# Electromagnetics Spring

2020 Homework 2

Deadline: 3.24 23:59 pm

说明:
全用英文作答;
每道题要对所有小问作答，要给出全部必要的推导过程，计算题要算出最终的数值结果，比如开根号之类的;
所有计算出来的结果如果是有单位的物理量，一定要写明单位;
每题的分数在括号中给出;
可以互相讨论，也可以上网查，但是不能抄袭，也不能找别人代做;
可以在电脑敲字解答，也可以手写解答，最后统一转换为 PDF 格式，按分组信息邮件或 BB 上提交;
邮件主题&附件命名规范:姓名_章节，不按规范发送扣除一半分数;
请在作业 PDF 的第一行写上姓名和学号;
有问题请给老师或助教发邮件;

Textbook: Fundamentals of Applied Electromagnetics, 7th edition

## Part I. Problems in textbook.

**3.27 (20 points)**
**3.28 (20 points)**
**3.29 (20 points)**
**3.36(a) (10 points)**
**3.36(c) (10 points)**
**3.36(g) (10 points)**
**3.38 (10 points)**
**3.42 (10 points)**
**3.44(a) (10 points)**
**3.44(f) (10 points)**
**3.44(i) (10 points)**
**3.47 (10 points)**
**3.50 (20 points)**
**3.58 (20 points)**

Part II.Problems in quiz.

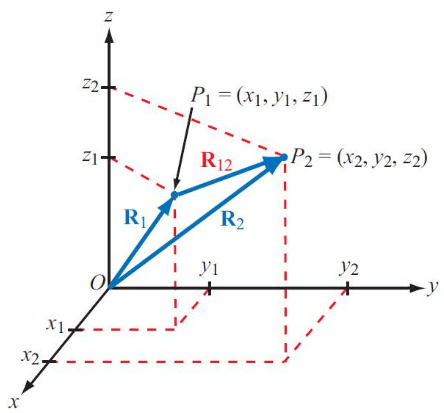
**1.** (**3 points**) Given $\mathbf{A} = \left(A_x, A_y, A_z\right)$ and $\mathbf{B} = \left(B_x, B_y, B_z\right)$, evaluate their dot product $\mathbf{A} \cdot \mathbf{B}$ and cross product $\mathbf{A} \times \mathbf{B}$.

**2.** (**6 points**) Evaluate the following six cross products.

(a) $\widehat{\boldsymbol{x}} \times -\widehat{\boldsymbol{y}} =$ (b) $\widehat{\boldsymbol{y}} \times \widehat{\boldsymbol{z}} =$ (c) $-\widehat{\boldsymbol{z}} \times -\widehat{\boldsymbol{x}} =$

(d) $\widehat{\boldsymbol{\rho}} \times \widehat{\boldsymbol{\phi}} =$ (e) $-\widehat{\boldsymbol{z}} \times \widehat{\boldsymbol{\rho}} =$ (f) $\widehat{\boldsymbol{\theta}} \times \widehat{\boldsymbol{R}} =$

**3.** (**2 points**) How to express the unit vector of the difference vector $\overrightarrow{P_1 P_2}$ shown below in terms of $\overrightarrow{\mathrm{R}_1}$ and $\overrightarrow{\mathrm{R}_2}$.



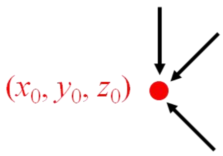

**4.** (**17 points**)

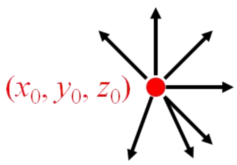
(a) (**4 points**) Describe the relationship between the gradient of a scalar $T(x, y, z)$ at point $(x_0, y_0, z_0)$ and the directional derivative at this point. Need to describe both magnitude and direction. Explain the relationship between gradient and directional derivative in your words.

(b) (**2 points**) Write out the expression of the gradient $\nabla\Phi$ of a scalar field $\phi(x, y, z)$ in the Cartesian coordinate systems.

(c) (**2 points**) Write out the expression of the divergence $\nabla \cdot \mathbf{A}$ of a vector field $\mathbf{A}(x, y, z)$ in the Cartesian coordinate systems.

(d) (**2 points**) For the two cases given below, which one indicates $\nabla \cdot \mathbf{A} > 0$ at the point $(x_0, y_0, z_0)$ and which one indicates $\nabla \cdot \mathbf{A} < 0$?




(e) (**2 points**) Write out the expression of the curl $\nabla \times \mathbf{A}$ of a vector field $\mathbf{A}(x, y, z)$ in the Cartesian coordinate systems.

(f) (**1 point**) If the curl of a vector field $\boldsymbol{A}$ at a point is not zero, what feature does the field $\boldsymbol{A}$ have at this point?

(g) (**2 points**) For the magnetic flux density $\boldsymbol{B}$ due to a line current along the $z$ axis, does $\boldsymbol{B}$ have nonzero curl at point $(x_0, y_0, z_0) = (0, 0, 7)$? If yes, please specify the direction of the curl.

(h) (**2 points**) Does $\boldsymbol{B}$ have nonzero curl at point $(x_0, y_0, z_0) = (1, 3, 2)$? If yes, please specify the direction of the curl.

**5.** (**3 points**) Write out the expression of the volume $dV$ of an infinitesimal cube when calculating a three-fold integration in the Cartesian coordinates, cylindrical coordinates and spherical coordinates, respectively.

**6.** (**16 points**)

(a) (**4 points**) In Cartesian coordinates, vector **A** points from the origin to point $P_1 = (2, -2, 1)$, and vector **B** is directed from $P_1$ to point $P_2 = (2, 2, 4)$. Find the vector **B**. Calculate the angle $\theta_{AB}$ between **A** and **B**.

(b) (**6 points**) In Cartesian coordinates, vector **A** points from $P_1 = (0, 0, 2)$ to $P_2 = (1, \sqrt{3}, 0)$. Express the vector **A** in cylindrical coordinates. Express the unit vector $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\phi}}$ at point $P_2$ in terms of the unit vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$ in Cartesian coordinates.

(c) (**6 points**) In Cartesian coordinates, vector **A** points from $P_1 = (0, 1, 1)$ to $P_2 = (0, 2, 2)$. Express the vector **A** in spherical coordinates. Express the unit vector $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\phi}}$ at point $P_2$ in terms of the unit vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$ in Cartesian coordinates.