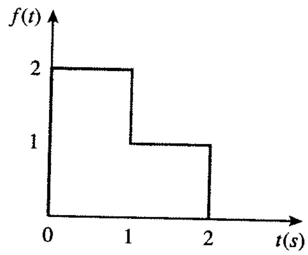
SOLUTION

ELE302

Quiz-4a

Name:.................................... Std.#:.............................. Sec:.......

Find the Laplace transform of $f(t)$ shown in the Figure below.



$$f(t) = 2\left[u(t) - u(t-1)\right] + 1\left[u(t-1) - u(t-2)\right]$$

$$\therefore F(s) = \frac{2}{s} - \frac{2}{s}e^{-1s} + \frac{1}{s}e^{-s} - \frac{1}{s}e^{-2s}$$

$$= \frac{2}{s} - \frac{1}{s}e^{-s} - \frac{1}{s}e^{-2s}$$

$$F(s) = \frac{1}{s}\left(2 - e^{-s} - e^{-2s}\right)$$

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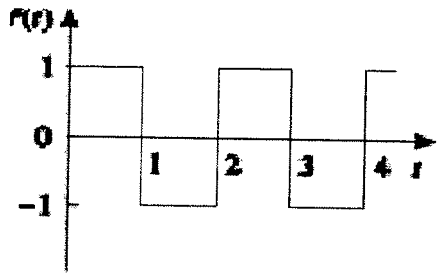
Quiz-4b

Name:.................................... Std.#:.............................. Sec:.......

Calculate the Laplace Transform of the function shown in Figure below.



$f(t)$ is a periodic function of $T = 2s$

1st period, $f_1(t) = 1[u(t) - u(t-1)] - 1[u(t-1) - u(t-2)]$

$$f_1(t) = u(t) - 2u(t-1) + u(t-2)$$

$$F_1(s) = \mathcal{L} f_1(t) = \frac{1}{s} - \frac{2}{s} e^{-s} + \frac{1}{s} e^{-2s}$$

$$= \frac{1}{s}\left(1 - 2e^{-s} + e^{-2s}\right)$$

$$F_1(s) = \frac{1}{s}\left(1 - e^{-s}\right)^2$$

For the periodic function, $F(s) = \dfrac{F_1(s)}{(1 - e^{-Ts})}$

$$F(s) = \frac{(1 - e^{-s})^2}{s(1 - e^{-2s})}$$

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Quiz-4c

Name:…………………………………. Std.#:………………………… Sec:…….

Find the Inverse Laplace of following function of I(s).

$$I(s)=\frac{12}{(s+2)^2(s+4)} = \frac{K_1}{(s+2)^2}+\frac{K_2}{(s+2)}+\frac{K_3}{(s+4)}$$

$$K_1 = (s+2)^2 I(s)\Big|_{s=-2} = \frac{12}{(s+4)}\Big|_{s=-2} = \frac{12}{-2+4} = 6$$

$$K_2 = \frac{1}{1!}\frac{d}{ds}\left[\frac{12}{(s+4)}\right]\Big|_{s=-2} = \frac{d}{ds}\left[12(s+4)^{-1}\right]\Big|_{s=-2}$$

$$= (-12)(s+4)^{-2}\Big|_{s=-2}$$

$$= \frac{-12}{(s+4)^2}$$

$$K_2 = \frac{-12}{(-2+4)^2} = \frac{-12}{4} = -3$$

$$\& \quad K_3 = (s+4)I(s)\Big|_{s=-4} = \frac{12}{(s+2)^2}\Big|_{s=-4} = \frac{12}{(-4+2)^2} = 3$$

$$\therefore \quad I(s) = \frac{6}{(s+2)^2} - \frac{3}{(s+2)} + \frac{3}{(s+4)}$$

$$\therefore \quad i(t) = \mathcal{L}^{-1}\frac{6}{(s+2)^2} - \mathcal{L}^{-1}\frac{3}{(s+2)} + \mathcal{L}^{-1}\frac{3}{(s+4)}$$

$$= 6te^{-2t} - 3e^{-2t} + 3e^{-4t}$$

$$i(t) = 3e^{-2t}\left[2t - 1 + e^{-2t}\right]\cdot u(t)$$

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Quiz-4d

Name:………………………………….. Std.#:………………………… Sec:…….

Find the inverse Laplace transform of:

$$V(s) = \frac{2s+26}{s\underbrace{(s^2+4s+13)}}$$

$$s^2+4s+13=0$$

$$s = \frac{-4 \pm \sqrt{(4)^2-4(13)}}{2}$$

$$s = -2 \pm j3$$

$$\therefore V(s) = \frac{2s+26}{s(s+2-j3)(s+2+j3)} = \frac{K_1}{s} + \frac{K_2}{(s+2-j3)} + \frac{K_2^*}{(s+2+j3)}$$

$$K_1 = sV(s)\Big|_{s=0} = \frac{2s+26}{(s^2+4s+13)}\Big|_{s=0} = \frac{26}{13} = 2$$

$$K_2 = (s+2-j3)V(s)\Big|_{s=-2+j3} = \frac{2s+26}{s(s+2+j3)} = \frac{2(-2+j3)+26}{(-2+j3)(-2+j3+2+j3)}$$

$$K_2 = \frac{-4+j6+26}{(-2+j3)(j6)}$$

$$K_2 = \frac{22+j6}{(-2+j3)(j6)}$$

$$= \frac{22.804\angle 15.26^\circ}{(3.606\angle 123.69^\circ)\,6\angle 90^\circ}$$

$$K_2 = 1.054\angle -198.43^\circ$$

$$\therefore V(s) = \frac{2}{s} + \frac{1.054\angle -198.43^\circ}{(s+2-j3)} + \frac{1.054\angle 198.43^\circ}{(s+2+j3)}$$

$$v(t) = \mathcal{L}^{-1}V(s) = 2 + 2(1.054)e^{-2t}\cos(3t-198.43^\circ)$$

$$v(t) = [2 + 2.108\,e^{-2t}\cos(3t-198.43^\circ)]\,u(t)$$