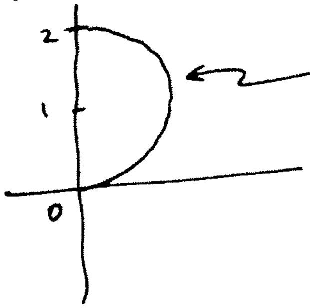
1. [5 marks] Find points on the surface Rewrite the given iterated integral

$$\int_0^2 \int_0^{\sqrt{2y-y^2}} (1 - x^2 - y^2)\, dx\, dy$$

in polar coordinate. **Do not eveluate the integral.**



$x = \sqrt{2y - y^2}$

$x^2 = 2y - y^2$

$x^2 + y^2 - 2y + 1 = 1$

$x^2 + (y-1)^2 = 1$

$\Rightarrow r^2 = 2r\sin\theta$

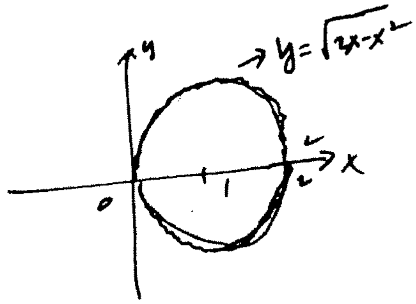
$\therefore r = 2\sin\theta$

$$\int_0^{\frac{\pi}{2}} \int_0^{2\sin\theta} (1 - r^2)\, r\, dr\, d\theta$$

2. [5 marks] Write a double integral to find the volume of the solid bounded by the graphs of the given equations.
$z = 4 - y^2,\ x^2 + y^2 = 2x,\ z = 0$. **Do not eveluate the integral.**

$x^2 - 2x + 1 + y^2 = 1$

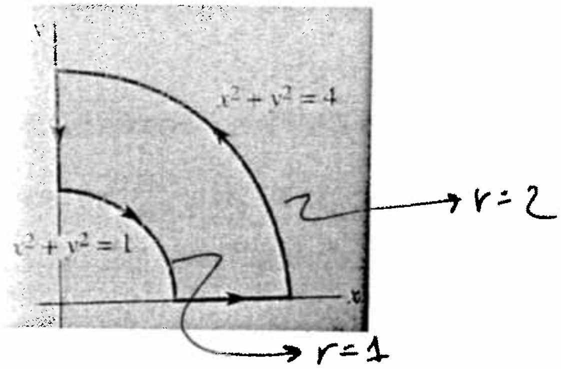
$(x-1)^2 + y^2 = 1$



$$V = 2\int_0^2 \int_0^{\sqrt{2x - x^2}} (4 - y^2)\, dy\, dx$$

or $$\int_0^2 \int_{-\sqrt{2x - x^2}}^{\sqrt{2x - x^2}} (4 - y^2)\, dy\, dx$$

3. [10 marks] Find the work done by the force $F(x, y) = -xy^2\,\vec{\mathbf{i}} + 3x^2y\,\vec{\mathbf{j}}$ around the closed curve :



$$P = -x^2y \longrightarrow \frac{\partial P}{\partial y} = -2xy$$

$$Q = 3x^2y \longrightarrow \frac{\partial Q}{\partial x} = 6xy$$

$$W = \oint_C F\cdot dr = \iint_R (6xy + 2xy)\,dA$$

$$= \int_0^{\frac{\pi}{2}}\int_1^2 8(r\cos\theta)(r\sin\theta)\cdot r\,dr\,d\theta$$

$$= \int_0^{\frac{\pi}{2}}\int_1^2 (8r^3\sin\theta\cdot\cos\theta)\,dr\,d\theta$$

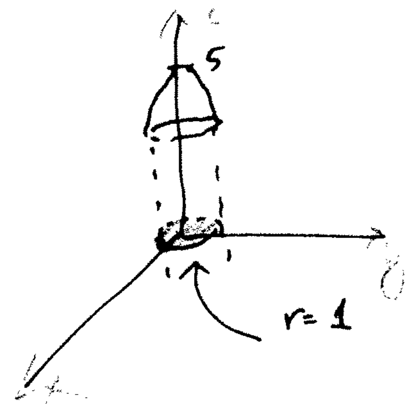
$$= \int_0^{\frac{\pi}{2}} 2r^4\Big]_1^2 \sin\theta\cos\theta\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} 30\cdot\sin\theta\cos\theta\,d\theta$$

$$= 30\cdot\frac{1}{2}\sin^2\theta\Big]_0^{\frac{\pi}{2}}$$

$$= \boxed{15}$$

1. [14 marks] Find the upward flux $\iint_S (\boldsymbol{F}\cdot\vec{\mathbf{n}})\,dS$ for the vector field $\boldsymbol{F}(x,y,z) = z\,\vec{\mathbf{k}}$ through the surface S that part of the paraboloid $z = 5 - x^2 - y^2$ inside the cylinder $x^2 + y^2 = 1$.



part1/ $x^2+y^2+z+5=0$

$\nabla g = 2x\,i + y\,j + k$

$\therefore \vec{n} = \dfrac{2x\,i + y\,j + k}{\sqrt{4x^2+4y^2+1}}$

$\vec{F}\cdot\vec{n} = \dfrac{z}{\sqrt{4x^2+4y^2+1}}$

part2/ $f_x = -2x,\ f_y = -2y.\ \therefore dS = \sqrt{1+4x^2+4y^2}\ dA$

part3/ $\text{Flux} = \iint_R \left(\dfrac{z}{\sqrt{}}\right)\cdot\sqrt{}\ dA$

$= \iint_R (5-x^2-y^2)\,dA$

$= \int_0^{2\pi}\int_0^1 (5-r^2)\,r\,dr\,d\theta$

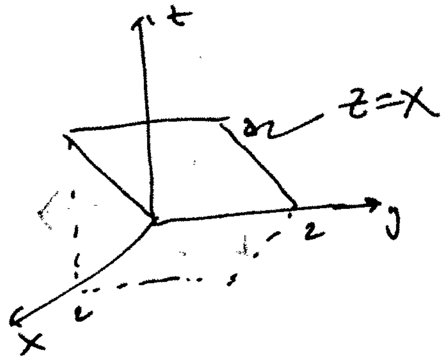
$= \int_0^{2\pi} \left.\tfrac{5}{2}r^2 - \tfrac{1}{4}r^4\right]_0^1 d\theta$

$= \int_0^{2\pi} \left(\tfrac{10}{4}-\tfrac{1}{4}\right)d\theta$

$= \dfrac{9}{4}\cdot 2\pi = \boxed{\dfrac{9}{2}\pi}$

5. [16 marks] Use Stoke's theorem to evaluate $\iint_S (curl\,\boldsymbol{F}\cdot\vec{\mathbf{n}})d\boldsymbol{S}$ for the vector field $\boldsymbol{F}(x,y,z) = 3x^2\,\vec{\mathbf{i}} + 8x^3y\,\vec{\mathbf{j}} + 3x^2y\,\vec{\mathbf{k}}$ through the surface S that portion of the plane $z = x$ that lies inside the rectangular cylinder defined by the planes $x = 0, y = 0, x = 2, y = 2$. S is oriented ~~up~~ward down

Do not use line integrals. There will be no mark if solved by line integrals.



part1/ $g(x,y,z) = x - z = 0$ (Not $-x+z=0$)

$\nabla g = \mathbf{i} - \mathbf{k}$

$\vec{n} = \dfrac{\nabla g}{|\nabla g|} = \dfrac{1}{\sqrt{2}}(\mathbf{i} - \mathbf{k})$

part2/ $\operatorname{curl} F = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2 & 8x^3y & 3x^2y \end{vmatrix}$

$= (3x^2 - 0)\mathbf{i} - (6xy - 0)\mathbf{j} + (24x^2y - 0)\mathbf{k}$

$\therefore \operatorname{curl} F \cdot \vec{n} = \frac{1}{\sqrt{2}}(3x^2 - 24x^2y)$

part3/ $dS = \sqrt{1+1+0}\,dA = \sqrt{2}\,dA$

part4/ $W = \iint_S (\operatorname{curl} F\cdot n)\,dS = \iint_R \frac{1}{\sqrt{2}}(3x^2 - 24x^2y)\cdot\sqrt{2}\,dA$

$= \int_0^2\int_0^2 (3x^2 - 24x^2y)\,dx\,dy = \int_0^2 x^3 - 8x^3y\Big|_0^2\,dy$

$= \int_0^2 (8 - 64y)\,dy = 8y - 32y^2\Big]_0^2 = 16 - 128 = \boxed{-112}$