Final Exam Review - ELE 202 W 2023

Q 1: [20 marks]

Q 1: Part a) The switch in Figure 1 a has been opened for a long time. At $t = 0$, the switch is closed.

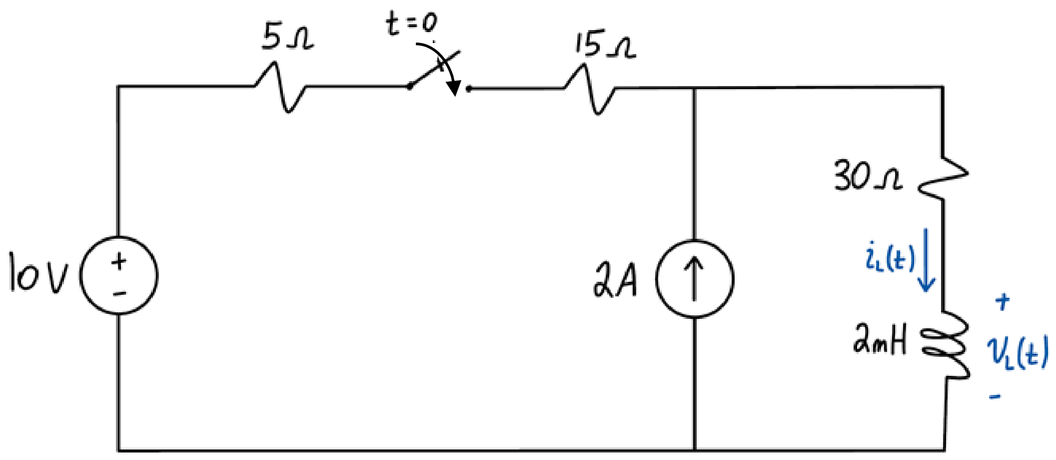
i) Calculate $i_L(0^+)$ and $i_L(\infty)$ **(Use Source Transformation only when solving for $i_L(\infty)$)** **[4 marks]**

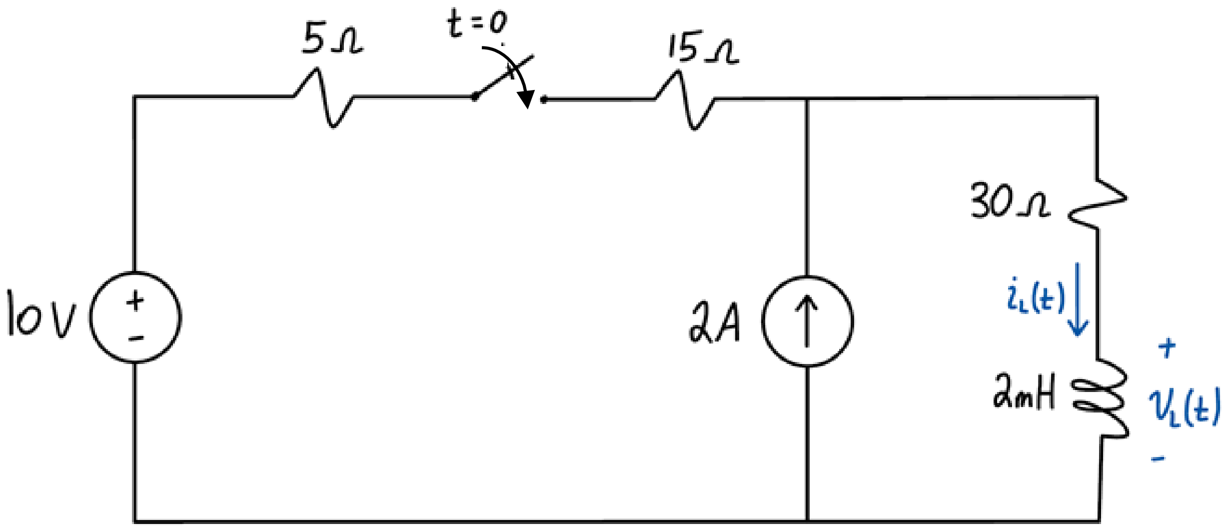
ii) Determine an expression for $i_L(t)$ for t>0 **[3 marks]**

iii) Determine an expression for $v_L(t)$ for t>0 **[2 marks]**

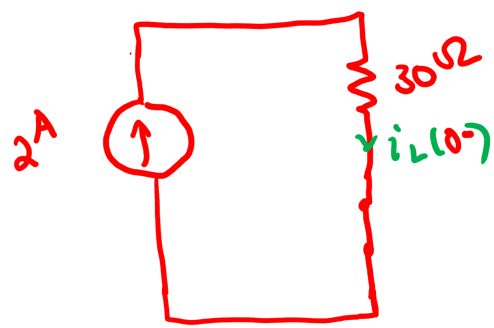
v) Draw $i_L(t)$ versus time **[2 marks]**

Hints: $\tau = \frac{L}{R}$, $y(t) = y(\infty) + [y(0^+) - y(\infty)]e^{-\frac{t}{\tau}}$





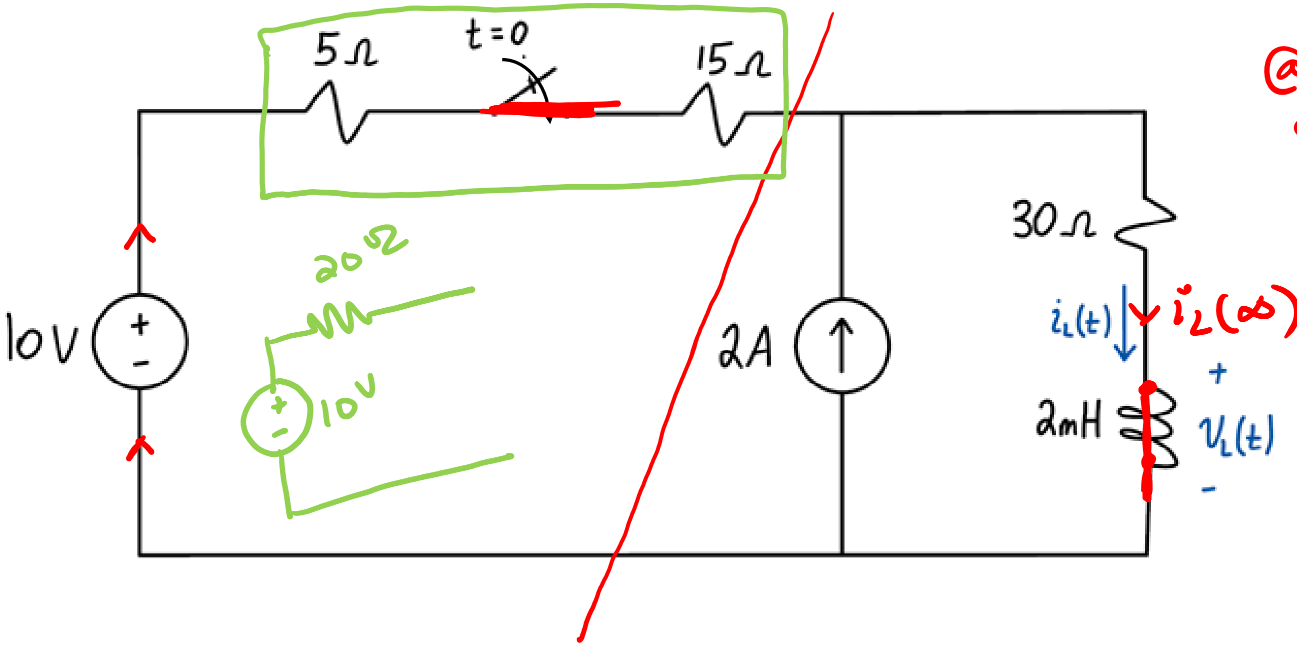
① circuit @ $t<0$: switch open

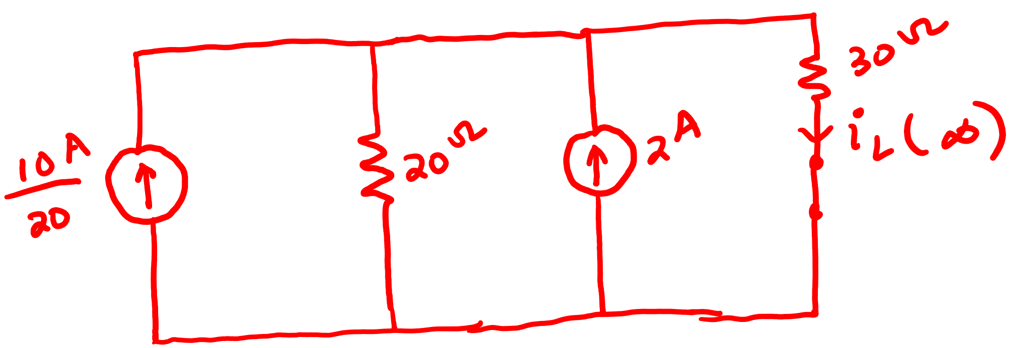


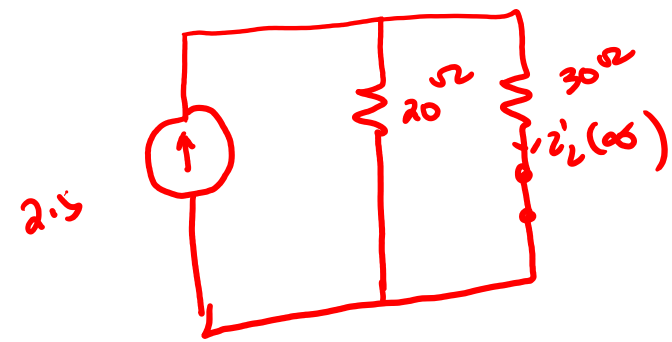
$$i_L(0^-) = 2A$$

$$i_L(0^+) = i_L(0^-) = 2A$$

@ t>0 : switch closed







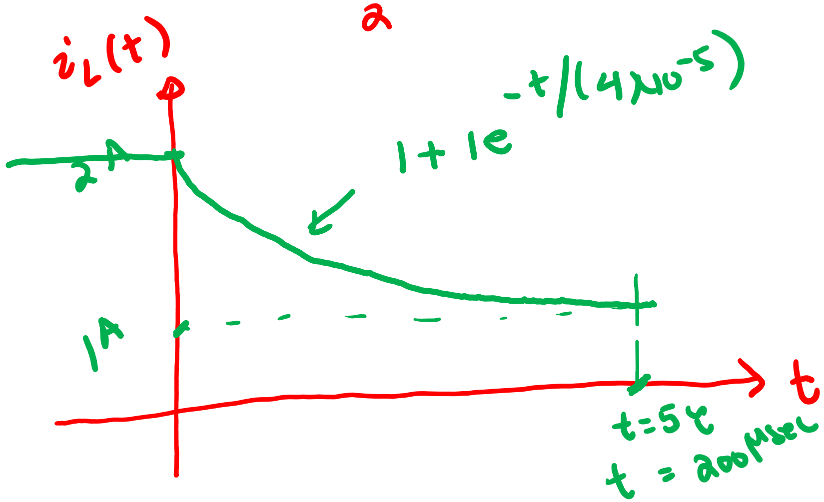
$$i_L(t) = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau}$$

$$i_L(t) = 1 + [2-1]e^{-t/(4\times10^{-5})} = 1 + 1e^{-t/(4\times10^{-5})}\ A \quad \text{for } t>0$$

$$v_L(t) = L\frac{d\,i_L(t)}{dt} = 2^{mH}\frac{d}{dt}\left[1 + 1e^{-t/(4\times10^{-5})}\right]$$

$$= 2^{mH}\left(0 + (1)\left(-\frac{1}{4\times10^{-5}}\right)e^{-t/(4\times10^{-5})}\right]$$

$$v_L(t) = -\cancel{2}\times10^{-3}\frac{1}{\underset{2}{\cancel{4}}\times10^{-5}}e^{-t/(4\times10^{-5})} = -50e^{-t/(4\times10^{-5})}\ V \quad t>0$$



Q 1: Part b)

The switch in Figure 1 b has been closed for a long time. At $t = 0$, the switch is opened.

i) Calculate $v_C(0^+)$ and $v_C(\infty)$ (**Use Superposition Principle only when solving for** $v_C(0^+)$**)** [**4 marks]**

ii) Determine an expression for $v_C(t)$ for t>0 **[3 marks]**

iii) Determine an expression for $i_C(t)$ for t>0 **[2 marks]**

Hints: $\tau = RC$, $y(t) = y(\infty) + [y(0^+) - y(\infty)]e^{-\frac{t}{\tau}}$

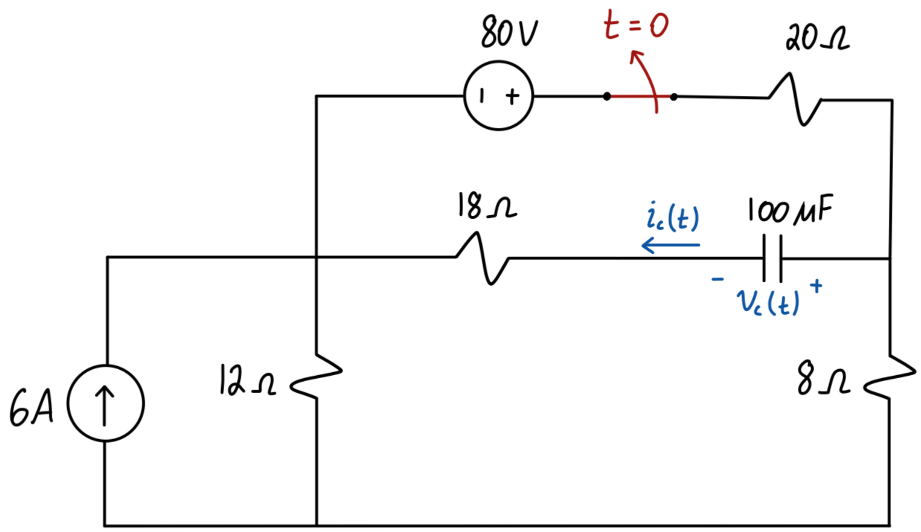
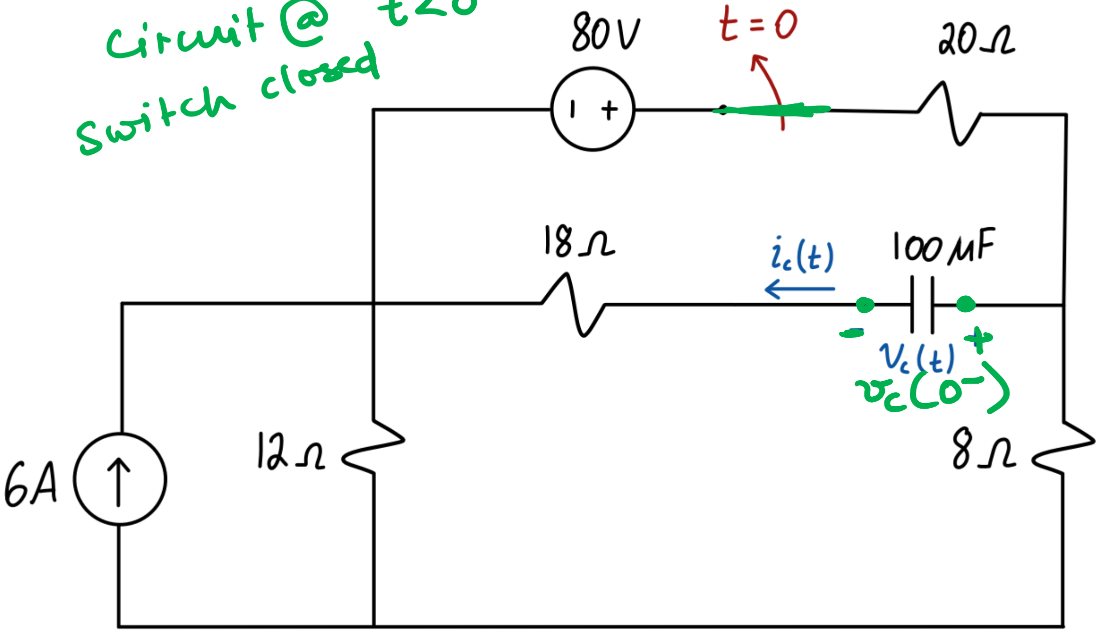
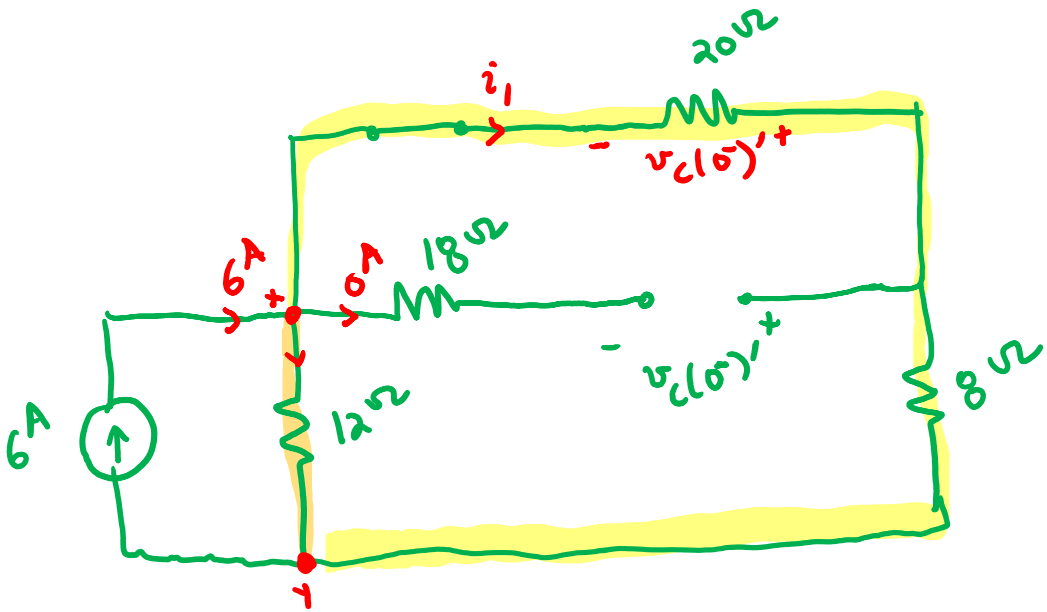


Figure 1 b: RC circuit with DC input

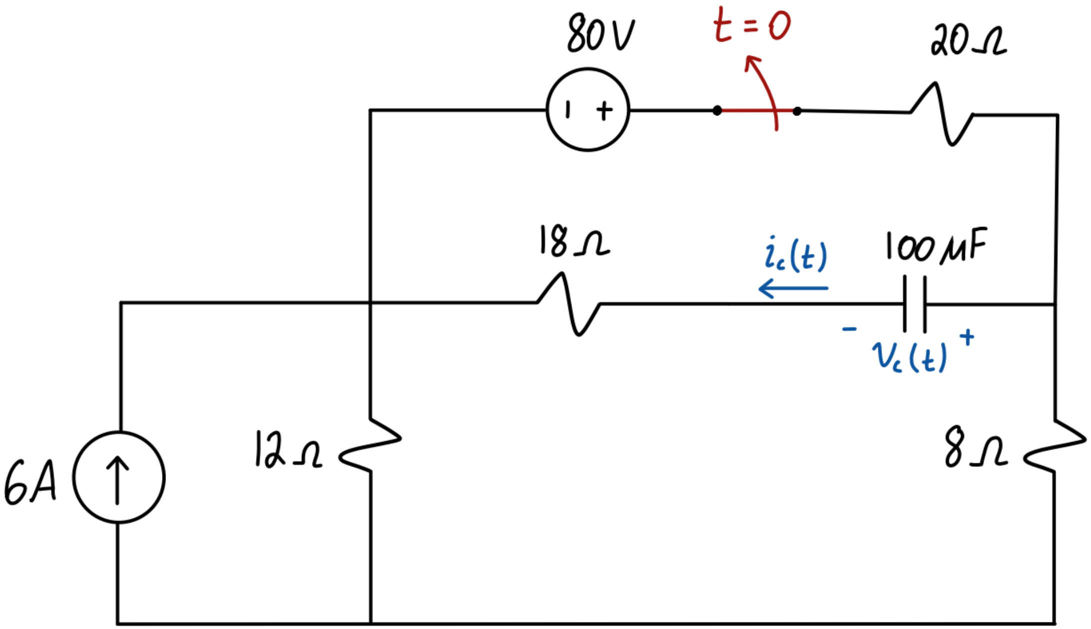
Circuit @ $t<0$
Switch closed

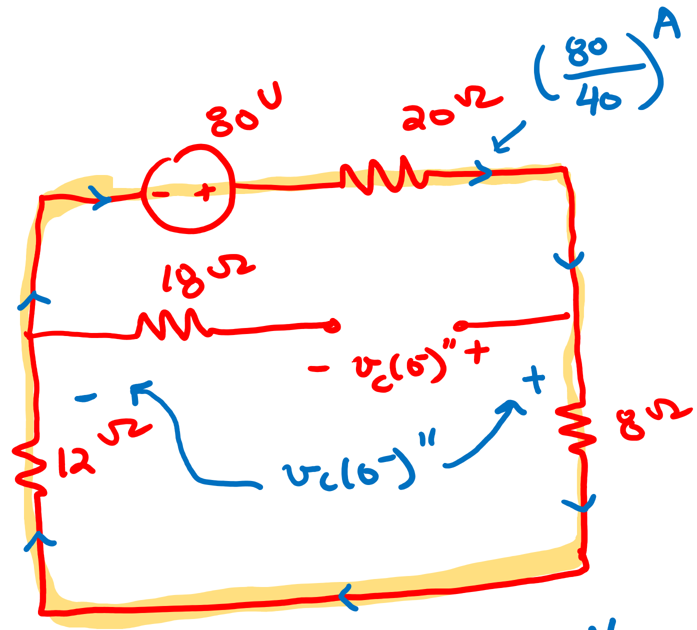


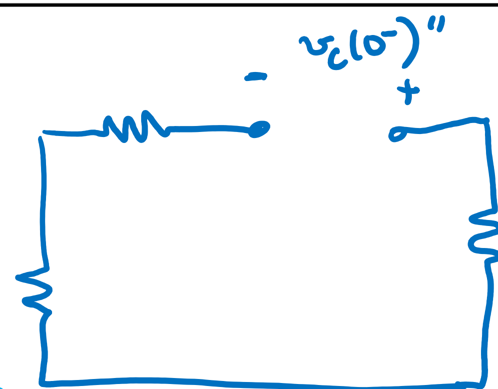


$$i_1 = \frac{12}{(12+20+8)}(6) = \frac{9}{5}\ \text{A}$$

$$v_c(0^-)' = -20 \times \frac{9}{5} = -36\ \text{V}$$



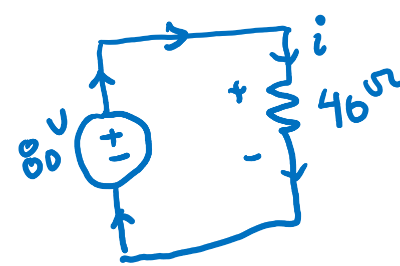




$$v_c(0^-)'' = 2^A \times 20^\Omega = 40^V$$

$$v_c(0^-) = v_c(0^-)' + v_c(0^-)''$$

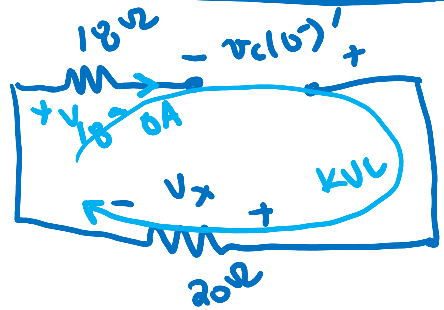
$$= -36^V + 40^V = 4^V$$



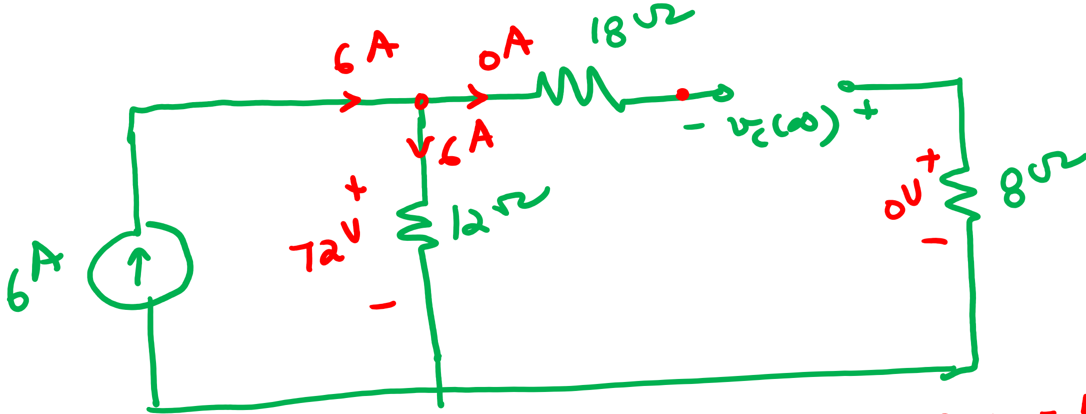
$$i = \frac{80^V}{40^\Omega}$$

$$-v_c(0^-)'' + V_x + \cancel{V_{18}}^{\,0} = 0$$

$$V_x = v_c(0^-)''$$

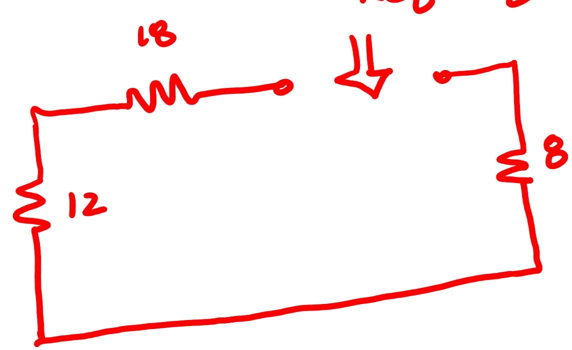


circuit $t > 0$: switch open



$v_c(\infty) = -72\,V$

$R_{eq} = \cancel{48}\ 38\ \Omega$



$\tau = R_{eq}\, C$

$= \cancel{48}\ 38 \times 100\ \mu sec$

$= 3.8\ msec$

$v_c(t) = -72 + [4 - (-72)]e^{-t/\tau} = -72 + 76\, e^{-t/3.8\times10^{-3}}\ V$ for $t>0$

$i_c(t) = C\dfrac{dv_c}{dt} = 100\times10^{-6} \times \left[76 \times \left(-\dfrac{1}{3.8\times10^{-3}}\right) e^{-t/\tau}\right]$

$i_c(t) = -2e^{-t/(3.8\times10^{-3})}$ A for $t>0$

Q 2: [20 marks] Solve the circuit given below using Nodal Analysis.

1. Write the nodal equations at all the non-reference nodes **[6 marks]**
2. Simplify the equations **[4 marks]**
3. Rearrange the equations in Matrix form **[2 marks]**
4. Show all the steps of Cramer's rule to find the Node voltage **[2 marks]**
5. Using a calculator, calculate the numerical values of the Node voltages at all the non reference nodes **[2 marks]**
6. Express $\bar{I}$ in terms of the node voltages and calculate its numerical value. **[2 marks]**
7. Draw a phasor diagram showing $\bar{V}_{s1}$ and $\bar{I}$ phasors. **[2 marks]**

Note: All sources are AC.

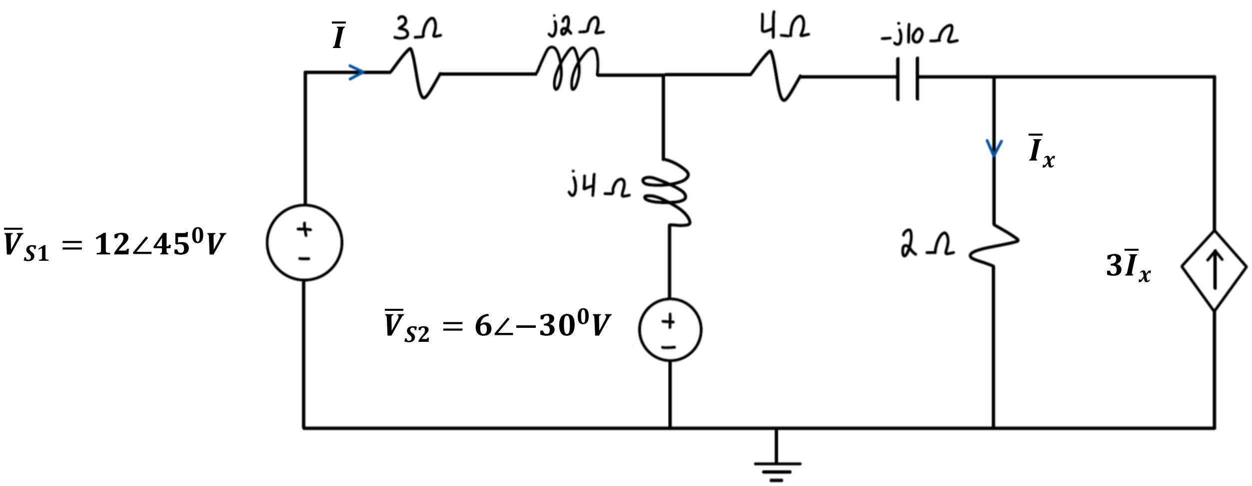
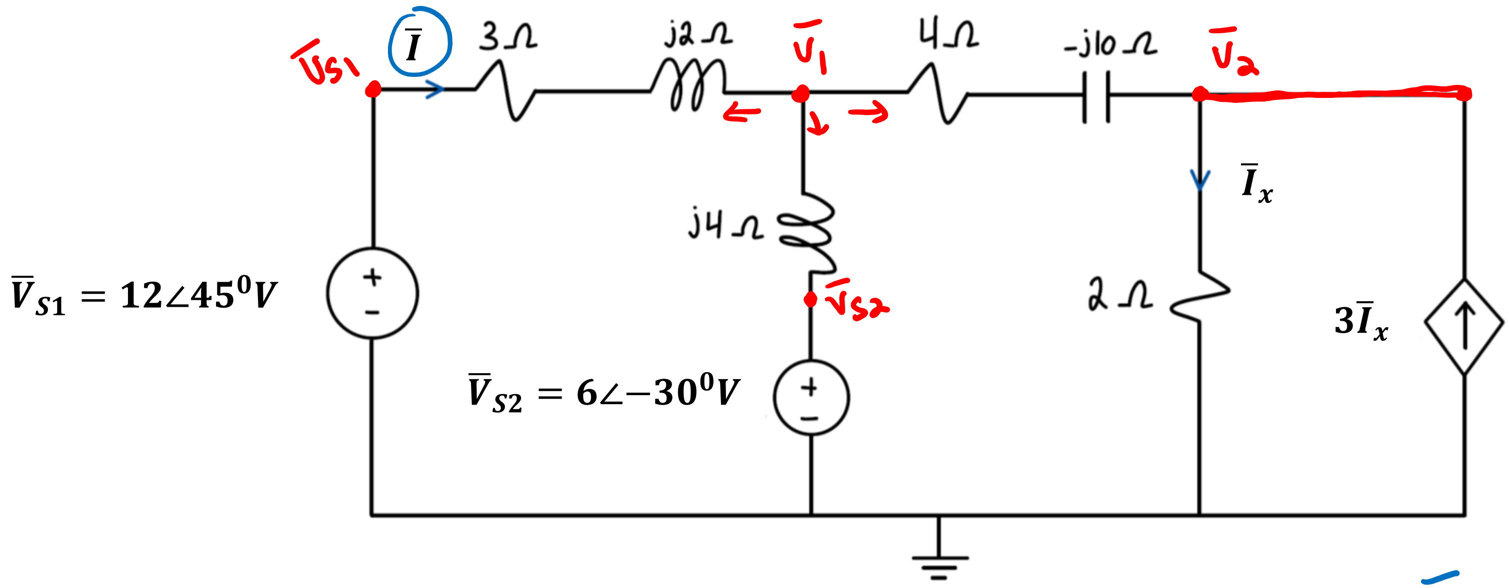


Figure 2: Circuit for Q 2



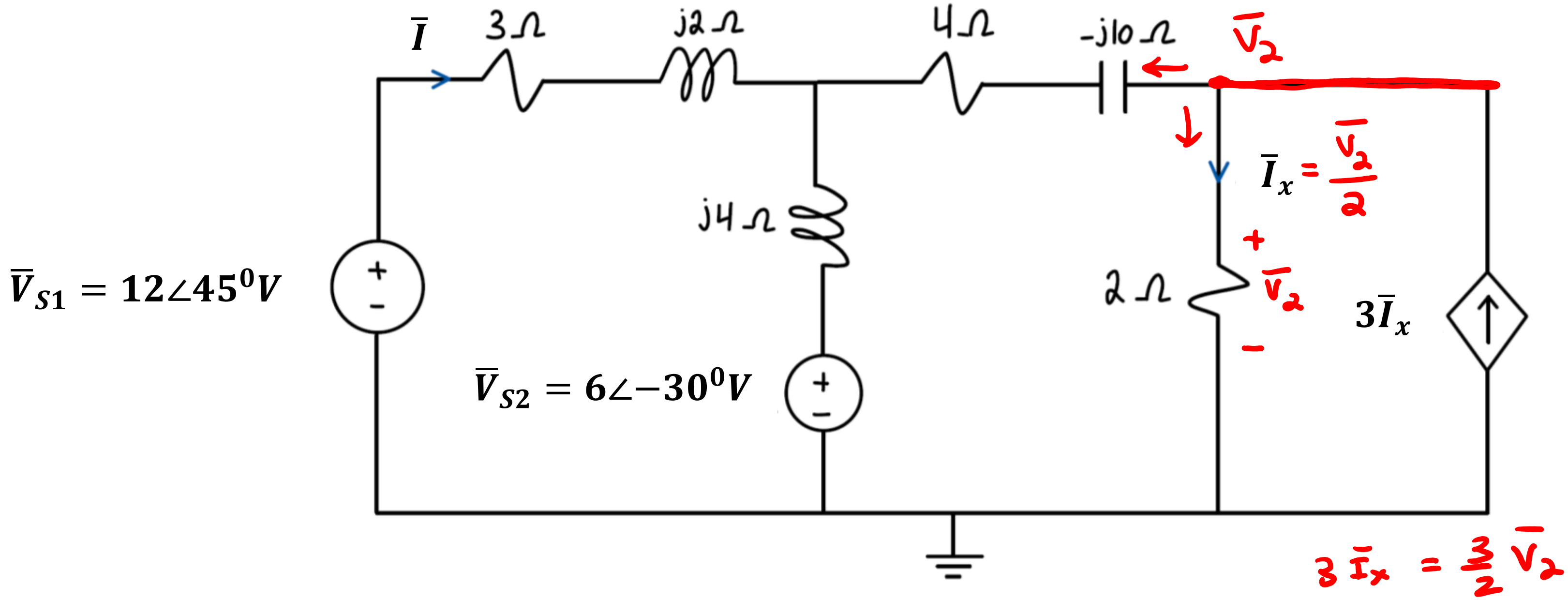
KCL @ node V_1

$$\frac{\bar V_{S1} - \bar V_1}{(3+j2)} = \bar I$$

$$\frac{\bar V_1 - 12\angle 45^\circ}{(3+j2)} + \frac{\bar V_1 - 6\angle -30^\circ}{j4} + \frac{\bar V_1 - \bar V_2}{(4-j10)} = 0$$

$$\left(\frac{1}{(3+j2)} + \frac{1}{j4} + \frac{1}{(4-j10)}\right]\bar V_1 - \frac{1}{(4-j10)}\bar V_2 = \frac{12\angle 45^\circ}{(3+j2)} + \frac{6\angle -30^\circ}{(j4)}$$

$$(0.2653 - j0.3176)\bar V_1 + (-0.0345 - j0.0862)\bar V_2 = 2.5136 - j0.6463 \quad \cdots (1)$$



KCL @ node $\bar{V}_2$

$$\frac{\bar{V}_2 - \bar{V}_1}{(4-j10)} + \frac{\bar{V}_2}{2} - \frac{3}{2}\bar{V}_2 = 0$$

$$-\frac{1}{(4-j10)}\bar{V}_1 + \left(\frac{1}{(4-j10)} + \frac{1}{2} - \frac{3}{2}\right)\bar{V}_2 = 0$$

$$(0.0345 + j0.0862)\bar{V}_1 + (0.9655 - j0.0862)\bar{V}_2 = 0 \quad --- \quad 2$$

$$\begin{vmatrix} (0.2653 - j0.3176) & (-0.0345 - j0.0862) \\ (0.0345 + j0.0862) & (0.9655 - j0.0862) \end{vmatrix} \begin{vmatrix} \bar{V}_1 \\ \bar{V}_2 \end{vmatrix} = \begin{vmatrix} (2.5136 - j0.6463) \\ 0 \end{vmatrix}$$

Cramer's Rule

$$\bar{V}_1 = \frac{\Delta_1}{\Delta} \qquad \bar{V}_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta = \quad$$

$$= 0.2225 - j0.3236$$

$$\Delta_1 = \det \begin{vmatrix} (2.5136 - j0.6463) & (-0.0345 - j0.0862) \\ 0 & (0.9655 - j0.0862) \end{vmatrix}$$

$$\Delta_1 = 2.3712 - j0.8467$$

$$\bar{V}_1 = 6.406 \angle 35.96^\circ \text{ V}$$

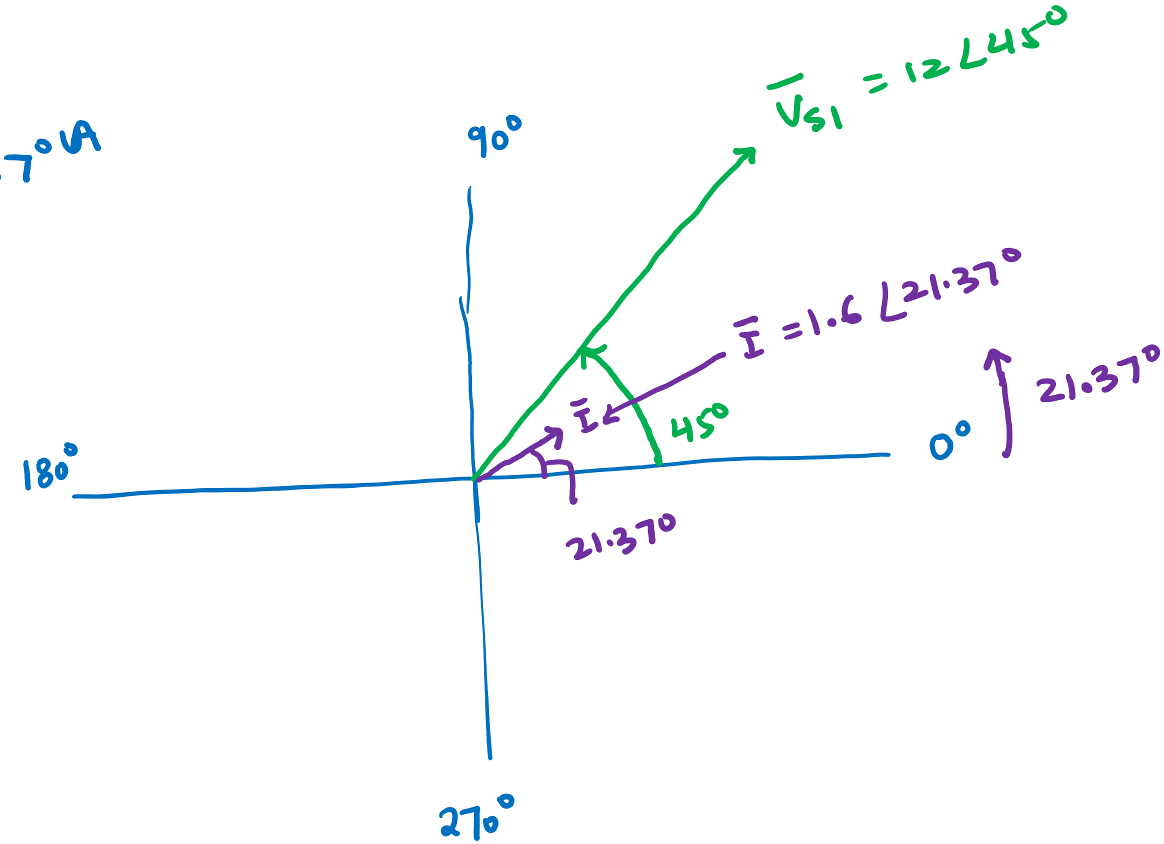
$$\Delta_2 = \begin{vmatrix} (0.2653 - j0.3176) & (2.5136 - j0.6463) \\ (0.0345 + j0.0862) & 0 \end{vmatrix}$$

$$\Delta_2 = -0.1424 - j0.1944$$

$$\bar{V}_2 = \frac{\Delta_2}{\Delta} = 0.614 \angle -70.75^\circ \text{ V}$$

$\bar{V}_{S1} = 12 \angle 45^\circ V$

$\bar{I} = 1.598 \angle 21.37^\circ A$



Q 3: **[20 marks]** Solve the circuit given below using Mesh Analysis.

1. Write all the mesh equations **[10 marks]**
2. Simplify the equations **[4 marks]**
3. Rearrange the equations in Matrix form **[2 marks]**
4. Show all the steps of Cramer's rule to find the mesh currents **[2 marks]**
5. Using a calculator, calculate the numerical value for the mesh currents **[2 marks]**

Note: All sources are AC.

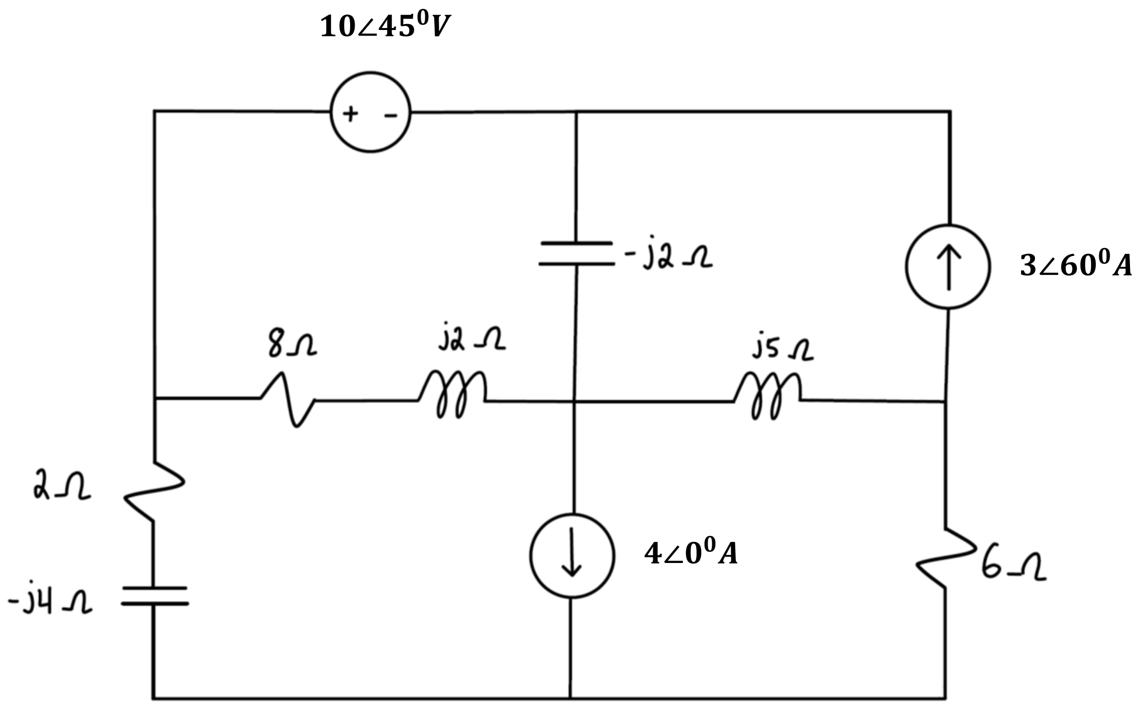
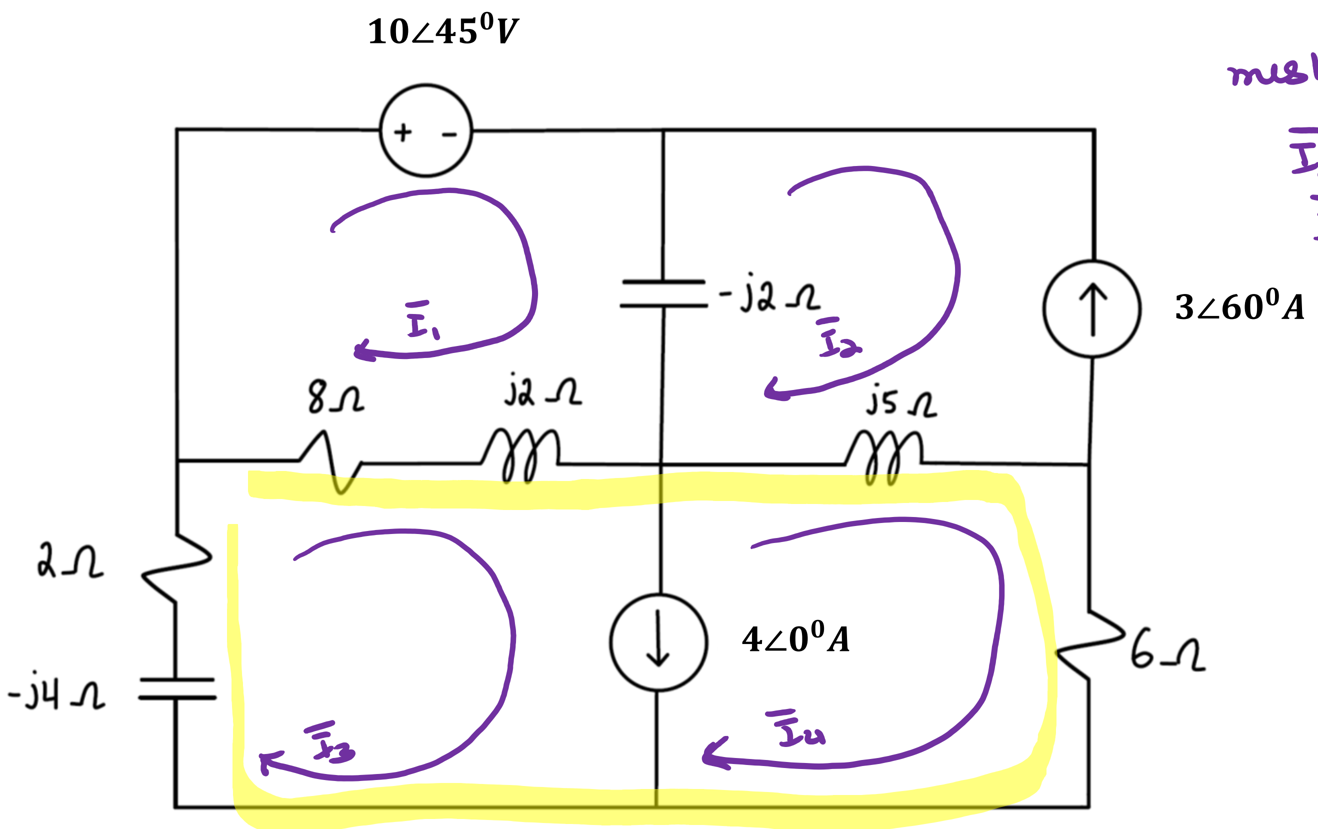


Figure 3: Circuit for Q 3



mesh 2

$\bar{I}_2 = -3\angle 60^\circ$ A

$\bar{I}_2 = 3\angle 240^\circ$ A

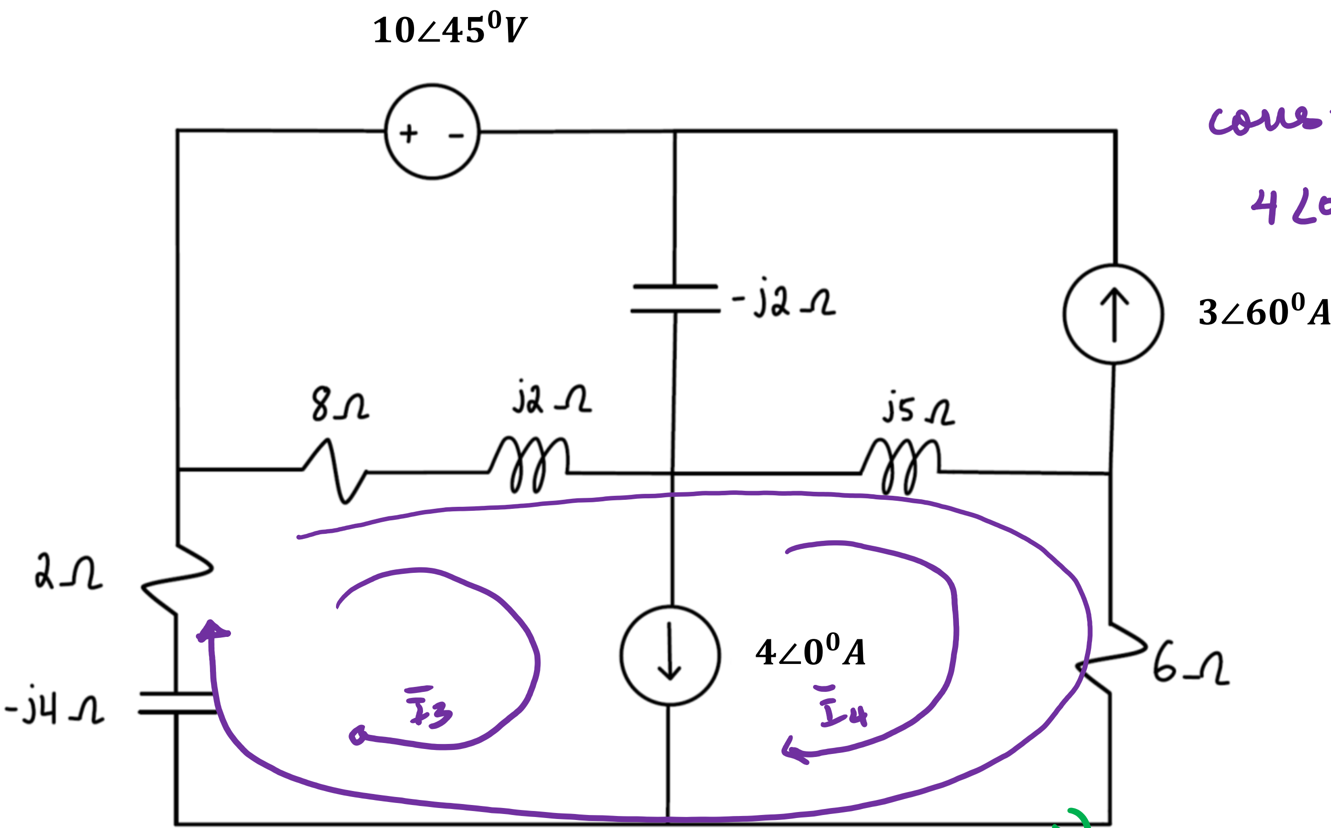
KVL @ mesh 1

$$+10\angle 45^\circ - j2(\bar{I}_1 - \bar{I}_2) + (8+j2)(\bar{I}_1 - \bar{I}_3) = 0$$

$$(8+j2-j2)\bar{I}_1 + j2(-3\angle 60^\circ) - (8+j2)\bar{I}_3 = -10\angle 45^\circ$$

$$8\bar{I}_1 - (8+j2)\bar{I}_3 = [+(3\angle 60^\circ \times 2\angle 90^\circ) - 10\angle 45^\circ]$$

$$= -12.267 - j4.071 \quad --- \quad (1)$$



constraint eqn

$4\angle 0^\circ = \bar{I}_3 - \bar{I}_4$ --- 2

$\bar{I}_3 = 4 + \bar{I}_4$

$\bar{I}_4 = \bar{I}_3 - 4$

KVL @ supermesh

$(8+j2)(\bar{I}_3 - \bar{I}_1) + j5(\bar{I}_4 - \cancel{\bar{I}_2}) + 6\bar{I}_4 - j4\bar{I}_3 + 2\bar{I}_3 = 0$ $\quad (-3\angle 60^\circ)$

$-(8+j2)\bar{I}_1 + (8+j2-j4+2)\bar{I}_3 + (j5+6)\bar{I}_4 = (3\angle 60^\circ)(5\angle 90^\circ)$

$-(8+j2)\bar{I}_1 + (10-j2)\bar{I}_3 + (6+j5)I_4 = (13-j7.5)$ --- 3

$$\bar{I}_1 = 1.65 \angle 46.36^\circ \text{ A}$$

$$\bar{I}_2 = -3 \angle 60^\circ = 3 \angle 240^\circ \text{ A}$$

$$\bar{I}_3 = 3.07 \angle 18.47^\circ \text{ A}$$

$$\bar{I}_4 = 1.46 \angle 138.09^\circ \text{ A}$$

Q 4: [20 marks] Determine the Thevenin equivalent at terminals $a - b$ of the circuit shown in Figure 4. The circuit is given in time domain. Please convert the component values to phasor domain before solving for the required variables.

1. Calculate the Thevenin equivalent voltage, V_{th}. Use the **source transformation only** as an analysis tool. If you use any other methods, marks will not be assigned even the value calculated is correct. **[8 marks]**
2. Calculate Z_{th}. **[8 marks]**
3. Calculate the value of Z_L for maximum power **[2 marks]**
4. Determine the maximum power absorbed by Z_L **[2 marks]**

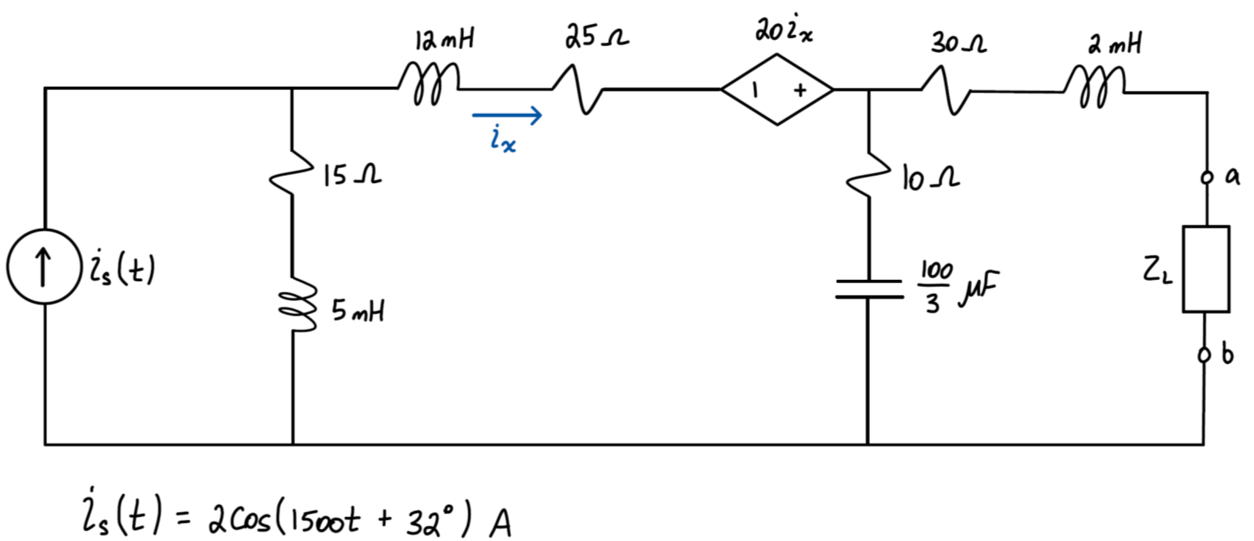
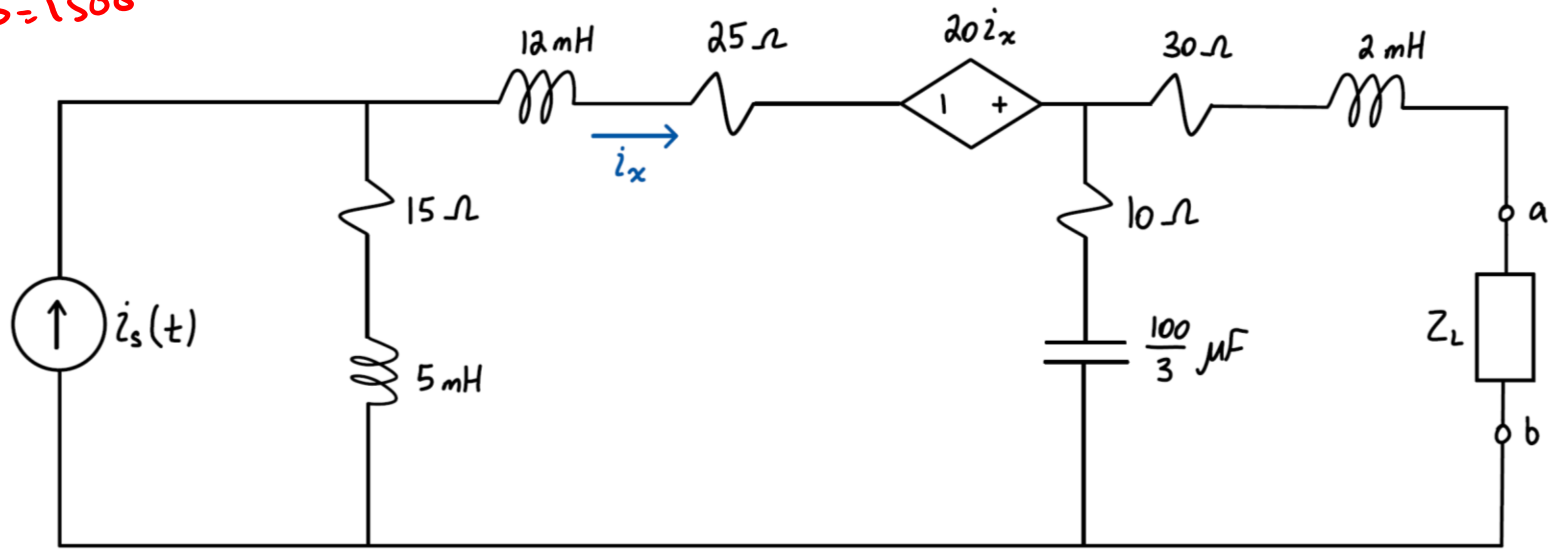


Figure 4: Circuit for Q 4 – Thevenin Theorem

$\omega = 1500$ rad/sec



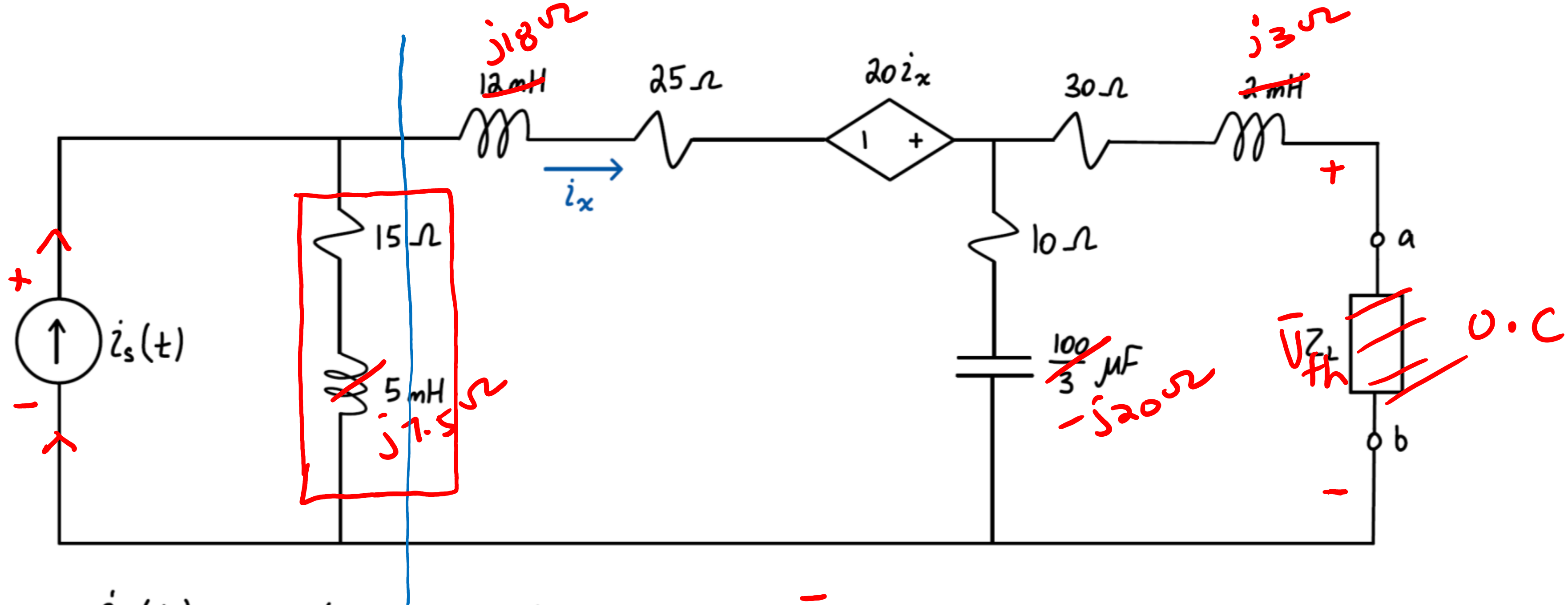
$i_s(t) = 2\cos(\underbrace{1500t}_{\omega t} + 32°)$ A $\Rightarrow$ $\bar{I}_s = 2\angle 32°$ A

5 mH $\rightarrow$ $j(1500 \times 5 \times 10^{-3}) = j7.5\,\Omega$

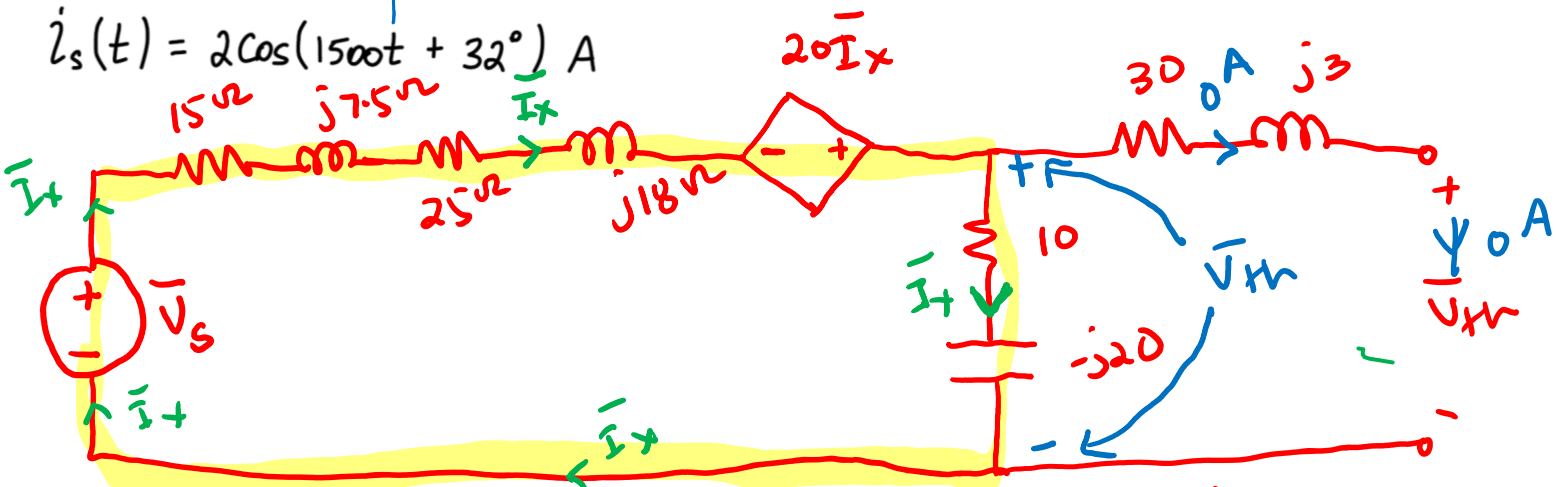
12 mH $\rightarrow$ $j(1500 \times 12 \times 10^{-3}) = j18\,\Omega$

2 mH $\rightarrow$ $j(1500 \times 2 \times 10^{-3}) = j3\,\Omega$

$\frac{100}{3}$ μF $\rightarrow$ $\dfrac{-j}{(1500 \times \frac{100}{3} \times 10^{-6})} = -j20\,\Omega$



$i_s(t) = 2\cos(1500t + 32°)$ A



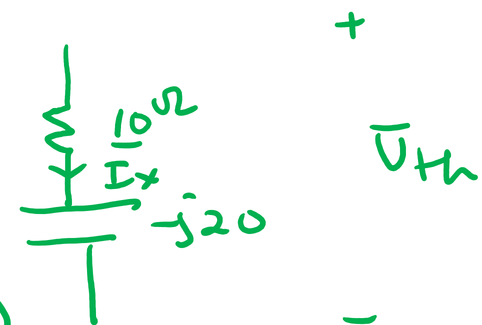
$\bar{V}_s = \bar{I}_s(15 + j7.5) = (2\angle 32°)(15 + j7.5) = 33.54\angle 58.56°$ V

$-33.54\angle 58.56° + (50 + j5.5)\bar{I}_x - 20\bar{I}_x = 0$

$$(30+j5.5)\bar{I}_x = 33.54\angle 58.56^\circ$$

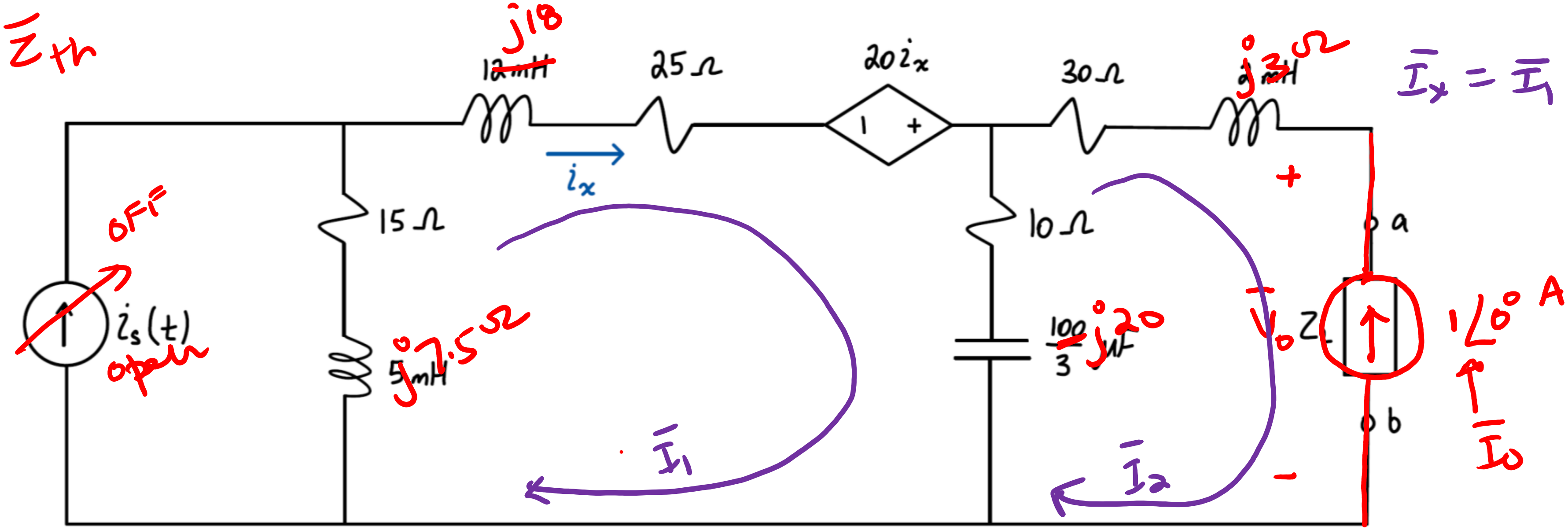
$$\bar{I}_x = \frac{33.54\angle 58.56}{(30+j5.5)} = 1.0997\angle 48.17^\circ A$$

$$\bar{V}_{th} = \bar{I}_x(10-j20)$$



$$\bar{V}_{th} = (10-j20)(1.0997\angle 48.17^\circ)$$

$$\bar{V}_{th} = 24.59\angle -15.26^\circ \text{ V}$$



$i_s(t) = 2\cos(1500t + 32°)$ A

KVL @ mesh 1

$$(15+j7.5)\bar{I}_1 + (25+j18)\bar{I}_1 - 20\bar{I}_1 + (10-j20)(\bar{I}_1-\bar{I}_2)=0$$

$$(15+j7.5+25+j18-20+10-j20)\bar{I}_1 - (10-j20)\bar{I}_2 = 0$$

$$(30+j5.5)\bar{I}_1 - (10-j20)\bar{I}_2 = 0 \quad \text{--- } 1$$

KVL for mesh 2

$$+\bar{V}_0 + (10-j20)(\bar{I}_2-\bar{I}_1) + (30+j3)\bar{I}_2 = 0$$

$$\bar{V}_0 = (10-j20)(\bar{I}_1-\bar{I}_2) - (30+j3)\bar{I}_2 \quad \text{--- } 3$$

mesh 2

$$\bar{I}_2 = -1\angle 0° \text{ A} \quad \text{--- } 2$$

$$(30+j5.5)\,\bar{I}_1 = (10-j20)(-1)$$

$$\bar{I}_1 = \frac{(-10+j20)}{(30+j5.5)}$$

$$\bar{I}_2 = -1 \ A$$

$$\bar{V}_0 = \left\{(10-j20)\left[\frac{(-10+j20)}{(30+j5.5)} + 1\right]\right\} + (30+j3)$$

$$\bar{V}_0 = (52.04 - j5.875) \ V$$

$$\bar{Z}_{th} = \frac{\bar{V}_0}{\bar{I}_0} = (52.04 - j5.875) \ \Omega$$

$\bar{Z}_L$ for maximum power transfer

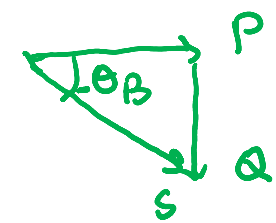
$$\bar{Z}_L = \bar{Z}_{th}^{*} = (\overset{R_{th}}{52.04} + j\,\overset{X_{th}}{5.875})\ \Omega$$

$$P_{max} = \frac{|V_{th}|^2}{8R_{th}} = \frac{(24.59)^2}{8(52.04)} = 1.45\ W$$

Q 5: [20 marks] For the circuit shown in Figure 5, complete the following:

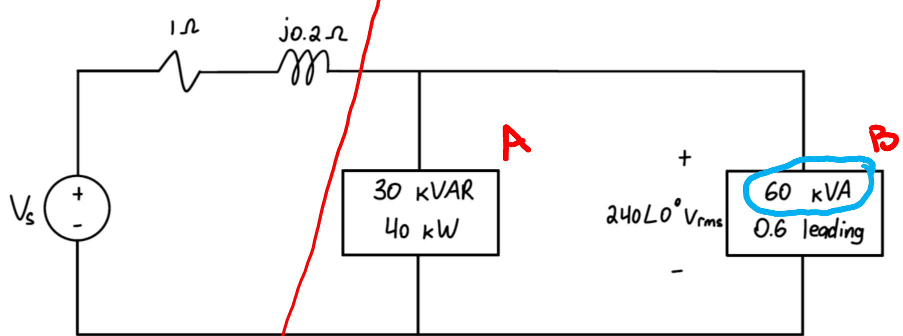
leading p.f. (capacitive)

1. Calculate the total apparent power **[3 marks]**
2. Calculate the total average power (real / active) **[3 marks]**
3. Calculate the total reactive power **[3 marks]**
4. Calculate the total power factor **[3 marks]**
5. Calculate the total impedance **[3 marks]**
6. Calculate the rms value of **Vs** (both magnitude and phase) **[5 marks]**



complex power

$\bar{S}_T = \bar{S}_A + \bar{S}_B$



$\bar{S}_A = (40 + j30)$ KVA

$\bar{S}_B = 60 \angle \theta_B$ KVA

$\cos\theta_B = 0.6$

$\theta_B = \cos^{-1}(0.6)$

$= 53.13°$

$\bar{S}_T = (40 + j30) + (60 \angle -53.13°)$

$\bar{S}_B = 60 \angle -53.13°$ KVA

$\bar{S}_T = (76 - j18)$ KVA $= 78.10 \angle -13.32°$ KVA

total apparent power

$P_T - jQ_T$

total average power

total reactive power

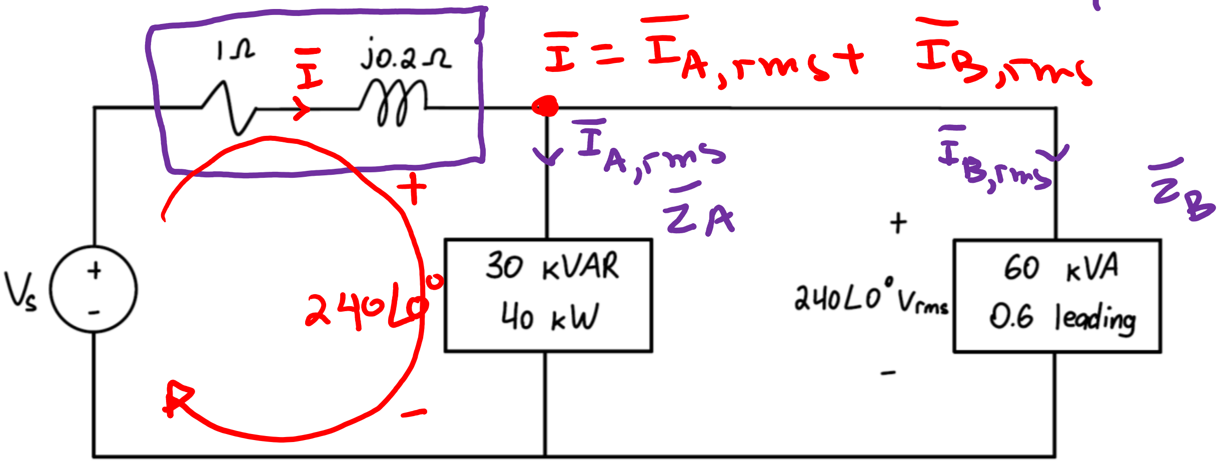
$|\bar{S}_T|$

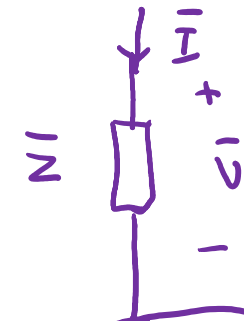
total p.f. $\cos(13.32) = 0.9731$ (leading)

$-\bar{V}_s + (1+j0.2)\bar{I} + 240\angle 0^\circ = 0$

$\bar{Z}_T = (1+j0.2) + [\bar{Z}_A // \bar{Z}_B]$

$\bar{I} = \bar{I}_{A,rms} + \bar{I}_{B,rms}$





$\bar{Z} = \dfrac{\bar{V}}{\bar{I}}$

$\bar{S}_A = \bar{V}_{rms} \, \bar{I}^*_{A,rms}$

$\bar{I}^*_{A,rms} = \dfrac{(40+j30)\,kVA}{240\angle 0^\circ\, V} = 208\angle 36.87^\circ\, A$

$\bar{I}_{A,rms} = 208\angle -36.87^\circ\, A$

$\bar{S}_B = \bar{V}_{rms} \, \bar{I}^*_{B,rms} \Rightarrow \bar{I}^*_{B,rms} = \dfrac{60\angle -53.13^\circ\, kVA}{240\angle 0^\circ\, V}$

$\bar{I}_{B,rms} = 250\angle 53.13^\circ\, A$

$$\bar{Z}_A = \frac{\bar{V}_{rms}}{\bar{I}_{rms}} = \frac{240\angle 0^\circ \text{ V}}{208\angle -36.87^\circ \text{ A}} = 1.154\angle 36.87\ \Omega$$

$$\bar{Z}_B = \frac{240\angle 0^\circ \text{ V}}{250\angle 53.13^\circ \text{ A}} = 0.96\angle -53.13^\circ\ \Omega$$

$$\bar{Z}_T = (1+j2) + \left[\frac{(1.154\angle 36.87)(0.96\angle -53.13^\circ)}{(1.154\angle 36.87^\circ + 0.96\angle -53.13^\circ)}\right]$$

$$\bar{Z}_T = 2.51\angle 46.8^\circ\ \Omega$$

$2.51\angle 46.8^\circ\ \Omega$

$2.5\angle 46.8^\circ\ \Omega$

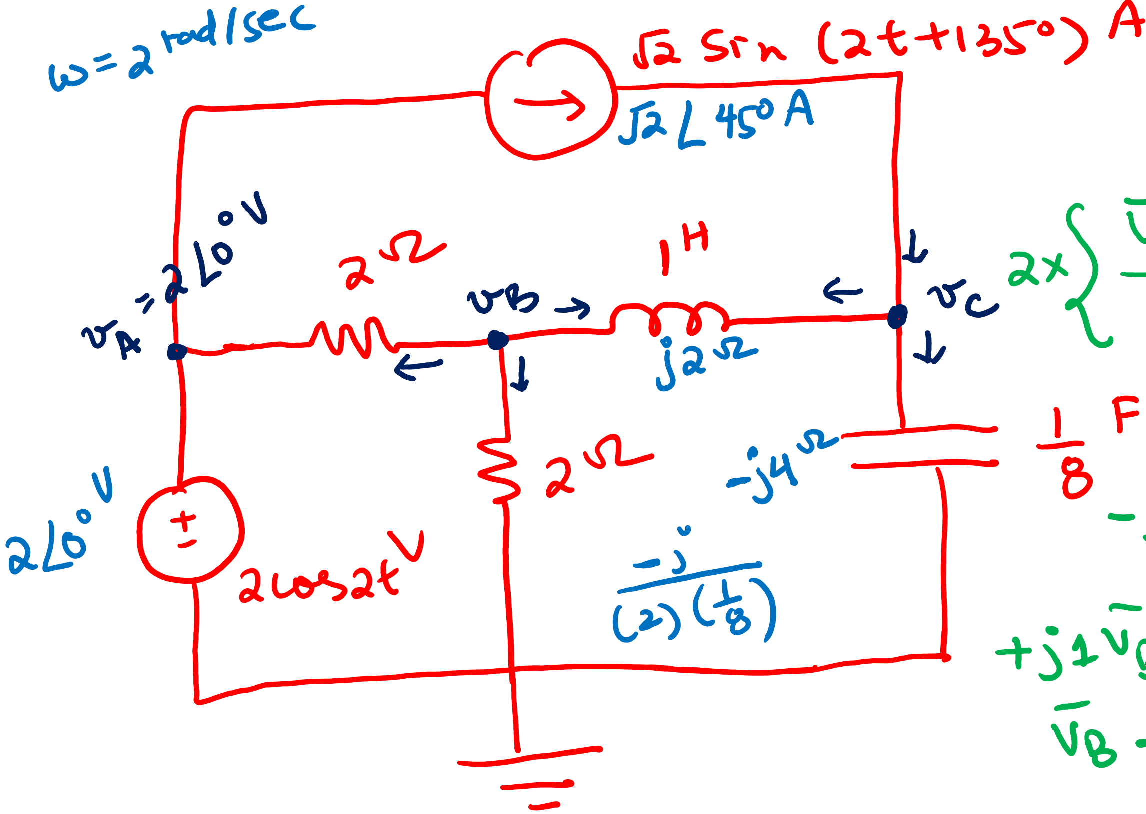
$$\bar{V}_s = (1+j0.2)\,\bar{I} + 240\angle 0^\circ$$

$$= \left[(1+j0.2)^{\Omega} \left(208\angle -36.87^{\circ\,A} + 250\angle 53.13^{\circ}\right)^{A} \right]$$

$$+ 240\angle 0^{\circ\,V}$$

$$\bar{V}_s = 558.19\angle 14.31^\circ \text{ V}$$

$\omega = 2$ rad/sec



KCL@ node $\bar{v}_C$

$$2\times\left\{\frac{\bar{V}_C-\bar{V}_B}{j2}+\frac{\bar{V}_C}{-j4}-\sqrt{2}\angle 45^\circ=0\right.$$

$$-j(\bar{V}_C-\bar{V}_B)+j\tfrac{1}{2}\bar{V}_C-2\sqrt{2}\angle 45^\circ=0$$

$$+j1\bar{V}_B-j\tfrac{1}{2}\bar{V}_C=2\sqrt{2}\angle 45^\circ$$

$$\bar{V}_B-\tfrac{1}{2}\bar{V}_C=2\sqrt{2}\angle -45^\circ \quad \text{---} \ 2$$

KCL@ node v_B

$$\left.\frac{\bar{V}_B-2\angle 0^\circ}{2}+\frac{\bar{V}_B}{2}+\frac{(\bar{V}_B-\bar{V}_C)}{j2}=0\right\}\times 2$$

$$(\bar{V}_B-2)+\bar{V}_B-j(\bar{V}_B-\bar{V}_C)=0$$

$$(1+1-j1)\bar{V}_B+j1\bar{V}_C=2 \quad \text{---} \ 1$$

$$\begin{array}{cc} A & B \\ \end{array}$$

$$\begin{vmatrix} (2-j1) & j1 \\ 1 & -\frac{1}{2} \end{vmatrix} \begin{vmatrix} \bar{V}_B \\ \bar{V}_C \end{vmatrix} = \begin{vmatrix} 2 \\ 2\sqrt{2}\angle -45^\circ \end{vmatrix}$$

$$\begin{array}{cc} C & D \end{array}$$

$AD - BC =$

$$\Delta = -\frac{1}{2}(2-j1) - j1 = -1 + j\frac{1}{2} - j1 = \left(-1 - j\frac{1}{2}\right)$$

$$\Delta_1 = \begin{vmatrix} 2 & j1 \\ 2\sqrt{2}\angle 45^\circ & -\frac{1}{2} \end{vmatrix} = -1 - j1(2\sqrt{2}\angle -45^\circ)$$

$$= -1 - (2\sqrt{2}\angle 45^\circ) = (-3 - j2)$$

$$\Delta_2 = \begin{vmatrix} (2-j1) & 2 \\ 1 & 2\sqrt{2}\angle -45^\circ \end{vmatrix} = \left[(2-j1)(2\sqrt{2}\angle -45^\circ)\right] - 2$$

$$= 6\angle -90^\circ$$

$$\bar{V}_B = \frac{(-3-j2)}{(-1-j\frac{1}{2})} = 3.23\angle 7.125^\circ \, V$$

$$\bar{V}_C = \frac{6\angle -90^\circ}{(-1-j\frac{1}{2})} = 5.367\angle 63.43^\circ$$

$v_A = 2\cos 2t \, V$

$v_B = 3.23\cos(2t + 7.125^\circ) \, V$

$v_C = 5.367\cos(2t + 63.43^\circ) \, V$