Fuzzy Logic

The term fuzzy refers to things that are not clear or are vague. In the real world many times we encounter a situation when we can't determine whether the state is true or false, their fuzzy logic provides very valuable flexibility for reasoning. In this way, we can consider the inaccuracies and uncertainties of any situation.

Fuzzy Logic is a form of many-valued logic in which the truth values of variables may be any real number between 0 and 1, instead of just the traditional values of true or false. It is used to deal with imprecise or uncertain information and is a mathematical method for representing vagueness and uncertainty in decision-making.

Fuzzy Logic is based on the idea that in many cases, the concept of true or false is too restrictive, and that there are many shades of gray in between. It allows for partial truths, where a statement can be partially true or false, rather than fully true or false.

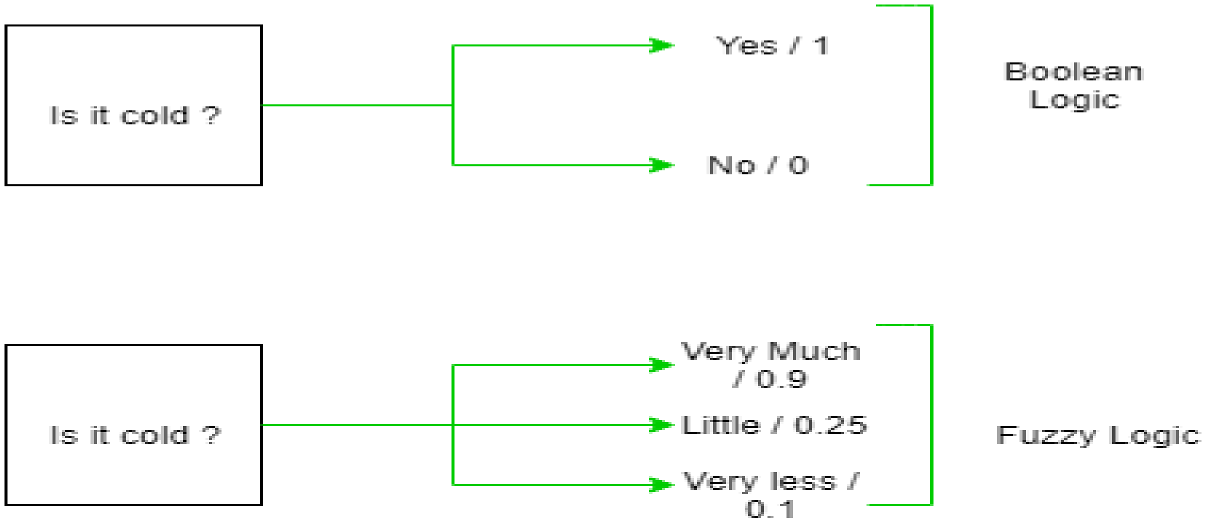
Fuzzy Logic is used in a wide range of applications, such as control systems, image processing, natural language processing, medical diagnosis, and artificial intelligence.

The fundamental concept of Fuzzy Logic is the membership function, which defines the degree of membership of an input value to a certain set or category. The membership function is a mapping from an input value to a membership degree between 0 and 1, where 0 represents non-membership and 1 represents full membership.

Fuzzy Logic is implemented using Fuzzy Rules, which are if-then statements that express the relationship between input variables and output variables in a fuzzy way. The output of a Fuzzy Logic system is a fuzzy set, which is a set of membership degrees for each possible output value.

In summary, Fuzzy Logic is a mathematical method for representing vagueness and uncertainty in decision-making, it allows for partial truths, and it is used in a wide range of applications. It is based on the concept of membership function and the implementation is done using Fuzzy rules.

In the boolean system truth value, 1.0 represents the absolute truth value and 0.0 represents the absolute false value. But in the fuzzy system, there is no logic for the absolute truth and absolute false value. But in fuzzy logic, there is an intermediate value to present which is partially true and partially false.



Crisp Value

A **crisp value** is a single, exact numerical value. In classical logic and traditional set theory, values are crisp because they do not have any degree of uncertainty or fuzziness. They are either fully included in a set or not included at all.

Example of a Crisp Value

- A temperature reading of exactly 25°C is a crisp value.
- A person's age of 30 years is a crisp value.

Crisp Set

A **crisp set** (or classical set) is a collection of distinct, well-defined elements where each element either belongs to the set or does not. There is no ambiguity or partial membership. The membership of an element in a crisp set is binary: it is either 1 (if the element is in the set) or 0 (if the element is not in the set).

Example of a Crisp Set

Consider the set of even numbers less than 10:

A={2,4,6,8}In this set:

- 2 is in the set A, so its membership value is 1.
- 3 is not in the set A, so its membership value is 0.

Comparison with Fuzzy Sets

In contrast to crisp sets, **fuzzy sets** allow for partial membership. Each element in a fuzzy set has a membership value (or degree of membership) between 0 and 1, inclusive. This value indicates the extent to which the element belongs to the set.

Example of a Fuzzy Set

Consider a fuzzy set B representing the concept of "warm temperatures":

B={(20°C,0.2),(25°C,0.5),(30°C,0.8),(35°C,1.0)} Here:

- 20°C has a membership value of 0.2 in the fuzzy set B
- 25°C has a membership value of 0.5 in the fuzzy set B.

Membership:

- **Crisp Set:** An element's membership is binary (either 0 or 1).
- **Fuzzy Set:** An element's membership is gradual and can take any value between 0 and 1.

Key Terminology in Fuzzy Logic

Fuzzy Set: A set without a crisp, clearly defined boundary. It can contain elements with only a partial degree of membership. For example, in a fuzzy set representing "tall people," individuals can belong to the set with varying degrees of membership depending on their height.

Membership Function: A function that defines the degree to which a given element belongs to a fuzzy set. It maps each element to a value between 0 and 1. For example, the membership function for "tall" might assign a height of 180 cm and a membership value of 0.8.

Linguistic Variable: A variable whose values are words or sentences rather than numerical values. For instance, "temperature" can be a linguistic variable with terms such as "cold," "warm," and "hot."

Fuzzy Rule: A conditional statement in the form "If A, then B," where A and B are linguistic values defined by fuzzy sets. For example, "If the temperature is high, then the fan speed should be fast."

Fuzzification: The process of converting a crisp input value into a fuzzy value based on membership functions. For example, converting a precise temperature reading into fuzzy terms like "cold," "warm," or "hot."

Defuzzification: The process of converting a fuzzy output back into a crisp value. Common defuzzification methods include the centroid method, which computes the center of gravity of the fuzzy set, and the maximum method, which selects the output with the highest membership value.

Inference System: The component of a fuzzy logic system that applies fuzzy rules to the fuzzified inputs to produce fuzzy outputs. The inference mechanism often involves techniques like the Mamdani or Sugeno methods.

Fuzzy Operator (AND, OR, NOT): Operations that combine fuzzy sets. The AND operator typically takes the minimum value of the membership functions, the OR operator takes the maximum value, and the NOT operator calculates the complement (1 minus the membership value).

Fuzzy Control System: A control system based on fuzzy logic, used to handle complex processes where classical control methods are difficult to apply. Common applications include temperature control, speed control, and decision-making systems.

Example to Illustrate Fuzzy Logic

Consider a temperature control system:

- **Linguistic Variables**: Temperature (input) and Fan Speed (output).
- **Fuzzy Sets**:
    - Temperature: "Low," "Medium," "High."
    - Fan Speed: "Slow," "Medium," "Fast."
- **Membership Functions**:
    - Low temperature might have a membership function that assigns a high degree of membership to temperatures below 15°C.
    - High temperature might have a membership function that assigns a high degree of membership to temperatures above 25°C.
- **Fuzzy Rules**:
    - If the temperature is low, then the fan speed is slow.
    - If the temperature is medium, then the fan speed is medium.
    - If the temperature is high, then the fan speed is fast.
- **Fuzzification**: A measured temperature of 20°C might be fuzzified into "Low" with a membership value of 0.2 and "Medium" with a membership value of 0.8.
- **Inference**: Using fuzzy rules to determine the fuzzy output.
- **Defuzzification**: Converting the fuzzy output (e.g., fan speed) into a crisp value to set the actual fan speed.

Set Theoretic Fuzzy Operations

Set theoretic operations in fuzzy logic extend the classical set operations to handle fuzzy sets, which are characterized by membership functions that assign a degree of membership to each element in the set. Here are the primary set theoretic operations for fuzzy sets:

1. Fuzzy Union (OR)
2. Fuzzy Intersection (AND)
3. Fuzzy Complement (NOT)
4. Fuzzy Difference
5. Fuzzy Containment (Subset)

1. Fuzzy Union (OR)

The fuzzy union of two fuzzy sets A and B is a fuzzy set C where the membership function of C at any element x is the maximum of the membership functions of A and B at x.

$\mu C(x)=\max(\mu A(x),\mu B(x))$

2. Fuzzy Intersection (AND)

The fuzzy intersection of two fuzzy sets A and B is a fuzzy set C where the membership function of C at any element x is the minimum of the membership functions of A and B at x.

$\mu C(x)=\min(\mu A(x),\mu B(x))$

3. Fuzzy Complement (NOT)

The fuzzy complement of a fuzzy set A is a fuzzy set A′ where the membership function of A′ at any element x is one minus the membership function of A at x.

$\mu A'(x)=1-\mu A(x)$

4. Fuzzy Difference

The fuzzy difference (or fuzzy relative complement) of two fuzzy sets A and B is a fuzzy set C where the membership function of C at any element x is given by the minimum of the membership function of A and one minus the membership function of B.

$\mu C(x)=\min(\mu A(x),1-\mu B(x))$

5. Fuzzy Containment (Subset)

A fuzzy set A is a subset of a fuzzy set B if and only if the membership function of A at any element x is less than or equal to the membership function of B at x.

$A \subseteq B \Leftrightarrow \mu A(x) \leq \mu B(x)\ \forall\ x$

Example Illustrations

Consider fuzzy sets A and B defined over a universe of discourse X with the following membership functions:

- $A=\{(x1,0.3),(x2,0.7),(x3,0.9)\}$
- $B=\{(x1,0.5),(x2,0.4),(x3,0.8)\}$

Fuzzy Union

$\mu A \cup B(x)=\max(\mu A(x),\mu B(x))$, $\mu A \cup B(x1)=\max(0.3,0.5)=0.5$ $\mu A \cup B(x2)=\max(0.7,0.4)=0.7$ $\mu A \cup B(x3)=\max(0.9,0.8)=0.9$ Thus, $A \cup B=\{(x1,0.5),(x2,0.7),(x3,0.9)\}$

Fuzzy Relations

Fuzzy relation defines the mapping of variables from one fuzzy set to another. Like crisp relations, we can also define the relation over fuzzy sets.

Let A be a fuzzy set on universe X and B be a fuzzy set on universe Y, then the Cartesian product between fuzzy sets A and B will result in a fuzzy relation R which is contained with the full Cartesian product space or it is a subset of the cartesian product of fuzzy subsets. Formally, we can define fuzzy relation as,

$$\underline{R} = \underline{A} \times \underline{B}$$

and

$$\underline{R} \subset (X \times Y)$$

where the relation $\underline{R}$ has a membership function,

$$\mu_{\underline{R}}(x, y) = \mu_{\underline{A} \times \underline{B}}(x, y) = \min(\mu_{\underline{A}}(x), \mu_{\underline{B}}(y))$$

Let $\underline{A} = \{a_1, a_2, \ldots, a_n\}$ and $\underline{B} = \{b_1, b_2, .., b_m\}$, then the fuzzy relation between $\underline{A}$ and $\underline{B}$ is described by the fuzzy relation matrix as,

$$\begin{bmatrix} \mu_{R(a_1,b_1)} & \mu_{R(a_1,b_2)} & \cdot & \cdot & \mu_{R(a_1,b_m)} \\ \mu_{R(a_2,b_1)} & \mu_{R(a_2,b_2)} & \cdot & \cdot & \mu_{R(a_2,b_m)} \\ \cdot & \cdot & & \cdot\ \cdot\ \cdot & \\ \cdot & \cdot & & \cdot\ \cdot\ \cdot & \\ \mu_{R(a_n,b_1)} & \mu_{R(a_n,b_2)} & \cdot & \cdot & \mu_{R(a_n,b_m)} \end{bmatrix}$$

Fuzzy relation matrix

We can also consider fuzzy relation as a mapping from the cartesian space (X, Y) to the interval [0, 1]. The strength of this mapping is represented by the membership function of the relation for every tuple $\mu_{\underline{R}(x,\ y)}$

Example:

Given $\underline{A}$ = { (a1, 0.2), (a2, 0.7), (a3, 0.4) } and $\underline{B}$ = { (b1, 0.5), (b2, 0.6)}, find the relation over $\underline{A}$ x$\underline{B}$

Operations on Fuzzy Relations

1. **Max-Min Composition**
2. **Max-Product Composition**
3. **Projection**

Operations on fuzzy relations

For our discussion, we will be using the following two relation matrices:

Union:

$$\bar{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 0.8 & 0.1 & 0.1 & 0.7 \\ 0.0 & 0.8 & 0.0 & 0.0 \\ 0.9 & 1.0 & 0.7 & 0.8 \end{bmatrix} \end{array}$$

$$\bar{S} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 0.4 & 0.0 & 0.9 & 0.6 \\ 0.9 & 0.4 & 0.5 & 0.7 \\ 0.3 & 0.0 & 0.8 & 0.5 \end{bmatrix} \end{array}$$

Union:

R ∪ S = { (a, b), μA ∪ B(a, b) }

μR ∪ S(a, b) = max(μR(a, b), μS(a, b))

μR ∪ S(x1, y1) = max(μR(x1, y1), μS(x1, y1))

= max(0.8, 0.4) = 0.8

μR ∪ S(x1, y2) = max(μR(x1, y2), μS(x1, y2))

= max(0.1, 0.0) = 0.1

μR ∪ S(x1, y3) = max(μR(x1, y3), μS(x1, y3))

= max(0.1, 0.9) = 0.9

μR ∪ S(x1, y4) = max(μR(x1, y4), μS(x1, y4))

max(0.7, 0.6) = 0.7

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μR ∪ S(x3, y4) = max(μR(x3, y4), μS(x3, y4))

= max(0.8, 0.5) = 0.8

Thus, the final matrix for union operation would be,

$$\bar{R} \cup \bar{S} = \begin{array}{c c} & \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.8 & 0.1 & 0.9 & 0.7 \\ 0.9 & 0.8 & 0.5 & 0.7 \\ 0.9 & 1.0 & 0.8 & 0.8 \end{bmatrix} \end{array}$$

Intersection:

R ∩ S = { (a, b), μA ∩ B(a, b) }

μR ∩ S(a, b) = min(μR(a, b), μS(a, b))

μR ∩ S(x1, y1) = min(μR(x1, y1), μS(x1, y1))

= min(0.8, 0.4) = 0.4

μR ∩ S(x1, y2) = min(μR(x1, y2), μS(x1, y2))

= max(0.1, 0.0) = 0.0

μR ∩ S(x1, y3) = min(μR(x1, y3), μS(x1, y3))

= max(0.1, 0.9) = 0.1

μR ∩ S(x1, y4) = min(μR(x1, y4), μS(x1, y4))

μR ∩ S(x3, y4) = min(μR(x3, y4), μS(x3, y4))

= max(0.8, 0.5) = 0.5

$$\bar{R} \cap \bar{S} = \begin{array}{c c} & \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.4 & 0.0 & 0.1 & 0.6 \\ 0.0 & 0.4 & 0.0 & 0.0 \\ 0.3 & 0.0 & 0.7 & 0.5 \end{bmatrix} \end{array}$$

Complement:

The complement of relation R would be,

$$\bar{R}^c = \begin{array}{c c} & \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.2 & 0.9 & 0.9 & 0.3 \\ 1.0 & 0.2 & 1.0 & 1.0 \\ 0.1 & 0.0 & 0.3 & 0.2 \end{bmatrix} \end{array}$$

Projection:

The projection of R on X :

∏X(x) = sup(R(x, y) | y ∈ Y)

The projection of R on Y :

∏Y(y) = sup(R(x, y) | x ∈ X)

sup: Supremum of the set

$$\bar{R} = \begin{array}{c c} & \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.8 & 0.1 & 0.1 & 0.7 \\ 0.0 & 0.8 & 0.0 & 0.0 \\ 0.9 & 1.0 & 0.7 & 0.8 \end{bmatrix} \end{array}$$

The projection of R on X :

∏X(x1) = 0.8

∏X(x2) = 0.8

∏X(x3) = 1.0

The projection of R on Y :

∏Y(y1) = 0.9

∏Y(y2) = 1.0

∏Y(y3) = 0.7

∏Y(y4) = 0.8

1. Max-Min Composition

Given two fuzzy relations $R \subseteq X \times Y$ and $S \subseteq Y \times Z$, the max-min composition $T \subseteq X \times Z$ is defined as:

$$\mu_T(x, z) = \max_{y \in Y} \min(\mu_R(x, y), \mu_S(y, z))$$

Max-Min Composition of Fuzzy Relations

Example

Consider the following fuzzy relations represented as matrices:

Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2, y_3\}$, and $Z = \{z_1, z_2\}$.

The fuzzy relation $R \subseteq X \times Y$:

$$R = \begin{bmatrix} 0.2 & 0.5 & 0.3 \\ 0.8 & 0.6 & 0.4 \end{bmatrix}$$

The fuzzy relation $S \subseteq Y \times Z$:

$$S = \begin{bmatrix} 0.3 & 0.7 \\ 0.6 & 0.4 \\ 0.5 & 0.9 \end{bmatrix}$$

We want to find the fuzzy relation $T \subseteq X \times Z$ using the Max-Min Composition.

The element $\mu_T(x_1, z_1)$ is calculated as:

$$\mu_T(x_1, z_1) = \max(\min(0.2, 0.3), \min(0.5, 0.6), \min(0.3, 0.5))$$

$$\mu_T(x_1, z_1) = \max(0.2, 0.5, 0.3) = 0.5$$

The element $\mu_T(x_1, z_2)$ is calculated as:

$$\mu_T(x_1, z_2) = \max(\min(0.2, 0.7), \min(0.5, 0.4), \min(0.3, 0.9))$$

$$\mu_T(x_1, z_2) = \max(0.2, 0.4, 0.3) = 0.4$$

The element $\mu_T(x_2, z_1)$ is calculated as:

$$\mu_T(x_2, z_1) = \max(\min(0.8, 0.3), \min(0.6, 0.6), \min(0.4, 0.5))$$

$$\mu_T(x_2, z_1) = \max(0.3, 0.6, 0.4) = 0.6$$

$$\mu_T(x_2, z_1) = \max(0.3, 0.6, 0.4) = 0.6$$

The element $\mu_T(x_2, z_2)$ is calculated as:

$$\mu_T(x_2, z_2) = \max(\min(0.8, 0.7), \min(0.6, 0.4), \min(0.4, 0.9))$$

$$\mu_T(x_2, z_2) = \max(0.7, 0.4, 0.4) = 0.7$$

Thus, the resulting fuzzy relation T is:

$$T = \begin{bmatrix} 0.5 & 0.4 \\ 0.6 & 0.7 \end{bmatrix}$$

Max-Product Composition of Fuzzy Relations

Given two fuzzy relations $R \subseteq X \times Y$ and $S \subseteq Y \times Z$, the Max-Product Composition $T \subseteq X \times Z$ is defined as:

$\mu_T(x, z) = \max_{y \in Y}(\mu_R(x, y) \cdot \mu_S(y, z))$

Example

Consider the following fuzzy relations represented as matrices:

Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2, y_3\}$, and $Z = \{z_1, z_2\}$.

The fuzzy relation $R \subseteq X \times Y$:

$$R = \begin{bmatrix} 0.2 & 0.5 & 0.3 \\ 0.8 & 0.6 & 0.4 \end{bmatrix}$$

The fuzzy relation $S \subseteq Y \times Z$:

$$S = \begin{bmatrix} 0.3 & 0.7 \\ 0.6 & 0.4 \\ 0.5 & 0.9 \end{bmatrix}$$

We want to find the fuzzy relation $T \subseteq X \times Z$ using the Max-Product Composition.

The element $\mu_T(x_1, z_1)$ is calculated as:

$$\mu_T(x_1, z_1) = \max(\mu_R(x_1, y_1) \cdot \mu_S(y_1, z_1), \mu_R(x_1, y_2) \cdot \mu_S(y_2, z_1), \mu_R(x_1, y_3) \cdot \mu_S(y_3, z_1))$$

$$\mu_T(x_1, z_1) = \max(0.2 \cdot 0.3, 0.5 \cdot 0.6, 0.3 \cdot 0.5) = \max(0.06, 0.3, 0.15) = 0.3$$

The element $\mu_T(x_1, z_2)$ is calculated as:

$$\mu_T(x_1, z_2) = \max(\mu_R(x_1, y_1) \cdot \mu_S(y_1, z_2), \mu_R(x_1, y_2) \cdot \mu_S(y_2, z_2), \mu_R(x_1, y_3) \cdot \mu_S(y_3, z_2))$$

$$\mu_T(x_1, z_2) = \max(0.2 \cdot 0.7, 0.5 \cdot 0.4, 0.3 \cdot 0.9) = \max(0.14, 0.2, 0.27) = 0.27$$

The element $\mu_T(x_2, z_1)$ is calculated as:

$$\mu_T(x_2, z_1) = \max(\mu_R(x_2, y_1) \cdot \mu_S(y_1, z_1), \mu_R(x_2, y_2) \cdot \mu_S(y_2, z_1), \mu_R(x_2, y_3) \cdot \mu_S(y_3, z_1))$$

$$\mu_T(x_2, z_1) = \max(0.8 \cdot 0.3, 0.6 \cdot 0.6, 0.4 \cdot 0.5) = \max(0.24, 0.36, 0.2) = 0.36$$

The element $\mu_T(x_2, z_2)$ is calculated as:

$$\mu_T(x_2, z_2) = \max(\mu_R(x_2, y_1) \cdot \mu_S(y_1, z_2), \mu_R(x_2, y_2) \cdot \mu_S(y_2, z_2), \mu_R(x_2, y_3) \cdot \mu_S(y_3, z_2))$$

$$\mu_T(x_2, z_2) = \max(0.8 \cdot 0.7, 0.6 \cdot 0.4, 0.4 \cdot 0.9) = \max(0.56, 0.24, 0.36) = 0.56$$

Thus, the resulting fuzzy relation T is:

$$T = \begin{bmatrix} 0.3 & 0.27 \\ 0.36 & 0.56 \end{bmatrix}$$

Fuzzy Rule and Fuzzy Reasoning

Fuzzy logic extends classical logic to handle the concept of partial truth, which can range between completely true and completely false. This is particularly useful in situations where information is imprecise or uncertain. Fuzzy rules and fuzzy reasoning are fundamental concepts in fuzzy logic systems.

Fuzzy Rules

A fuzzy rule is a conditional statement in the form:

IF <fuzzy proposition> THEN <fuzzy proposition>

These rules are used to model the relationships between inputs and outputs in a fuzzy system. Each rule typically includes:

- Antecedent (IF part): Describes a condition using fuzzy sets and linguistic variables.
- Consequent (THEN part): Describes the result or output using fuzzy sets and linguistic variables.

Example

Consider a fuzzy system for controlling a heating system. A fuzzy rule might look like this:

IF temperature IS cold THEN heater IS high

In this rule:

- "temperature" is a linguistic variable.
- "cold" is a fuzzy set defining the temperature.
- "heater" is another linguistic variable.
- "high" is a fuzzy set defining the heater's setting.

Fuzzy Reasoning (Fuzzy Inference)

Fuzzy reasoning (or fuzzy inference) is the process of formulating the mapping from a given input to an output using fuzzy logic. The process involves several steps:

1. **Fuzzification:** Convert crisp inputs into fuzzy sets using membership functions.

2. **Rule Evaluation:** Apply fuzzy rules to the fuzzified inputs to determine the fuzzy outputs.
3. **Aggregation:** Combine the fuzzy outputs from all rules into a single fuzzy set.
4. **Defuzzification:** Convert the aggregated fuzzy set into a crisp output.

Example

Let's take the previous heating system example and consider two fuzzy rules:

1. IF temperature IS cold THEN heater IS high
2. IF temperature IS warm THEN heater IS low

Assume the following membership functions:

Assume the following membership functions:

- "cold" is defined as: $\mu_{\text{cold}}(x) = \max(0, \min(1, \frac{20-x}{10}))$
- "warm" is defined as: $\mu_{\text{warm}}(x) = \max(0, \min(1, \frac{x-20}{10}))$
- "high" and "low" are binary sets for simplicity.

Given a temperature input of $x = 15$:

1. **Fuzzification:** Determine the degree of membership:
   - $\mu_{\text{cold}}(15) = \max(0, \min(1, \frac{20-15}{10})) = 0.5$
   - $\mu_{\text{warm}}(15) = \max(0, \min(1, \frac{15-20}{10})) = 0$
2. **Rule Evaluation:**
   - Rule 1: $\mu_{\text{cold}}(15) = 0.5$ implies $\mu_{\text{high}} = 0.5$
   - Rule 2: $\mu_{\text{warm}}(15) = 0$ implies $\mu_{\text{low}} = 0$
3. **Aggregation:** Combine the fuzzy outputs. In this case, we only have one output fuzzy set "high" with a membership value of 0.5.
4. **Defuzzification:** Convert the fuzzy output to a crisp value. One common method is the **centroid method**, which finds the center of gravity of the fuzzy set. For simplicity, assume "high" corresponds to 100% heater power and "low" to 0%. The output would be:

$$\text{Heater setting} = 0.5 \times 100\% = 50\%$$