RYERSON UNIVERSITY
DEPARTMENT OF MATHEMATICS

MTH314 — DISCRETE MATHEMATICS FOR ENGINEERS
MIDTERM TEST

March 5, 2013

INSTRUCTIONS

1. Duration: 1.5 hours item You are allowed **one** 8.5" × 11" formula sheet (two-sided). The information on the sheet must be hand-written, with your own hand-writing.
2. Marks (out of) are shown in brackets.
   (a) All the answers on the test must be accompanied by a full explanation. Marks will not be given solely on a short answer. Show all your work.
   (b) Write your solutions in the space provided. If you need more space, use the back of the page. Indicate this fact on the original page, making sure that your solution cannot be confused with any rough work which may be there.
3. Do not separate the sheets.
4. Make sure that your test paper is complete; there are 8 questions on 7 pages.
5. Have your student card available on your desk.

Last Name (Print): ____________________

First Name (Print): ____________________

Student I.D. ______________________

Signature ______________________

Grade /50

[3 marks] **(1)** Construct the truth table for $p \to (\sim q \vee r)$.

p	q	r	$\sim q$	$\sim q \vee r$	
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	T

[4 marks] **(2)** Show that $p \to (q \wedge r)$ is equivalent to $(\sim q \vee \sim r) \to \sim p$. Do **not** use the truth table.

$$LHS = p \to (q \wedge r) \equiv \sim(q \wedge r) \to \sim p$$

$$\equiv (\sim q \vee \sim r) \to \sim p \equiv RHS$$

[5 marks] **(3)** Show that the following argument is valid:

$$\begin{array}{l} (1)\ p \vee \sim s \\ (2)\ p \rightarrow (q \vee \sim s) \\ (3)\ u \rightarrow (\sim q \vee \sim r) \\ (4)\ s \wedge r \\ \hline \therefore q \wedge \sim u \end{array}$$

by using standard argument forms (Modus Ponens, Modus Tollens, etc.) and logical equivalences. Be sure to justify each step, making clear which of the standard valid forms or logical equivalences you have used. You do **not** need to provide the names.

(4) implies s

(4) implies r

(1) and s implies p

(2) and p implies $q \vee \sim s$

$q \vee \sim s$ and s implies q

(3) is equivalent to $\sim(\sim q \vee \sim r) \rightarrow \sim u$

$\equiv (q \wedge r) \rightarrow \sim u$

r and q imply $r \wedge q$

$(q \wedge r) \rightarrow \sim u$ and $r \wedge q$ imply $\sim u$

$\therefore q \wedge \sim u$

[4+4=8 marks] **(4)** Prove that the following statements are true

(a)

$$\forall x \in \mathbb{R}\ \exists y \in \mathbb{Z} \text{ such that } x > 2y$$

$y < \frac{x}{2}$

Let $x \in \mathbb{R}$.

Take $y = \lfloor \frac{x}{2} \rfloor - 11 \in \mathbb{Z}$

Clearly, $2y = 2\left(\lfloor \frac{x}{2} \rfloor - 11\right) \leq 2\left(\frac{x}{2} - 11\right)$
$= x - 22 < x$

(b)

$$\sim (\exists y \in \mathbb{R} \text{ such that } \forall x \in \mathbb{R}\ x > y)$$

(Hint: negate the statement first.)

$\forall y \in \mathbb{R}\ \exists x \in \mathbb{R}$ s.t. $x \leq y$

Let $y \in \mathbb{R}$.

Take $x = y \in \mathbb{R}$

Clearly, $x = y \leq y$

[5+5=10 marks] **(5)** Prove or disprove. Clearly state your answer: true/false.

(a) $2(a+3)-7$ is odd for *any* integer a.

True $\quad 2(a+3)-7 = 2(a+3)-8+1$

$= 2(a+3-4)+1$

$= 2(a-1)+1$

and $a-1 \in \mathbb{Z}$. So $2(a+3)-7$ is odd

(b) If n is even, then $(n = 4k$ for some integer k or $n = 4k+1$ for some integer $k.)$

False $\quad$ Counterexample $n=2$.

n is even but it cannot be expressed as $4k$ or $4k+1$ $(k \in \mathbb{Z})$. Why?

$2 = 4k$	$2 = 4k+1$
$k = 1/2 \notin \mathbb{Z}$	$k = 1/4 \notin \mathbb{Z}$

[5 marks] **(6)** Prove by contraposition that if $\underset{P}{xy \text{ is odd}}$, then $\underset{Q}{\text{both } x \text{ and } y \text{ are odd}}$.

Instead of proving $p \to q$, we will prove that $\sim q \to \sim p$ (equivalent!), that is, x is even <u>or</u> y is even implies xy is even

<u>Proof</u>: WLOG, we may assume that x is even, that is, $x = 2k$, $k \in \mathbb{Z}$.

Then, $xy = (2k)y = 2(ky)$ and $ky \in \mathbb{Z}$.
So xy is even.

[1+1+1+1+1=5 marks] **(7)** Given $A = \{-1, 0, 1, 2, 3, 4\}$

$$A = \{x \in \mathbb{Z} \mid -1 \leq x < 5\}, \qquad B = \{x \in \mathbb{Z} \mid x \text{ is divisible by } 3\}, \qquad C = \{1, 2, 3\},$$

find the following sets. Give your answers in the standard set notation. You may assume that $U = \mathbb{Z}$.

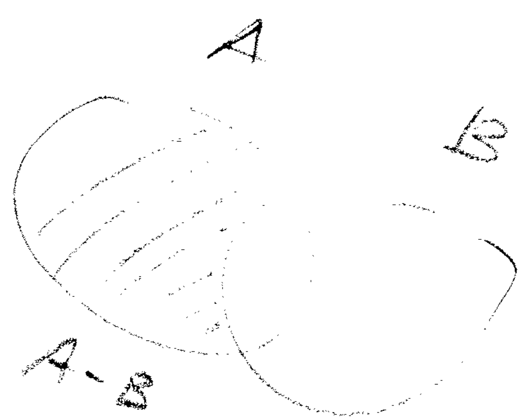
(a) $A^c = \{\ldots, -4, -3, -2,\} \cup \{5, 6, 7, 8, \ldots\}$

(b) $A \cap B = \{0, 3\}$

(c) $A \cup C = A = \{-1, 0, 1, 2, 3, 4\}$

(d) $A - (B - C) = \{-1, 1, 2, 3, 4\}$

(e) $C - (A - B) = \{3\}$



$A - B = \{-1, 1, 2, 4\}$

$A - B = A \cap B^C$

$A - B = A - (A \cap B)$

[5+5=10 marks] **(8)** Prove or disprove (by giving a counterexample). Clearly state your answer: true/false.

(a) If $A \subseteq B \cup C$ and $A \cap B = \emptyset$, then $A \subseteq C$.

TRUE

Suppose that $A \subseteq B \cup C$ and $A \cap B = \emptyset$. Take $x \in A$.

Since $A \subseteq B \cup C$ and $x \in A$, $x \in B \cup C$, that is, $x \in B$ or $x \in C$.

Since $A \cap B = \emptyset$ and $x \in A$, $x \notin B$.

Hence, $x \in C$ which implies that $A \subseteq C$.

(b) If $A \subseteq B$, then $A \setminus C = B \setminus C$.

FALSE

Counterexample: $A = \{1\}$

$B = \{1, 2\}$

$C = \{3\}$

Clearly, $A \subseteq B$ but $A \setminus C = A \neq B = B \setminus C$.