PCS 125 Lab 1 : Simple Harmonic Motion

***Instructor**: Dr. Yuan*

***TA**: Meiyun Cao*

***Section**: 45*

Ahad Atiq
&
Sayeed Ahamad
&
Samuel Signorelli

***Student Number**:*
501185010 & 501209136 & 501179049

Contents

1 Introduction

The main objective of this lab is to study and examine the properties of simple harmonic waves. This will be done by graphing the position of the hanging mass vs time.

The resulting graphs shall be analyzed by using the LoggerPro software and by using Hooke's law to analyze the spring-mass systems vertical strech, amplitude and period of oscillation.

2 Theory

Before beginning this investigation we must know the methods and the theory behind the investigation and the due course of action.

2.1 Hooke's Law

Hooke's Law refers to the restoration force that acts on an object that is a part of a spring-mass system. It is described mathematically as,

$$F_s = -kx$$

Where, k is the spring constant and x is the initial displacement of the mass from the spring's equilibrium point. Spring force is always opposite to the direction of the object's displacement.

As change in the displacement can be measured as a instantaneous function of displacement. Mathematically,

$$x\left(t\right) = x_i \cos\left(\omega t + \phi\right)$$

Where x_i is the initial displacement of the mass on the spring-mass system, $\omega = \sqrt{k/m}$ is the angular frequency of the system and ϕ is the phase of the system (zero in our experiment).

Alternatively, we may mathematically represent the instantaneous spring restoring force as,

$$F_s(t) = -kx_i \cos(\omega t + \phi)$$

And because our experiment ensures that our system is in phase, $\phi = 0$. Therefore,

$$F_s(t) = -kx_i \cos(\omega t)$$

2.2 Energy of a Spring-Mass System

Acceleration is defined as a change in velocity, either in magnitude or direction . Because the direction of the velocity changes constantly in uniform circular motion, there is always an acceleration, even if the speed is constant.

2.2.1 Potential Energy

We know that potential energy is defined mathematically as,

$$U = W = \int_{x_i}^{x_f} F dx = \int_{x_i}^{x_f} -kx dx = -\left[\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right] = U_i - U_f$$

More generally defined as,

$$U = \frac{1}{2}kx^2$$

Where k is the spring constant and x is the spring-mass system's displacement from the eqilibrium position.

2.2.2 Kinetic Energy

We know for any real general mechanical system, the kinetic energy is mathematically defined as,

$$K = \frac{1}{2}mv^2$$

Where v is the instantaneous velocity of the spring-mass system.

2.2.3 Total Energy

Total energy is defined as the instantaneous sum of potential and kinetic energies. Mathematically,

$$E = U + K$$

Therfore, using the SHM equations for instantaneous position and velocity we have,

$$E = \frac{1}{2}kA^2$$

3 Materials Required

- Stand, Rod & Clamp
- Spring
- 5, 50g Hanging Masses
- Metal Cage (Protection for the Motion sensor)
- Meter Scale
- Vernier Motion Detector
- Vernier Logger Lab Pro Software
- Graphical Analysis Software

4 Procedure

4.1 Procedure I - In Equilibrium

1. Connect the spring to the horizontal rod that is attached to the stand. Make sure that it is fastened and secured. Hang the hanger (50g) to the end of the spring.
2. Continue to add the masses to the hanger until the spring begins to extend a bit. Once the masses is hanging freely, measure the distance to the bottom of the of the hager from the top of the table.

3. Repeat these steps for each increasing mass measurement. Make sure to record all data and uncertainties.

4.2 Procedure II - In Oscillation

1. Set-up the Equipment.
   (a) Hang 200g of the masses in the spring. Make sure that everything is secure so it doesn't come loose and fall onto the sensor.
   (b) Align the sensor directly below the spring. Place a metal cage over the sensor to protect it.
   (c) Connect the motion detector to the DIG/SONIC1 channel of the interface.
   (d) Practice a few attempts to get an understanding on how it should work. Lift up the hanger a couple centimeters and release it. This should cause the hanger to oscillate vertically.
   (e) Click on the button "Collect" to start taking data. It will automatically stop after 5 seconds.
   (f) A sinusoidal graph should appear. Click the button "Zoom All" if not close enough. If any irregular spikes occur, data may have to be re taken.
2. As the mass is stabally hanging, zero the sensor by clicking the button on the bottom right side of your screen that shows the current position.
3. Lift the mass about 5 centimeters above its current location and release. Like in practice, it should oscillate vertically.
4. Click the button "Collect" and a graph should appear. The graph represents the mass's position as a function of time.
5. Click on each maximum and minimum points. Calculate the average amplitude of the graph by adding all these points up and dividing by how many there are. Make sure to include uncertainty.
6. With the position graph that was created, determine the period of the oscillation. Do this by dragging the mouse across two points.
7. Repeat the procedure from steps 2-6 for the same mass that was used, except use a larger amplitude of oscillation. If increasing it is not possible, reduce the amplitude.

8. Repeat the procedure from steps 2-6 except use a total mass of 300g. Make sure to record all data and uncertainties. This graph that will be created will be used for analysis II.

5 Experimental Data

Mass (kg)	Distance (m)
0.050.0005 kg	0.450.005 m
0.100.0005 kg	0.440.005 m
0.150.0005 kg	0.420.005 m
0.200.0005 kg	0.390.005 m
0.250.0005 kg	0.360.005 m

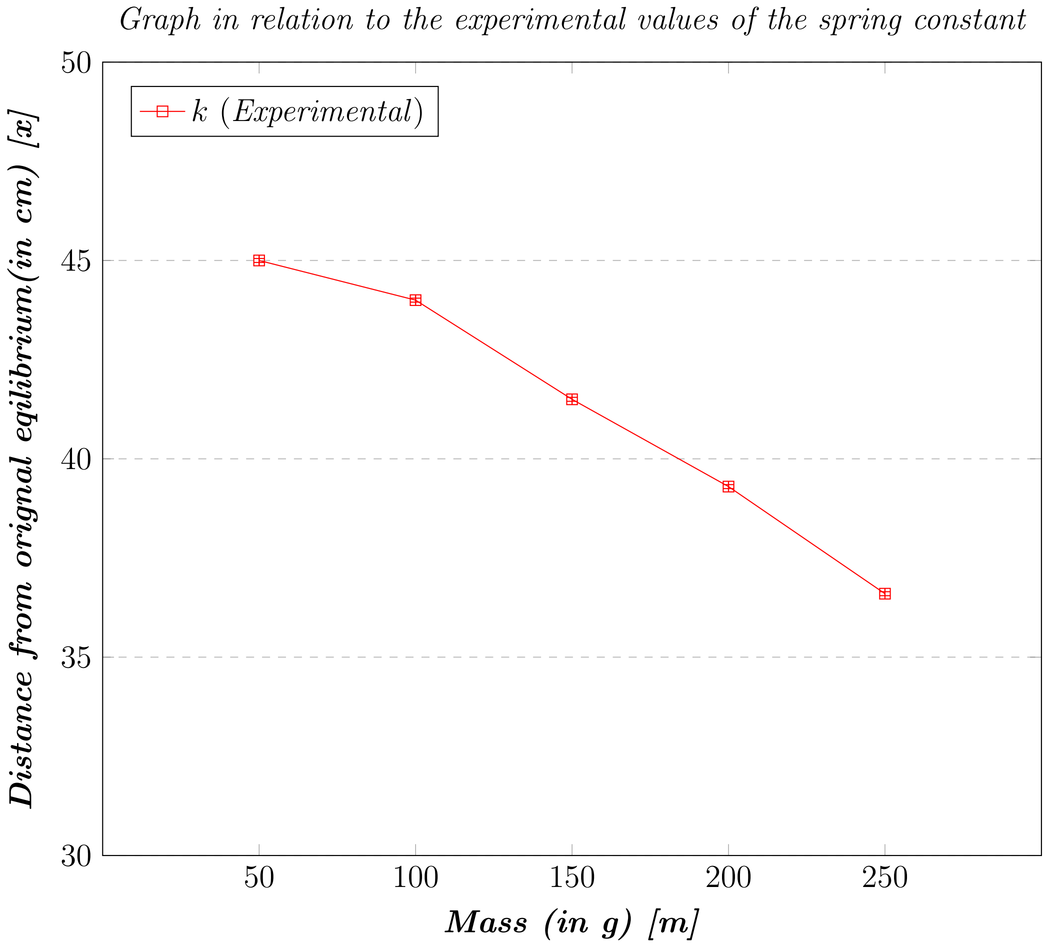
Table 5.1: Measurements of spring with mass at rest

Mass (kg)	Compression (m)	Average Amplitude (m)	Average Period (s)
0.200.0005 kg	0.05 m	0.0455 ± 0.0005 m	0.650.005 s
0.200.0005 kg	0.10 m	0.0296 ± 0.0005 m	0.650.005 s
0.300.0005 kg	0.05 m	0.0388 ± 0.0005 m	0.800.005 s

Table 5.2: Measurements of spring with oscillating mass

6 Analysis

6.1 Part I



We know that when the mass is attached to the spring, results in the spring being streched from its equilibrium position, resulting in an new equilibrium position. This is due to the force acting on the spring-mass system due to gravity. Therfore we have,

$$F_g = mg$$

We also know that the Spring force is mathematically represented as,

$$F_s = -kx$$

As we know that the spring eventually comes to an equilibrium length, where the forces equal, we have,

$$F_s = F_g$$

Therfore,

$$mg = -kx \implies k = -\frac{m}{x}g$$

Therefore from the above graph, we have,

$$k \approx 1.246 \text{ N/m}$$

6.2 Part II

We know that,

$$x(t) = A\cos(\omega t + \phi)$$

We also know that,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.8} = 7.85 \text{ rad/s}$$

Therfore,

$$y(t) = A\cos(7.85t + \phi)$$

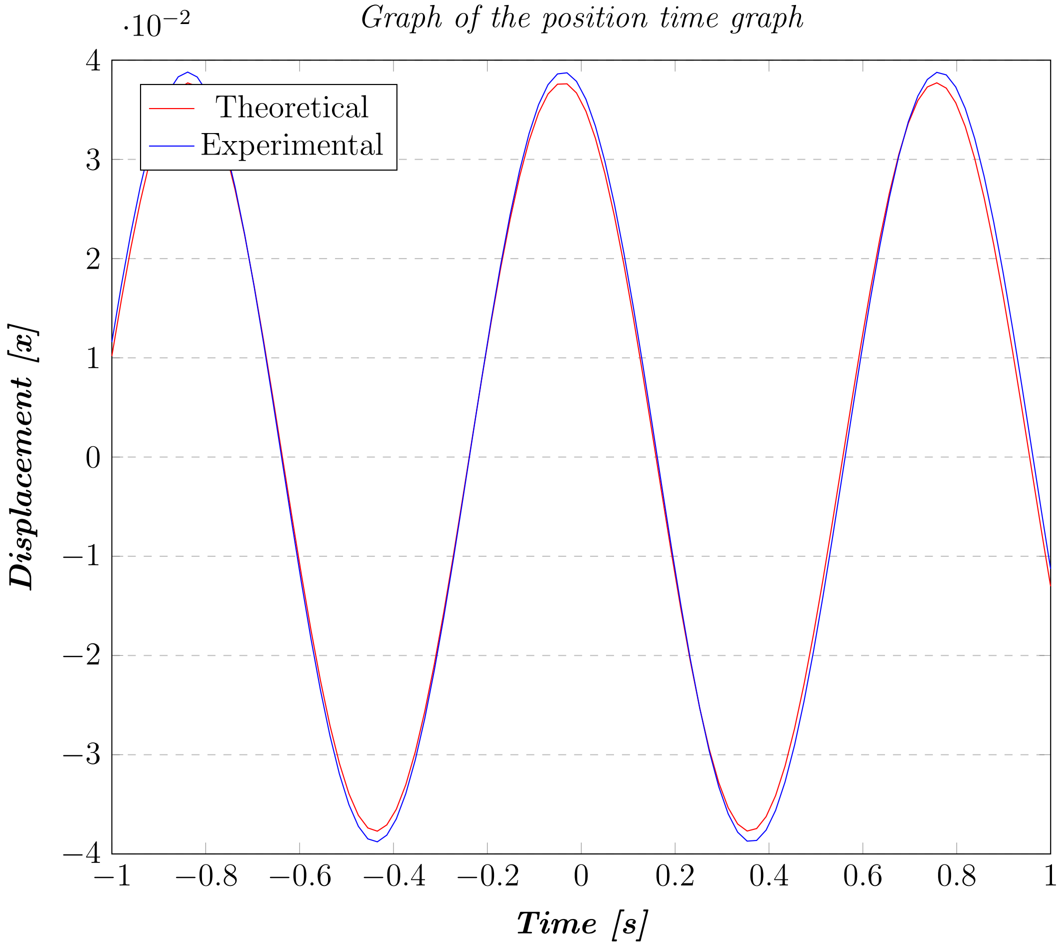
Also,

$$y(0) = A\cos(\phi)$$

Therfore,

$$\phi = \ 1.87 \text{ rad}$$

This implies that the phase of the system which is ϕ is 1.87 rad.



After using the Sine curve of the form, $A \sin(Bx + C)$ to fit the data from the above graph, the values for A, B & C are,

Expected Value	Measured Value
$A = 0.03772$ m	$A = 0.03885$ m
$B = 7.894$ rad/s	$B = 7.85$ rad/s
$C = 1.884$ rads	$C = 1.87$ rads

Table 6.1: Values of A, B & C in the form $A \sin(Bx + C)$ that describes the motion of the system

We now know that, $\omega = B$, therefore $\omega = 7.85$ rad/s

Therefore,

$$\omega = 7.85 = \sqrt{\frac{k}{m}}$$

$$\implies k = 18.49 \text{ N/m}$$

7 Discussion

1. Based on your data, is the oscillation period affected by the amplitude of the oscillation? Explain your reasoning in a sentence or two ?

Answer: Based on the results from the experiment, the oscillation period is not affected by the amplitude of oscillation. The only difference between our first trial and second trial was the change in mass, which in the end did not affect the oscillation period.

2. If you combine and plot the sum $\frac{1}{2}ky^2 + \frac{1}{2}mv^2$, what do you expect the graph to look like? Explain in words ?

Answer: If the sum of $\frac{1}{2}ky^2 + \frac{1}{2}mv^2$ is combined and plotted, expectations are that the graph would show that potential and kinetic energy levels would be opposite to each other. This is because when one is high, the other is low. Overally the graph is expected to be a straight line that is fixed about a constant value.

3. If you examine the motion of the mass for many (more than 20) oscillations, what happens to the amplitude of the oscillation? What is the physical reason for this ?

Answer: If the motion of the mass for about 20 oscillations were examined, a slight decrease in amplitude of the oscillation would occur. This is because air resistance would slow down the oscillation. In theory, oscillations without friction/air resistance can go on forever with a constant amplitude. If this was tested, air resistance would affect the weight and cause it to slow down over time, resulting in a smaller amplitude.

8 Conclusion

There are many things that can be taken away from this experiment such as theory vs. practical and potential vs kinetic energy. With the experiment only taking place within 5 seconds, not much can interfere with the results. Calculations were done to find different values using the data observed. If the experiment was done with a larger time interval, the calculations could have been different. This is due to air resistance which would have made the oscillations slow down.

On the topic of theory, this experiment supports the theory that potential energy and kinetic energy are opposite to each other. When the graph was completed, the lines that represent each type of energy were always opposite to each other. This is because potential energy transforms into kinetic energy and vice versa.

Bibliography

[1] Serway, R. A., Jewett, J. W. (2018). Physics for Scientists and Engineers. Cengage Learning.