Ryerson University
Department of Electrical and Computer Engineering

ELE202: Electric Circuits Analysis
Final Examination, April, 29 2009
Duration: 3 hours

Solutions

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NOTES:

1. Please **TICK** mark against the name of your professor.

2. This is a **Closed Book** examination. No aids other than the approved calculators are allowed.

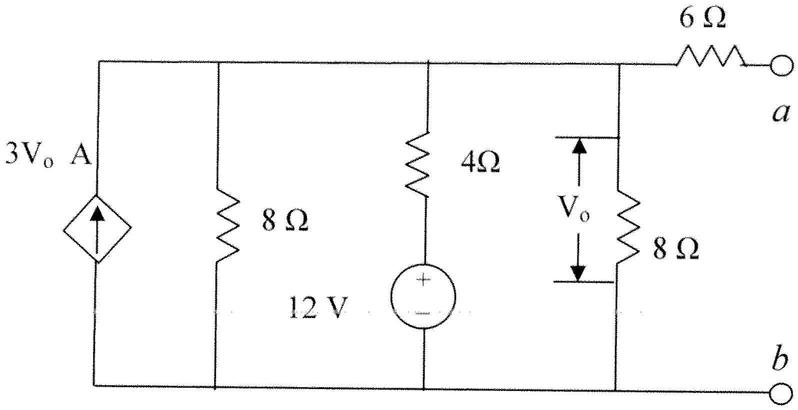
3. Answer all **SIX** questions.

4.

Question No.	Mark for each question	Mark obtained
Q1	15	
Q2	15	
Q3	15	
Q4	20	
Q5	15	
Q6	20	
	Total (100)	

Problem 1 (15 marks):

Find the Thevenin equivalent, V_{th} and R_{th}, at terminal *a-b* of the following circuit.



or: $-3V_o + \frac{V_o}{8} + \frac{V_o - 12}{4} + \frac{V_o}{8} =$

$-12V_o + V_o + V_o - 12 = 0$

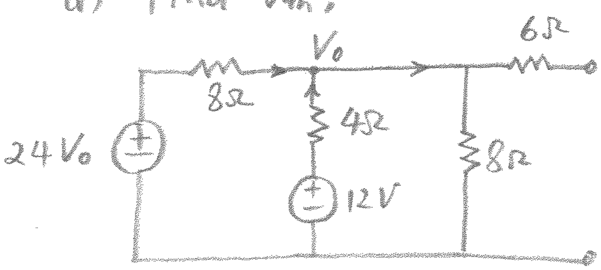
$-10V_o = 12$

$V_o = -1.2$

$V_{th} = V_o = -1.2\ V$

$(V_b > V_a)$

a) Find V_{th}.



The node voltage is the same as V_o, using nodal analysis

$$\frac{24V_o - V_o}{8} + \frac{12 - V_o}{4} = \frac{V_o}{8} \quad 3$$

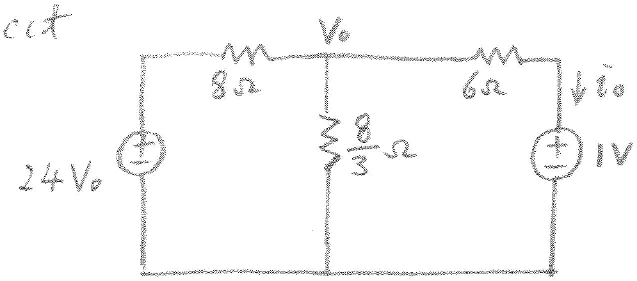
$$\frac{23V_o}{8} - \frac{3V_o}{8} = -3$$

$$V_o = \frac{-24}{20} = -1.2\ V = V_{th} \quad 2$$

1

b) Find R_{th}, apply 1V source across terminal a-b, and combine 4Ω and 8Ω branches

2 cct



$$\frac{24V_o - V_o}{8} - \frac{3V_o}{8} = \frac{V_o - 1}{6} \quad 2$$

$$\frac{23V_o - 3V_o}{8} - \frac{V_o}{6} = \frac{-1}{6}$$

$$V_o\left(\frac{20}{8} - \frac{1}{6}\right) = \frac{-1}{6}$$

$$V_o \frac{60 - 4}{24} = \frac{-1}{6}$$

$$V_o = \frac{-4}{56} = \frac{-1}{14} \quad 2$$

$$i_o = \frac{-\frac{1}{14} - 1}{6} = \frac{-15}{14 \times 6} = \frac{-5}{28} = -0.179A$$

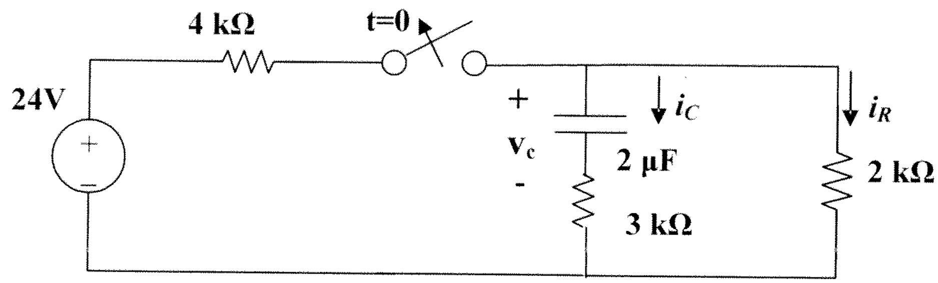
3 $\quad R_{th} = \frac{1}{i_o} = \frac{28}{5} = 5.6\ \Omega$

answers: $\begin{cases} V_{th} = -1.2\ V & (V_b > V_a) \\ R_{th} = 5.6\ \Omega \end{cases}$

Problem 2 (15 marks):

In the circuit below, the switch was closed for a long time and is opened at t = 0.

Find $i_R(t)$, $i_C(t)$ and $v_C(t)$ for t < 0 and t > 0.



For $t < 0$, switch is closed.

(1) $i_c = 0$

(2) $i_R = \dfrac{24}{(4+2)\times 10^3} = 4\ mA$

(2) $V_c = 24 \cdot \dfrac{2}{(4+2)} = 8\ V$ Notice that 3 KΩ Resistor doesn't play a role in V_c calculation

For $t > 0$, The right Loop forms a sourceless cct,

(1) $\tau = RC = (3+2)\times 10^3 \times 2\times 10^{-6} = 10^{-2} = 0.01$ sec

(1) $V_c(0) = 8V$,

(1) $V_c(\infty) = 0\ V$

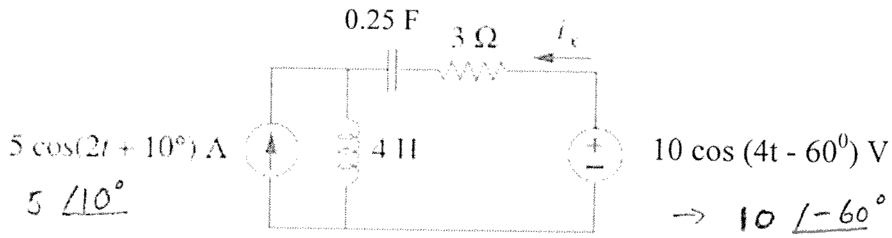
(2) $V_c(t) = V_c(0)\, e^{-t/\tau} = 8e^{-100t}$

(2) $i_c(t) = C\dfrac{dV_c}{dt} = 2\times 10^{-6} \cdot 8 \times (-100)\, e^{-100t} = -1.6\, e^{-100t}$ mA, (cw) (1)

(2) $i_R(t) = -i_c(t) = 1.6\, e^{-100t}$ mA

Problem 3 (15 marks):

Using **superposition principle**, find $i_x(t)$ in the circuit below.



0,5 $5 \angle 10^\circ$ $\rightarrow 10 \angle -60^\circ$ 0,5

Let $i_x = i_1 + i_2$,

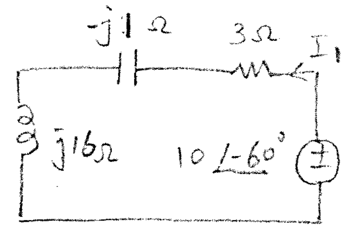
i_1 — due to voltage source
i_2 — " current source.

2

$\omega = 2$
$4H \rightarrow j8$
$0.25\,F \rightarrow -j2$

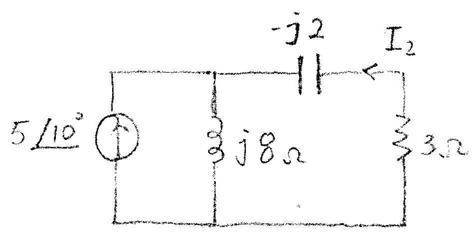
$\omega = 4$
$4H \rightarrow j16$
$0.25\,F \rightarrow -j1$

2



$$I_1 = \frac{10\angle -60^\circ}{3 + j16 - j} = \frac{10\angle -60^\circ}{3 + j15} = \frac{10\angle -60^\circ}{15.297\angle 78.69^\circ}$$

$\underline{I_1 = 0.6537 \angle -138.69^\circ}$ 4 $(-0.491 - j0.4314)$



0,5 $i_1(t) = 0.6537 \cos(4t - 138.69^\circ)$ A

use current division :

$$I_2 = 5\angle 10^\circ \frac{-j8}{3 - j2 + j8} = \frac{-40j \angle 10}{3 + j6}$$

$$= \frac{-40j \angle 10}{6.708 \angle 63.43} = 5.963 \angle -90^\circ - 10^\circ - 63.43$$

$\underline{I_2 = 5.963 \angle -143.43^\circ}$ 4 $(-4.79 - j3.553)$

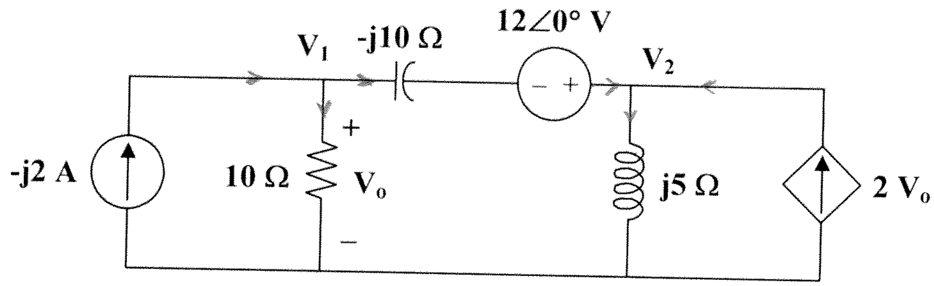
0,5 $i_2(t) = 5.963 \cos(2t - 143.43^\circ)$ A

1 $i_x = i_1(t) + i_2(t)$

$= 0.6537 \cos(4t - 138.69^\circ)$
$+ 5.963 \cos(2t - 143.43^\circ)$ A

Problem 4 (20 marks):

Using nodal analysis, find the phasor voltage V_1 and V_2 in the following circuit.



at V_1

$$-j2 = \frac{V_1}{10} + \frac{V_1 - V_2 + 12}{-j10} \quad 5$$

$\times -j10$, $-20 = -jV_1 + V_1 - V_2 + 12$

$$(1-j)V_1 - V_2 = -32 \quad 2$$

at V_2

$$\frac{V_1 - V_2 + 12}{-j10} + 2V_1 = \frac{V_2}{j5} \quad 5$$

$\times -j10$, $V_1 - V_2 + 12 - 20jV_1 = -2V_2$

$$(1-20j)V_1 + V_2 = -12 \quad 2$$

add two Eqs. $(2-21j)V_1 = -44$

$$V_1 = \frac{-44}{2-21j} = \frac{-44}{21.095\angle -84.56^\circ}$$

$= -2.086\angle 84.56^\circ$

$= 2.086\angle 84.56^\circ - 180^\circ$

$= 2.086\angle -95.44^\circ$ V

$= -0.1978 - j2.077$ 3

substitute V_1 into above eq.

$V_2 = -12 - (1-20j) \times 2.086\angle -95.44^\circ$

$= -12 - 20.025\angle -87.14 \times 2.086\angle -95.44^\circ$

$= -12 - 41.772\angle -182.58$

$= -12 + 41.772\angle -2.58^\circ$

$= -12 + 41.772(0.999 - j0.0450)$

$= -12 + 41.73 - j1.8797$

$= 29.73 - j1.8797$

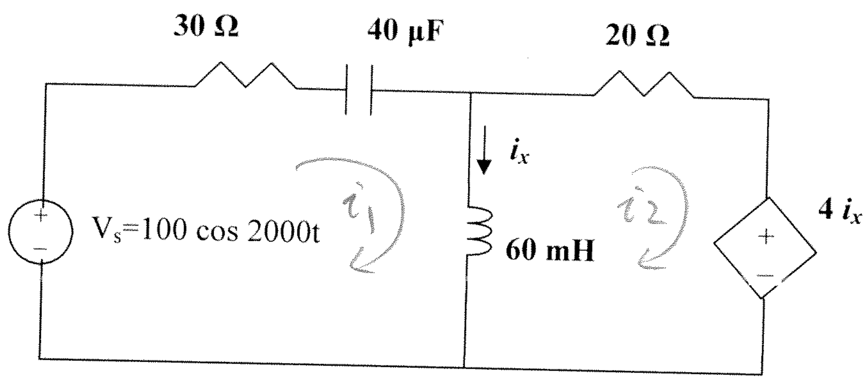
$= 29.789\angle -3.618^\circ$ V

3

$= 29.729 - j1.8797$

Problem 5 (15 marks):

I n the circuit below: find the time-domain expression for current $i_x(t)$ **using mash analysis**.



$\omega = 2000$ rad/s

$40\mu F \rightarrow \frac{1}{j\omega C} = -j12.5\ \Omega$

$60\,mH \rightarrow j\omega L = j120\ \Omega$

$V_s = 100\angle 0^\circ$

From Left loop:

(3) $-100 + i_1(30 - j12.5 + j120) - j120\, i_2 = 0$

(2) $(30 + j107.5)\, i_1 - j120\, i_2 = 100$

Substitute i_1 into above Eq.

$(30 + j107.5)(0.9945 - j0.1665)\, i_2 - j120\, i_2 = 100$

$(29.835 - j4.995 + j106.91 + 17.90 - j120)\, i_2 = 100$

(1) $i_2 = \dfrac{100}{47.735 - j18.085} = \dfrac{100}{51.046\angle -20.75^\circ}$

$= 1.959\angle 20.75^\circ$ A

$= 1.832 + j0.694$

(1) $i_1 = 1.975\angle 11.25^\circ$

$= 1.937 + j0.385$ A

From right Loop:

(3) $i_2(20 + j120) + 4i_x - j120\, i_1 = 0$

$i_x = i_1 - i_2$ (1)

$(200 + j120)\, i_2 + 4i_1 - 4i_2 - j120\, i_1 = 0$

$(4 - j120)\, i_1 + (16 + j120)\, i_2 = 0$

(2) $(1 - j30)\, i_1 + (4 + j30)\, i_2 = 0$

$i_1 = \dfrac{-(4 + j30)}{(1 - j30)}\, i_2 = \dfrac{i_2}{901}(896 - j150)$

$= (0.9945 - j0.1665)\, i_2$

$= 1.0083\angle -9.504^\circ\, i_2$

(1)

(0.5) $i_x = i_1 - i_2 = 0.105 - j0.309$

$= 0.326\angle -71.23^\circ$ A

$i_x(t) = 0.326\cos(2000t - 71.23^\circ)$ A

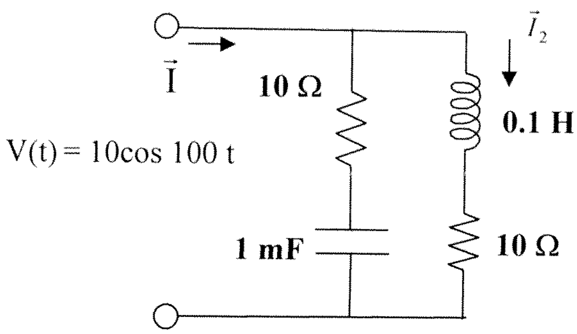
(0.5)

Problem 6 (20 marks):

A sinusoidal voltage source with V(t) = 10cos 100 t V is connected to a load as shown below.

Find: a) the load impedance of inductive branch, Z_2; phase current, $\vec{I}_2$, complex power, $\vec{S}_2$ and power factor pf_2 (indicate leading or lagging pf). Draw its power triangle.

b) load impedance, Z_{tot}; phase current, $\vec{I}$ and complex power, $\vec{S}$ for the source.



$\omega = 100$ rad/s

$0.1\,H \rightarrow j\omega L = \underline{j10\,\Omega}$ (1)

$1\,mF \rightarrow \frac{-j}{\omega C} = \underline{-j10\,\Omega}$ (1)

$\vec{V} = 10\underline{/0^\circ}$

a)	Z_2	$10+j10\ \Omega$	(2)
	$\vec{I}_2$	$0.707\underline{/-45^\circ}$ A	(2)
	$\vec{S}_2$	$2.5+j2.5$	(3)
	pf_2	0.707 Lagging	(2)
b)	Z_{tot}	10 Ω	(2)
	$\vec{I}$	$1\underline{/0^\circ}$ A.	(2)
	$\vec{S}$	5 W	(2)

a) $Z_2 = 10 + j10 = 10\sqrt{2}\underline{/45^\circ}\ \Omega$

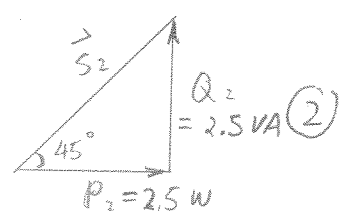
$\vec{I}_2 = \frac{\vec{V}}{Z_2} = \frac{10}{10+j10} = \frac{1}{1+j} = \frac{1}{\sqrt{2}\underline{/45^\circ}}$

$= 0.707\underline{/-45^\circ} = \underline{0.5 - j0.5\ A}$

$\vec{S}_2 = \frac{1}{2}\vec{V}\cdot\vec{I}^* = \frac{10}{2} \times 0.707\underline{/45^\circ} = 3.54\underline{/45^\circ}$

$= \frac{1}{2}(5 + j5) = 2.5\,(W) + j2.5\,(V\cdot A)$

$pf_2 = \cos 45^\circ = 0.707$ $\underline{(\text{Lagging})}$ (1)



b) The left branch of the Load:

$Z_1 = 10 - j10\ \Omega$

$Z_{tot} = Z_1 \parallel Z_2 = \frac{10(1-j)\cdot 10(1+j)}{10-j10+10+j10}$

$= \frac{100 \times 2}{20} = 10\,\Omega.$

$\vec{I} = \frac{\vec{V}}{Z_{tot}} = \frac{10\underline{/0^\circ}}{10} = 1\underline{/0^\circ}\ A.$

$\vec{S} = \frac{1}{2}\vec{V}\cdot\vec{I}^* = \frac{1}{2}\times 10 \times 1 = 5,$ $\quad \vec{S} = P = 5\,W$