Numerical Optimization, 2021 Fall
Homework 6

Your Name
Your ID

Due 23:59 (CST), Nov. 27, 2021
(NOTE: Homework will not be accepted after this due for any reason.)

1 Convex Basics

Prove that $f : \mathbb{R}^n \to \mathbb{R}$ is affine if and only if f is both convex and concave.

2 Newton's Method

Consider minimizing $\frac{1}{2}x^T Ax + b^T x + c$, A positive definite. Prove that Newton's method will converge to the optimal in one step starting at an arbitrary initial point x_0.

3 Coding

In this example we want to compute the minimum of the non-convex bivariate Rosenbrock function "banana function"

$$f(x, y) = (1 - x)^2 + 100(y - x^2)^2.$$

It is notorious in optimization examples because of the slow convergence most methods exhibit when trying to solve this problem. And the domain of the above Rosenbrock function is $x \in [-2, 2]$, $y \in [-2, 2]$.

1. Show the gradient of $f(x, y)$.
2. Plot the 3D-surface of $f(x, y)$ on its domain.
3. Use Gradient Descent method with *Armijo Backtracking Line Search* to minimize $f(x, y)$.
4. Use Newton's method with *Armijo Backtracking Line Search* to minimize $f(x, y)$.

Please start at the point $(x, y) = (-1.2, 1)$ to initiate your algorithms. For each case, draw the curves of the step size α^k and $\|\nabla f(x^k, y^k)\|_\infty$ of the two algorithms, with respect to the iteration number k.

Your program may be implemented with Python or MATLAB. Please paste your curves down below, then put your code in an independent file (We encourage you to submit a *.mlx (by MATLAB Live Editor) or *.ipynb (by Jupyter) file to answer the question. Others are fine, of course) and submit it to Blackboard alongside the PDF.

Hint: You may set initial step size $\alpha_0 = 1$, constant $c_1 = 10^{-4}$, and termination condition as $\|\nabla f(x^k, y^k)\|_\infty \leq 10^{-4}$.