# Multi-Stage Amplifiers



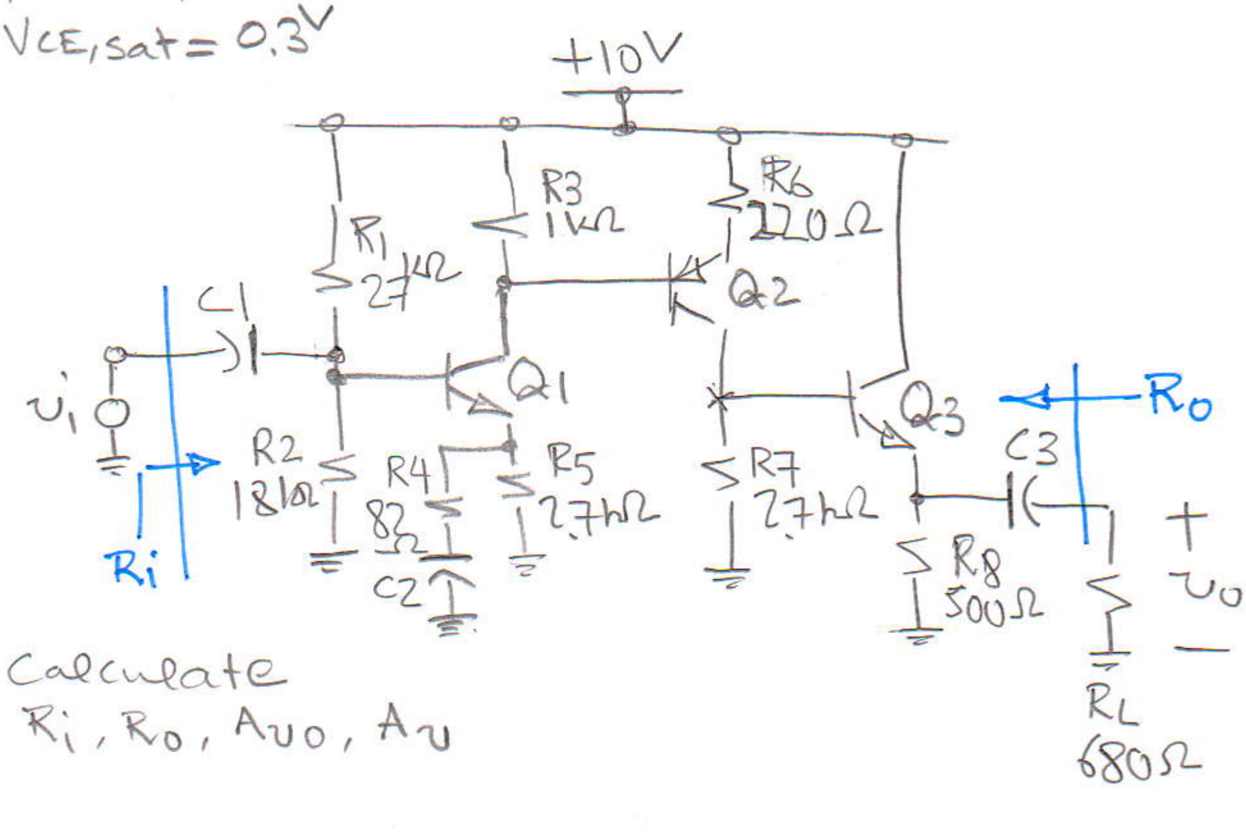
Amplifiers typically consist of more than one amplification stage (transistor). The reason is, almost always, to offer a large (enough) input resistance, and a small (enough) output resistance, while a large (enough) voltage gain is ensured. This lecture provides illustrative examples for presenting the methodology for analyzing multi-stage amplifiers.

Example #1

Analyze the following amplifier

$\beta_1 = \beta_2 = \beta_3 = 100$ ; $V_{A1} = V_{A2} = V_{A3} = \infty$ ; $V_{BE,on} = 0.7^V$ for all.

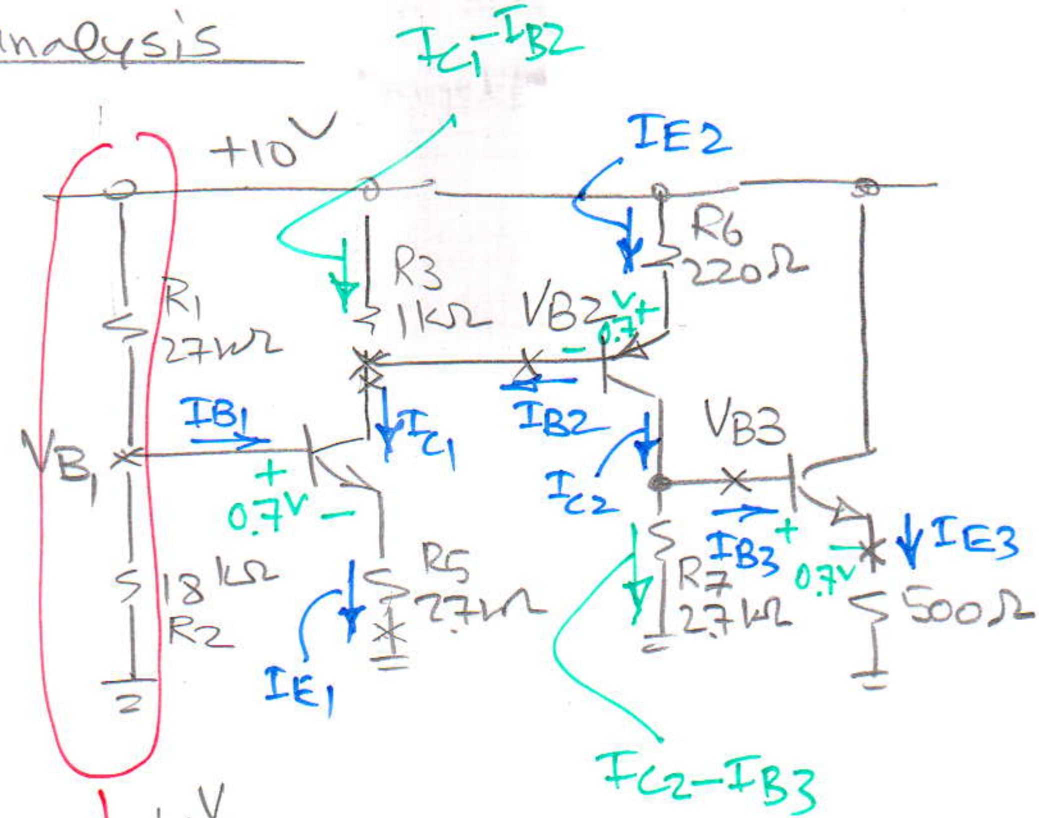
$V_{CE,sat} = 0.3^V$

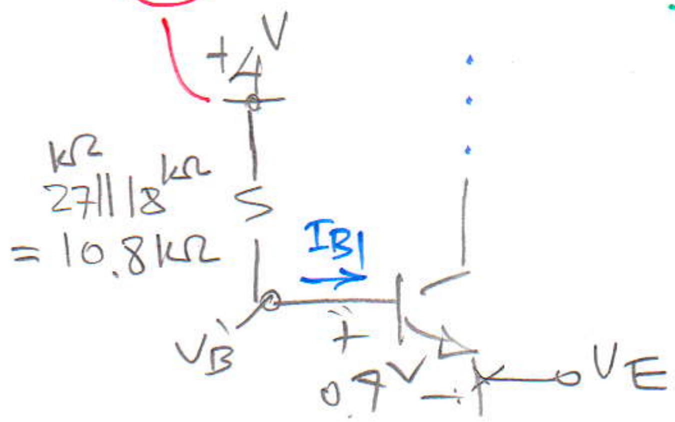



Calculate

$R_i, R_o, A_{vo}, A_v$

## DC analysis




KVL 18 $\quad +4^{V} - 10.8^{k\Omega} I_{B1} - 0.7^{V} - 2.7 I_{E1} = 0$

$$I_{B1} = \frac{1}{\beta} I_{C1}, \quad I_{E1} = \frac{\beta+1}{\beta} I_{C1}$$

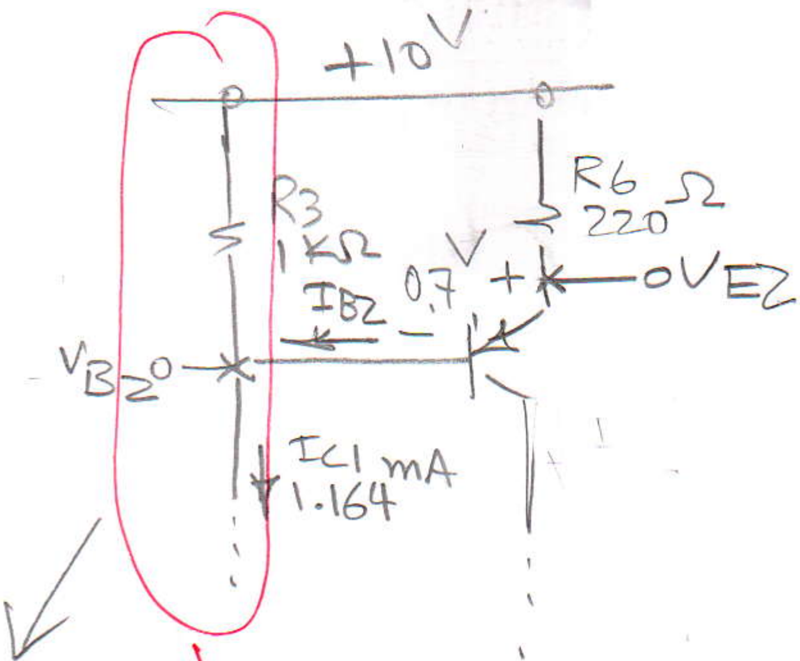
$$I_{C1} = \frac{(4-0.7)\times 100}{10.8 + (100+1)\,2.7} = 1.164^{mA}$$

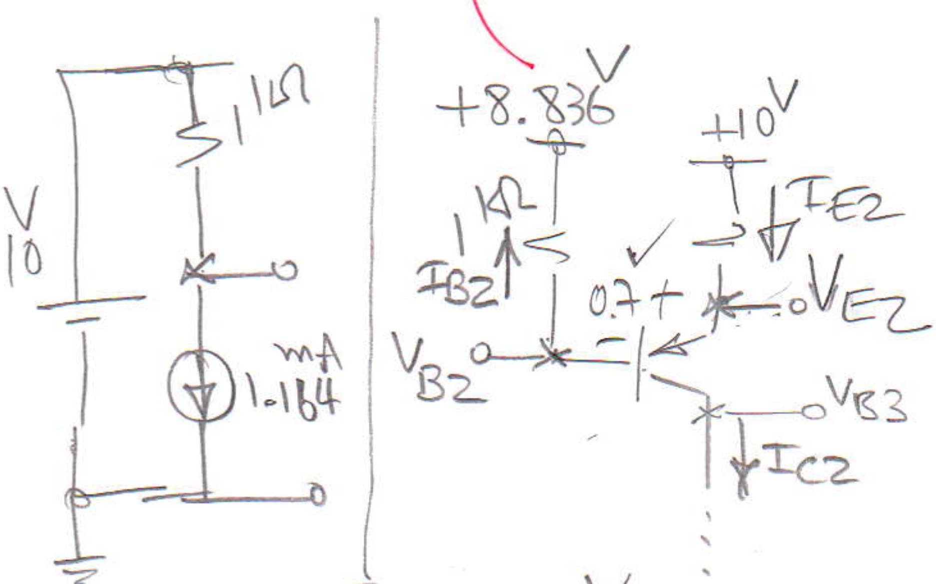
$$I_{E1} = \frac{101}{100} \times 1.164 = 1.175\ mA$$

$$V_{E1} = 2.7 \times 1.175 = 3.17^{V}$$

$$V_{B1} = V_E + 0.7 = 3.87V$$

DC analysis (cont.) 




KVL: $8.836^{V} + I_{B2} \times 1^{k\Omega} + 0.7^{V} + 0.22 I_{E_2} = 10^{V}$

$I_{B2} = \frac{1}{\beta} I_{C2}$; $I_{E2} = \frac{\beta+1}{\beta} I_{C2}$

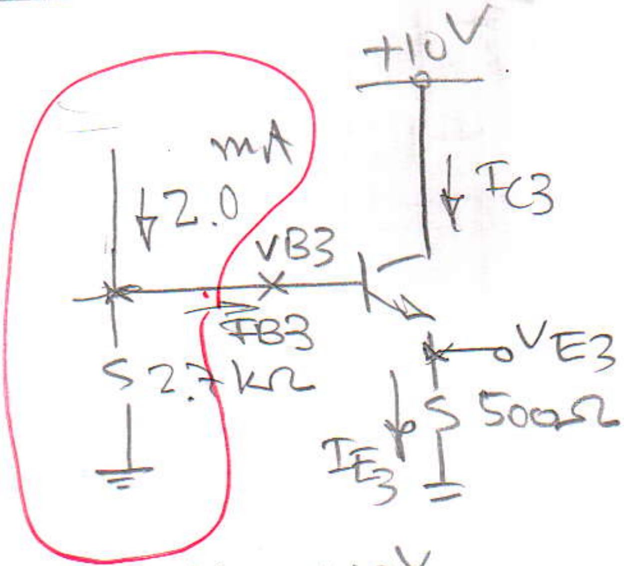
$$\Rightarrow I_{C2} = \frac{(10 - 0.7 - 8.836) \times 100}{1 + (100+1) \times 0.22} = 1.998^{mA} \simeq 2.0^{mA}$$

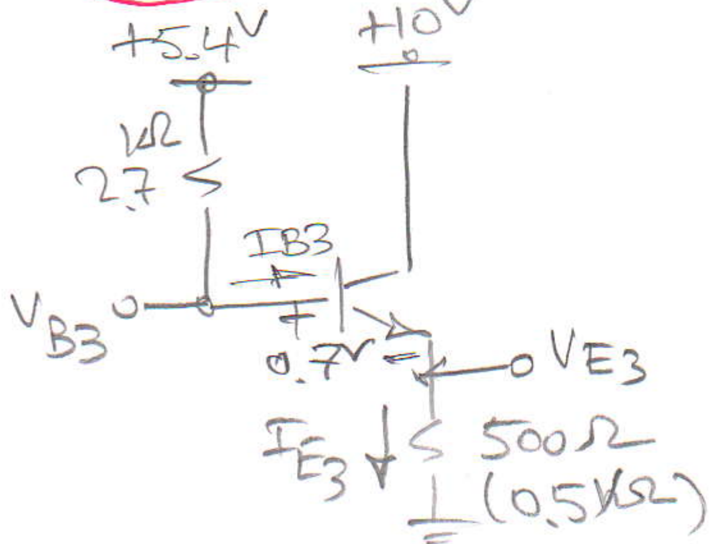
$I_{E_2} = \frac{\cancel{100}}{100} \times 2.0 = 2.02^{mA}$

$V_{E_2} = 10 - 0.22^{k\Omega} \times 2.02 = 9.56^{V}$

$V_{B2} = V_{E_2} - 0.7 = 8.86^{V}$

## DC analysis




$$KVL: 5.4 - 2.7 \times I_{B3} - 0.7 - 0.5 I_{E3} = 0$$

$$I_{B3} = \frac{1}{\beta} I_{C3} \; ; \; I_{E3} = \frac{\beta+1}{\beta} I_{C3}$$

$$I_{C3} = \frac{(5.4 - 0.7) \times 100}{2.7 + 0.5 \times (100+1)} = 8.83^{mA}$$

$$I_{E3} = \frac{101}{100} \times 8.83 = 8.92 \, mA$$

$$V_{E3} = 8.92 \times 0.5 = 4.46^V$$

$$VB3 = 4.46^V + 0.7^V = 5.16V$$

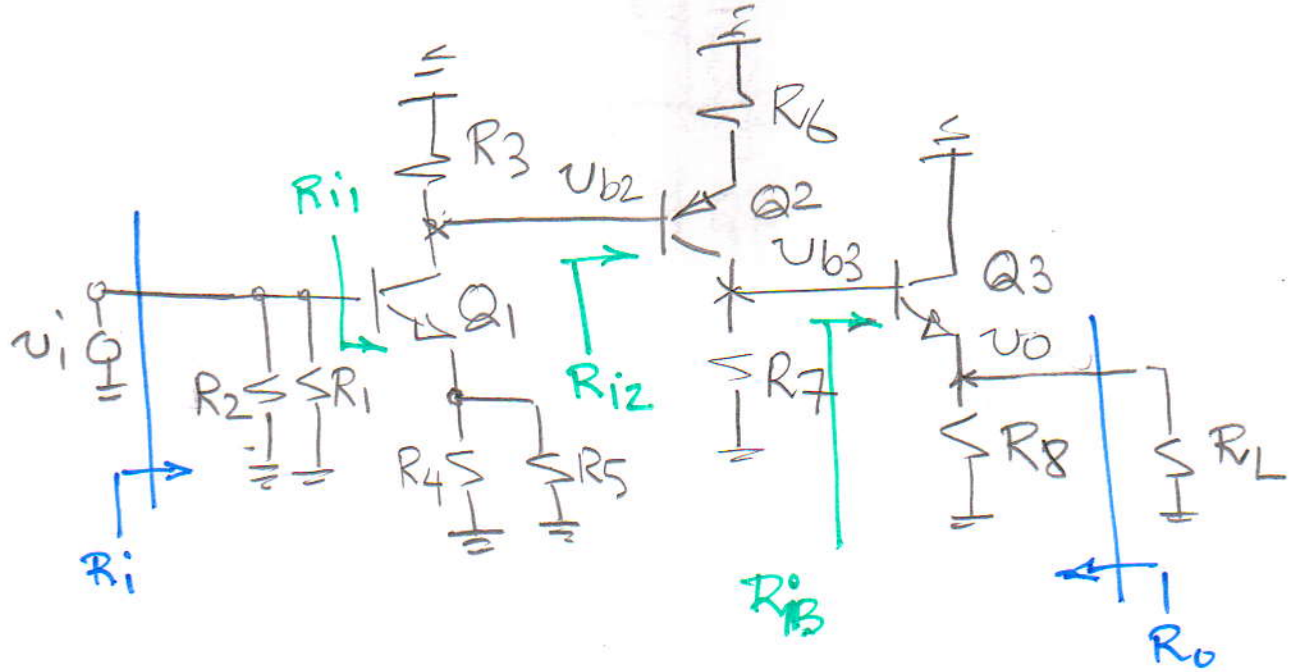
$$V_{CE_1} = V_{B2} - V_{E_1} = 8.86 - 3.17 > 0.3$$

$$V_{EC_2} = V_{E_2} - V_{B_3} = 9.95 - 5.16 > 0.3$$

$$V_{CE_3} = V_{C3} - V_{E_3} = 10 - 4.46 > 0.3$$

All Three ~~#~~ transistors are in the active mode.

## ac analysis




$$R_{i3} = r_{\pi 3} + (\beta+1)(R_8 \| R_L)$$

$$R_{i2} = r_{\pi 2} + (\beta+1) R_6$$

$$R_{i1} = r_{\pi 1} + (\beta+1)(R_4 \| R_5)$$

$$\boxed{R_i = R_{i1} \| R_1 \| R_2}$$

$$\boxed{\frac{v_{b2}}{v_i} = \frac{-g_{m1}(R_3 \| R_{i2})}{1 + g_{m1}(R_4 \| R_5)}}$$

$$\boxed{\frac{v_{b3}}{v_{b2}} = \frac{-g_{m2}(R_7 \| R_{i3})}{1 + g_{m2} R_6}}$$

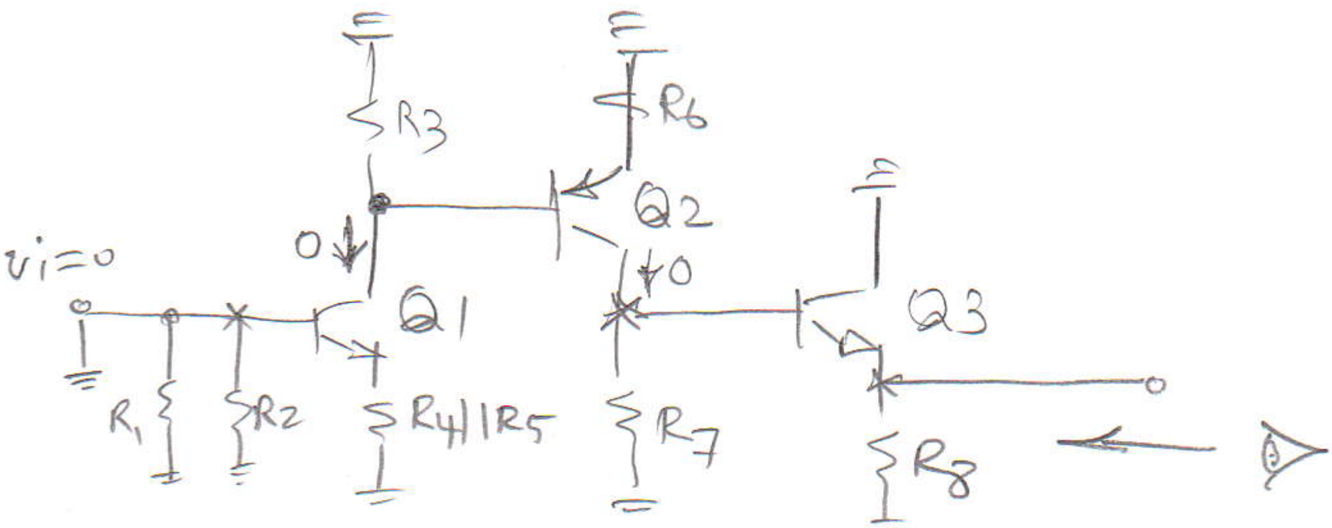
$$\boxed{\frac{v_o}{v_{b3}} = \frac{g_{m3}(R_8 \| R_L)}{1 + g_{m3}(R_8 \| R_L)}}$$

$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_{b3}} \times \frac{v_{b3}}{v_{b2}} \times \frac{v_{b2}}{v_i} \qquad A_{vo} = A_v \Big|_{R_L \to \infty}$$

# ac analysis (cont.)

Calculation of $R_o$:

$v_i = 0 \Rightarrow$ resistance seen when one looks into the amp. from the output terminal.

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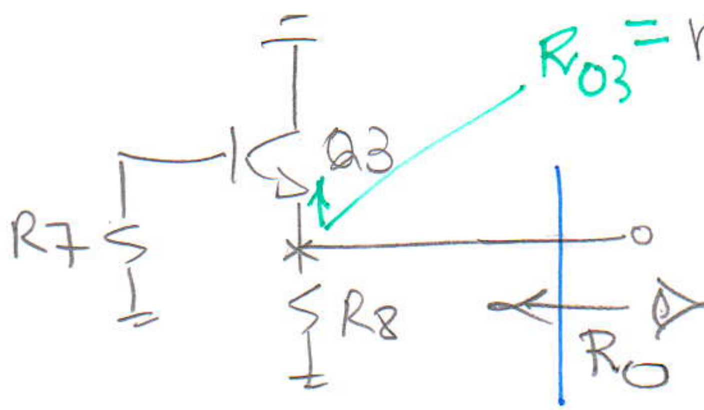



$$R_{O3} = r_{e3} + \frac{R_7}{\beta+1}$$

$$R_O = R_8 \,||\, R_{O3}$$

$$= R_8 \,||\, \left[ r_{e3} + \frac{R_7}{\beta+1} \right]$$

## Numerical Calculations



$$g_{m1} = 40 \times 1.164 = 46.56\ mS$$

$$r_{\pi 1} = \frac{\beta}{g_{m1}} = \frac{100}{46.56} = 2.15\ k\Omega \Rightarrow r_{e1} = \frac{r_{\pi 1}}{\beta+1} = 0.021\ k\Omega$$

$$g_{m2} = 40 \times 2.0 = 80\ mS$$

$$r_{\pi 2} = \frac{\beta}{g_{m2}} = \frac{100}{80} = 1.25\ k\Omega \Rightarrow r_{e2} = \frac{1.25}{101} = 0.012\ k\Omega$$

$$g_{m3} = 40 \times 8.83 = 353.2\ mS$$

$$r_{\pi 3} = \frac{100}{353.2} = 0.283\ k\Omega \Rightarrow r_{e3} = 0.0028\ k\Omega$$

$$R_{i1} = r_{\pi 1} + (\beta+1)(R_4 \| R_5)$$
$$= 2.15 + 101 \times (\underbrace{0.082 \| 2.7}) = 10.18\ k\Omega$$

$$R_{i2} = r_{\pi 2} + (\beta+1) R_6$$
$$= 1.25 + 101 \times 0.22 = 23.47\ k\Omega$$

$$R_{i3} = r_{\pi 3} + (\beta+1)(\underbrace{R_8 \| R_L}_{0.288}) = 0.283\ k\Omega + 101 \times (0.5 \| 0.68)\ k\Omega$$
$$= 29.38\ k\Omega$$

$$\frac{v_{b2}}{v_i} = \frac{-46.56 \times (1\ k\Omega \| 23.47\ k\Omega)}{1 + 46.56 \times (\underbrace{0.082 \| 2.7}_{0.079})} \cong -9.54\ \frac{V}{V}$$

$$\frac{v_{b3}}{v_i} = \frac{-80\times(\overbrace{2.7^{k\Omega} \| 29.38^{k\Omega}}^{2.47^{k\Omega}})}{1+80\times 0.22} = -10.63\,\frac{V}{V}$$

$$\left.\frac{v_o}{v_{b3}}\right|_{R_L=\infty} = \frac{353.2\times(0.5^{k\Omega}\|\infty)}{1+353.2\times(0.5^{k\Omega}\|\infty)}$$

$$= 0.994\,\frac{V}{V}$$

$$\left.\frac{v_o}{v_{b3}}\right|_{R_L=0.68^{k\Omega}} = \frac{353.2\times(\overbrace{0.5\|0.68}^{0.288})}{1+353.2\times(0.5\|0.68)}$$

$$= 0.990\,\frac{V}{V}$$

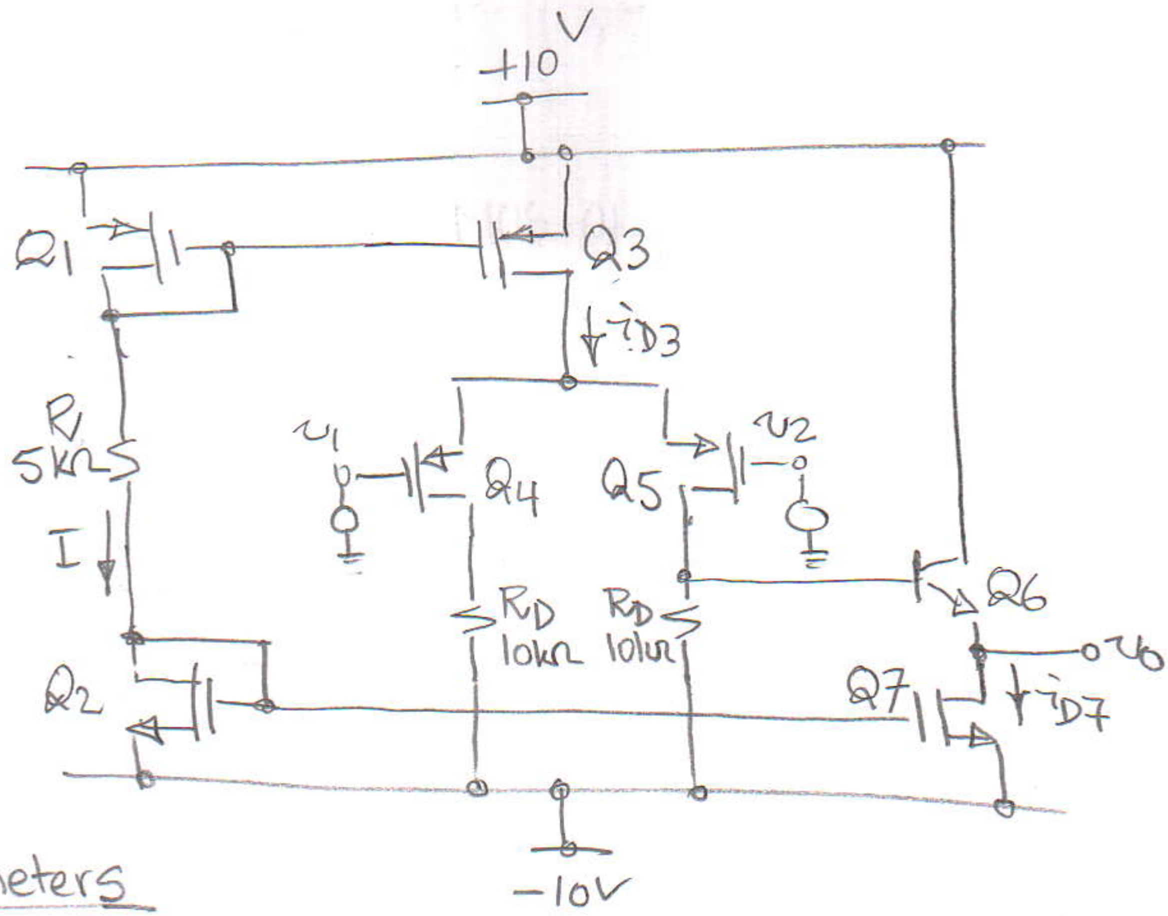
$$\left.\begin{aligned} A_{v_o} &= (-9.54)\times(-10.63)\times 0.994 \cong 100.8\,\tfrac{V}{V} \\ A_v &= (-9.54)\times(-10.63)\times 0.990 = 100.4\,\tfrac{V}{V} \end{aligned}\right\}\text{ practically the Same!}$$

$$R_o = \left\{0.5^{k\Omega} \,\|\, \Big[\underbrace{0.0028 + \frac{2.7}{101}}_{0.0295}\Big]\right\} \cong 0.028^{k\Omega}$$

or $\underline{28\,\Omega}$

That is why the amplifier does not care when the 680-Ω load is connected to it!

# Example#2: A BiCMOS OP-AMP




## Parameters

$K_n' = 2K_p' = 0.2\ mA/V^2$ ; $V_{tn} = 0.8^V$ ; $V_{tp} = -1.0^V$

$\left(\frac{W}{L}\right)_{1,2,4,5} = 10$ ; $V_{An} = |V_{Ap}| = V_{A6} = \infty$

$\beta = 50$ ; $V_{BE,on} = 0.7^V$ ; $V_{CE,sat} = 0.3^V$

a) Determine $\left(\frac{W}{L}\right)_3$ and $\left(\frac{W}{L}\right)_7$ for $I_{D3} = 2.2^{mA}$ and $I_{D7} = 1.5\ mA$.

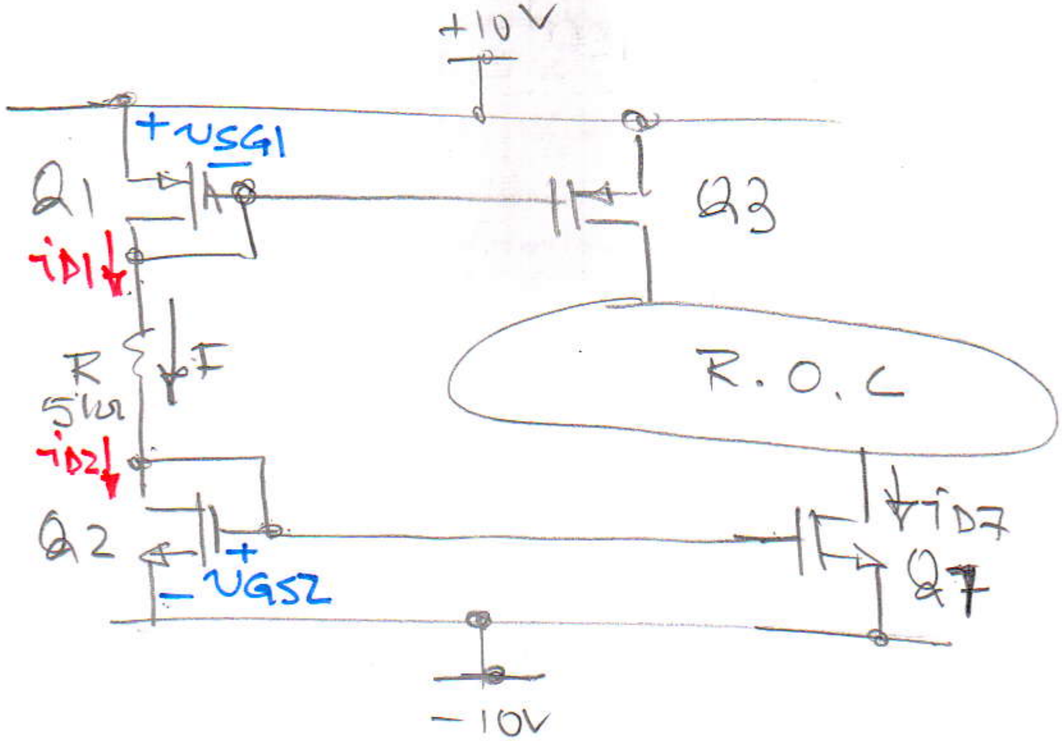
b) Determine all node voltages and branch currents for $v_1 = v_2 = 0$

c) Determin the Common-Mode input voltage range

d) Determine the differential-mode gain $A_d = v_0/(v_1 - v_2)$.

e) Repeat if $V_{An} = 40^V$; $V_{Ap} = -20^V$; and $V_A = 80^V$

# DC Analysis




First task: to calc. I.

$$i_{D1} = \frac{1}{2} K_1 V_{OV_1}^2 = \frac{1}{2} \times \overbrace{0.1}^{k'_p} \times \underbrace{10}_{(W/L)_1} \times V_{OV_1}^2 = 0.5 V_{OV_1}^2$$

$$\Rightarrow I = 0.5 V_{OV_1}^2 \Rightarrow \boxed{V_{OV_1} = \sqrt{2I}} \quad (1)$$

$$i_{D2} = \frac{1}{2} K_2 V_{OV_2}^2 = \frac{1}{2} \times \underbrace{0.2}_{k'_n} \times \underbrace{10}_{(W/L)_2} \times V_{OV_2}^2 = V_{OV_1}^2$$

$$\Rightarrow \boxed{V_{OV_2} = \sqrt{I}} \quad (2)$$

$$\text{KVL: } 10^V - V_{SG_1} - RI - V_{GS2} = -10^V$$

$$10^V - \left(V_{OV_1} + |V_{tp}|\right) - 5I - \left(V_{OV_2} + V_{tn}\right) = -10$$

$$\boxed{I = 3.64 - 0.2\left(V_{OV_1} + V_{OV_2}\right)} \quad (3)$$

~~So~~ let us resort to iterations to solve:

$$V_{ov_1} = V_{ov_2} = 0 \xRightarrow{(3)} I = 3.64\,mA$$

$$I = 3.64\,mA \xRightarrow{(1)\&(2)} V_{ov_1} \cong 2.7^V \;\&\; V_{ov_2} \cong 1.91^V$$

$$V_{ov_1} = 2.7^V \;\&\; V_{ov_2} = 1.91^V \xRightarrow{(3)} I = 2.72\,mA$$

$$I = 2.72^{mA} \xRightarrow{(1)\&(2)} V_{ov_1} \simeq 2.33^V \;\&\; V_{ov_2} \simeq 1.65^V$$

$$V_{ov_1} \simeq 2.33^V \;\&\; V_{ov_2} \simeq 1.65^V \xRightarrow{(3)} I = 2.844^{mA}$$

$$I = 2.844^{mA} \xRightarrow{(1)\&(2)} V_{ov_1} \simeq 2.38^V \;\&\; V_{ov_2} \cong 1.69^V$$

$$V_{ov_1} = 2.38^V \;\&\; V_{ov_2} = 1.69^V \xRightarrow{(3)} I = 2.83^{mA}$$

$$I = 2.83^{mA} \xRightarrow{(1)\&(2)} V_{ov_1} \simeq 2.38^V \;\&\; V_{ov_2} \simeq 1.68^V$$

Values are repeating wrt the previous iteration. Stop it!

$$\underline{I \cong 2.83^{mA}} \;;\; V_{ov_1} \simeq 2.38^V \;;\; V_{ov_2} \simeq 1.68^V$$

$$V_{SG_1} = 2.38 + |-1.0| = 3.38^V$$

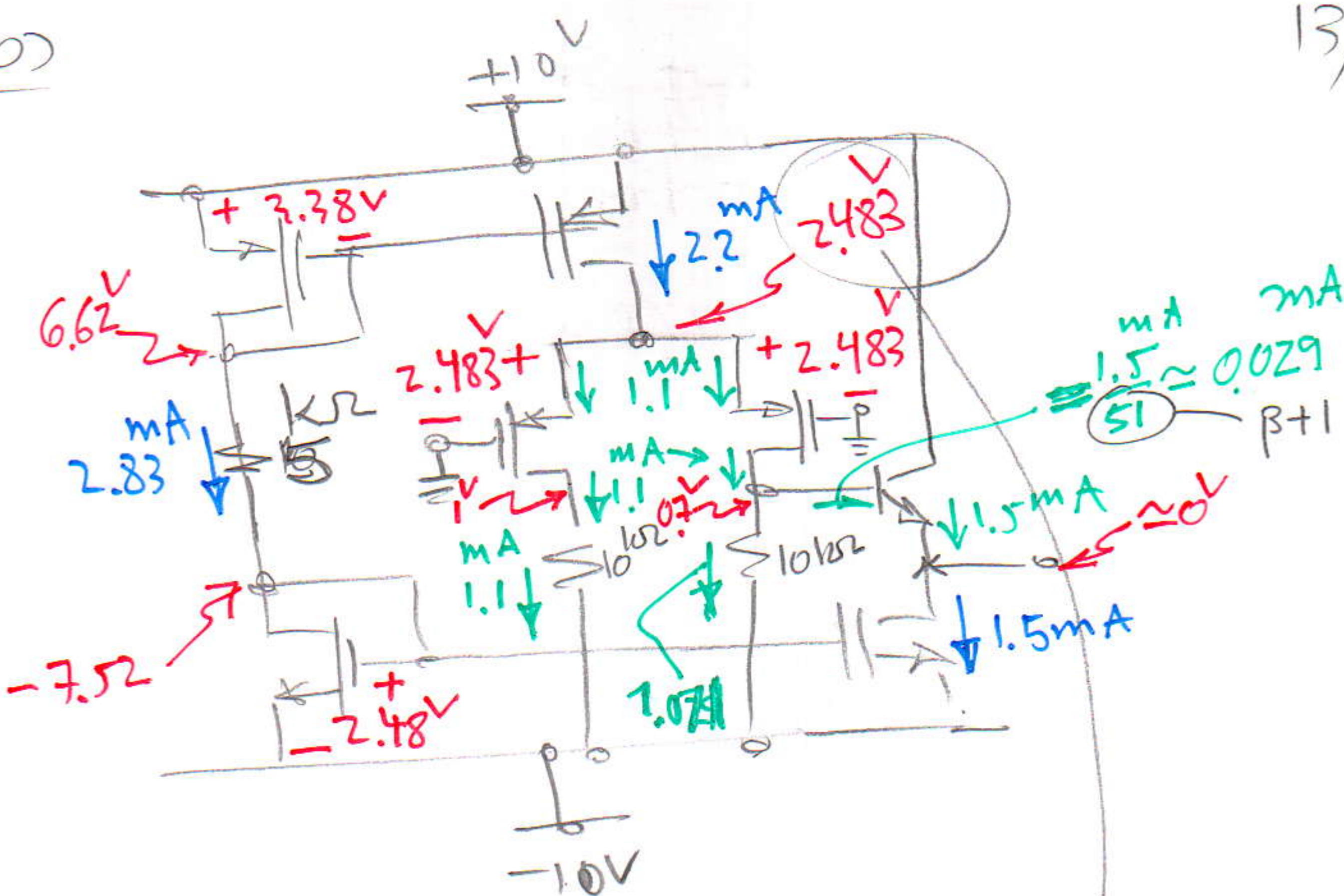
$$V_{GS_2} = 1.68 + 0.8 = 2.48^V$$

$$\frac{i_{D3}}{i_{D1}} = \frac{(\frac{W}{L})_3}{(\frac{W}{L})_1} \Rightarrow \frac{2.2}{2.83} = \frac{(\frac{W}{L})_3}{10} \Rightarrow \boxed{\left(\frac{W}{L}\right)_3 = 7.77}$$

$$\frac{i_{D7}}{i_{D2}} = \frac{(\frac{W}{L})_7}{(\frac{W}{L})_2} \Rightarrow \frac{1.5}{2.83} = \frac{(\frac{W}{L})_7}{10} \Rightarrow \boxed{\left(\frac{W}{L}\right)_7 = 5.3}$$

b)






$i_{D5} = \frac{1}{2} k_p' \left(\frac{W}{L}\right)_5 V_{OV5}^2$

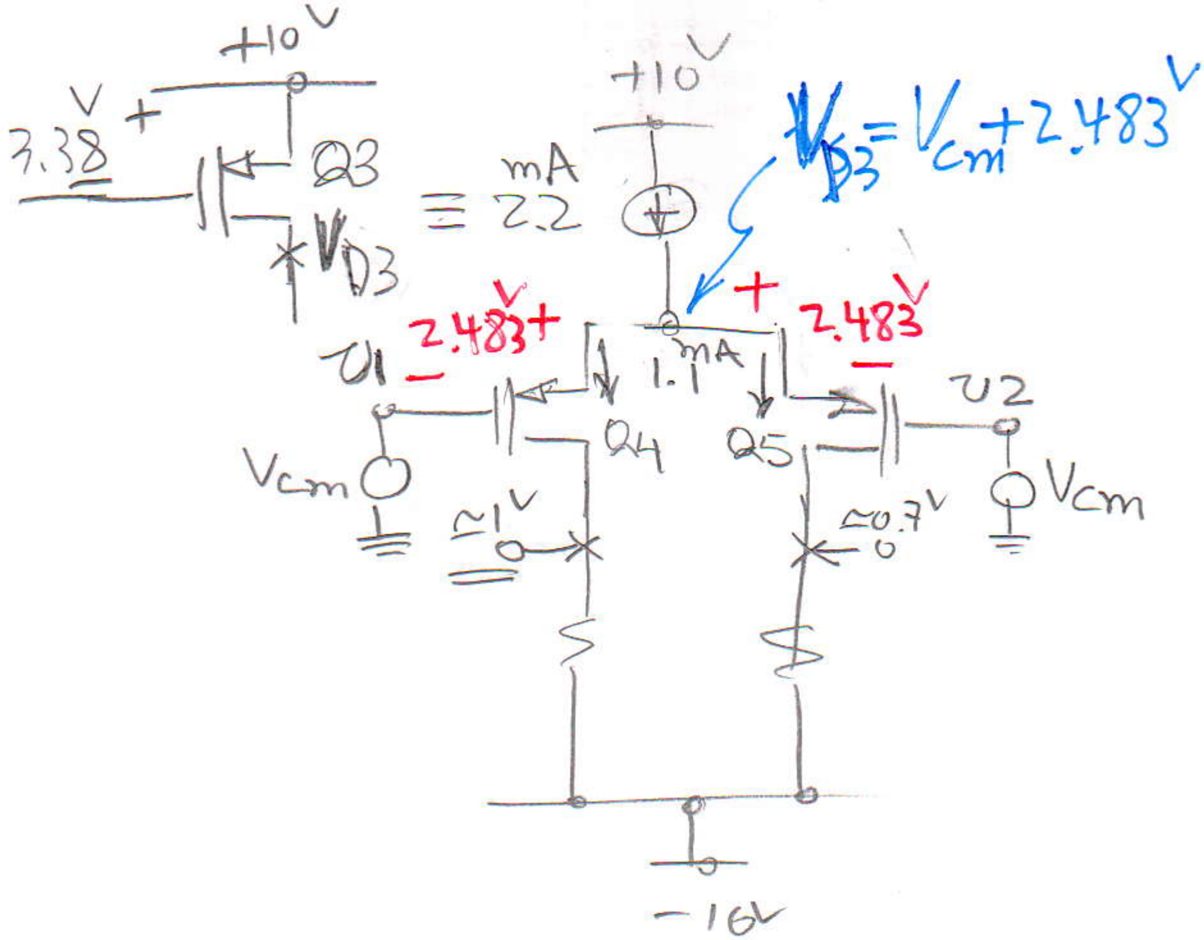
$1.1 = \frac{1}{2} \times 0.1 \times 10 \times V_{OV5}^2 \Rightarrow V_{OV5} = 1.483^V$

$V_{SG5} = V_{OV5} + |V_{tp}| = 1.483 + 1.0 = 2.483^V$

$V_{S3} = \cancel{V_i}^{\,0} + V_{SG5}$ or $V_i + V_{SG4}$

$= 2.483V$

## c) Common-Mode Range




Find range of Vcm over which no transistor goes into the triode mode.

For Q3 to remain in sat. mode:

$$V_{SD3} \geq V_{SG3} - |V_{tp}|$$

$$10 - V_{D3} \geq 3.38 - |-1.0|$$

$$10 - (V_{cm} + 2.483) \geq 2.38$$

$$\Rightarrow \boxed{V_{cm} \leq 5.137^{V}}$$

c (cont.)

For Q4 & Q5 to be in sat. mode:

$$v_{SD4} \geq v_{SG4} - |V_{tp}|$$

$$v_{S4} - 1.0^{V} \geq 2.483 - |-1.0|$$

$$\Downarrow$$

$$V_{D3} - 1.0 \geq \cancel{1}.483$$

$$\Downarrow$$

$$V_{cm} + 2.483 \geq 2.483$$

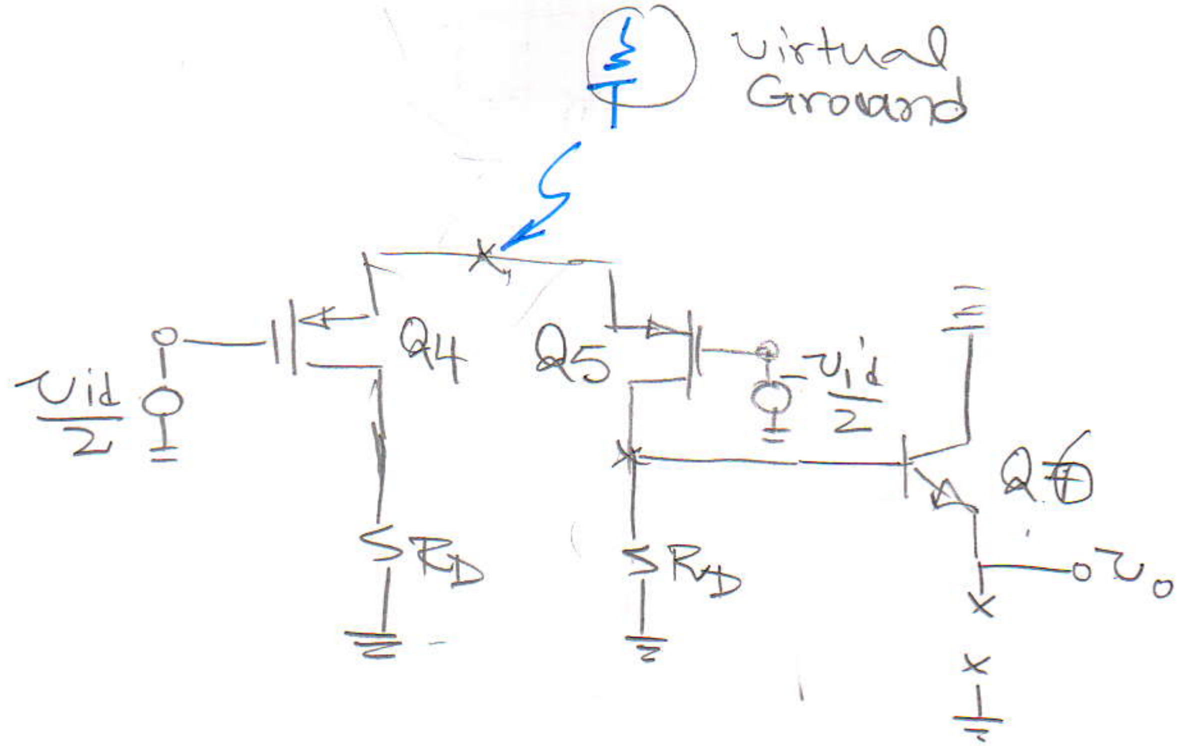
$$\Rightarrow \boxed{V_{cm} \geq 0}$$

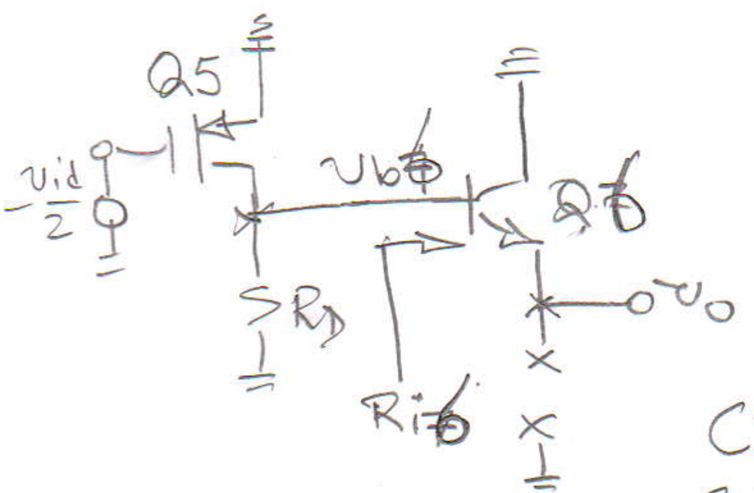
Overall,

$$\boxed{0 \leq V_{cm} \leq 5.137^{V}}$$

## d) Differential Gain

ac analysis




$$R_{i7} = r_{\pi 7} + (\beta+1)\infty = \infty$$

CS:

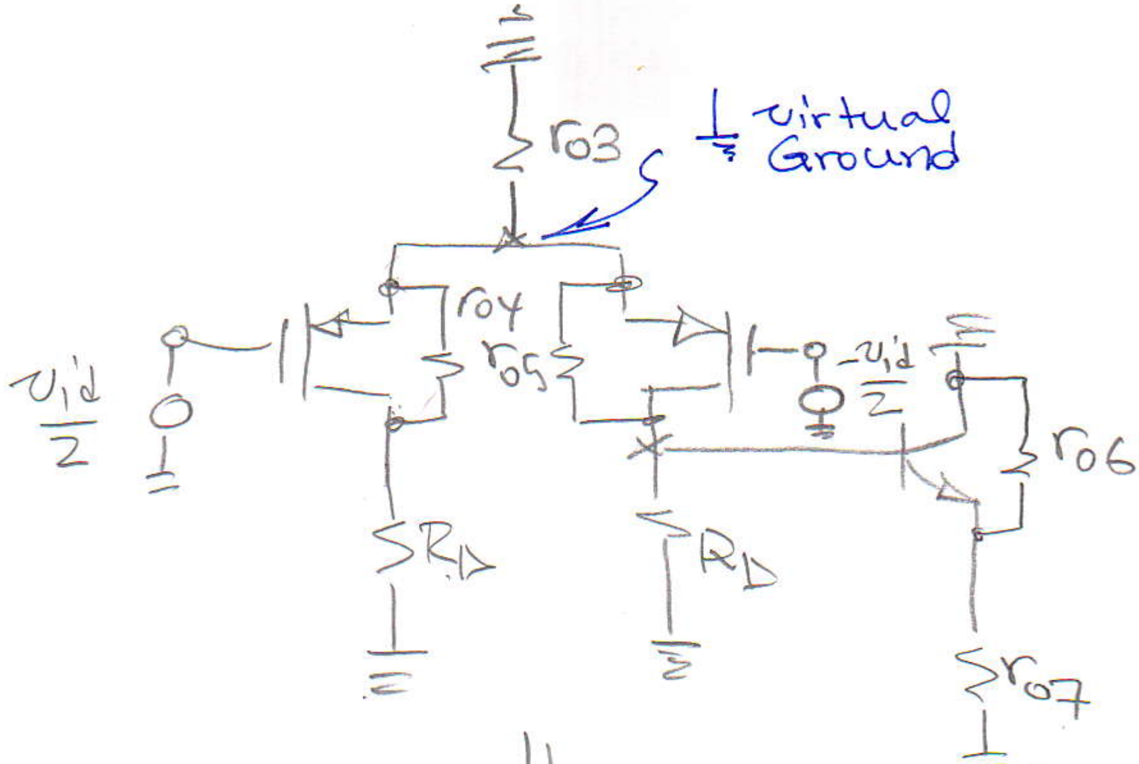
$$\frac{v_{b7}}{-\frac{v_{id}}{2}} = -g_{m5}\left(R_D \,\|\, \underbrace{R_{i7}}_{\infty}\right) = -g_{m5} R_D$$

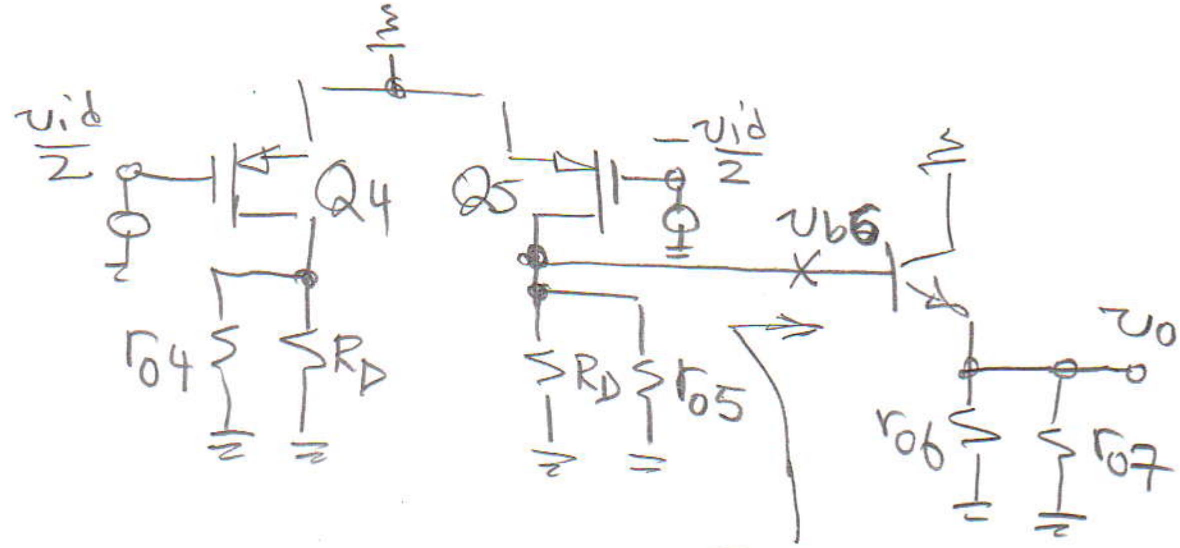
$$\Rightarrow \boxed{\frac{v_{b7}}{v_{id}} = \frac{1}{2} g_{m5} R_D}$$

CC $\left( \frac{v_o}{v_{b7}} = \frac{g_{m7} R_E}{1 + g_{m7} R_E} = 1 \right)$

$$\boxed{\frac{v_o}{v_{id}} = \frac{v_o}{v_{b7}} \cdot \frac{v_{b7}}{v_{id}} = \frac{1}{2} g_{m5} R_D}$$

e) With Early effect considered




$R_{i6} = r_{\pi 6} + (\beta+1)(r_{o6} \| r_{o7})$

CS: $\frac{v_{b6}}{\left(-\frac{v_{id}}{2}\right)} = -g_{m5}(R_D \| r_{o5} \| R_{i6}) \Rightarrow \boxed{\frac{v_{b6}}{v_{id}} = \frac{1}{2} g_{m5}(R_D \| r_{o5} \| R_{i6})}$

CC $\boxed{\frac{v_o}{v_{b6}} = \frac{g_{m6}(r_{o6} \| r_{o7})}{1 + g_{m6}(r_{o6} \| r_{o7})}}$ $\boxed{\frac{v_o}{v_{id}} = \frac{v_o}{v_{b6}} \times \frac{v_{b6}}{v_{id}}}$