Fuzzy Inference System

A Fuzzy Inference System (FIS) is a framework for modeling complex systems using fuzzy set theory, fuzzy if-then rules, and fuzzy reasoning. Unlike traditional binary logic, which operates with clear true/false values, fuzzy logic allows for degrees of truth, representing data with a range of values between 0 and 1. This approach is particularly useful for dealing with uncertainties and ambiguities in real-world scenarios.

Key Components of a Fuzzy Inference System

1. **Fuzzification**: Converts crisp input values into fuzzy sets. This is done using membership functions, which define how each point in the input space is mapped to a membership value between 0 and 1.
2. **Knowledge Base**: Contains the fuzzy rules and the membership functions. The fuzzy rules are typically in the form of "if-then" statements that describe the relationship between input and output variables.
3. **Inference Engine**: Applies the fuzzy rules to the fuzzified inputs to produce fuzzy outputs. This involves evaluating the degree to which each rule applies to the given inputs.
4. **Aggregation**: Combines the fuzzy outputs from all the rules into a single fuzzy set. This step ensures that all possible rules that apply to the inputs contribute to the final decision.
5. **Defuzzification**: Converts the aggregated fuzzy output back into a crisp value. This is necessary because the final output of the system needs to be a precise, actionable decision.

Steps in Fuzzy Inference Process

1. **Input Fuzzification**: Transform input variables into fuzzy sets using appropriate membership functions.
2. **Rule Evaluation**: Apply fuzzy logical operators (such as AND, OR, NOT) to evaluate the if-then rules based on the fuzzified inputs.
3. **Aggregation of Rule Outputs**: Combine the results of all rules to form a single fuzzy set.
4. **Output Defuzzification**: Convert the aggregated fuzzy set into a crisp output value using defuzzification methods like the centroid method, bisector method, etc.

Types of Fuzzy Inference Systems

1. **Mamdani FIS**: Uses fuzzy sets and linguistic variables for both the input and output, and is widely used due to its simplicity and interpretability. The output is also a fuzzy set which needs defuzzification.
2. **Sugeno FIS (Takagi-Sugeno-Kang)**: Uses fuzzy sets for the input but the output is a crisp value, which is usually a linear or constant function of the input variables. It simplifies the defuzzification process.

Example of a Fuzzy Inference System

Suppose you are designing a FIS to control the speed of a car based on the distance to the car ahead and the current speed.

1. **Fuzzification**:
   - Input 1: Distance (Close, Medium, Far)
   - Input 2: Speed (Slow, Moderate, Fast)
2. **Rule Base**:
   - Rule 1: If distance is Close and speed is Fast, then decelerate fast.
   - Rule 2: If distance is Medium and speed is Moderate, then maintain speed.
   - Rule 3: If distance is Far and speed is Slow, then accelerate.
3. **Inference**: Apply the input values to these rules to obtain the fuzzy output.
4. **Aggregation**: Combine the outputs of all rules to get a final fuzzy output.
5. **Defuzzification**: Convert the fuzzy output into a crisp value to determine the precise acceleration or deceleration needed.

Applications of FIS

- **Control Systems**: Industrial process control, home appliances (like washing machines and air conditioners), automotive systems.
- **Decision Making**: Financial forecasting, risk assessment, medical diagnosis.
- **Pattern Recognition**: Image processing, handwriting recognition, speech recognition.

Fuzzy Inference Systems provide a robust way to model complex systems where human reasoning and decision-making processes can be encoded in a machine-interpretable format.

Example of fuzzy inference system

Let's create a simple example of a Fuzzy Inference System (FIS) with two fuzzy sets. We'll use an example where we determine the level of comfort in a room based on two input variables: temperature and humidity.

Step-by-Step Example

1. Define Input Variables and Fuzzification

- Input 1: Temperature
  - Low: Defined by a membership function that could look like a triangular shape.
  - High: Defined by another triangular membership function.
- Input 2: Humidity
  - Low: Defined by a triangular membership function.
  - High: Defined by another triangular membership function.

For simplicity, let's assume the membership functions for Temperature (Temp) and Humidity (Hum) are defined as follows:

- Temperature:
  - Low: $\mu Low(x)=\max(0,\min((25-x)/10,1))$
  - High: $\mu High(x)=\max(0,\min((x-15)/10,1))$
- Humidity:
  - Low: $\mu Low(y)=\max(0,\min((60-y)/20,1))$
  - High: $\mu High(y)=\max(0,\min((y-40)/20,1))$

2. Define Output Variable and Fuzzification

- Output: Comfort Level
  - Uncomfortable: Defined by a triangular membership function.
  - Comfortable: Defined by another triangular membership function.

Assume the membership functions for Comfort (Comfort) are defined as:

- Comfort:
  - Uncomfortable: $\mu Uncomfortable(z)=\max(0,\min((70-z)/20,1))$
  - Comfortable: $\mu Comfortable(z)=\max(0,\min((z-50)/20,1))$

3. Define the Rule Base

We create fuzzy rules to determine the comfort level based on temperature and humidity.

- Rule 1: If Temperature is Low and Humidity is Low, then Comfort is Comfortable.
- Rule 2: If Temperature is High and Humidity is High, then Comfort is Uncomfortable.
- Rule 3: If Temperature is Low and Humidity is High, then Comfort is Uncomfortable.
- Rule 4: If Temperature is High and Humidity is Low, then Comfort is Uncomfortable.

4. Fuzzification of Inputs

Let's take an example with the following inputs:

- Temperature = 20°C
- Humidity = 50%

Calculate the membership values for the inputs:

- μLow(20)=max(0,min((25−20)/10,1))=0.5
- μHigh(20)=max(0,min((20−15)/10,1))=0.5
- μLow(50)=max(0,min((60−50)/20,1))=0.5
- μHigh(50)=max(0,min((50−40)/20,1))=0.5

5. Apply the Fuzzy Rules

Evaluate each rule using the membership values:

- **Rule 1**: Min($\mu_{Low}(20)$, $\mu_{Low}(50)$) = Min(0.5, 0.5) = 0.5 for Comfort (Comfortable).
- **Rule 2**: Min($\mu_{High}(20)$, $\mu_{High}(50)$) = Min(0.5, 0.5) = 0.5 for Comfort (Uncomfortable).
- **Rule 3**: Min($\mu_{Low}(20)$, $\mu_{High}(50)$) = Min(0.5, 0.5) = 0.5 for Comfort (Uncomfortable).
- **Rule 4**: Min($\mu_{High}(20)$, $\mu_{Low}(50)$) = Min(0.5, 0.5) = 0.5 for Comfort (Uncomfortable).

6. Aggregation of Rule Outputs

Combine the results of the rules to get a single fuzzy output set for each possible output value:

- For Comfortable: μComfortable(z)=max(0.5)

- For Uncomfortable: μUncomfortable(z)=max(0.5,0.5,0.5)=0.5

7. Defuzzification

Convert the fuzzy output into a crisp value. One common method is the centroid method, which calculates the center of gravity of the aggregated fuzzy set.

Assume the membership functions for comfort are centered around certain values, for example:

- Comfortable: Center around 70
- Uncomfortable: Center around 50

Calculate the crisp value z using the weighted average:

$$z = \frac{0.5\times70+0.5\times50}{0.5+0.5} = \frac{35+25}{1} = 60$$

So, the comfort level in the room is determined to be 60 on a scale of 0 to 100, where higher values indicate more comfort.

Fuzzification and Defuzzification Methods

Fuzzification and defuzzification are key processes in a Fuzzy Inference System (FIS). They transform crisp values into fuzzy sets and vice versa, enabling the system to handle uncertainties and imprecisions. Here’s a detailed look at various methods used in fuzzification and defuzzification.

Fuzzification Methods

Fuzzification is the process of converting crisp input values into fuzzy sets by determining their degree of membership in each relevant fuzzy set. The main methods used for fuzzification are:

1. **Membership Functions**:
    - **Triangular Membership Function**: A Triangular Membership Function (TMF) is one of the simplest and most commonly used membership functions in fuzzy logic. It is defined by three parameters and has a triangular shape. Defined by three parameters a, b, and c where a≤b≤c

The function increases linearly from a to b and decreases linearly from b to c.

$$\mu(x;a,b,c) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x < b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0, & x > c \end{cases}$$

a: The start point of the triangle where the membership value begins to rise from 0.

b: The peak point of the triangle where the membership value is 1.

c: The end point of the triangle where the membership value decreases back to 0.

Let's consider an example where we define a triangular membership function for "temperature":

- **a** (start point): 10°C
- **b** (peak point): 20°C
- **c** (end point): 30°C

The membership function μtemperature(x;10,20,30)

$$\mu_{\text{temperature}}(x;10,20,30) = \begin{cases} 0 & \text{if } x \leq 10 \\ \frac{x-10}{20-10} & \text{if } 10 < x \leq 20 \\ \frac{30-x}{30-20} & \text{if } 20 < x \leq 30 \\ 0 & \text{if } x > 30 \end{cases}$$

Example Calculation

For specific temperature values:

- At $x = 10$: $\mu(10) = 0$
- At $x = 15$: $\mu(15) = \frac{15-10}{20-10} = \frac{5}{10} = 0.5$
- At $x = 20$: $\mu(20) = 1$
- At $x = 25$: $\mu(25) = \frac{30-25}{30-20} = \frac{5}{10} = 0.5$
- At $x = 30$: $\mu(30) = 0$

Trapezoidal Membership Function: Defined by four parameters a, b, c, and d where a≤b≤c≤d. The function is a trapezoid shape with a flat top between b and c.

$$\mu(x; a, b, c, d) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x < b \\ 1, & b \leq x \leq c \\ \frac{d-x}{d-c}, & c \leq x \leq d \\ 0, & x > d \end{cases}$$

Gaussian Membership Function: Defined by two parameters c (the mean) and σ (the standard deviation).

$$\mu(x; c, \sigma) = e^{-\frac{(x-c)^2}{2\sigma^2}}$$

Bell-Shaped Membership Function: Defined by three parameters a, b, and c.

$$\mu(x; a, b, c) = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}}$$

Defuzzification Methods

Defuzzification is the process of converting fuzzy sets resulting from the inference process into a single crisp output value. The main methods used for defuzzification are:

1. **Centroid Method (Center of Gravity)**:
    - This method finds the center of the area under the curve. It is the most commonly used defuzzification technique.

$$z = \frac{\int_{\text{all } x} x \cdot \mu(x)\,dx}{\int_{\text{all } x} \mu(x)\,dx}$$

where:

- z is the variable on the horizontal axis (the output variable).
- μ(z) is the membership function of the fuzzy set.
- a and b are the bounds of the universe of discourse for z.

Bisector Method:

- The value z that divides the area under the curve into two equal parts.

$$\int_a^z \mu(x)\,dx = \int_z^b \mu(x)\,dx$$

Mean of Maximum (MOM):

- This method finds the average of the maximum values of the output fuzzy set.

$$z = \frac{\sum_{x \in \text{MOM}} x}{|\text{MOM}|}$$

- Where MOM is the set of values x at which μ(x) reaches its maximum.

Smallest of Maximum (SOM):

- This method takes the smallest value at which the output fuzzy set reaches its maximum membership value.

Largest of Maximum (LOM):

- This method takes the largest value at which the output fuzzy set reaches its maximum membership value.

Example Illustration

Consider an example where we have an input temperature value of 20°C and we want to fuzzify it using triangular membership functions and then defuzzify it using the centroid method.

Fuzzification

- **Membership Functions for Temperature**:
  - Low: $\mu Low(x)=\max(0,\min((25-x)/10,1))$
  - High: $\mu High(x)=\max(0,\min((x-15)/10,1))$

For a temperature of 20°C:

- $\mu Low(20)=\max(0,\min((25-20)/10,1))=0.5$
- $\mu High(20)=\max(0,\min((20-15)/10,1))=0.5$

Defuzzification

Assume we have an aggregated output fuzzy set for comfort with membership values:

- $\mu Comfortable(z)=0.5$ for $z=70$
- $\mu Uncomfortable(z)=0.5$for $z=50$

Using the centroid method:

$$z=\frac{(70\times 0.5)+(50\times 0.5)}{0.5+0.5}=\frac{35+25}{1}=60$$

Thus, the defuzzified output is 60, indicating the comfort level on a scale of 0 to 100.

Fuzzy Based Expert System

A Fuzzy-Based Expert System (FES) is a type of expert system that uses fuzzy logic instead of classical Boolean logic to reason about data and make decisions. This is particularly useful when dealing with uncertain, imprecise, or vague information. Here's a comprehensive example to illustrate the concepts and operations of a Fuzzy-Based Expert System.

Example: Medical Diagnosis System

Objective: Diagnose the likelihood of a patient having the flu based on symptoms like fever and cough intensity.

Step-by-Step Process

1. **Define Input Variables and Membership Functions**:

- **Fever (°C)**:
  - Low: $\mu_{Low}(x) = \max(0, \min((37 - x)/2, 1))$
  - Medium: $\mu_{Medium}(x) = \max(0, \min((x - 36)/1, (38 - x)/1))$
  - High: $\mu_{High}(x) = \max(0, \min((x - 37)/2, 1))$
- **Cough Intensity**:
  - Mild: $\mu_{Mild}(y) = \max(0, \min((5 - y)/5, 1))$
  - Moderate: $\mu_{Moderate}(y) = \max(0, \min((y - 3)/2, (7 - y)/2))$
  - Severe: $\mu_{Severe}(y) = \max(0, \min((y - 5)/5, 1))$

2. Define Output Variable and Membership Functions:

- **Flu Likelihood**:
  - Low: $\mu_{Low}(z) = \max(0, \min((50 - z)/50, 1))$
  - Medium: $\mu_{Medium}(z) = \max(0, \min((z - 30)/20, (70 - z)/20))$
  - High: $\mu_{High}(z) = \max(0, \min((z - 50)/50, 1))$

3.Define Rule Base:

- Rule 1: If Fever is High and Cough Intensity is Severe, then Flu Likelihood is High.
- Rule 2: If Fever is Medium and Cough Intensity is Moderate, then Flu Likelihood is Medium.
- Rule 3: If Fever is Low and Cough Intensity is Mild, then Flu Likelihood is Low.
- Rule 4: If Fever is High and Cough Intensity is Mild, then Flu Likelihood is Medium.
- Rule 5: If Fever is Low and Cough Intensity is Severe, then Flu Likelihood is Medium.

4.Fuzzification:

- Suppose Fever = 38°C and Cough Intensity = 6.

- $\mu_{Low}(38) = \max(0, \min((37 - 38)/2, 1)) = 0$
- $\mu_{Medium}(38) = \max(0, \min((38 - 36)/1, (38 - 38)/1)) = 0$
- $\mu_{High}(38) = \max(0, \min((38 - 37)/2, 1)) = 0.5$
- $\mu_{Mild}(6) = \max(0, \min((5 - 6)/5, 1)) = 0$
- $\mu_{Moderate}(6) = \max(0, \min((6 - 3)/2, (7 - 6)/2)) = 0.5$
- $\mu_{Severe}(6) = \max(0, \min((6 - 5)/5, 1)) = 0.2$

5. Rule Evaluation:

- **Rule 1**: If Fever is High (0.5) and Cough Intensity is Severe (0.2), then Flu Likelihood is High.
  - $\text{Min}(0.5, 0.2) = 0.2$
- **Rule 2**: If Fever is Medium (0) and Cough Intensity is Moderate (0.5), then Flu Likelihood is Medium.
  - $\text{Min}(0, 0.5) = 0$
- **Rule 3**: If Fever is Low (0) and Cough Intensity is Mild (0), then Flu Likelihood is Low.
  - $\text{Min}(0, 0) = 0$
- **Rule 4**: If Fever is High (0.5) and Cough Intensity is Mild (0), then Flu Likelihood is Medium.
  - $\text{Min}(0.5, 0) = 0$
- **Rule 5**: If Fever is Low (0) and Cough Intensity is Severe (0.2), then Flu Likelihood is Medium.
  - $\text{Min}(0, 0.2) = 0$

6. Aggregation:

- Combine the results of the rules to get a single fuzzy output set for Flu Likelihood.

- $\mu_{High}(z) = 0.2$
- $\mu_{Medium}(z) = 0$
- $\mu_{Low}(z) = 0$

Defuzzification:

- Using the centroid method:
  - Suppose the centers of the membership functions are at 30 for Low, 50 for Medium, and 70 for High.
  - The aggregated fuzzy set for Flu Likelihood has a maximum membership value of 0.2 at High.
  - Using the weighted average of maximum method:

$$z = \frac{30 \times 0 + 50 \times 0 + 70 \times 0.2}{0 + 0 + 0.2} = \frac{14}{0.2} = 70$$

Thus, the system indicates a high likelihood (70 on a scale of 0 to 100) that the patient has the flu based on the given symptoms.