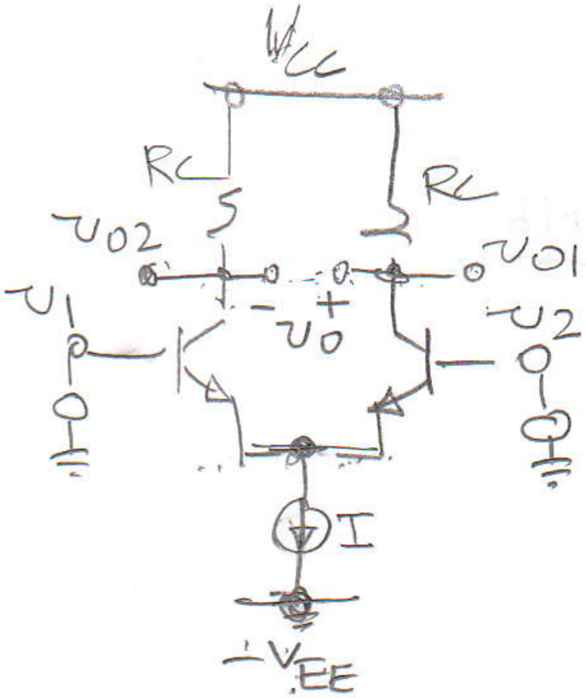
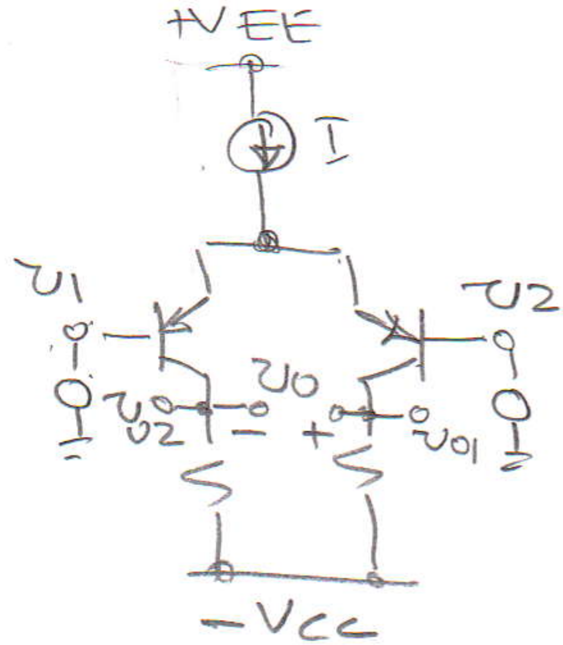
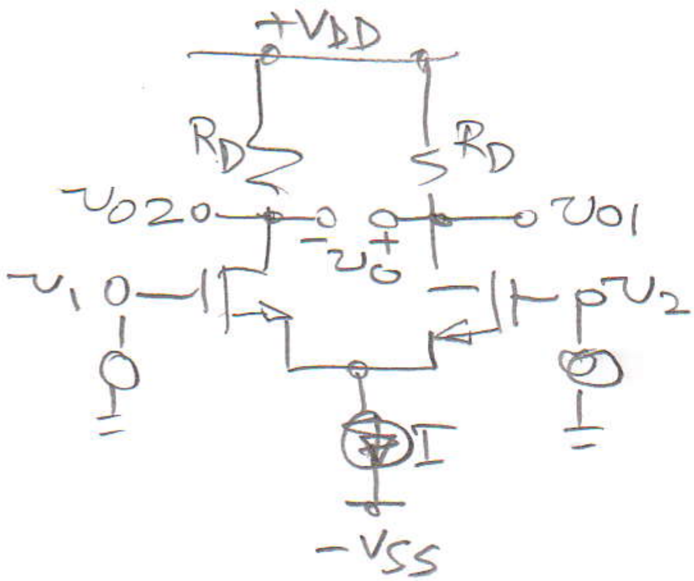
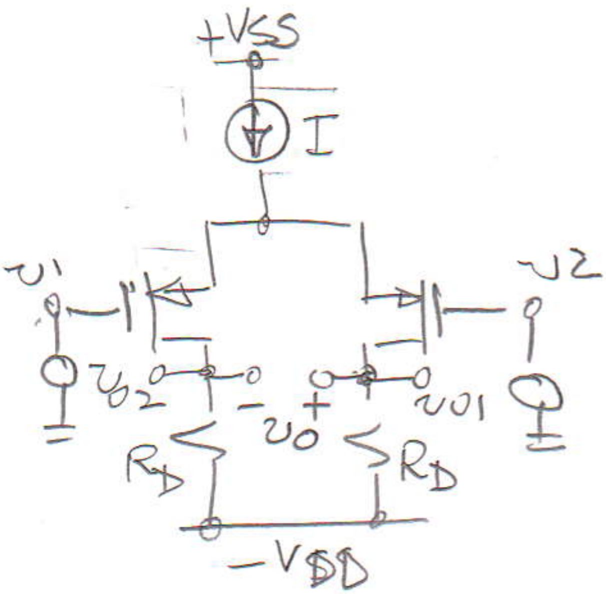
Differential Amplifiers









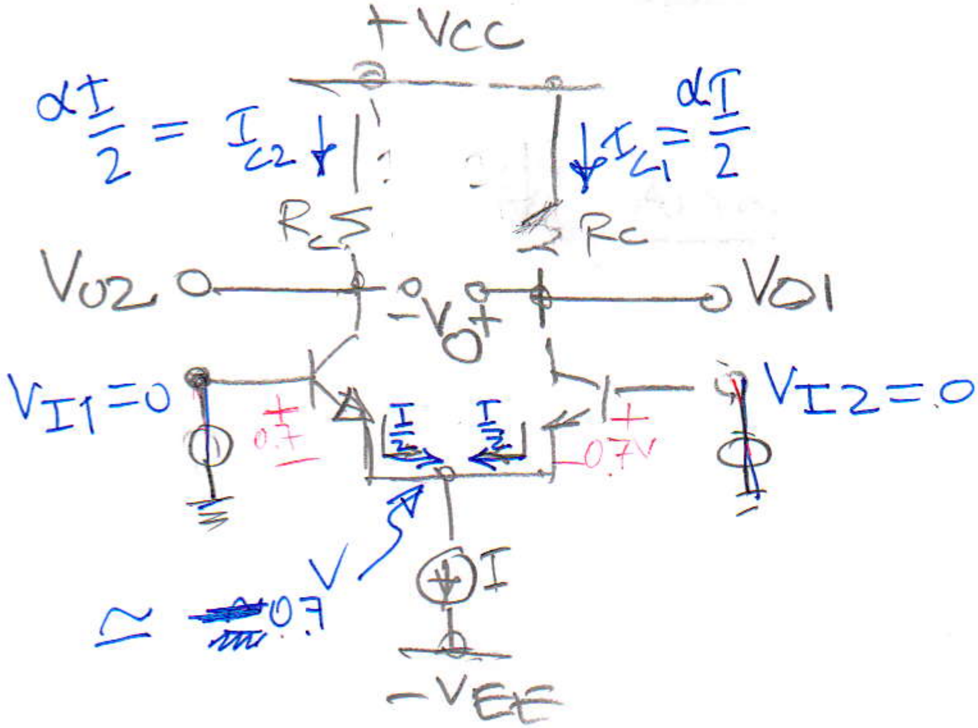
Objective

- to only amplify the difference between the two inputs

assumptions

- transistors identical
- current source ideal
- transistors are in active mode (saturation for MOS).

DC Analysis



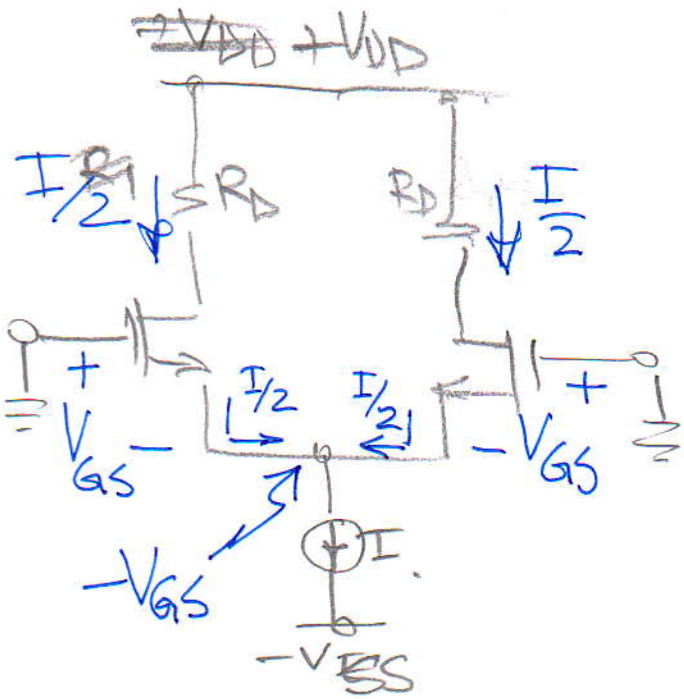
$$V_{O1} = V_{C1} = V_{CC} - R_C \frac{\alpha I}{2}$$

$$V_{O2} = V_{C2} = V_{CC} - R_C \frac{\alpha I}{2}$$

$$V_O = V_{O1} - V_{O2} = 0$$

$$g_m \triangleq g_{m1} = g_{m2} \cong 40 \frac{\alpha I}{2} \approx \underline{20 I}$$

DC Analysis (cont'd)



$$I_D = \frac{1}{2} k V_{ov}^2 \Rightarrow \frac{I}{2} = \frac{1}{2} k V_{ov}^2$$

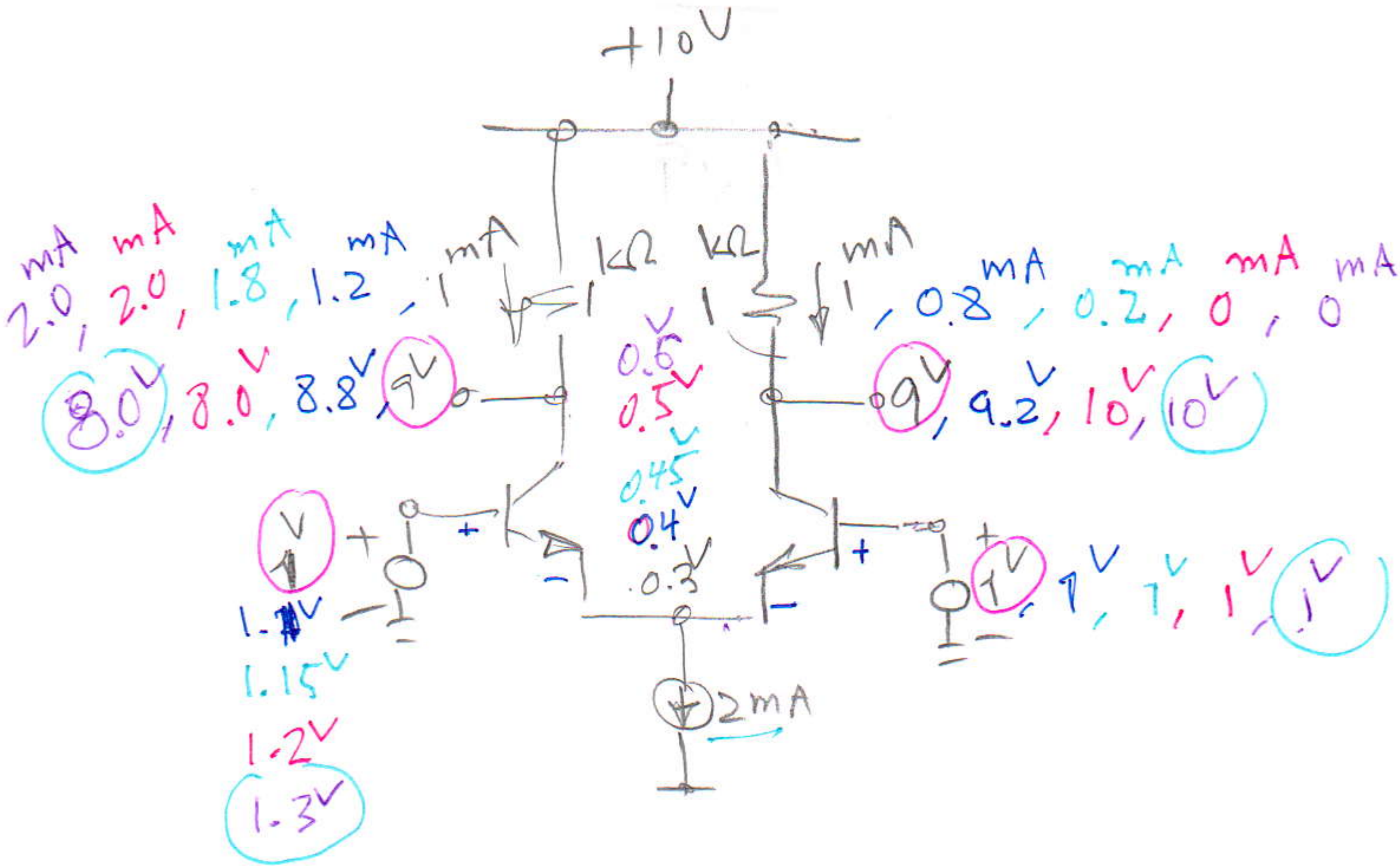
$$\Rightarrow \quad V_{ov} = \sqrt{I/k}$$

$$\Rightarrow V_{GS} = V_{ov} + V_t = \sqrt{\frac{I}{k}} + V_t$$

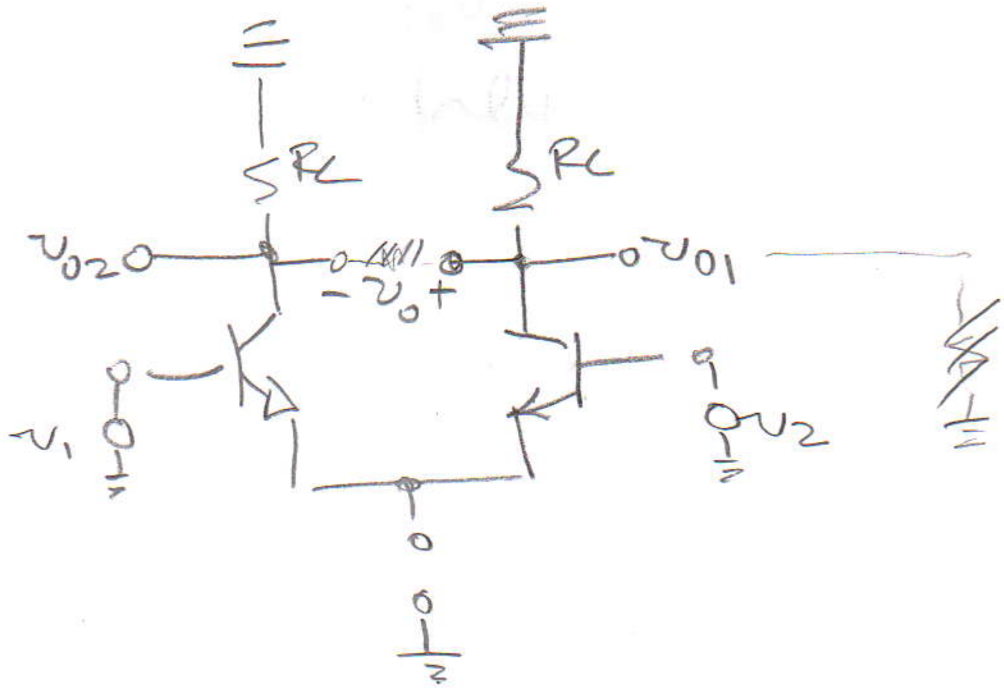
$$g_m \triangleq g_{m1} = g_{m2} = \sqrt{2 k I_D} =$$

$$= \sqrt{2kI/2} = \sqrt{kI}$$

Big Picture


+10V
1 kΩ
1 kΩ
2.0 mA, 2.0 mA, 1.8 mA, 1.2 mA, 1 mA
1 mA, 0.8 mA, 0.2 mA, 0 mA, 0 mA
8.0V, 8.0V, 8.8V, 9V
9V, 9.2V, 10V, 10V
0.6V
0.5V
0.45V
0.4V
0.3V
1V
1.1V
1.15V
1.2V
1.3V
1V, 1V, 1V, 1V, 1V
2mA

AC Analysis



$$A_d = \frac{v_{O1}}{v_1 - v_2} = \frac{v_{O1}}{v_d}, \text{ where } v_d \triangleq v_1 - v_2$$

$$A_d = \frac{v_{O2}}{v_1 - v_2} = \frac{v_{O2}}{v_d},$$

$$A_d = \frac{v_{O1} - v_{O2}}{v_1 - v_2} = \frac{v_O}{v_d}$$

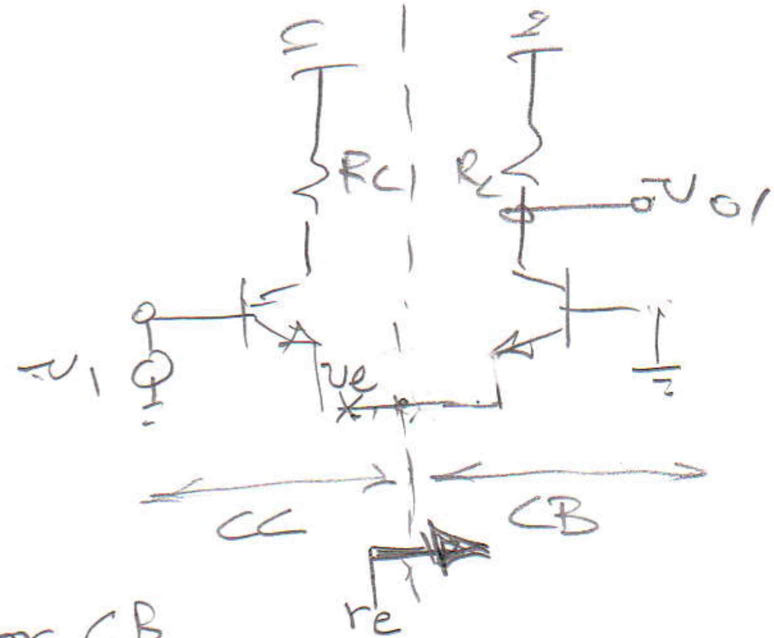
How to do that?

— Superposition -

Sure, But tedious!

Superposition

#1 v_1: on v_2: off

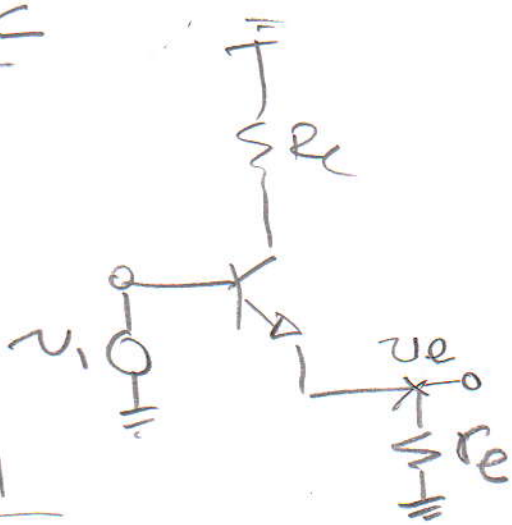


$$\frac{v_{o1}}{v_1} = ?$$

for CB

$$\frac{v_{o1}}{v_e} = g_m R_C$$

for CC



~~v_{o2}~~

R_L

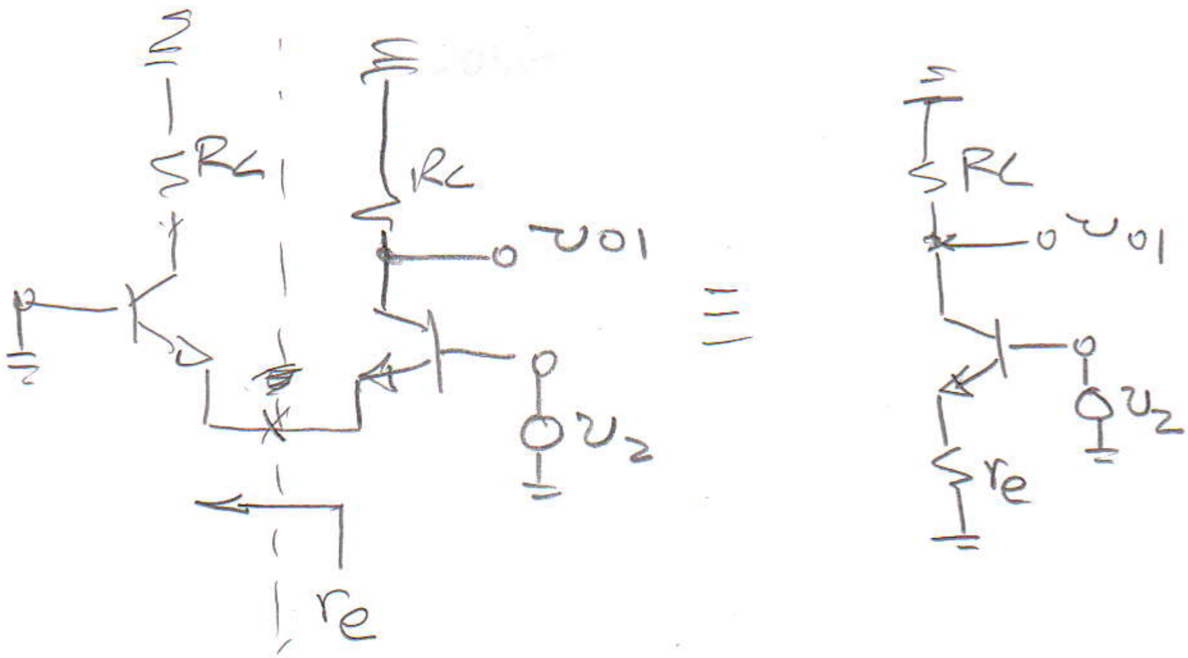
$$\frac{v_e}{v_1} = \frac{g_m r_e}{1+g_m r_e} \approx \frac{1}{2}$$

Overall

$$\frac{v_{o1}}{v_1} = \frac{v_{o1}}{v_e} \times \frac{v_e}{v_1} = g_m R_C \times \frac{1}{2} \Rightarrow \boxed{\frac{v_{o1}}{v_1} = \frac{1}{2} g_m R_C}$$

Superposition

#2: v_1: off, v_2: ON



CE

$$\frac{v_{o1}}{v_2} = \frac{-g_m R_C}{1 + g_m r_e} = \boxed{-\frac{1}{2} g_m R_C}$$

R_E

Therefore,

#1 $\frac{v_{o1}}{v_1} = \frac{1}{2} g_m R_C \Rightarrow v_{o1} = \left(\frac{1}{2} g_m R_C\right) v_1$

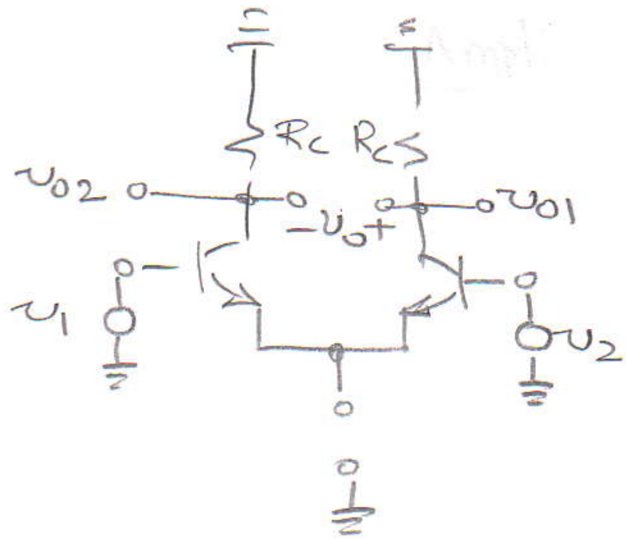
#2 $\frac{v_{o1}}{v_2} = -\frac{1}{2} g_m R_C \Rightarrow v_{o1} = \left(-\frac{1}{2} g_m R_C\right) v_2$

$\Rightarrow v_{o1} = \left(\frac{1}{2} g_m R_C\right) v_1 + \left(-\frac{1}{2} g_m R_C\right) v_2$

$\Rightarrow v_o = \left(\frac{1}{2} g_m R_C\right)(v_1 - v_2) \Rightarrow \boxed{A_d = \frac{1}{2} g_m R_C}$

Differential Amplifiers (cont'd)

Calculated the differential gain (last lecture) using superposition:



$$A_d = \frac{v_{O1}}{v_1 - v_2} = \frac{1}{2} g_m R_C \quad (\text{if } v_{O1} \text{ is the output})$$

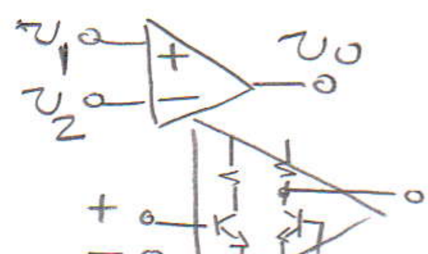
$$A_d = \frac{v_{O2}}{v_1 - v_2} = \frac{-1}{2} g_m R_C \quad (\text{if } v_{O2} \text{ is the output})$$

$$A_d = \frac{v_O}{v_1 - v_2} = \frac{v_{O1} - v_{O2}}{v_1 - v_2} = g_m R_C \quad (\text{if } v_O = v_{O1} - v_{O2} \text{ is the output})$$

Let us assume that v_{O1} is the output:

$$\Rightarrow v_{O1} = \underbrace{\left(\frac{1}{2} g_m R_C\right)}_{A_d > 0}(v_1 - v_2)$$

$$= A_d v_1 + (-A_d) v_2$$



$\Rightarrow v_{O1} \uparrow\downarrow$ if $v_1 \uparrow\downarrow \Rightarrow v_1$: non-inverting input

$v_{O1} \downarrow\uparrow$ if $v_2 \uparrow\downarrow \Rightarrow v_2$: inverting input

A Couple of Definitions

$v_d \triangleq v_1 - v_2$: differential input

$v_{cm} \triangleq \frac{v_1 + v_2}{2}$: common-mode input

$$v_O = A_d v_d + A_{cm} v_{cm}$$ superposition

A_d : differential gain $= \left.\frac{v_O}{v_d}\right|_{v_{cm}=0}$

A_{cm} : common-mode gain $= \left.\frac{v_O}{v_{cm}}\right|_{v_d=0}$

An ideal differential amplifier features $A_{cm}=0 \Rightarrow$ the signal component common to both v_1 and v_2, e.g., the exogenous ~~will~~ interference, will not appear in the output.

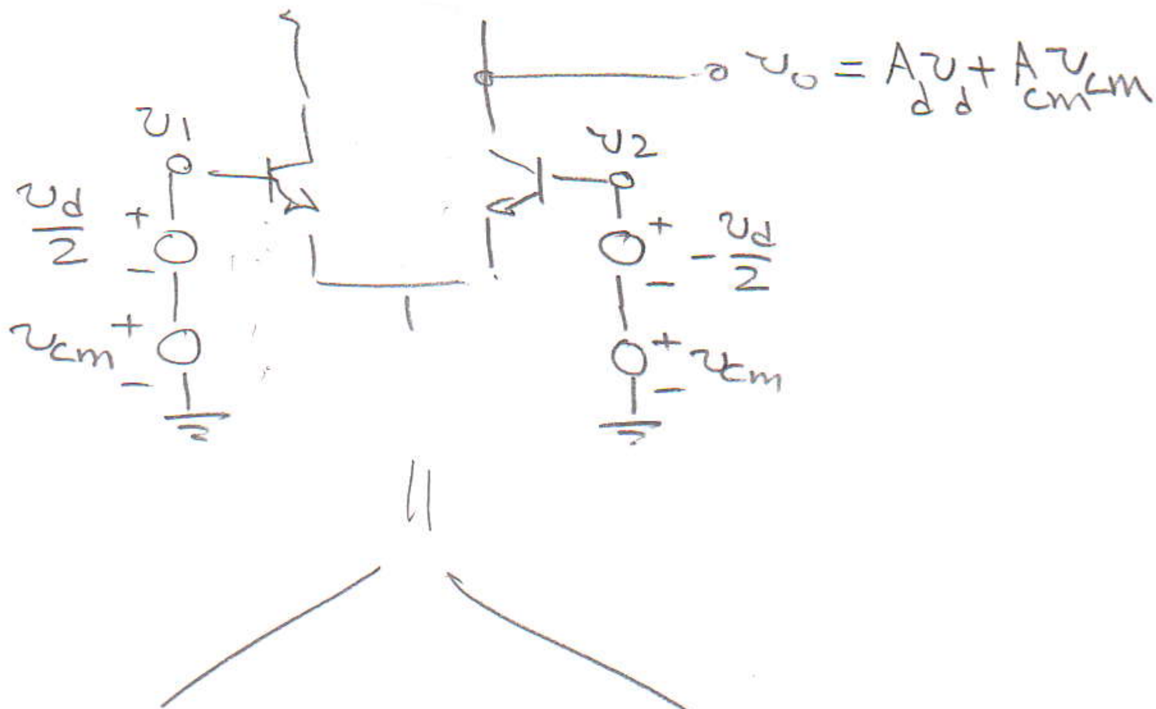
In practice, $A_{cm} \neq 0 \Rightarrow v_O$ will also be a function of $\frac{v_1+v_2}{2}$. However, A_{cm} is very small (by proper design), in a commercial op-amp.

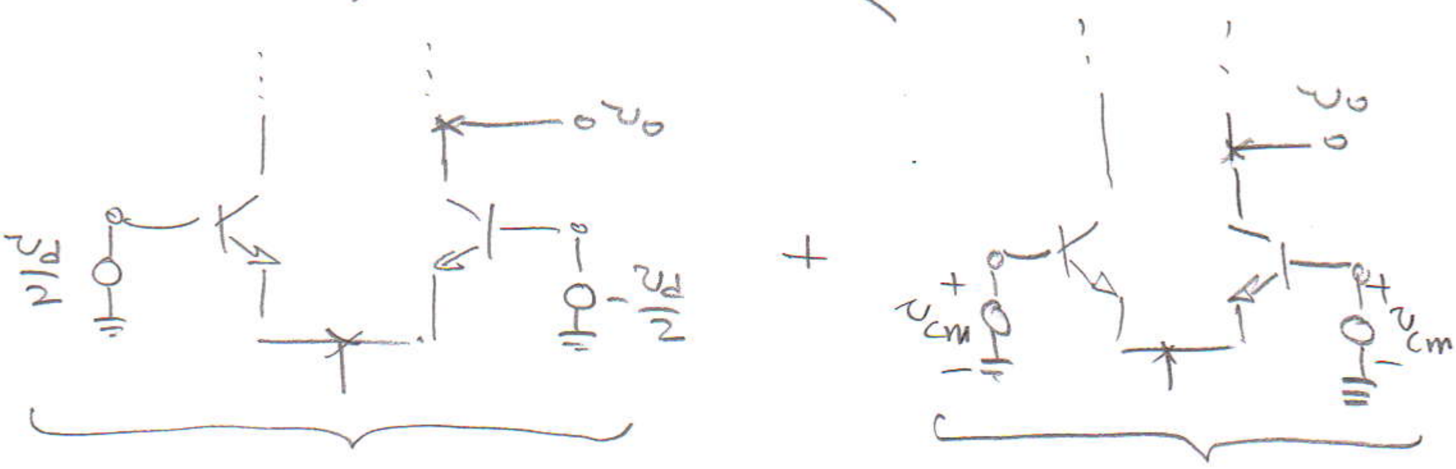
CMRR: Common-Mode Rejection Ratio $\triangleq \left|\frac{A_d}{A_{cm}}\right|$

CMRR is very large in a commercial amp.

$CMRR(dB) = 20 \log \left|\frac{A_d}{A_{cm}}\right|$

$$\left.\begin{aligned} v_d &\triangleq v_1 - v_2 \\ v_{cm} &\triangleq \tfrac{1}{2} v_1 + \tfrac{1}{2} v_2 \end{aligned}\right\} \Rightarrow \begin{aligned} v_1 &= \frac{v_d}{2} + v_{cm} \\ v_2 &= -\frac{v_d}{2} + v_{cm} \end{aligned}$$

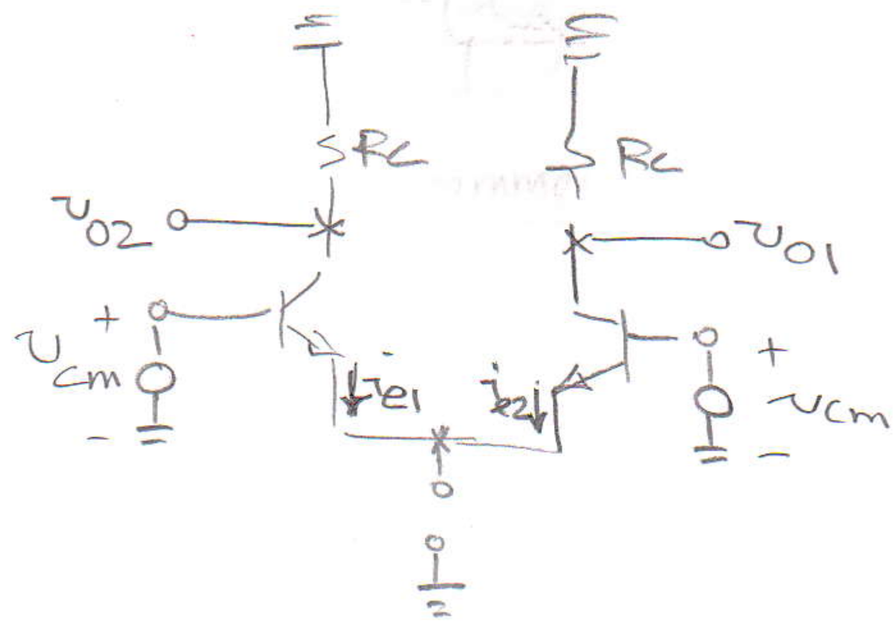




$v_o = A_d v_d$

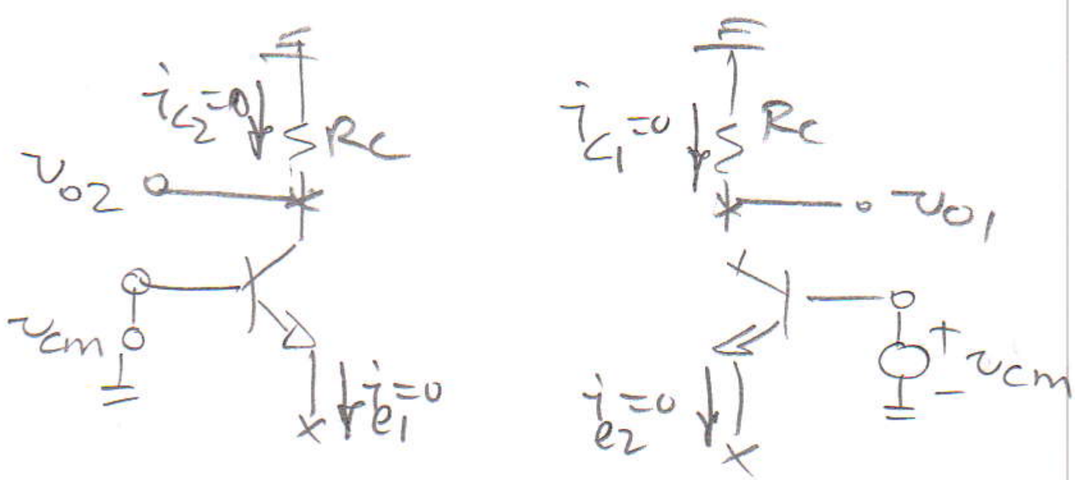
$v_o = A_{cm} v_{cm}$

Calc. Common-mode gain



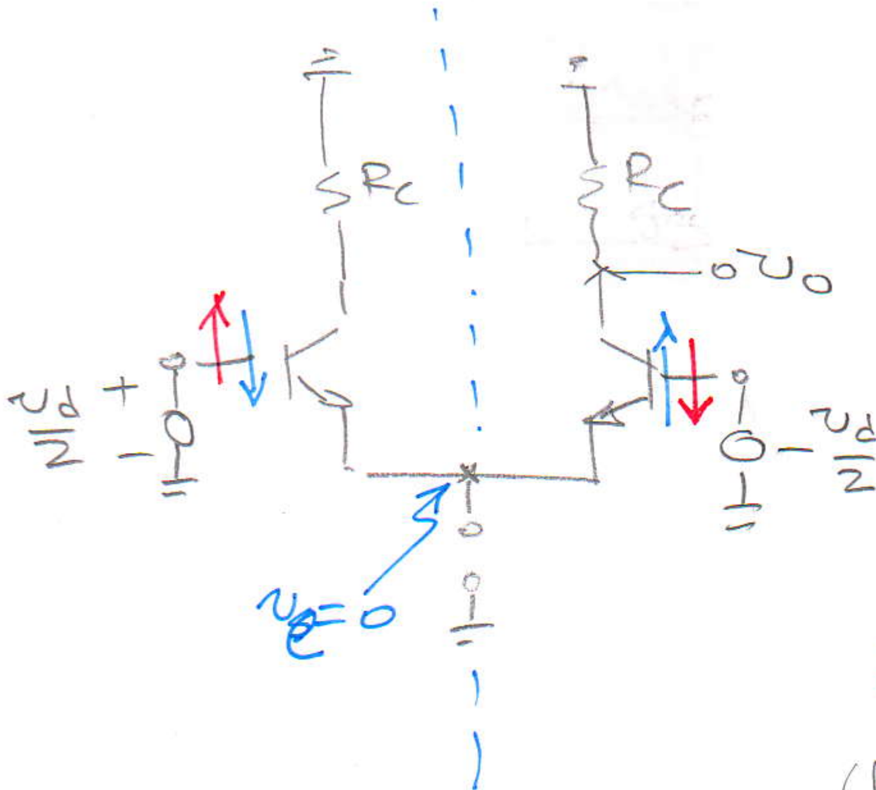
$$\left.\begin{aligned} &i_{e1} = i_{e2} \text{ (because the circuit is symmetrical)} \\ &i_{e1} + i_{e2} = 0 \quad \text{(by KCL)} \end{aligned}\right\} \Rightarrow i_{e1} = i_{e2} = 0$$

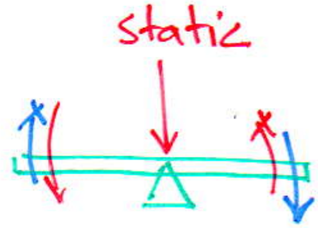
⇒ the link between E1 & E2 can be cut open!



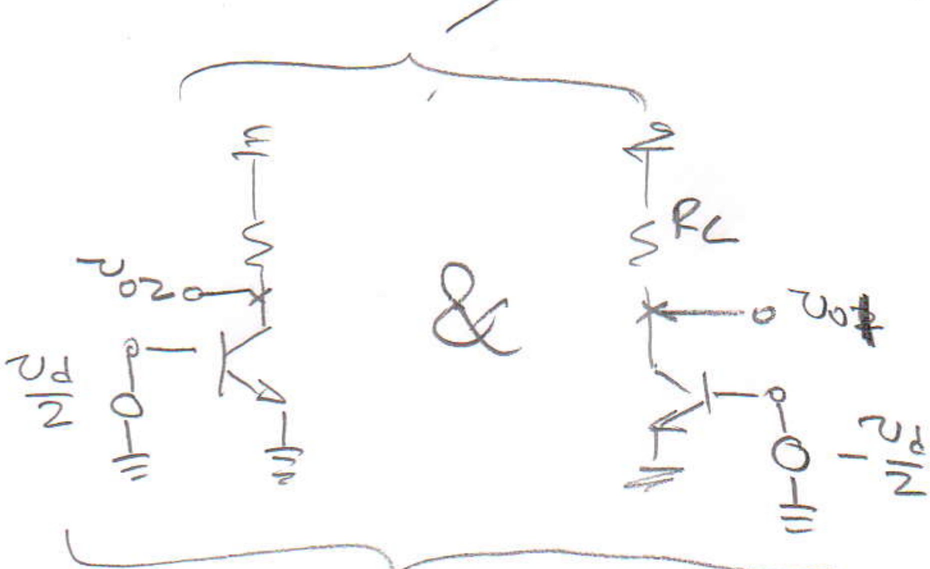
$$\left.\begin{aligned} v_{o1} &= 0 \\ v_{o2} &= 0 \end{aligned}\right\} \Rightarrow \underline{A_{cm} = 0}$$

Calc. Diff. Gain





(Mechanical Analogy: The Seesaw)

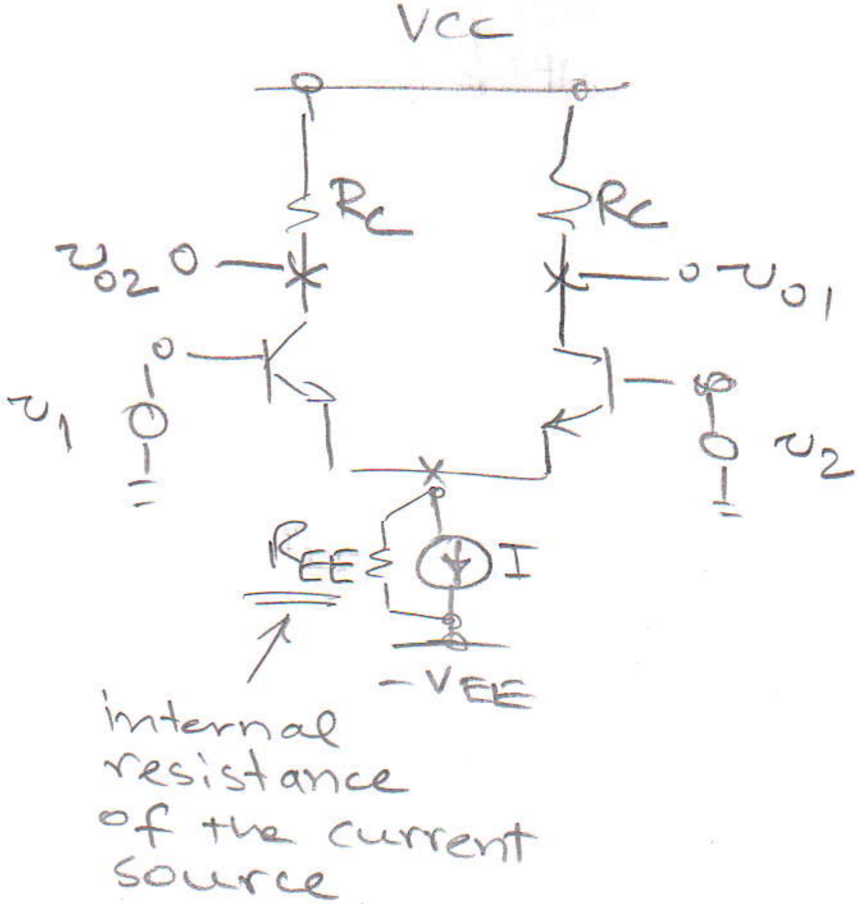


Half-Circuits

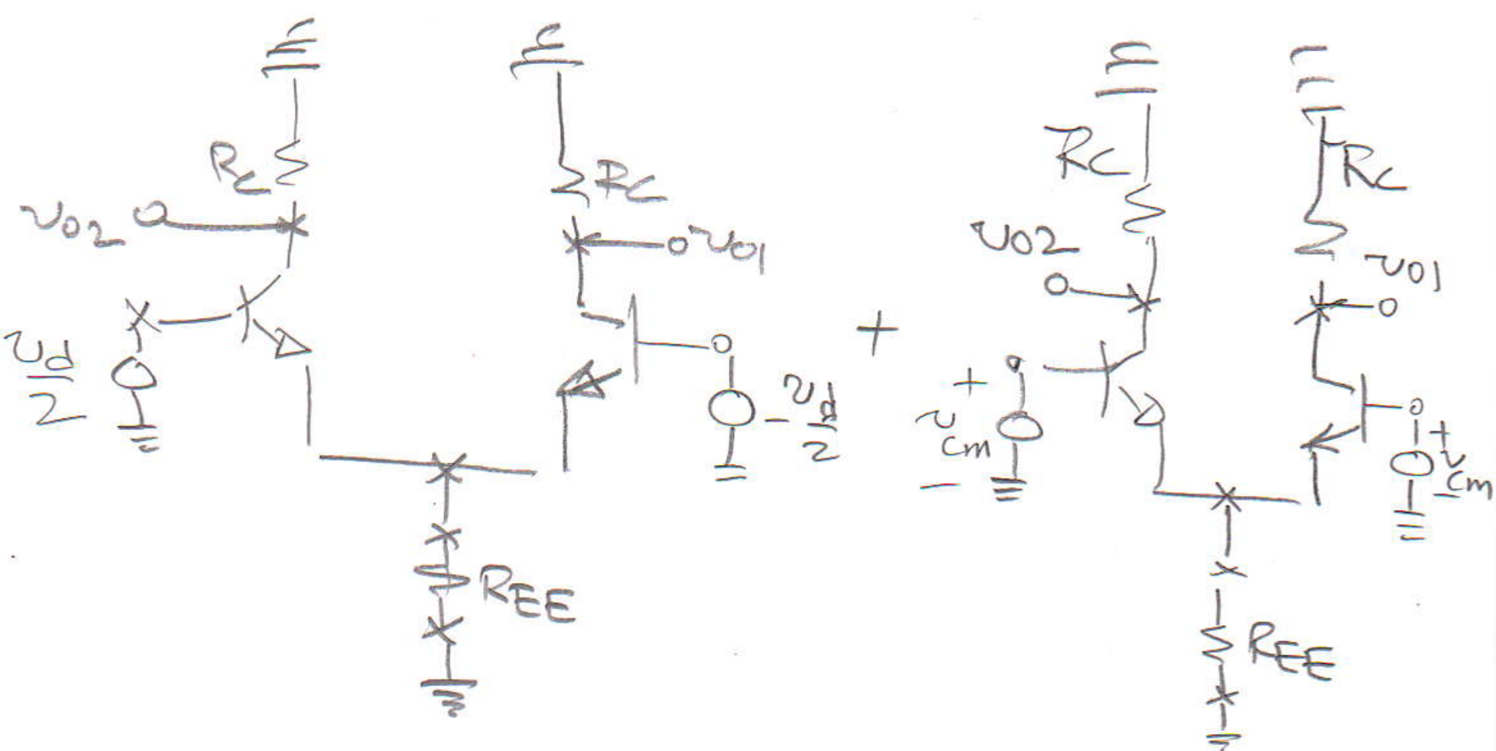
$$\frac{v_{o1}}{\left(-\frac{v_d}{2}\right)} = -g_m R_C \Rightarrow \boxed{\frac{v_{o1}}{v_d} = \frac{1}{2} g_m R_C}$$

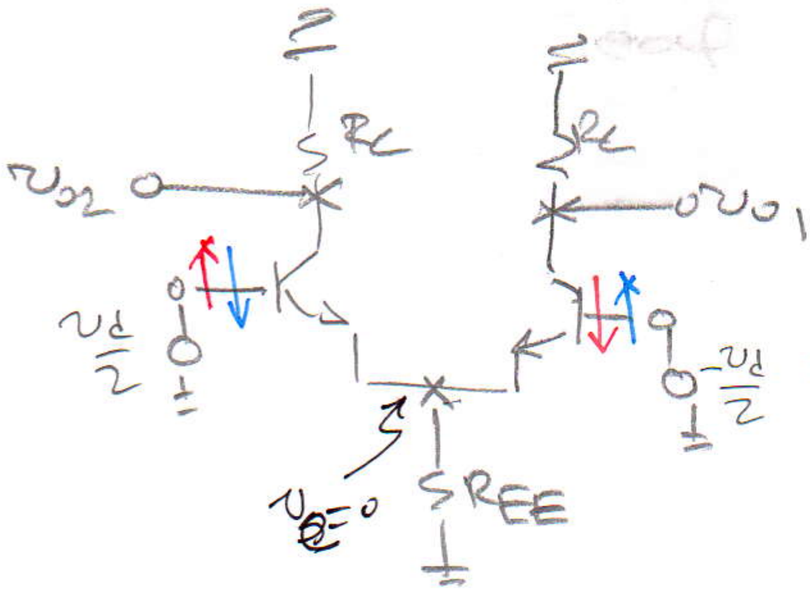
$$\frac{v_{o2}}{\left(\frac{v_d}{2}\right)} = -g_m R_C \Rightarrow \boxed{\frac{v_{o2}}{v_d} = -\frac{1}{2} g_m R_C}$$

Situation for the Case with a non-ideal Current Source



⇓



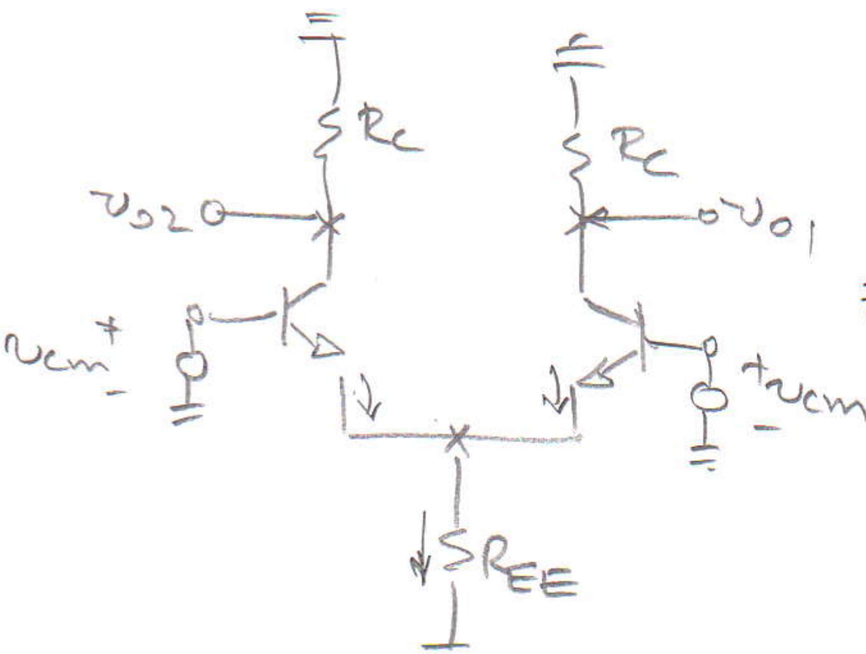


$$\Rightarrow \frac{v_{o1}}{\left(-\frac{v_d}{2}\right)} = -g_m R_C$$

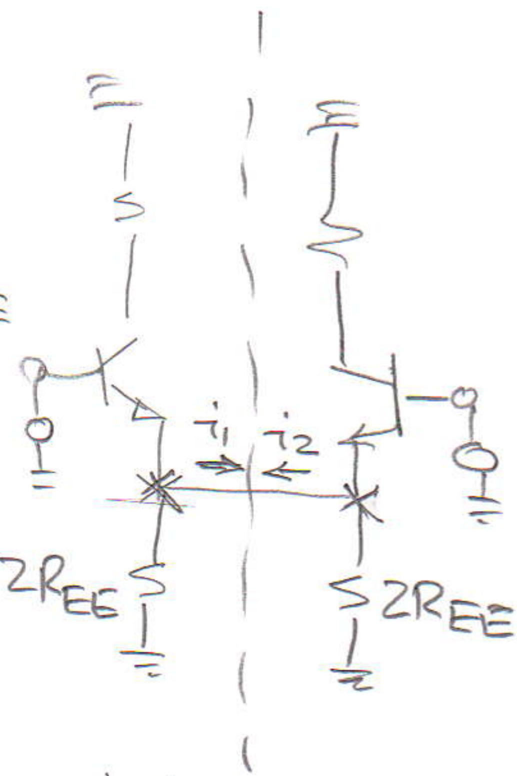
$$\Rightarrow \boxed{\frac{v_{o1}}{v_d} = \frac{1}{2} g_m R_C}$$

$$\frac{v_{o2}}{\left(\frac{v_d}{2}\right)} = -g_m R_C$$

$$\Rightarrow \boxed{\frac{v_{o2}}{v_d} = -\frac{1}{2} g_m R_C}$$



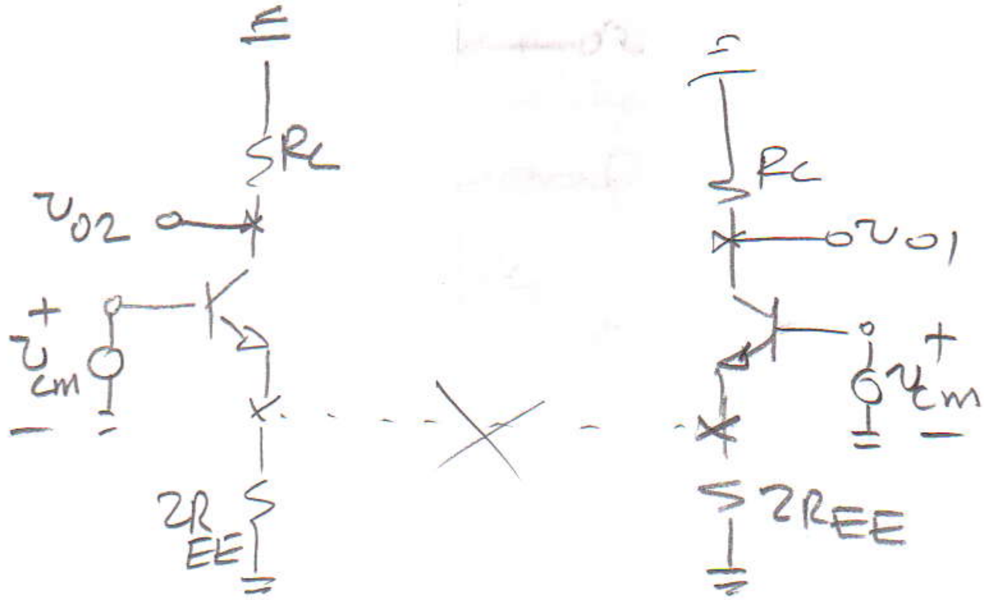
$\equiv$



$i_1 = i_2$ (symmetry)

$i_1 + i_2 = 0$ (by KCL)

$\Rightarrow i_1 = i_2 = 0$



$$A_{cm} = \frac{v_{o1}}{v_{cm}} = \frac{-g_m R_C}{1 + g_m(2R_{EE})}$$

$$A_{cm} = \frac{v_{o2}}{v_{cm}} = \frac{-g_m R_C}{1 + g_m(2R_{EE})}$$

If $v_o \triangleq v_{o1} - v_{o2}$ was taken as the input. then $\underline{A_{cm} = 0}$

<u>Example</u>

$\Rightarrow$ $I = 2mA \rightarrow g_m = 40mS$

$R_C = 1 k\Omega$

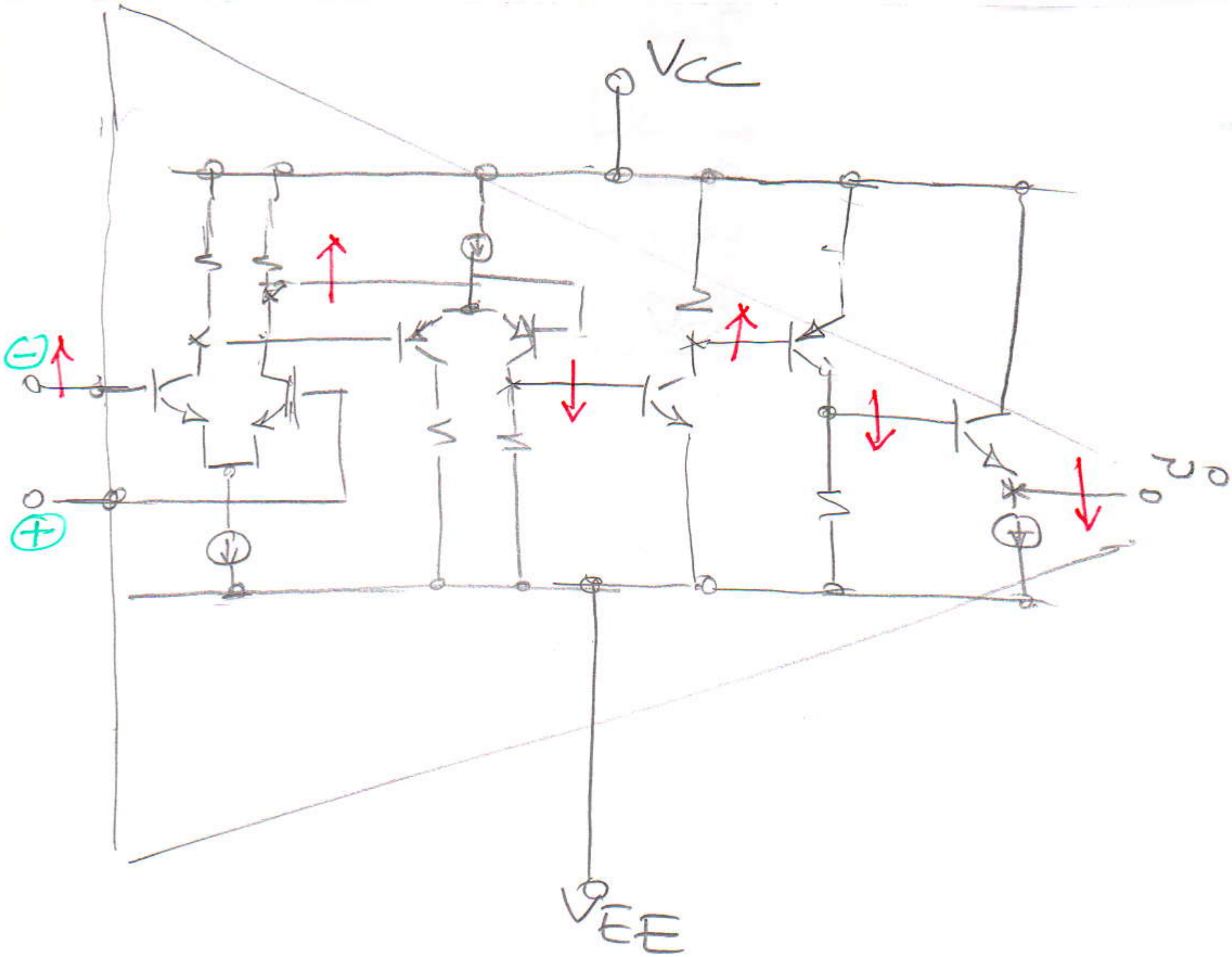
$R_{EE} = 20 k\Omega$

$$A_d = \tfrac{1}{2} \times 40 \times 1 k\Omega = 20 \; \frac{V}{V}$$

$$A_{cm} = \frac{-40 \times 1}{1 + 40 \times 40} \simeq -\frac{1}{40} = -2.5 \times 10^{-2} \; \underline{\frac{V}{V}}$$

$$CMRR = \left| \frac{20}{-2.5 \times 10^{-2}} \right| = \frac{2000}{2.5} = \frac{8000}{10} = \underline{800}$$

$CMRR(dB) \simeq 58 dB$



An increase in the top input has resulted in a decrease in the output. Therefore, the top input is the inverting input (−) whereas the bottom input is the non-inverting input (+).