Friot: A Functional Reactive Language for IoT Programs with Dependent Type-and-Effect System (Report)

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I. Type checking trees for function delay

```
delay t =
   if t == 0 then ev (LightUp)
   else ev (Tick);
        delay (t - 1);
```

$\tau_{\texttt{delay}} \;=\; \texttt{t} : \texttt{Int} \to (\texttt{Unit} \;\&\; \Phi_{\texttt{delay}})$
$\Phi_{\texttt{delay}} \;=\; (\texttt{t} \geq 0 \wedge \underline{\textbf{Tick}}^{\texttt{t}} \cdot \underline{\textbf{LightUp}}) \vee (\texttt{t} < 0 \wedge \underline{\textbf{Tick}}^{\omega})$

delay

$$\dfrac{\dfrac{\Phi = \underline{\textbf{LightUp}}}{\Gamma' \vdash \texttt{ev (LightUp)} : \texttt{Unit} \;\&\; \underline{\textbf{LightUp}}}\;\text{(T-Event)} \qquad \dfrac{\dfrac{\Phi = \underline{\textbf{Tick}}}{\Gamma' \vdash \texttt{ev (Tick)} : \texttt{Unit} \;\&\; \underline{\textbf{Tick}}}\;\text{(T-Event)} \qquad \textbf{A}}{\Gamma' \vdash (\texttt{ev (Tick); delay (t - 1);}) : \texttt{Unit} \;\&\; (\underline{\textbf{Tick}} \cdot \Phi_{\texttt{delay}}^{(\texttt{t}-1)})}\;\text{(T-Let)}}{\dfrac{\Gamma' \vdash (\texttt{if t == 0 then ev (LightUp) else ev (Tick); delay (t - 1);}) : \texttt{Unit} \;\&\; (\texttt{t} == 0 \wedge \underline{\textbf{LightUp}}) \vee (\texttt{t} \neq 0 \wedge \underline{\textbf{Tick}} \cdot \Phi_{\texttt{delay}}^{(\texttt{t}-1)})}{\dfrac{\textit{... (Effects Computation I) ...}}{\Gamma \vdash (\texttt{delay t = if ... then ... else ...}) : (\texttt{t} : \texttt{Int}) \to (\texttt{Unit} \;\&\; (\texttt{t} \geq 0 \wedge \underline{\textbf{Tick}}^{\texttt{t}} \cdot \underline{\textbf{LightUp}}) \vee (\texttt{t} < 0 \wedge \underline{\textbf{Tick}}^{\omega}))}\;\text{(T-Fun)}}}\;\text{(T-If)}$$

$\Gamma' = \Gamma,\ \texttt{delay} : \tau_{\texttt{delay}}$
$\Phi_{\texttt{delay}}^{(\texttt{t}-1)} = (\texttt{t} \geq 1 \wedge \underline{\textbf{Tick}}^{\texttt{t - 1}} \cdot \underline{\textbf{LightUp}}) \vee (\texttt{t} < 1 \wedge \underline{\textbf{Tick}}^{\omega})$

A:

$$\dfrac{\dfrac{\texttt{sty}(\Gamma'(\texttt{delay})) \in \to}{\Gamma' \vdash \texttt{delay} : (\texttt{t} : \texttt{Int}) \to (\texttt{Unit} \;\&\; (\texttt{t} \geq 0 \wedge \underline{\textbf{Tick}}^{\texttt{t}} \cdot \underline{\textbf{LightUp}}) \vee (\texttt{t} < 0 \wedge \underline{\textbf{Tick}}^{\omega}))}\;\text{(T-VaF)} \qquad \dfrac{\texttt{sty}(\Gamma'(\texttt{t})) = \texttt{Int}}{\Gamma' \vdash (\texttt{t - 1}) : \texttt{int}}\;\text{(T-Var, T-Op)}}{\Gamma' \vdash \texttt{delay (t - 1)} : (\texttt{t - 1} : \texttt{int}) \to (\texttt{Unit} \;\&\; (\texttt{t} \geq 1 \wedge \underline{\textbf{Tick}}^{\texttt{t - 1}} \cdot \underline{\textbf{LightUp}}) \vee (\texttt{t} < 1 \wedge \underline{\textbf{Tick}}^{\omega}))}\;\text{(T-App, S-Base)}$$

I:

$$\begin{aligned}
&(\texttt{t} == 0 \wedge \underline{\textbf{LightUp}}) \vee (\texttt{t} \neq 0 \wedge \underline{\textbf{Tick}} \cdot \Phi_{\texttt{delay}}^{(\texttt{t}-1)}) \\
\equiv\;& (\texttt{t} == 0 \wedge \underline{\textbf{LightUp}}) \vee (\texttt{t} \neq 0 \wedge \underline{\textbf{Tick}} \cdot ((\texttt{t} \geq 1 \wedge \underline{\textbf{Tick}}^{\texttt{t - 1}} \cdot \underline{\textbf{LightUp}}) \vee (\texttt{t} < 1 \wedge \underline{\textbf{Tick}}^{\omega}))) \\
\equiv\;& (\texttt{t} == 0 \wedge \underline{\textbf{LightUp}}) \vee ((\texttt{t} \neq 0 \wedge \texttt{t} \geq 1) \wedge \underline{\textbf{Tick}} \cdot \underline{\textbf{Tick}}^{\texttt{t - 1}} \cdot \underline{\textbf{LightUp}}) \vee ((\texttt{t} \neq 0 \wedge \texttt{t} < 1) \wedge (\underline{\textbf{Tick}} \cdot \underline{\textbf{Tick}}^{\omega})) \\
\equiv\;& (\texttt{t} == 0 \wedge \underline{\textbf{LightUp}}) \vee (\texttt{t} \geq 1 \wedge \underline{\textbf{Tick}} \cdot \underline{\textbf{Tick}}^{\texttt{t - 1}} \cdot \underline{\textbf{LightUp}}) \vee (\texttt{t} < 0 \wedge (\underline{\textbf{Tick}} \cdot \underline{\textbf{Tick}}^{\omega})) \\
\equiv\;& (\texttt{t} == 0 \wedge \underline{\textbf{LightUp}}) \vee (\texttt{t} \geq 1 \wedge \underline{\textbf{Tick}}^{\texttt{t}} \cdot \underline{\textbf{LightUp}}) \vee (\texttt{t} < 0 \wedge \underline{\textbf{Tick}}^{\omega}) \\
\equiv\;& (\texttt{t} \geq 0 \wedge \underline{\textbf{Tick}}^{\texttt{t}} \cdot \underline{\textbf{LightUp}}) \vee (\texttt{t} < 0 \wedge \underline{\textbf{Tick}}^{\omega}) \\
\equiv\;& \Phi_{\texttt{delay}}
\end{aligned}$$

II. Type checking trees for function door_control

```
motion_sensor =
   if * then
      ev (Active)
   else
      ev (Passive);
      motion_sensor;
```

```
door_control =
   ev (Close);
   motion_sensor;
   ev (Open);
   ev (Delay);
   door_control;
```

$\tau_{\texttt{motion\_sensor}} \;=\; \texttt{Unit} \to (\texttt{Unit} \;\&\; \Phi_{\texttt{motion\_sensor}})$
$\Phi_{\texttt{motion\_sensor}} \;=\; \underline{\textbf{Passive}}^{*} \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^{\omega}$

$\tau_{\texttt{door\_control}} = \texttt{Unit} \to (\texttt{Unit} \,\&\, \Phi_{\texttt{door\_control}})$
$\Phi_{\texttt{door\_control}} = (\underline{\textbf{Close}} \cdot (\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}})^\omega$

door_control

$$\dfrac{\dfrac{\dfrac{\Phi = \underline{\textbf{Close}}}{\Gamma' \vdash \texttt{ev (Close)} : \texttt{Unit} \,\&\, \underline{\textbf{Close}}}\,(\texttt{T-Event}) \quad \textbf{A} \quad \dfrac{\Phi = \underline{\textbf{Open}}}{\Gamma' \vdash \texttt{ev (Open)} : \texttt{Unit} \,\&\, \underline{\textbf{Open}}}\,(\texttt{T-Event}) \quad \dfrac{\Phi = \underline{\textbf{Delay}}}{\Gamma' \vdash \texttt{ev (Delay)} : \texttt{Unit} \,\&\, \underline{\textbf{Delay}}}\,(\texttt{T-Event}) \quad \textbf{B}}{\Gamma' \vdash \texttt{(ev (Close); motion\_sensor; ev (Open); ev (Delay); door\_control;)} : \texttt{Unit} \,\&\, (\underline{\textbf{Close}} \cdot \Phi_{\texttt{motion\_sensor}} \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}} \cdot \Phi_{\texttt{door\_control}})}\,(\texttt{T-If})}{\dots \textit{ (Effects Computation } \textbf{\textit{I}}\textit{) } \dots}$$
$$\dfrac{}{\Gamma \vdash \texttt{(door\_control = ...)} : \texttt{Unit} \to (\texttt{Unit} \,\&\, (\underline{\textbf{Close}} \cdot (\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}})^\omega)}\,(\texttt{T-Fun})$$

$\Gamma' = \Gamma,\ \texttt{door\_control} : \tau_{\texttt{door\_control}}$

A:
$$\dfrac{\texttt{sty}(\Gamma'(\texttt{motion\_sensor})) \in \to}{\Gamma' \vdash \texttt{motion\_sensor} : \texttt{Unit} \to (\texttt{Unit} \,\&\, \Phi_{\texttt{motion\_sensor}})}\,(\texttt{T-VaF})$$

B:
$$\dfrac{\texttt{sty}(\Gamma'(\texttt{door\_control})) \in \to}{\Gamma' \vdash \texttt{door\_control} : \texttt{Unit} \to (\texttt{Unit} \,\&\, \Phi_{\texttt{door\_control}})}\,(\texttt{T-VaF})$$

I:
$$\begin{aligned}
&\underline{\textbf{Close}} \cdot \Phi_{\texttt{motion_sensor}} \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}} \cdot \Phi_{\texttt{door_control}} \\
\equiv\ &\underline{\textbf{Close}} \cdot (\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}} \cdot ((\underline{\textbf{Close}} \cdot (\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}})^\omega) \\
\equiv\ &(\underline{\textbf{Close}} \cdot (\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}})^{\omega+1} \\
\equiv\ &(\underline{\textbf{Close}} \cdot (\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}})^\omega \\
\equiv\ &\Phi_{\texttt{door_control}}
\end{aligned}$$

III. Type checking trees for function temperature_control

```
device_control t =
   if t < 20 then ev (CloseBoth)
   else if 20 <= t < 30
         then ev (Fan)
   else if 30 <= t < 40
         then ev (Alternate)
   else if 40 <= t < 50
         then ev (AC)
   else ev (OpenBoth);
```

```
temperature_control =
   device_control (get_temp ());
   temperature_control;
```

$\tau_{\texttt{device\_control}} = \texttt{t} : \texttt{Int} \to (\texttt{Unit} \,\&\, \Phi_{\texttt{device\_control}})$
$\Phi_{\texttt{device\_control}} = (\texttt{t} < 20 \wedge \underline{\textbf{CloseBoth}}) \vee (20 \leq \texttt{t} < 30 \wedge \underline{\textbf{Fan}}) \vee (30 \leq \texttt{t} < 40 \wedge \underline{\textbf{Alternate}}) \vee (40 \leq \texttt{t} < 50 \wedge \underline{\textbf{AC}}) \vee (50 \leq \texttt{t} \wedge \underline{\textbf{OpenBoth}})$
$\tau_{\texttt{get\_temp}} = \texttt{Unit} \to \texttt{Int}$
$\tau_{\texttt{temperature\_control}} = \texttt{Unit} \to (\texttt{Unit} \,\&\, \Phi_{\texttt{temperature\_control}})$
$\Phi_{\texttt{temperature\_control}} = ((\texttt{t} < 20 \wedge \underline{\textbf{CloseBoth}}) \vee (20 \leq \texttt{t} < 30 \wedge \underline{\textbf{Fan}}) \vee (30 \leq \texttt{t} < 40 \wedge \underline{\textbf{Alternate}}) \vee (40 \leq \texttt{t} < 50 \wedge \underline{\textbf{AC}}) \vee (50 \leq \texttt{t} \wedge \underline{\textbf{OpenBoth}}))^\omega$

device_control

$$\dfrac{\dfrac{\dfrac{\Phi = \underline{\textbf{CloseBoth}}}{\Gamma' \vdash \texttt{ev (CloseBoth)} : \texttt{Unit} \;\&\; \underline{\textbf{CloseBoth}}}\,\text{(T-Event)} \quad (\underline{\textbf{Fan}}) \quad (\underline{\textbf{Alternate}}) \quad (\underline{\textbf{AC}}) \quad \dfrac{\Phi = \underline{\textbf{OpenBoth}}}{\Gamma' \vdash \texttt{ev (OpenBoth)} : \texttt{Unit} \;\&\; \underline{\textbf{OpenBoth}}}\,\text{(T-Event)}}{\Gamma' \vdash (\texttt{if t} < 20 \texttt{ then ev (CloseBoth) else if ... ... else ev (OpenBoth);}) : \texttt{Unit} \;\&\; (\Phi_1 \vee \Phi_2 \vee \Phi_3 \vee \Phi_4 \vee \Phi_5)}\,\text{(T-If)}}{\Gamma \vdash (\texttt{device\_control t = if ...}) : (\texttt{t} : \texttt{Int}) \to (\texttt{Unit} \;\&\; \Phi_{\texttt{device\_control}})}\,\text{(T-Fun)}$$

*... (Effects Computation **I**) ...*

$\Gamma' = \Gamma,\ \texttt{device\_control} : \tau_{\texttt{device\_control}}$
$\Phi_1 = (\texttt{t} < 20 \wedge \underline{\textbf{CloseBoth}})$
$\Phi_2 = (20 \leq \texttt{t} < 30 \wedge \underline{\textbf{Fan}})$
$\Phi_3 = (30 \leq \texttt{t} < 40 \wedge \underline{\textbf{Alternate}})$
$\Phi_4 = (40 \leq \texttt{t} < 50 \wedge \underline{\textbf{AC}})$
$\Phi_5 = (50 \leq \texttt{t} \wedge \underline{\textbf{OpenBoth}})$

I:

$$\begin{aligned}
&\underline{\textbf{Close}} \cdot \Phi_{\texttt{motion_sensor}} \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}} \cdot \Phi_{\texttt{door_control}} \\
\equiv\ & \underline{\textbf{Close}} \cdot (\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}} \cdot ((\underline{\textbf{Close}} \cdot (\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}})^\omega) \\
\equiv\ & (\underline{\textbf{Close}} \cdot (\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}})^{\omega+1} \\
\equiv\ & (\underline{\textbf{Close}} \cdot (\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}})^\omega \\
\equiv\ & \Phi_{\texttt{door_control}}
\end{aligned}$$

temperature_control

$$\dfrac{\dfrac{\dfrac{\texttt{sty}(\Gamma'(\texttt{temperature\_control})) \in \to}{\Gamma' \vdash \texttt{temperature\_control} : \texttt{Unit} \to (\texttt{Unit} \;\&\; \Phi_{\texttt{temperature\_control}})}\,\text{(T-VaF)} \quad \textbf{A}}{\Gamma' \vdash (\texttt{device\_control get\_temp; temperature\_control;}) : \texttt{Unit} \;\&\; (\Phi_{\texttt{device\_control}} \cdot \Phi_{\texttt{temperature\_control}})}\,\text{(T-Let)}}{\Gamma \vdash (\texttt{temperature\_control = ...}) : \texttt{Unit} \to (\texttt{Unit} \;\&\; \Phi_{\texttt{temperature\_control}})}\,\text{(T-Fun)}$$

*... (Effects Computation **II**) ...*

$\Gamma' = \Gamma,\ \texttt{temperature\_control} : \tau_{\texttt{temperature\_control}}$

A:

$$\dfrac{\dfrac{\texttt{sty}(\Gamma'(\texttt{device\_control})) \in \to}{\Gamma' \vdash \texttt{device\_control} : (\texttt{t} : \texttt{Int}) \to (\texttt{Unit} \;\&\; \Phi_{\texttt{device\_control}})}\,\text{(T-VaF)} \quad \dfrac{\dfrac{\texttt{sty}(\Gamma'(\texttt{get\_temp})) \in \to}{\Gamma' \vdash \texttt{get\_temp} : \texttt{Unit} \to \texttt{Int}}\,\text{(T-VaF)} \quad \Gamma' \vdash () : \texttt{Unit}}{\Gamma' \vdash \texttt{get\_temp ()} : \texttt{Int}}\,\text{(T-App)}}{\Gamma' \vdash \texttt{device\_control (get\_temp ())} : \texttt{Unit} \;\&\; (\Phi_{\texttt{device\_control}})}\,\text{(T-App)}$$

II:

$$\begin{aligned}
&(\Phi_{\texttt{device_control}} \cdot \Phi_{\texttt{temperature_control}}) \\
\equiv\ & ((\texttt{t} < 20 \wedge \underline{\textbf{CloseBoth}}) \vee (20 \leq \texttt{t} < 30 \wedge \underline{\textbf{Fan}}) \vee (30 \leq \texttt{t} < 40 \wedge \underline{\textbf{Alternate}}) \vee (40 \leq \texttt{t} < 50 \wedge \underline{\textbf{AC}}) \vee (50 \leq \texttt{t} \wedge \underline{\textbf{OpenBoth}})) \\
& \cdot((\texttt{t} < 20 \wedge \underline{\textbf{CloseBoth}}) \vee (20 \leq \texttt{t} < 30 \wedge \underline{\textbf{Fan}}) \vee (30 \leq \texttt{t} < 40 \wedge \underline{\textbf{Alternate}}) \vee (40 \leq \texttt{t} < 50 \wedge \underline{\textbf{AC}}) \vee (50 \leq \texttt{t} \wedge \underline{\textbf{OpenBoth}}))^\omega \\
\equiv\ & ((\texttt{t} < 20 \wedge \underline{\textbf{CloseBoth}}) \vee (20 \leq \texttt{t} < 30 \wedge \underline{\textbf{Fan}}) \vee (30 \leq \texttt{t} < 40 \wedge \underline{\textbf{Alternate}}) \vee (40 \leq \texttt{t} < 50 \wedge \underline{\textbf{AC}}) \vee (50 \leq \texttt{t} \wedge \underline{\textbf{OpenBoth}}))^{\omega+1} \\
\equiv\ & ((\texttt{t} < 20 \wedge \underline{\textbf{CloseBoth}}) \vee (20 \leq \texttt{t} < 30 \wedge \underline{\textbf{Fan}}) \vee (30 \leq \texttt{t} < 40 \wedge \underline{\textbf{Alternate}}) \vee (40 \leq \texttt{t} < 50 \wedge \underline{\textbf{AC}}) \vee (50 \leq \texttt{t} \wedge \underline{\textbf{OpenBoth}}))^\omega \\
\equiv\ & \Phi_{\texttt{temperature_control}}
\end{aligned}$$

IV. Type checking trees for function surveillance

```
camera_on = ev (10Mins)
permission t f =
   if * then return ev (Legal)
   else if t == 0 then camera_on ()
   else
      ev (Tick);
      permission (f t) f
```

```
surveillance t =
   motion_sensor;
   permission t (\x -> x - 1);
   ev (CameraOff);
   surveillance t;
```

$\tau_{\texttt{camera\_on}} = \texttt{Unit} \to (\texttt{Unit} \;\&\; \underline{\textbf{10Mins}})$
$\tau_{\texttt{permission}} = \texttt{t} : \texttt{Int} \to \texttt{f} : (\texttt{x} : \texttt{Int} \to \{\texttt{u} : \texttt{Int} \mid \texttt{u} = \texttt{x - 1}\}) \to (\texttt{Unit} \;\&\; \Phi_{\texttt{permission}})$
$\Phi_{\texttt{permission}} = (\underline{\textbf{Legal}} \vee ((\texttt{t}{<}0 \wedge \underline{\textbf{Tick}}^\omega) \vee (\texttt{t}{\geq}0 \wedge \underline{\textbf{Tick}}^{\texttt{t}} \cdot \underline{\textbf{10Mins}}))$
$\tau_{\texttt{surveillance}} = \texttt{t} : \texttt{Int} \to (\texttt{Unit} \;\&\; \Phi_{\texttt{surveillance}})$
$\Phi_{\texttt{surveillance}} = ((\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot (\underline{\textbf{Legal}} \vee ((\texttt{t}{<}0 \wedge \underline{\textbf{Tick}}^\omega) \vee (\texttt{t}{\geq}0 \wedge \underline{\textbf{Tick}}^{\texttt{t}} \cdot \underline{\textbf{10Mins}}))) \cdot \underline{\textbf{CameraOff}})^\omega$

permission

$$\dfrac{\dfrac{\Phi = \underline{\textbf{Legal}}}{\Gamma' \vdash \texttt{ev (Legal)} : \texttt{Unit} \,\&\, \underline{\textbf{Legal}}}\,(\texttt{T-Event}) \quad \dfrac{\dfrac{\texttt{sty}(\Gamma'(\texttt{camera\_on})) \in \rightarrow}{\Gamma' \vdash \texttt{camera\_on} : \texttt{Unit} \rightarrow (\texttt{Unit} \,\&\, \underline{\textbf{10Mins}})}\,(\texttt{T-VaF}) \quad \Gamma' \vdash () : \texttt{Unit}}{\Gamma' \vdash \texttt{camera\_on ()} : (\texttt{Unit} \,\&\, \underline{\textbf{10Mins}})}\,(\texttt{T-App}) \quad \textbf{A}}{\Gamma' \vdash (\texttt{if } * \texttt{ then return ev (Legal) else if t == 0 then camera\_on () else ev (Tick); permission (f t) f;}) : \texttt{Unit} \,\&\, (\Phi_1 \vee (\Phi_2 \vee \Phi_3))}\,(\texttt{T-If})$$

$$\dfrac{\textit{... (Effects Computation } \textbf{\textit{I}}\textit{) ...}}{\Gamma \vdash (\texttt{permission t f = if ... }) : \texttt{t} : \texttt{Int} \rightarrow \texttt{f} : (\texttt{x} : \texttt{Int} \rightarrow \{\texttt{u} : \texttt{Int} \mid \texttt{u} = \texttt{x - 1}\}) \rightarrow (\texttt{Unit} \,\&\, \Phi_{\texttt{permission}})}\,(\texttt{T-Fun})$$

$\Gamma' = \Gamma,\ \texttt{permission} : \tau_{\texttt{permission}},\ \texttt{t} : \texttt{Int},\ \texttt{f} : (\texttt{x} : \texttt{Int} \rightarrow \{\texttt{u} : \texttt{Int} \mid \texttt{u} = \texttt{x - 1}\})$
$\Phi_1 = \underline{\textbf{Legal}}$
$\Phi_2 = \texttt{t==0} \wedge \underline{\textbf{10Mins}}$
$\Phi_3 = \texttt{t}\neq\texttt{0} \wedge (\underline{\textbf{Legal}} \vee \underline{\textbf{Tick}} \cdot ((\texttt{t} \geq 1 \wedge \underline{\textbf{Tick}}^{\texttt{t}-1} \cdot \underline{\textbf{10Mins}}) \vee (\texttt{t} < 1 \wedge \underline{\textbf{Tick}}^{\omega})))$

A:
$$\dfrac{\dfrac{\Phi = \underline{\textbf{Tick}}}{\Gamma' \vdash \texttt{ev (Tick)} : \texttt{Unit} \,\&\, \underline{\textbf{Tick}}}\,(\texttt{T-Event}) \quad \dfrac{\dfrac{\texttt{sty}(\Gamma'(\texttt{permission})) \in \rightarrow}{\Gamma' \vdash \texttt{permission} : \tau_{\texttt{permission}}}\,(\texttt{T-VaF}) \quad \textbf{B} \quad \textbf{C}}{\Gamma' \vdash \texttt{permission (f t) f} : \texttt{Unit} \,\&\, \Phi^{\texttt{t}-1}_{\texttt{permission}}}\,(\texttt{T-App})}{\Gamma' \vdash (\texttt{ev (Tick); permission (f t) f;}) : \texttt{Unit} \,\&\, (\underline{\textbf{Tick}} \cdot \Phi^{\texttt{t}-1}_{\texttt{permission}})}\,(\texttt{T-Let})$$

$\Phi^{\texttt{t}-1}_{\texttt{permission}} = \underline{\textbf{Legal}} \vee (\texttt{t} \geq 1 \wedge \underline{\textbf{Tick}}^{\texttt{t}-1} \cdot \underline{\textbf{10Mins}}) \vee (\texttt{t} < 1 \wedge \underline{\textbf{Tick}}^{\omega})$

B:
$$\dfrac{\dfrac{\texttt{sty}(\Gamma'(\texttt{f})) \in \rightarrow}{\Gamma' \vdash \texttt{f} : (\texttt{x} : \texttt{Int} \rightarrow \{\texttt{u} : \texttt{Int} \mid \texttt{u} = \texttt{x - 1}\}) \rightarrow \texttt{Int}}\,(\texttt{T-VaF}) \quad \Gamma' \vdash \texttt{t} : \texttt{Int}}{\Gamma' \vdash \texttt{f t} : \{\texttt{u} : \texttt{Int} \mid \texttt{u} = \texttt{t - 1}\}}\,(\texttt{T-App})$$

C:
$$\dfrac{\texttt{sty}(\Gamma'(\texttt{f})) \in \rightarrow}{\Gamma' \vdash \texttt{f} : (\texttt{x} : \texttt{Int} \rightarrow \{\texttt{u} : \texttt{Int} \mid \texttt{u} = \texttt{x - 1}\}) \rightarrow \texttt{Int}}\,(\texttt{T-VaF})$$

I:
$$\begin{aligned}
&(\Phi_1 \vee (\Phi_2 \vee \Phi_3)) \\
\equiv\ &\underline{\textbf{Legal}} \vee (\texttt{t==0} \wedge \underline{\textbf{10Mins}}) \vee (\texttt{t}\neq\texttt{0} \wedge (\underline{\textbf{Legal}} \vee \underline{\textbf{Tick}} \cdot ((\texttt{t} \geq 1 \wedge \underline{\textbf{Tick}}^{\texttt{t}-1} \cdot \underline{\textbf{10Mins}}) \vee (\texttt{t} < 1 \wedge \underline{\textbf{Tick}}^{\omega})))) \\
\equiv\ &(\underline{\textbf{Legal}} \vee ((\texttt{t}<\texttt{0} \wedge \underline{\textbf{Tick}}^{\omega}) \vee (\texttt{t}\geq\texttt{0} \wedge \underline{\textbf{Tick}}^{\texttt{t}} \cdot \underline{\textbf{10Mins}})) \\
\equiv\ &\Phi_{\texttt{permission}}
\end{aligned}$$