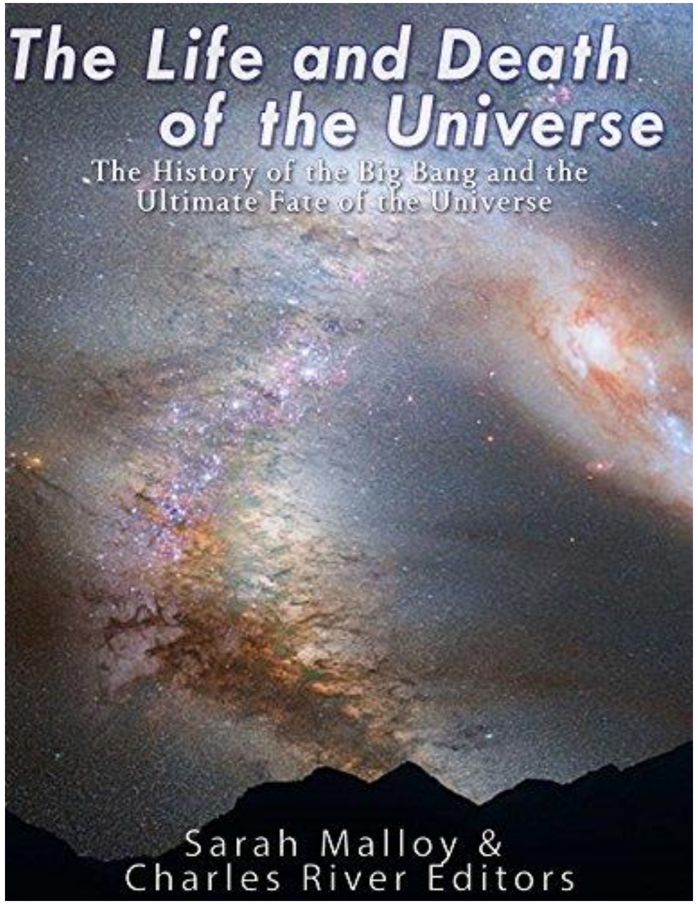
$\boldsymbol{a}_2$:法向加速度，改变速度的方向



$$a_1=\frac{dv}{dt}\qquad a_2=v\frac{d\theta}{dt}=v\frac{ds}{dt}\frac{d\theta}{ds}=\frac{v^2}{\rho}$$

其中，$\rho=\frac{ds}{d\theta}=\left|\frac{ds}{d\theta}\right|$　为质点处轨迹的曲率半径，始终为正

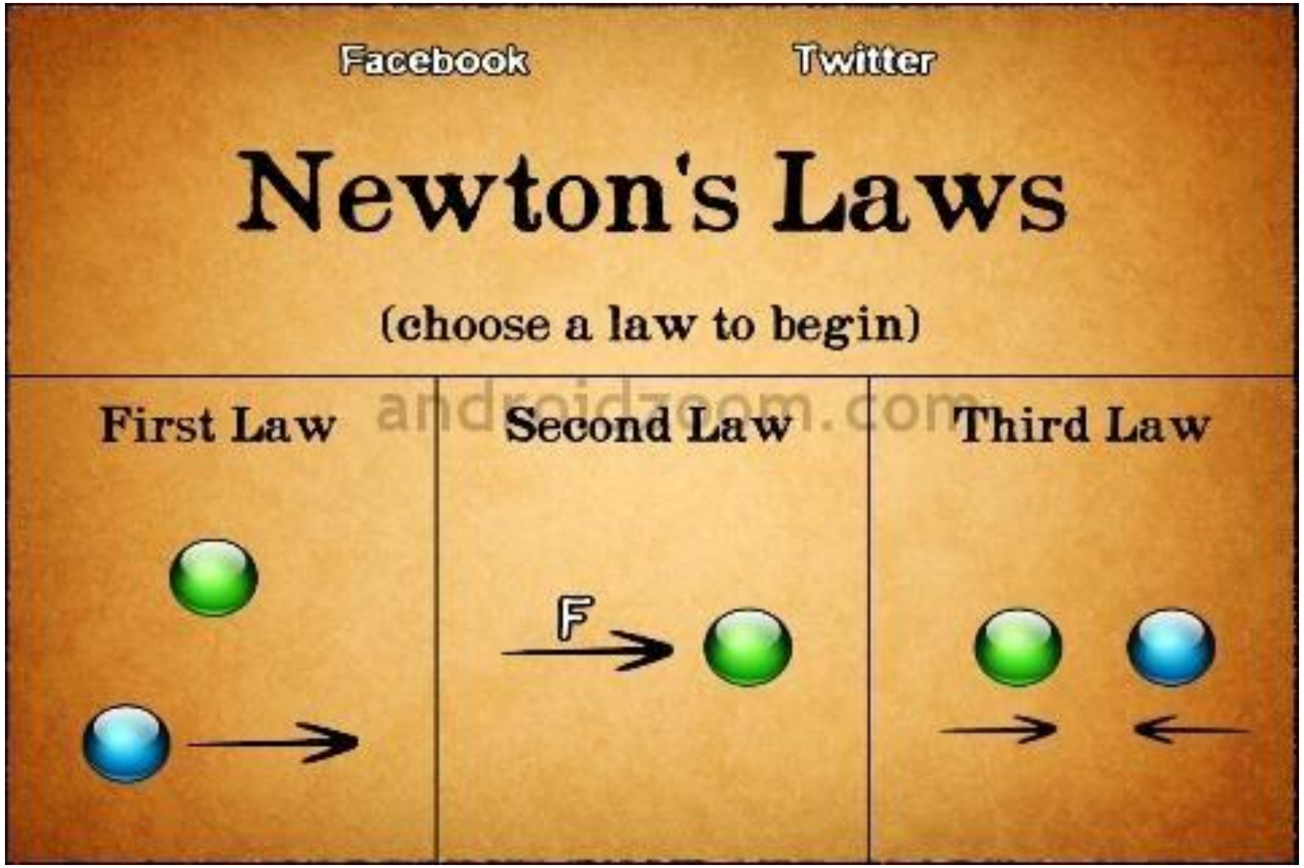
运动-力-相互作用



我们的宇宙生于相互作用。

没有相互作用，等于死寂。

牛顿三大运动定律



你不推，我不变　　你一推，我就动　　你动我，我动你

1. Every body persists in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by force impressed.

2. The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

3. To every action there is always opposed an equal reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

基本的概念

动力学，描述运动的“原因” ，即受力与运动之间的相互关系。

- Newton’s laws of motion: 牛顿运动定律
- Force: 力
- Inertial frame of reference: 惯性参考系

力与运动关系的探索

亚里士多德：力是物体运动的原因
伽利略：力是改变物体运动的原因

伽利略：力不是维持运动的原因。

↓

笛卡尔：无外力，等速直线运动

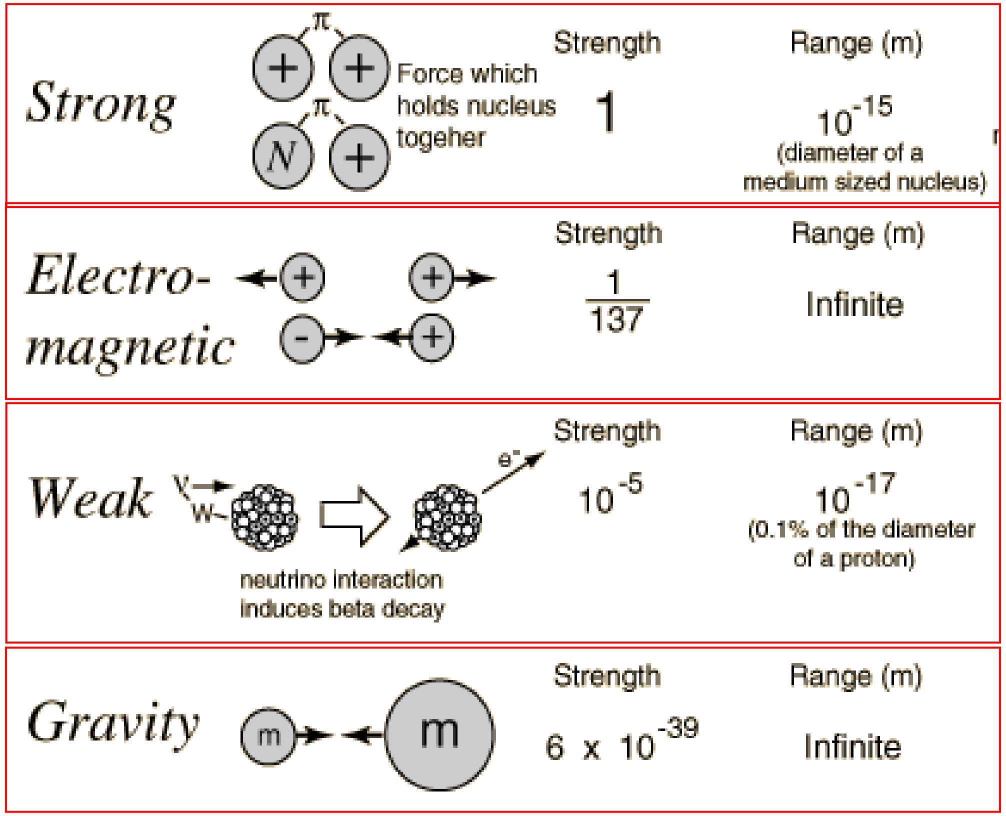
↓

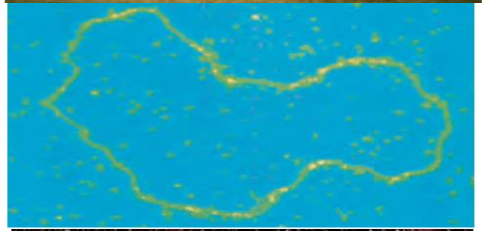
牛顿：无外力，匀速直线运动或静止；直到有外力改变这个状态。

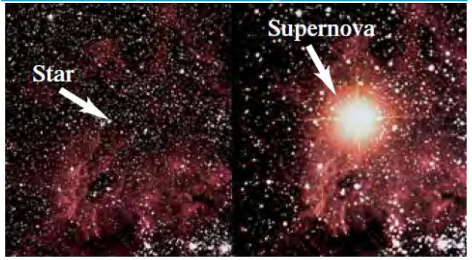
力：Force

描述物体之间的相互作用的量。
兼有大小和方向两个属性， 因此为矢量。

四种基本的相互作用







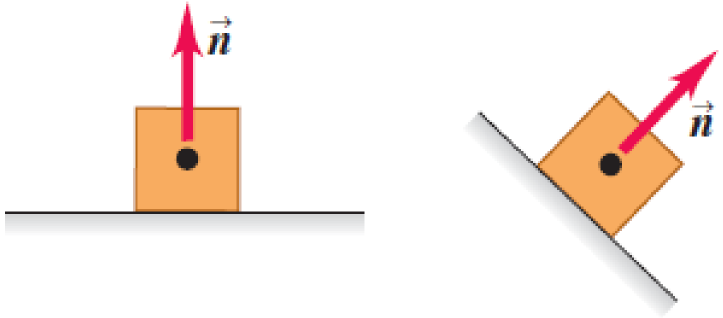
力都有多大？

TABLE 4.1 Typical Force Magnitudes

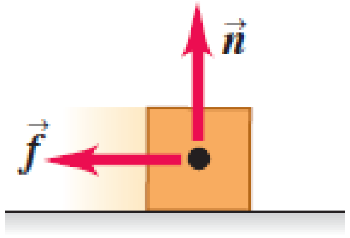
Sun's gravitational force on the earth	3.5×10^{22} N
Weight of a large blue whale	1.9×10^{6} N
Maximum pulling force of a locomotive	8.9×10^{5} N
Weight of a 250 lb linebacker	1.1×10^{3} N
Weight of a medium apple	1 N
Weight of the smallest insect eggs	2×10^{-6} N
Electric attraction between the proton and the electron in a hydrogen atom	8.2×10^{-8} N
Weight of a very small bacterium	1×10^{-18} N
Weight of a hydrogen atom	1.6×10^{-26} N
Weight of an electron	8.9×10^{-30} N
Gravitational attraction between the proton and the electron in a hydrogen atom	3.6×10^{-47} N

四种常见的力 Four common types of force

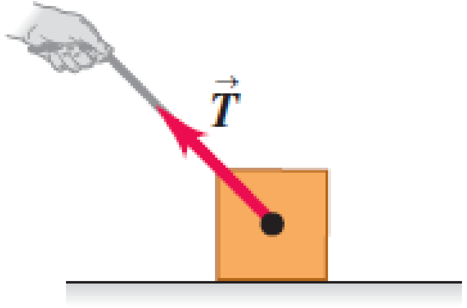
(a) **Normal force $\vec{n}$:** When an object rests or pushes on a surface, the surface exerts a push on it that is directed perpendicular to the surface.



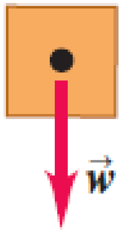
(b) **Friction force $\vec{f}$:** In addition to the normal force, a surface may exert a friction force on an object, directed parallel to the surface.



(c) **Tension force $\vec{T}$:** A pulling force exerted on an object by a rope, cord, etc.



(d) **Weight $\vec{w}$:** The pull of gravity on an object is a long-range force (a force that acts over a distance).

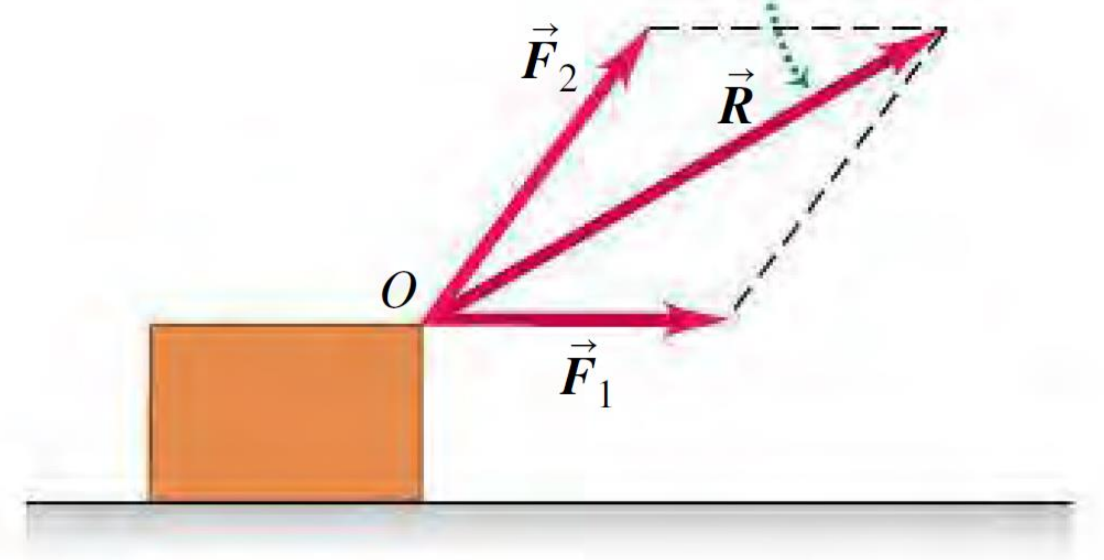


合力 Superposition of forces

The net force acting on an object ... $\vec{R} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$

... is the vector sum, or resultant, of all individual forces acting on that object.

Two forces $\vec{F}_1$ and $\vec{F}_2$ acting on a body at point O have the same effect as a single force $\vec{R}$ equal to their vector sum.



2D:

$$R_x = \sum F_x \qquad R_y = \sum F_y$$

$$R = \sqrt{R_x^2 + R_y^2}$$

3D:

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

牛顿第一定律（惯性定律）

NEWTON'S FIRST LAW OF MOTION An object acted on by no net external force has a constant velocity (which may be zero) and zero acceleration.

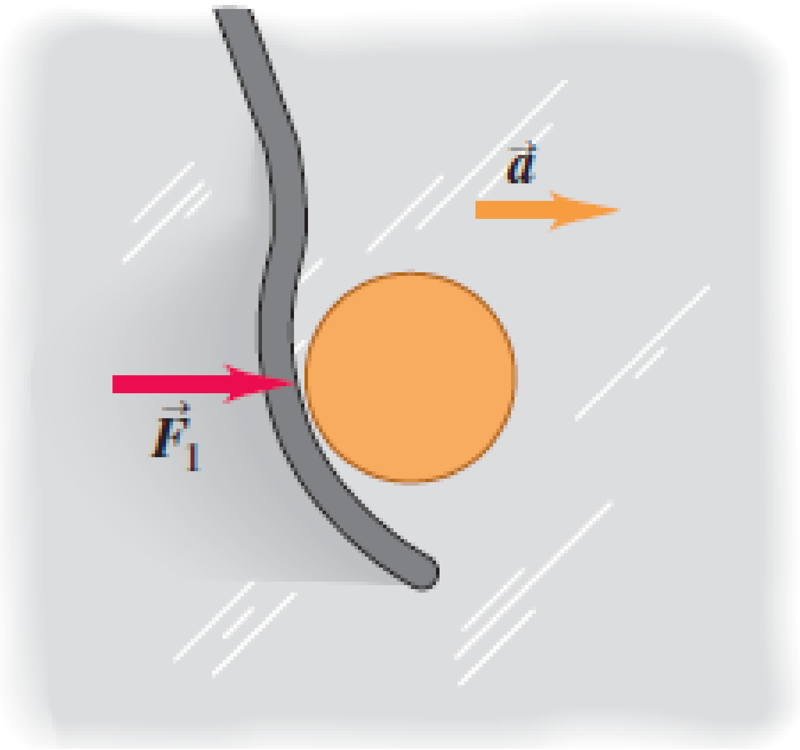
任何物体如果没有外力的作用，都将保持静止或作匀速直线运动的状态。

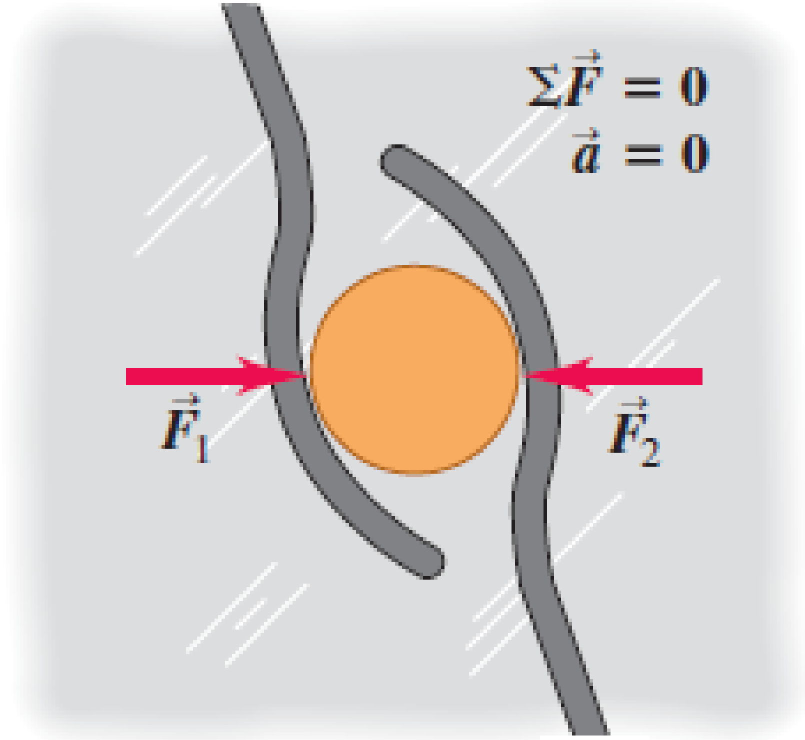
现代版本的描述：

自由粒子永远保持静止或匀速直线运动的状态。

牛顿第一定律的数学表达：

Newton's first law:
Net external force on an object ... $\sum \vec{F} = 0$ ... must be zero if the object is in **equilibrium.**

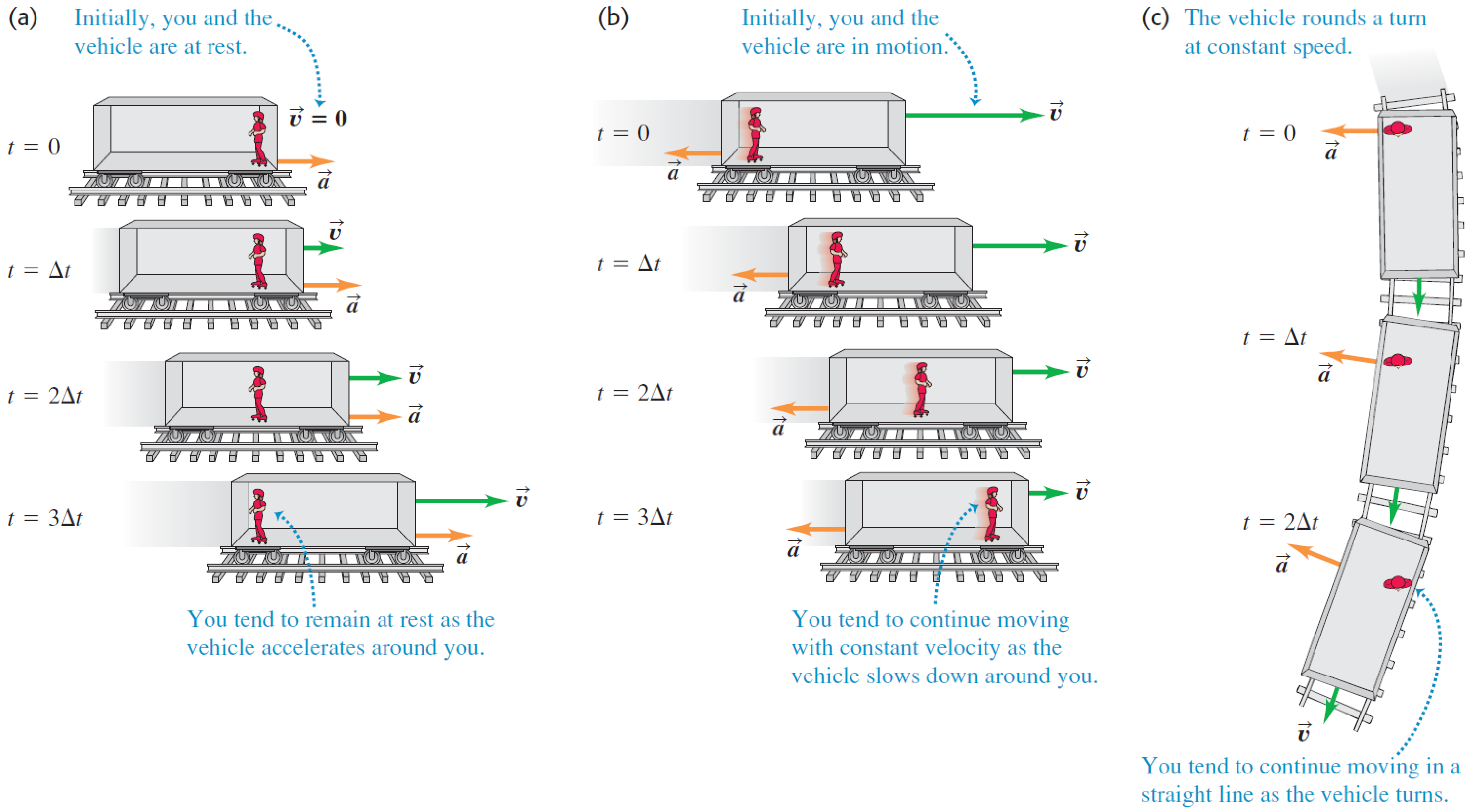




注意：牛顿第一定律特指外力 External force

近似匀速直线运动：冰球 Hockey puck（无阻力）

牛顿第一定律定义惯性参考系（inertial frame of reference）



牛顿第一定律的意义

(1) 定义了惯性参考系 ← 在牛顿第一定律适用的参考系才是惯性参考系

物体静止或匀速直线运动，相对哪个参照系?

——惯性参考系

(2) 定义了物体的惯性和力的概念

- 物体保持运动状态的特性——惯性
- 改变物体运动状态的原因——力 （物体间的相互作用）

惯性参考系：inertial frame of reference

我们接触到的惯性参考系：地球表面（近似）

我们接触到的非惯性参考系：加速减速的车厢，过弯道的火车，启动停止的电梯等

惯性参考系：inertial frame of reference

惯性坐标系不唯一： 如果参考系B是惯性坐标系，坐标系A相对于坐标系B的速度是恒定的，那么B也是惯性坐标系。

常量 constant

$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$$

相对于惯性参考系静止或者匀速直线运动的参考系，均为惯性参考系。

因此，惯性定律中描述的静止和匀速直线运动，并没有差别。

牛顿第二定律:

NEWTON'S SECOND LAW OF MOTION If a net external force acts on an object, the object accelerates. The direction of acceleration is the same as the direction of the net external force. The mass of the object times the acceleration vector of the object equals the net external force vector.

Newton's second law: If there is a net external force on an object ...

$$\sum \vec{F} = m\vec{a}$$

Mass of object

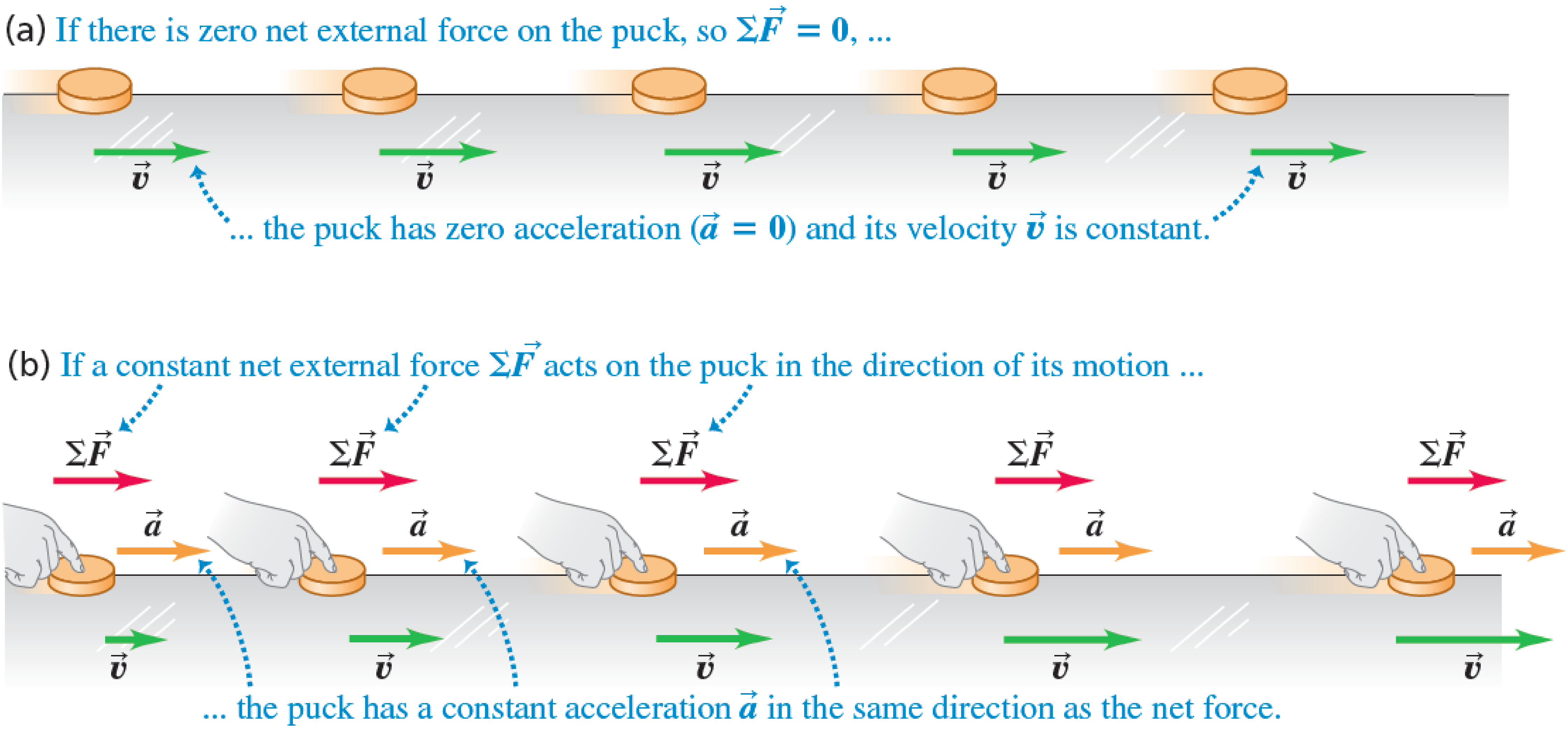
... the object accelerates in the same direction as the net external force.

在外力作用下，质点的加速度方向 $\vec{a}$ 与外力方向相同，大小等于外力除以质量。

例子：冰球 Hockey puck on ice

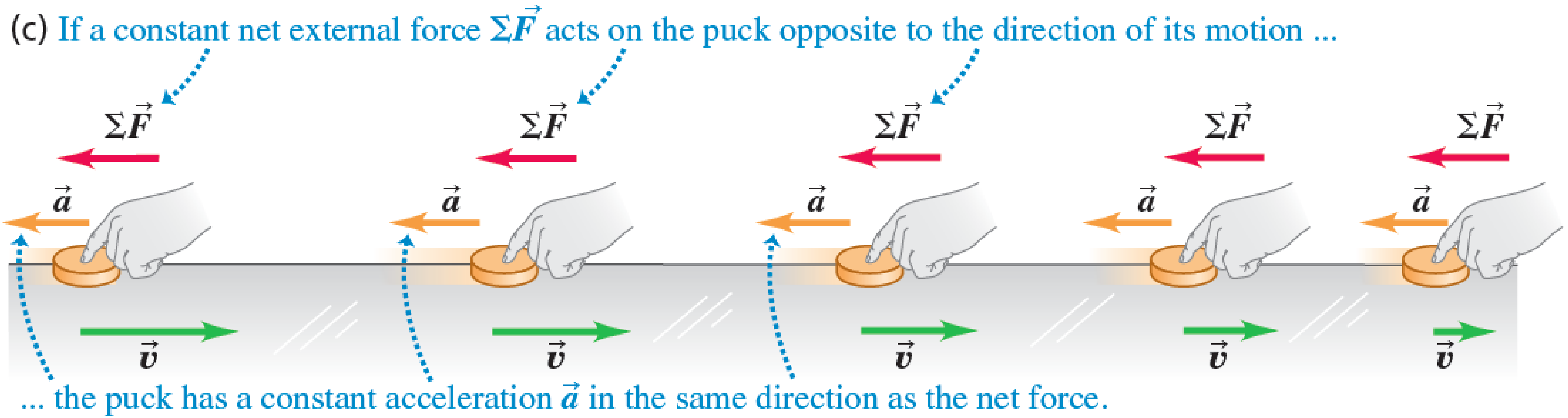
匀速

加速



例子：冰球 Hockey puck on ice

减速



牛顿第二定律

第二定律同时定义（a）力的量度

(b）物体（惯性）质量

力的量度和物体质量通过第二定律协调定义。

在国际单位制中：$\vec{F}$, N; m, kg; a, m/s^2

根据牛顿第二定律定义 $\Rightarrow \vec{a} = \dfrac{\sum_i \vec{F}_i}{m}$

你们熟悉的F=ma, 矢量的形式

力、质量、加速度的常用单位

TABLE 4.2 Units of Force, Mass, and Acceleration

System of Units	Force	Mass	Acceleration
SI	newton (N)	kilogram (kg)	m/s^2
cgs	dyne (dyn)	gram (g)	cm/s^2
British	pound (lb)	slug	ft/s^2

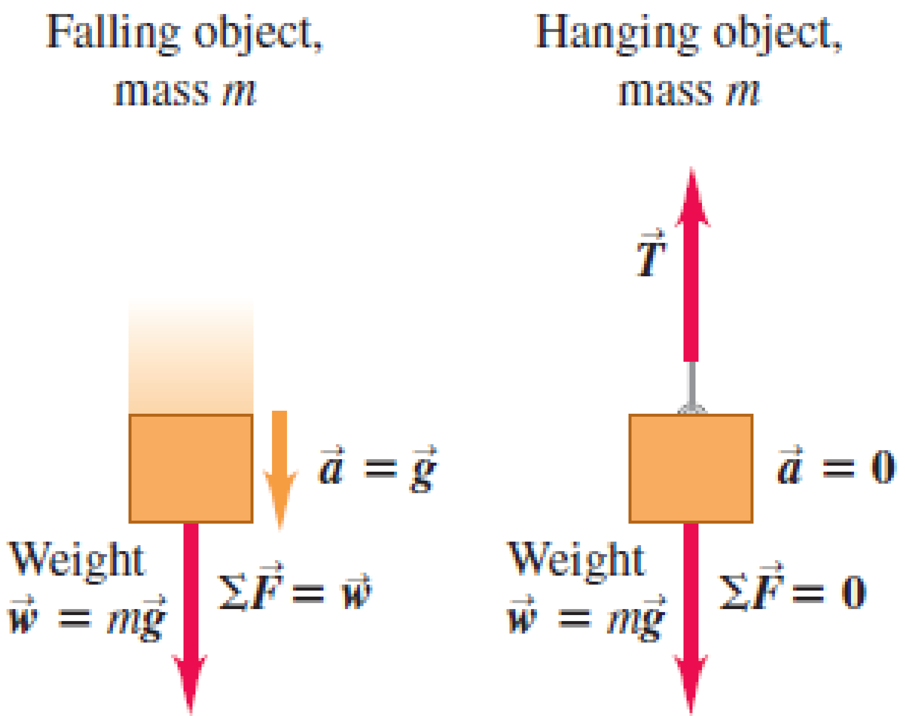
重量（w）和质量（m）

Magnitude of weight of an object

$$w = mg$$

Mass of object

Magnitude of acceleration due to gravity



- The relationship of mass to weight: $\vec{w} = m\vec{g}$.
- This relationship is the same whether an object is falling or stationary.

牛顿第二定律的特征

- 牛顿第二定律的瞬时性

质点的加速度与其所受的力同时出现或同时消失！

（然而力的传递需要时间）

- 牛顿第二定律的矢量性

在直角坐标系中：$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$

$$= m\vec{a}$$

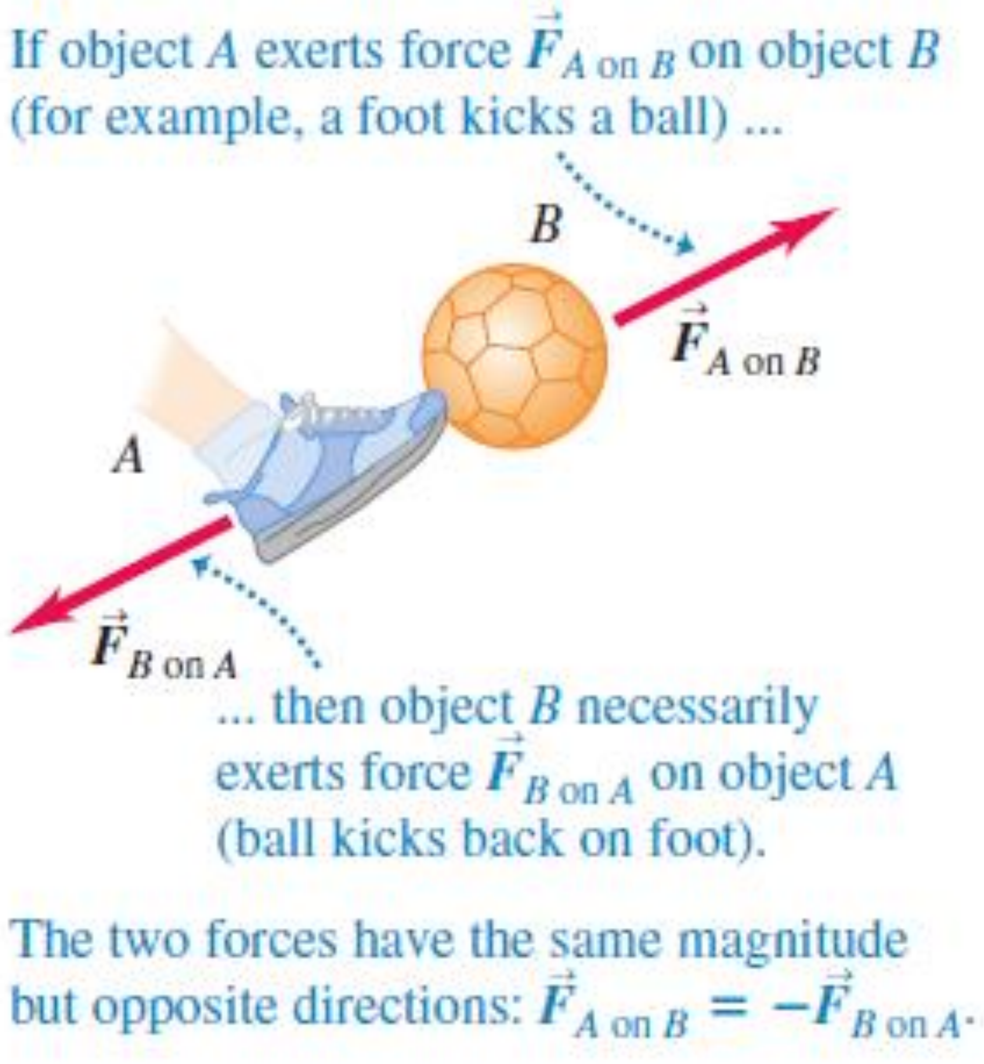
$$= ma_x\vec{i} + ma_y\vec{j} + ma_z\vec{k}$$

$$\Rightarrow$$

$$F_x = ma_x = m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} \quad F_y = ma_y = m\frac{\mathrm{d}^2 y}{\mathrm{d}t^2}$$

$$F_z = ma_z = m\frac{\mathrm{d}^2 z}{\mathrm{d}t^2}$$

牛顿第三定律（作用力与反作用力定律）



NEWTON's THIRD LAW OF MOTION: If object A exerts a force on object B (an "action"), then object B exerts a force on object A (a "reaction"). These two forces have the same magnitude but are opposite in direction. These two forces act on different objects.

A作用于B的力大小，等于B作用于A的力大小，但方向相反，且两个力作用在不同物体上。

牛顿第三定律的特点

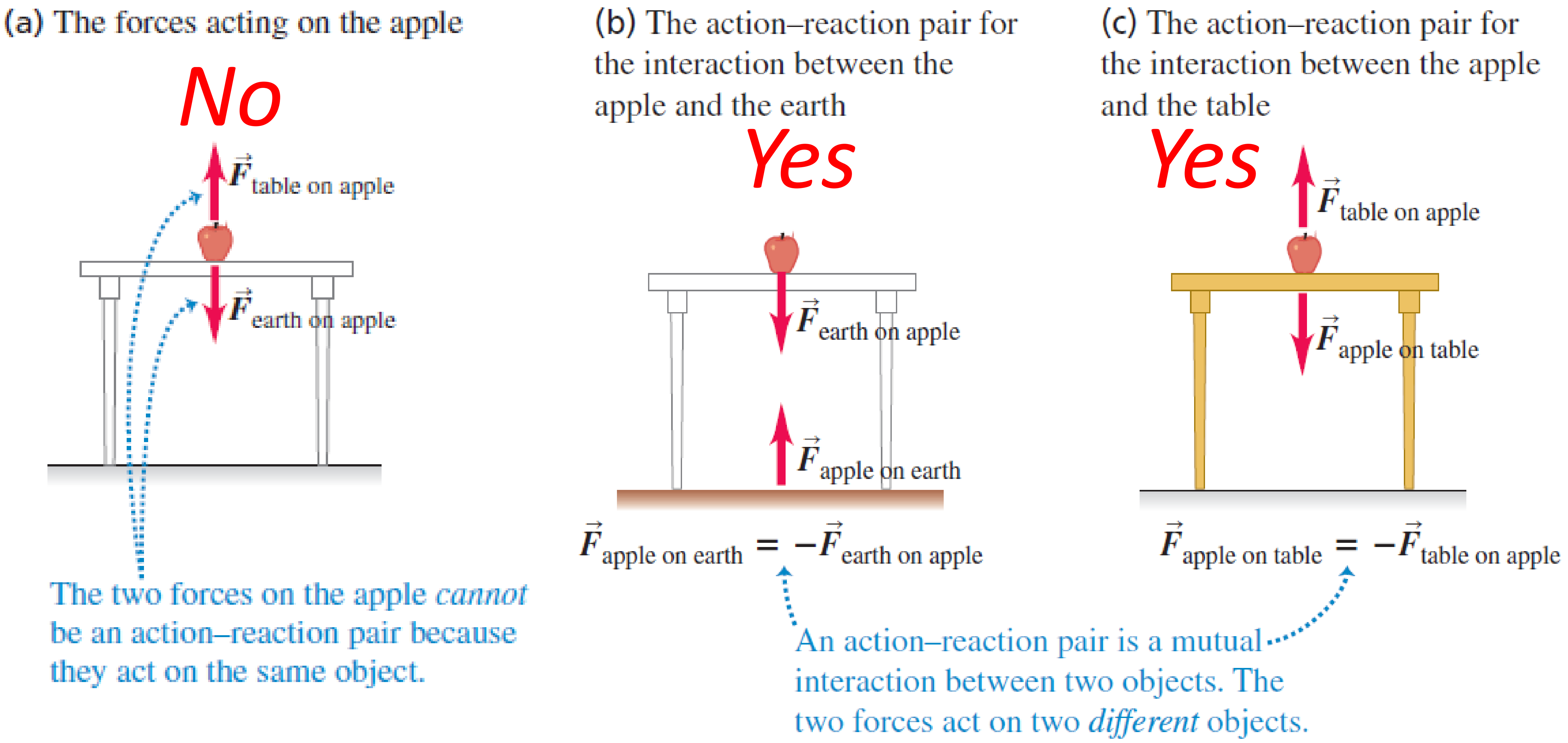
牛顿第三定律定义了相互作用的性质!

除作用在不同物体上以外

作用力与反作用力本质相同

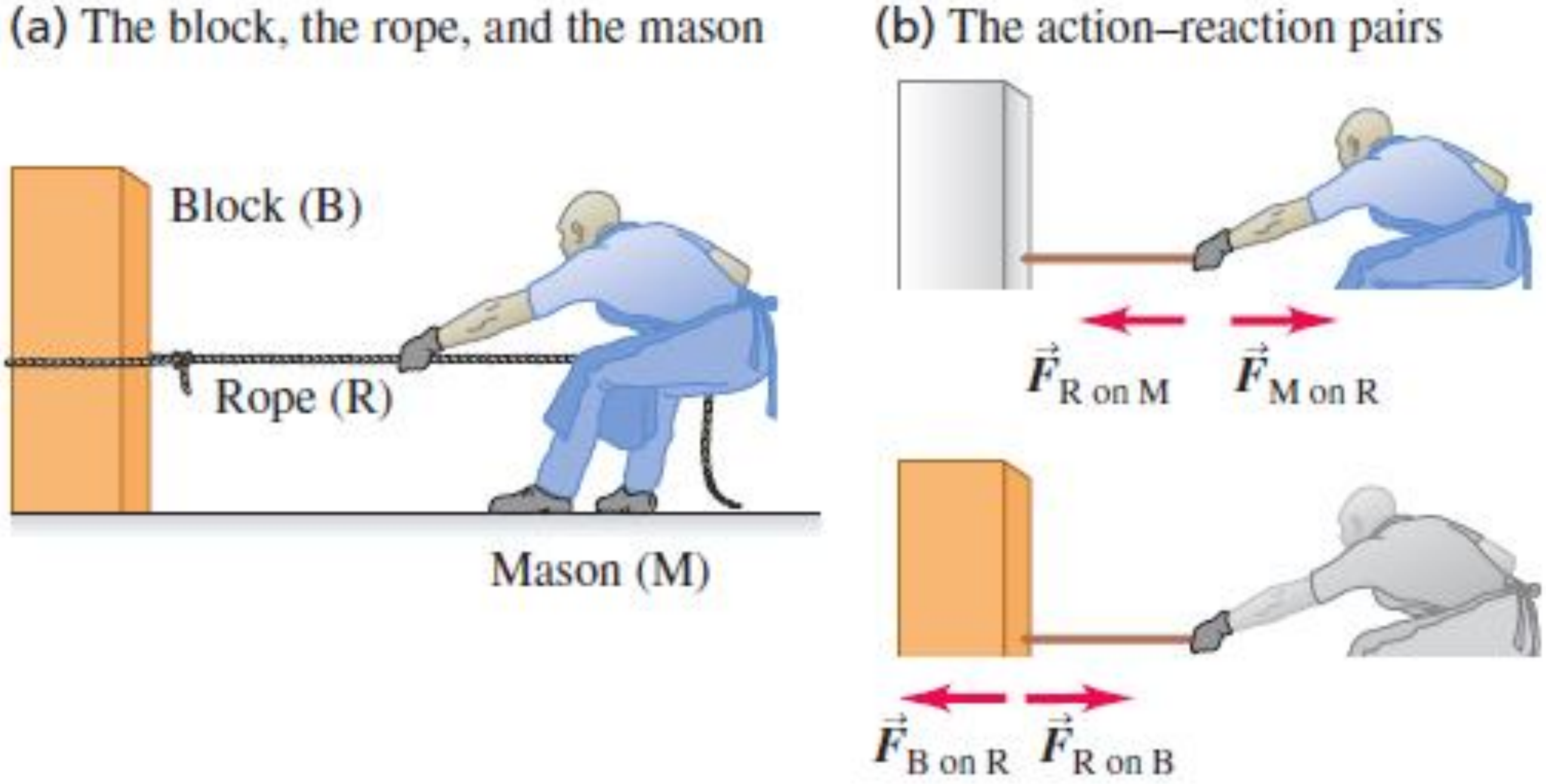
* 注意作用力和反作用力与平衡力的差别。

作用力和反作用力举例



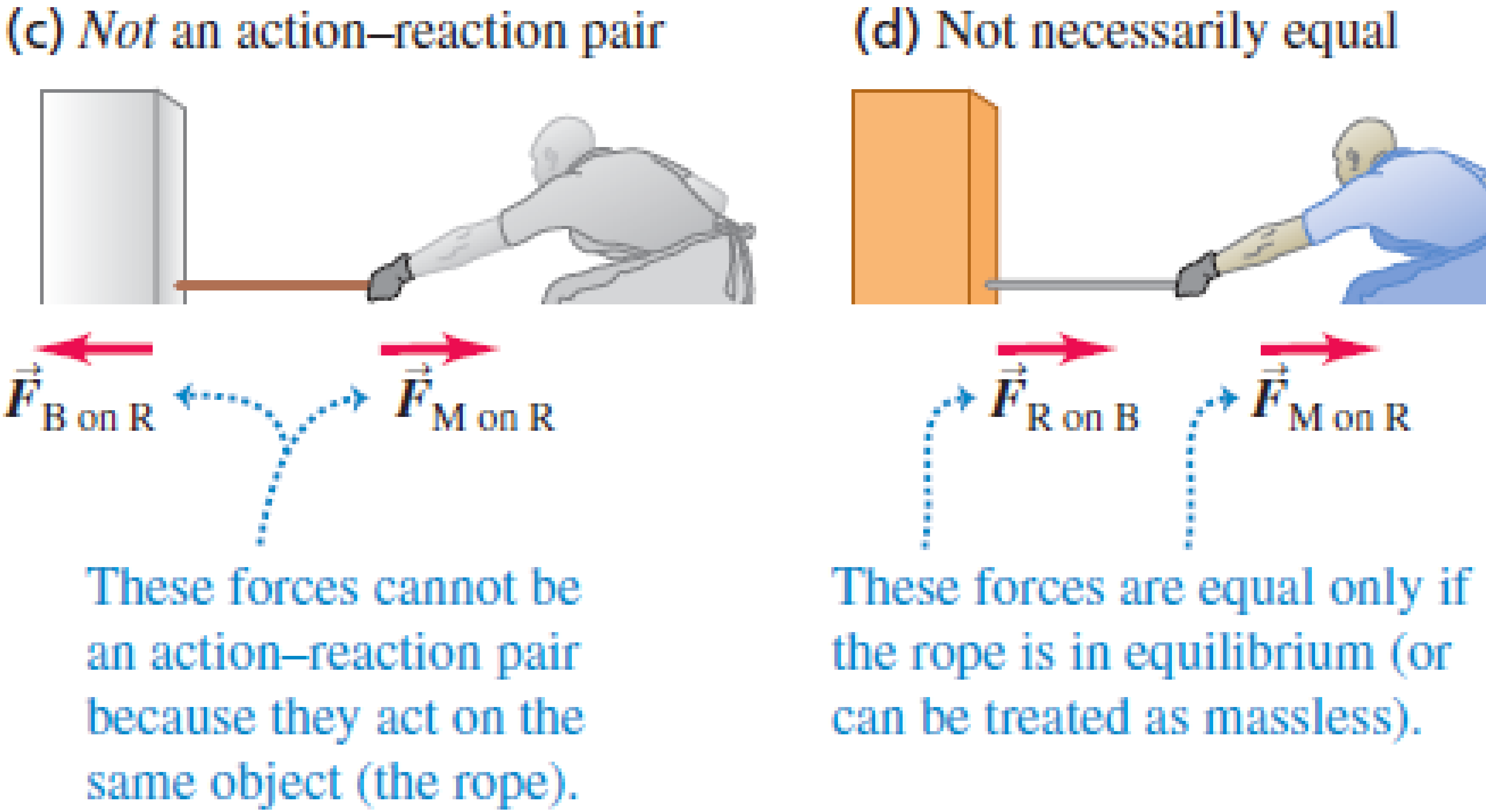
(a) The forces acting on the apple
No
$\vec{F}_{\text{table on apple}}$
$\vec{F}_{\text{earth on apple}}$
The two forces on the apple *cannot* be an action–reaction pair because they act on the same object.
(b) The action–reaction pair for the interaction between the apple and the earth
Yes
$\vec{F}_{\text{earth on apple}}$
$\vec{F}_{\text{apple on earth}}$
$\vec{F}_{\text{apple on earth}} = -\vec{F}_{\text{earth on apple}}$
(c) The action–reaction pair for the interaction between the apple and the table
Yes
$\vec{F}_{\text{table on apple}}$
$\vec{F}_{\text{apple on table}}$
$\vec{F}_{\text{apple on table}} = -\vec{F}_{\text{table on apple}}$
An action–reaction pair is a mutual interaction between two objects. The two forces act on two *different* objects.

作用力和反作用力举例



(a) The block, the rope, and the mason
Block (B)
Rope (R)
Mason (M)
(b) The action–reaction pairs
$\vec{F}_{\mathrm{R\ on\ M}}$
$\vec{F}_{\mathrm{M\ on\ R}}$
$\vec{F}_{\mathrm{B\ on\ R}}$
$\vec{F}_{\mathrm{R\ on\ B}}$

作用力和反作用力举例



(c) *Not* an action–reaction pair
(d) Not necessarily equal
$\vec{F}_{\text{B on R}}$
$\vec{F}_{\text{M on R}}$
$\vec{F}_{\text{R on B}}$
$\vec{F}_{\text{M on R}}$
These forces cannot be an action–reaction pair because they act on the same object (the rope).
These forces are equal only if the rope is in equilibrium (or can be treated as massless).

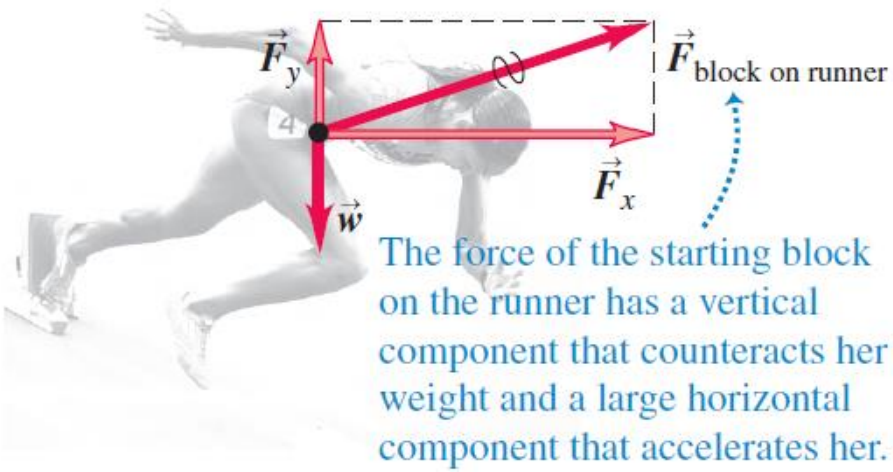
自由体受力图 Free-body diagrams

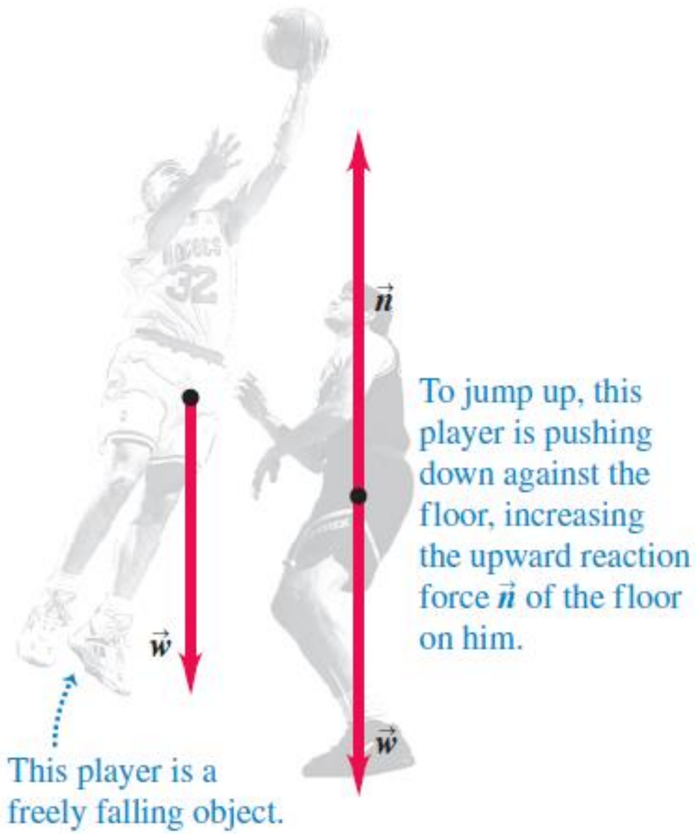
1. Newton’s first and second laws apply to a specific object.

2. Only forces acting on the object matter.

3. Free-body diagrams are essential to help identify the relevant forces. A free-body diagram shows the chosen object by itself, “free” of its surroundings, with vectors drawn to show the magnitudes and directions of all the forces that act on the object.

自由体受力图 Free-body diagrams

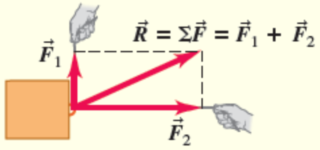




CHAPTER 4 SUMMARY

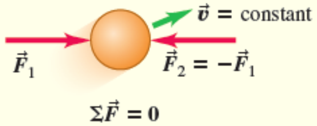
Force as a vector: Force is a quantitative measure of the interaction between two objects. It is a vector quantity. When several external forces act on an object, the effect on its motion is the same as if a single force, equal to the vector sum (resultant) of the forces, acts on the object. (See Example 4.1.)

$$\vec{R} = \Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots \quad (4.1)$$



The net external force on an object and Newton's first law: Newton's first law states that when the vector sum of all external forces acting on a object (the *net external force*) is zero, the object is in equilibrium and has zero acceleration. If the object is initially at rest, it remains at rest; if it is initially in motion, it continues to move with constant velocity. This law is valid in inertial frames of reference only. (See Examples 4.2 and 4.3.)

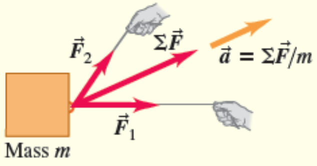
$$\Sigma\vec{F} = \mathbf{0} \quad (4.3)$$



Mass, acceleration, and Newton's second law: The inertial properties of an object are characterized by its *mass.* Newton's second law states that the acceleration of an object under the action of a given set of external forces is directly proportional to the vector sum of the forces (the *net force*) and inversely proportional to the mass of the object. Like Newton's first law, this law is valid in inertial frames of reference only. In SI units, the unit of force is the newton (N), equal to $1\ \text{kg}\cdot\text{m/s}^2$. (See Examples 4.4 and 4.5.)

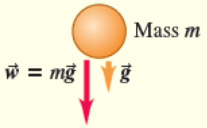
$$\Sigma\vec{F} = m\vec{a} \quad (4.6)$$

$$\Sigma F_x = ma_x \qquad \Sigma F_y = ma_y \qquad \Sigma F_z = ma_z \quad (4.7)$$



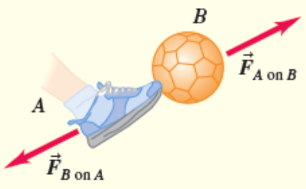
Weight: The weight $\vec{w}$ of an object is the gravitational force exerted on it by the earth. Weight is a vector quantity. The magnitude of the weight of an object at any specific location is equal to the product of its mass m and the magnitude of the acceleration due to gravity g at that location. The weight of an object depends on its location; its mass does not. (See Examples 4.6 and 4.7.)

$$w = mg \quad (4.8)$$



Newton's third law and action–reaction pairs: Newton's third law states that when two objects interact, they exert forces on each other that are equal in magnitude and opposite in direction. These forces are called action and reaction forces. Each of these two forces acts on only one of the two objects; they never act on the same object. (See Examples 4.8–4.11.)

$$\vec{F}_{A\text{ on }B} = -\vec{F}_{B\text{ on }A} \quad (4.10)$$



Homework （Due 2 weeks）

4.36 ••• **CP** An advertisement claims that a particular automobile can "stop on a dime." What net force would be necessary to stop a 850 kg automobile traveling initially at 45.0 km/h in a distance equal to the diameter of a dime, 1.8 cm?

4.47 ••• **CP** A small rocket with mass 20.0 kg is moving in free fall toward the earth. Air resistance can be neglected. When the rocket is 80.0 m above the surface of the earth, it is moving downward with a speed of 30.0 m/s. At that instant the rocket engines start to fire and produce a constant upward force F on the rocket. Assume the change in the rocket's mass is negligible. What is the value of F if the rocket's speed becomes zero just as it reaches the surface of the earth, for a soft landing? (*Hint:* The net force on the rocket is the combination of the upward force F from the engines and the downward weight of the rocket.)

Homework （Due 2 weeks）

4.52 ••• **CALC** The position of a training helicopter (weight 2.75×10^5 N) in a test is given by $\hat{r} = (0.020\ \text{m/s}^3)t^3\hat{\imath} + (2.2\ \text{m/s})t\hat{\jmath} - (0.060\ \text{m/s}^2)t^2\hat{k}$. Find the net force on the helicopter at $t = 5.0$ s.

4.55 ••• **CP CALC** A block of mass 2.00 kg is initially at rest at $x = 0$ on a slippery horizontal surface for which there is no friction. Starting at time $t = 0$, a horizontal force $F_x(t) = \beta - \alpha t$ is applied to the block, where $\alpha = 6.00$ N/s anwd $\beta = 4.00$ N. (a) What is the largest positive value of x reached by the block? How long does it take the block to reach this point, starting from $t = 0$, and what is the magnitude of the force when the block is at this value of x? (b) How long from $t = 0$ does it take the block to return to $x = 0$, and what is its speed at this point?

4.56 ••• **CALC** An object of mass m is at rest in equilibrium at the origin. At $t = 0$ a new force $\vec{F}(t)$ is applied that has components

$$F_x(t) = k_1 + k_2 y \qquad F_y(t) = k_3 t$$

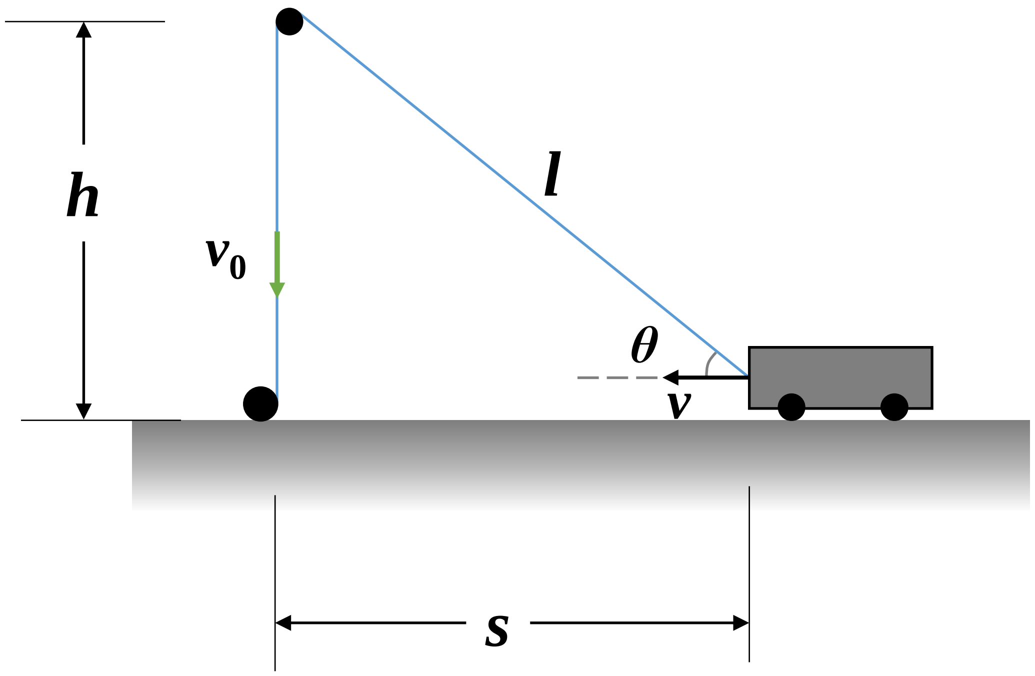
where k_1, k_2, and k_3 are constants. Calculate the position $\vec{r}(t)$ and velocity $\vec{v}(t)$ vectors as functions of time.

Homework （Due 2 weeks）

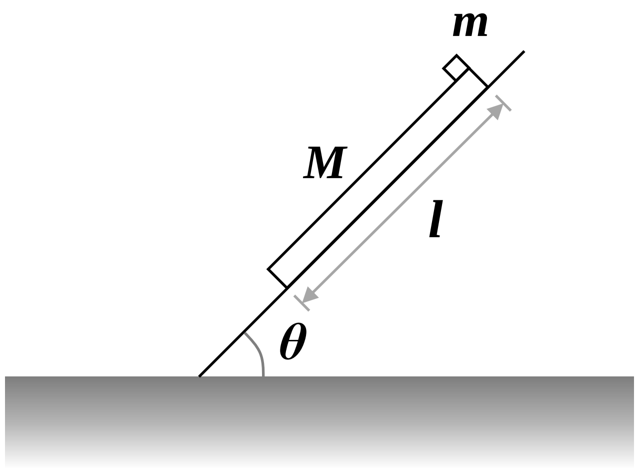
6、从地面以角度$\boldsymbol{\theta_0}$、初速度$\boldsymbol{v_0}$，发射一质量为$\boldsymbol{m}$的炮弹，炮弹在运动中始终受到与速度成正比的空气阻力$\boldsymbol{F} = \boldsymbol{kv}$，阻力系数$k$为常数。求炮弹的运动方程。

Homework （ Due 2 weeks ）

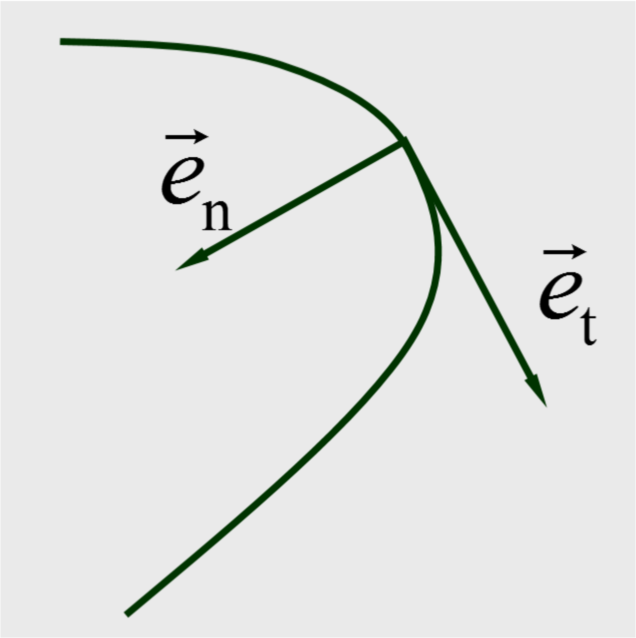
7、水平直地面上有一辆小车，地面有一电机，正上方有一滑轮，电机以匀速率 v_0 收绳，小车被绳拉着在地面上移动(如图所示)，问（1）用s、h、v_0给出小车运动速度v的方程。（2）当牵引绳与水平方向夹角为 θ 的瞬时，小车的速度v多大?



8、在倾角为$\theta = 45°$ 的斜面上放一质量 $\boldsymbol{M} = 1$千克，长 $\boldsymbol{l} = 1.4$米的板，板上端放一质量 $\boldsymbol{m} = 0.5$ 千克的小方块。设板与斜面间的摩擦系数 $\boldsymbol{\mu}$ 等于(a) 0.7，(b) 0.5，分别求方块从板上滑下的时间。方块和板间的摩擦忽略不计，起始时刻方块与板都静止不动。



牛顿第二定律的分量形式：自然坐标系



$$\vec{F} = F_t \vec{e}_t + F_n \vec{e}_n$$

$$\vec{a} = \frac{dv}{dt}\vec{e}_t + \frac{v^2}{\rho}\vec{e}_n$$

$$F_t = ma_t = m\frac{dv}{dt}$$

$$F_n = ma_n = m\frac{v^2}{\rho}$$