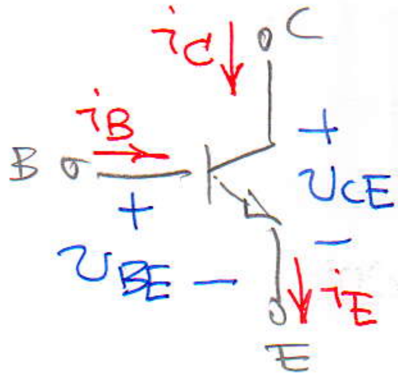
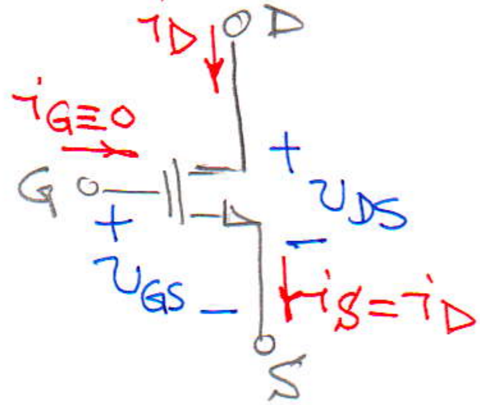
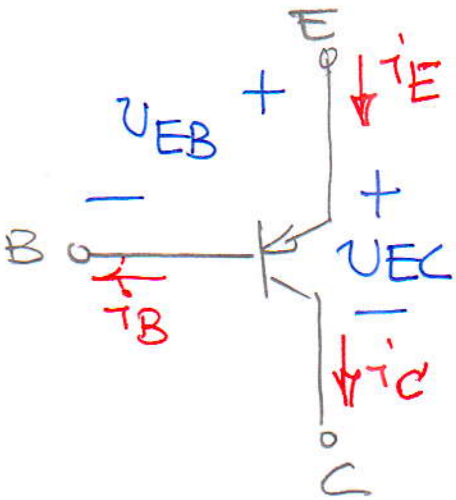
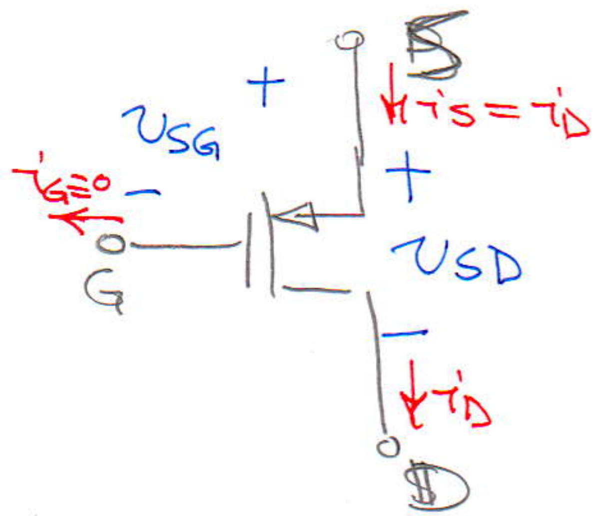
MOS and MOS Amplifiers





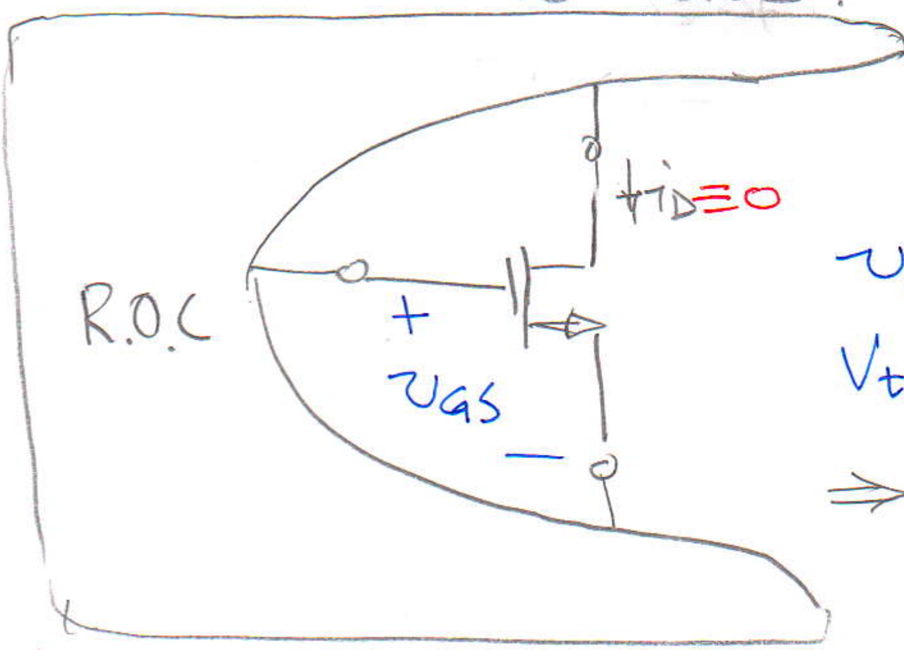




Modes of Operation

① Cut-off mode (corresponding to the cut-off mode in BJT)

Focus on NMOS Device:



$v_{GS} \leq V_{tn}$

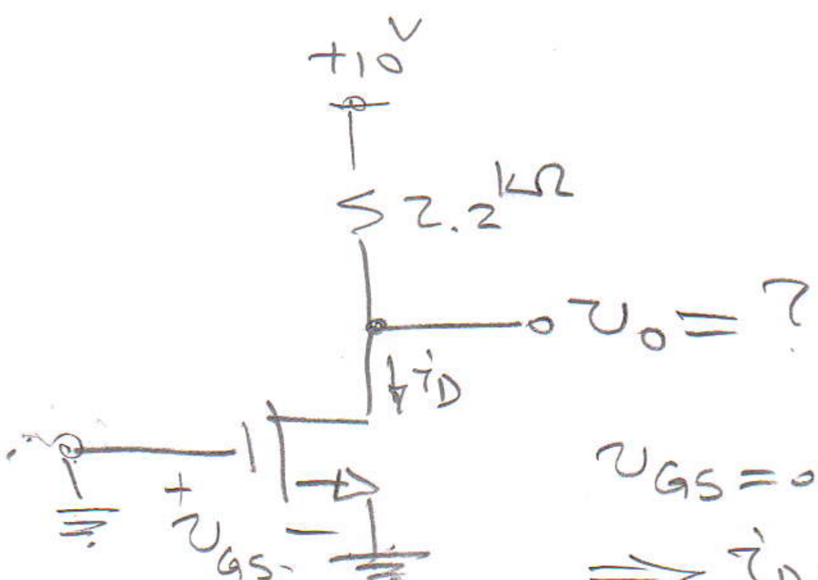
V_{tn}: Threshold voltage

$\Rightarrow$ MOS is "off"

$\Rightarrow i_D \equiv 0$

V_{tn}: from 0.5^V — 1.5^V

Example#1



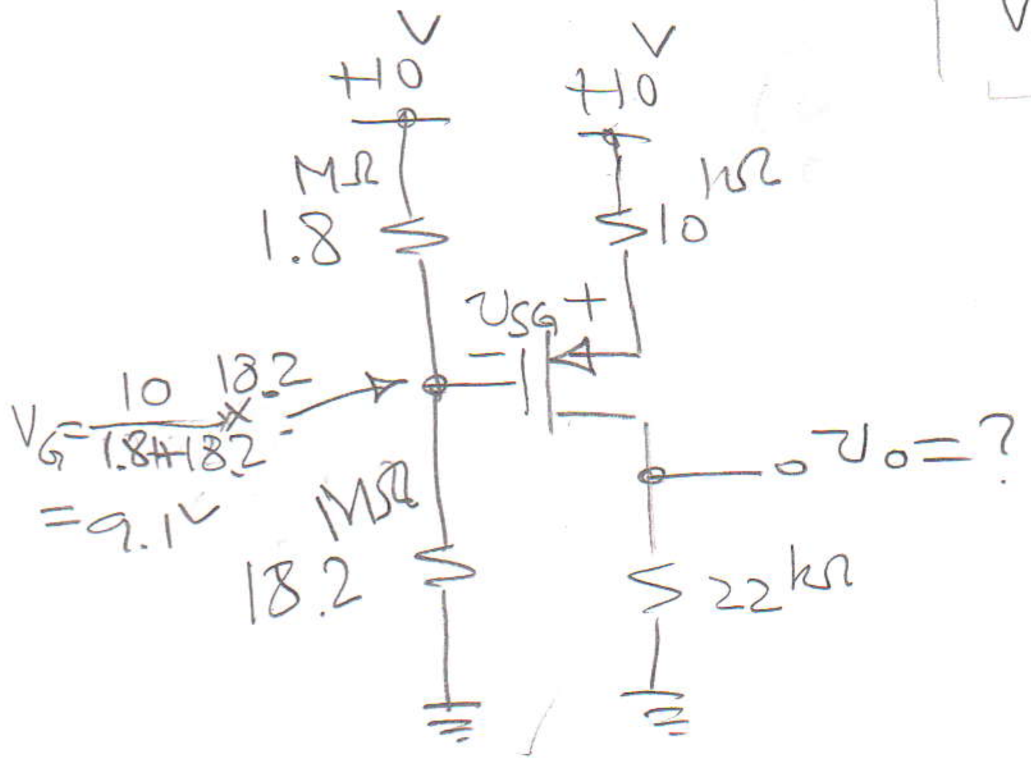
$v_{GS} = 0 \Rightarrow v_{GS} < V_{tn}$

$\Rightarrow i_D = 0$

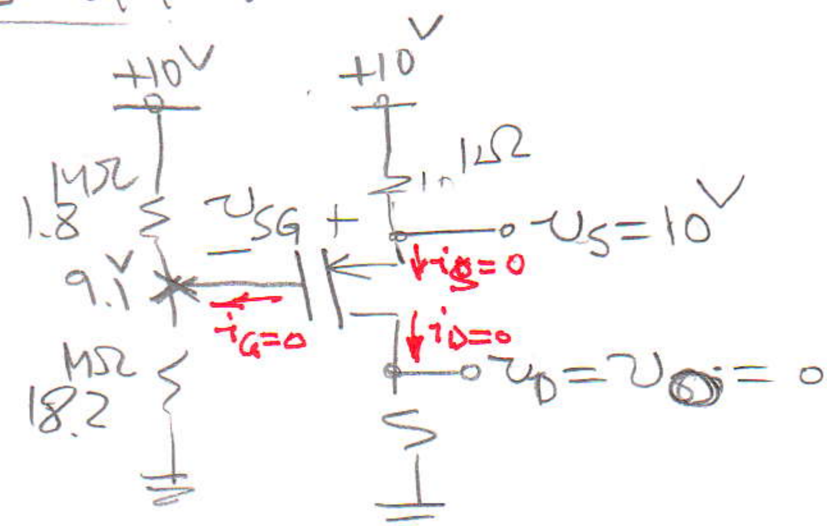
$\Rightarrow \boxed{v_O = 10V}$

Example#2.

$V_{tp} = -1.1^V$



Assume off :

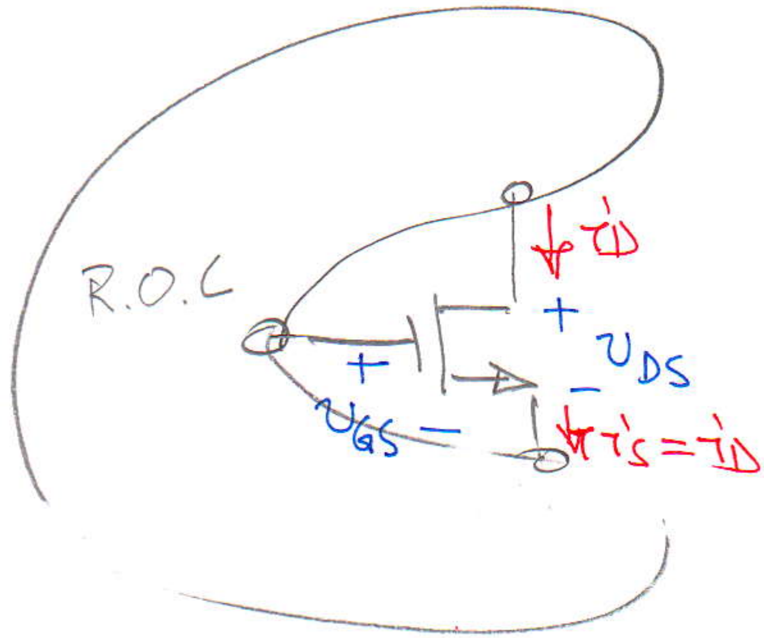


$v_{SG} = v_S - v_G = 10 - 9.1 = 0.9^V$

$v_{SG} \geq |V_{tp}|$?

No! $V_{SG} = 0.9^V$, $|V_{tp}| = 1.1^V$

2 Triode mode of operation (corresponding to Saturation mode in the BJT)



$$\left.\begin{array}{l} v_{GS} > V_{tn} \\ v_{DS} \leqslant \underbrace{v_{GS} - V_{tn}}_{V_{ov}} \end{array}\right\} \Rightarrow i_D = K_n\Big[\overbrace{(v_{GS} - V_t)}^{V_{ov}} v_{DS} - \frac{v_{DS}^2}{2}\Big]$$

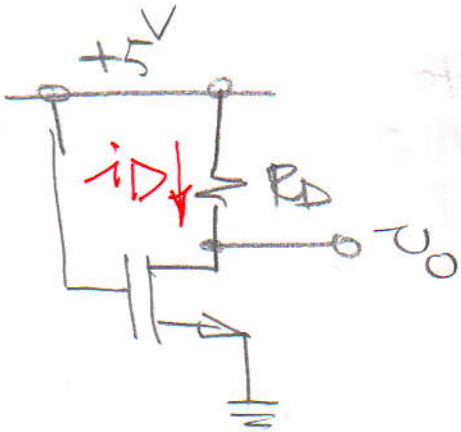
$$= K_n\Big[\underbrace{(v_{GS} - v_t)}_{V_{ov}} - \frac{v_{DS}}{2}\Big] v_{DS}$$

where K_n is called "transconductance parameter" in mA/V2

$$v_{GS} - V_t \triangleq v_{OV} \Rightarrow v_{GS} = V_t + V_{OV}$$

v_{OV}: over-drive voltage (typically 0.5^V — ..)

Example #3



$k_n = 1.0 \ mA/V^2$

$V_{tn} = 1.0V$

Find R_D if

$v_O = 0.1V$

$v_G = 5^V$; $v_S = 0^V \Rightarrow v_{GS} = 5^V$

$v_{GS} > V_{tn}$? ; Yes! $v_{GS} = 5^V$
$V_{tn} = 1.0^V$

$v_{DS} = v_D - v_S = 0.1 - 0 = 0.1^V$

$\underbrace{v_{DS}}_{0.1} \leq \underbrace{v_{GS} - V_{tn}}_{V_{OV} = 5-1 = 4V}$? ; Yes!

Transistor is in triode mode.

$$i_D = K_n\left[\underbrace{(v_{GS} - V_t)}\, v_{DS} - \frac{1}{2} v_{DS}^2\right]$$

$$= 1.0\left[4 \times 0.1 - \frac{1}{2} \times (0.1)^2\right]$$

$$= \underline{0.395 mA}$$

$$R_D = \frac{5^V - 0.1^V}{0.395} = 12.4 k\Omega$$

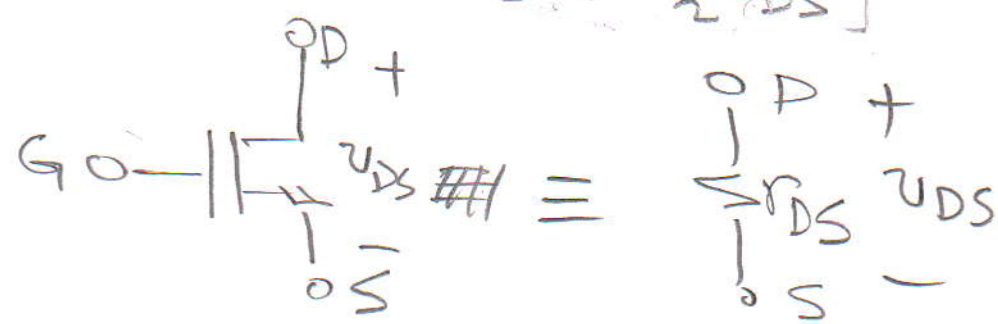
$$i_D = K_n\left[(\underbrace{v_{GS} - V_{tn}}_{V_{OV}})v_{DS} - \frac{1}{2}v_{DS}^2\right]$$

$$i_D = K_n\left[V_{OV} - \frac{1}{2}v_{DS}\right]v_{DS}$$

$$\Rightarrow \frac{v_{DS}}{i_D} = \frac{1}{K_n\left[V_{OV} - \frac{1}{2}v_{DS}\right]}$$

of resistance dimension

$$r_{DS} \triangleq \frac{1}{K_n\left[V_{OV} - \frac{1}{2}v_{DS}\right]}$$

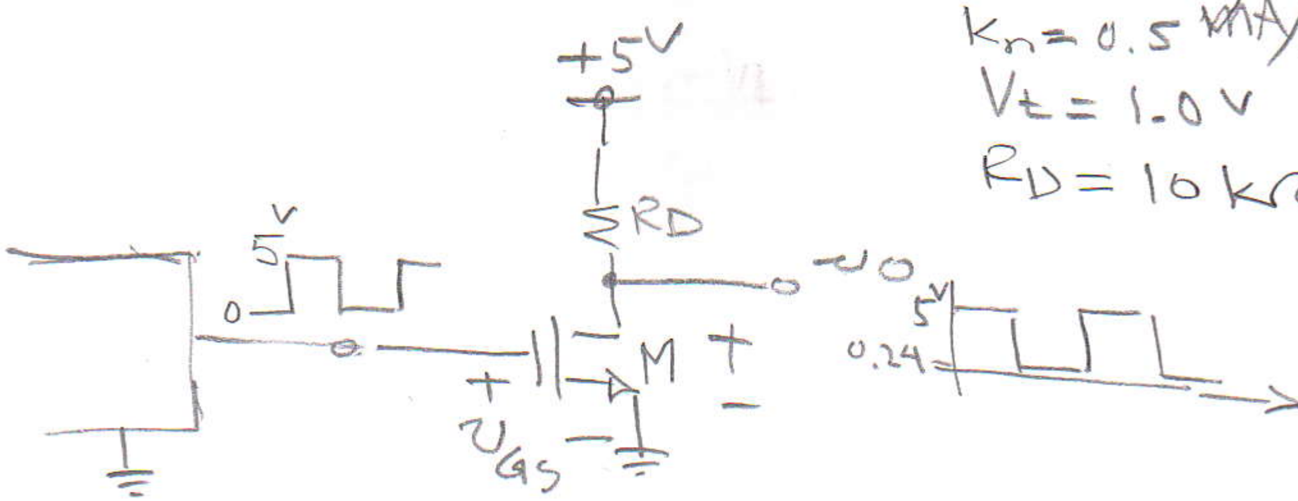


~~If~~ r_{DS} is nonlinear since its value depends on the voltage across it, i.e., v_{DS}.

If $v_{DS} \ll 2V_{OV} \Rightarrow r_{DS} \simeq \dfrac{1}{K_n \underbrace{V_{OV}}_{V_{GS}-V_t}}$

A NOT GATE

$K_n = 0.5\ mA/V^2$
$V_t = 1.0\ V$
$R_D = 10\ k\Omega$



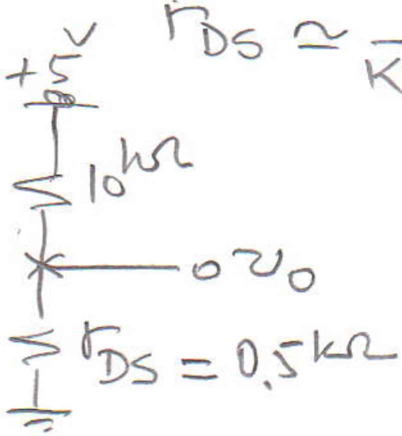
if $v_{GS} = 0 \Rightarrow M: \text{off} \rightarrow i_D = 0 \rightarrow v_0 = 5^V$ High

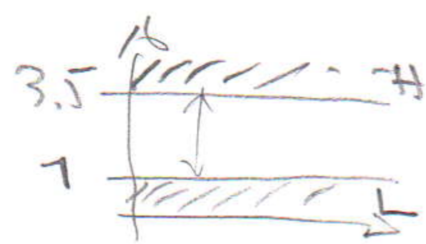
if $v_{GS} = 5^V \rightarrow M$: triode mode $\rightarrow$

$$i_D = k\left[V_{OV} - \frac{1}{2}V_{DS}\right]V_{DS}$$

But we know:

$$r_{DS} \simeq \frac{1}{KV_{OV}} = \frac{1}{0.5\times(5-1)} = 0.5^{k\Omega}$$

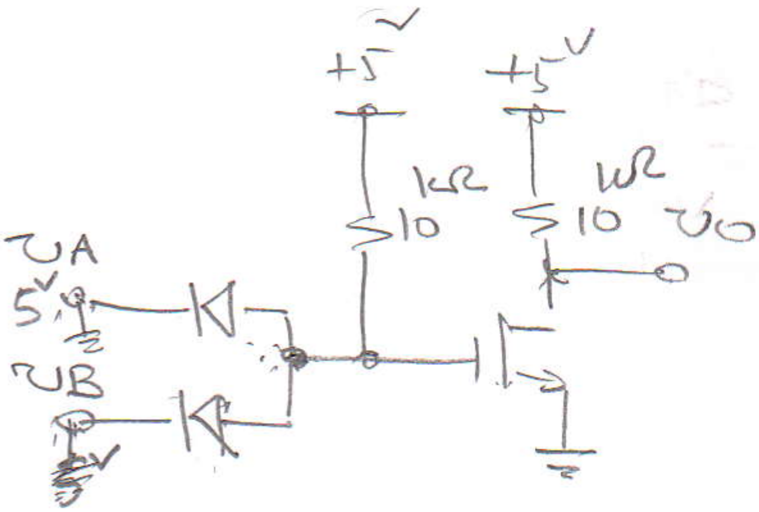




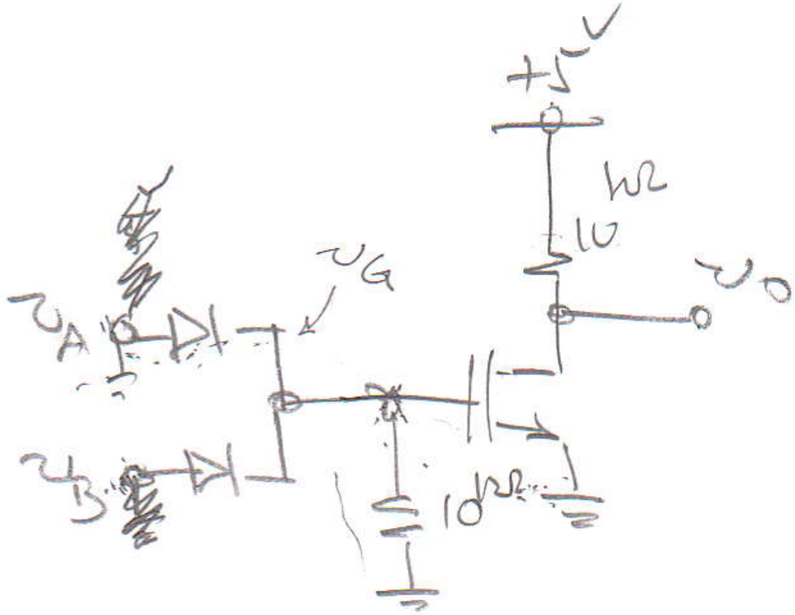
$$v_O = \frac{5}{10.5}\times 0.5 = \underline{0.24^V} \rightarrow \underline{\text{LOW}}$$

$v_{DS} = 0.24^V$, $v_{OV} = 4^V$ $\quad v_{DS} < v_{OV}$ ✓ triode is correct

NAND Gate

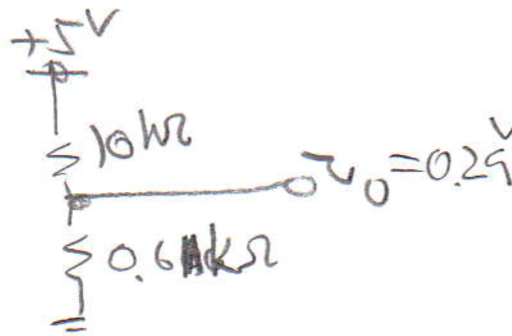


v_A	v_B	v_O
0	0	5
0	1	5
1	0	5
1	1	0.24^V

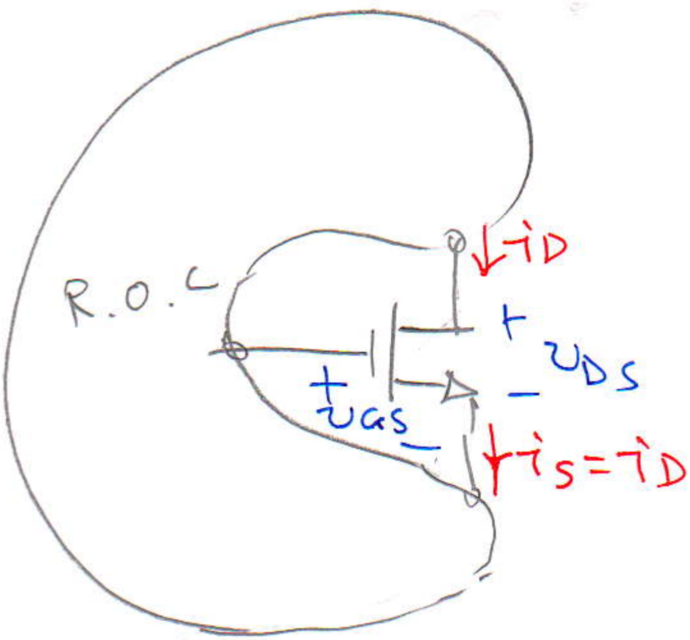


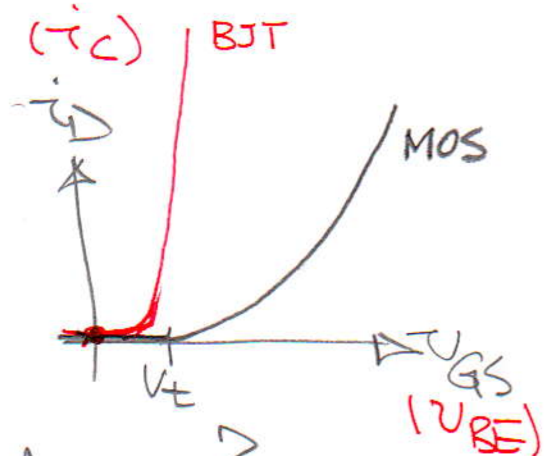
v_A	v_B	v_O
0	0	5^V
0	1	0.29
1	0	0.29
1	1	0.29

$$v_G = 4.3^V \rightarrow r_{DS} = \frac{1}{0.5\times(4.3-1)} = 0.61^V$$



③ Saturation mode of operation (equivalent to the active mode in the BJT) ⇒ good for amplification.





$v_{GS} > V_t$

$v_{DS} > \underbrace{v_{GS} - V_t}_{V_{OV}}$ $\rightarrow$ $i_D = \frac{1}{2} K_n v_{OV}^2$ ignoring Early effect.

$$i_D = \frac{1}{2} K_n v_{OV}^2 \left(1 + \frac{v_{DS}}{V_A}\right)$$

where $v_{OV} = v_{GS} - V_t$

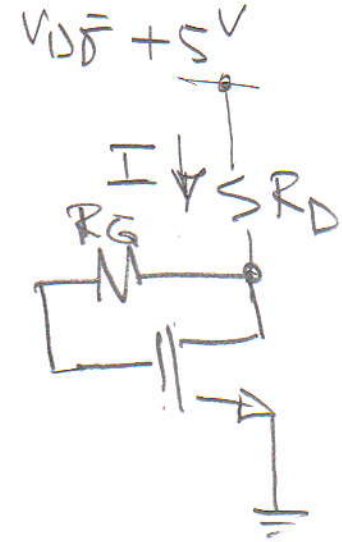
For the BJT:

$i_C = I_S e^{v_{BE}/V_T}$ W.O.E.E

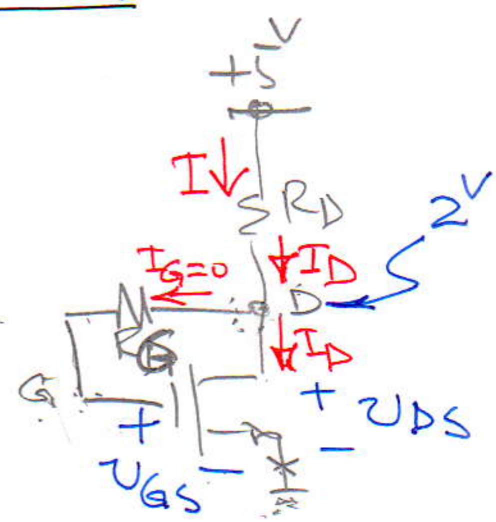
$i_C = I_S e^{v_{BE}/V_T} \left(1 + \frac{v_{CE}}{V_A}\right)$ W.E.E

Example

$K_n = 1.0\ mA/V^2$, $V_t = 1.0^V$, $\lambda = 0$ $\left(\lambda = \frac{1}{V_A} \Rightarrow V_A = \infty\right)$
$V_{DD} = 5^V$, $I_D = 0.5\ mA$; Find R_D.



Solution



Goes without saying that MOS is in the Saturation mode:

If $v_{GS} = v_{DS}$ (KVL)

$\Rightarrow v_{DS} \geq v_{GS} - V_t$

~~KVL~~

$$0.5^{mA} = \frac{1}{2} \times 1.0 \times V_{ov}^2 \Rightarrow \underline{V_{ov} = 1^V}$$

$$V_{GS} = V_{ov} + V_t = 1 + 1 = 2^V$$

$V_{DS} = V_{GS} \Rightarrow V_{DS} = 2^V \Rightarrow V_D = 2.0^V$

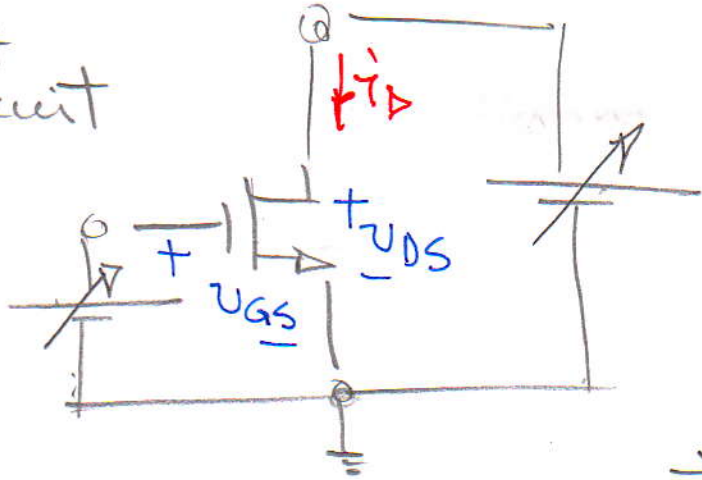
$$R_D = \frac{5-2}{0.5} = 6.0\ k\Omega$$

E12	E24
1.0	
1.2	
1.8	
1.8	
2.2	
2.7	
3.3	
3.9	
4.7	
5.6	
6.8	
8.2	
10	
12	
15	
.	
.	
.	

There are 12 values in every decade...

Small-Signal Model of the MOSFET

Test Circuit



Device: in the Saturation mode (active mode in in the BJT).

$$v_{DS} \geq v_{GS} - V_t$$

$$\Rightarrow i_D = \frac{1}{2} k v_{ov}^2$$

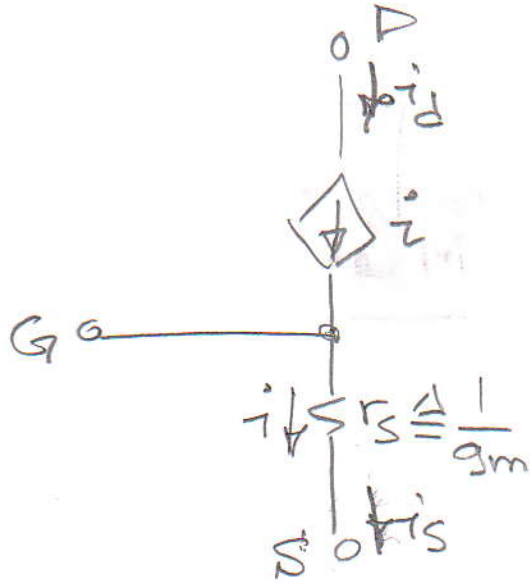
$$\Rightarrow i_D = \frac{1}{2} k \underbrace{(v_{GS} - V_t)}_{v_{ov}}^2$$

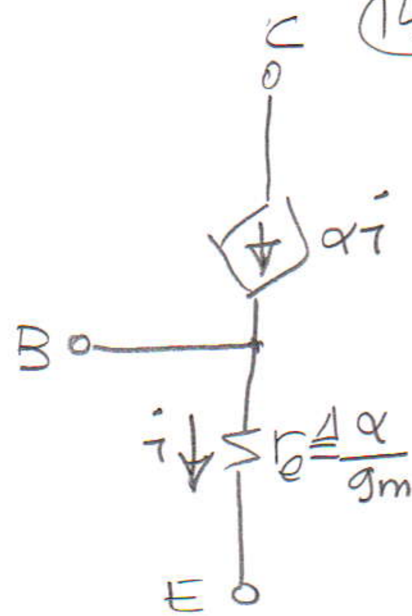
(ignoring Early effect)

Let $v_{GS} = \underbrace{V_{GS}}_{\text{large \& constant}} + \underbrace{v_{gs}}_{\text{small \& variable}}$ $\quad |v_{gs}| \ll V_{GS}$

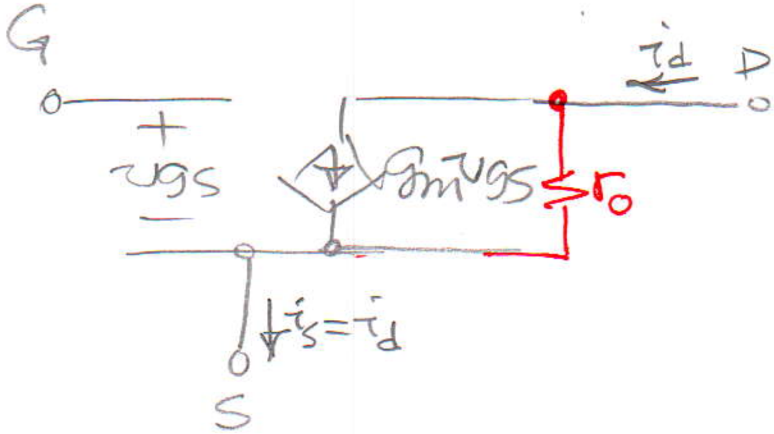
$$i_D = I_D + i_d$$

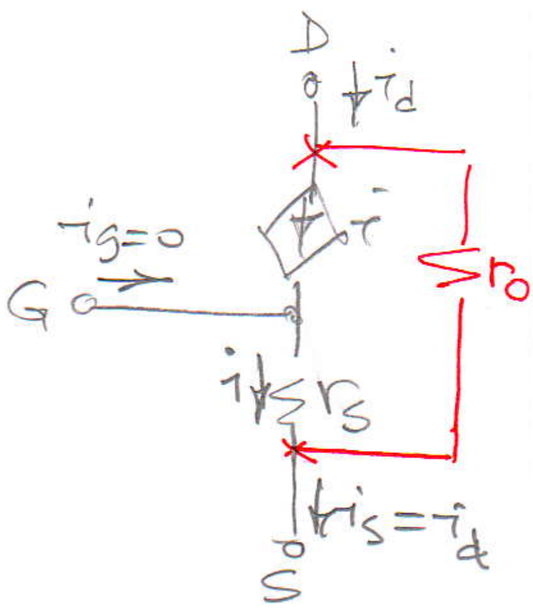
Question: How is i_d related to v_{gs}.





Including "Channel-length modulation" effect or "Early effect".





$$r_o = \frac{|V_A|}{I_D'}$$

I_D' = Drain current ignoring Early eff.

$$I_D' = \frac{1}{2} k V_{ov}^2$$

$$\boxed{g_m = K V_{ov}}$$

But $I_D \cong \frac{1}{2} K V_{ov}^2 \Rightarrow V_{ov} = \sqrt{\frac{2I_D}{K}}$

$$\Rightarrow \boxed{g_m \cong \sqrt{2KI_D}}$$

Also,

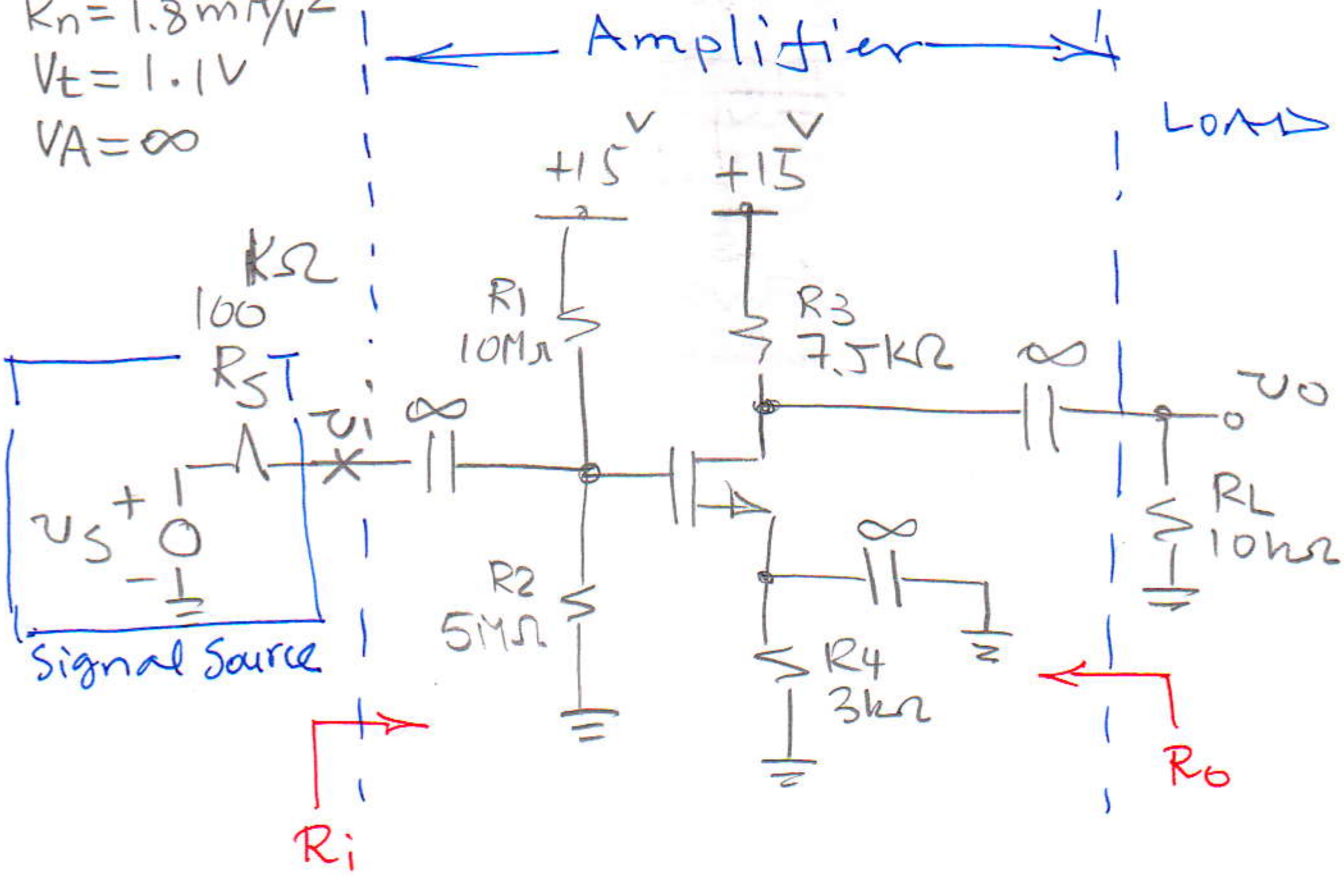
$$K = \frac{2I_D}{V_{ov}^2}$$

$$\Rightarrow \boxed{g_m \cong \frac{2I_D}{V_{ov}}}$$

Amplifiers

BJT	CE	CC	CB
MOS	CS	CD	CG

$K_n = 1.8\,mA/V^2$
$V_t = 1.1V$
$V_A = \infty$



$$A_{vo} = \left.\frac{v_o}{v_i}\right|_{R_L=\infty}$$

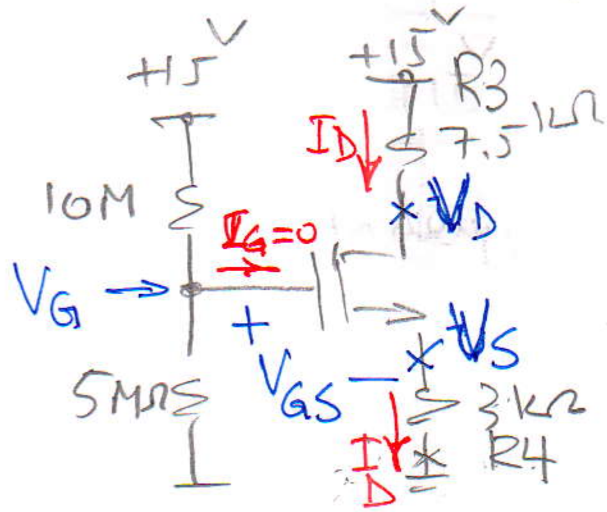
R_i :

R_O :

$$A_v = \left.\frac{v_o}{v_i}\right|_{R_L \neq \infty}$$

$$A_{vs} = \left.\frac{v_o}{v_s}\right|_{R_L \neq \infty}$$

DC Analysis



$$V_G = \frac{15}{10+5} \times 5 = 5^V$$

KVL: $V_G - V_{GS} - R_4 I_D = 0$

$$5 - V_{GS} - 3 I_D = 0 \quad (1)$$

Ass. sat. mode:

$$I_D = \frac{1}{2} K (V_{GS} - V_t)^2$$

$$I_D = \frac{1}{2} \times 1.8 (V_{GS} - 1.1)^2$$

$$= 0.9 (V_{GS} - 1.1)^2$$

$$= 0.9 (V_{GS}^2 - 2.2 V_{GS} + 1.21)$$

(2)

Eliminate I_D between (1) & (2):

$$5 - V_{GS} - 3 \times 0.9 (V_{GS}^2 - 2.2 V_{GS} + 1.21) = 0$$

$$2.7 V_{GS}^2 - 4.94 V_{GS} - 1.733 = 0 \Rightarrow V_{GS} \begin{cases} -3.01^V \text{ ✗} \\ 2.13^V \text{ ✓} \end{cases}$$

~~$I_D = \frac{1}{2} \times$~~

$$V_{ov} = V_{GS} - V_t =$$
$$= 2.13 - 1.1 = \underline{1.03^V}$$

$$I_D = \frac{1}{2} K V_{ov}^2 = \frac{1}{2} \times 1.8 \times (1.03)^2$$
$$= 0.9 \times 1.06$$
$$= 0.954\,mA$$

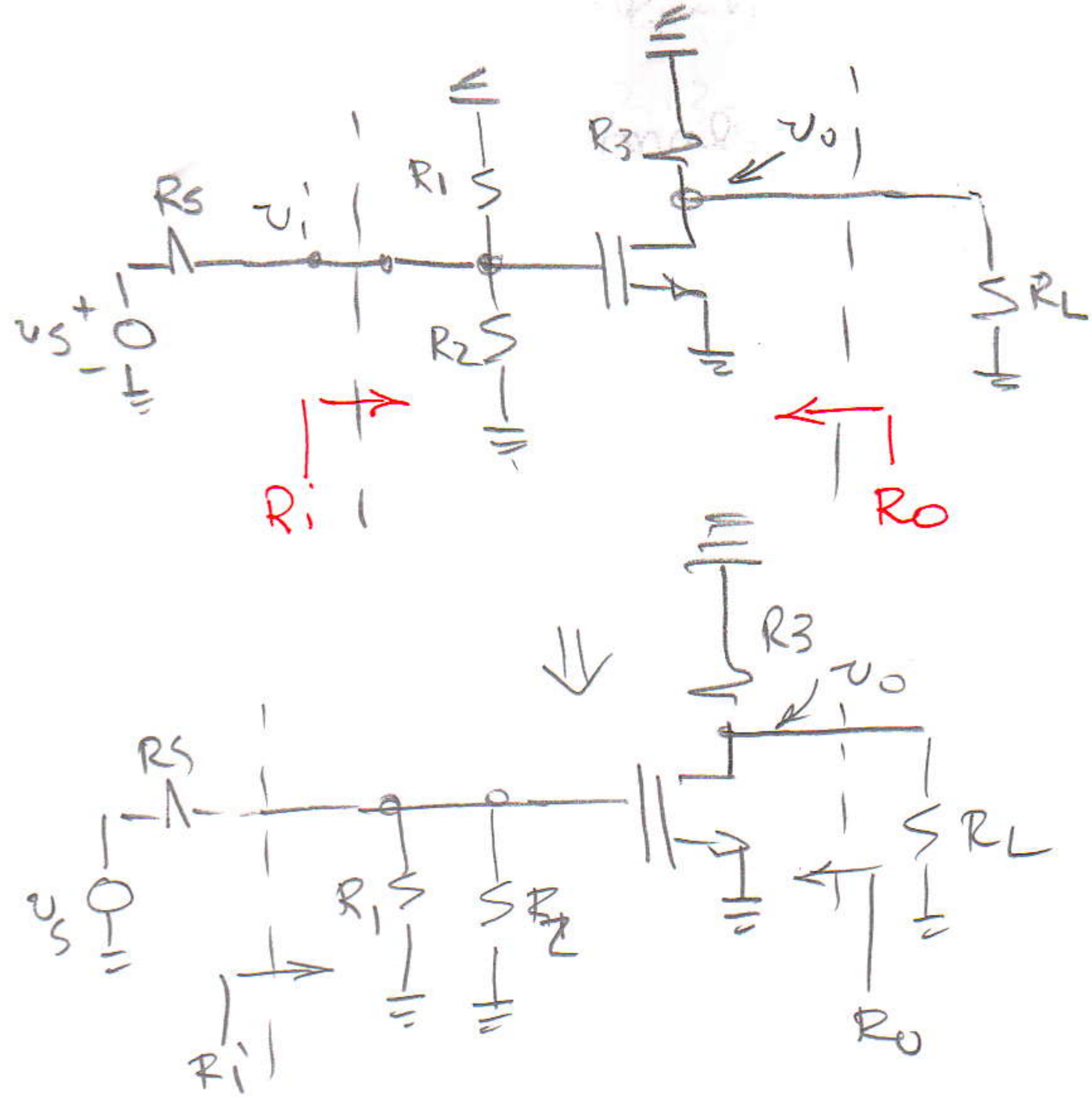
$$V_D = +15^V - R_3 I_D$$
$$= +15 - 7.5 \times 0.954 = 7.84^V$$
$$V_S = R_4 I_D = 3 \times 0.954 = 2.86^V$$

$$V_{DS} = V_D - V_S = 7.84 - 2.86 = 4.98^V$$
$$V_{OV} = 1.03^V$$

$\Rightarrow V_{DS} > V_{ov}$ ✓ sat. mode is the right mode.

$$g_m = K V_{OV} = 1.8 \times 1.03$$
$$= 1.854\,mS$$

Ac Analysis



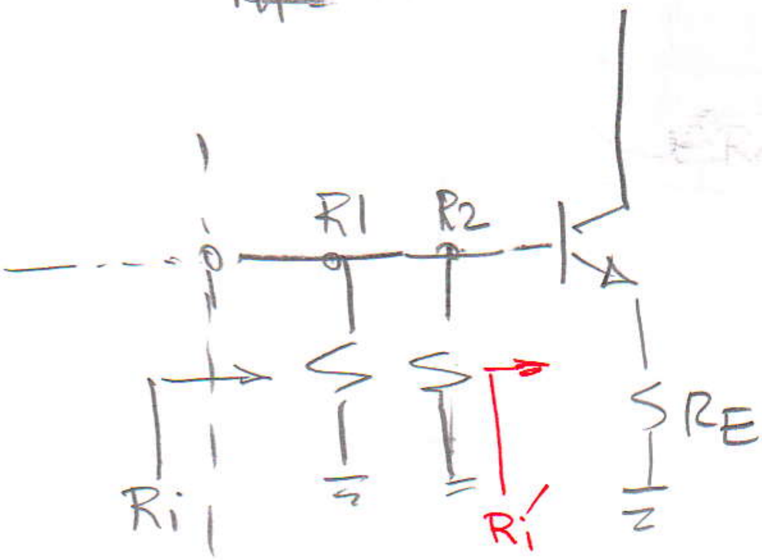
CS amp. (equivalent to CE).

BJT	MOS
$\frac{v_o}{v_i} = \frac{-g_m R_C}{1+g_m R_E}$	$\frac{v_o}{v_i} = \frac{-g_m R_D}{1+g_m R_S}$

$$A_{vo} = \frac{v_o}{v_i}\Big|_{R_L=\infty} = \frac{-g_m R_3}{1+g_m \times 0} = -g_m R_3$$

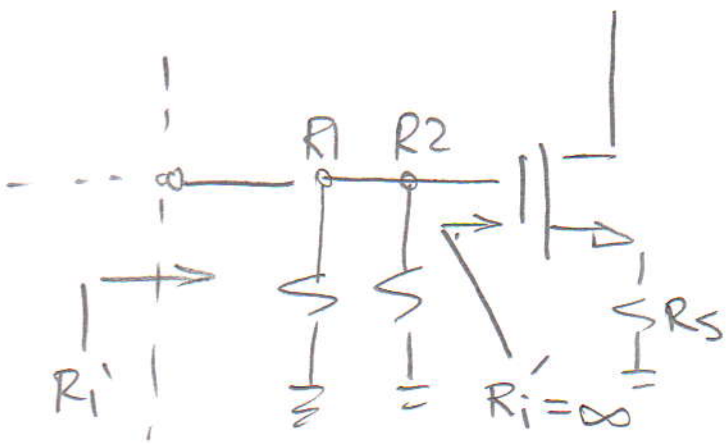
$$A_v = \frac{v_o}{v_i}\Big|_{R_L\neq\infty} = \frac{-g_m (R_3 \| R_L)}{1+g_m \times 0} = -g_m (R_3 \| R_L)$$

Input Resis.



$$R_i' = r_\pi + (\beta+1) R_E$$

$$R_i = R_1 \| R_2 \| R_i'$$



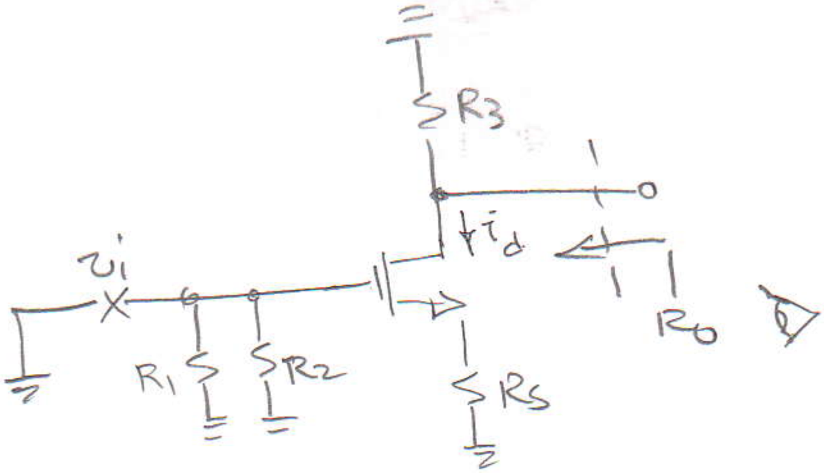
$$R_i' = \infty$$

$$R_i = R_1 \| R_2 \| \infty$$

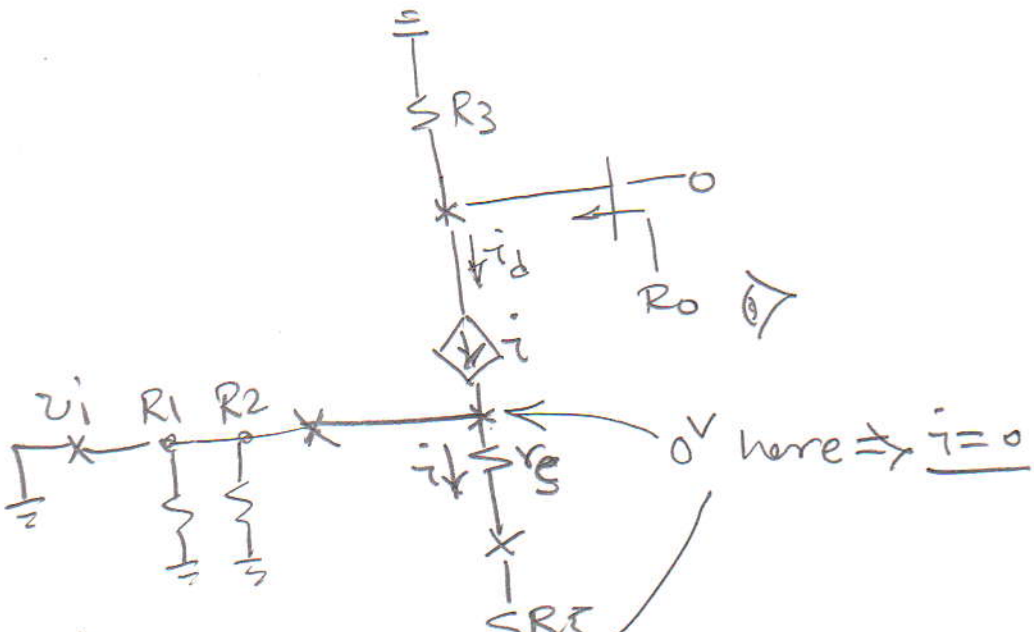
$$= R_1 \| R_2$$

~~Finally~~

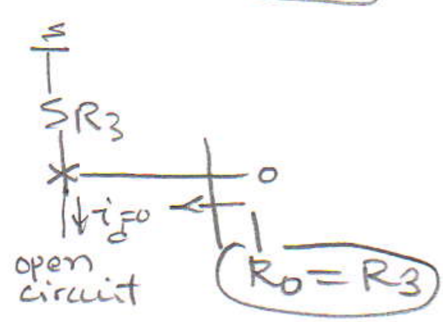
Output Resistance



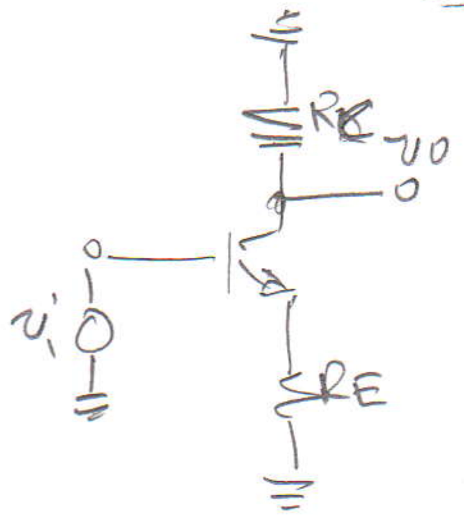
⇓



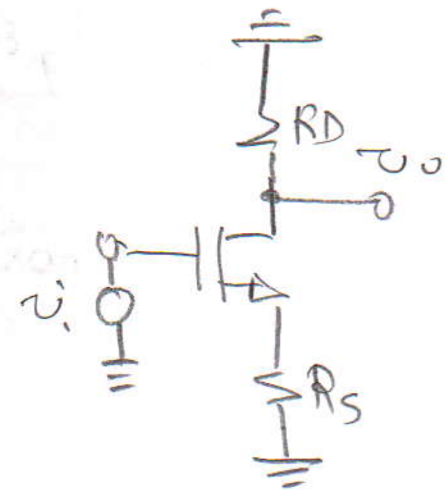
$i = 0 \Rightarrow i_d = 0$



DESIGN Example



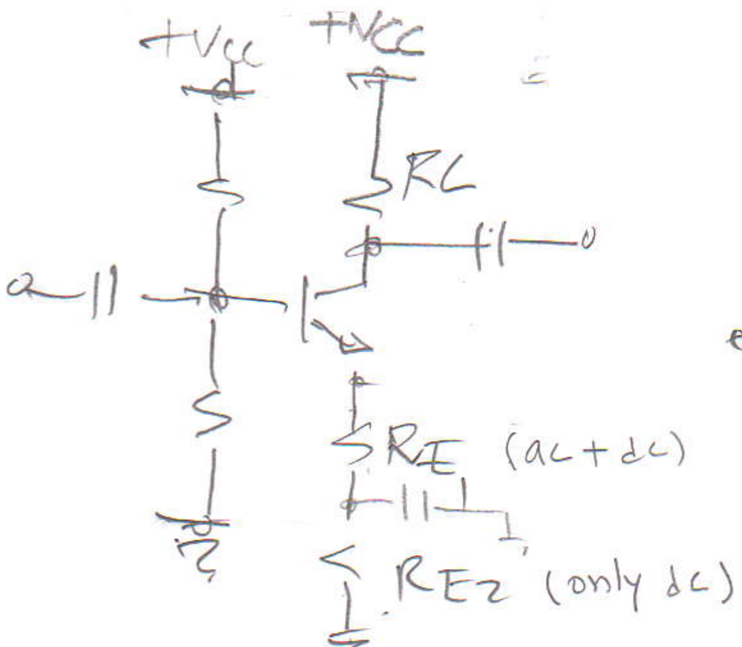
CE

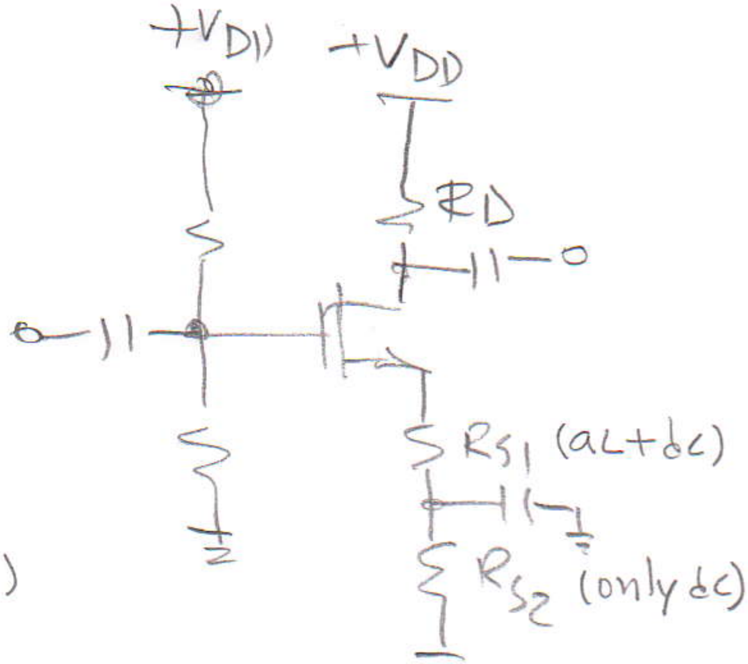


CS

$$A_{v0} \simeq \frac{-g_m R_C}{1+g_m R_E}$$

$$A_{v0} = \frac{-g_m R_D}{1+g_m R_S}$$





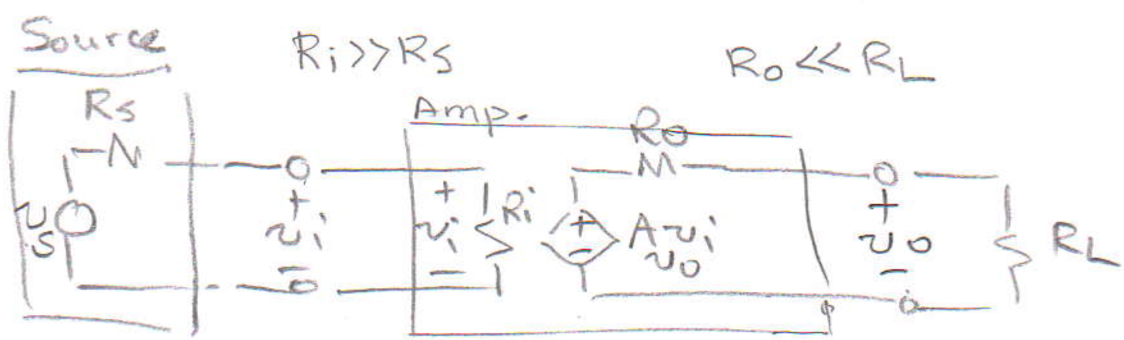
$$A_{v0} = \frac{-g_m R_C}{1+g_m R_{E1}}$$

$$A_{v0} = \frac{-g_m R_D}{1+g_m R_{S1}}$$

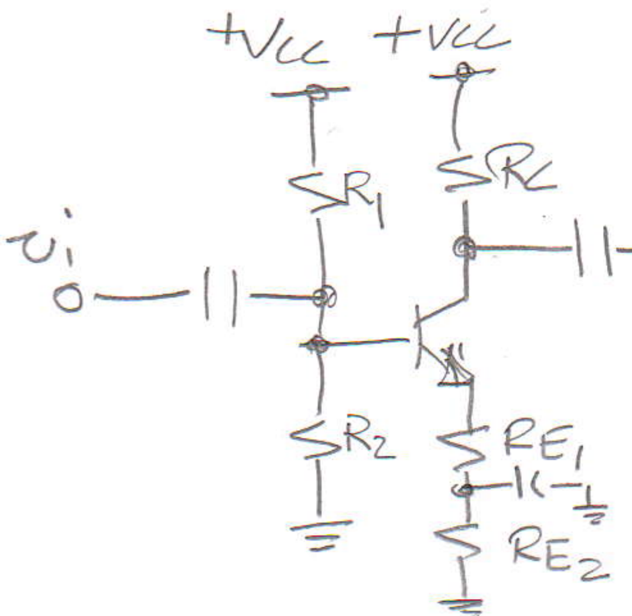
Design (using BJT, and may be other things . . .) such that:

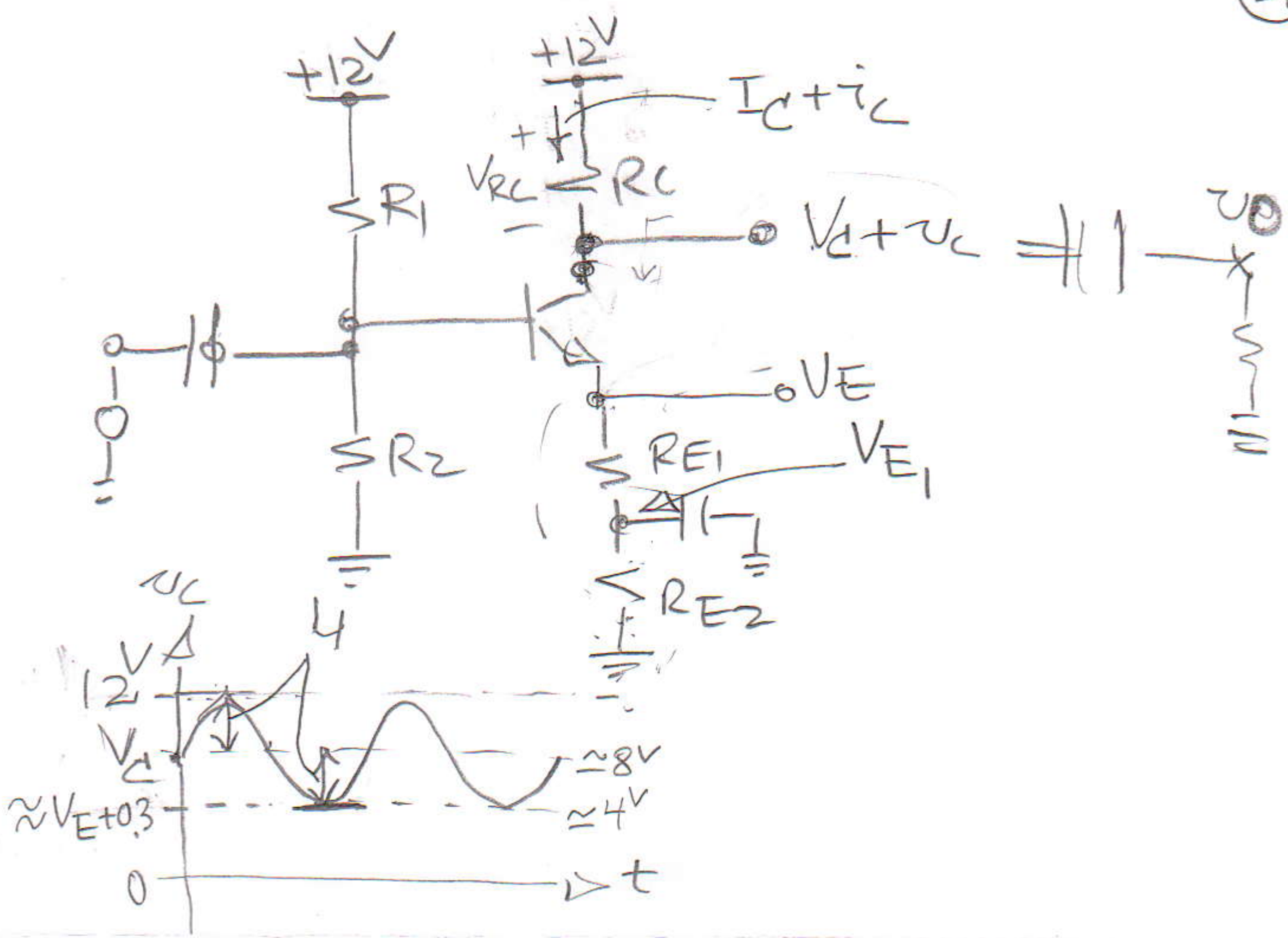
1. – $|A_{vo}| = 25\,V$
2. – $R_{out} = 5\,k\Omega$
3. – $R_{in} \geq 15\,k\Omega$
4. – $V_{CC} = +12\,V,\ 0$
5. – Reasonable swing is achieved

$\boxed{\beta = 100}$



Candidate Design: CE amp.





Rule of Thumb:

Try $V_{RC} = V_{CE} = V_E \simeq \frac{V_{CC}}{3} = 4^V$
for a reasonable swing!

On the other hand, we know that (in CE):

$$R_{out} = R_C \Rightarrow \underline{R_C = 5^{k\Omega}}$$

$$I_C = \frac{V_{RC}}{R_C} = \frac{4}{5^{k\Omega}} = \underline{0.8\,mA}$$

$$\Rightarrow g_m \simeq 40 \times 0.8 = 32\,mS$$

$$\Rightarrow g_m R_C = 32 \times 5 = 160 \ \frac{V}{V}$$

$$|A_{vo}| = \frac{g_m R_C}{1 + g_m R_{E_1}} = 25 \Rightarrow$$

$$g_m R_{E_1} = \frac{160}{25} - 1$$

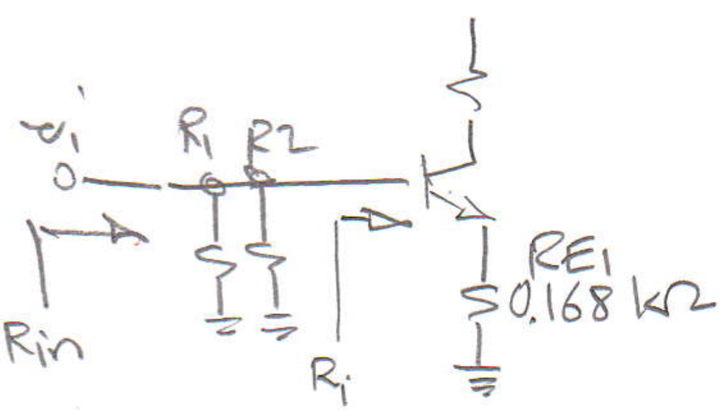
$$\Rightarrow R_{E_1} = \underline{0.168 \ k\Omega}$$

$$\text{or } \underline{168}\ \Omega$$

$$V_E = 4^V, \quad I_C = 0.8^{mA} \Rightarrow I_E \simeq I_C = 0.8^{mA}$$

$$R_{E_1} + R_{E_2} = \frac{4^V}{0.8} = 5^{k\Omega}$$

$$R_{E_2} = 5 - .168 \cong \underline{4.83}\ k\Omega$$



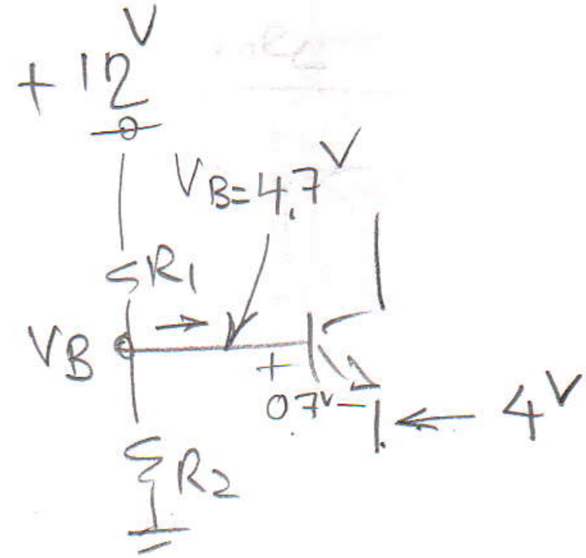
$$R_i = r_\pi + (\beta + 1) R_{E_1}$$

$$= \frac{100}{32} + 101 \times 0.168$$

$$\simeq 20 k\Omega$$

$$R_{in} = \underline{\underline{R_i}} \| R_1 \| R_2 \geq 15^{k\Omega}$$

$$\Rightarrow R_1 \| R_2 \geq 60^{k\Omega}$$



Ignore I_B (as an approx.)

$$V_B \simeq \frac{+12^V}{R_1+R_2} \times R_2$$

$$\Rightarrow 4.7 = \frac{12}{R_1+R_2} R_2$$

$$\rightarrow \frac{R_1}{R_2} = \frac{12}{4.7} - 1 \Rightarrow \underline{R_1 \cong 1.55 R_2}$$

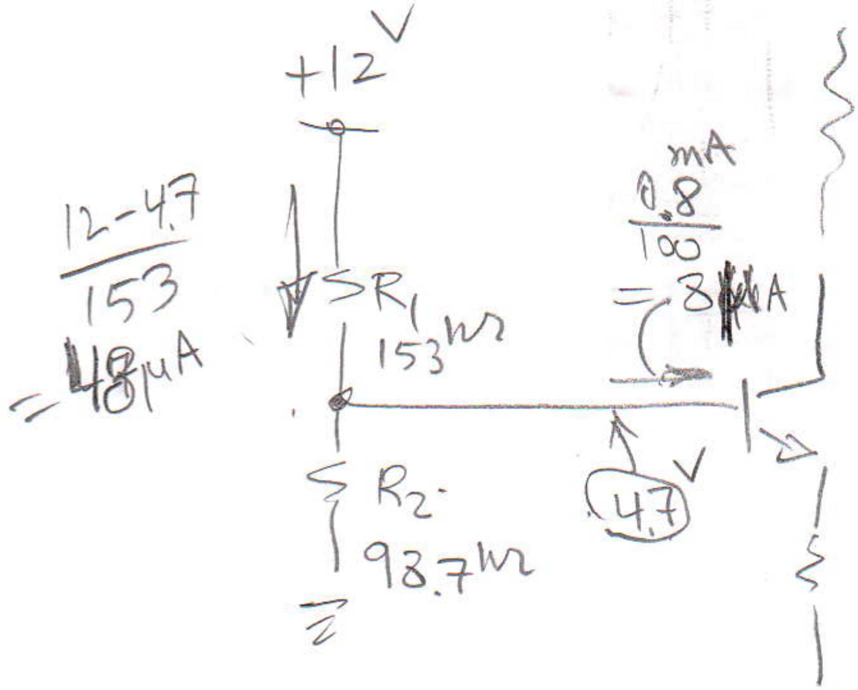
On the other hand,

$$\frac{R_1 R_2}{R_1+R_2} \geq 60 \quad \cancel{\text{let}}$$

let $\dfrac{R_1 R_2}{R_1+R_2} = 60$

$$\frac{1.55 R_2^2}{2.55 R_2} = 60 \Rightarrow R_2 = 98.7^{K\Omega}$$

$$\Rightarrow \underline{R_1 = 153 K\Omega}$$



If Rin was required to be so large (e.g. ≥ 40kΩ) that no value of $R_1 \| R_2$ would be feasible, we would bring a CC stage as Buffer: