Mathematics IA Ver 1.01

The Mathematics of Signal processing

What is the relationship between the Laplace transform, the Fourier transform and the Fourier series of a function?

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Contents

Abstract

We shall discuss the ***Laplace Transform*** *extensively and the effects it has on functions when its is applied on them, and also on how an fundamental intuitive mathematical idea of* ***superposition*** *gave rise to one of the most important concepts and fields of study in* ***calculus****,* ***number theory****,* ***combinatorics*** *and how* ***information*** *is* ***transferred*** *and* ***processed*** *globally. We shall also discuss the* ***optimal methods*** *through which the transform can be applied to yield the most* ***accurate solutions*** *and* ***values*** *of a function.*

1 *Introduction*

*I have chosen this research question because I intend to study **Electrical Engineering** and further study **Mechatronics**. The fundamental concepts of the **Laplace transform**, **Fourier transform** and the **Fourier series** form the **backbone** of a subfield in Electrical Engineering, that is "**Signal processing**", without which any sort of **communication**, through any **electrical device** would not be possible, which implies that **modern day computers** and **computational technology** that exists today would also **not be existent**.*

*The very idea that all of computational technology depends on the concepts laid out **150** to **200 years ago**, deeply interests me, and thus has led me to **study the complex relationships** within **various mathematical concepts**.*

2 *Aim*

*To **collect experimental raw data** relating to the variation in **radial displacement**, the variation in **angular displacement**, the variation in **angular frequency** and the variation in **absolute frequency** in a lab controlled setting of a **damped elastic harmonic oscillator** and to compare it with **computer simulations** of the same and hence study its **different states** and further to study its **sensitivity** to **initial conditions** and calculate the value the **inconsistencies** in **accuracy** of the **mathematically modeled***

computer simulation *to that of the* ***actual experiment*** *and vice-versa.*

3 *Background Research*

3.1 *Types of Domains*

3.1.1 *Time Domain (Real Domain)*

The time domain, or the real domain is the domain in which we classically deal with functions, usually the x plane. In practical applications, every physical quantity is measured relatively against time, therefore the name as, the time domain.

3.1.2 *Frequency Domain (Complex Domain)*

The frequency domain, or the complex domain is the domain which is usually expanded mathematically from the real domain, with the aid of imaginary numbers and complex analysis, usually dealt in the z plane.

In practical applications when a function or otherwise known as a signal is transformed mathematically, it aids the observer to study with the concept of super-positioning to understand how much of each signal as a part summed up with other parts make up the original function.

In terms of physical quantities, this is done by studying the frequencies of each counterpart of a signal, thus its name as, the frequency domain.

3.2 *The Laplace Transform*

The Laplace Transform is a integral transform that transforms a function fro the time domain to the complex frequency-domain. The transform is mathematically defined as,

$$\mathcal{L}\{f\}(s) = F(s) = \int_0^\infty f(t)e^{-st}dt = \int_0^\infty f(t)e^{-\sigma t}e^{-iwt}dt$$

Where $s = \sigma + iw$. Therefore the transform is defined as,

$$\mathcal{L}\{f\}(s) = F(s) = \int_0^\infty f(t)e^{-\sigma t}e^{-iwt}dt$$

The Laplace Transform is a linear operator, therefore it is only applicable and can only be applied to linear polynomials and functions, which implies that this transform can be used to compute the solutions of differential equations of any order, provided that they are linear.

3.3 *The Fourier Transform*

The Fourier Transform is also an integral transform that transforms a function from the time domain to the complex frequency-domain. The Fourier transform differs from the Laplace transform because the Fourier transform is a slice of the Laplace transform, a component of the Laplace transform. The transform represents for one single value of the Laplace transform.

The transform is mathematically defined as,

$$\mathcal{F}\{f\}(s) = F(s) = \int_0^\infty f(t)e^{-iwt}dt$$

The Fourier Transform is also a linear operator, therefore it is only applicable and can only be applied to linear polynomials and functions, which implies that this transform can be used to compute the solutions of differential equations of any order, provided that they are linear.

3.4 *The Fourier Series*

The Fourier Series is a infinite summation representation of sinusoidal functions of a function. Mathematically,

$$f(x) = \sum_{n=0}^{\infty} (a_n \cdot \cos(nx) + b_n \cdot \sin(nx))$$

Alternatively,

$$f(x) = \sum_{n=0}^{\infty} a_n \cdot \cos(nx) + \sum_{n=0}^{\infty} b_n \cdot \sin(nx)$$

Or,

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cdot \cos(nx) + \sum_{n=1}^{\infty} b_n \cdot \sin(nx)$$

Where,

$$a_n = \frac{2}{P} \cdot \int_P s(x) \cdot \cos\left(\frac{2\pi}{P} \cdot nx\right) dx$$

$$b_n = \frac{2}{P} \cdot \int_P s(x) \cdot \sin\left(\frac{2\pi}{P} \cdot nx\right) dx$$

4 *Effects of the transforms & Series on various functions*

4.1 *Sine & Cosine functions*

a

4.2 *Exponential functions*

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4.3 ****

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5 *Limitations of Study*

There are various limitations in this study/investigation as we have placed forth, certain strict initial conditions and other conditions that make this system so constrained and disables us to expand our researching capability of this chaotic phenomenon. Conditions such as,

- *Restricting the value of* $\theta(0)$ *to* $\pi/2$
- *Restricting the value of* $x(0)$ *to* 0
- *Restricting the value of* l_0 *to 1 m*
- *Restricting the value of the time domain from 0 to 20 seconds*
- *Restricting the value of the radius of the mass employed to* 5×10^{-2} *m*
- *Restricting the value of the spring stiffness constant to 100 N/m*

6 *Conclusion*

In this paper, I have shown the effects of changes in the ***mass*** *in an* ***elastic pendulum*** *oscillating in a fluid medium (Air(Damped motion)) on the* ***radial displacement****,* ***angular displacement****,* ***angular frequency*** *and* ***absolute frequency*** *by collecting raw data relating to the above parameters, under certain controlled conditions as so to completely study the motion/dynamics* *of the elastic pendula motion. I have also shown the possible* ***uncertainties in measurement*** *of the* ***radial displacement****,* ***angular displacement****,* ***angular frequency*** *and* ***absolute frequency****.*

Bibliography

[1] "Density of Liquids vs. Pressure and Temperature Change." *Engineering Tool Box*, Engineering Tool Box, www.engineeringtoolbox.com/fluid-density-temperature-pressure-d_309.html. Accessed 10 Oct. 2021.

[2] Cleynen, Olivier. "Streamlines Past a Sphere at Very Low Reynolds Numbers (Stokes or Creeping Flow)." *Wikimedia.Org*, Wikimedia, May 2014, commons.wikimedia.org/wiki/File:Flow_patterns_around_a_sphere_at_very_low_Reynolds_numbers.svg.

[3] John Deyst, and Jonathan How. *16.61 Aerospace Dynamics*. Spring 2003. Massachusetts Institute of Technology: MIT OpenCourseWare, https://ocw.mit.edu. License: Creative Commons BY-NC-SA.

[4] Bellomo, Nicola, et al. "Mechanics and Dynamical Systems With Mathematica®." *Modeling and Simulation in Science, Engineering and Technology*, 2000, pp. 341–408. *Crossref*, doi:10.1007/978-1-4612-1338-3.

[5] "1. History of Dynamics; Motion in Moving Reference Frames." *YouTube*, uploaded by MIT OpenCourseWare, 3 Sept. 2013, www.youtube.com/watch?v=GUvoVvXwoOQ.

[6] "Classical Mechanics — Lecture 3." *YouTube*, uploaded by Stanford, 16 Dec. 2011, www.youtube.com/watch?v=3apIZCpmdls.

[7] "Simple Pendulum with Air Resistance — Classical Mechanics — LetThereBeMath —." *YouTube*, uploaded by LetThereBeMath, 25 Oct. 2017, www.youtube.com/watch?v=erveOJD_qv4.

[8] Simionescu, P. *Computer-Aided Graphing and Simulation Tools for AutoCAD Users*. Abingdon, United Kingdom, Taylor Francis, 2014.

[9] Kreyszig, Erwin. *Advanced Engineering Mathematics*. Hoboken, NJ, United States, Wiley, 1972.

[10] Hayek, Sabih I. "Mechanical Vibration and Damping." *Digital Encyclopedia of Applied Physics*, 2003. *Crossref*, doi:10.1002/3527600434.eap231.

[11] Fowles, Grant R., and George L. Cassiday. *Analytic Mechanics*. 5th ed., Fort Worth, Saunders College, 1986.

[12] Serway, Raymond, and John Jewett. *Physics for Scientists and Engineers*. Thomson-Brooks/Cole, 2004.

[13] Tipler, Paul. *Physics for Scientists and Engineers*. W. H. Freeman, 1998.

[14] Wylie, Clarence Raymond. *Advanced Engineering Mathematics*. 4th ed., McGraw-Hill, 1975.

[15] Rowell, Derek. "State-Space Representation of LTI Systems." *2.14 Analysis and Design of Feedback Control Systems*, D. Rowell, 2002, pp. 1–18.

[16] Goldstein, Herbert. *Classical Mechanics (Addison-Wesley Series in Physics*. 2nd ed., Addison-Wesley, 1980.

[17] J. Vandiver, and David Gossard. 2.003SC Engineering Dynamics. Fall 2011. Massachusetts Institute of Technology: MIT OpenCourseWare, `https://ocw.mit.edu`. License: Creative Commons BY-NC-SA.