# *MTH 141 HW 1*

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# 1 Problem 3

$$\overrightarrow{v} = \begin{bmatrix} q_1 \\ q_2 \\ t_1 \\ t_2 \\ f \end{bmatrix} = \begin{bmatrix} 70 \\ 85 \\ 80 \\ 75 \\ 90 \end{bmatrix}$$

# 2 Problem 4

$$\overrightarrow{a} = \begin{pmatrix} x - 2y \\ 2x - y \\ 2z \end{pmatrix}; \overrightarrow{b} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$

if $\overrightarrow{a} = \overrightarrow{b}$, then,

$$\begin{pmatrix} x - 2y \\ 2x - y \\ 2z \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$

Converting this in matrix notation we have,

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

By Row Elimination we have,

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \\ 1 \end{bmatrix}$$

Therefore by back-substitution we have,

$$\{z = \frac{1}{2}; y = -2; x = -2\} \in \mathbb{R}$$

# 3 Problem 7

P(1,$x^2$), Q(3,4), $P_1(4,5)$ & $Q_1(6,1)$ $\forall\, x \in \mathbb{R}$

$$\overrightarrow{PQ} = \sqrt{(3-1)^2 + (4-x^2)^2} = \sqrt{20 + x^4 - 8x^2}$$

$$\overrightarrow{P_1Q_1} = \sqrt{(6-4)^2 + (1-5)^2} = \sqrt{20}$$

When equating $\overrightarrow{PQ} = \overrightarrow{P_1Q_1}$ we have,

$$20 = 20 + x^4 - 8x^2$$

After Simplification we have,

$$x = \{-2\sqrt{2}, 0, 2\sqrt{2}\} \in \mathbb{R}$$

# 4 Problem 10

$$\overrightarrow{a} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \overrightarrow{b} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}, \overrightarrow{c} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}; a \in \mathbb{R}$$

i) $2\overrightarrow{a} - \overrightarrow{b} + 5\overrightarrow{c}$

$$2\overrightarrow{a} - \overrightarrow{b} + 5\overrightarrow{c} = \begin{pmatrix} -2 \\ 4 \\ 6 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} + \begin{pmatrix} 15 \\ 0 \\ -5 \end{pmatrix} = \begin{pmatrix} 11 \\ 6 \\ 1 \end{pmatrix}$$

ii) $4\overrightarrow{a} + \alpha\,\overrightarrow{b} - 2\overrightarrow{c}$

$$4\overrightarrow{a} + \alpha\,\overrightarrow{b} - 2\overrightarrow{c} = \begin{pmatrix} -4 \\ 8 \\ 12 \end{pmatrix} + \alpha \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} + \begin{pmatrix} -6 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -10 + 2\alpha \\ 8 - 2\alpha \\ 14 \end{pmatrix}$$

# 5 Problem 12

$$\overrightarrow{u} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 2 \end{pmatrix}; \overrightarrow{v} = \begin{pmatrix} -2 \\ -1 \\ 1 \\ -4 \end{pmatrix} \& \overrightarrow{w} = \begin{pmatrix} 3 \\ 2 \\ -1 \\ 0 \end{pmatrix}$$

a) $\overrightarrow{a} \in \mathbb{R} : 2\overrightarrow{u} - 3\overrightarrow{v} - \overrightarrow{a} = \overrightarrow{w}$

$$\therefore \begin{pmatrix} 2 \\ -2 \\ 0 \\ 4 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \\ -3 \\ 12 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ -1 \\ 0 \end{pmatrix} = \overrightarrow{a}$$

$$\implies \overrightarrow{a} = \begin{pmatrix} 11 \\ 3 \\ -4 \\ 16 \end{pmatrix}$$

# 6 Problem 13

$$\overrightarrow{a} = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 2 \end{pmatrix}; \overrightarrow{b} = \begin{pmatrix} -3 \\ 0 \\ 3 \\ 6 \end{pmatrix}$$

if $\overrightarrow{a} \parallel \overrightarrow{b}$ then, $\overrightarrow{b} = k\overrightarrow{a}$, where k is an arbitary constant.

$$\therefore \begin{pmatrix} -3 \\ 0 \\ 3 \\ 6 \end{pmatrix} = k \begin{pmatrix} -1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$$

$$\implies k = 3$$