$\int u\,dv = uv - \int v\,du$

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1. **[10 Marks]** (a) Why is $u = \cos(x)$ and $dv = x\,dx$ a poor choice for evaluating

$\int x\cos(x)\,dx$. 3 marks

a) ~~&~~ Attempting to solve the integral by the given substitution & Integration by parts would only lead us to more integrals in the solution such the the powers of x; increase with the simplification of each integral, making the integral solution never algebraically absolute in terms of its solution. (Solution would never algebraically terminate)

(b) Consider the following integral.

$\int 2\cos(x)\ln(\sin(x))\,dx$

Let $w = \sin(x)$.

i) Express the above integral in terms of w. 2 marks

ii) Hence or otherwise evaluate the given integral. 5 marks

b) i) $\int 2\cos(x)\ln(\sin(x))\,dx = 2\int \ln(w)\,dw$

[if $w = \sin(x)$]

$\Rightarrow dw = \cos(x)\,dx$

ii) $\therefore\ 2\int \ln(w)\,dw$; $u = \ln(w) \Leftrightarrow du = \frac{1}{w}\,dw$

$dv = dw \Leftrightarrow v = w$

$$\therefore\ 2\int \ln(w)\,dw = 2\left[w\ln(w) - \int w\cdot\frac{1}{w}\,dw\right] + C$$

$$= 2\left[w\ln(w) - \int dw\right] = 2\left[w\ln(w) - w\right] + C$$

$$= \underline{2w\left[\ln(w) - 1\right] + C}$$

$$\therefore\ \int 2\cos(x)\ln(\sin(x))\,dx = 2\int \ln(w)\,dw = 2w\left[\ln(w)-1\right] + C$$

$$= \underline{\left[2\sin(x)\right]\cdot\left[\ln(\sin(x) - 1\right] + C}$$