Physics IA Ver 1.02

The Thermodynamics of a Spherical Object

How does Temperature of the fluid medium in laminar flow affect the drag force on a spherical body in linear motion?

***Word Count**: 2000*

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Contents

Abstract

We will discuss the effects of change in the magnitude of ***temperature*** *on the* ***Air Drag/Fluid Resistance*** *of a* ***spherical*** *body. We shall accomplish this by collecting raw data on the measurement of* ***drag force*** *with respect to* ***temperature*** *under certain controlled spaces with* ***defined standard initial conditions****.*

1 *Introduction*

I have chosen this research question because the fundamental relation between the ***mechanics of fluid flow****, its* ***complex dynamics*** *and its* ***excruciating difficulty in possibly modeling*** *this behavior has led me to explore this field, as this phenomenon is exciting, unique and wonderful, with ample scope to carry out research and collect data both* ***quantitatively*** *and* ***qualitatively****.*

2 *Aim*

To ***collect experimental raw data*** *relating to the variation in* ***drag force*** *with the variation in* ***temperature*** *in a lab controlled setting of a* ***fluid medium*** *flow in a* ***spherical object*** *and to compare it with* ***computer simulations*** *of the same and hence study its* ***different states*** *and calculate the value the* ***inconsistencies*** *in* ***accuracy*** *of the* ***mathematically modeled computer simulation*** *to that of the* ***actual experiment*** *and vice-versa.*

3 *Hypothesis*

The ***drag force*** *on the* ***spherical mass*** *would act* ***inversely proportional*** *to the the change in* ***temperature****, which means that with increase in temperature, the eminent drag force in impact on the spherical mass must decrease,****drag motion should be an inverse exponential compared to changes in temperature****.*

4 *Background Research*

Before beginning this investigation we must first know some important facts, formulae, laws and be familiar with the concepts that are to be incorporated in this investigation.

4.1 *Air Drag/Fluid Resistance*

Air Drag/Fluid Resistance is the force acting opposite to the relative motion of any object moving in any fluid medium. Drag force is proportional to the ***square of velocity****, as we are dealing with relatively high-speeds, which can be inferred from the small Reynolds's number.*

Drag forces decrease the fluid velocity relative to the solid mass in the fluid's path.

The type of Drag in play in this system is that of an underdamped ($\zeta < 1$) oscillator with viscous drag.

The general Drag equation is mathematically defined as,

$$F_D = \frac{1}{2}\rho v^2 C_D A$$

Where F_D is the ***Air/Fluid resistance*** *between the mass and the fluid, ρ is the* ***density of the fluid****, v is the* ***speed of the object*** *relative to the fluid, C_D is* ***velocity decay constant*** *(damping constant) and A is the* ***cross sectional area****.*

Note*: The* ***velocity decay constant****, C_D for the particular case that we are investigating, that is on* ***spherical bodies*** *has a set defined value of* ***0.47****.*

4.2 *Temperature dependence on density*

We must know the fundamental relation between change in ***temperature*** *on change in* ***density****.*

We know that,

$$\rho = \frac{m}{V}$$

Where, ρ is the ***density*** *of a particular substance, m is its* ***mass*** *and V is its* ***volume****.*

Therefore, we have

$$\rho \propto \frac{1}{V}$$

Therefore, we infer that, ***density*** *is* ***inversely proportional*** *to* ***volume*** *of the substance, here the* ***fluid****.*

We have a equation for the temperature dependence on density [1]. That is,

$$\rho = \frac{\rho_0}{1 + \beta \cdot \Delta T}$$

Where ρ *is the* ***current density*** *of a particular substance,* ρ_0 *is the* ***initial density*** *of a particular substance,* β *is the* ***volumetric/cubic thermal expansion coefficient*** *and* ΔT *is the change in the temperature from the initial state.*

4.3 *Computer Simulation Software*

To ***model*** *the* ***drag force*** *on a* ***sphere in fluid flow****, we would need the aid of* ***computational technology****. Software's such MATLAB, Mathematica or some simple, eccentric computer programming language code in a emulatable script format, that would compute and yield solutions for necessary simulations that are required.*

5 *Materials Required*

- *Huge Rectangular Glass Container*
- *Spherical Masses*
- *Thermometer (Digital/Analog)*
- *Fluid -* ***Ethanol***

Note*: In theory, the fluid utilized in this research could of arbitrary any substance with fluid properties. For the purpose of this investigation, we shall specifically use* ***ethanol as the flow fluid****.*

6 *Variables*

Physical Quantity	*Symbol*
Flow velocity	v
Drag coefficient	C_D
Temperature	T
Radius of spherical mass	r

Table 1: *General physical quantities employed in this investigation*

Note*: In theory, the radius of the spherical mass and the flow velocity incorporated in research could of any arbitrary value. For the purpose of this investigation, we shall specifically use masses of radius* 5×10^{-2} *m and the flow velocity shall be set constant at 10 m/s.*

Independent Variable	*Dependent Variable*	*Controlled Variable*
Temperature	*Drag Force*	*Fluid medium*
-	-	*Radius of the spherical mass*
-	-	*Flow velocity*

Table 2: *Segregation of employed variables as IV, DV or CV*

Note*: The flow velocity, type of fluid medium and the radius of the mass is no longer variable as we have defined a set value to it.*

7 *Procedure*

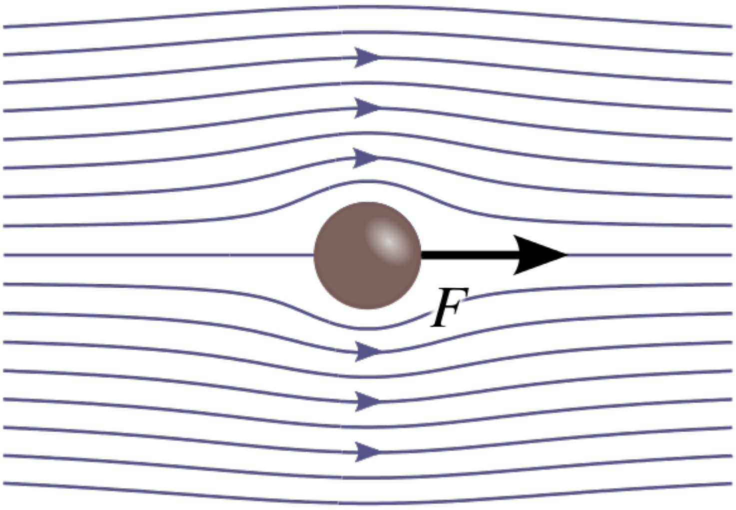


Figure 1: *Diagram of the experiment carried down in this investigation*

Using the materials specified in chapter 5, arrange the components as specified in figure 1 and make sure that the spherical mass is kept about at a fixed point.

Following which, pass through the fluid in a rectangular chamber while varying temperatures and obtain the drag force eminent on the spherical mass.

After the prior setup and during the experiment, the fluid must interact with the spherical mass in a manner depicted in figure 1.

8 *Equations*

8.1 *Equations of Motion*

We know that,

$$F_D = \frac{1}{2}\rho v^2 C_D A$$

Also,

$$\rho = \frac{\rho_0}{1 + \beta \cdot \Delta T}$$

Substituting the second equation in the first equation we have,

$$F_D = \frac{1}{2}\left(\frac{\rho_0}{1+\beta\cdot\Delta T}\right)\cdot v^2 C_D A$$

This equation can be rewritten as,

$$F_D = \frac{1}{2}\left(\frac{\rho_0}{1+\beta\cdot(T-T_0)}\right)\cdot v^2 C_D A$$

When further reducing the variables to constants the following equation reduces to,

$$F_D = \frac{1356661}{36752222}\cdot\left(\frac{789}{1+0.00109\cdot(T-298.16)}\right)\cdot 10^2$$

Note*: The initial density and temperature taken into account is 789* kg/m^3 *and* $25°C$ *or 298.16 K.*

8.2 ***Fundamental Derived Equation***

The ***fundamental derived equation*** *we shall be using in this investigation are:*

$$F_D = \frac{1356661}{36752222}\cdot\left(\frac{789}{1+0.00109\cdot(T-298.16)}\right)\cdot 100 \tag{1}$$

9 *Data*

9.1 *Simulation Data*

Observation number (x_i)	*Temperature of Fluid ($^\circ C$)*	*Drag Force (kN)*
1	20	2.92845
2	40	2.86564
3	60	2.80546
4	80	2.74776
5	100	2.69239

Table 3: *Simulation data relating to changes in drag force relative to changes in the temperature of the fluid*

9.2 *Experimental Data*

Observation number (x_i)	*Temperature of Fluid ($^{\circ}C$)*	*Drag Force (kN)*
1	20	2.92765
2	20	2.92925
3	20	2.92685
4	20	2.92765
5	20	2.92885
6	40	2.86426
7	40	2.86540
8	40	2.86626
9	40	2.86882
10	40	2.86462
11	60	2.80486
12	60	2.80548
13	60	2.80662
14	60	2.80826
15	60	2.80248
16	80	2.74770
17	80	2.74678
18	80	2.74472
19	80	2.74874
20	80	2.74272
21	100	2.69240
22	100	2.69438
23	100	2.69626
24	100	2.69822
25	100	2.70046

Table 4: *Experimental data on drag force on the object against temperature of the fluid*

9.3 *Complete Data*

Temperature of Fluid ($^\circ C$)	*Avg. Exp. Drag Force (kN)*	*Sim. Drag Force (kN)*
20	2.92805	2.92845
40	2.86587	2.86564
60	2.80554	2.80546
80	2.74613	2.74776
100	2.69634	2.69239

Table 5: *Simplified version of complete data derived from tables 4 and 3*

10 *Observations*

Let $F_{D_{Exp}}$ *be the experimental values and* $F_{D_{Sim}}$ *be the simulation values respectively for each and every case that we are investigating.*

If we define F_{D_n} *as the uncertainty in measurement in the experimental values of* F_D*, then* F_{D_n} *is mathematically defined as* $\left|F_{D_{Sim}} - F_{D_{Exp}}\right|$

By using the above definitions we have,

10.1 *Observation from Case 1 where* $T = 20^\circ C$

By using equation 1, we see that the experimental and simulation value for F_{D_1} *is 2.92805 kN and 2.92845 kN.*

Therefore uncertainty in measurement for F_{D_1} *in this case is* $\pm$*0.4 N.*

10.2 *Observation from Case 2 where* $T = 40^\circ C$

By using equation 1, we see that the experimental and simulation value for F_{D_2} *is 2.86587 kN and 2.86564 kN.*

Therefore uncertainty in measurement for F_{D_2} *in this case is* $\pm$*0.23 N.*

10.3 *Observation from Case 3 where* $T = 60^\circ C$

By using equation 1, we see that the experimental and simulation value for F_{D_3} *is 2.80554 kN and 2.80546 kN.*

Therefore uncertainty in measurement for F_{D_3} *in this case is* $\pm$*0.08 N.*

10.4 *Observation from Case 4 where $T = 80°C$*

By using equation 1, we see that the experimental and simulation value for F_{D_1} is 2.74613 kN and 2.74776 kN.

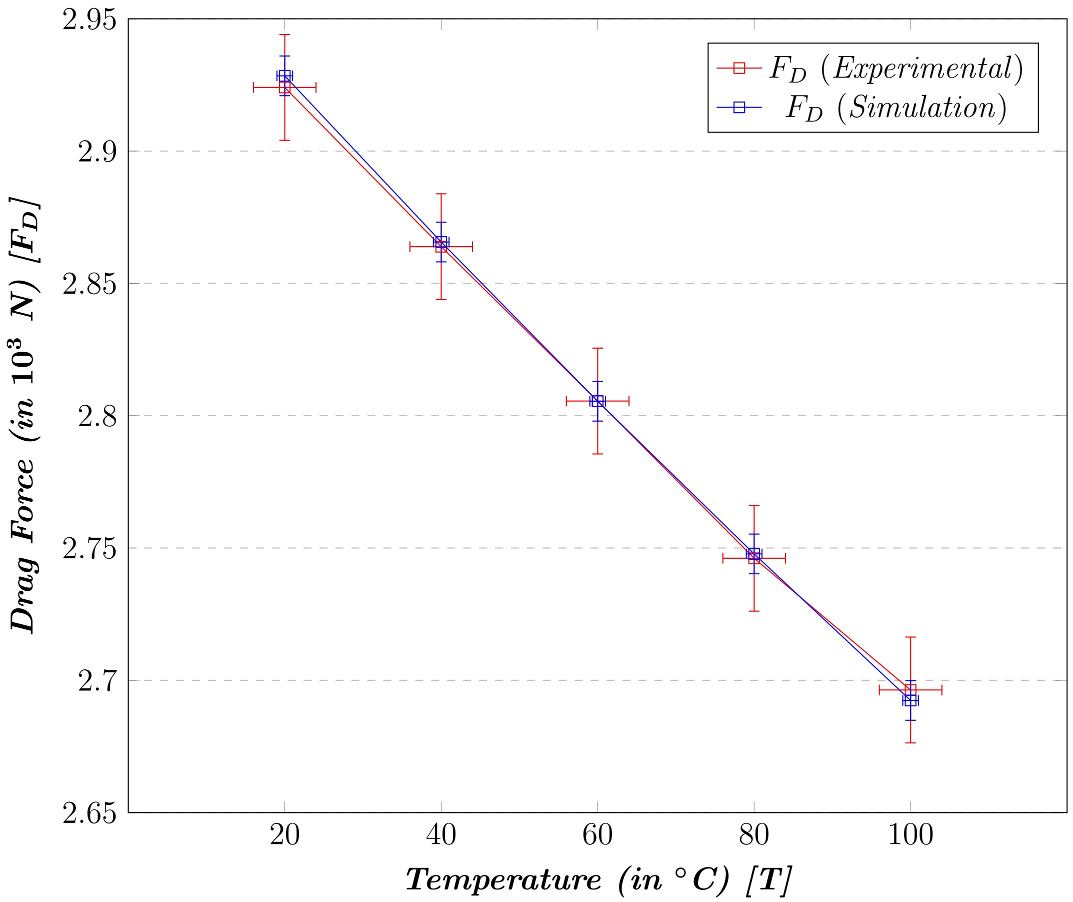
Therefore uncertainty in measurement for F_{D_1} in this case is ±1.63 N.

10.5 *Observation from Case 5 where $T = 100°C$*

By using equation 1, we see that the experimental and simulation value for F_{D_5} is 2.69634 kN and 2.69239 kN.

Therefore uncertainty in measurement for F_{D_5} in this case is ±3.95 N.

11 *Analysis*



Upon close visual observation, we see that the difference in the plotted values *of that of the* ***simulation*** *and* ***experimental values*** *of the* ***drag force*** *versus* ***temperature*** *from the* ***F-T*** *graph are very* ***minute*** *to the extent that it would be right to say and consider that the experimental values are both* ***accurate*** *and* ***precise*** *in relation to that of the* ***literature/theoretical/simulation values****.*

It is evident from studying the system ***numerically*** *and* ***graphically*** *that the system exhibits an* ***linear decay*** *with time for the parameter that is being investigated. This points out to* ***one conclusion****, that the* ***parameter of the system undergoes linear decay not exponential decay****, according to their various* ***inert energies****.*

This ***partially contradicts*** *the* ***initial hypothesis*** *laid out prior to beginning the investigation as if, the system had to model* ***similarly*** *to the inverse exponential, then the linear behavior that we observe would have not existed, but we see that this is not the case.*

It would be right to say that the ***initial hypothesis*** *that was laid out prior, beginning the experimentation was* ***partially correct*** *and is a* ***valid statement****.*

12 *Evaluation*

We further define, the average uncertainty in measurement across all cases that have been investigated to be,

$$\overline{F_{D_n}} = \frac{\sum_{n=1}^{n} F_{D_n}}{n} = \frac{F_{D_1} + F_{D_2} + F_{D_3} + F_{D_4} + F_{D_5}}{5}$$

Therefore we have,

$\overline{F_{D_n}} = 1.258N = 1.258 \times 10^{-3}kN$

Upon observation, we see that the value of $\overline{F_{D_n}}$ *we have found is not equal to 1, but is relatively very close, so we can say that we have some errors in calculating the* ***drag force versus temperature****.*

Percentage uncertainty *in measurement of* ***drag force versus temperature*** *is* $\left|1 - \overline{F_{D_n}}\right| \cdot 100\% =$ ***0.1258%*** $\approx$ ***0.13%***

13 *Limitations of Study*

There are various limitations in this study/investigation as we have placed forth, certain strict conditions that make this system so constrained and *disables us to expand our researching capability of this chaotic phenomenon. Conditions such as,*

- *Restricting the value of temperature domain from* $20°$ *Celsius to* $100°$ *Celsius*
- *Restricting the value of radius of the spherical body employed to* 5×10^{-2} *meters*
- *Restricting the the fluid type to only ethanol*

14 *Safety Measures*

The Safety Measures that were taken during the experimentation as as follows:

- *Proper Laboratory equipment was utilized to conduct and collect data*
- *All Laboratory equipment was used in the presence of Lab instructors and Lab personnel*
- *The experiment was performed at a distance from the observer so as to, limit or eliminate the chances of any possible physical harm to the observer*
- *Any and all lab equipment was thoroughly examined for any defects that could potentially lead to safety hazards, before initiating experimentation*
- *It was made sure the experiment shall not be performed to highly flammable materials that would result in combustion from the frictional force onto the fluid medium*

15 *Sources of Error*

There are various ways through which errors might have crept into our raw and ordered data, some of the possible sources of errors are:

1. *Raw experimental data presented here in the investigation report is collected through lab experimentation, and there are chances that the data collected may have slight discrepancy in it*
2. *Insignificant random human errors by the observer, ie. parallax errors*
3. *Uncertainties that cannot be minimized due to lack of highly sophisticated equipment and materials used in the experiments in this investigation*

4. *Assumptions and certain conditions put forth on the the system to model its chaotic behavior*

16 *Conclusion*

In this paper, I have shown the effects of changes in the ***temperature*** *on an* ***spherical body*** *in a fluid medium in laminar flow on the* ***drag force*** *by collecting raw data relating to the above parameters, under certain controlled conditions as so to completely study the* ***motion/dynamics of the laminar fluid flow on a spherical body****.*

I have also investigated the ***validity*** *of our* ***initial hypothesis*** *and have come to a conclusion that our initial hypothesis was an* ***partially valid statement****.*

I have also shown the possible ***uncertainties in measurement*** *of* ***drag force*** *with respect to* ***temperature****.*

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