Simple Harmonic Motion

Physics Topics

If necessary, review the following topics and relevant textbook sections from Serway / Jewett "Physics for Scientists and Engineers", 10th Ed.

- Motion of an Object Attached to a Spring (Serway, Sec. 15.1)
- Analysis Model: Particle in Simple Harmonic Motion (Serway, Sec. 15.2)
- Energy of the Simple Harmonic Oscillator (Serway, Sec. 15.3)

Introduction

The simple harmonic oscillator (a mass oscillating on a spring) is the most important system in physics. There are several reasons behind this remarkable claim:

- *Any* system which is in stable equilibrium and disturbed slightly will undergo oscillations. (Think of a ball sitting at the bottom of a valley; moving it slightly will cause it to roll back and forth around the minimum. But this also applies to other systems like buildings disturbed by a gust of wind, or molecules jostled by an electric field).
- The mathematics of oscillations - sines and cosines - can be used to describe *any* system which behaves in a regular, repeating way. For example, most astrophysical systems exhibit periodic motion (rotations of stars, orbits of planets, etc...) and can be described as a 2D oscillation.
- The mathematics of oscillations - sines and cosines - can be used to describe waves. Sound, light, (and even, in some sense, subatomic particles) are waves!

In this lab, you will use Hooke's law, and the theory of simple harmonic motion to predict and examine the motion of a mass on a spring.

Pre-Lab Questions

Please complete the following questions prior to coming to lab. They will help you prepare for both the lab and the pre-lab quiz (Found on D2L).

1.) Read through the entire lab writeup before beginning

2.) What is the **specific** goal of this lab? Exactly what question(s) are you trying to answer? Be as specific as possible. ("To learn about topic X..." is **not** specific!)

3.) What **specific** measurements or observations will you make in order to answer this question?

4.) Mass and spring at rest

(a) Draw two diagrams side by side: one with a spring hanging vertically (with no weight attached to it), and one which shows the same spring stretched downward by a distance Δy after a mass M is hung on it.

(b) Choose a coordinate system. Pick a convenient direction to be the positive y direction, and label this on your sketch.

(c) For the part of the diagram where the mass is attached, draw a free body diagram for the hanging mass.

(d) According to Newton's Second Law, what must be true about the forces acting on the mass assuming it is in equilibrium?

(e) Write an equation relating the forces acting on the mass. .

5.) Oscillating mass and spring

(a) Let's choose the origin of our coordinate system $y = 0$ at the *new* equilibrium position where the mass hangs at rest. Label this position on your sketch.

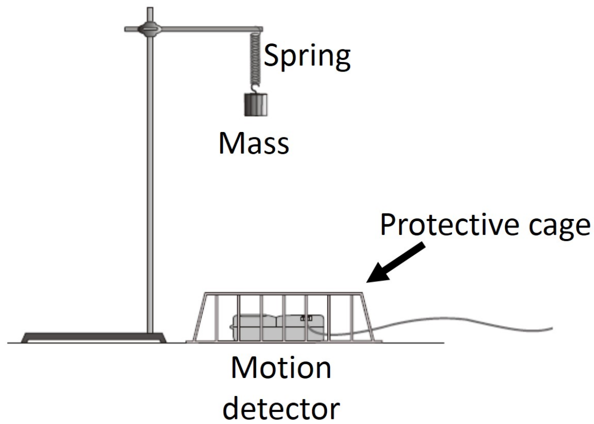
(b) Suppose the system from the previous question has a mass of 100g, is displaced upward from the new equilibrium position by 10cm and then released from this position at time $t = 0$. Suppose the period of oscillation is 1 second. Draw a careful graph of the mass' position $y(t)$. Be as quantitative as possible by labeling your axes, include units and tick marks.

(c) Now suppose the system is identical to the previous question, but is displaced upward from the new equilibrium position by **20cm** and then released from this position at time $t = 0$. Predict the motion of the mass by drawing another careful sketch of the mass's position $y(t)$ on the same set of axes. Be as quantitative as possible by labeling your axes, include units and tick marks.

(d) Finally, suppose the hanging mass is increased to 200g and is allowed to come to its new equilibrium position (which we again call $y = 0$). It is then displaced upward from the new equilibrium position by **10cm** and then released from this position at time $t = 0$. Predict the motion by drawing another careful sketch of the mass's position $y(t)$ on a separate set of axes. Be as quantitative as possible by labeling your axes, include units and tick marks.

(e) You should now have three separate graphs. Use your knowledge of simple harmonic motion to write a *specific* equation for $y(t)$ next to each graph. Be as specific as possible, inserting numbers for all known quantities in each case. Include proper units in your equation.

Apparatus



- Computer
- Vernier computer interface (LabPro)
- Graphical Analysis software
- Vernier motion detector
- 50 g hanger, and 50 g masses (5)
- Stand, rod and clamp
- Spring
- Metal cage
- Metre Stick

Procedure I - In equilibrium

1.) Attach the spring to the horizontal rod connected to the stand. Securely fasten the rod to the stand. Hang the 50 g hanger on the end of the spring.

2.) Place enough mass on the 50 g hanger such that it begins to stretch an observable amount (this may already occur with just the hanger). When the mass hangs freely, measure the distance from the top of the table to the bottom of the hanger.

3.) Repeat the measurement above for increasing masses (of 50 g), recording all data and uncertainties.

Analysis I - In Equilibrium

1.) Use Excel (or any program you choose) to plot your data in such a way so that you can determine the spring constant k from the slope of a line. Record your value of k determined using this method.

Procedure II - Oscillations

1.) Equipment setup

(a) Hang a total of 200g mass on the spring. Ensure the spring, mass, and rod are secured so the mass will not fall and damage the sensor.

(b) Position the motion detector below the mass. To protect the motion detector, place a metal cage around the sensor.

(c) Connect the motion detector to the DIG/SONIC1 channel of the interface.

(d) Take a preliminary run to make sure things are setup correctly. Lift the mass upward a few centimeters and release. Take care that the mass does not fall off the spring as it could damage the sensor below. The mass should oscillate vertically only.

(e) Click COLLECT to begin data collection. After 5 seconds, data collection will stop.

(f) The position graph should show a clean sinusoidal curve. If you cannot see any data on the position graph, click the "Zoom All" button  below the graph to automatically zoom in on the data. If necessary, you can also zoom into a particular region of the graph by selecting a region and then clicking . The program will zoom into the region of the **dark** grey rectangle you selected.

If the position curve has flat regions or spikes, reposition the motion detector and try again, taking care that there are no objects in between the mass and the sensor. You may need to adjust the cage slightly. Once you are satisfied that everything is positioned properly, proceed to the next part.

2.) With the mass hanging from the spring at rest, click the position reading Position: **-0.010 m** in the bottom right corner and zero the sensor. From now on, all distances will be measured relative to this position. When the mass moves closer to the detector than this position, the position will be reported as negative.

3.) Raise the mass about 5 cm above the equilibrium position and release it. The mass should oscillate along a vertical line only.

4.) Click COLLECT to create a graph of the mass' position as a function of time. Once the 5s collection time has passed you can begin to analyze your data.

Note: By clicking a point on a graph the program will display it's coordinates.

5.) Determine the average amplitude A_{avg} of the motion by averaging the amplitude of several oscillations together. It is best to examine both the maximum and minimum position to determine the amplitude! If there is a significant difference between the maximum and minimum displacements, you need to re-zero your sensor (see step 2). Record the average amplitude, including uncertainty.

6.) Using the position graph, determine the period of oscillation. Note that if you drag the mouse from one part of the graph to another, the time at both ends of the highlighted area will be displayed, use this to calculate Δt. Record the period with an estimate of uncertainty.

7.) Repeat steps 2 - 6 for the same mass, but with a larger amplitude of oscillation ($\sim$ 10 cm). Record your measurements of the average amplitude and period including uncertainty. **Note: depending on the spring, you may not be able to reach 10 cm. Do your best to obtain an oscillation amplitude different from your previous measurement.**

8.) Hang a total of 300 g of mass from the spring and repeat steps 2 - 6, recording all data and uncertainties. Keep this last position graph on the screen, you will use it in the analysis section.

Analysis II - Oscillations

1.) For the last run currently displayed on the screen, determine and record the initial phase ϕ of the measured oscillation. Here are some hints for doing so:

- Using Serway's equations for simple harmonic motion, we have $y(t) = A\cos(\omega t + \phi)$ and $v(t) = -A\omega\sin(\omega t + \phi)$.
- Plugging in $t = 0$ into the simple harmonic motion equations give $y_0 = A\cos(\phi)$ and $-v_0 = A\omega\sin(\phi)$.
- Remember that when you take an inverse trig function there are two solutions, even though you calculator only gives you one. (For example, $\cos^{-1}(0.5) = \pi/3$ or $2\pi/3$). You will need to decide which solution is the correct one.
- Here is another method you can use: determine what fraction F of (a cosine) period had occurred before you started taking data. The phase $\phi = F \cdot 2\pi$.

2.) Graphical Analysis allows you to fit your data to a sine curve using the following procedure.

(a) Click the  icon to the lower left of the position graph

(b) Choose Apply Curve Fit

(c) Select sine function from the General Equation list

(d) The sine equation is of the form $y = A\sin(Bt + C) + D$.

(e) Click Apply.

3.) Given your measured values of amplitude, period and phase, what values would you expect for the parameters A, B and C? Are the actual values found from the fit consistent with your expectations? [Hint: remember the phase ϕ you found above assumed a cosine function whereas here we are using a sine function. How is your ϕ related to C?]

4.) Using your fitted parameters, calculate the value of the spring constant k. Is your result consistent with the value you found in part I (using the mass at rest)?

5.) Save your Graphical Analysis file and submit it with your report.

6.) Export all of your data (so you can work with it outside of Graphical Analysis). Unhide all of your saved runs, click data, export *File*, *Export as*, Text or CSV.

Wrap Up - Complete In Lab

The following questions are designed to make sure that you understand the physics implications of the experiment and also to extend your knowledge of the concepts covered. Your report should seamlessly answer these questions in their noted sections.

1.) [Results] Based on your data, is the oscillation period affected by the amplitude of the oscillation? Explain your reasoning in a sentence or two.

2.) [Discussion]If you combine and plot the sum $\frac{1}{2}ky^2 + \frac{1}{2}mv^2$, what do you expect the graph to look like? Explain in words.

(a) Graphical Analysis allows you to make this plot.Make sure you have saved and exported your data.

i. Click the "View Options" button  in the top right corner and set Data Table to visible. You will be creating a set of calculated columns, one for kinetic energy, one for elastic potential and one for their sum.

ii. If one of your data sets is well centered around zero (the sensor was zeroed at the equilibrium position) you can use it for this analysis. If not, re-zero the motion sensor and collect another data set.

iii. **Kinetic Energy:** Scroll to your most recent data set and click the three dots beside one of the columns. You will be presented with a drop down, select "Add Calculated Column". Name this new column (Kinetic energy), provide it's units and click insert expression. Select A*X^B , input $\frac{1}{2}m$ as your A value, select velocity as your Y value and input 2 for your B value. Then click apply.

iv. **Potential Energy:** Follow the same procedure you did for Kinetic energy, you will only need to change what you input for A and select position as your Y.

v. **Summing:** Create one final column for E_{tot}, this column will use the expression A*X+B*Y. Let A, B = 1, X = Kinetic Energy, Y = Elastic Potential Energy. Then click apply.

vi. **Plotting:** Click the label on the y-axis of one of your graphs. Turn off everything that is currently being plotted. Scroll to your final data set and select your data for elastic potential energy, kinetic energy and sum of energies. The plot should now display these three data sets as a function of time.

vii. Save your Graphical Analysis file and submit it along with the rest of your data. You should also include your energy graph as a Figure in your lab report.

(b) Comment on the appearance of the plot. Does the graph agree with your expectations?

3.) [Discussion] If you examine the motion of the mass for *many* (more than 20) oscillations, what happens to the amplitude of the oscillation? What is the physical reason for this?

4.) Extra challenge (ungraded): Can you guess how to modify the simple cosine/sine equations to account for the effect dicussed in the previous question?

Report

Here is a brief guide for writing the report for the lab. The report should include the following sections:

- **Title Page**
  - Include: Report Title, Your Name, Course, Section Number, Instructor, TA Name, and Date of Submission.
- **Introduction**
  - What is the experiment's objective?
- **Theory**
  - Derivations of the physics being investigated, or reference to a source that provides a description/equation representing the physics being investigated.
  - Providing graphs that illustrate or predict how the system under study is expected to behave.
- **Procedure**
  - Briefly explain the systematic steps taken for the experiment.
- **Results and Calculation**
  - Tabulate the measurements in an organized manner.

- Based on the procedure, one should have a sense of how the tables will look like prior to taking measurements.
- Graph the main results.
- Provide examples of any calculations.

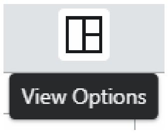
- **Discussion and Conclusion**
  - Discuss the main observations and outcomes of the experiment.
  - Summarize any significant conclusions.

Appendix A - Using Graphical Analysis

Graphical Analysis Interface Details

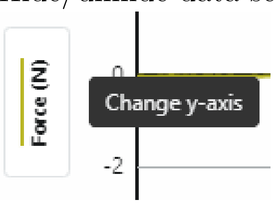
Here is a quick reference to using the Graphical Analysis. For a complete guide, see Venier Graphical Analysis User Manual

- View Data Tables by using the View Option button in the top right.

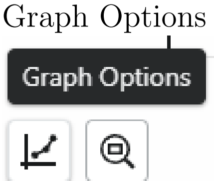
  Important: Unlike LoggerPro, Graphical Analysis saves every single measurement collected (including unwanted sets). This means it is very important to keep track of your data sets. Be sure to rename data sets (so you can identify them) and delete unwanted data sets immediately to keep your data organized! Renaming or deleting is achieved by clicking $\boxed{\cdots}$ next to the data set name.

- Hide/unhide data sets by clicking on the y-axis title.

- Single clicking anywhere on the plot will reveal the single value of at that position.

- For analysis, click and drag a region of interest, and select an analysis option under Graph Options in the lower left. The zoom button is to the right of it.

  Important: Unlike LoggerPro, Graphical Analysis will apply the curve fit to all data sets shown. It is recommended to curve fit a single data set and record the results before proceeding to the next data set.

- Save your experiment using File Menu in the top left. The file format is .gambl which can be opened using Graphical Analysis. You can download a free version of Graphical Analysis to use at home