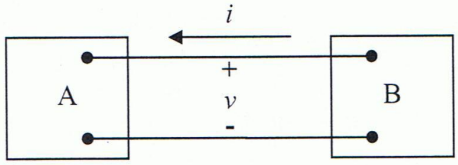
Q1(i):

Two electric circuits represented by boxes A and B are connected as shown in the following figure. For each of the following sets of numerical values, calculate the power in the interconnection and state whether the power is flowing from A to B or vice versa (3 marks each).

(a) $i = 10$ A, $v = 125$ V
(b) $i = 5$ A, $v = -240$ V
(c) $i = -12$ A, $v = 480$ V
(d) $i = -25$ A, $v = -660$V



(a) $10 * 125 = 1250$ watts

(b) $5 * (-240) = -1200$ watts

(c) $-12 * 480 = -5760$ watts

(d) $-25 * (-660) = 16500$ watts.

(a) 1250 W B → A

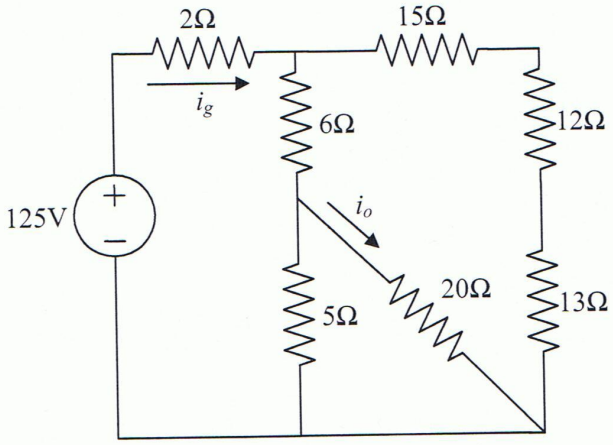
(b) 1200 W A → B

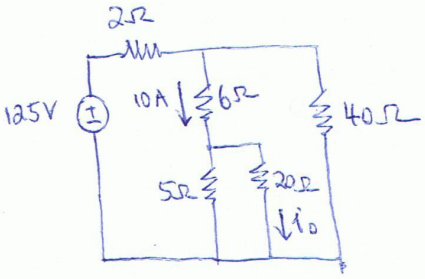
(c) 5760 W A → B

(d) 16500 W B → A

Q1(ii):

Find i_o and i_g in the following circuits (6.5 marks for each current).





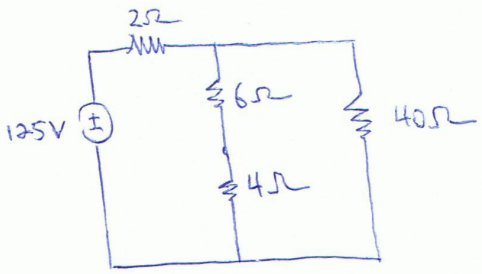
$$i_g = \frac{125V}{10\Omega} = 12.5A$$

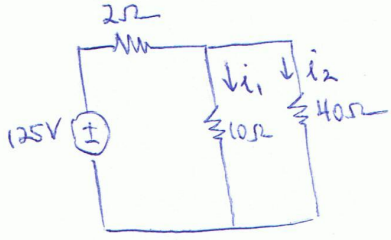
$$i_1 = 12.5 * \frac{40}{10+40}$$

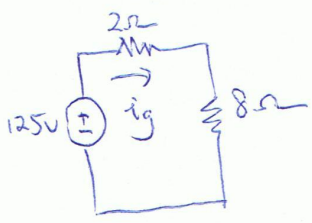
$$= 10A$$

$$i_o = 10 * \frac{20}{5+20}$$

$$= 8A$$



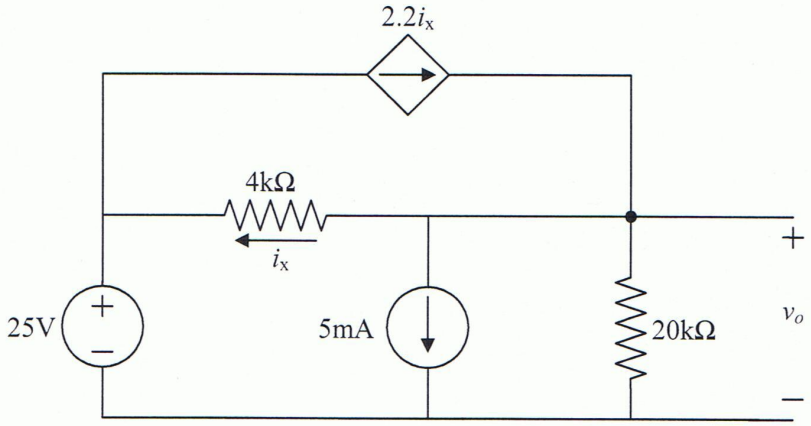




i_g= 12.5 A
i_o= 8 A .

Q2:

Using superposition to find v_o in the following circuit.



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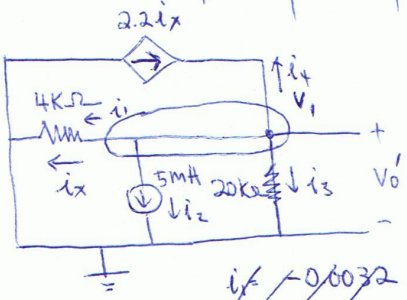

$i_1 + i_2 + i_3 + i_4 = 0$

$$\begin{cases} \frac{V_1}{4000} + 5m + \frac{V_1}{20000} - 2.2 i_x = 0 \\ i_x = \frac{V_1}{4000} \end{cases}$$

$$\frac{V_1}{4000} + 0.005 + \frac{V_1}{20000} - \frac{2.2V_1}{4000} = 0$$

$$5V_1 + 0.005 * 20000 + V_1 - 5 \cdot 2.2V_1 = 0$$

$$5V_1 + 100 + V_1 - 5 * 2.2V_1 = 0$$

~~$(5+1-5*2.2)V_1 = -105$~~

~~$8.000 V_1 = 105$~~

~~$V_1 = 13.8$ volts.~~

$(5+1-5*2.2)V_1 + 100 = 0$

$-(6-11)V_1 = 100$

$V_1 = \frac{100}{5}$

$= 20$ volts.

$i_1 = \frac{20}{4000} = 0.005$

$i_2 = 0.005$

$i_3 = \frac{20}{20000} = 0.001$

$i_4 = -0.005 * 2.2$

$= -0.011$

$i_1 + i_2 + i_3 + i_4 = 0.005 + 0.005 + 0.001 - 0.011$

$= 0.010 + 0.001 - 0.011$

$= 0.$

$V_o' = V_1 = 20$ volts.

$v_o = V_o' + V_o'' = 20 + 30$ volts $= 50$ volts

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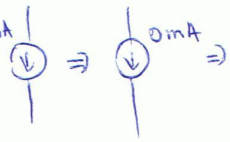
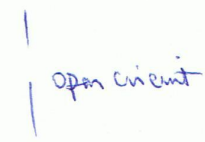
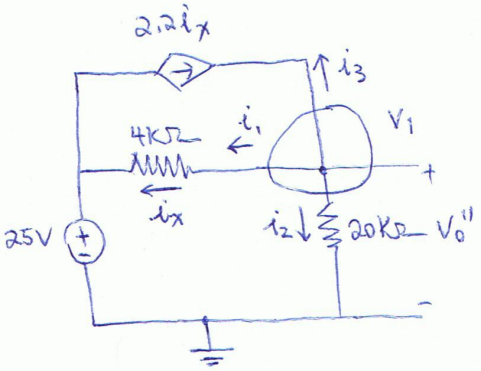
Disable 5mA   Open circuit



$i_1+i_2+i_3=0$

$V_o''=V_1=30$ volts

$$\frac{V_1-25}{4000}+\frac{V_1}{20000}-2.2\cdot i_x=0$$

$$i_x=i_1=\frac{V_1-25}{4000}$$

$$\frac{V_1-25}{4000}+\frac{V_1}{20000}-\frac{2.2(V_1-25)}{4000}=0$$

$$5V_1-125+V_1-11V_1+275=0$$

$$(5+1-11)V_1+150=0$$

$$-5V_1+150=0$$

$$V_1=30 \text{ volts}$$

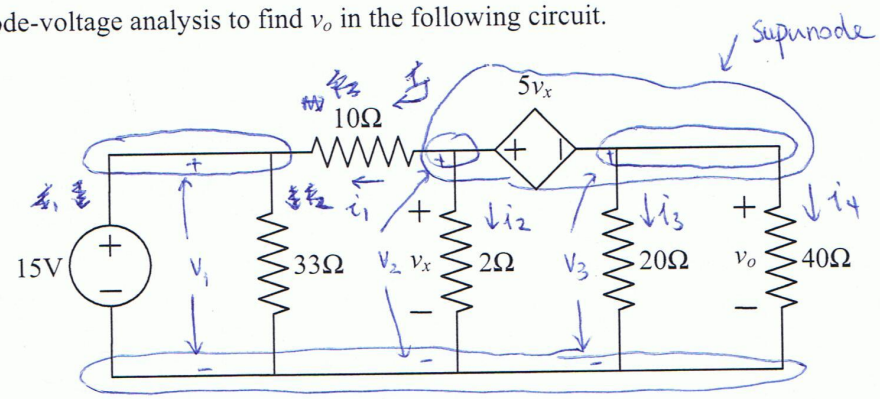
$$i_1=\frac{30-25}{4000}=0.00125\ A$$

$$i_2=\frac{30}{20000}=0.0015\ A$$

$$i_3=-2.2\cdot 0.00125=-0.00275$$

Q3:

Use node-voltage analysis to find v_o in the following circuit.



~~$i_1+i_2+i_3=0$~~

$V_1=15$

$i_1+i_2+i_3+i_4=0$

$$\frac{V_2-15}{10}+\frac{V_2}{2}+\frac{V_3}{20}+\frac{V_3}{40}=0$$

$4V_2-60+20V_2+2V_3+V_3=0$

$24V_2+3V_3=60$

$8V_2+V_3=20$ (1)

Sub (3) into (1)

$8V_2+(-4V_2)=20$

$4V_2=20$

$V_2=5$ ~~(V)~~ volts (4)

Sub (4) into (3)

$V_3=-4*5$

$V_3=-20$ volts

$V_3+5V_x=V_2$

$V_2=V_x$

$V_3+5V_2=V_2$

$V_3+4V_2=0$

$4V_2+V_3=0$ (2)

$-4V_2=V_3$ (3)

$$i_1=\frac{5-15}{10}=-1A$$

$$i_2=\frac{5}{2}=2.5A$$

$$i_3=\frac{-20}{20}=-1A$$

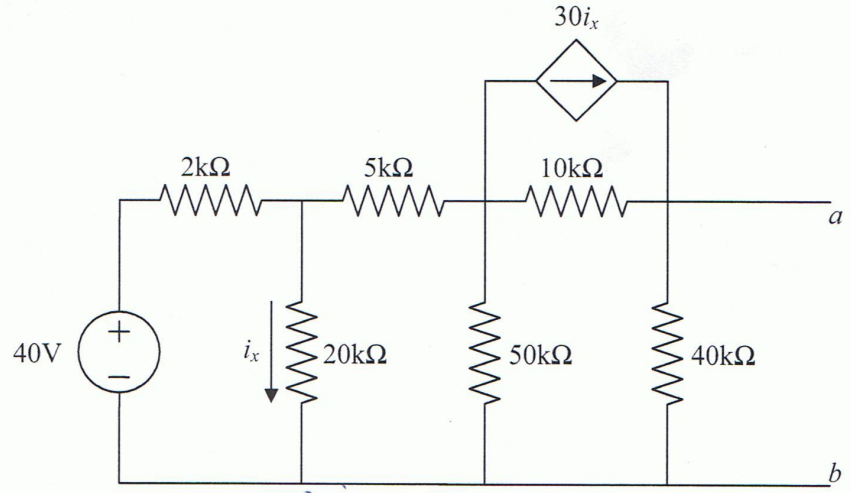
$$i_4=\frac{-20}{40}=-0.5A$$

$-1+2.5-1-0.5=0$ check.

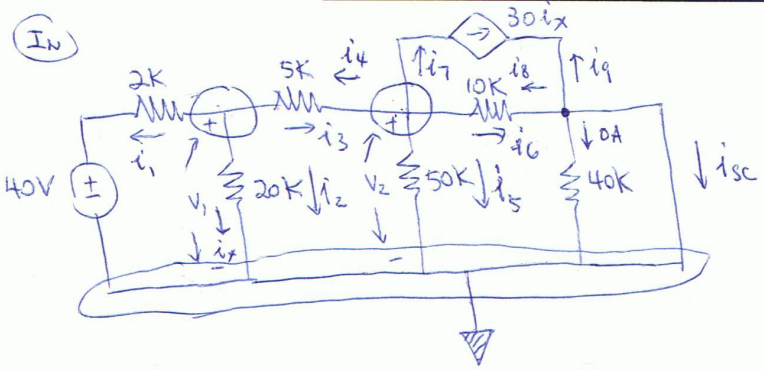
$V_o=V_3=-20$ volts.

Q4:

Find the Norton equivalent between terminals *a*-*b* of the following circuit:



I_N



$i_{sc} + i_8 + i_9 = 0$

$i_{sc} + \frac{0+52}{10000} - 30 \cdot i_x = 0$ → $0.0192A$

$i_{sc} + \frac{52}{10000} - 30 \cdot \frac{12.8}{20000} = 0$

$+0.0052$

$i_{sc} = 0.014$ Amps

$i_4 + i_5 + i_6 + i_7 = 0$

$\frac{V_2 - V_1}{5000} + \frac{V_2}{50000} + \frac{V_2}{10000} + 30 i_x = 0$

$i_x = \frac{V_1}{20000}$

$\frac{V_2 - V_1}{5000} + \frac{V_2}{50000} + \frac{V_2}{10000} + 30 \frac{V_1}{20000} = 0$

$20V_2 - 20V_1 + 2V_2 + 10V_2 + 150V_1 = 0$

$130V_1 + 32V_2 = 0$ (2)

$i_4 = \frac{-52-12.8}{5000} = -0.01296A$

$i_5 = \frac{-52}{50000} = -0.00104A$

$i_6 = \frac{-52}{10000} = -0.0052A$

$i_7 = 30 \cdot \frac{12.8}{20000} = 0.0192A$

$i_1 + i_2 + i_3 = 0$

$\frac{V_1 - 40}{2000} + \frac{V_1}{20000} + \frac{V_1 - V_2}{5000} = 0$

$10V_1 - 400 + V_1 + 4V_1 - 4V_2 = 0$

$15V_1 - 4V_2 = 400$ (1)

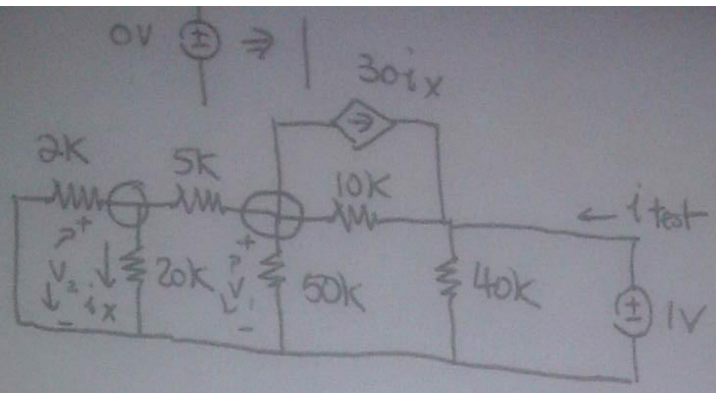
$V_1 = 12.8$ volts

$V_2 = -52$ volts

$i_1 = \frac{12.8 - 40}{2000} = -0.0136$

$i_2 = \frac{12.8}{20000} = 0.00064$

$i_3 = \frac{12.8 + 52}{5000} = 0.01296$ ✓



$$\frac{V_1-V_2}{5} + \frac{V_1-1}{10} + 30i_x + \frac{V_1}{50} = 0$$

$$\frac{V_1-V_2}{5000} + \frac{V_1-1}{10000} + 30i_x + \frac{V_1}{50000} = 0$$

$$50000\left(\frac{V_1-V_2}{5000} + \frac{V_1-1}{10000} + 30i_x + \frac{V_1}{50000}\right) = 0$$

$$10(V_1-V_2) + 5(V_1-1) + 50000 \cdot 30i_x + V_1 = 0$$

$$10V_1 - 10V_2 + 5V_1 - 5 + 1500000i_x + V_1 = 0$$

$$16V_1 - 10V_2 + 1500000i_x = 5 \quad (1)$$

$$\frac{V_2}{2000} + \frac{V_2}{20000} + \frac{V_2-V_1}{5000} = 0$$

$$\frac{V_2}{2} + \frac{V_2}{20} + \frac{V_2-V_1}{5} = 0$$

$$20\left(\frac{V_2}{2} + \frac{V_2}{20} + \frac{V_2-V_1}{5}\right) = 0$$

$$10V_2 + V_2 + 4V_2 - 4V_1 = 0$$

$$4V_1 = 15V_2$$

$$\frac{4}{15}V_1 = V_2 \quad (2)$$

$$i_x = \frac{V_2}{20000}$$

$$i_x = \frac{4}{15}\frac{V_1}{20000} \quad (3)$$

Sub (2) & (3) into (1)

$$16V_1 - 10\left(\frac{4}{15}V_1\right) + \frac{100}{1500{,}000}\cdot\frac{4}{15}\frac{5}{20{,}000}\cdot V_1 = 5$$

$$16V_1 - \frac{40}{15}V_1 + 20V_1 = 5$$

$$16V_1 - \frac{8}{3}V_1 + 20V_1 = 5$$

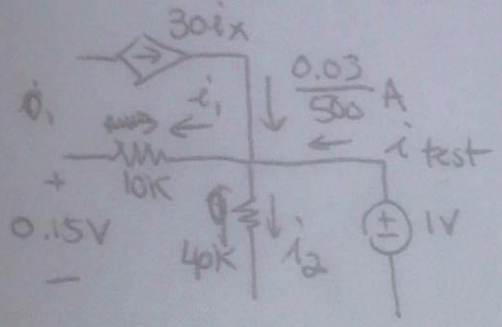
$$48V_1 - 8V_1 + 60V_1 = 15$$

$$40V_1 + 60V_1 = 15$$

$$100V_1 = 15$$

$$V_1 = 0.15 \Rightarrow i_x = \frac{4}{15}\frac{0.15}{20{,}000} = \frac{4\cdot 0.01}{20{,}000}$$

$$= \frac{0.01}{5{,}000}\ A$$



$$30i_x = 30\cdot\frac{0.01}{5{,}000}$$

$$= \frac{0.03}{500}\ A$$

$$i_1 = \frac{1-0.15}{10K} = \frac{0.85}{10K}$$

$$i_2 = \frac{1}{40K}$$

$$\frac{40000}{500} = 80$$

$$\frac{0.03}{500} + i_{test} = \frac{0.85}{10K} + \frac{1}{40K}$$

$$40000\left(\frac{0.03}{500} + i_{test}\right) = 40000\left(\frac{0.85}{10000} + \frac{1}{40000}\right)$$

$$80\cdot 0.03 + 40{,}000\, i_{test} = 4\cdot 0.85 + 1$$

$$2.4 + 40{,}000\, i_{test} = 4.4$$

$$40{,}000\, i_{test} = 4.4 - 2.4$$

$$i_{test} = \frac{2}{40{,}000}$$

$$R_{th} = \frac{1}{i_{test}} = \frac{40{,}000}{2}$$

$$= 20{,}000$$

$$= 20K.$$

Answer $R_{th} = 20k$.