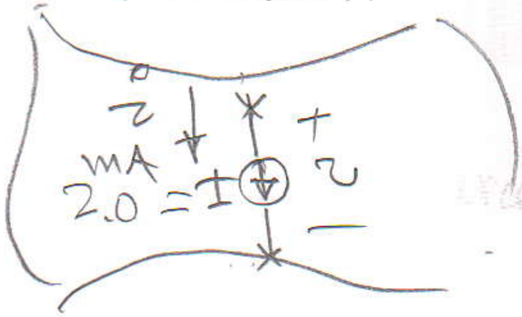
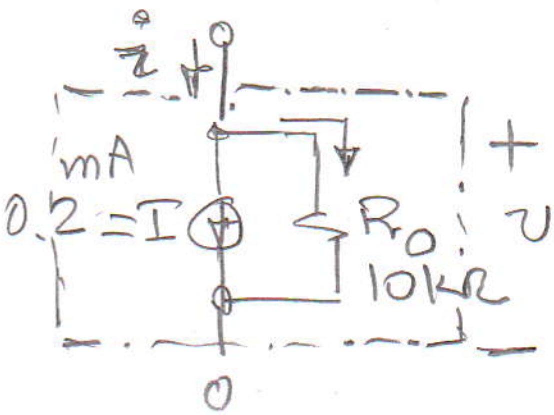
Current Sources & Current Sinks

Ideal current source.



$i = I$, say 2.0 mA irrespective of v

ROC



$$i = I + \frac{v}{R_o}$$

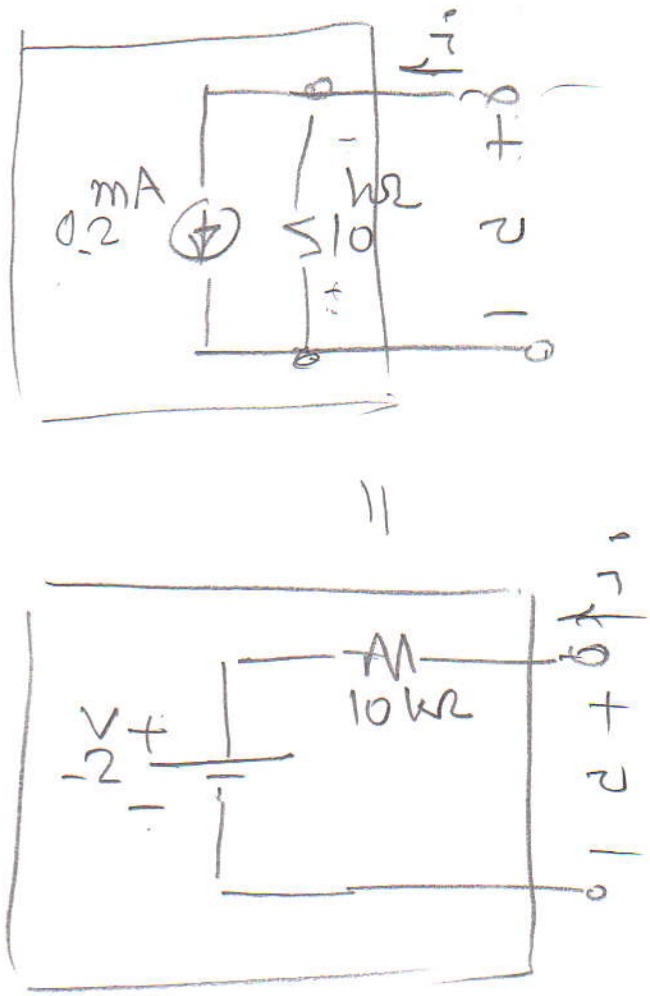
i depends on v

If $R_o \to \infty \Rightarrow \underline{i = I}$

R_o: internal resistance, or,
output resistance, or,
Northon resistance.

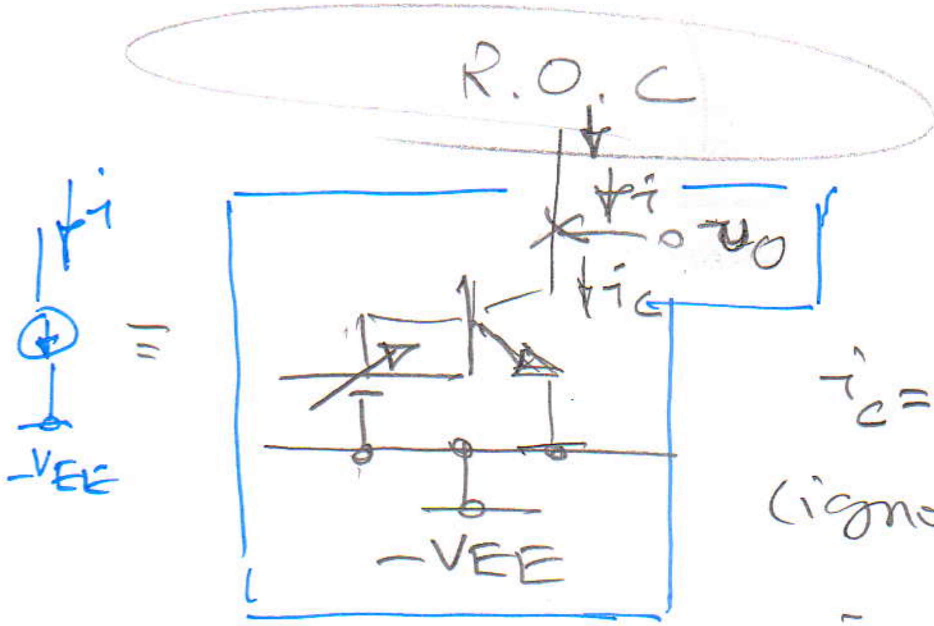
$$\Delta i = \frac{\Delta v}{R_o} \quad (\text{since } I \text{ is constant})$$

$$\Rightarrow \boxed{\frac{\Delta i}{\Delta v} = \frac{1}{R_o}}$$


i
0.2 mA
10 kΩ
+
v
−
||
i
V
2
+
−
10 kΩ
+
v
−

Implement

Current Sink



$i_c = I_s e^{v_{BE}/V_T}$

(ignoring Early eff.)

$\Rightarrow i = i_c \Rightarrow$

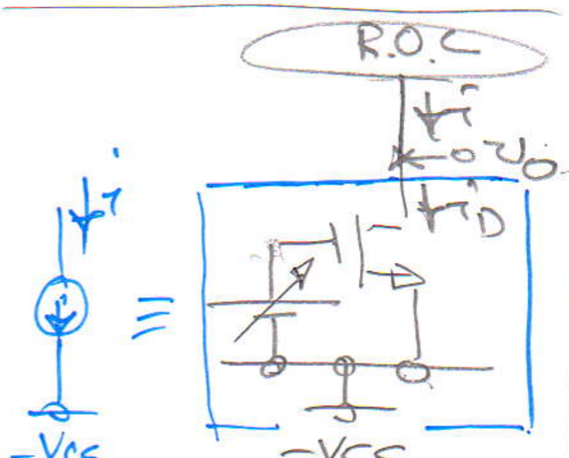
$i = I_s e^{v_{BE}/V_T}$

if $v_{CE} \geq 0.3^V$ (active mode)

$v_O - (-V_{EE}) \geq 0.3^V$

$\Rightarrow \underline{v_O \geq -V_{EE} + 0.3^V}$

condition for the current sink to remain a current sink aka "<u>compliance</u>".



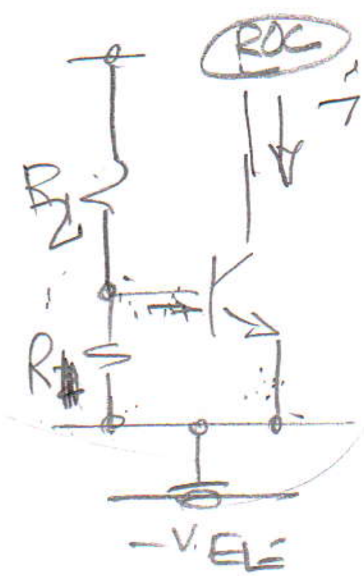
$i = i_D = \frac{1}{2}K(V_{GS} - V_t)^2$

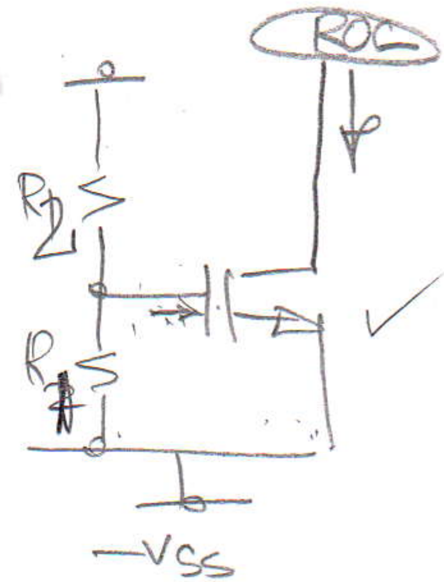
(ignoring E. Eff).

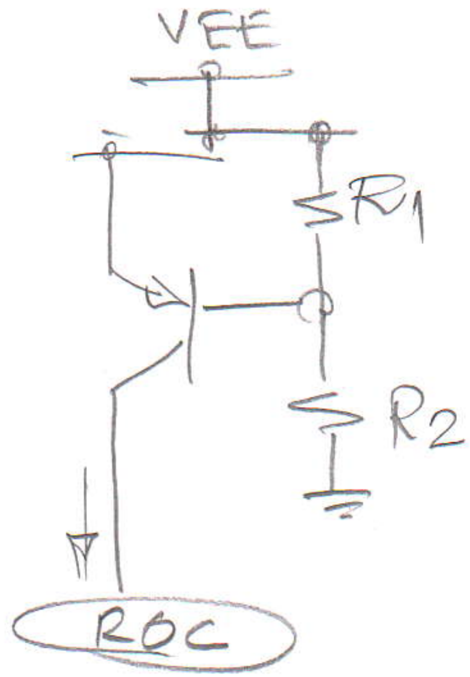
if $v_{DS} \geq V_{GS} - V_t$,

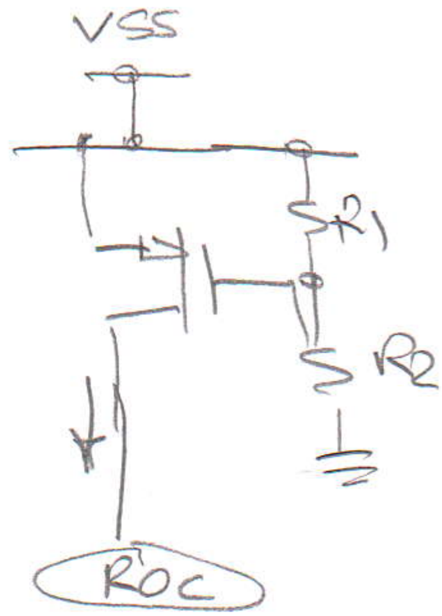
$\Rightarrow v_O - (-V_{SS}) \geq V_{GS} - V_t$

⋮


ROC
R2
−VEE


ROC
R2
−VSS


VEE
R1
R2
ROC


VSS
R1
R2
ROC

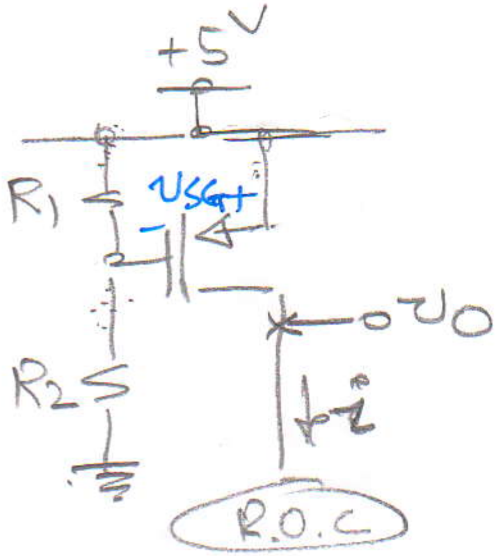
Example #1

$K = 1.0\ mA/V^2$

$V_t = -0.8V$

$V_A = \infty$

- Current Source
- $I \simeq 0.125$
- $+5^V, 0^V$



$i_D = \frac{1}{2} K V_{OV}^2$

$0.125 = \frac{1}{2} \times 1.0 \times V_{OV}^2$

$\Rightarrow V_{OV} = 0.5^V$

$V_{SG} = V_{OV} + |V_t|$

$= 0.5 + |0.8|$

$= 1.3\ V$

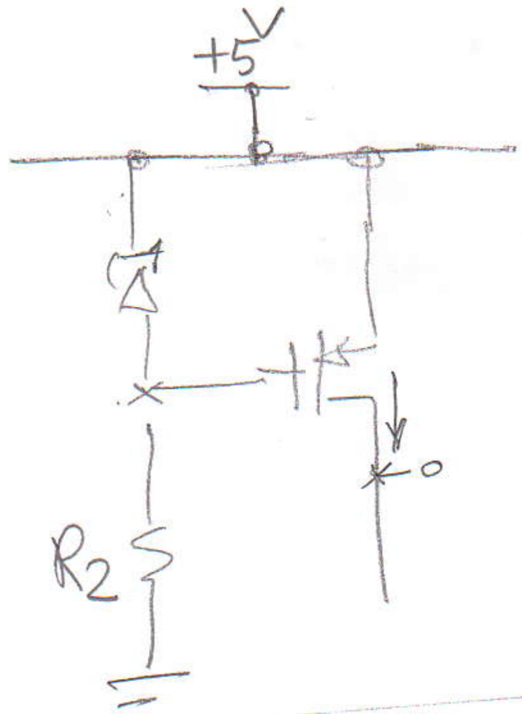
$V_{SG} = \frac{+5^V}{R_1 + R_2} \times R_1 \Rightarrow 1 + \frac{R_2}{R_1} = \frac{5}{1.3}$

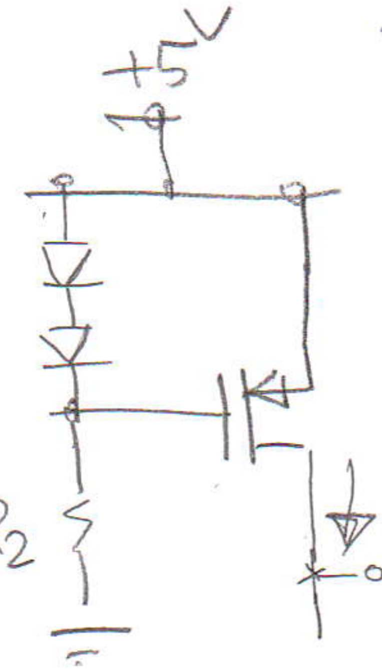
$\Rightarrow R_2/R_1 = 2.846$

$\Rightarrow \boxed{R_2 = 2.846 R_1}$

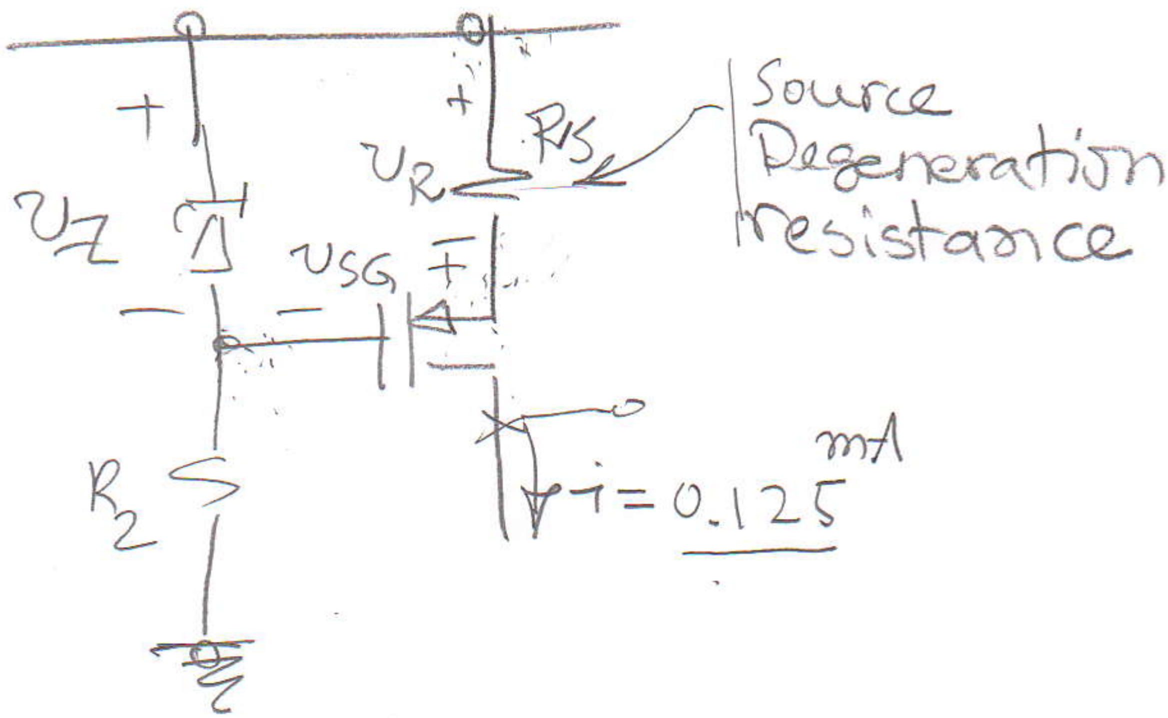
$v_{SD} \geq V_{OV} \Rightarrow 5 - v_O \geq 0.5^V$

$\Rightarrow v_O \leq 4.5^V$





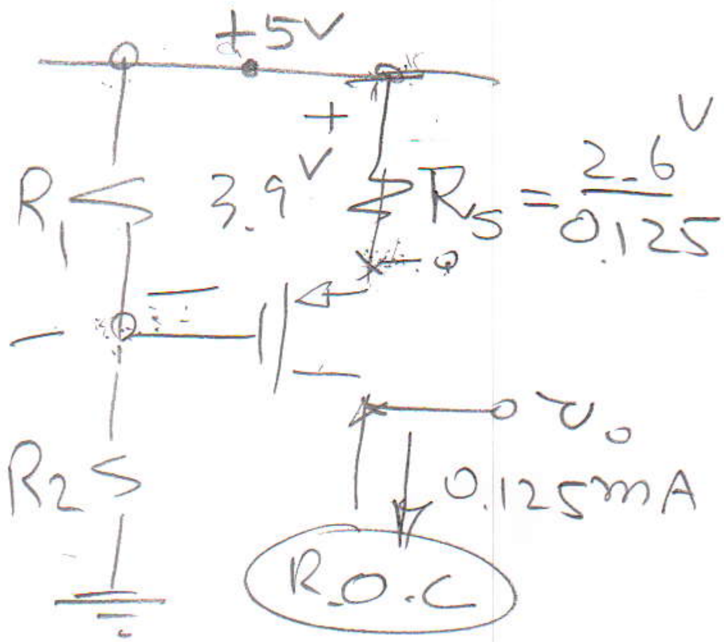
Can we do better?



Previous Example

$i = 0.125 \Rightarrow V_{SG} \simeq 1.3^V$

let $V_R = \underbrace{2 V_{CE}}_{2.6^V}$ (the larger the better)



$R_S = \frac{2.6^V}{0.125} = 20.8^{k\Omega}$

$3.9 = \frac{+5^V}{R_1 + R_2} R_1 \Rightarrow$

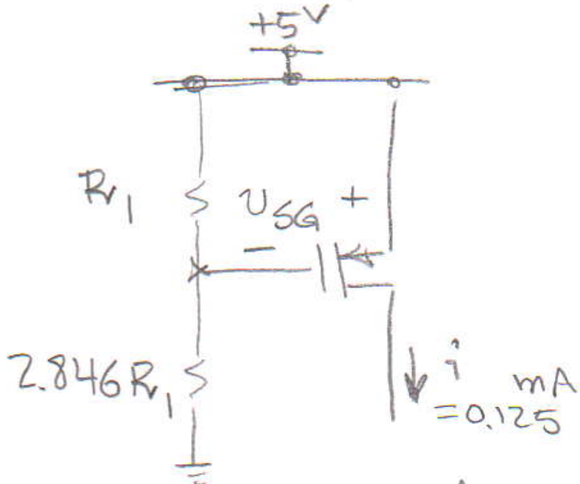
$\frac{R_2}{R_1} = 0.282 \Rightarrow \boxed{R_2 = 0.282 R_1}$

$v_{SD} \geq \underbrace{v_{SG} - |V_t|}_{V_{ov}} \Rightarrow \underbrace{(5-2.6)}_{v_S} - v_o \geq 0.5^V$

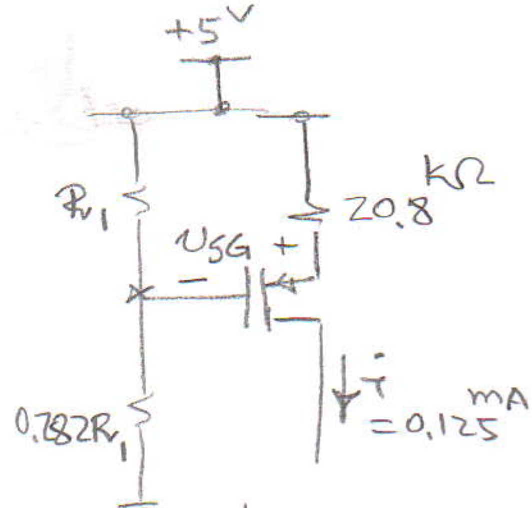
$\Rightarrow v_o \cancel{\leq} 1.9^V$

Sensitivity to k

$$\left| \begin{array}{l} k = 1.0 \, mA/V^2 \\ V_t = -0.8 \, V \end{array} \right\} \text{— nominal parameters}$$



$V_{SG} = 1.3^V$ @ 0.125^{mA}



$V_{SG} = 1.3^V$, $V_{RS} = 2.6^V$

Now, if $k = 1.1 \, mA/V^2$

$$i = \frac{1}{2} \times 1.1 \times (1.3 - 0.8)^2$$
$$= 0.1375 \, mA$$

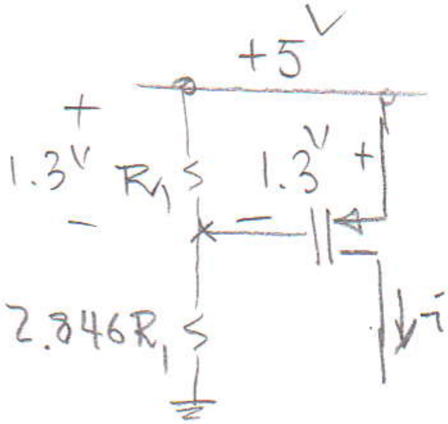
without Source Degeneration.

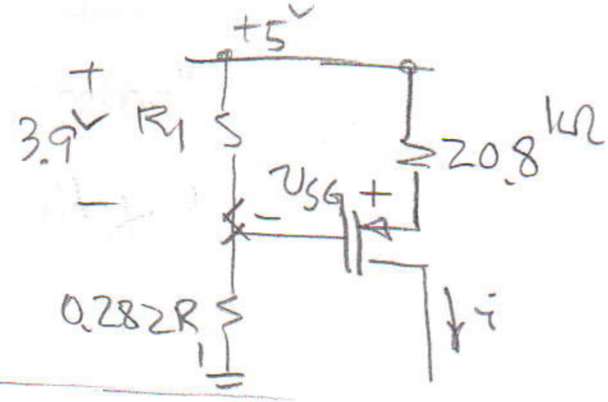
The change in current, relative to the designed value of 0.125 mA is 10%

$$v_{SG} + 20.8\, i = 3.9^V$$
$$v_{SG} + 20.8 \times \frac{1}{2} \times 1.1 (v_{SG} - 0.8)^2$$
$$v_{SG} + 11.44 (v_{SG}^2 - 1.6 v_{SG} + 0.64) = 3.9$$
$$11.44 v_{SG}^2 - 17.3 v_{SG} + 3.42 = 0$$
$$v_{SG} = \frac{17.3 \pm \sqrt{(17.3)^2 - 4 \times 11.44 \times 3.42}}{2 \times 11.44}$$
$$= 1.278 \, V$$
$$\Rightarrow i = \frac{1}{2} \times 1.1 \times (1.278 - 0.8)^2$$
$$= 0.1257 \, mA$$

with Source Degeneration. The change in the current, relative to 0.125 mA is only 0.56%

Sensitivity to V_t





If $V_t = -0.85^V$,

$$i = \frac{1}{2} \times 1.0 \times (1.3 - 0.85)^2$$

$$= \underline{0.101\ mA}$$

−19.2% change in the current due to the change in V_t from -0.8^V to -0.85^V.

$$v_{SG} + 20.8\, i = 3.9^V$$

$$v_{SG} + 20.8 \times \frac{1}{2} \times 1.0 \times (v_{SG} - 0.85)^2 = 3.9$$

$$v_{SG} + 10.4\,(v_{SG}^2 - 1.7 v_{SG} + 0.7225) = 3.9$$

$$10.4 v_{SG}^2 - 16.68 v_{SG} + 3.614 = 0$$

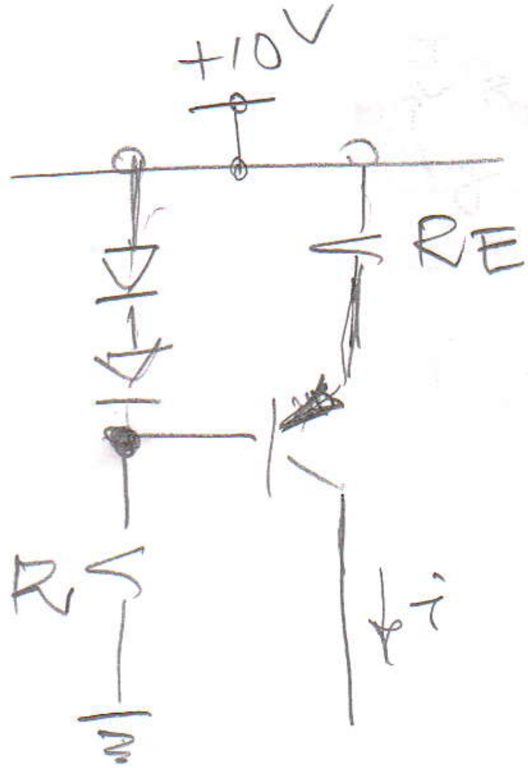
$$v_{SG} = \frac{16.68 \pm \sqrt{(16.68)^2 - 4 \times 10.4 \times 3.614}}{2 \times 10.4}$$

$$v_{SG} = 1.345\ V$$

$$i = \frac{1}{2} \times 1 \times (1.345 - 0.85)^2$$

$$= \underline{0.122\ mA}$$

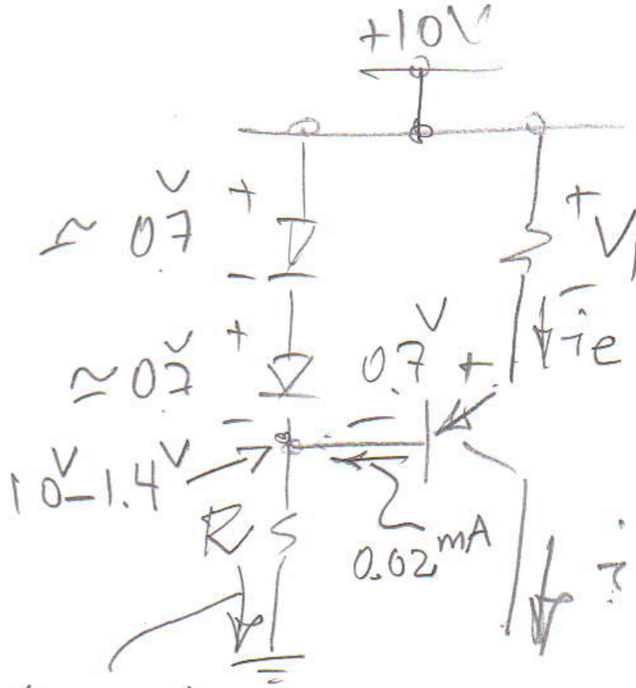
−2.4% change in current due to the change ~~from~~ in V_t from -0.8^V to -0.85^V.



$\beta = 50$

$i \simeq 1.0\,\text{mA}$

<u>choose R_E & R</u>



$V_R = 2\times 0.7 - 0.7 \simeq 0.7\,\text{V}$

$i \simeq \alpha i_E \simeq \frac{50}{51} i_E$

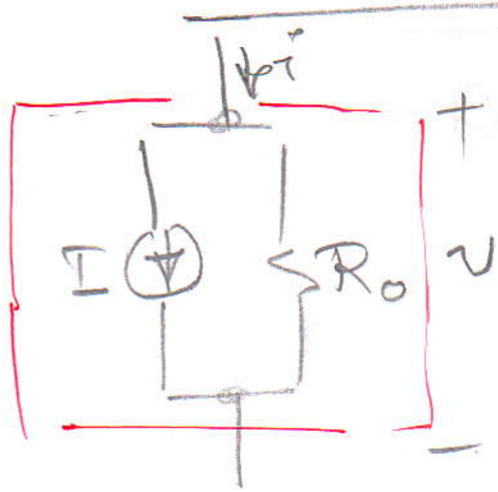
$\Rightarrow i_E = \frac{51\times 1}{50} = 1.02\,\text{mA}$

$i \approx 10\, i_B = 0.2\,\text{mA}$

$R = \frac{10^V - 1.4^V}{0.2} = \underline{43}\,\text{k}\Omega$

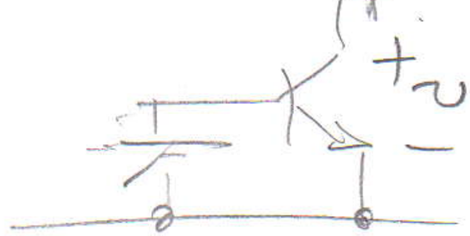
$R_E \approx \frac{0.7^V}{1.02} = 0.686\,\text{k}\Omega$ or $\underline{680}\,\Omega$

Internal Resistance



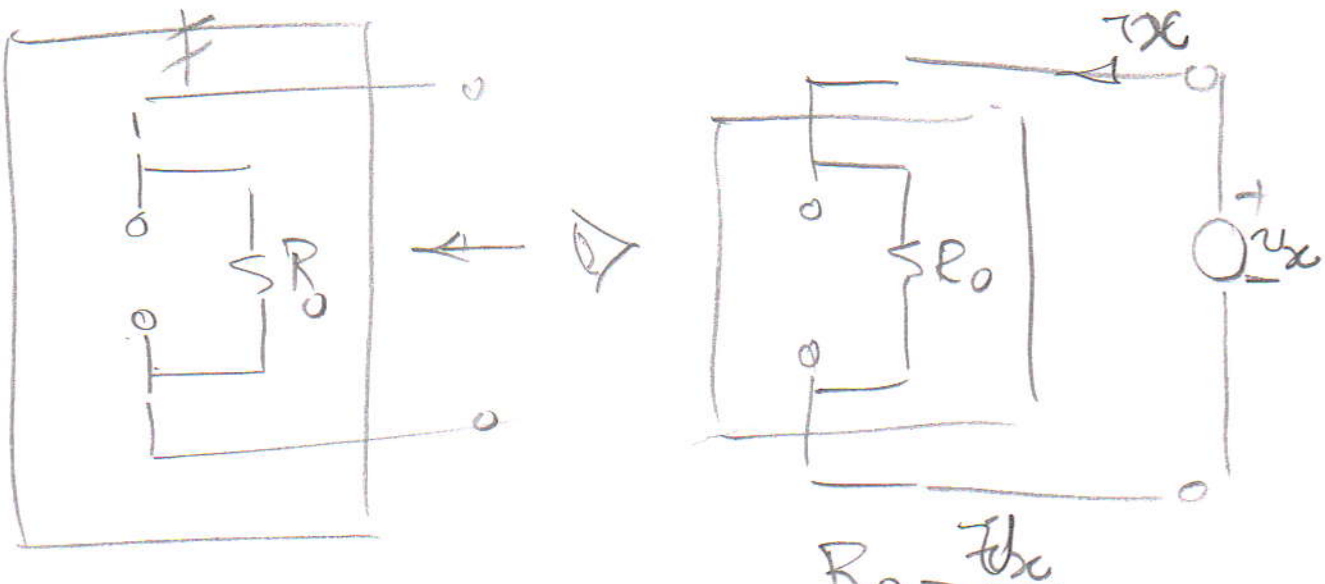
Physical nature of Ro: Early effect

$$i = I_s e^{v_{BE}/V_T}\left(1 + \frac{v_{CE}}{V_A}\right)$$



How to find Ro

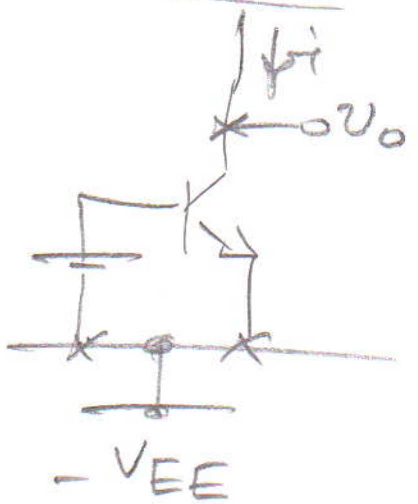
Do small-signal (ac) analysis



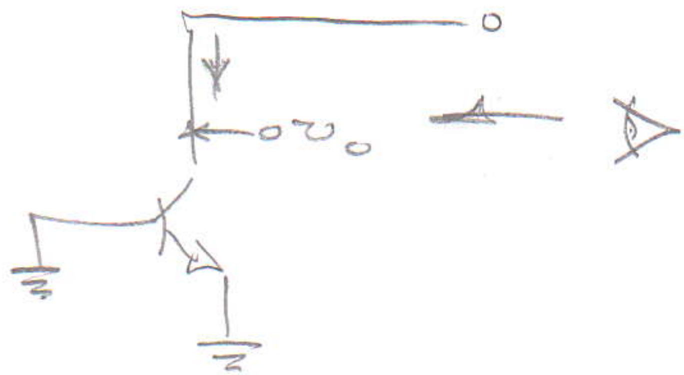
$$R_o = \frac{v_x}{i_x}$$

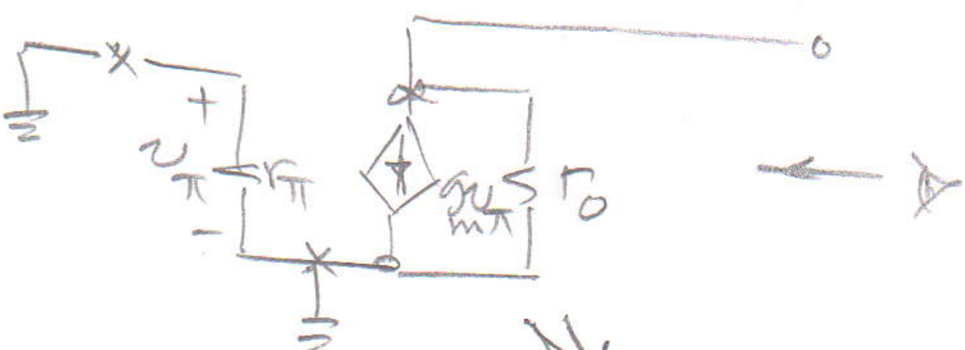
R.O.C

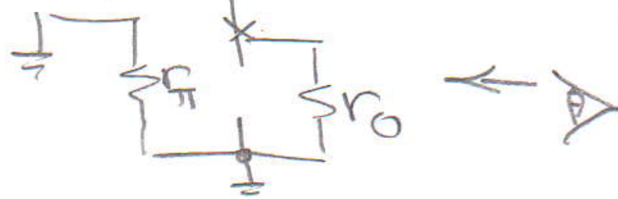
find R_o



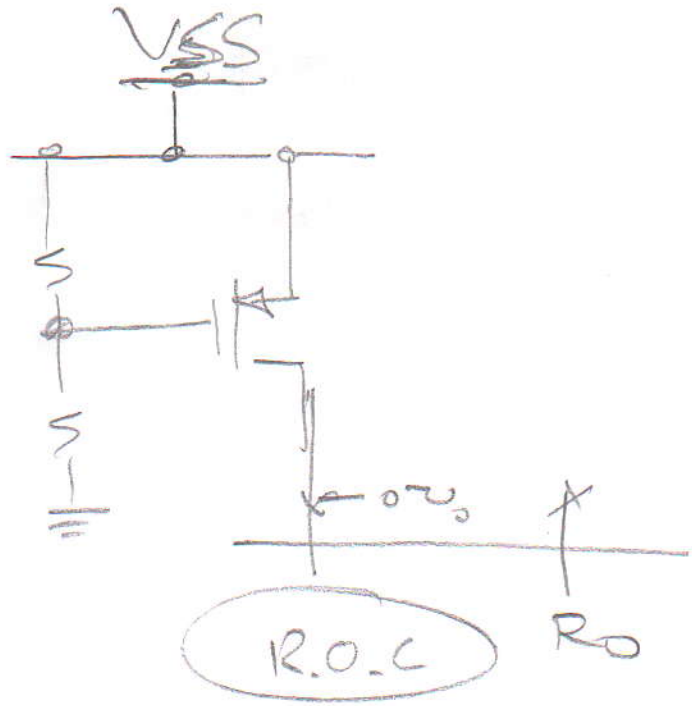
ac analysis



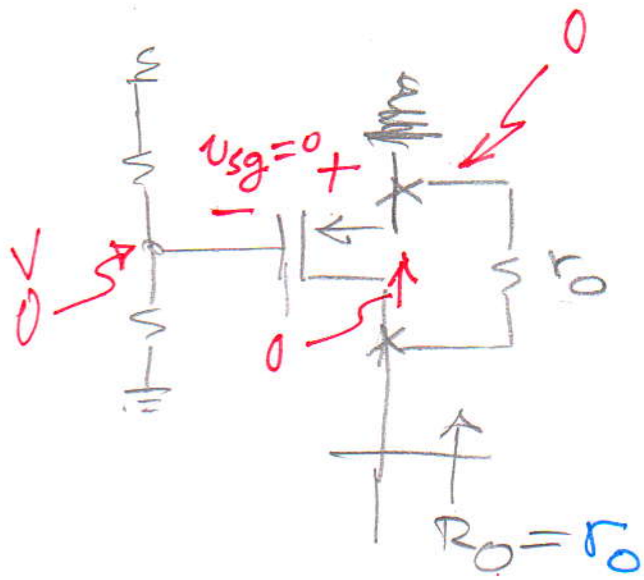




$\boxed{R_o = r_o}$

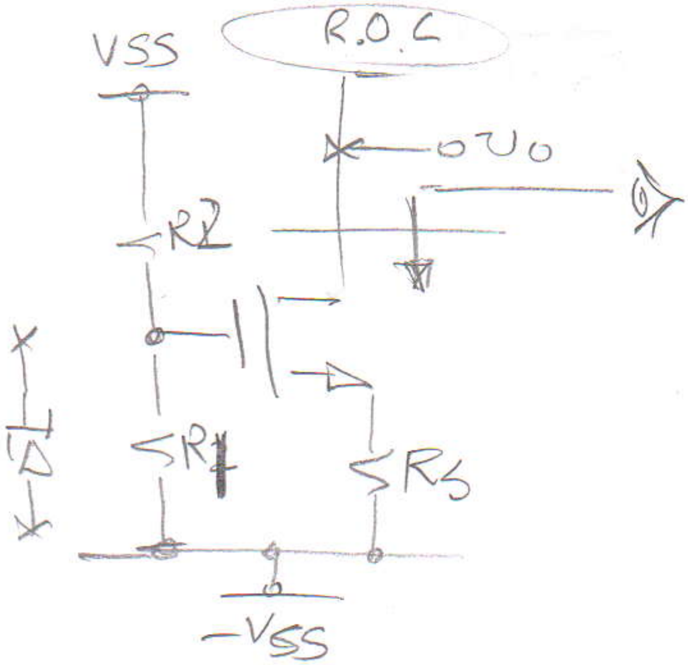


ac analysis

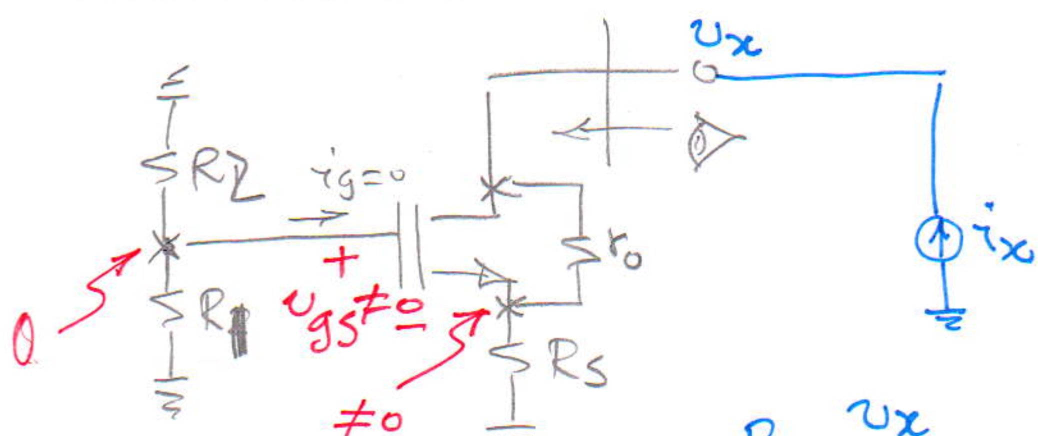


In general, $R_O = r_o$ if the emitter or source is grounded (ac wise).

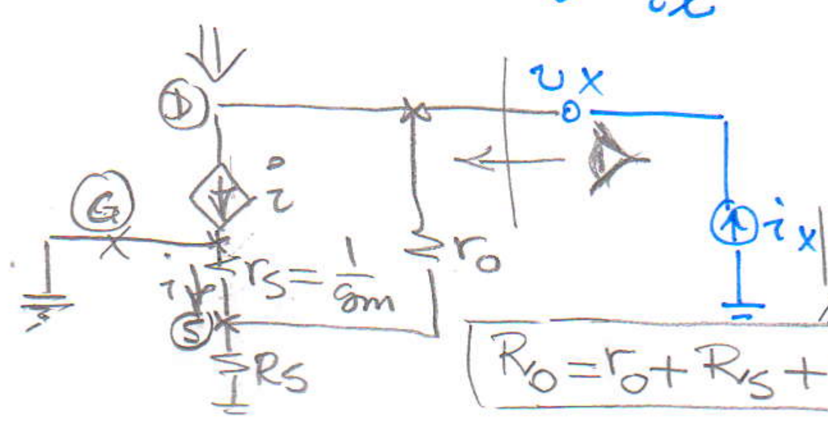
Cases with Emitter Degeneration (Source Degeneration) Resistance



ac analysis



$$R_o = \frac{v_x}{i_x}$$



$$\boxed{R_o \simeq (1 + g_m R_s) r_o}$$

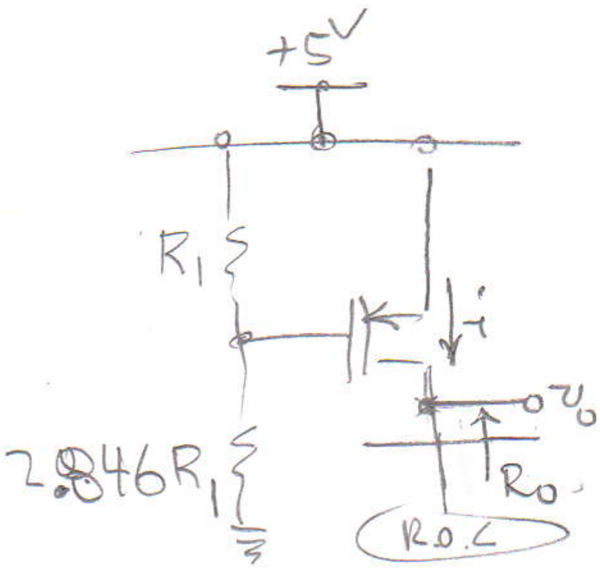
$$\boxed{R_o = r_o + R_s + g_m r_o R_s}$$

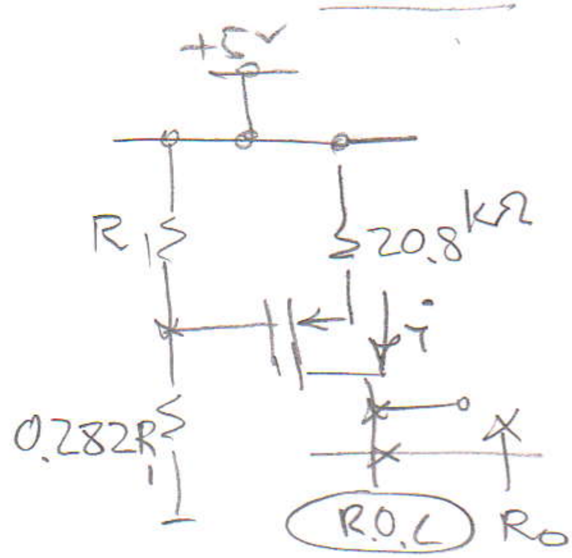
$K = 1.0\,mA/V^2$ $\quad V_A = -20^V$

$I = 0.125\,mA$ $\quad r_o = \frac{|V_A|}{I} = \frac{20}{0.125} = 160^{k\Omega}$

$$\Rightarrow g_m = \sqrt{2KI_D} = \sqrt{2\times1\times0.125}$$

$$= \sqrt{0.25} = 0.5\,mS$$





$R_o = r_o$

$= 160^{k\Omega}$

$$\frac{\Delta i}{\Delta v_o} = \frac{1}{R_o} = \frac{1}{160} = 6.25\,\mu A/V$$

$R_o = r_o + R_S + g_m r_o R_S$

$= 160 + 20.8 + \underbrace{0.5\times20.8}_{10.4}\times160$

$= 1844.8\,k\Omega$

$= 1.84\,M\Omega$

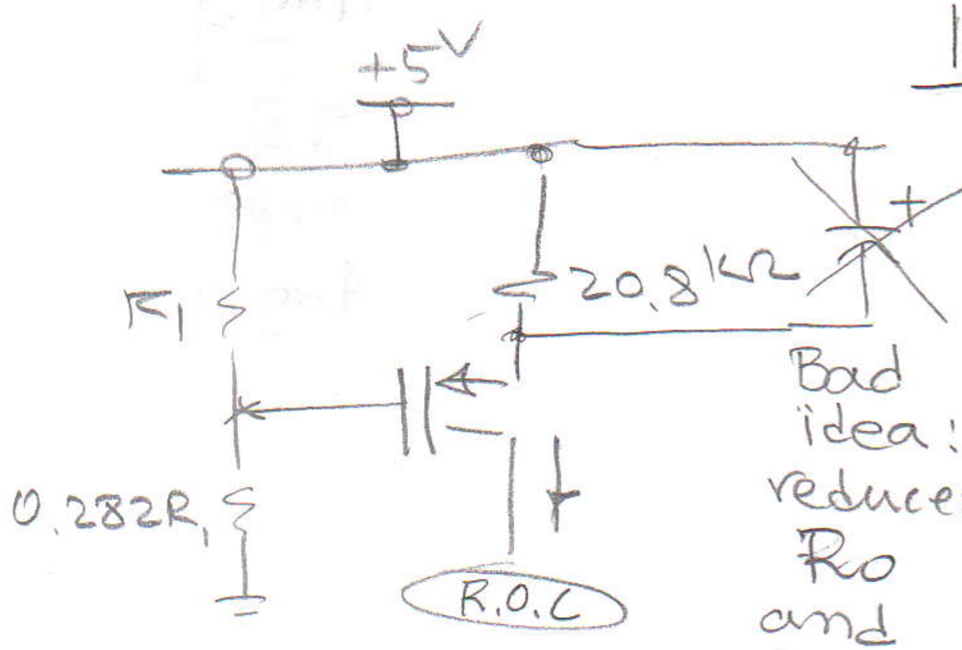
$R_o \simeq (1+g_m R_S)r_o$

$= (1+10.4)\times160$

$= 1824\,k\Omega$

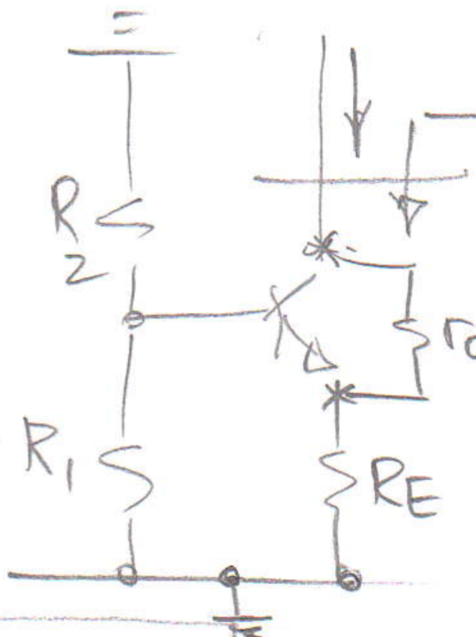
$= 1.82\,M\Omega$

$$\frac{\Delta i}{\Delta v_o} = \frac{1}{R_o} = \frac{1}{1.84} = 0.54\,\mu A/V$$



Bad idea: reduces Ro and increases line regulation

For BJT



$R_o \approx \left[1 + g_m (r_\pi \| R_E)\right] r_o$

$\beta = 50 \quad V_A = 50^V$

$I = 1.0\,mA$

$R_E = 0.686\,k\Omega$

$r_o = \frac{50}{1.0} = 50^{k\Omega}$

$g_m \cong 40 I_C = 40 \times 1.0 = 40\,mS$

$r_\pi = \frac{\beta}{g_m} = \frac{50}{40} = 1.25\,k\Omega$

$R_o \simeq [1 + \underbrace{40(1.25 \| 0.686)}_{17.7}] \times 50 = 936^{k\Omega}$

$\boxed{\frac{\Delta i}{\Delta v_o} = \frac{1}{936} \simeq 1.1\,\mu A/V}$

Temperature Invariant Sources