On the Coverage Problem in Device-to-Device Relay Networks

Wanyue Qu, Geng Li, and Yuping Zhao

***Abstract*—The device-to-device (D2D) relay technology is one of the key technologies in 5G networks, and its coverage problem is important as it determines key properties in network management. However, the traditional relay coverage theories (e.g., based on the received signal-to-interference-plus-noise ratio) are not optimal for D2D relay due to the dynamicity in backhaul resources of D2D relays. In this paper, we study the coverage problem in D2D relay networks with theoretical derivation and experimental simulation using stochastic geometry. In particular, we formulate the coverage problem to maximize the overall system downlink rate, and then the domain optimization problem is transformed into a 0-1 integer programming problem. We propose a novel algorithm to obtain the optimal solution by greedily including the best locations into the coverage. Simulation results verify the theoretical solution and show that the optimal coverage is surprisingly in an irregular shape that excludes the D2D relay. The results also obtain the best relay location and show the performance gain than the existing scheme.**

***Index Terms*—Device-to-device (D2D) relay, coverage problem, stochastic geometry.**

I. Introduction

The new release of Third Generation Partnership Project (3GPP) specifies that, a 5G system shall support a user equipment (UE) to connect to the network either directly or using another UE as a relay [1], [2]. In accordance with the 5G requirement, the *device-to-device (D2D) relay technology*, based on D2D communication, has been recently proposed [3], where a D2D-enabled UE can assist cellular transmission by acting as a relay between the base station (BS) and some other UEs. Different from traditional relays, the D2D relays provide higher flexibility for networking by dynamic deployment and resource allocation according to the traffic demand.

Existing work of D2D relays is mainly focusing on the user pairing problem [3], [4], resource allocation problem [5], [6] and transmission capacity analysis [7]. There is only limited research on the coverage problem of D2D relay, which is a fundamental issue and non-trivial to network management, such as relay selection and user association.

The coverage probability is studied using stochastic geometry in [8]. They adopt a simple scheme which assumes that a UE is within the coverage of a D2D relay if the received signal-to-interference-plus-noise ratio (SINR) is above a threshold. The SINR-based scheme is commonly used in the traditional relay networks, and performs well since the deployment and backhaul resources of the traditional relays are fixed [9]. But in D2D relay networks, every D2D relay is a normal UE until configured by the BS to become a relay. The backhaul resources (e.g., bandwidth) of each D2D relay are dynamically changed; i.e., as a new UE is associated with the relay, the UE's transmission resources are devoted to the relay's backhaul resources [4]. In addition, D2D relays and the served UEs adopt D2D communication, whose resource allocation also affects the coverage area. Thus the criterion of received SINR is not optimal in D2D relay networks. *How to determine the coverage of a D2D relay is a new problem.*

In this paper, we tackle the coverage problem in D2D relay networks using stochastic geometry [10], and provide the optimal solution with theoretical derivation and experimental simulation. Specifically, the goal is to find the optimal coverage that maximizes the overall system downlink rate with consideration of both the received SINR and the resource dynamicity. We first formulate the D2D relay coverage problem, as a domain optimization problem, which is then converted to a 0-1 integer programming problem with an equivalent transformation. We propose a novel algorithm to obtain the optimal solution by greedily including the best locations into the coverage, and the optimality is proved. Extensive simulation results verify the theoretical solution, and show that the proposed scheme leads to better performance than the existing SINR-based scheme [8]. Surprisingly, the optimal coverage of a D2D relay is in an irregular shape (i.e., ellipse-like) that excludes the relay and changes along with the relay's location. We also obtain the best D2D relay location that maximizes the system downlink rate, which can be used for relay selection in D2D relay networks.

II. Network Model

In order to find the theoretical solution, we model the network based on stochastic geometry [10]. Without loss of generality, we consider a single macro cell with area $\mathbb{R}^2$. The UEs are spatially distributed according to a bi-dimensional homogeneous Poisson Point Processes (PPP) with density λ_u; i.e., the total number of UEs in the cell is $\int_{\mathbb{R}^2} \lambda_u dx$. In this paper, we concentrate on the *downlink transmission* which is commonly used for system performance [4], and the results can be easily extended to the uplink case.

In a D2D-relay-enabled network, UEs can form into clusters according to the traffic demand in hot-spot areas, with the following definitions.

Definition 1: A *cluster* is a temporary transmission set of UEs. One certain UE working as a D2D relay is defined as *cluster head (CH)*, while the other UEs in the cluster coverage are defined as *cluster members* and served by the CH in a two-hop fashion to communicate with the macro BS. We use CH to denote the D2D relay in the rest of the paper. As shown in Fig. 1, the radio link between the BS and a CH is referred to

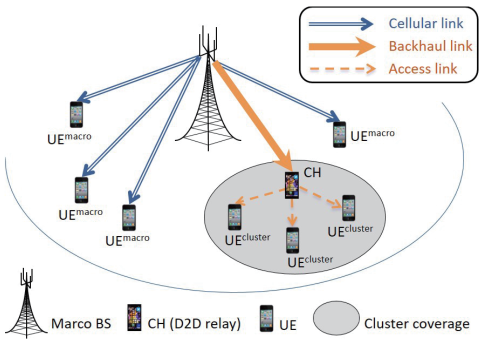


Fig. 1. Illustration of a D2D-relay-enabled network.

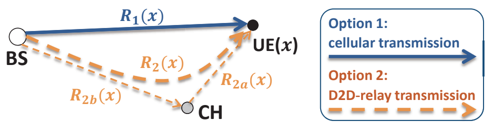


Fig. 2. Two options for downlink transmission.

as a *backhaul link*, that between a CH and a cluster member is an *access link*, and that between the BS and an ordinary UE out of any clusters is a *cellular link*.

For simple illustration, we consider one cluster with the CH at location $y \in \mathbb{R}^2$ in our model, and the case with more CHs can be solved by the same token. For an arbitrary UE (except for the CH), there are two options for performing downlink transmission, namely the *cellular transmission* (Option 1) and the *D2D-relay transmission* (Option 2), as shown in Fig. 2.

With regard to the spectrum usage, we consider a LTE Frequency Division Duplex (FDD) system where the downlink and uplink carrier frequencies are f_1 and f_2 respectively. Both of them encompass a usable bandwidth of W. A portion of the uplink frequency band is dedicated to D2D communications as specified in literature [11]. In summary, the cellular links and backhaul links work on the downlink frequency f_1 with a total bandwidth of W, while the access links work on the uplink frequency f_2 with a bandwidth denoted by W_{D2D}. (The results could be extended to full-duplex D2D relays in future work.) Regarding to the resource allocation, each UE needs a portion of f_1 spectrum, which is used for either the cellular transmission (Option 1) or the backhaul transmission by its CH (Option 2). The macro BS evenly divides its available bandwidth amongst all the UEs. Thus the resource bandwidth at f_1 acquired by each UE is given by $B_0 = W / \int_{\mathbb{R}^2} \lambda_u dx$.

For a UE at location $x \in \mathbb{R}^2$, we quantify the end-to-end downlink data rates in the two transmission options separately.

Option 1: Cellular transmission.

When the UE is directly communicating with the macro BS, the data rate can be calculated by the Shannon capacity as

$$R_1(x) = B_0 \cdot \log_2(1 + SINR_x^{macro}), \tag{1}$$

where $SINR_x^{macro}$ is the received SINR at location x from the macro BS.

Option 2: D2D-relay transmission.

When taking the CH as its D2D relay, the UE devotes its belonging bandwidth B_0 to the CH for its data transmission in the backhaul link. Thus, the effective user data rate in the backhaul link dedicated to the UE can be expressed as

$$R_{2b}(x) = B_0 \cdot \log_2(1 + SINR_y^{macro}). \tag{2}$$

In order to maximize the system downlink rate, the CH allocates a certain bandwidth $B(x)$ at f_2 to the UE for D2D communication. The computation of $B(x)$ will be described in Section III. B. Then the rate of the UE in the access link can be given by

$$R_{2a}(x) = B(x) \cdot \log_2(1 + SINR_x^{CH}), \tag{3}$$

where $SINR_x^{CH}$ is the received SINR at location x from the CH.

Since working on different carrier frequencies, the backhaul link and access link support simultaneous downlink tramsmission. Based on the max-flow min-cut theorem for the link capacity [12], the end-to-end data rate experienced by the UE at location x in Option 2 is the minimum of two values, expressed as follows:

$$R_2(x) = \min(R_{2b}(x), R_{2a}(x)). \tag{4}$$

III. Coverage Problem of D2D Relays

Our goal is to find the theoretical optimal cluster coverage that maximizes the system downlink rate. In this section, we formulate the coverage problem as a *domain optimization problem* which can be converted to a *0-1 integer programming problem* with an equivalent transformation. Then we propose a novel algorithm that greedily selects the best locations into the coverage to derive the theoretical optimal solution, and the optimality is proved.

A. Problem Formulation

Let $\Omega \subseteq \mathbb{R}^2$ denote the domain of the cluster coverage, where the UEs choose Option 2 for transmission, and the UEs out of Ω adopt Option 1. Therefore, the downlink rate of the UE at location x, denoted by $R(x)$ is

$$R(x) \ = \begin{cases} R_2(x), & x \in \Omega, \\ R_1(x), & x \notin \Omega. \end{cases} \tag{5}$$

In our stochastic model, the expectation of the system downlink rate is computed by integration as

$$\begin{aligned} \mathbb{E}[R_{sum}] &= \int_{\mathbb{R}^2} \lambda_u R(x) dx \\ &= \int_{\Omega} \lambda_u R_2(x) dx + \int_{\mathbb{R}^2 - \Omega} \lambda_u R_1(x) dx, \end{aligned} \tag{6}$$

where the first term is the sum rate of the UEs in the cluster coverage, and the second term is that of the rest UEs out of the coverage.

Given the CH's location y and the dedicated D2D bandwidth W_{D2D}, our goal is to find the optimal cluster coverage Ω^* that maximizes the system downlink rate. The problem can be formulated as a *domain optimization problem* as below:

Problem 1:

$$\begin{aligned} \max \quad & \int_{\Omega} \lambda_u R_2(x) dx + \int_{\mathbb{R}^2 - \Omega} \lambda_u R_1(x) dx, \\ \text{s.t.} \quad & \int_{\Omega} \lambda_u B(x) dx \le W_{D2D}. \end{aligned} \tag{7}$$

The direct formulation of a *domain optimization problem* poses a major challenge due to high complexity. To cope with that, we introduce a *domain eigen function* $\mathcal{X}_\Omega(x)$, indicating whether location x is in Ω or not. That is, $\mathcal{X}_\Omega(x) = 1$ if $x \in \Omega$, and $\mathcal{X}_\Omega(x) = 0$ otherwise. Accordingly, the expectation of the system downlink rate can be expressed as

$$\begin{aligned}&\mathbb{E}[R_{sum}]\\&= \int_{\mathbb{R}^2} [\lambda_u R_2(x) \cdot \mathcal{X}_\Omega(x) + \lambda_u R_1(x) \cdot (1 - \mathcal{X}_\Omega(x))]\, dx\\&= \lambda_u \int_{\mathbb{R}^2} (R_2(x) - R_1(x)) \cdot \mathcal{X}_\Omega(x) dx + \lambda_u \int_{\mathbb{R}^2} R_1(x) dx,\end{aligned} \tag{8}$$

where λ_u and $\int_{\mathbb{R}^2} R_1(x)dx$ are constant and irrelevant to $\mathcal{X}_\Omega(x)$. Therefore, we can equivalently transform the *domain optimization problem* in *Problem 1* into a *0-1 integer programming problem* as the following final simplified form:

Problem 2:

$$\begin{aligned}\max \quad & \int_{\mathbb{R}^2} V(x) \cdot \mathcal{X}_\Omega(x) dx,\\ \text{s.t.} \quad & \int_{\mathbb{R}^2} B(x) \cdot \mathcal{X}_\Omega(x) dx \leq B',\end{aligned} \tag{9}$$

where $V(x) = R_2(x) - R_1(x)$ represents the rate increase at location x by D2D-relay transmission (Option 2) over cellular transmission (Option 1), and $B' = W_{D2D}/\lambda_u$.

B. Greedy-based Solution

Proposition 1: The *integer programming problem* in *Problem 2* is equivalent to a *continuous knapsack problem* in a continuous region, where

- the *"item"* is x,
- the *"profit of item x"* is $V(x)$,
- the *"weight of item x"* is $B(x)$.

Proof: See Appendix A. ■

Based on the above proposition, we propose a greedy-based algorithm to obtain the optimal solution of *Problem 2*. Specifically, we define a *efficiency value* of location x as $\beta(x) = \frac{V(x)}{B(x)}(bits/s/Hz)$, which represents the rate increase per dedicated D2D bandwidth by D2D-relay transmission (Option 2) over cellular transmission (Option 1). Intuitively, given the limited D2D resources, the larger the efficiency value is, the more rate increase can be achieved. However, the value of $B(x)$ depending on dynamic allocation is unknown, and how to decide its value is also part of our solution.

1) Computation of $B(x)$: Since the rate of D2D-relay transmission is $R_2(x) = \min(R_{2b}(x), R_{2a}(x))$ as Equation (4), to maximize the efficiency value $\beta(x)$ of a location x, $B(x)$ should be obtained by solving $R_{2b}(x) = R_{2a}(x)$. The intuition here is to achieve the highest rate without wasting any bandwidth. The simplification is omitted due to space limitation, and the final solution of $B(x)$ is expressed as below:

$$B(x) = R_{2b}(x)/(\log_2(1 + SINR_x^{CH})). \tag{10}$$

Fig. 3 provides a concrete example of the efficiency value distribution and its contour lines. In this figure, we notice the following features.

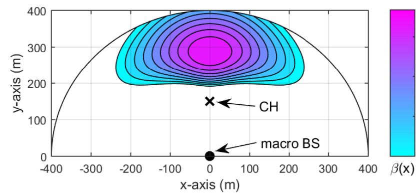


Fig. 3. An example of the distribution and contour lines of efficiency value $\beta(x)$. The dot mark represents the location of BS and the cross mark represents the location of CH.

- The efficiency values of locations close to the CH are relatively low. This is because the rate increase is limited; i.e., the numerator $V(x)$ is too small in $\beta(x)$.
- The efficiency values of locations far away from the CH are also low. This is because the cost of rate increase is pretty high; i.e., the denominator $B(x)$ is too large in $\beta(x)$.

2) Solution of $\mathcal{X}_\Omega^*(x)$: Based on the distribution of the efficiency value $\beta(x)$, we greedily select the locations with the largest $\beta(x)$ into the domain of cluster coverage Ω until the D2D resources are run out; i.e., $\int_\Omega B(x)dx > B'$. We express the critical efficiency value as β^*, which can be given by

$$\beta^* = \inf_\beta \{\int_{\{x \in \mathbb{R}^2 | \beta(x) \geq \beta\}} B(x)dx \leq B'\}. \tag{11}$$

As a result, we can obtain our greedy-based solution $\mathcal{X}_\Omega^*(x)$ to *Problem 2*, where the locations with the efficiency values greater than β^* will be included into the domain,

$$\mathcal{X}_\Omega^*(x) = \begin{cases} 1, & \beta(x) \geq \beta^* \\ 0, & \beta(x) < \beta^* \end{cases}, x \in \mathbb{R}^2. \tag{12}$$

Finally, the optimal domain of cluster coverage Ω^* can be expressed as below:

$$\Omega^* = \{x \in \mathbb{R}^2 | \mathcal{X}_\Omega^*(x) = 1\}. \tag{13}$$

Lemma 1 **(Optimality)**: The solution obtained by the proposed greedy-based algorithm is optimal.

Proof: The detailed proof can be found in Appendix B. Intuitively, 1) the greedy-based algorithm is optimal for *continuous knapsack problems*; and 2) the value of $B(x)$ is obtained by maximizing the efficiency value $\beta(x)$ in the greedy-based algorithm. ■

To find β^* as quickly as possible and to reduce the complexity of implementation, we propose an approximation approach to speed up the greedy-based algorithm as shown in **Algorithm 1**. Here we assume the maximum efficiency value is β_{max}, which can be easily found on the extension line determined by the BS and the CH. We use $B' - \varepsilon$ to approximate B', and ε is small enough. Then we adopt dichotomic search to find β^*; i.e., the interval in which β^* lies is gradually narrowed until B' is left ε-approached.

IV. Simulation Results

In this section, we verify the theoretical solution by extensive simulation to provide visualized and numerical results of the D2D relay coverage. We consider the downlink of a

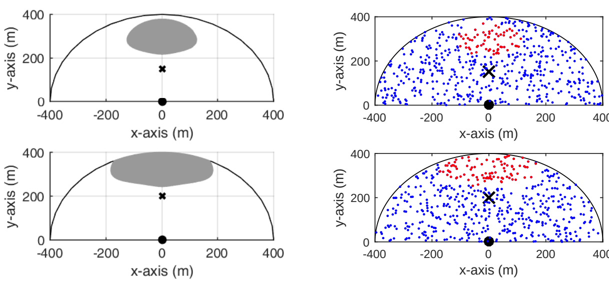
Algorithm 1 Greedy-based algorithm by dichotomic search

Initialization:

Set $\beta_1 = 0$; $\beta_2 = \beta_{max}$;

Iteration:

while 1 **do**
  $\beta = \frac{\beta_1+\beta_2}{2}$;
  $temp = \int_{\{x\in\mathbb{R}^2|\beta(x)\geq\beta\}} B(x)dx$;
  if $temp > B'$ **then**
    $\beta_1 = \beta$;
    Continue;
  end if
  if $temp < B' - \varepsilon$ **then**
    $\beta_2 = \beta$;
    Continue;
  end if
  if $B' - \varepsilon \leq temp \leq B'$ **then**
    Break;
  end if
end while
$\beta^* = \beta$;



(a) Theoretical (b) Simulated

Fig. 4. Shape of the optimal cluster coverage with 15dBm transmit power of CH. (a) Theoretical shape in grey color. (b) Simulations. The red dots represent the UEs joining the cluster, and the blue dots represent macro UEs.

network in an area of a circle with radius $400m$, consisting of one macro BS and one CH. UEs are randomly distributed with density $\lambda_u = 2000users/km^2$. The dedicated D2D resource W_{D2D} takes 10% proportion of the total uplink bandwidth. The settings of antenna configuration and channel model conform to "3GPP TR 36.814", in which only the path loss is considered.

1) The shape of the cluster coverage: Fig. 4 illustrates the shape of the optimal cluster coverage with different CH locations. Unlike the coverage of traditional fixed relays, the cluster coverage excludes the CH, i.e., with a certain distance from the CH. This is because the rate increase of UEs close to the CH is relatively small, so this part with low efficiency value is not included into the coverage. In addition, we can tell the shape is ellipse-like and changes with the CH location. As the CH moves outwards, the cluster spreads out at the cell edge. The irregular shape stems from the distribution of the efficiency value discussed in Section III.

2) The area of the cluster coverage: Fig. 5 shows the area of the optimal cluster coverage as a function of CH location with different CH transmit power denoted as P_c. As the CH moves outwards, the coverage area becomes larger. This is due to the fact that the required bandwidth of each UE for D2D communication gets less, and consequently the given dedicated D2D resources can serve more UEs. In addition,

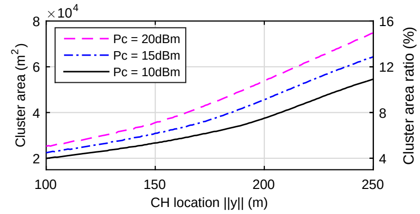


Fig. 5. Area of the optimal cluster coverage. The left y-axis indicates the area value, and the right y-axis indicates the ratio of the cluster coverage area over the cell area $\mathbb{R}^2$.

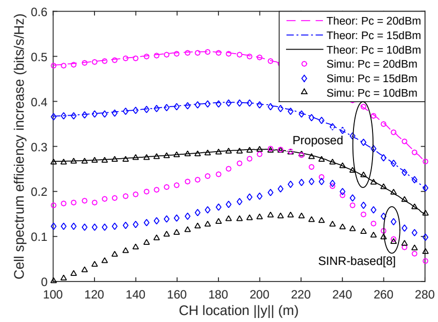


Fig. 6. Spectrum efficiency increase of the cell.

we can observe that the transmit power can be efficiently transformed into cluster coverage.

3) The spectrum efficiency improvement: Fig. 6 shows the cell spectrum efficiency increase in contrast to the traditional cellular network, with respect to CH location for different CH transmit power denoted as P_c. The simulated and theoretical results match well. As the CH location grows, the spectrum efficiency increase rises first and then goes down. The reason is that the cluster covers more UEs and enhances their data rate by the better two-hop transmission as the CH moves outwards. But the channel condition of the backhaul link also deteriorates, thus the spectrum efficiency increase goes down at last. We can obtain the best CH location that maximizes the system downlink rate, e.g., $\|y\| = 175m$ for $P_c = 20dBm$. Moreover, our proposed scheme performs better than the existing SINR-based scheme [8]. This is because it takes into account not only the received SINR, but also the resource limit and dynamicity. Lastly, we can see that the transmit power can be efficiently transformed into spectrum efficiency.

V. Conclusion

This paper studies the coverage problem in D2D relay networks using stochastic geometry. With the objective of maximizing the system downlink rate, we formulate the coverage problem, and convert it to a 0-1 integer programming problem with a novel transformation. A greedy-based algorithm is proposed to obtain the optimal relay coverage. Simulation results validate the theoretical solution and show the performance gain than the existing scheme. The results of this paper can be used for the guidance of network management such as D2D relay selection and user association in D2D relay networks.

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Appendix A
Proof of Proposition 1

The definition of the *knapsack problem* is given as below.

Definition 2: Given a set of n items, each with weight w_i ($w_i \geq 0$) and profit v_i ($v_i \geq 0$) ($i \in \{1, 2, ...n\}$), and one knapsack whose load capacity is C. If x_i ($0 \leq x_i \leq 1$) part of item i is loaded into the knapsack, $v_i x_i$ profit is obtained. A *knapsack problem* is to determine the part of each item to include into the knapsack that maximize the total profit under the total capacity constraint, expressed as follows:

$$\begin{aligned} \max \quad & \sum\nolimits_{i=1}^{n} v_i x_i, \\ \text{s.t.} \quad & \sum\nolimits_{i=1}^{n} w_i x_i \leq C. \end{aligned} \tag{14}$$

It is called a *continuous knapsack problem* if $0 \leq x_i \leq 1, \exists i \in \{1, 2, ...n\}$, and *0-1 knapsack problem* if $x_i = \{0, 1\}, \forall i \in \{1, 2, ...n\}$.

Firstly, the *integer programming problem* in *Problem 2* can be transformed into a *0-1 knapsack problem* in the two-dimension plane with functional analysis [13]. Intuitively, any point x in the plane can be regarded as an "item", with its "profit" and "weight". The "profit" represents the rate increase $V(x)$ by D2D-relay transmission (Option 2) over cellular transmission (Option 1), and the "weight" indicates the bandwidth $B(x)$ allocated to it by the CH for D2D transmission in the access link. Whether to include a point into the cluster domain is equivalent to whether to load the "item" into the knapsack.

Furthermore, since "item" x is a point in the two-dimension plane, whose size is infinitely small, the process of loading the "items" into the knapsack is continuous rather than discrete. Consequently, the *0-1 knapsack problem* is equivalent to a *continuous knapsack problem*.

In summary, the *integer programming problem* in *Problem 2* is equivalent to a *continuous knapsack problem*.

Appendix B
Proof of Lemma 1

The proof is provided in two parts with two lemmas below. Firstly, we prove the optimality of the greedy-based algorithm for *continuous knapsack problems*. Secondly, we prove that the allocation of $B(x)$ as Equation (10) maximizes the efficiency value $\beta(x)$, which is the fundamental factor in the greedy-based algorithm.

Lemma 2: In a *continuous knapsack problem*, the solution obtained by the greedy-based algorithm is optimal, where the item with the largest efficiency value $\xi_i = v_i / w_i$ amongst the remaining items is selected to load into the knapsack each time; i.e., the items are loaded in a non-increasing order of the efficiency value.

Proof: Firstly, the items are sorted by the efficiency value in an non-increasing order, that is $\xi_1 \geq \xi_2 \geq ...\xi_n$. Suppose $X = (x_1, ...x_n)$ is the solution obtained by the greedy-based algorithm. The corresponding total profit is $V(X) = \sum_{i=1}^{n} v_i x_i$. If $x_i = 1, \forall i \in \{1, 2, ...n\}$, it is evidently the optimal solution. We focus on the case that $x_i \neq 1, \exists i \in \{1, 2, ...n\}$. Assume j is the minimum subscript which satisfies $x_j \neq 1$; i.e., $x_i = 1$ for $i < j$, $x_i = 0$ for $j < i \leq n$, and $0 \leq x_j < 1$.

We derive the proof by contradiction. Assuming that X is not the optimal solution, there must exist a feasible solution $Y = (y_1, ...y_n)$, with the total profit $V(Y) = \sum_{i=1}^{n} v_i y_i > V(X)$. Without loss of generality, we assume k is the minimum subscript satisfying $y_k \neq x_k$. Three possible cases are discussed in the following, i.e., $k < j$, $k = j$ and $k > j$.

1) If $k < j$ or $k = j$, due to $x_i = 0$ for $j < i \leq n$ and $x_i = y_i$ for $i < k$, we can calculate the total profit of solution X as

$$\begin{aligned} V(X) &= \sum\nolimits_{i=1}^{j} v_i x_i \\ &= \sum\nolimits_{i=1}^{k-1} v_i x_i + \sum\nolimits_{i=k}^{j} v_i x_i \\ &= \sum\nolimits_{i=1}^{k-1} v_i y_i + \sum\nolimits_{i=k}^{j} v_i x_i \\ &= \sum\nolimits_{i=1}^{k-1} v_i y_i + \sum\nolimits_{i=k}^{j} (\xi_i w_i) x_i. \end{aligned} \tag{15}$$

With respect to the total weight, because solution X obtained by the greedy-based algorithm can fill up the knapsack, we have $\sum_{i=1}^{n} w_i x_i \geq \sum_{i=1}^{n} w_i y_i$. Besides, due to $x_i = y_i$ for $i < k$, we further have

$$w_k x_k + \sum\nolimits_{i=k+1}^{n} w_i x_i \geq w_k y_k + \sum\nolimits_{i=k+1}^{n} w_i y_i, \tag{16}$$

which can be deformed into

$$w_k x_k - w_k y_k \geq \sum\nolimits_{i=k+1}^{n} (w_i y_i - w_i x_i). \tag{17}$$

Furthermore, as $\xi_k \geq \xi_{k+1} \geq ...\xi_n \geq 0$, we have

$$\xi_k (w_k x_k - w_k y_k) \geq \sum\nolimits_{i=k+1}^{n} \xi_i (w_i y_i - w_i x_i), \tag{18}$$

which can be deformed into

$$\sum\nolimits_{i=k}^{n} (\xi_i w_i) x_i \geq \sum\nolimits_{i=k}^{n} (\xi_i w_i) y_i. \tag{19}$$

Moreover, with $x_i = 0$ for $j < i \leq n$, we further have

$$\sum\nolimits_{i=k}^{j} (\xi_i w_i) x_i \geq \sum\nolimits_{i=k}^{n} (\xi_i w_i) y_i. \tag{20}$$

Based on (20) and (15), the following inequality can be derived:

$$\begin{aligned} V(X) &= \sum\nolimits_{i=1}^{k-1} v_i y_i + \sum\nolimits_{i=k}^{j} (\xi_i w_i) x_i \\ &\geq \sum\nolimits_{i=1}^{k-1} v_i y_i + \sum\nolimits_{i=k}^{n} (\xi_i w_i) y_i \\ &= V(Y), \end{aligned} \tag{21}$$

which is in contradiction to our assumption $V(Y) > V(X)$.

2) If $k > j$, $x_k = 0$. Because k is the minimum subscript satisfying $y_k \neq x_k$, we have $y_k > 0$. Thus, with $y_k > 0$, $x_i = y_i$ for $i < k$, and $x_i = 0$ for $j < i \leq n$, we can derive that

$$\begin{aligned} \sum\nolimits_{i=1}^{k} w_i y_i &> \sum\nolimits_{i=1}^{k-1} w_i y_i \\ &= \sum\nolimits_{i=1}^{k-1} w_i x_i \\ &\geq \sum\nolimits_{i=1}^{j} w_i x_i \\ &= C, \end{aligned} \tag{22}$$

which is in contradiction to the premise $\sum_{i=1}^{k} w_i y_i \leq C$.

In conclusion, there exists no feasible solution superior to the solution obtained by the greedy-based algorithm. The greedy-based algorithm solves *continuous knapsack problems* to optimality. ■

Lemma 3: The allocation of the access link resource $B(x)$ by solving $R_{2b}(x) = R_{2a}(x)$ as Equation (10) maximizes the efficiency value $\beta(x)$.

Proof: Let $B(x)$ denote the bandwidth obtained by solving $R_{2b}(x) = R_{2a}(x)$. The efficiency value in the greedy-based algorithm is calculated as $\beta(x) = \frac{V(x)}{B(x)} = \frac{R_2(x) - R_1(x)}{B(x)}$.

Assume the CH actually allocates $B^*(x) = \sigma B(x)$ bandwidth to the UE at location x for D2D transmission in the access link. In this case, the data rate of the UE in the access link can be expressed as

$$R_{2a}^*(x) = B^*(x) \cdot \log_2(1 + SINR_x^{CH}) = \sigma R_{2a}(x). \tag{23}$$

Two cases when $B^*(x) < B(x)$ and $B^*(x) > B(x)$ are discussed respectively as below.

1) When $B^*(x) < B(x)$, i.e., $0 \leq \sigma < 1$, the end-to-end data rata in D2D-relay transmission (Option 2) is given by

$$R_2^*(x) = \min(R_{2b}(x), R_{2a}^*(x)) = \sigma R_2(x). \tag{24}$$

The corresponding efficiency value $\beta^*(x)$ in the greedy-based algorithm is

$$\begin{aligned} \beta^*(x) &= \frac{R_2^*(x) - R_1(x)}{B^*(x)} \\ &= \frac{\sigma R_2(x) - R_1(x)}{\sigma B(x)} \\ &< \frac{R_2(x) - R_1(x)}{B(x)} \\ &= \beta(x). \end{aligned} \tag{25}$$

2) When $B^*(x) > B(x)$, i.e., $\sigma > 1$, the end-to-end data rata in D2D-relay transmission (Option 2) is given by

$$R_2^*(x) = \min(R_{2b}(x), R_{2a}^*(x)) = R_2(x). \tag{26}$$

The corresponding efficiency value $\beta^*(x)$ in the greedy-based algorithm is

$$\begin{aligned} \beta^*(x) &= \frac{R_2^*(x) - R_1(x)}{B^*(x)} \\ &= \frac{R_2(x) - R_1(x)}{\sigma B(x)} \\ &< \frac{R_2(x) - R_1(x)}{B(x)} \\ &= \beta(x). \end{aligned} \tag{27}$$

Based on the above analysis, *Lemma 3* is proved. ■

In summary, *Lemma 2* proves the optimality of the greedy-based algorithm for *continuous knapsack problems*. According to the greedy-based algorithm, the larger the efficiency value is, the more rate increase can be achieved with the given D2D resources. Then *Lemma 3* proves the allocation of $B(x)$ maximizes the efficiency value. Thus *Lemma 1* is proved.