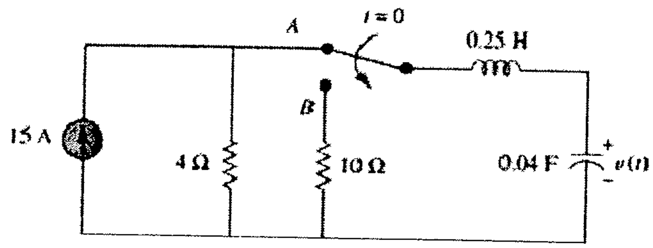
SOLUTION

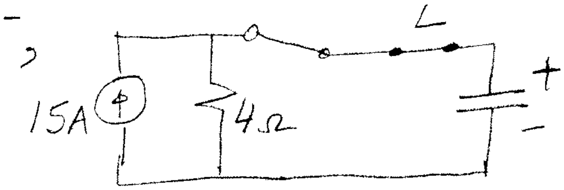
QUIZ-2a

Name:... Std.No.............................. Sec:.....

Q: In the circuit of Fig. 8.71, the switch instantaneously moves from position A to B at $t = 0$. Find $v(t)$ for all $t \geq 0$.



At $t = 0^-$,

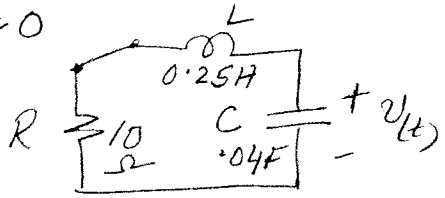


$v_{c(0^-)} = 4(15) = 60V = v_{c(0^+)}$

$60V = v_{(0^+)}$

$i_{L(0^-)} = 0 = i_L(0^+)$

At $t \geq 0$



$v_{(t)} = \cancel{v_{ss}}^{\,0} + v_{tran} = K_1 e^{s_1 t} + K_2 e^{s_2 t}$

$S = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$, for the series circuit $\alpha = \frac{R}{2L} = \frac{10}{2(.25)} = 20$

and $\omega_0^2 = \frac{1}{LC} = \frac{1}{(.25)(.04)} = 100$

$S = -20 \pm \sqrt{(20)^2 - 100} = -20 \pm 17.321$

$\therefore S_1 = -37.321, \quad S_2 = -2.68$

$\therefore v_{(t)} = K_1 e^{-37.32t} + K_2 e^{-2.68t}$

At $t = 0^+$; $v_{(0^+)} = 60 = K_1 + K_2 \Rightarrow K_2 = 60 - K_1$ ...... (1)

$i_c = c\frac{dv_c}{dt} = c\frac{dv}{dt} \quad \therefore \frac{dv}{dt} = \frac{i_c}{C} = \frac{i_L}{c}$ for the series circuit at $t \geq 0^-$

$\therefore \frac{dv}{dt}(0) = \frac{i_L(0)}{c} = \frac{0}{c} = 0$

$\frac{dv}{dt} = -37.32 K_1 e^{-37.32t} - 2.68 K_2 e^{-2.68t} \quad \therefore \frac{dv}{dt}(0^+) = -37.32 K_1 - 2.68 K_2 = 0$ ...(2)

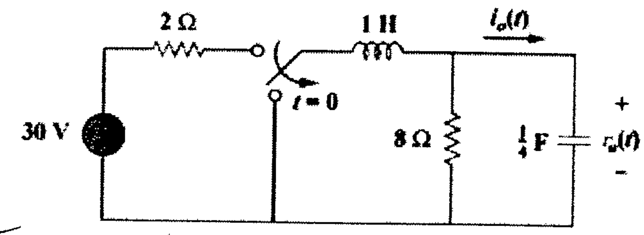
From (1) & (2), $-37.32 K_1 - 2.68(60 - K_1) = 0 \Rightarrow -34.64 K_1 = 160.8 \Rightarrow K_1 = -4.64$ & $K_2 = 64.64$

$\therefore v_{(t)} = 64.64 e^{-2.68t} - 4.64 e^{-37.32t}$

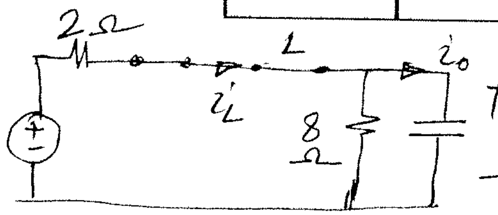
QUIZ-2b

Name:... Std.No.............................. Sec:.....

Q: In the circuit , calculate $i_o(t)$ and $v_o(t)$ for $t > 0$.



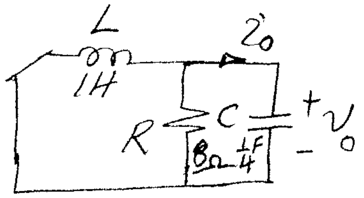
At $t=0^-$,



$i_L(0^-) = i_L(0^+) = \frac{30}{10} = 3A$

$i_o = i_c = 0$

At $t \geq 0$



$v_c(0^-) = v_c(0^+) = 8 \times i_L(0^-) = 8 \times 3 = 24V$

$$v_o(t) = \cancel{v_{ss}}^{\,0} + v_{tran} = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

$S = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$, for the parallel circuit $\alpha = \frac{1}{2RC} = \frac{1}{2(8)(\frac{1}{4})} = \cdot 25$

& $\omega_0^2 = \frac{1}{LC} = \frac{1}{1(\frac{1}{4})} = 4$

$\therefore\ s = -0.25 \pm \sqrt{(\cdot 25)^2 - 4} = 0.25 \pm j1.984$ ← complex roots

$\therefore\ v_o(t) = e^{-\alpha t}[A \cos \omega_d t + B \sin \omega_d t] = e^{-\cdot 25t}[A \cos 1.98t + B \sin 1.98t]$

At $t=0^+$, $v_o(0^+) = 24 = A$

$\therefore\ v_o(t) = e^{-0.25t}[24 \cos 1.98t + B \sin 1.98t]$

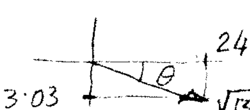
$\frac{dv_o}{dt} = -0.25 e^{-\cdot 25t}(24 \cos 1.98t) + 24 \times 1.98 \times e^{-\cdot 25t}(-\sin 1.98t)$
$+ (-\cdot 25 e^{-\cdot 25t}) B \sin 1.98t + 1.98 B e^{-\cdot 25t} \cos 1.98t$

$i_o = i_c = C\frac{dv_o}{dt}$ $\therefore$ at $t=0^+$, $i_c = 0$ $\therefore \frac{dv_o}{dt}(0^+) = 0$

$0 = (-\cdot 25)(24) + 1.98B$ $\therefore B = 3\cdot 03$

$\therefore\ v_o(t) = e^{-0.25t}[24 \cos 1.98t + 3.03 \sin 1.98t]$

$v_o(t) = 24.19 e^{-0.25t} \cos(1.98t - 7.2°)$



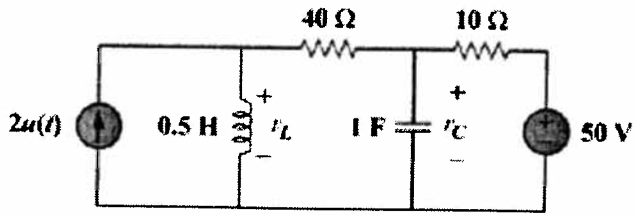
$\theta = \tan^{-1}\frac{3.03}{24} = 7.2°$

$\sqrt{(3.03)^2 + (24)^2} = 24.19$

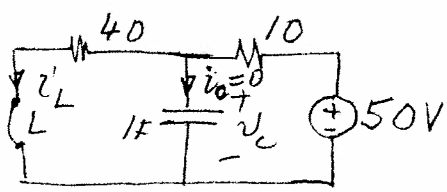
QUIZ-2c

Name:.. Std.No.............................. Sec:.....

Q: Find $v_L(0+)$ and $v_C(0+)$ in the given circuit, where u(t) is the unit step function i.e. its value is u(t) = 0 when t ≤ 0- , and u(t) = 1 when t ≥0+ .



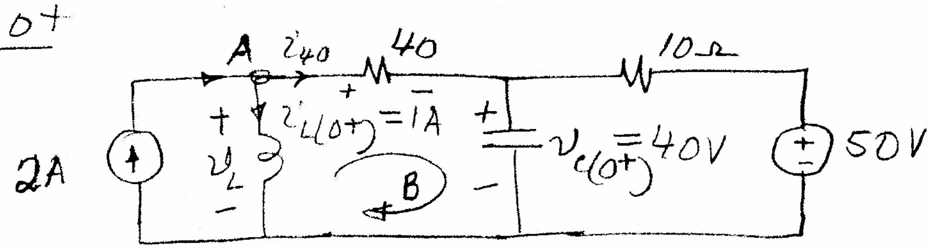
At $t = 0^-$;



$$i_{L(0^-)} = i_{L(0^+)} = \frac{50}{(40+10)} = 1A$$

$$v_{c(0^-)} = 40(1) = 40V = v_{c(0^+)}$$

At $t = 0^+$



$$\boxed{v_{c(0^+)} = 40V}$$

KCL at node-A : $2 = i_{L(0^+)} + i_{40(0^+)}$ $\therefore$ $i_{40(0^+)} = 2 - i_{L(0^+)}$

$i_{40(0^+)} = 2 - 1 = 1A$

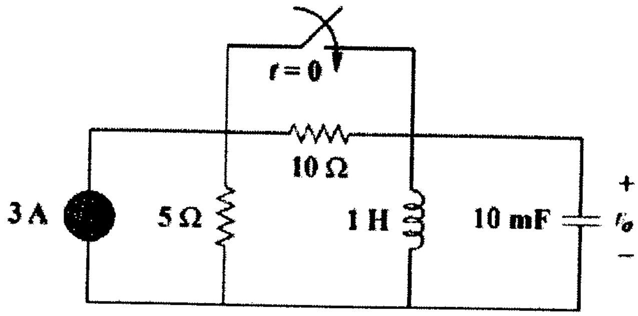
KVL in Mesh-B : $-v_{L(0^+)} + 40\,i_{40(0^+)} + v_{c(0^+)} = 0$

$\therefore$ $$\boxed{v_{L(0^+)} = 40\,i_{40(0^+)} + v_{c(0^+)} = 40(1) + 40 = 80V}$$

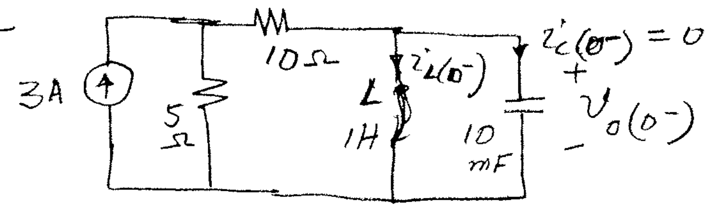
QUIZ-2d

Name:... Std.No.............................. Sec:.....

Q: Find the output voltage $v_o(t)$ in the circuit.



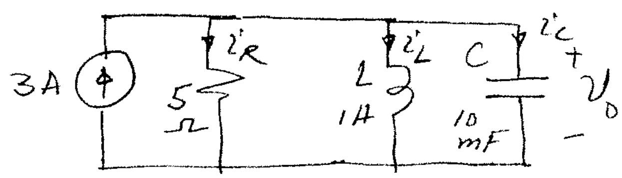
At $t=0^-$



$i_L(0^-)$ by current division principle, $i_{L(0^-)} = 3 \times \frac{5}{(5+10)} = 1\,A = i_{L(0^+)}$

$v_{C(0^-)} = 0 = v_{C(0^+)}$

At $t \geq 0^+$:



At $t=\infty$, $v_{o(ss)} = 0$

$\therefore v_o = \cancel{v_{ss}}^{\,0} + v_{tran} = K_1 e^{s_1 t} + K_2 e^{s_2 t}$

$s = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$, for the RLC parallel circuit, $\alpha = \frac{1}{2RC}$

$\therefore \alpha = \frac{1}{2(5)(10\times10^{-3})} = 10$, $\omega_o^2 = \frac{1}{LC} = \frac{1}{1\times 10\times10^{-3}} = 100$

$\therefore s = -10 \pm \sqrt{(10)^2 - 100} = -1 = s_1, s_2$ Identical roots, critically damped circuit

$\therefore v_o = (A+Bt)e^{st} = (A+Bt)e^{-1t}$

At $t=0^+$, $v_{o(0^+)} = v_{c(0^+)} = 0 = A$

$\therefore v_{o(t)} = Bt\,e^{-t}$

$\frac{dv_o}{dt} = Be^{-t} + Bt(-1)e^{-t} = \frac{i_c}{C}$, By KCL $3 = i_{R(0^+)} + i_{L(0^+)} + i_{c(0^+)}$

$3 = \frac{0}{R} + 1 + i_{c(0^+)} \therefore i_{c(0^+)} = 2$

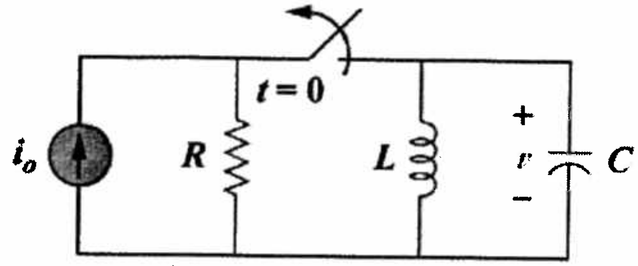
$\therefore \frac{dv_o}{dt}(0) = B = \frac{i_{c(0^+)}}{C} = \frac{2}{10\times10^{-3}} = 200$

$\therefore \boxed{v_{o(t)} = 200t\,e^{-t}}$

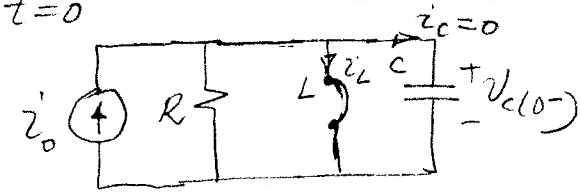
QUIZ-2e

Name:……………………………………… Std.No………………………… Sec:…..

Q: Find $v(t)$ for $t > 0$ in the circuit.



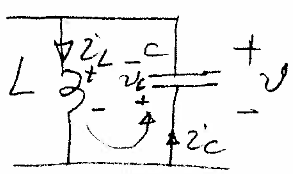
At $t=0^-$



$v_{c(0^-)} = v_{c(0^+)} = 0$

$i_{L(0^-)} = i_{L(0^+)} = i_o$

At $t \geq 0$



Note: $v = -v_c$

$v_L + v_c = 0$

$L\dfrac{di_L}{dt} + v_c = 0$

$L\dfrac{di_c}{dt} + v_c = 0$

$L\dfrac{d\left[C\frac{dv_c}{dt}\right]}{dt} + v_c = 0$

$LC\dfrac{d^2v_c}{dt^2} + v_c = 0$

$\dfrac{d^2v_c}{dt^2} + \dfrac{1}{LC}v_c = 0$

Characteristic Eqn $s^2 + \frac{1}{LC} = 0$ $\therefore$ $s = \pm\frac{1}{\sqrt{-LC}} = \pm\frac{1}{j\sqrt{LC}} = \pm j\frac{1}{LC} = \cancel{\alpha}^{\,0} \pm j\omega_d$

$\therefore$ $v_c = e^{-\alpha t}\left[A\cos\omega_d t + B\sin\omega_d t\right] = 1.\left[A\cos\frac{1}{\sqrt{LC}} + B\sin\frac{1}{\sqrt{LC}}\right]$

At $t = 0^+$, $v_{c(0^+)} = 0 = A$

$\therefore$ $v_c = B\sin\frac{1}{\sqrt{LC}}$

$\dfrac{dv_c}{dt} = B\left(\frac{1}{\sqrt{LC}}\right)\cos\frac{1}{\sqrt{LC}}t = \dfrac{i_c}{C}$ $\therefore$ $\dfrac{dv_c}{dt}(0^+) = \dfrac{B}{\sqrt{LC}} = \dfrac{i_{c(0^+)}}{C} = \dfrac{i_o}{C}$

$\therefore$ $B = \dfrac{i_o\sqrt{LC}}{C} = i_o\sqrt{\dfrac{L}{C}}$

$\therefore$ $v_c = i_o\sqrt{\frac{L}{C}}\sin\frac{1}{\sqrt{LC}}t = -v$ $\therefore$ $v_{(t)} = i_o\sqrt{\frac{L}{C}}\sin\left(\frac{t}{\sqrt{LC}} - 180^\circ\right)$