Standing Waves on a String

Physics Topics

If necessary, review the following topics and relevant textbook sections from Serway / Jewett "Physics for Scientists and Engineers", 10th Ed.

- Analysis Model: Waves in Interference (Serway 17.1)
- Standing Waves (Serway 17.2)
- Boundary Effects: Reflection and Transmission (Serway 17.3)
- Waves Under Boundary Conditions & Resonance (Serway 17.4, 17.5)

Introduction

Imagine two sinusoidal traveling waves with equal amplitudes and frequencies moving in opposite directions. These waves are described mathematically by the expressions.

$$y_1(x,t) = A\sin(kx - \omega t) \tag{1}$$

$$y_2(x,t) = A\sin(kx + \omega t) \tag{2}$$

Where k is the wave number, related to the wavelength as $k = 2\pi/\lambda$, and ω is the angular frequency, related to the frequency by $\omega = 2\pi f$. If these two waves interfere with each other and have the same amplitude and frequency, we can add them using the principle of superposition.

$$y_{\text{standing}}(x,t) = y_1(x,t) + y_2(x,t) = A\left[\sin(kx - \omega t) + \sin(kx + \omega t)\right] \tag{3}$$

$$= 2A\sin(kx)\cos(\omega t) \tag{4}$$

We call the result a "standing wave". This standing wave does not appear to travel one direction or the other and has the presence of "nodes" (points on the string which do not move at all). This interference between two waves traveling in opposite directions is exactly what happens if you send two wave pulses down a string which is fixed at one end. Waves will reflect off of the fixed end, travel back toward the source and interfere with the waves which have yet to hit the fixed end. In this lab, you will be able to vibrate a string with an sinusoidal oscillator at a frequency and amplitude you control using the function generator. A pulley and set of masses allow you to vary the tension in the string which affects the speed of the waves.

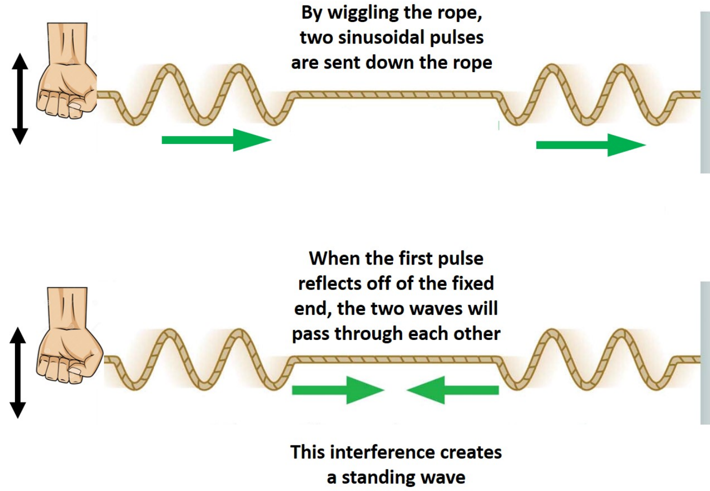


Figure 1: Adapted from Knight "Physics for Scientists and Engineers", 4th Ed. ©2017, Pearson Education

Pre-Lab Questions

Please complete the following questions prior to coming to lab. They will help you prepare for both the lab and the pre-lab quiz (Found on D2L).

1.) Read through the entire lab manual before beginning

2.) What is the **specific** goal of this lab? Exactly what question(s) are you trying to answer? Be as specific as possible. ("To learn about topic X..." is **not** specific!)

3.) What **specific** measurements or observations will you make in order to answer the identified question(s)?

4.) Equation (4) given in the Introduction is valid for any standing wave. In this experiment, we have a string of length L which is fixed at both ends, which means both ends must be nodes. [Actually the oscillator moves up and down slightly, but the amplitude of the oscillation is so small that we will ignore it and treat the string as fixed at $x = 0$ and $x = L$.] How is it possible to have $y_{\text{standing}}(L, t) = 0$ at **any** time t? Write a condition in the form of an equation which must be enforced for this to be true. .

5.) Rewrite the condition you found in the previous part to relate the wavelength λ to the length L. You should find that only certain values of λ are allowed, labeled by an integer n. .

6.) Draw pictures of the first three allowed standing waves which have nodes at each end, label them as $n = 1, n = 2, n = 3$.

7.) In this lab, you will measure the frequency instead of wavelength. Write an equation relating frequency to wavelength and use it to eliminate λ from your equation.

8.) In this lab, you cannot measure the speed of waves directly, but you can determine the tension straightforwardly. Write an equation relating the speed of waves on the string to the tension in the string T and other physical properties of the string. Explain all symbols appearing in your formula. Substitute this equation into your result from the previous step to remove v in favor of the tension T.

9.) In the setup shown in Fig. 2, if the total mass hanging from the string is M, what is the tension in the string? It may help to draw a free body diagram on the mass M.

10.) Use your equation from the previous question to finalize your prediction. You should now have an equation relating the frequencies which lead to standing waves f_n, the length of the string L, the hanging mass M, properties of the string, and the "standing wave mode number" n which is the number of antinodes in the standing wave. Check the units of your equation to make sure they work out.

11.) In this lab, we vary the tension T and f_n (which results in a change in the "mode number", n). How can you plot your data so that the result is a straight line? What will be the slope of this line?

Apparatus

- Weight set and hanger ($\sim$ 300g total)
- Variable frequency oscillator
- Sinusoidal Function Generator
- Low friction pulley
- String
- Wooden bridge (to act as one of the fixed ends of the string)
- Rigid support or clamp (the other fixed end of string)

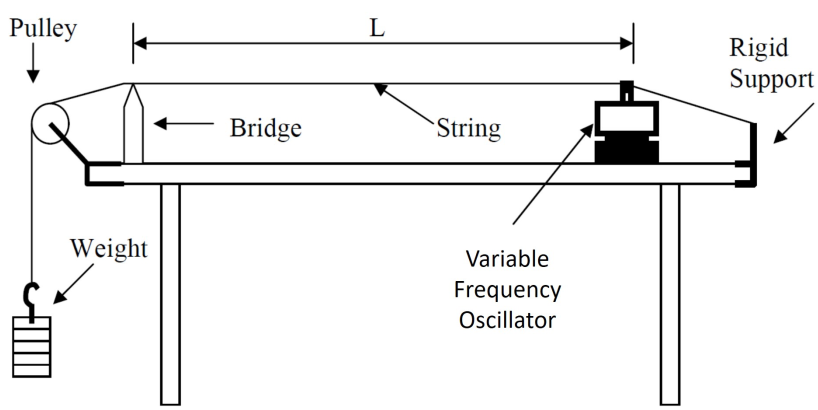


Figure 2: Standing Wave Apparatus

Procedure

1.) Set up the apparatus as shown in Figure 2. Slide the wooden bridge under the string to adjust the portion of the string which will oscillate. The length L should be between 1 and 1.5 metres. Be sure to carefully record this length, including uncertainty. Set the hanging mass to be 100g.

2.) Turn on the function generator, and adjust the amplitude (output level) so that motion is visible, but the noise it makes is not too loud. Adjust the frequency of the oscillator to get a standing wave pattern with $n = 1$ and record the value of the frequency. If you are using the *BK Precision Generator*, wait a few seconds for the display to update.

3.) Try varying your frequency slightly; at what range of frequencies is the standing wave visible? Use this range to estimate the uncertainty in f_n and record this uncertainty.

4.) Increase the frequency until the standing waves with $n = 2, 3, 4, 5$ are visible and record the frequency each time including uncertainty.

5.) Increase the hanging mass to 200g and repeat the previous two steps, measuring the first five resonant frequencies including uncertainty.

6.) Increase the hanging mass to 300g and repeat the procedure, measuring the first five resonant frequencies including uncertainty.

Analysis

1.) Plot your data set(s) in such a way that the result is a straight line. Include all data on the same plot.

2.) Include error bars on your plot. You may need to review the rules for propagating uncertainty posted in the Labs section on D2L.

3.) Fit your data with a line, and use it to determine the linear mass density μ of the string from the results of your plot.

4.) Directly measure the linear mass density of the string using a balance. Since you are only interested in the mass density, you are free to weigh a length of string which is much longer than the one you used for your experiment. Be sure to record the data include uncertainty.

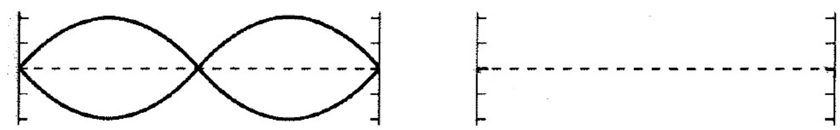
5.) **Quantitatively** compare your measured value of μ to the one you obtained with from plot. Comment on the consistency of these two values.

Wrap Up

The following questions are designed to make sure that you understand the physics implications of the experiment and also to extend your knowledge of the concepts covered. Your report should seamlessly answer these questions in their noted sections.

1.) [Theory] The note heard on a musical string instrument (e.g. a guitar, violin, etc...) is related to its fundamental frequency. A guitarist tunes their instrument by turning pegs on the bridge causing the tension in the string to increase or decrease. If a guitarist notices that a certain string is sounding “flat” (meaning it is playing a note with a lower frequency than desired), should she tighten or loosen the string? .

2.) [Discussion] The figure shows a standing wave on a string.



Original wave, tension T Tension $4T$

(a) Draw the resulting standing wave if the tension is quadrupled while the frequency is held constant.

(b) Suppose that, instead of being quadrupled, the tension is only doubled. Will there be a standing wave? If so, how many antinodes will it have? If not, explain why not.

3.) [Discussion] Suppose that for a particular setup, the hanging mass is 200g, and the $n = 3$ mode is found to have a driving frequency of 80Hz. If the mass is changed to 300g, what frequency will lead to the $n = 4$ mode with this heavier mass?

4.) [Discussion] Suppose that instead of both ends being fixed, the end of the string at $x = L$ is free to move up and down. Then, the point at $x = L$ is an antinode and not a node. Start with the equation for a standing wave $y_{\text{standing}}(x, t) = 2A\sin(kx)\cos(\omega t)$ and derive the condition for the resonant frequencies f_n in terms of n, v and L?

Report

Here is a brief guide for writing the report for the lab. The report should include the following sections:

- **Title Page**
  - Include: Report Title, Your Name, Course, Section Number, Instructor, TA Name, and Date of Submission.

- **Introduction**
  - What is the experiment's objective?
- **Theory**
  - Derivations of the physics being investigated, or reference to a source that provides a description/equation representing the physics being investigated.
  - Providing graphs that illustrate or predict how the system under study is expected to behave.
- **Procedure**
  - Briefly explain the systematic steps taken for the experiment.
- **Results and Calculation**
  - Tabulate the measurements in an organized manner.
  - Based on the procedure, one should have a sense of how the tables will look like prior to taking measurements.
  - Graph the main results.
  - Provide examples of any calculations.
- **Discussion and Conclusion**
  - Discuss the main observations and outcomes of the experiment.
  - Summarize any significant conclusions.