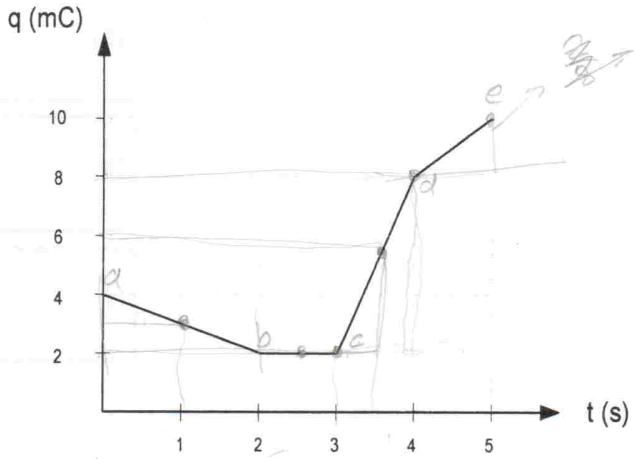
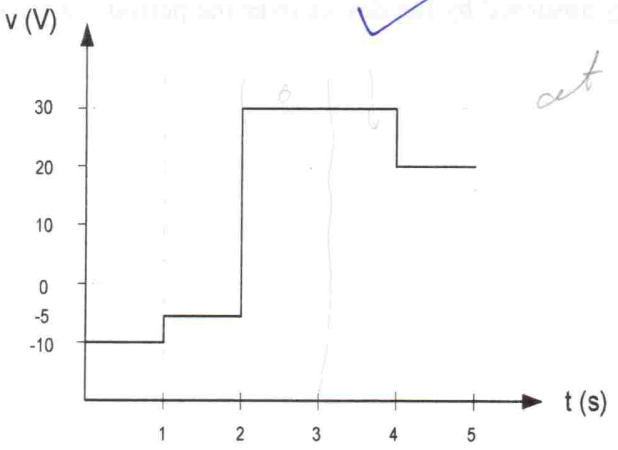
**Question 1(i):**

The following figures show the charge entering; and voltage across a certain element:




Find the current through the element at:

(a) t =1 sec
(b) t = 2.5 sec
(c) t = 3.5 sec

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at 1 sec

a → b

$m = \frac{-2}{2} = -1$

$q = -1 * t + c$ (4)

$q = -t + 4$

$\frac{dq}{dt} = -1$ mA

line eq

$y = mx + c$

$q = -mt + c$

$i = -1$,

at 3,5 sec

c → d

$m = \frac{8-2}{1} = 6$

$y = mx + c$

$q = 6t + c$

$q = 6t$

$\frac{dq}{dt} = 6$ mA

at 4-5 sec

d → e

$m = \frac{2}{1} = 2$

$q = 2t + c$

$\frac{dq}{dt} = 2$ mA

b → c

at 2,5 sec

$m = 0$

$y = c$

$q = 2$

$\frac{dq}{dt} = 0$

0 → 5 sec

power $= (-1 * -10) + (-1 * -5) + 0 + (6 * 30) + (2 * 20) =$

$15 + 180 + 40 = 235$ mW

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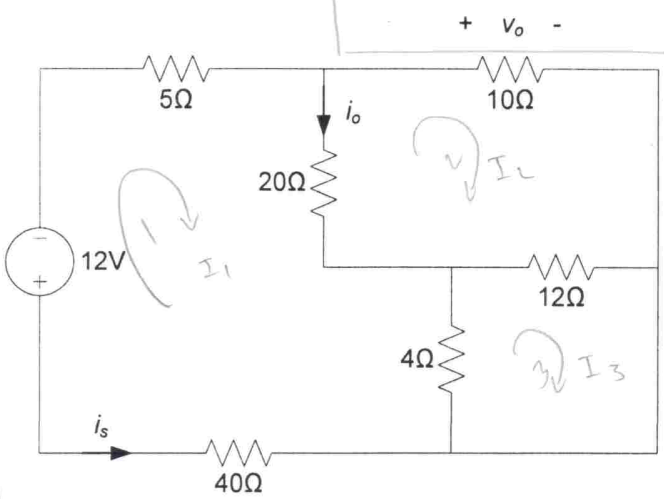
$$E = \int p\,dt$$

$$= \int vi\,dt \quad J$$

**Question 1(ii):**

Find $i_s$ , $i_o$ and $v_o$ in the following circuit:




or simplify


$i_s = \frac{(40 \,\|\, 12\,\Omega)}{((23 \,\|\, 10)+5)}$

$= 0,2309$ A

$i_o = i_s \frac{10}{33}$

$= 0,07$ A

$V_o = 10 * \left(i_s \frac{23}{33}\right)$

$= 1,6909$ V

at (1)

$12 + 5I_1 + 20(I_1 - I_2) + 4(I_1 - I_3) + 40 I_1 = 0$

$69 I_1 - 20 I_2 - 4 I_3 = -12$ (1)

at (2)

$10 I_2 + 12(I_2 - I_3) + 20(I_2 - I_1) = 0$

$-20 I_1 + 42 I_2 - 12 I_3 = 0$ (2)

at (3)

$4(I_3 - I_1) + 12(I_3 - I_2) = 0$

$-4 I_1 - 12 I_2 + 16 I_3 = 0$ (3)

$\begin{bmatrix} 69 & -20 & -4 \\ -20 & 42 & -12 \\ -4 & -12 & 16 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -12 \\ 0 \\ 0 \end{bmatrix}$

$I_1 = -0,2309$ A

$I_2 = 0,1609$ A

$I_3 = 0,178$ A

$i_o = i_1 - i_2 = -0,07$ A

$i_s = -I_1 = 0,2309$ A +

$V_o = I_2 * 10 = 1,609$ V

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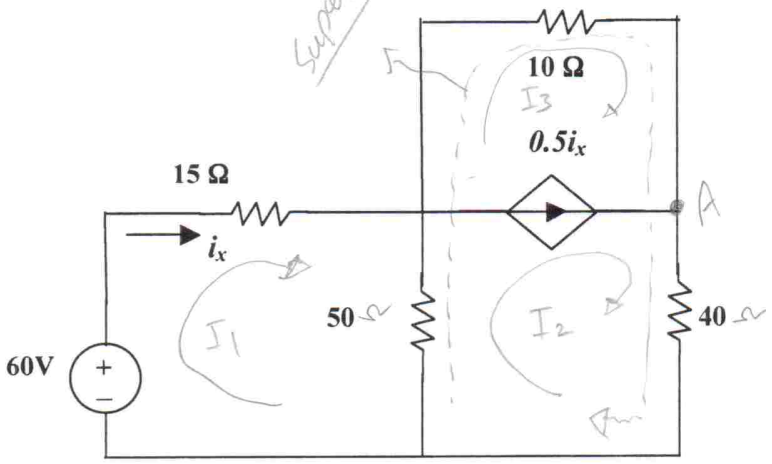
**Question 2**:

Use mesh analysis to solve for $i_x$ in the following circuit:




at ①

$-60 + 15I_1 + 50(I_1 - I_2) = 0$

$65I_1 - 50I_2 = 60$ ①

$$\begin{bmatrix} 65 & -50 & 0 \\ -50 & 90 & 10 \\ 0.5 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \\ 0 \end{bmatrix}$$

at supermesh

$50(I_2 - I_1) + 10(I_3) + 40(I_2) = 0$

$-50I_1 + 90I_2 + 10(I_3) = 0$ ②

$I_1 = -1.6$ A

$i_x = I_1 = -1.6$A

KCL at A

$I_3 + 0.5i_x - I_2 = 0$ $\quad \{ i_x = I_1$

$I_3 + 0.5I_1 - I_2 = 0$

$0.5I_1 - I_2 + I_3 = 0$ ③

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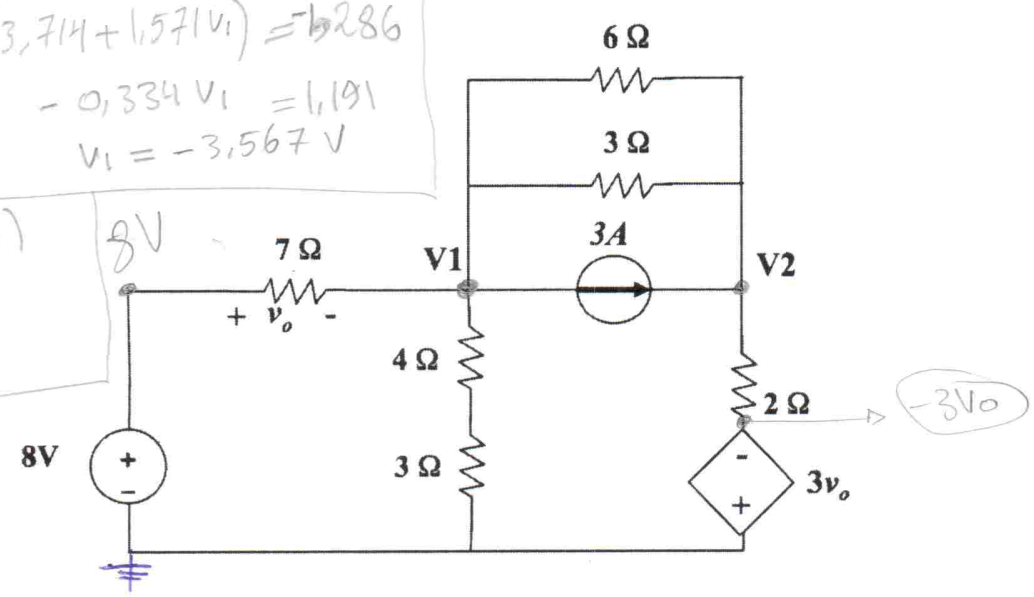
$V_2 = -1,888$

**Question 3**:

Using nodal analysis, determine **V1** and **V2** in the following circuit:




Sub $V_2$ in (2)

$0,714V_1 - 0,667(3,714 + 1,571V_1) = -1,286$

$-0,334V_1 = 1,191$

$V_1 = -3,567\ V$

$V_2 = 3,714 + 1,571(V_1)$

$V_2 = -1,889\ V$

8V

at $V_1$

$$\frac{8-V_1}{7} + \left(\frac{-V_1}{7}\right) - 3 + \frac{V_2-V_1}{3} + \frac{V_2-V_1}{6} = 0$$

$$\left(-\frac{1}{7} - \frac{1}{7} - \frac{1}{3} - \frac{1}{6}\right)V_1 + \left(\frac{1}{3} + \frac{1}{6}\right)V_2 = 3 - \frac{8}{7}$$

$$-0,7857V_1 + 0,5V_2 = 1,857 \quad (1)$$

$V_2 = 3,714 + 1,571V_1$

$v_o = 8 - V_1$

$\frac{3V_o}{2} = 3\left(\frac{8-V_1}{7}\right) = \frac{24-3V_1}{14}$ $\quad V_o = \frac{8-V_1}{7}$

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at $V_2$

$$3 + \frac{V_1-V_2}{3} + \frac{V_1-V_2}{6} + \left(\frac{-3V_o-V_2}{2}\right) = 0$$

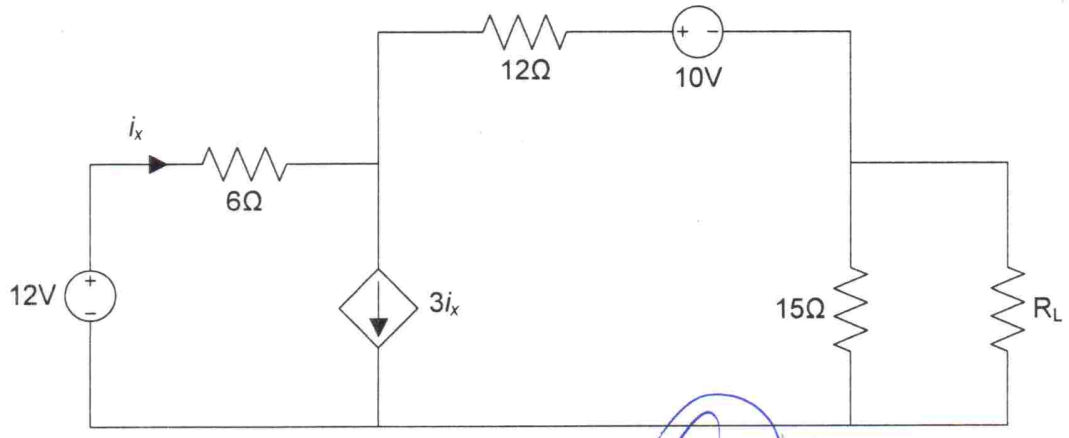
$$3 + \frac{V_1-V_2}{3} + \frac{V_1-V_2}{6} + \left(\frac{24-3V_1}{14}\right) - \frac{V_2}{2} = 0$$

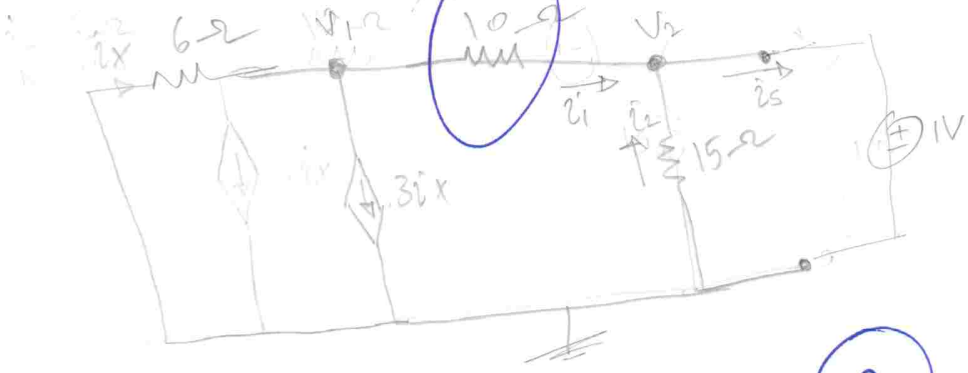
$$\left(\frac{1}{3} + \frac{1}{6} + \frac{3}{14}\right)V_1 + \left(-\frac{1}{6} - \frac{1}{2}\right)V_2 = -3 + \frac{24}{14}$$

$$0,714V_1 - 0,667V_2 = -1,286 \quad (2)$$

**Question 4**:

Find the maximum power that can be transferred to the resistor $R_L$ in the following circuit:




KCL at $V_1$

$$\frac{-V_1}{6} - 3i_x + \frac{V_2 - V_1}{10} = 0$$

$$i_x = \frac{-V_1}{6}$$

$$\frac{-V_1}{6} + \frac{V_1}{2} + \frac{V_2}{10} - \frac{V_1}{10} = 0$$

$$\left(\frac{-1}{6} + \frac{1}{2} - \frac{1}{10}\right)V_1 + \frac{1}{10}V_2 = 0$$

$$0{,}233\,V_1 + 0{,}1\,V_2 \quad (1)$$

$V_2 = 1\ V$

$$0{,}233\,V_1 = -0{,}1$$

$$V_1 = 0{,}4291$$

$$i_s = i_1 + i_2$$

$$= \left(\frac{V_1 - V_2}{10}\right) + \left(\frac{-V_2}{15}\right)$$

$$= \frac{0{,}429}{10} - \frac{1}{10} - \frac{1}{15}$$

$$= -0{,}1237\ A$$

$$R_{th} = \frac{1}{0{,}1237} = 8{,}080\ \Omega$$

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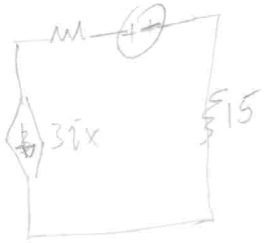
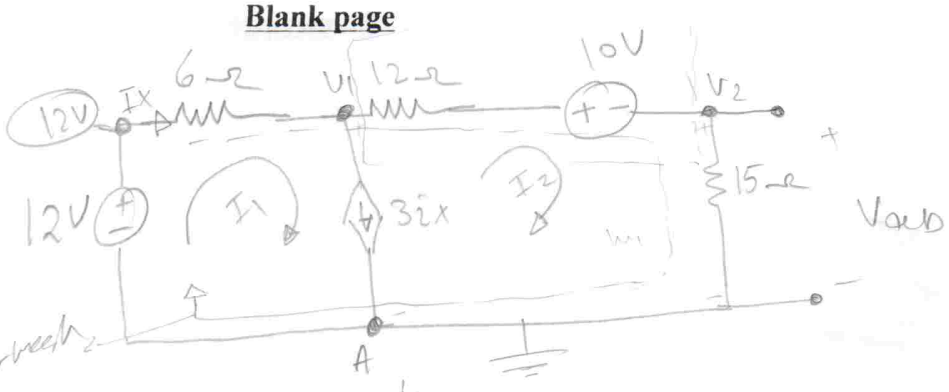



at supermesh

$$6i_1 + 12i_2 + 10 + 15i_2 - 12 = 0$$

$$6i_1 + 27i_2 = 2 \quad (1)$$

KCL at A

$$I_2 + 3i_x - I_1 = 0 \qquad , i_x = I_1$$

$$2I_1 + I_2 = 0 \quad (2)$$

$$I_2 = -2I_1$$

Sub in (1)

$$6i_1 + 27(-2i_1) = 2$$

$$48i_1 = 2$$

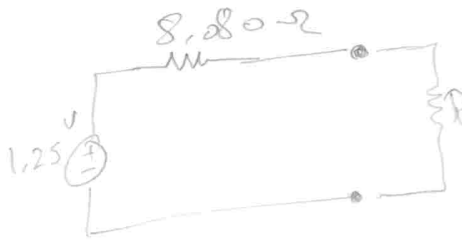
$$i_1 = 0{,}041667\ A$$

$$I_2 = -2I_1 = 0{,}0833$$

$$V_{ab} = 15 * 0{,}0833 = 1{,}25\ V$$

✓

thevenin's model




max power occurs when

$R_{th} = R_L$

$$P_{max} = \frac{V_{th}^2}{4R_{th}}$$

$$= 48{,}34\ mW$$

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