SOLUTIONS, Midterm, F08

Department of Electrical and Computer Engineering

Ryerson University

ELE302: Electric Networks

Midterm Examination

Duration: 1.5 hours

October 14, 2008

Student's Name:..

Student Number:..

Section:..............................

Dr.Karim

Dr.Venkatesh

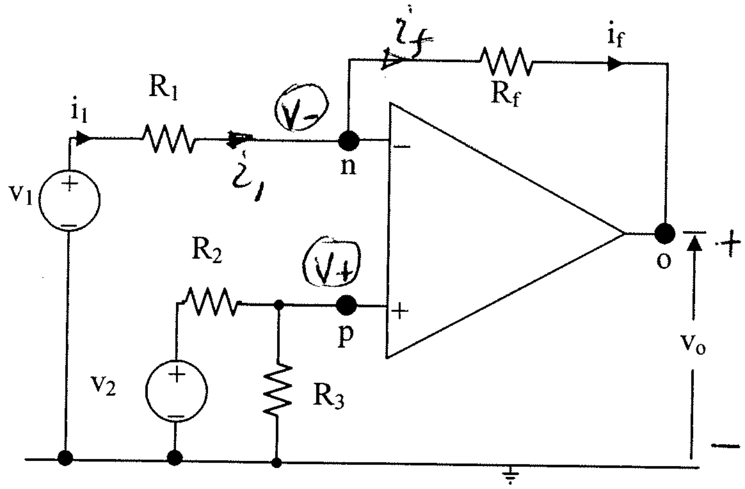
Note:

1. Please check your professor's name above.
2. **Answer all questions**

Questions	Marks	Marks obtained
Q1	20	
Q2	15	
Q3	15	
Q4	20	
Total	70	

Mid-Term, F08

Q1. Derive an expression for v_o in terms of v_1 and v_2.



Given:
$R_1 = 10\ k\Omega$
$R_2 = 10\ k\Omega$
$R_3 = 10\ k\Omega$
$R_f = 100\ k\Omega$

Solution: By voltage division principle, $V_+ = \frac{R_3}{(R_2+R_3)} \cdot V_2$

(5) For the ideal opamp $V^- = V^+ = \frac{R_3}{(R_2+R_3)} \cdot V_2$

By KCL: $i_1 = i_f$

(5)

$$\frac{V_1 - V^-}{R_1} = \frac{V^- - V_o}{R_f}$$

$$R_f(V_1 - V^-) = R_1(V^- - V_o) = R_1 V^- - R_1 V_o$$

$$\therefore\ V_o = \frac{1}{R_1}(R_f + R_1)V^- - \left(\frac{R_f}{R_1}\right) \cdot V_1$$

$$V_o = \frac{1}{R_1}(R_f + R_1) \cdot \frac{R_3}{(R_2+R_3)} V_2 - \left(\frac{R_f}{R_1}\right) V_1$$

$$V_o = \frac{R_3}{R_1} \cdot \frac{(R_f + R_1)}{(R_2 + R_3)} V_2 - \left(\frac{R_f}{R_1}\right) V_1$$

$$V_o = \frac{10}{10} \cdot \frac{(100+10)}{(10+10)} V_2 - \left(\frac{100}{10}\right) V_1$$

(10)

$$V_o = 5.5 V_2 - 10 V_1$$

Q2. Draw Bode Plots (both Amplitude and Phase plots) for the following transfer function:

$$H(j\omega) = \frac{1000.(j\omega)}{(5 + j\omega).(20 + j\omega)}$$

Use to semi-log sheet for your sketch

Solution: $$H(j\omega) = \frac{1000 . j\omega}{5\left(1 + j\frac{\omega}{5}\right) 20\left(1 + j\frac{\omega}{20}\right)} = \frac{10\cancel{0}\cancel{0} . j\omega}{1\cancel{0}\cancel{0}\left(1 + j\frac{\omega}{5}\right)\left(1 + j\frac{\omega}{20}\right)}$$

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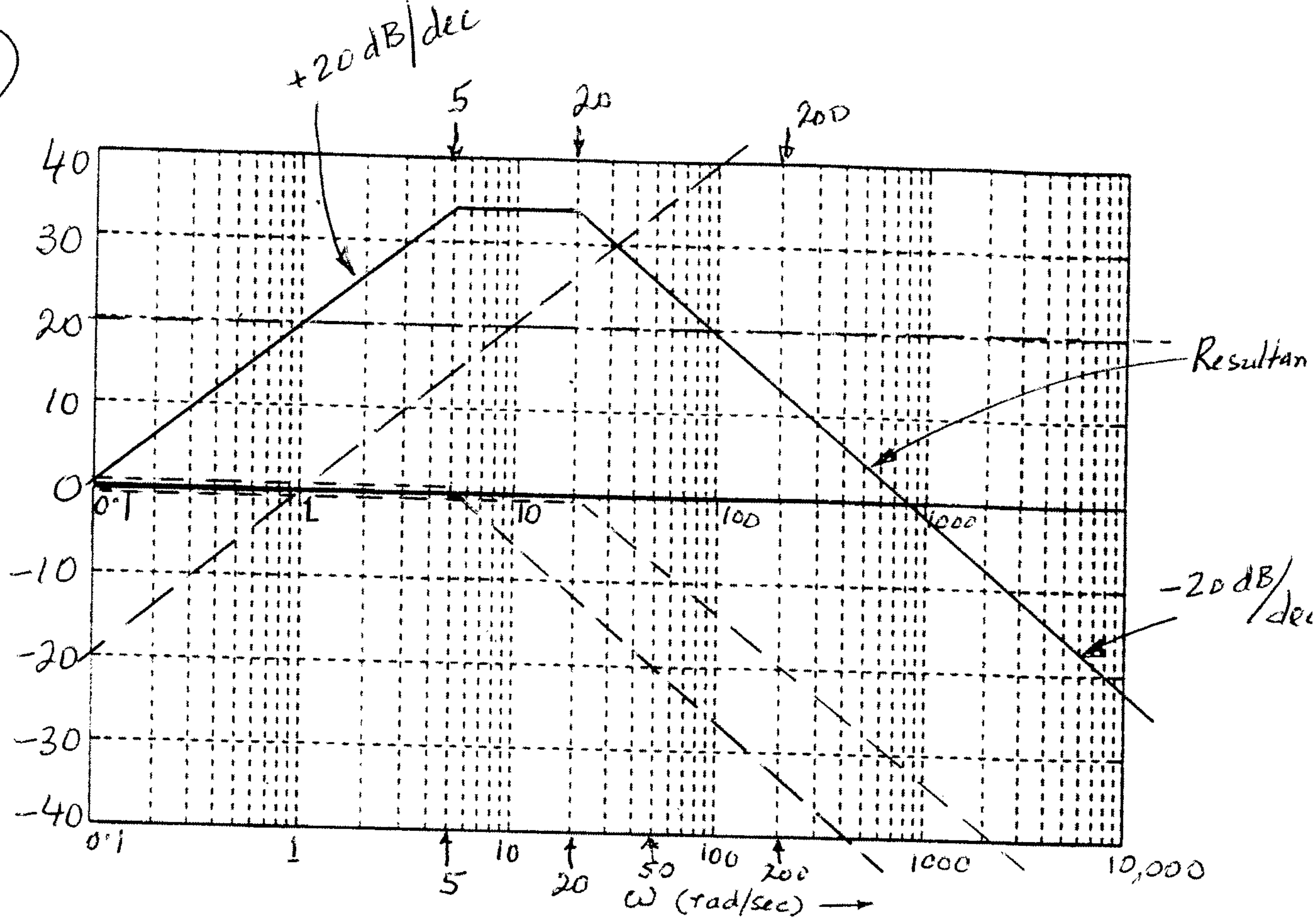
$$H(j\omega) = \frac{10 . j\omega}{\left(1 + j\frac{\omega}{5}\right)\left(1 + j\frac{\omega}{20}\right)}$$

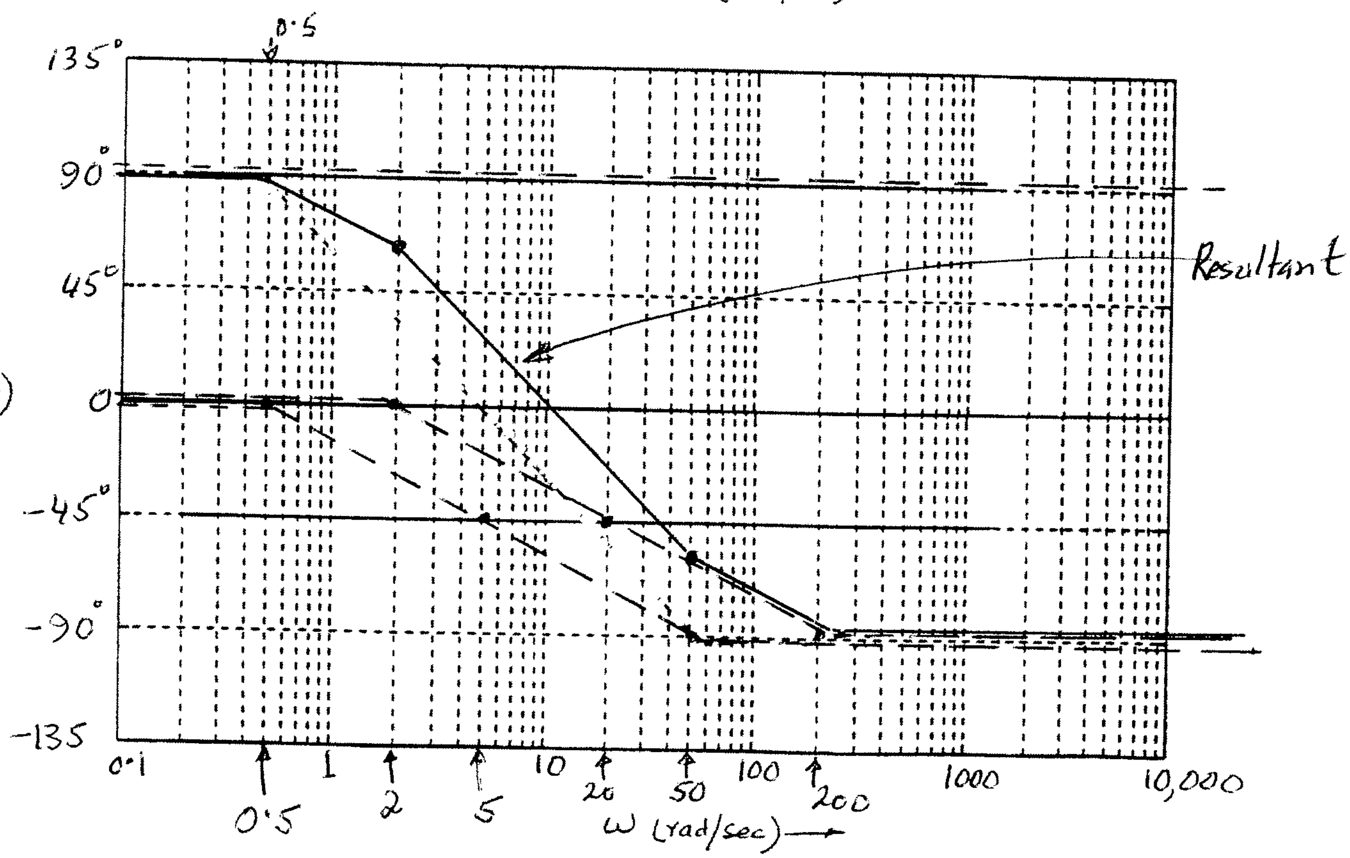
$$10 \Rightarrow 20 \log_{10} 10 = 20 \text{ dB}.$$

Q2 (contd)



+20 dB/dec
5
20
200
40
30
20
10
0
-10
-20
-30
-40
dB
0·1
1
10
100
1000
10,000
5
20
50
200
ω (rad/sec)
Resultan
-20 dB/dec

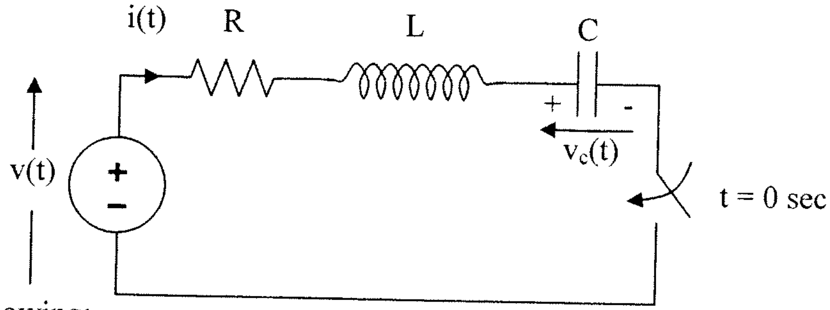
(5)



0·5
135°
90°
45°
0
-45°
-90°
-135
θ (deg)
Resultant
0·1
1
10
100
1000
10,000
0·5
2
5
20
50
200
ω (rad/sec)

(5)

Q3. R= 200 Ω; L = 10 H, C = 10 μF and v(t) = 10V. For the circuit shown above, it is given that $v_c(0^-) = 5V$.



Answer the following:

a) write a differential equation of *i*(t).

$$V = Ri + L\frac{di}{dt} + V_c(0) + \frac{1}{C}\int_0^t i\,dt$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} = \frac{1}{L}\frac{dV}{dt}$$ 5

$$R/L = 20, \quad \frac{1}{LC} = 10^{+4}, \quad \frac{1}{L} = 0.1$$

b) Write characteristic equation and its natural solution.

$$s^2 + 20s + 10^4 = 0 \qquad \alpha = 10;\ \omega_0 = 100$$ 2

$$s_1, s_2 = -10 \pm \sqrt{10^2 - 10^4}$$

$$= -10 \pm j\sqrt{9900}$$ 3

$$i_N = A e^{-10t} \cos(\sqrt{9900}\,t + \phi)$$

c) determine initial conditions and $\quad i_L(0) = 0; \quad V_C(0) = 5V$

1

$$10 = R \cdot i_L(0) + L\frac{di}{dt} + V_C(0)$$

$$10 - 5 = 10 \cdot \frac{di}{dt} \quad \therefore \quad \frac{di}{dt} = 0.5\ A/s$$ 1

d) find $i(t)$, $t \geq 0$

$$i = i_n + i_F = A \cdot e^{-10t} \cos(\omega t + \phi)$$

$$i(0) = \cancel{10A}\ A \cos\phi = 0$$ 1

$$\frac{di}{dt} = -A \cdot 10 \cos(\phi) - A \cdot \sin(\phi) \cdot \omega = 0.5$$

1

$$\phi = 90^\circ.$$

$$-A \sin\phi \cdot \omega = 0.5$$

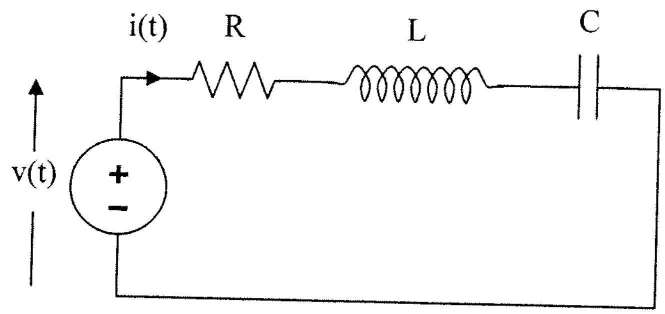
$$\therefore \quad A = \frac{-0.5}{\sqrt{9900}}$$

1

$$i = -\frac{0.5}{\sqrt{9900}} e^{-10t} \cos(\sqrt{9900}\, t + 90^\circ)$$

$$= (0.5/\sqrt{9900})\, e^{-10t} \sin(\sqrt{9900}\, t)$$

Q4. For the circuit shown below



Given: $R = 10\Omega$, $L = 0.01$; $C = 0.01$ F

Answer the following:

a) Find resonant frequency (ω_o)

$$\omega_o = \frac{1}{\sqrt{LC}} = 100 \text{ rad/sec.}$$

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b) Find two half power frequencies (ω_1, ω_2)

$$\omega_1, \omega_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + 1/LC}$$

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$$= 9.9020 \, , \, 1009.9 \text{ rad/se}$$

c) Find quality factor (Q) $= \omega_o / (R/L)$

$= 0.1$

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d) Find band width (B) $= \omega_2 - \omega_1 = 1000 \frac{\text{rad}}{\text{sec}}$

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