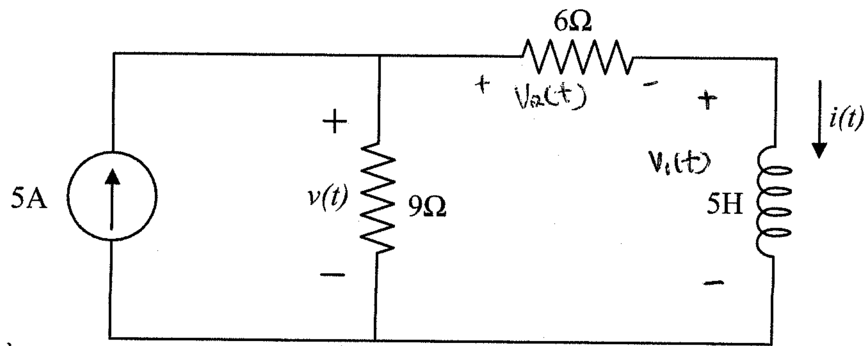
Q1: In the circuit shown below, the current through the inductor, *i(t)*, is governed by the following equation:

$$i(t) = 3 + 2e^{-3t} \text{ A for } t \geq 0$$

Determine *v(t)* for $t \geq 0$.



$V_1(t) = L\dfrac{d\,i(t)}{dt}$

$= 5 \cdot \dfrac{d(3+2e^{-3t})}{dt}$

$= 5 \cdot \left(\dfrac{d}{dt}(3) + 2\dfrac{d}{dt}e^{-3t}\right)$

$= 5 \cdot (2 \cdot (-3)e^{-3t})$

$= -30e^{-3t}$

8

$V_2(t) = 6\, i(t)$

$= 6 \cdot (3+2e^{-3t})$

$= 18 + 12e^{-3t}$

6

$v(t) = V_1(t) + V_2(t)$

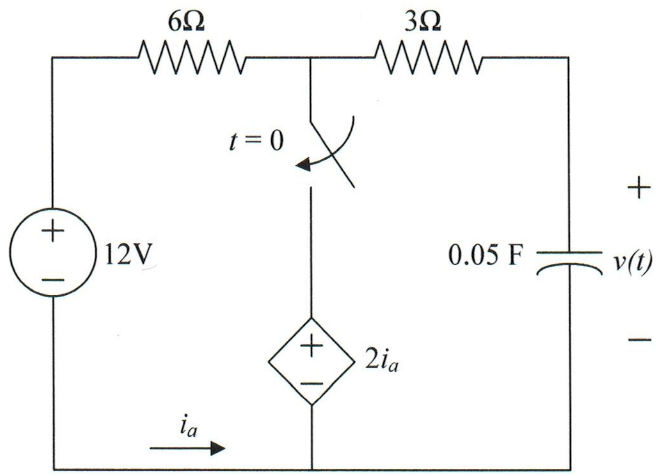
$= -30e^{-3t} + 18 + 12e^{-3t}$

$= 18 - 18e^{-3t}$.

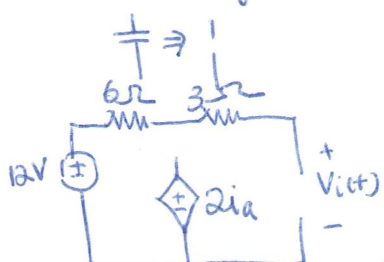
6

$v(t) = 18 - 18e^{-3t}$

Q2: The following circuit is at steady state before the switch closes at time t=0. Determine the capacitor voltage, v(t), for $t \geq 0$.



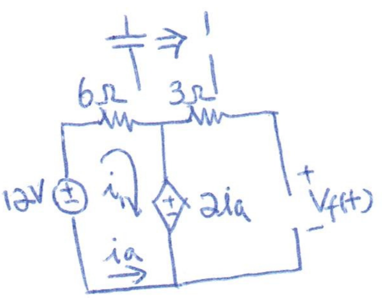
Initial voltage



$V_i(t) = 12$ volts

5 marks

Final voltage



$12 - 6i_1 - 2i_a = 0$

$i_1 = -i_a$

$12 - 6i_1 - 2(-i_1) = 0$

$12 - 6i_1 + 2i_1 = 0$

$12 = 4i_1$

$i_1 = 3$ Amps

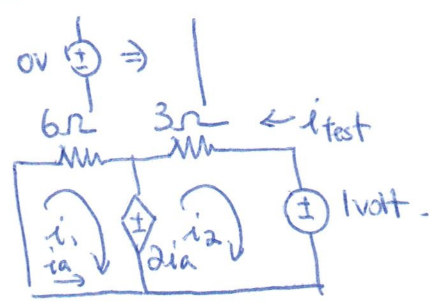
$V_f = 12 - 6 \cdot 3$

$= 12 - 18$

$= -6$ volts

6 marks

Rth seen by the Capacitor for $t \geq 0$



$0 - 6i_1 - 2i_a = 0$ (1)

$i_1 = -i_a$ (2)

$2i_a - 3i_2 - 1 = 0$ (3)

Sub (2) into (1)

$0 - 6i_1 - 2(-i_1) = 0$

$-6i_1 + 2i_1 = 0$

$i_1 = 0$ Amp.

$i_a = 0$ Amp (4)

Sub(4) into (3)

$2(0) - 3i_2 - 1 = 0$

$-3i_2 = 1$

$i_2 = -\frac{1}{3}$ Amp

$i_{test} = \frac{1}{3}$ Amp $= -i_2$

$R_{th} = \frac{1}{i_{test}} = \frac{1}{\frac{1}{3}} = 3\Omega$

$R_{th} = 3 \cdot 0.05 = 0.15$ seconds.

v(t) = $-6 + 18e^{-t/0.15}$ volts

3 marks

$V(t) = V_f(t) + [V_i(t) - V_f(t)] \cdot e^{-t/\tau}$

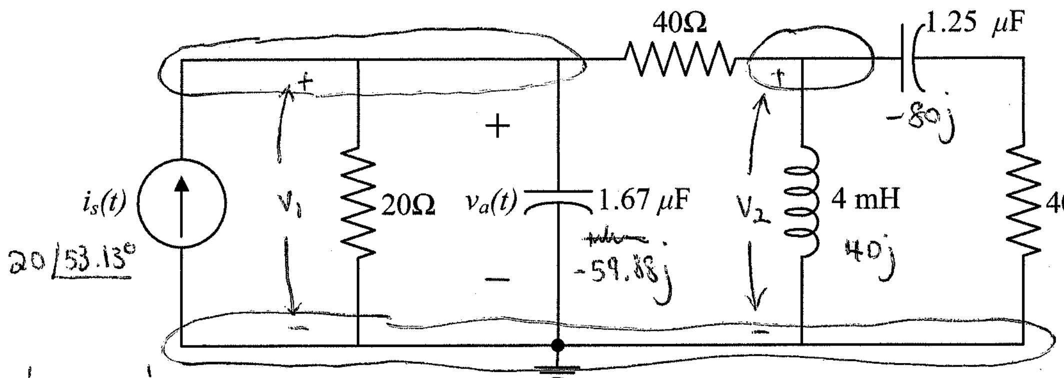
$= -6 + [12 - (-6)]e^{-t/0.15}$

$= -6 + 18e^{-t/0.15}$ volts

using mesh current analysis

Q3: Determine the voltage $v_a(t)$ for the following circuit when

$i_s(t) = 20 \cos(\omega t + 53.13°)$A and $\omega = 10^4$ rad/s.



$$Z_{C1} = \frac{1}{j\omega C_1} = \frac{1}{j \times 10^4 \times 1.67 \times 10^{-6}} = -59.88j$$

$$Z_{C2} = \frac{1}{j\omega C_2} = \frac{1}{j \times 10^4 \times 1.25 \times 10^{-6}} = -80j$$

3

$$Z_L = j\omega L = j \cdot 0.004 \times 10^4 = 40j$$

$$-20\angle 53.13° + \frac{V_1}{20} + \frac{V_1}{-59.88j} + \frac{V_1 - V_2}{40} = 0$$

$$\left(\frac{1}{20} + \frac{1}{-59.88j} + \frac{1}{40}\right)V_1 + \left(-\frac{1}{40}\right)V_2 = 20\angle 53.13°$$

6

$$(0.076837\angle 12.553°)V_1 - 0.025V_2 = 20\angle 53.13° \quad (2)$$

$$\frac{V_2 - V_1}{40} + \frac{V_2}{40j} + \frac{V_2}{40-80j} = 0$$

6

$$\frac{1}{40}V_1 - \left(\frac{1}{40} + \frac{1}{40j} + \frac{1}{40-80j}\right)V_2 = 0$$

$$0.025V_1 - 0.03354\angle -26.565\, V_2 = 0$$

$$V_1 = \frac{0.03354\angle -26.565}{0.025}V_2 = 1.3416\angle -26.565\, V_2 \quad (1)$$

Sub (1) into (2)

$$(1.3416\angle -26.565)(0.076837\angle 12.553°)V_2 - 0.025V_2 = 20\angle 53.13°$$

$$0.07906\angle -18.403\, V_2 = 20\angle 53.13°$$

$$V_2 = 252.972\angle 71.533°$$

3

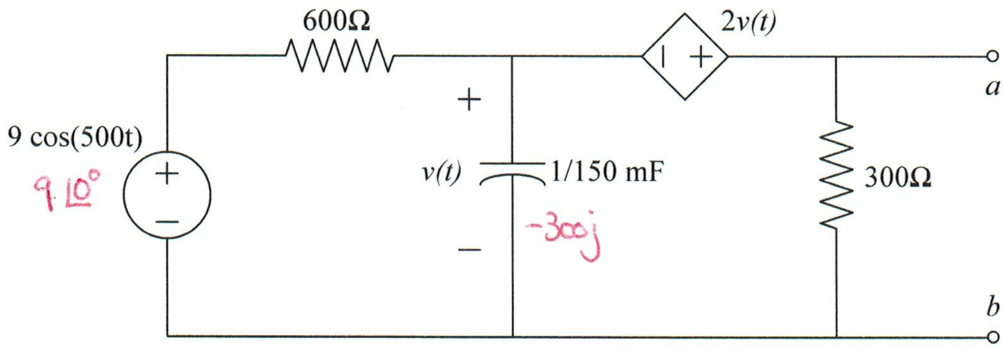
$v_a(t) =$ 339.3878 cos$(10^4 t + 44.968°)$ volts.

$$V_1 = 1.3416\angle -26.565 \cdot 252.972\angle 71.533°$$

$$= 339.3878\angle 44.968°$$

2

Q4: Find the Thevenin equivalent circuit between point *a* and *b* for the following circuit:



$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 500 \cdot \frac{1}{150} \cdot 10^{-3}}$$
$$= -300j$$

$P(v(t)) = V$

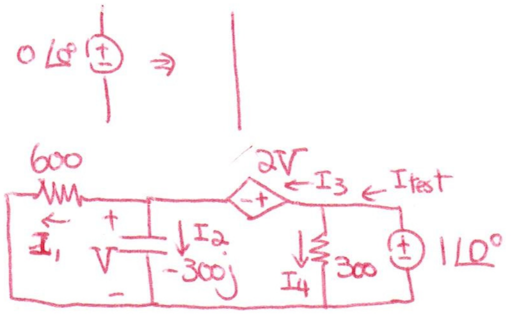
$$\frac{V-9}{600} + \frac{V}{-300j} + \frac{V+2V}{300} = 0$$

$$\frac{V}{600} - \frac{9}{600} + \frac{V}{-300j} + \frac{3V}{300} = 0$$

$$\left(\frac{1}{600} + \frac{1}{-300j} + \frac{1}{100}\right) V = \frac{9}{600}$$

$$V = 1.2362\angle{-15.945^\circ}$$

$$V_{th} = 1.2362\angle{-15.945^\circ} * 3$$
$$= 3.7087\angle{-15.945^\circ}$$



$$1\angle 0^\circ = 3V$$
$$V = 0.333\angle 0^\circ \text{ Volts}$$

$$I_1 = \frac{0.333\angle 0^\circ}{600}$$
$$= 0.556\angle 0^\circ \text{ mA}.$$

$$I_2 = \frac{0.333\angle 0^\circ}{-300j}$$
$$= 1.11\angle 90^\circ \text{ mA}$$

$$I_3 = I_1 + I_2$$
$$= 0.556\angle 0^\circ + 1.11\angle 90^\circ = 1.241\angle 63.39^\circ \text{ mA}$$

$$I_4 = \frac{1\angle 0^\circ}{300}$$
$$= 3.333\angle 0^\circ \text{ mA}$$

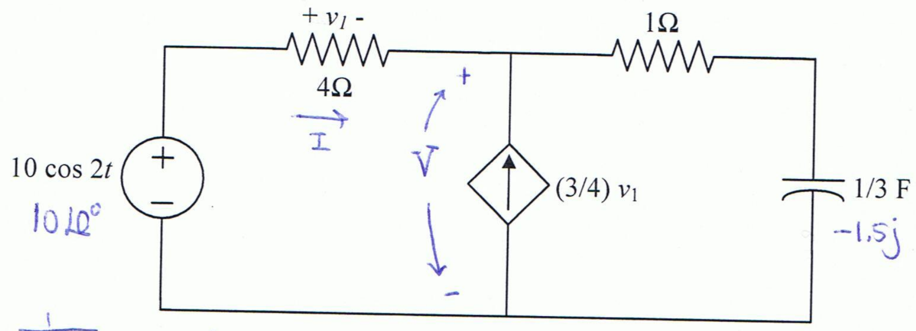
$$I_{test} = I_3 + I_4$$
$$= 1.241\angle 63.39^\circ + 3.333\angle 0^\circ$$
$$= 4.044\angle 15.924^\circ \text{ mA}.$$

$$R_{th} = \frac{1\angle 0^\circ}{4.044\angle 15.924^\circ} \text{ k}\Omega$$
$$= 247.28\angle{-15.924^\circ}$$

$V_{th} = 3.7087\angle{-15.945^\circ}$

$Z_{th} = 247.28\angle{-15.924^\circ}$

Q5: Find the complex power delivered by the voltage source and the power factor seen by the voltage source for the following ~~source~~ circuit:



$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 2 \cdot \frac{1}{3}} = -1.5j$$

$$\frac{V - 10\angle 0^\circ}{4} + \frac{V}{1 - 1.5j} - \frac{3}{4}V_1 = 0 \quad (1)$$

$$V_1 = 10\angle 0^\circ - V \quad (2)$$

Sub (2) into (1)

$$\frac{V - 10\angle 0^\circ}{4} + \frac{V}{1-1.5j} - \frac{3}{4}(10\angle 0^\circ - V) = 0$$

$$\frac{V}{4} - \frac{10\angle 0^\circ}{4} + \frac{1}{1-1.5j}V + \frac{3}{4}V - \frac{3}{4}10\angle 0^\circ = 0$$

$$\left(\frac{1}{4} + \frac{3}{4} + \frac{1}{1-1.5j}\right)V = \frac{10\angle 0^\circ}{4} + \frac{3}{4}10\angle 0^\circ$$

$$1.38675\angle 19.44\, V = 10\angle 0^\circ$$

$$V = 7.211\angle -19.44^\circ$$

$$I = \frac{10\angle 0^\circ - 7.211\angle -19.44^\circ}{4} = 1\angle 36.869^\circ$$

$$S = \frac{1}{2}VI^* = \frac{1}{2}10\angle 0^\circ (1\angle 36.869^\circ)^* = 5\angle 36.869^\circ$$

Power factor

$\cos(36.869^\circ) = 0.8$

Leading

S = $5\angle 36.869^\circ$
pf = 0.8 leading