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1. **[10 Marks]** Evaluate the following integrals.

   (a) $\int \cos(11x)\cos(10x)\,dx$.

   (b) $\int \sec^6(x)\tan^5(x)\,dx$.

a)

$$\cos x \cos y = \frac{1}{2}\left[\cos(x+y) + \cos(y-x)\right]$$

$$\therefore \cos(11x)\cos(10x) = \frac{1}{2}\left[\cos(21x) + \cos(x)\right]$$

$$\therefore \int \cos(11x)\cos(10x)\,dx = \frac{1}{2}\int \left[\cos(21x) + \cos(x)\right] dx$$

$$= \frac{1}{2}\left[\frac{1}{21}\sin(21x) + \sin(x)\right] + C = \frac{1}{42}\sin(21x) + \frac{1}{2}\sin(x) + C$$

b)

$$\int \sec^6(x)\tan^5(x)\,dx = \int \sec^5(x)\left(\tan^4(x)\right)\tan x\,dx$$

$$= \int \sec^5(x)\left(\sec^2(x) - 1\right)^2 \sec(x)\tan(x)\,dx \quad \left[\because \sec^2 x - 1 = \tan^2(x)\right]$$

let $u = \sec(x) \Rightarrow du = \sec(x)\tan(x)\,dx$

$$\therefore \int \sec^5(x)\left(\sec^2(x) - 1\right)^2 \sec(x)\tan(x)\,dx = \int u^5(u^2-1)^2\,du$$

$$\cancel{= \int (u^7 - u^5)\,du = \frac{u^8}{8} - \frac{u^6}{6} + C = \frac{\sec^8(x)}{8} - \frac{\sec^6(x)}{6}}$$

$$= \int u^5(u^4 - 2u^2 + 1)\,du = \int (u^9 - 2u^7 + u^5)\,du = \frac{u^{10}}{10} - \frac{u^8}{4} + \frac{u^6}{6} + C$$

$$= \frac{\sec^{10}(x)}{10} - \frac{\sec^8(x)}{4} + \frac{\sec^6(x)}{6} + C$$