Physics HL
Internal Assessment

How does Temperature of the fluid medium in laminar flow affect the drag force on a spherical body in linear motion?

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1 Research Question

How does Temperature of the fluid medium in laminar flow affect the drag force on a spherical body in linear motion ?

2 Introduction

This particular research question/topic of interest has appealed to me because the essential relation between the **mechanics of fluid flow**, its **complex dynamics** and the **extreme difficulty in possibly modelling** this phenomenon has prompted me to investigate this field.

3 Aim

To **gather experimental raw data** correlating the changes in **drag force** with the changes in **temperature** in a controlled lab setting with the **fluid flow** on a **spherical object**. Therefore, comparing it to **computer models** of the same, investigating its **many states**, and evaluating the value of the **inaccuracies in the precision** of the **numerically modelled computer model** to that of the **experiment conducted**, and vice versa.

4 Hypothesis

The **drag force** on the **spherical mass** would act **inversely proportional** to the the change in **temperature**, which means that with increase in temperature, the eminent drag force in impact on the spherical mass must decrease,**drag motion should be an inverse exponential compared to changes in temperature**.

5 Background Research

Before we begin our investigation, we must first understand some important facts, equations, and laws, as well as be familiar with the ideas involved.

5.1 Air Drag/Fluid Resistance

The force working in opposition to a moving object in a fluid medium is known as air drag or fluid resistance. The drag force is proportional to the **velocity squared**. This is due to the fact that we will be dealing and working with relatively high speeds, as evidenced by the small Reynold's number.

Drag forces gradually reduce the velocity of the fluid with respect to the solid mass in its path.

The general Drag equation is mathematically defined as,

$$F_D = \frac{1}{2}\rho v^2 C_D A$$

Where F_D is the **Air/Fluid resistance** between the mass and the fluid, ρ is the **fluid density**, v is the **object speed** relative to the fluid, C_D is **velocity decay constant** (damping constant) and A is the **area of cross section**.

Note: The **velocity decay constant**, C_D for the particular case that we are investigating, that is on **spherical bodies** has a set defined value of **0.47**.

5.2 Temperature dependence of thermal expansion coefficient

The thermal expansion coefficient is defined as,

$$\alpha_L = \frac{1}{L} \cdot \frac{\partial L}{\partial T}$$

Where, L is the **length measurement**, α_L is the **thermal expansion coefficient** in the dimension of the length measurement and T is the **temperature**.

Because, the length measurement that we are dealing with is Volume, the above equation reduces to,

$$\alpha_V = \frac{1}{V} \cdot \frac{dV}{dT}$$

This clearly indicates that α_V or the cubic expansion coefficient is a function dependent of temperature.

The below table encompasses the values of the cubic expansion coefficient of water ("Volumetric (Cubic) Thermal Expansion") at certain temperatures.

Temperature $°C$	Cubic thermal expansion coefficient $1/°C$
0	-0.000050
10	0.000088
20	0.000207
30	0.000303
40	0.000385
50	0.000457
60	0.000522
70	0.000582
80	0.000640
90	0.000695

5.3 Temperature dependence of density

We must know the fundamental relation between change in **temperature** on change in **density**.

We know that,

$$\rho = \frac{m}{V}$$

Where, ρ is the **density** of a particular substance, m is its **mass** and V is its **volume**.

Therefore, we have

$$\rho \propto \frac{1}{V}$$

Therefore, we infer that, **density** is **inversely proportional** to **volume** of the substance, here the **fluid**.

We have a equation for the temperature dependence on density from ("Liquids - Densities vs. Pressure and Temperature Change"). That is,

$$\rho = \frac{\rho_0}{1+\gamma \cdot \Delta T}$$

Where ρ is the **current density** of a particular substance, ρ_0 is the **initial density** of a particular substance, γ is the **volumetric/cubic thermal expansion coefficient** and ΔT is the change in the temperature from the initial state.

As $(1+\gamma \cdot \Delta T)^{-1}$ is of the form $(1+x)^{-1}$ we have,

$$(1+\gamma \cdot \Delta T)^{-1} = 1-(\gamma \cdot \Delta T)+(\gamma \cdot \Delta T)^2-(\gamma \cdot \Delta T)^3 \cdots$$

If we ignore the higher order terms of $(\gamma \cdot \Delta T)$, as they are negligibly small, we have,

$$\rho = \rho_0 (1-\gamma \cdot \Delta T)$$

Because γ is a function of T, $\gamma = \gamma_T$, therefore we have,

$$\rho = \rho_0 (1-\gamma_T \cdot \Delta T)$$

5.4 Computer Simulation Software

To model the **drag force** on a **sphere in fluid flow**, we will need the help of computer technology. Softwares such as MATLAB and Mathematica would compute and yield solutions for necessary simulations.

This study uses a **MATLAB** script to model and simulates the system.

6 Materials Required

- Huge Rectangular Glass Container
- Spherical Mass
- Thermometer (Digital)
- Fluid - **Water**

Note: In theory, the fluid utilized in this research could of arbitrary any substance with fluid properties. For the purpose of this investigation, we shall specifically use **water as the flow fluid**.

7 Variables

Physical Quantity	Symbol
Flow velocity	v
Drag coefficient	C_D
Temperature	T
Radius of spherical mass	r

Table 1: Physical quantities used in this study

Note: Theoretically, the radius and flow velocity of the spherical mass used in the study can have any arbitrary values. In this study, we use specific masses with a radius of 2.5×10^{-2} m, and we set the flow velocity constant at 0.1 m/s.

Independent Variable	Dependent Variable	Controlled Variable
Temperature	Drag Force	Fluid medium
-	-	Radius of the spherical mass
-	-	Flow velocity

Table 2: Separation of variables used for the study as IV, DV, and CV

Note: The flow velocity, type of fluid medium and the radius of the mass is no longer variable as we have defined a set value to it.

Controlled Variable	Method of Control
Fluid Medium	Only water shall be used as a fluid medium
Radius of the spherical mass	Radius of mass is set constant at 2.5×10^{-2}
Flow velocity	Flow velocity is set constant at 0.1 m/s

Table 3: Employed variables that are controlled by using a method of control

8 Procedure

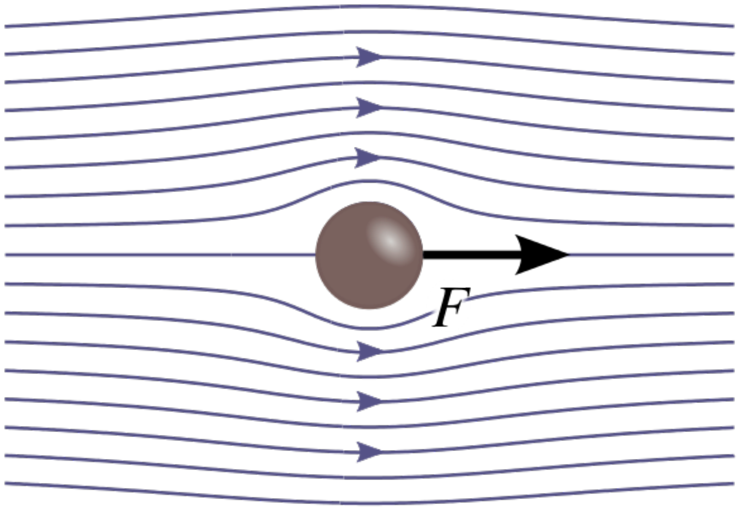


Figure 1: Diagram (Cleynen and Olivier) of the experiment conducted for this study

By utilizing the materials specified in chapter 6, arrange the materials as seen in figure 2 and make sure that the spherical mass is kept at a fixed point.

Following which, pass through the fluid in a rectangular chamber while varying temperatures and obtain the drag force eminent on the spherical mass.

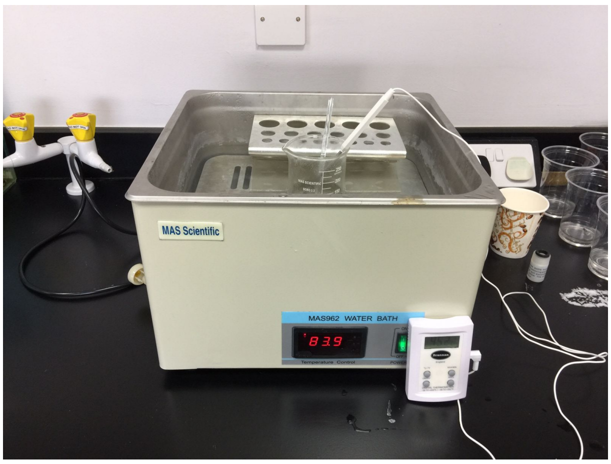


Figure 2: Experiment apparatus used to heat the fluid used for this study

After the prior setup and during the experiment, the fluid must interact with the spherical mass in a manner depicted in figure 2.

9 Equations

9.1 Drag Equation for the Study

We know that,

$$F_D = \frac{1}{2}\rho v^2 C_D A$$

Also,

$$\rho = \rho_0 (1 - \gamma \cdot \Delta T)$$

Substituting the second equation in the first equation we have,

$$F_D = \frac{1}{2}\rho_0 v^2 C_D A \cdot (1 - \gamma \cdot \Delta T)$$

This equation can be rewritten as,

$$F_D = \frac{1}{2}\rho_0 v^2 C_D A \cdot (1 - \gamma \cdot (T - T_0))$$

When further reducing the variables to constants the following equation reduces to,

$$F_D = 4.61 \cdot 10^{-3} \cdot (1 - \gamma \cdot T)$$

Note: The initial density and temperature taken into account is 999.83 kg/m^3 and $0°C$ or 271.16 K.

Note: This equation is derived considering the Celsius unit for temperature difference as the least count for both the units (Celsius and Kelvin) are the same.

9.2 Equation used for further Investigation

The **equation used for further investigation** we shall be using in this investigation is:

$$F_D = 4.61 \cdot 10^{-3} \cdot (1 - \gamma_T \cdot T) \tag{1}$$

Where γ_T is the cubic thermal expansion coefficient of water and T is the magnitude of the temperature of water.

10 Data

10.1 Experimental Data

Observation number (x_i)	Temperature of Fluid (°C)	Drag Force (10^{-3}N)
1	0	4.6100
2	10	4.6000
3	20	4.5906
4	30	4.6310
5	40	4.5270
6	50	4.5030
7	60	4.4606
8	70	4.4246
9	80	4.3666
10	90	4.3166

Table 4: Simulation data relating to changes in drag force relative to changes in the temperature of the fluid

Note: The above experimental data is the averaged mean of all data points from the data sets of the raw experimental data that can be found in the appendix A.

10.2 Simulation Data

Observation number (x_i)	Temperature of Fluid (°C)	Drag Force (10^{-3}N)
1	0	4.61
2	10	4.60
3	20	4.59
4	30	4.56
5	40	4.53
6	50	4.50
7	60	4.46
8	70	4.42
9	80	4.37
10	90	4.32

Table 5: Simulation data relating to changes in drag force relative to changes in the temperature of the fluid

10.3 Processed Data

Temperature of Fluid (°C)	Avg. Exp. Drag Force (10^{-3}N)	Sim. Drag Force (10^{-3}N)
0	4.6100	4.61
10	4.6000	4.60
20	4.5906	4.59
30	4.6310	4.56
40	4.5270	4.53
50	4.5030	4.50
60	4.4606	4.46
70	4.4246	4.42
80	4.3666	4.37
90	4.3166	4.32

Table 6: Simplified version of processed data derived from tables 4 and 5

Note: The above data is the consolidated form of all data collected, by simulation and experiment throughout all data points from the data set.

11 Evaluation

Let $F_{D_{Exp}}$ be the experimental values and $F_{D_{Sim}}$ be the simulation values correspondingly for each and every situation we are looking into.

If we define F_{D_n} as the uncertainty in measurement in the experimental values of F_D, then F_{D_n} is mathematically defined as $\left|F_{D_{Sim}} - F_{D_{Exp}}\right|$

By using the above definitions we have,

Case (n_i)	Temperature (°C)	Exp. Force (mN)	Sim. Force (mN)	F_{D_n}
1	0	4.61	4.61	$\pm$ 0
2	10	4.6	4.6	$\pm$ 0
3	20	4.631	4.59	$\pm$ 0.0006
4	30	4.5906	4.56	$\pm$ 0.071
5	40	4.527	4.53	$\pm$ 0.297
6	50	4.503	4.5	$\pm$ 0.03
7	60	4.4606	4.46	$\pm$ 0.0006
8	70	4.4246	4.42	$\pm$ 0.0046
9	80	4.3666	4.37	$\pm$ 0.0534
10	90	4.3166	4.32	$\pm$ 0.0034

Table 7: Data relating to the uncertainty (F_{D_n}) of the data collected with respect to the simulation

We further define, the average uncertainty in measurement across all cases that have been investigated to be,

$$\overline{F_{D_n}} = \frac{\sum_{n=1}^{n} F_{D_n}}{n} = \frac{F_{D_1} + F_{D_2} + F_{D_3} + F_{D_4} + F_{D_5} + F_{D_6} + F_{D_7} + F_{D_8} + F_{D_9} + F_{D_{10}}}{10}$$

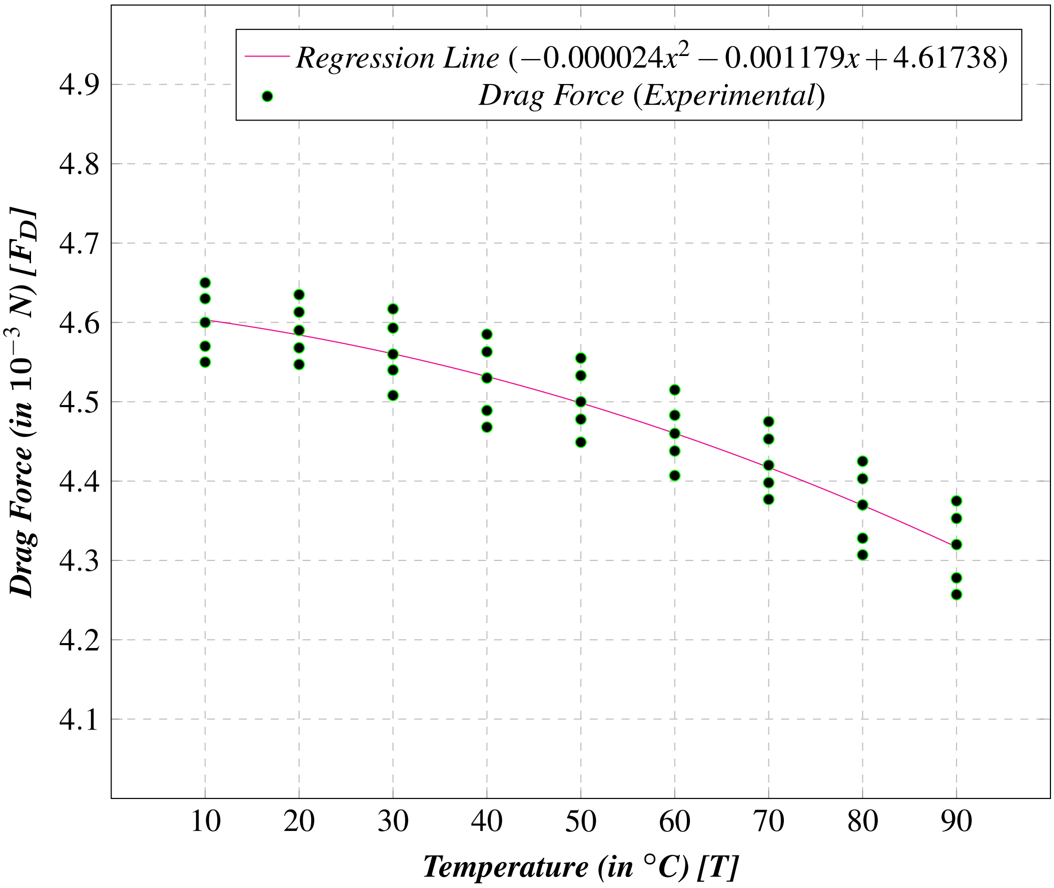
Therefore we have,

$\overline{F_{D_n}} = 0.046008mN = 4.6008 \times 10^{-6}N$

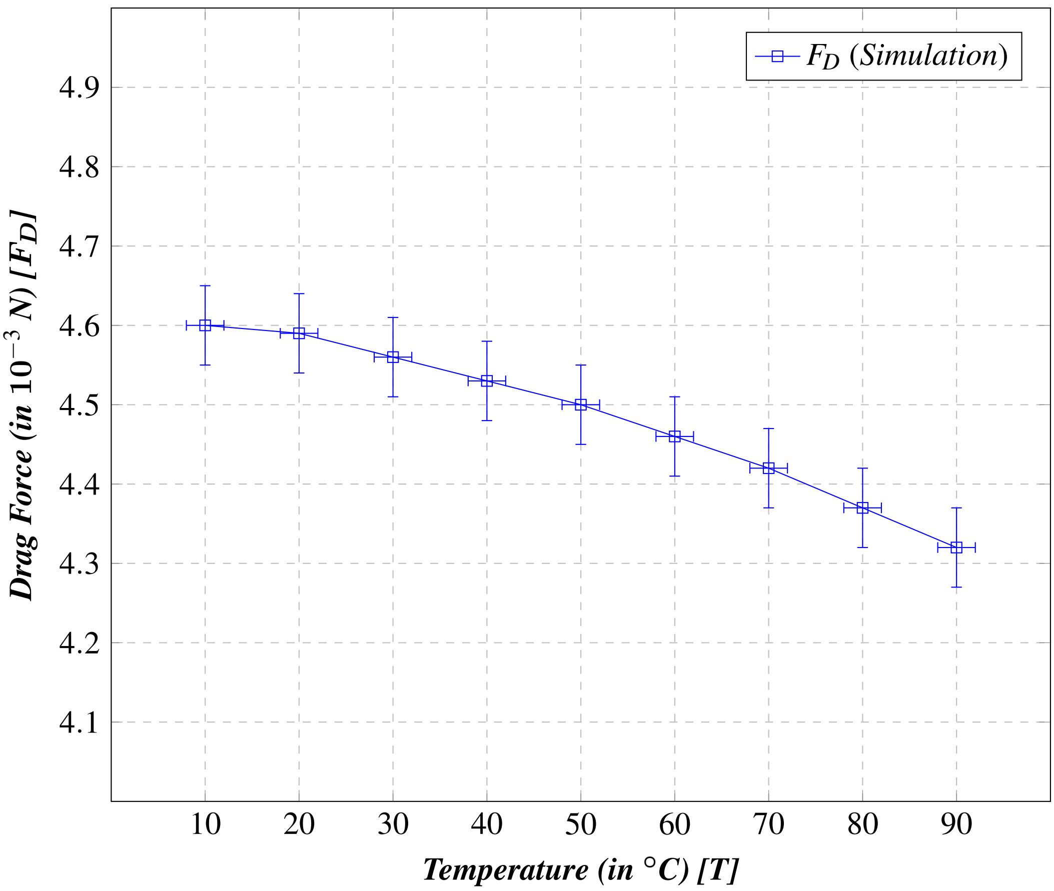
Upon observation, It is evident that the magnitude of $\overline{F_{D_n}}$ that we found is just not equivalent to 1, although it is incredibly close, indicating that there are certain flaws in our **temperature dependence on drag force** calculations.

Percentage uncertainty in measurement of **drag force versus temperature** is $\left|\frac{1-\overline{F_{D_n}}}{\overline{F_{D_n}}}\right| \cdot 100\% = \mathbf{4.6008\%} \approx \mathbf{4.60\%}$

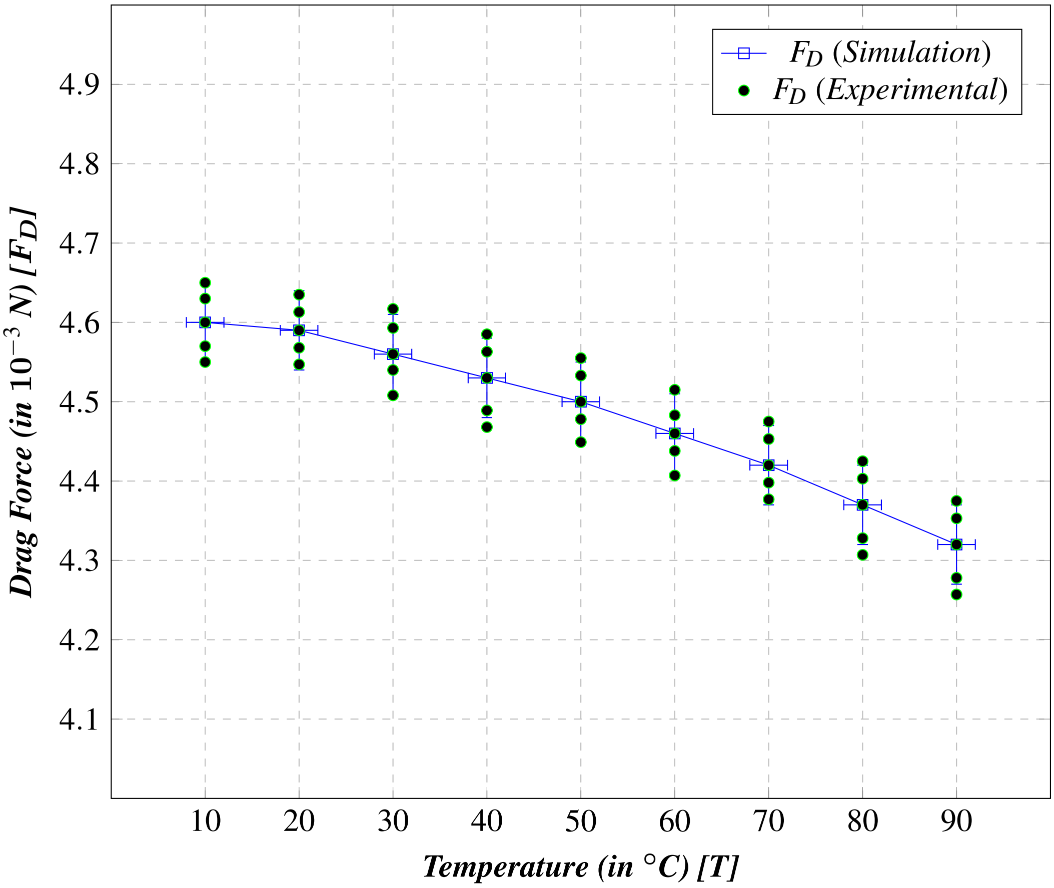
12 Analysis



The above graph is the **scatter plot** of all **data points** within the **data set** of **experimentally collected values** with the **quadratic regression** of the same. A **quadratic regression** was utilized to plot the **line of best fit**, because a regression creates an **accurate model** of the system according to the data points and is a **mathematical function of each and every data point**. This was used rather than plotting an simple line between the **range** of the **maximum and minimum** values of the data points, as it would be similar to guesswork.



The above graph is the collection of single data points at specific intervals created with accordance to a simulation, based accurately on the **mathematical model** with **zero uncertainty**. On observation it looks evidently **similar to the quadratic regression line** from the previous graph, this is an **evidence that the experimental data collected was of least uncertainty and least errors** to an extent that was possible. The above graph also contains **ranges of values** in the form of **error bars** that show that the data points can range from a particular minimum to a particular maximum.



The above graph is a **culmination** of all the **experimentally collected data points** and the **data points collected via a simulation**. The **matching similarity** of the underlying graphs shows that the data is of much **reliability** and **devoid of errors**. According to the graph from the simulation, the error bars range lower than the maximum and minimum values (for most cases) of the experimentally collected values, indicating that such values are **outliers** and indicates that there has been **some errors and uncertainties** in the collection of data.

Careful analysis reveals that the difference between the plotted readings of the **simulations** and the **experiment** for the **temperature dependence of drag force** from the **F-T** plot is so small that it is reasonable to infer that perhaps the experimentally measured readings are both accurate and precise in comparison to the literature/theoretical/simulation readings.

In order to model the system with accuracy, a **quadratic regression model** was utilized to model the **line of best-fit**.

The system exhibits a **logarithmic decay** with time with respect to the parameter that is being researched, as evidenced by the **numerical** and **graphical** analysis of the system. This leads to **one conclusion**, that depending on their various **inert energies**, the **parameter of the system exhibit logarithmic decay rather than linear decay**.

Note: A **quadratic regression model** has been used rather than a **logarithmic regression model** due to resource constraints, although the phase of the model we'll be examining exhibits logarithmic decay with respect to temperature and is **quite identical** with the quadratic regression model when limited to the 10 $^\circ$ C to 90 $^\circ$ C range.

These findings **contradict** the **initial hypothesis**, which said that if the system had to model in accordance with the **inverse exponential function**, the **logarithmic phenomenon we have found wouldn't have existed**.

It would be right to say that the **initial hypothesis** that was laid out prior, beginning the experimentation was **incorrect** and is **not** a valid statement.

13 Limitations of Study

As we have outlined before, this study/investigation has some limitations and some strict conditions that limit this system and prevents us from increasing our research capabilities on this phenomenon. Conditions like,

- Restriction on the value of the temperature domain from 10°Celsius to 90°Celsius
- Restriction on the value of radius of the spherical body employed to 2.5×10^{-2} meters
- Restricting the the fluid used to only water

14 Safety Measures

The following are the safety precautions that were implemented during the experiment:

- It was decided that the investigation experiment would not be conducted upon highly combustible materials, as this might result in combustion, caused by frictional forces acting on the fluid medium
- It was under the supervision of lab teachers and staff that the laboratory equipment was utilized
- Appropriate laboratory equipment has been used to perform and collect data

- Before beginning the experiment, all laboratory apparatus was rigorously evaluated for any faults that could result in a safety problem.
- The experiment was conducted out such that the observer was at a distance in order to reduce or eliminate the possibility of physical injury to the observer.

15 Sources of Error

Uncertainties & Errors can enter our raw data and processed data in various of ways, Some of the sources of uncertainties & errors are:

1. The unprocessed experimental data that is shared in this research report was collected by laboratory experiments, and the collected data may show slight variations
2. Assumptions and specific conditions imposed on the system to model the behavior of the system
3. Owing to the unavailability of highly advanced tools and materials employed within the experiments in this study, there are uncertainties & errors that cannot be minimized.
4. Random human errors by made by the observer, ie. parallax errors

16 Conclusion

In this investigation, I showed the effect of **temperature** change on a **sphere/spherical body** in a fluid medium in laminar flow on the **drag force** by collecting raw data related to the above parameters under specific controlled conditions to study the **laminar fluid flow on the sphere/spherical body** thoroughly.

I also examined the **validity** of our first hypothesis and concluded that our first hypothesis was a **partially valid statement**.

I also showed possible **uncertainties in drag force measurement's** with respect to **temperature**.

Works Cited

[1] "Density of Liquids vs. Pressure and Temperature Change." *Engineering Tool Box*, Engineering Tool Box, `www.engineeringtoolbox.com/fluid-density-temperature-pressure-d_309.html`. Accessed 10 Oct. 2021.

[2] Cleynen, Olivier. "Streamlines Past a Sphere at Very Low Reynolds Numbers (Stokes or Creeping Flow)." *Wikimedia.Org*, Wikimedia, May 2014, `commons.wikimedia.org/wiki/File:Flow_patterns_around_a_sphere_at_very_low_Reynolds_numbers.svg`.

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Appendices

A Raw Experimental Data

Temperature of Fluid (°C) ± 0.2	Trials (x_i)	Drag Force (mN) ± 0.0005
10	1	4.595
	2	4.598
	3	4.6
	4	4.602
	5	4.605
20	1	4.5856
	2	4.5886
	3	4.5906
	4	4.5926
	5	4.5956
30	1	4.305
	2	4.308
	3	4.31
	4	4.312
	5	4.315

Table 8: *Raw Experimental data on the drag force from the temperature range of 10°C to 30°C*

Temperature of Fluid (°C) ±0.2	Trials (x_i)	Drag Force (mN) ±0.0005
40	1	4.522
	2	4.525
	3	4.527
	4	4.529
	5	4.532
50	1	4.498
	2	4.501
	3	4.503
	4	4.505
	5	4.508
60	1	4.455
	2	4.458
	3	4.46
	4	4.462
	5	4.465

Table 9: *Raw Experimental data on the drag force from the temperature range of 40°C to 60°C*

Temperature of Fluid (°C) ±0.2	Trials (x_i)	Drag Force (mN) ±0.0005
70	1	4.4196
	2	4.4226
	3	4.4246
	4	4.4266
	5	4.4296
80	1	4.3616
	2	4.3646
	3	4.3666
	4	4.3686
	5	4.3716
90	1	4.3116
	2	4.3146
	3	4.3166
	4	4.3186
	5	4.3216

Table 10: *Raw Experimental data on the drag force from the temperature range of 70°C to 90°C*