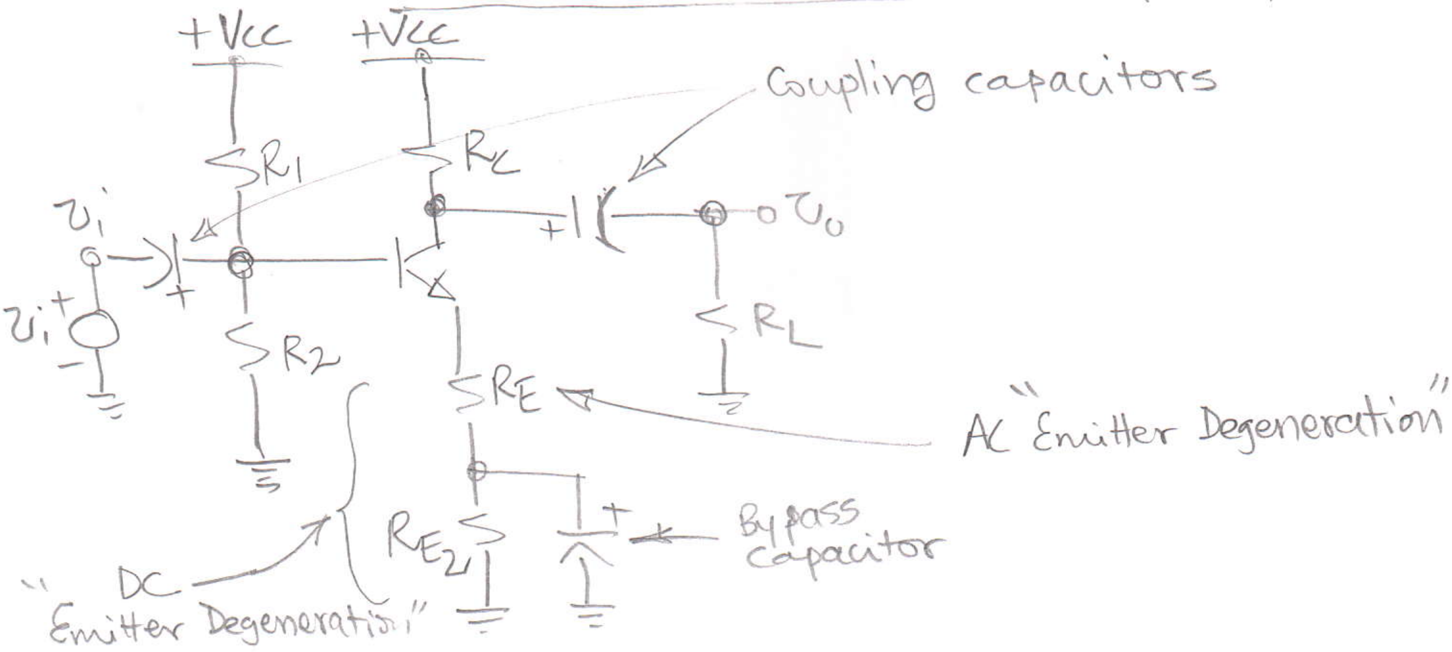
Use of BJT as an Amplifier



Analysis involves ① DC Analysis and ② AC Analysis

DC Analysis

- Objectives :
  1- to confirm transistor is in the active mode
  2- to calculate the quiescent (DC bias) values of the node voltages and branch currents. especially I_C.

$$I_C \Rightarrow g_m \simeq 40\, I_C \,,\; r_e = \frac{\alpha}{g_m} \simeq \frac{1}{g_m} \,,\; r_\pi = \frac{\beta}{g_m} = (\beta+1) r_e \simeq \beta r_e$$

- Procedure
  1- turn off signal source(s), i.e., any AC source
  2- replace capacitors with 'open' links; $X_C = \frac{1}{C\omega} = \infty$ if $\omega = 0$
  3- Assume active mode and solve circuit
  4- check active mode
  5- calculate I_C, g_m, r_e, and r_π

AC Analysis

- Objectives:

1- to calculate gain (the ratio of output voltage to input voltage).

2- to calculate input resistance of the amplifier.

3- to calculate output resistance of the amplifier.

- Procedure:

1- turn off power supplies, i.e., any thing DC

2- replace capacitors with short links; $X_C \approx 0$ if $C\omega \gg$

3- replace BJT with its small-signal (π or T) model and solve the circuit for the required ac parameter (gain, input resistance, output resistance, etc.)

Remark

If an amplifier fits into any of the three so-called "basic amplifiers", we do step 3 once symbolically and use the result thereafter, to save time and effort. More on this later.

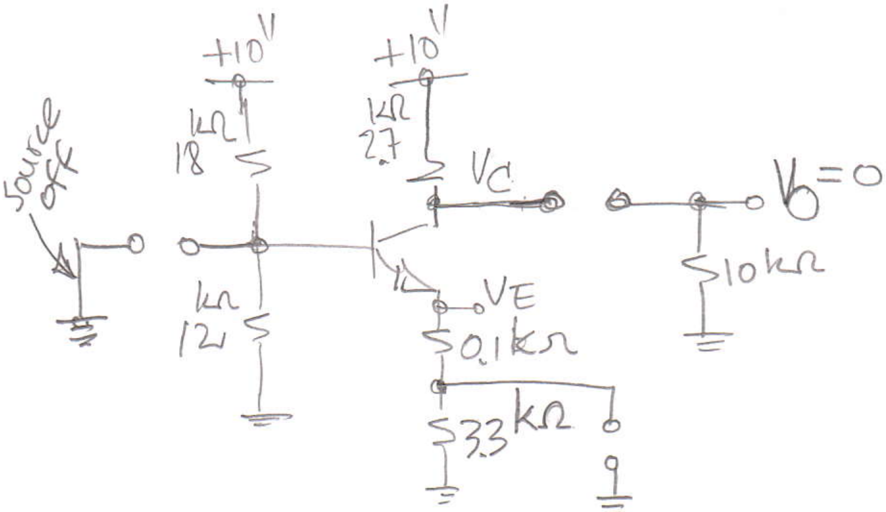
Example #1

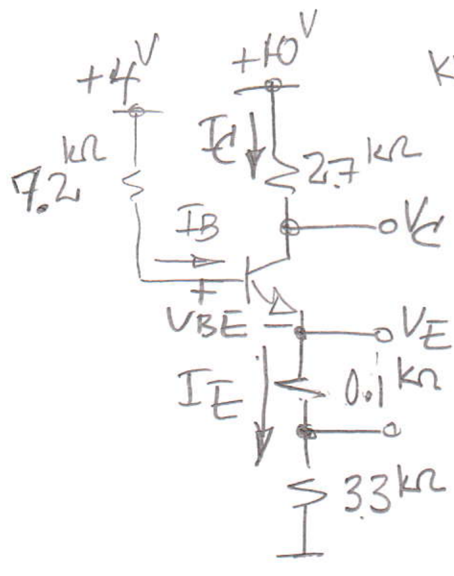
The circuit on Page 1, with the following parameters:

$V_{CC}=10^V$, $R_1=18\,k\Omega$, $R_2=12\,k\Omega$, $R_C=2.7\,k\Omega$, $R_{E}=0.1\,k\Omega$, $R_{E_2}=3.3\,k\Omega$, $\beta=100$, $R_L=10\,k\Omega$

$V_{BE_{on}}=0.7^V$, $V_{CE_{sat}}=0.3^V$

① DC Analysis





KVL: $4^V - 7.2 I_B - V_{BE} - (0.1+3.3) I_E = 0$

$4^V - 7.2 \frac{1}{\beta} I_C - 0.7^V - 3.4 \frac{\beta+1}{\beta} I_C = 0$

$\Rightarrow I_C = \dfrac{4-0.7}{\frac{7.2}{100} + 3.4\frac{101}{100}} = 0.94^{mA}$

$I_E = \frac{\beta+1}{\beta} I_C = 1.01 \times 0.94 \cong 0.95^{mA}$

$V_E = (0.1+3.3)\times I_E = 3.4\times 0.95 = 3.23^V$

$V_B = V_E + 0.7 = 3.93^V$

$V_C = 10^V - 2.7^{k\Omega} \times I_C = 10 - 2.7\times 0.94 \cong 7.46^V$

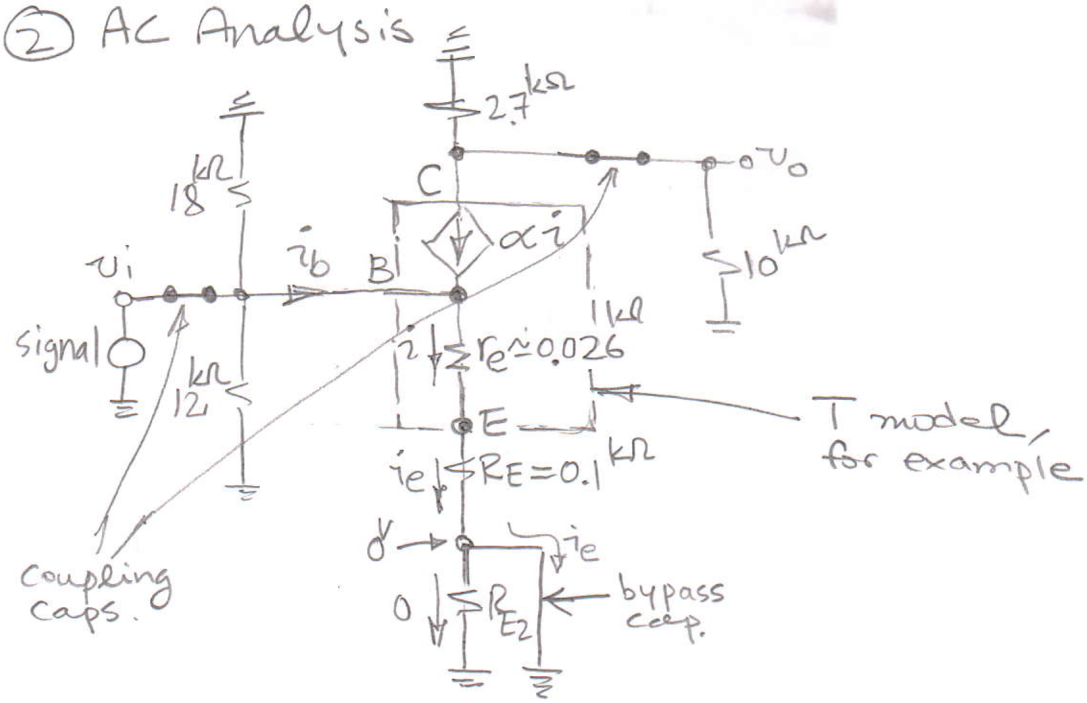
$V_{CE} = V_C - V_E = 7.46 - 3.23 = 4.23^V > V_{CE_{sat}} = 0.3^V \Rightarrow$ BJT in active mode.

$g_m \cong 40 I_C = 40\times 0.94 = 37.6\ mS$

$r_e \simeq \frac{1}{g_m} = 0.026\ k\Omega$, $r_\pi = \frac{\beta}{g_m} = 2.66\ k\Omega$

Example #1 (cont.)

2 AC Analysis



let us calculate the voltage gains $A_{vo} = \left.\frac{v_o}{v_i}\right|_{R_L=\infty}$ and $A_v = \frac{v_o}{v_i}$

For $R_L = \infty$, $v_o = (-\alpha i) \times 2.7^{k\Omega} = \underbrace{\frac{-\beta}{\beta+1}}_{0.99} \times 2.7^{k} \times i = -2.67\, i \left[\frac{V}{mA}\right]$ (1)

For $R_L = 10^{k\Omega}$, $v_o = -\alpha i \times (2.7 \| 10) = -0.99 \times 2.12 \times i = -2.1\, i \left[\frac{V}{mA}\right]$ (2)

But $i = \frac{v_i}{r_e + R_E} = \frac{v_i}{0.026 + 0.1} \Rightarrow i = 7.94\, v_i \left[\frac{mA}{V}\right]$ (3)

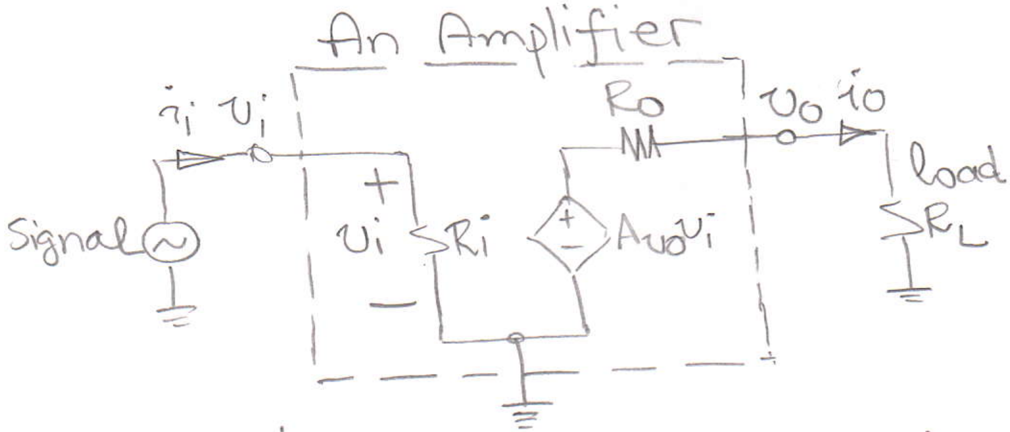
$$A_{vo} = \left.\frac{v_o}{v_i}\right|_{R_L=\infty} = \frac{-2.67\, i}{v_i} = \frac{-2.67 \times 7.94 v_i}{v_i} = -21.2\ \frac{V}{V}$$

$$A_v = \left.\frac{v_o}{v_i}\right|_{R_L=10k\Omega} = \frac{-2.1\, i}{v_i} = \frac{-2.1 \times 7.94 v_i}{v_i} = -16.7\ \frac{V}{V}$$

negative signs mean an inverting amplifier.

At this point, let us introduce the box (two-port) representation of an amplifier.

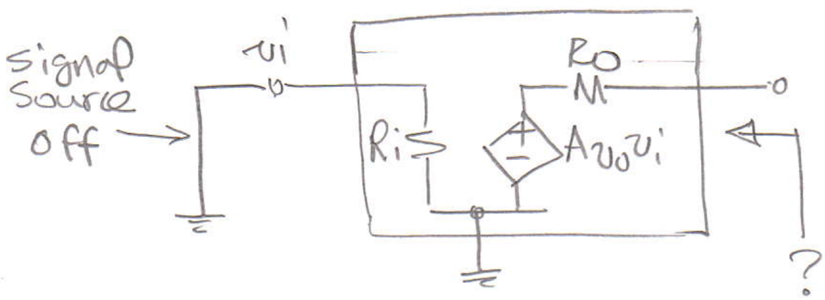
Box Representation of An Amplifier

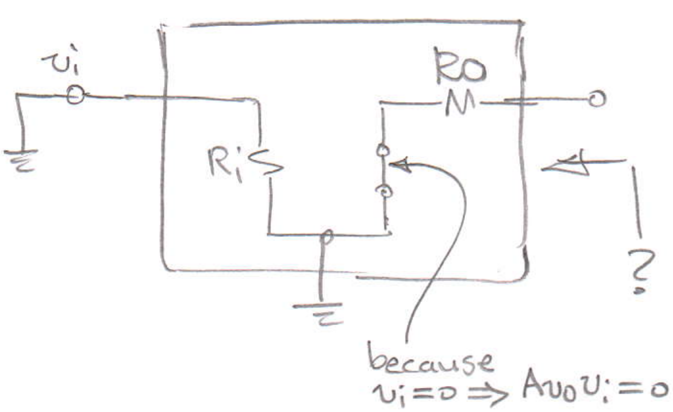


$$A_{vo} = \left.\frac{v_o}{v_i}\right|_{R_L=\infty} = \text{no-load voltage gain of amp.}$$

$$R_i = \frac{v_i}{i_i} = \text{input resistance (impedance) of amp.}$$

R_o = output resistance (impedance) of amp. It is the resistance one sees looking into the amp. from the output terminal(s), if the signal source is off. Note that the load is not a part of the amplifier and, therefore, must be excluded.



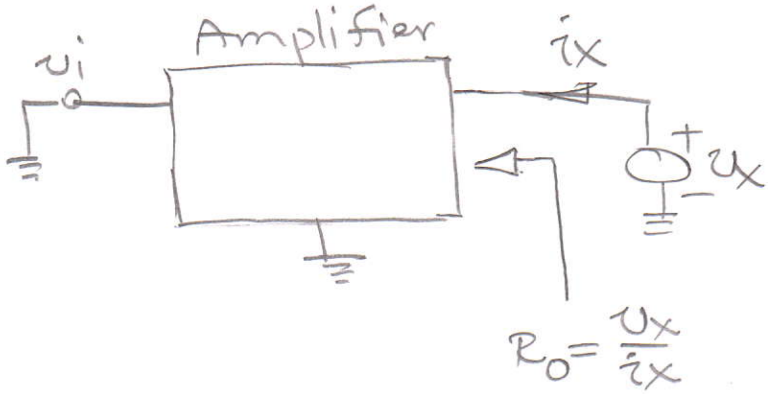


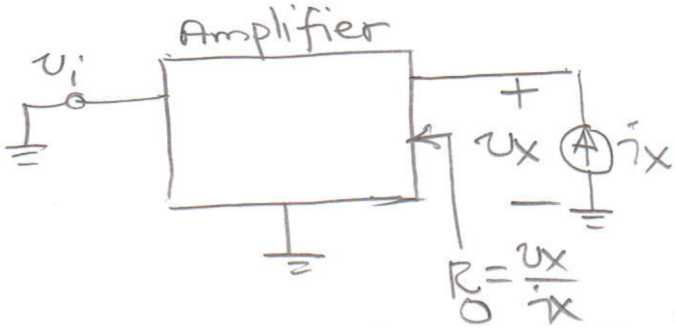
$\Rightarrow$ Therefore, the resistor seen from the output terminal to the ground inside the amp. is R_o.

Box Representation of an amp. (cont.)

How do we determine the output resistance?

Apply a test voltage and formulate the current, or apply a test current and formulate the voltage:



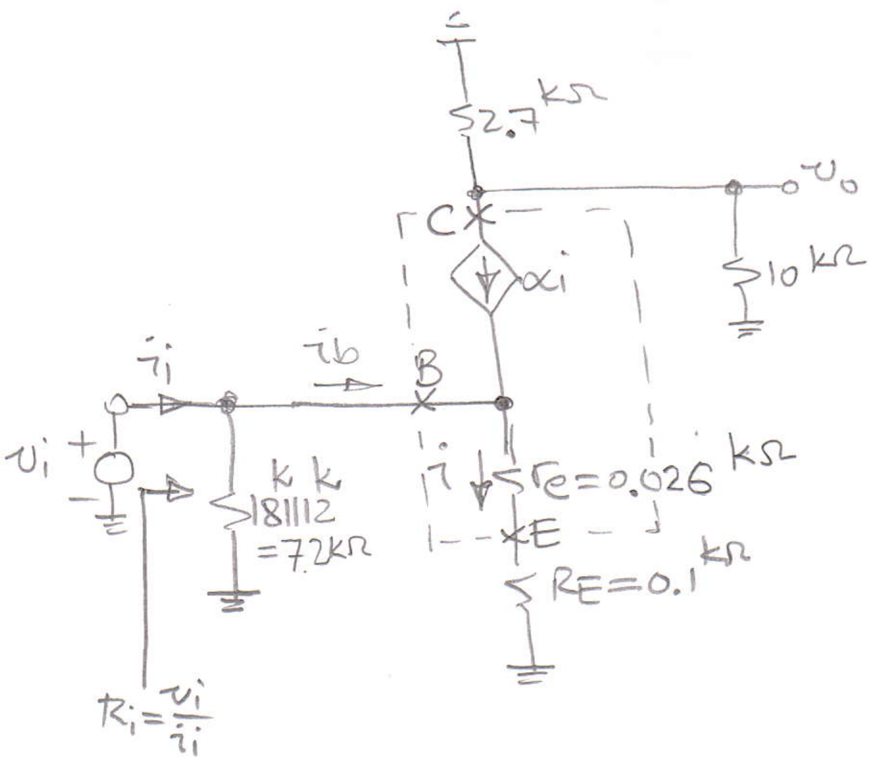


Again, remember to exclude the load when you do this.

Let us now get back to Example #1 and calculate its input and output resistances, based on what we learned.

Example #1 (cont.)

input resistance, R_i



$$i_i = \frac{v_i}{7.2\,k\Omega} + i_b \quad (1)$$

$$i_b = i - \alpha i = (1-\alpha) i = \left(1 - \frac{\beta}{\beta+1}\right) i = \frac{1}{\beta+1} i \quad (2)$$

$$i = \frac{v_i}{r_e + R_E} = \frac{v_i}{0.126} \quad (3)$$

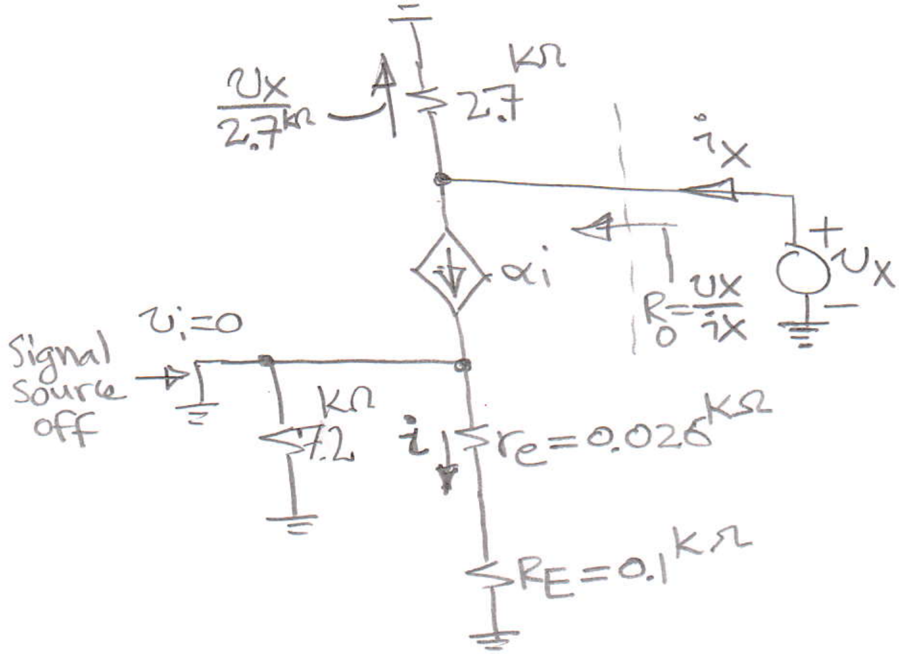
$$(2)\ \&\ (3) \Rightarrow \quad i_b = \frac{1}{101} \times \frac{v_i}{0.126} = \frac{v_i}{12.73} \quad (4)$$

$$(1)\ \&\ (4) \Rightarrow \quad i_i = \frac{v_i}{7.2} + \frac{v_i}{12.73} = 0.217\, v_i$$

$$\Rightarrow \boxed{R_i = \frac{v_i}{i_i} = \frac{v_i}{0.217\, v_i} \cong 4.6\ k\Omega}$$

Example #1 (cont.)

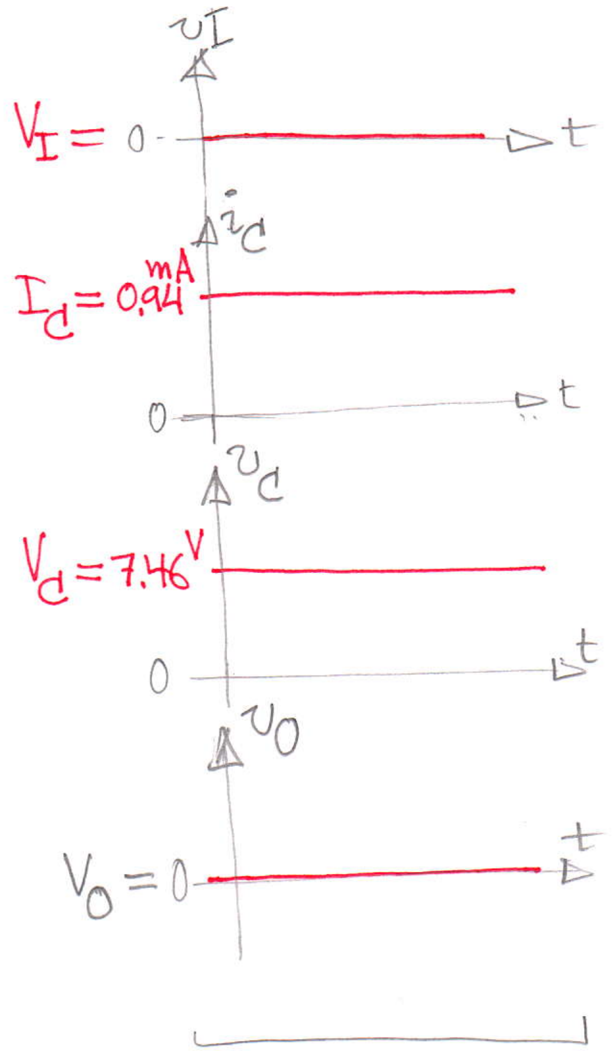
output resistance, R_o



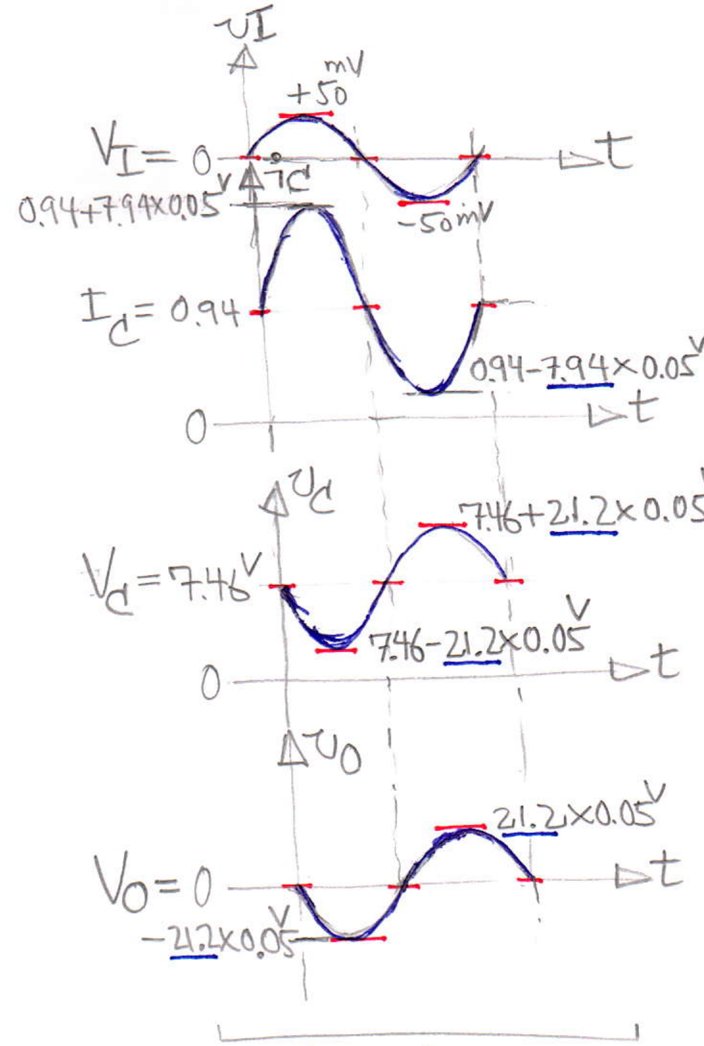
$$v_i = 0 \Rightarrow i = 0 \Rightarrow \alpha i = 0$$

$$i_x = \frac{v_x}{2.7} + \alpha i = \frac{v_x}{2.7k\Omega} \Rightarrow \boxed{R_o = \frac{v_x}{i_x} = 2.7^{k\Omega}}$$

What happens in reality



No signal;
only DC Power



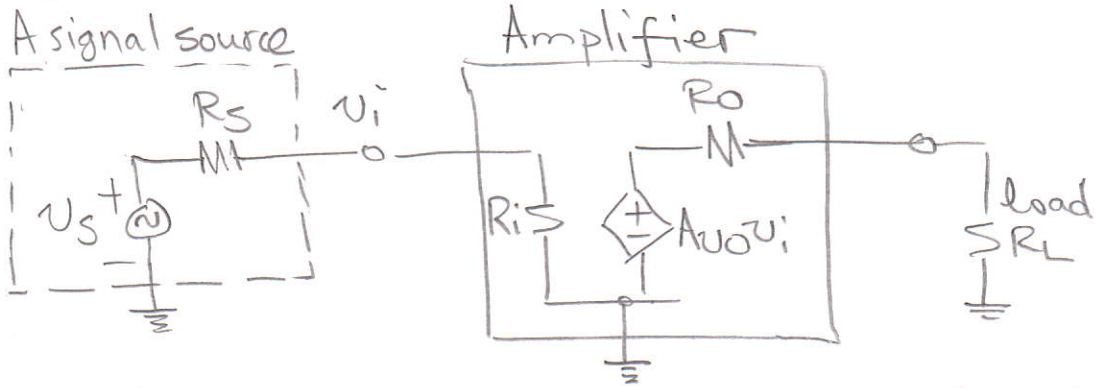
Signal +
DC power

Note

- The waveforms are not drawn to scale
- The waveforms correspond to Example #1 for $R_L = \infty$. The case of $R_L = 10\,k\Omega$ will have the same waveforms except that the gain 21.2 shall be replaced with 16.7.
- The input-output inversion property of this amplifier is evident from the waveforms.

Significance of R_i and R_O

Signal sources are not ideal; they have internal resistance (impedance). R_i helps one calculate how much signal actually gets to the amplifier's input. See the following diagram



Obviously, not the entire signal v_s reaches the amplifier. In fact,

$$\underbrace{v_i = \frac{v_S}{R_S + R_i} R_i}_{\text{Voltage Division}} \Rightarrow v_i = \underbrace{\frac{R_i}{R_S + R_i}}_{\substack{\text{fraction of } v_S \\ \text{that reaches} \\ \text{the amp. as } v_i}} v_S$$

Note that $\frac{R_i}{R_S + R_i} < 1 \Rightarrow v_i < v_S$

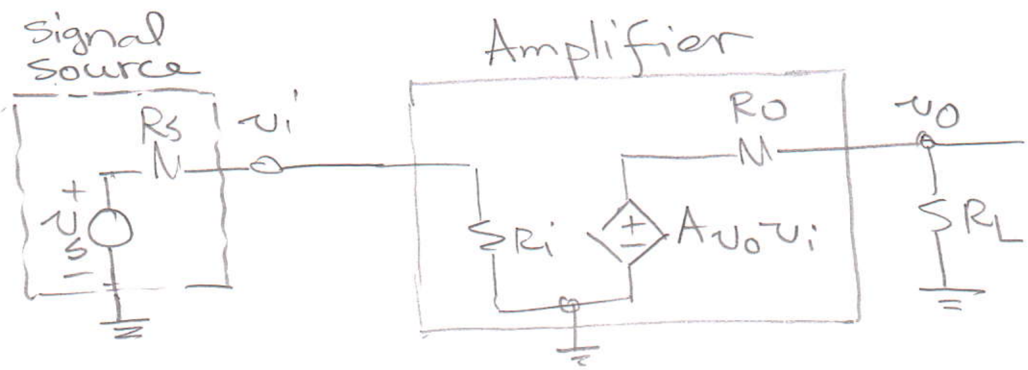
That is the amplifier "loads" the signal source.

A good design is one that is insensitive to source resistance variations and has $v_i \approx v_S$ (such that no signal is lost!). To achive this, one must ensure that $R_i >> R_S$

$$R_i >> R_S \Rightarrow \frac{R_i}{R_S + R_i} \approx 1 \Rightarrow v_i \approx v_S$$

Significance of R_i and R_o (cont.)

On the other hand, the output voltage of an amplifier drops when a load is connected to it; in other words, the load "loads" the amplifier. R_o is used to calculate the mentioned drop. See the following diagram:



$$v_o = \underbrace{\underbrace{\frac{R_L}{R_L + R_o}}_{\substack{\text{fraction of} \\ \text{the no-load} \\ \text{output voltage} \\ \text{that reaches} \\ \text{the load}}} \underbrace{A_{vo} v_i}_{\substack{\text{no-load} \\ \text{output} \\ \text{voltage}}}}_{\text{voltage division}}$$

A good design is insensitive to load variations and also has $v_o \approx \underbrace{A_{vo} v_i}_{\substack{\text{no-load} \\ \text{output} \\ \text{voltage}}}$. This can be

achieved if $R_o << R_L$:

$$R_o << R_L \Rightarrow \frac{R_L}{R_L + R_o} \approx 1 \Rightarrow v_o \approx A_{vo} v_i$$

Example #2

The amplifier of Example #1 is driven by a signal source v_s with an internal resistance of $R_S = 1.0\ k\Omega$, and drives a load of $R_L = 1.0\ k\Omega$.

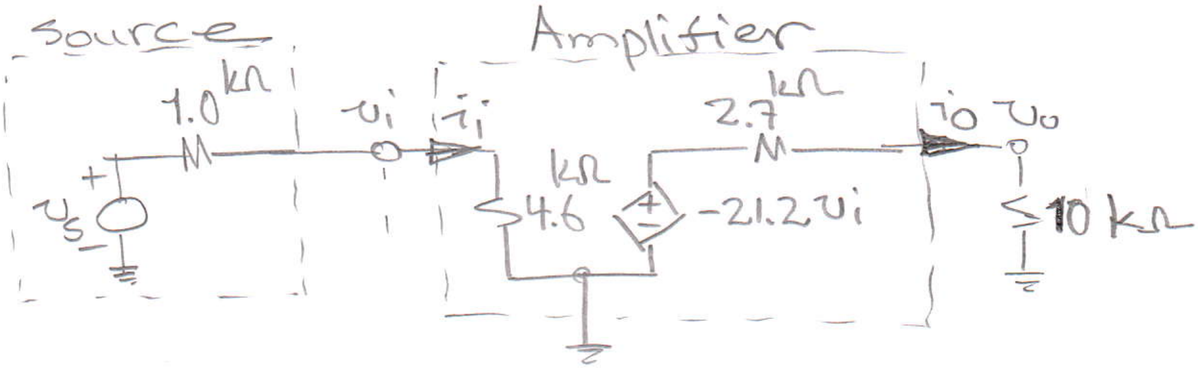
Calculate its

1) overal voltage gain $A_{vs} = \dfrac{v_o}{v_s}$
2) current gain $A_i = \dfrac{i_o}{i_i}$
3) power gain $A_p = \dfrac{p_o}{p_i}$

Solution

We have already shown (in Example #1) that $R_i = 4.6\ k\Omega$, $R_o = 2.7\ k\Omega$, and $A_{vo} = \left.\dfrac{v_o}{v_i}\right|_{R_L=\infty} = -21.2\ \dfrac{V}{V}$.

The rest can be calculated as:



$$\frac{v_i}{v_s} = \frac{4.6}{1.0+4.6} = 0.821 \quad (82.1\% \text{ of } v_s \text{ shows as } v_i)$$

$$\frac{v_o}{A_{vo} v_i} = \frac{10}{2.7+10} = 0.787 \quad (78.7\% \text{ of no-load voltage appears as } v_o)$$

$$A_{vs} = \frac{v_o}{v_s} = \left(\frac{v_o}{A_{vo} v_i}\right) \times A_{vo} \times \left(\frac{v_i}{v_s}\right) = 0.787 \times -21.2 \times 0.821 = -13.7\ \frac{V}{V}$$

It is interesting to calulate $A_v = \dfrac{v_o}{v_i}$:

$$A_v = \frac{v_o}{v_i} = \left(\frac{v_o}{A_{vo} v_i}\right) \times A_{vo} = 0.787 \times -21.2 = -16.7\ \frac{V}{V},$$

which is the same value we found in Example #1.

Example #2 (cont.)

current gain:

$$A_i = \frac{i_o}{i_i} = \frac{\frac{v_o}{R_L}}{\frac{v_i}{R_i}} = \frac{v_o}{v_i} \times \frac{R_i}{R_L}$$

$$A_i = -16.7 \times \frac{4.6}{10} = -7.68 \ A/A$$

Power gain:

$$A_p = \frac{P_o}{P_i} = \frac{v_o i_o}{v_i i_i} = \left(\frac{v_o}{v_i}\right)\left(\frac{i_o}{i_i}\right) = A_v A_I$$

$$= (-16.7)(-7.68)$$

$$= 128.2 \ \frac{W}{W}$$

The gains can also be expressed in dB:

$$A_{v_{dB}} = 20 \log |A_v| = 20 \log |-16.7| = 24.45 \text{ dB}$$

$$A_{i dB} = 20 \log |A_i| = 20 \log |-7.68| = 17.7 \text{ dB}$$

$$A_{p dB} = 10 \log |A_p| = 10 \log |128.2| = 21.08 \text{ dB}$$

It is interesting to note that

$$A_{p dB} = \frac{A_{v dB} + A_{i dB}}{2}$$

$$A_{p dB} = \frac{24.45 + 17.7}{2} = 21.07 \text{ dB}$$

Simpler methods of Analysis

Question: is this what we must do every time we analyze an amplifier, i.e., to use the T or π model and solve the AC circuit using basic circuit analysis techniques? What if the amplifier has multiple transistors?

Answer: it depends! Many times an amplifier is of one of the "basic configurations" or it can be resolved into a chain (cascade) of basic configurations. Then, if we already know the properties of the basic configurations (because we have already analyzed them once for all, using the T or π model and basic circuit analysis techniques), we can quickly use those properties and save a lot of time and effort.

However, if an amplifier cannot be resolved into one or more basic amplifiers, we have to use the T or π model to analyze it.

There are three basic amplifiers:

1- Common-Emitter (CE)
2- Common-Collector (CC)
3- Common-Base (CB)

We learn about them next.