Mathematics IA Ver 1.01

Laplace Transform & Fourier Series on a System of Ordinary Differential Equations

How does the Laplace Transform and the Fourier Series makes it easier to compute the solutions of complex functions?

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Abstract

We shall discuss the ***Laplace Transform*** *extensively and the effects it has on functions when its is applied on them, and also on how an fundamental intuitive mathematical idea of* ***superposition*** *gave rise to one of the most important concepts and fields of study in* ***calculus****,* ***number theory****,* ***combinatorics*** *and how* ***information*** *is* ***transferred*** *and* ***processed*** *globally. We shall also discuss the* ***optimal methods*** *through which the transform can be applied to yield the most* ***accurate solutions*** *of a function.*

1 *Introduction*

I have chosen this research question because the fundamental relation between the ***Laplace transform*** *and its effects on the the* ***applied function*** *and its excruciating difficulty in possibly modeling this behavior has led me to explore this field, as this mathematical concept is exciting, unique and wonderful, with ample scope to study and analyze.*

2 *Aim*

To ***collect experimental raw data*** *relating to the variation in* ***radial displacement****, the variation in* ***angular displacement****, the variation in* ***angular frequency*** *and the variation in* ***absolute frequency*** *in a lab controlled setting of a* ***damped elastic harmonic oscillator*** *and to compare it with* ***computer simulations*** *of the same and hence study its* ***different states*** *and further to study its* ***sensitivity*** *to* ***initial conditions*** *and calculate the value the* ***inconsistencies*** *in* ***accuracy*** *of the* ***mathematically modeled***

computer simulation *to that of the* ***actual experiment*** *and vice-versa.*

3 *Background Research*

3.1 *Generalized form of a Differential Equation*

An Ordinary Differential Equation in one variable is generally mathematically defined as,

$$a_n\left(\overset{n}{\dot{y}}\right)^p + a_{n-1}\left(\overset{n-1}{\dot{y}}\right)^q + a_{n-2}\left(\overset{n-2}{\dot{y}}\right)^r + \cdots + a_2\left(\ddot{y}\right)^e + a_1\left(\dot{y}\right)^f + a_0\left(y\right)^g = c$$

Where y is a function of a base variable, ie. x and $\overset{n}{\dot{y}}$ is the nth derivative of y, the principal function.

Or in alternate matrix representation, this can be expressed as,

$$\begin{bmatrix} a_n & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & a_{n-1} & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & a_{n-2} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_0 \end{bmatrix} \begin{bmatrix} \left(\overset{n}{\dot{y}}\right)^p \\ \left(\overset{n-1}{\dot{y}}\right)^q \\ \left(\overset{n-2}{\dot{y}}\right)^r \\ \vdots \\ \left(\ddot{y}\right)^e \\ \left(\dot{y}\right)^f \\ \left(y\right)^g \end{bmatrix} = c$$

Where n is the order of the differential equation, a_n, a_{n-1}, $a_{n-2}\cdots$ are some arbitrary constants known as the coefficients of the differential equation and

p, q, $r \cdots$ are also some arbitrary constants, of which the highest constant in value is the degree of the differential equation. It is these constants/functions differentiate differential equation with other forms of the differential equations. c is a fixed constant in the equation, usually zero. In theory these constants can be any function or a constant, but for every type of function that takes place, the values of p, q , r , etc. create some additional segregation and constraints in solving the differential equation.

3.2 *Types of Ordinary Differential Equations*

There are various types of Ordinary Differential Equations based on the specific values/functions that replace the the values of the coefficients of the differential equation, order and degree of the equation.

3.2.1 *Linear Ordinary Differential Equations*

Linear Ordinary Differential Equations are differential equations where, the degree/exponent of the differential equation is 1 and the coefficients of the differential equation are some arbitrary differential functions in the base variable.

3.2.2 *Non-Linear Ordinary Differential Equations*

An Ordinary Differential equation is said to be non-linear when the degree/exponent of the differential equation is anything but 1 and the co-efficient of the the differential equation is a non-differentiable function of in the principal function, ie. $\sin y$ *as* a_k *or* $\begin{pmatrix} k \\ \dot{y} \end{pmatrix}^2$ *for some value k.*

3.2.3 *Homogeneous Ordinary Differential Equations*

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3.2.4 *Heterogeneous Ordinary Differential Equations*

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3.3 *The Laplace Transform*

The Laplace Transform is a integral transform that transforms a function fro the time domain to the complex frequency-domain. The transform is mathematically defined as,

$$\mathcal{L}\{f\}(s) = F(s) = \int_0^\infty f(t)e^{-st}dt = \int_0^\infty f(t)e^{-\sigma t}e^{-iwt}dt$$

Where $s = \sigma + iw$. *Therefore the transform is defined as,*

$$\mathcal{L}\{f\}(s) = F(s) = \int_0^\infty f(t)e^{-\sigma t}e^{-iwt}dt$$

The Laplace Transform is a linear operator, therefore it is only applicable and can only be applied to linear polynomials and functions, which implies that this transform can be used to compute the solutions of differential equations of any order, provided that they are linear.

3.4 *The Fourier Transform*

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3.5 *The Fourier Series*

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4 *General Methods to solve a Differential Equations*

4.1 *General Analytically Methods*

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4.2 *General Numerical Methods*

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5 *Effects of the transforms & Series on various functions*

5.1 *Sine & Cosine functions*

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5.2 *Exponential functions*

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5.3 ****

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6 *Limitations of Study*

There are various limitations in this study/investigation as we have placed forth, certain strict initial conditions and other conditions that make this system so constrained and disables us to expand our researching capability of this chaotic phenomenon. Conditions such as,

- *Restricting the value of* $\theta(0)$ *to* $\pi/2$
- *Restricting the value of* $x(0)$ *to* 0
- *Restricting the value of* l_0 *to 1 m*
- *Restricting the value of the time domain from 0 to 20 seconds*
- *Restricting the value of the radius of the mass employed to* 5×10^{-2} *m*
- *Restricting the value of the spring stiffness constant to 100 N/m*

7 *Conclusion*

In this paper, I have shown the effects of changes in the ***mass*** *in an* ***elastic pendulum*** *oscillating in a fluid medium (Air(Damped motion)) on the* ***radial displacement****,* ***angular displacement****,* ***angular frequency*** *and* ***absolute frequency*** *by collecting raw data relating to the above parameters, under certain controlled conditions as so to completely study the motion/dynamics of the elastic pendula motion. I have also shown the possible* ***uncertainties in measurement*** *of the* ***radial displacement****,* ***angular displacement****,* ***angular frequency*** *and* ***absolute frequency****.*

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