Financial Markets

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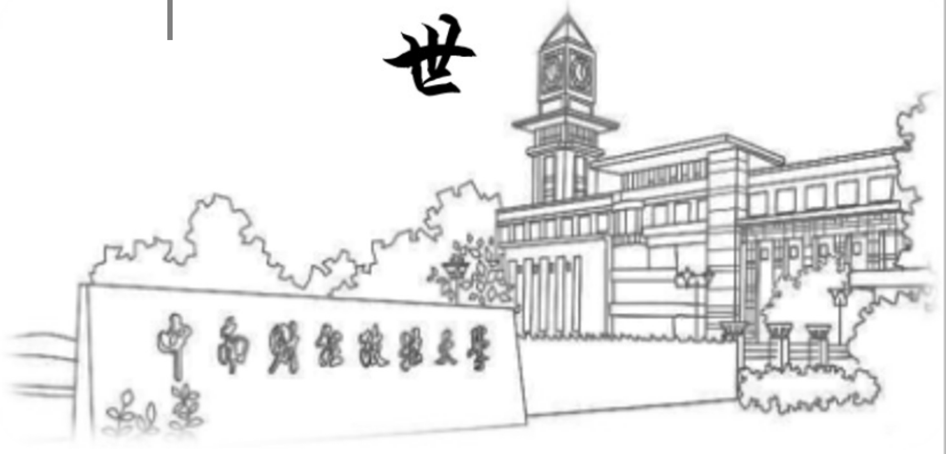


Chapter 4: The Meaning of Interest Rates

Chapter Roadmap

- Calculate the present value of future cash flows and the yield to maturity on the four types of credit market instruments.
- Recognize the distinctions among yield to maturity, current yield, rate of return, and rate of capital gain.
- Interpret the distinction between real and nominal interest rates.
- Calculate the duration of a security.

Measuring Interest Rates

- Different debt instruments have very different streams of cash payments (known as **cash flows**) and different timing to the holder.
- To evaluate different debt instruments, we need to introduce the concept of **present value**.
- The concept of present value is based on the commonsense that a dollar paid to you one year from now is less valuable than a dollar paid to you today.
  - Why: a dollar deposited today can earn interest and become $1 x (1+i) one year from today.

Present Value

Suppose you borrowed \$100 from your friend and you will pay her back one year later the principal of \$100 plus interest \$10.

In the case of a simple loan like this one, the interest payment divided by the amount of the loan is a natural and sensible way to measure the interest rate, we call this **simple interest rate**:

$$i = \frac{\$10}{\$100} = 0.1 = 10\%$$

Suppose you lend out the \$110 at the same interest rate, at the end of the send year you would have:

$$\$110 \times (1 + 0.1) = 121$$

For the third year, you will have:

$$\$121 \times (1 + 0.1) = 133 = \$100 \times (1 + 0.1)^3$$

Hence, if you can get \$133 three years late, how much will you value this \$133 today?

$$\frac{\$133}{(1 + 0.1)^3} = \$100$$

This is how we get the present value of \$133.

Simple Present Value

Let us generalize what we saw in the last slide.

The formula of calculating the present value of dollars received in the future is:

$$PV = \frac{CF}{(1+i)^n}$$

PV: present value

CF: cash flow

n: number of years

How much is that lottery worth?

Assume that you just won a $20 million lottery, which promises you a payment of $1 million every year for the next 20 years. Have you really won $20 million? How much did you really win?

$$\frac{\$1\ million}{(1+0.1)} = \$909{,}090$$

$$\frac{\$1\ million}{(1+0.1)^2} = \$826{,}446$$

$$\frac{\$1\ million}{(1+0.1)} + \frac{\$1\ million}{(1+0.1)^2} + \ldots + \frac{\$1\ million}{(1+0.1)^{20}} = \$9.4\ million$$

Quiz

1. If the interest rate is 10%, what is the present value of a security that pays you $1,100 next year?
2. If a security will pay you back $200 after two years and the present value of this security is $100, what is the interest rate?
3. If an investment will pay you $100 in the following 2 consecutive years? Can you calculate the present value of this investment given the interest rate is 10%?

Simple Present Value

Cannot directly compare payments scheduled in different points in the time line

	\$100	\$100	\$100	\$100
Year	0	1	2	n
PV	100	$100/(1+i)$	$100/(1+i)^2$	$100/(1+i)^n$

Four Types of Credit Market Instruments

- Simple Loan
- Fixed Payment Loan
- Coupon Bond
- Discount Bond

Simple Loan

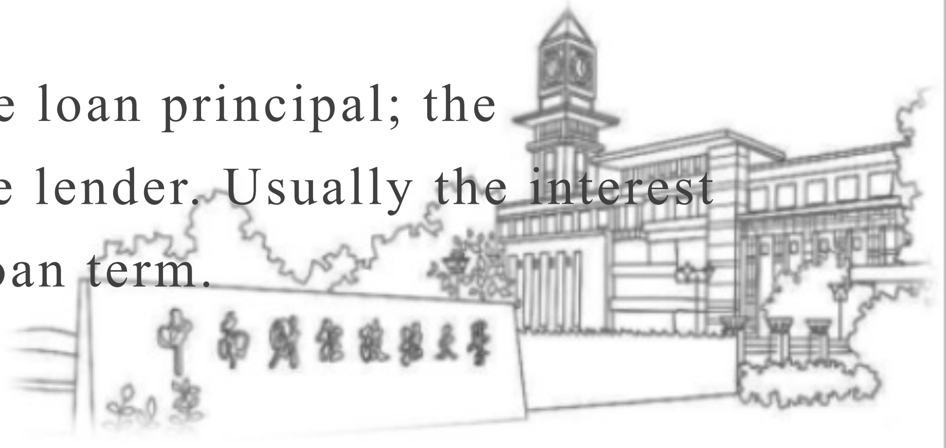
In a simple loan, the lender provides the borrower with an amount of funds that must be repaid to the lender at the maturity date, along with an additional payment for the interest.

Loan Principal: the amount of funds the lender provides to the borrower.

Maturity Date: the date the loan must be repaid; the **Loan Term** is from initiation to maturity date.

Interest Payment: the cash amount that the borrower must pay the lender for the use of the loan principal.

Simple Interest Rate: the interest payment divided by the loan principal; the percentage of principal that must be paid as interest to the lender. Usually the interest rate is expressed on an annual basis, irrespective of the loan term.



Fixed-payment Loan

A **fixed-payment loan** is also called a **fully amortized loan**

Lender provides the borrower with an amount of funds that the borrower must repay by making the same payment, consisting of part of the principal and interest, every period for a set number of years.

For example, if you borrow $1000, a fixed-payment loan might require you to pay $126 every year for 25 years.

Coupon Bond

A **coupon bond** pays the owner of the bond a fixed interest payment (**coupon payment**) every year until the maturity date, when a specified final amount (**face value** or **par value**) is repaid.

For example, a coupon bond with $1000 face value might pay you a coupon payment of $100 per year for 10 years, and then repay you the face value amount of $1000 at the maturity date.

Discount Bond

A **discount bond** is also called a **zero-coupon** bond

A discount bond is bought at a price below its face value (at a discount), and the face value is repaid at the maturity date.

For example, a one-year discount bond with a face value of $1000 might be bought for $900; in a year's time, the owner would be repaid the face value of $1000.

Yield to Maturity

How should we calculate interest rate? Or what is interest rate?

Yield to maturity: the interest rate that equates the present value of cash flow payments received from a debt instrument with its value today

Now we look at how yield to maturity is calculated for the four types of credit market instruments.

Yield to Maturity on a Simple Loan

For simple loans, the **simple interest rate** equals the **yield to maturity**.

The yield to maturity in a simple loan is:

$$PV = \frac{CF}{(1+i)^n}$$

where

$$PV = amount\ borrowed$$
$$CF = cash\ flow\ in\ each\ year$$
$$n = number\ of\ years$$
$$i = yield\ to\ maturity\ or\ simple\ interest\ rate$$

Yield to Maturity on a Fixed-Payment Loan

A fixed-payment loan has the same cash flow payment every period throughout the life of the loan.

The yield to maturity in a fixed-payment loan is:

$$LV = \frac{FP}{1+i} + \frac{FP}{(1+i)^2} + \cdots + \frac{FP}{(1+i)^n}$$

Where

$$LV = loan\ value$$
$$FP = fixed\ yearly\ payment$$
$$n = number\ of\ years\ until\ maturity$$
$$i = yield\ to\ maturity$$

Fixed-Payment Loan Example

You decide to purchase a new home and need a $100,000 mortgage. You take out a loan from the bank that has an interest rate of 7%. What is the yearly payment to the bank if you wish to pay off the loan in twenty years?

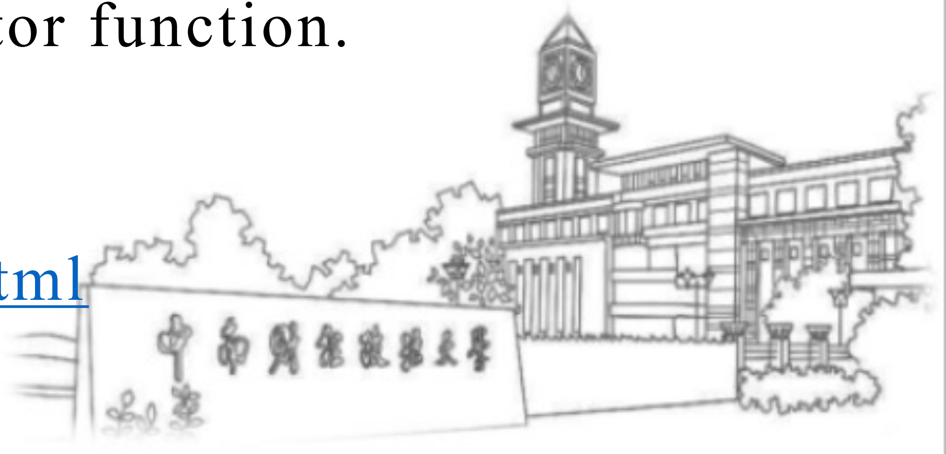
$$\$100{,}000 = \frac{FP}{1+0.07} + \frac{FP}{(1+0.07)^2} + \cdots + \frac{FP}{(1+0.07)^{20}}$$

Then we need to calculate FP. How?

A lot of software has the payment (PMT) calculator function.

Python, numpy-financial package:

https://numpy.org/numpy-financial/latest/index.html



Yield to Maturity on a Coupon Bond

The present value of a coupon bond = the sum of the present values of all the coupon payments + the present value of the final payment of the face value of the bond

The yield to maturity in a coupon bond is:

$$P = \frac{C}{1+i} + \frac{\mathrm{C}}{(1+\mathrm{i})^2} + \cdots + \frac{\mathrm{C}}{(1+\mathrm{i})^{\mathrm{n}}} + \frac{\mathrm{F}}{(1+\mathrm{i})^{\mathrm{n}}}$$

$$P = price\ of\ the\ coupon\ bond$$

$$C = yearly\ coupon\ payment$$

$$F = face\ value\ of\ the\ bond$$

$$n = years\ to\ maturity\ date$$

$$i = yield\ to\ maturity$$

Coupon Bond Example

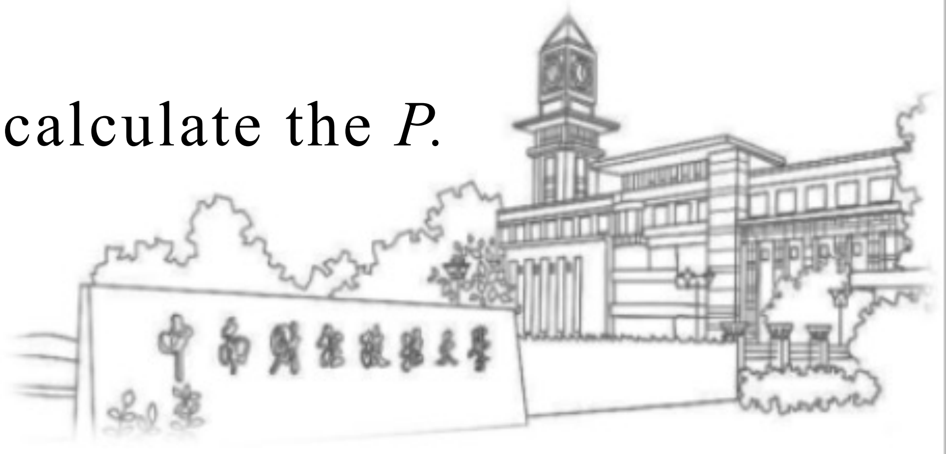
Find the price of a 10% coupon bond with a face value of $1000, a 12.25% yield to maturity and eight year to maturity.

The coupon rate is 10%, so the coupon payment is $1000 * 10% = $100

Hence, we want to calculate:

$$P = \frac{\$100}{1+0.1225} + \frac{\$100}{(1+0.1225)^2} + \cdots + \frac{\$100}{(1+0.1225)^8} + \frac{1000}{(1+0.1225)^8}$$

Still, we can use the numpy-financial package to calculate the P.



Coupon Bond

We can use python to calculate a series of yield to maturity and price of bond (see Table 1)

- When the coupon bond is priced at its face value, the yield to maturity equals the coupon rate.
- The price of a coupon bond and the yield to maturity are negatively related.
- The yield to maturity is greater than the coupon rate when the bond price is below its face value.

TABLE 1 Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = $1,000)

Price of Bond ($)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

Yield to Maturity on a Perpetuity

Perpetuity or **Consol**: a perpetual coupon bond with no maturity and on repayment of principal that makes fixed coupon payment of $C forever.

The price of a perpetuity is easy to calculate (why?):

$$P_c = \frac{C}{i_c}$$

Where

$$P_c = price\ of\ the\ perpetuity$$
$$C = yearly\ payment$$
$$i_c = yield\ ot\ maturity$$

Yield to Maturity on a Perpetuity

Also it is easy to calculate the yield to maturity of a perpetuity.

$$P_c = \frac{C}{i_c} \Rightarrow i_c = \frac{C}{P_c}$$

i_c here is very close to the yield to maturity for any long-term bond. For this reason, i_c has been given the name **current yield** and is frequently used as an approximation to describe interest rate (yields to maturity) on long-term bonds.

Yield to Maturity on a Discount Bond

The yield-to-maturity calculation for a discount bond is similar to that for a simple loan.

$$PV = \frac{CF}{(1+i)^n}$$

where

$$PV = amount\ borrowed$$

$$CF = cash\ flow\ in\ each\ year$$

$$n = number\ of\ years$$

$$i = yield\ to\ maturity$$

Yield to Maturity on a Discount Bond

For any one-year discount bond, the yield to maturity can be written as

$$i = \frac{F - P}{P}$$

$$F = face\ value\ of\ the\ discount\ bond$$

$$P = current\ price\ of\ the\ discount\ bond$$

An important feature of this equation is that it indicates that, for a discount bond, the yield to maturity is negatively related to the current bond price.

Actually, all these above examined bonds show that **current bond prices and interest rates are negatively related: when the interest rate rises, the price of the bond falls, and vice versa**.

The Distinction Between Interest Rates and Returns

Rate of Return: the amount of each payment to the owner plus the change in the security’s value, expressed as a fraction of its purchase price.

Example: You bought a \$1,000-face-value coupon bond with a coupon rate of 10% for \$1,000. You held it for one year, and then sold it for \$1,200. Your return for this bond:

$$\frac{\$100+\$200}{\$1,000}=0.3=30\%$$

In this case, the return is 30%, but the yield to maturity is 10%.

The return on a bond will not necessarily equal the yield to maturity on that bond.

Return On A Bond

Generally, the return on a bond held from time t to time t+1 can be written as:

$$R = \frac{C + P_{t+1} - P_t}{P_t}$$

where

$$R = return\ from\ holding\ the\ bond\ from\ time\ t\ to\ time\ t+1$$

$$P_t = price\ of\ the\ bond\ at\ time\ t$$

$$P_{t+1} = price\ of\ the\ bond\ at\ time\ t+1$$

$$C = coupon\ payment$$

Return On A Bond

The return formula can be rewritten as:

$$R = \frac{C + P_{t+1} - P_t}{P_t} = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

Here, the first term is the current yield: $i_c = \frac{C}{P_t}$

The second term is the **rate of capital gain**: $g = \frac{P_{t+1} - P_t}{P_t}$

Hence, we can also write:

$$R = i_c + g$$

This above equation tells us that even if the current yield i_c is an accurate measure of the yield to maturity, the return can still substantially differ from the interest rate due to capital gains or losses.

More on Capital Gain

Q: where does the capital gain come from?

Because the coupon rate is fixed, if the real world interest rate changes, then the present value of the coupon bond will change.

For example, if you purchased a $1,000 coupon bond with 10 years to maturity in 2020 and the coupon rate is 10%. However, in 2021, the interest rate raised from 10% to 20%, how will the present value of your bond change?

$$P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \cdots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

The Distinction Between Interest Rates and Returns

TABLE 2 One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price ($)	(4) Price Next Year* ($)	(5) Rate of Capital Gain (%)	(6) Rate of Return [col (2) + col (5)] (%)
30	10	1,000	503	−49.7	−39.7
20	10	1,000	516	−48.4	−38.4
10	10	1,000	597	−40.3	−30.3
5	10	1,000	741	−25.9	−15.9
2	10	1,000	917	−8.3	+1.7
1	10	1,000	1,000	0.0	+10.0

*Calculated with a financial calculator, using Equation 3.

Maturity and the Volatility of Bond Returns: Interest-Rate Risk

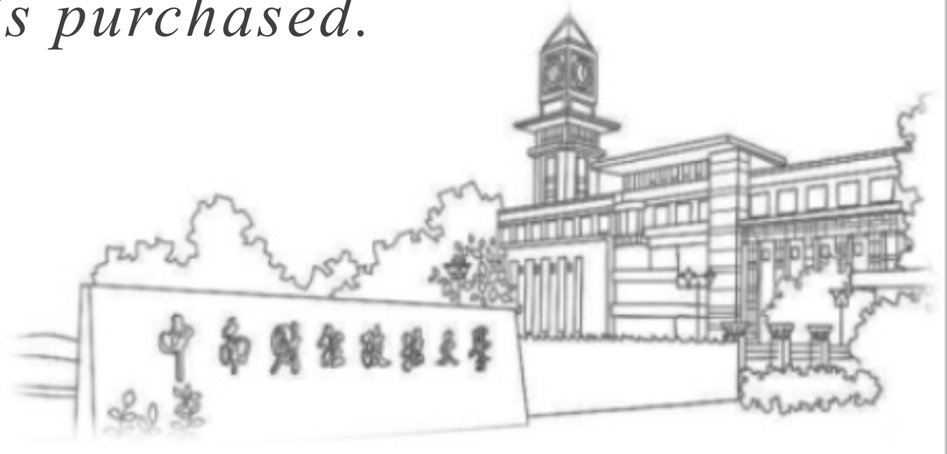
Several key findings from Table 2:

1. return = yield to maturity *only if* holding period = time to maturity
2. For bonds with maturity > holding period, if i goes up, then P goes down implying capital loss
3. Longer is maturity, greater is price change associated with interest rate change
4. Longer is maturity, larger return changes will be associated with interest rate changes
5. Bond with high initial interest rate can still have negative return if i goes up

Maturity and the Volatility of Bond Returns: Interest-Rate Risk

Prices and returns for long-term bonds are more volatile than those for shorter-term bonds.

There is no interest-rate risk for any bond whose time to maturity matches the holding period. *Because the change in interest rates have no effect on the price at the end of the holding period, and the return will be equal to the yield to maturity known at the time the bond is purchased.*



Reinvestment Risk

If an investor's holding period is longer than the term to maturity of the bond, the investor is exposed to a type of interest-rate risk called **reinvestment risk**.

Suppose John wants to purchase a bond, he has two options:

(a) purchase a $1,000 one-year, 10% coupon rate bond at face value and then purchase another one at the end of the first year

(b) purchase a $1,000 two-year, 10% coupon rate bond at face value

Which option should he choose? What if the interest rate changes at the end of the first year?

Reinvestment Risk

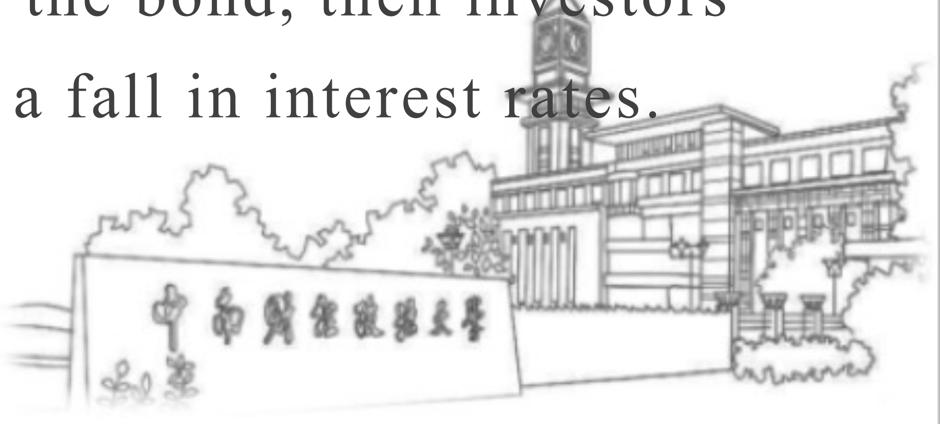
(1) if the interest rate on one-year bonds rises from 10% to 20%

First year, John will receive \$1,100, now he can buy \$1,100 face-value one-year bond and eventually get $\$1,100 \times 1.2 = \$1,320$. So John's total return is $\frac{\$1320-\$1000}{\$1000} = 0.32 = 32\%$, yearly return is 16%.

(2) if the interest rate on one-year bonds drops from 10% to 5%

Then John's total return is $\frac{\$1100 \times 1.05-\$1000}{\$1000} = 0.155 = 15.5\%$, yearly return is 7.75%.

Conclusion: holding period > term to maturity of the bond, then investors benefit from a rise in interest rates and is hurt by a fall in interest rates.



The Concept of Duration

When we discuss the interest-rate risk, we saw that when interest rates change, a bond with a longer term to maturity has a larger change in its price, and has higher interest-rate risk.

Q: If two bonds have the same term to maturity, does that mean they have the same interest-rate risk?

discount (zero-coupon) bond with 10 years to maturity VS 10% coupon bond with 10 years to maturity?

We need a concept of effective maturity to make discount bonds and 10% coupon bonds comparable in terms of interest-rate risk. That is why we introduce the concept **duration**.

Q: How to calculate duration?

We can measure the effective maturity of a coupon bond by treating a coupon bond as a set of zero-coupon discount bonds.

How to Calculate Duration?

TABLE 3.3 Calculating Duration on a \$1,000 Ten-Year 10% Coupon Bond When Its Interest Rate Is 10%

(1) Year	(2) Cash Payments (Zero-Coupon Bonds) (\$)	(3) Present Value (*PV*) of Cash Payments (*i* = 10%) (\$)	(4) Weights (% of total *PV* = *PV*/\$1,000) (%)	(5) Weighted Maturity (1 × 4)/100 (years)
1	100	90.91	9.091	0.09091
2	100	82.64	8.264	0.16528
3	100	75.13	7.513	0.22539
4	100	68.30	6.830	0.27320
5	100	62.09	6.209	0.31045
6	100	56.44	5.644	0.33864
7	100	51.32	5.132	0.35924
8	100	46.65	4.665	0.37320
9	100	42.41	4.241	0.38169
10	100	38.55	3.855	0.38550
10	1,000	385.54	38.554	3.85500
Total		1,000.00	100.000	6.75850

Formula for Duration

Duration is a weighted average of the maturities of the cash payment.

$$DUR = \sum_{t=1}^{n} t \frac{CP_t}{(1+i)^t} \Big/ \sum_{t=1}^{n} \frac{CP_t}{(1+i)^t}$$

Where

$$DUR = duration$$
$$t = years\ until\ cash\ payment\ is\ made$$
$$CP_t = cash\ payment\ (interest + pricipal) at\ time\ t$$
$$i = interest\ rate$$
$$n = years\ to\ maturity\ of\ the\ security$$

Properties of Duration

1. The longer the term to maturity of a band, everything else being equal, the greater its duration.
2. When interest rate rise, everything else being equal, the duration of a coupon bond falls.
3. The higher the coupon rate on the bond, everything else being equal, the shorter the bond's duration.
4. Duration is additive: the duration of a portfolio of securities is the weighted-average of the durations of the individual securities, with the weights equaling the proportion of the portfolio invested in each.

Calculating Duration 10-Year 10% Coupon Bond

TABLE 3.3 Calculating Duration on a $1,000 Ten-Year 10% Coupon Bond When Its Interest Rate Is 10%

(1) Year	(2) Cash Payments (Zero-Coupon Bonds) ($)	(3) Present Value (*PV*) of Cash Payments (*i* = 10%) ($)	(4) Weights (% of total *PV* = *PV*/$1,000) (%)	(5) Weighted Maturity (1 × 4)/100 (years)
1	100	90.91	9.091	0.09091
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7	100	51.32	5.132	0.35924
8	100	46.65	4.665	0.37320
9	100	42.41	4.241	0.38169
10	100	38.55	3.855	0.38550
10	1,000	385.54	38.554	3.85500
Total		1,000.00	100.000	6.75850

TABLE 3.4 Calculating Duration on a $1,000 Ten-Year 10% Coupon Bond When Its Interest Rate Is 20%

(1) Year	(2) Cash Payments (Zero-Coupon Bonds) ($)	(3) Present Value (*PV*) of Cash Payments (*i* = 20%) ($)	(4) Weights (% of total *PV* = *PV*/$580.76) (%)	(5) Weighted Maturity (1 × 4)/100 (years)
1	100	83.33	14.348	0.14348
2	100	69.44	11.957	0.23914
3	100	57.87	9.965	0.29895
4	100	48.23	8.305	0.33220
5	100	40.19	6.920	0.34600
6	100	33.49	5.767	0.34602
7	100	27.91	4.806	0.33642
8	100	23.26	4.005	0.32040
9	100	19.38	3.337	0.30033
10	100	16.15	2.781	0.27810
10	$1,000	161.51	27.808	2.78100
Total		580.76	100.000	5.72204

Duration and Interest-Rate Risk

$$\%\Delta P = -DUR \times \frac{\Delta i}{1+i}$$

$\%\Delta P = \frac{P_{t+1}-P_t}{P_t}$ = percentage change in the price of the security from t to t+1 = rate of capital gain

DUR = Duration

i = interest rate

Suppose we have two 10-year coupon bonds. Their coupon rates are 10% and 20%. If i goes up from 10% to 11%:

(1) 10% coupon bond:

$$\%\Delta P \approx -6.76\times\frac{0.01}{1+0.1} = -6.15\%$$

Duration and Interest-Rate Risk

year	cash payments	present value of cash payments	weights	weighted maturity
1	200	181.8181818	0.1126188	0.112618803
2	200	165.2892562	0.10238073	0.20476146
3	200	150.2629602	0.09307339	0.279220172
4	200	136.6026911	0.08461217	0.338448693
5	200	124.1842646	0.07692016	0.384600788
6	200	112.894786	0.06992742	0.419564496
7	200	102.6316236	0.06357038	0.444992647
8	200	93.30147604	0.05779125	0.462330023
9	200	84.81952367	0.0525375	0.472837524
10	200	77.10865789	0.04776137	0.47761366
10	1000	385.5432894	0.23880683	2.388068301
Total		1614.456711	1	5.985056567

(2) 20% coupon bond:

$$\%\Delta P \approx -5.98 \times \frac{0.01}{1+0.1} = -5.4\%$$

Duration and Interest-Rate Risk

The greater is the duration of a security, the greater is the percentage change in the market value of the security for a given change in interest rates.

Therefore, the greater is the duration of a security, the greater is its interest-rate risk

The Distinction Between Real and Nominal Interest Rates

- **Nominal interest rate** makes no allowance for inflation.
- **Real interest rate** is adjusted for changes in price level so it more accurately reflects the cost of borrowing.
    - ➢Ex ante real interest rate is adjusted for expected changes in the price level
    - ➢Ex post real interest rate is adjusted for actual changes in the price level

Fisher Equation

$$i = r + \pi^e$$

where

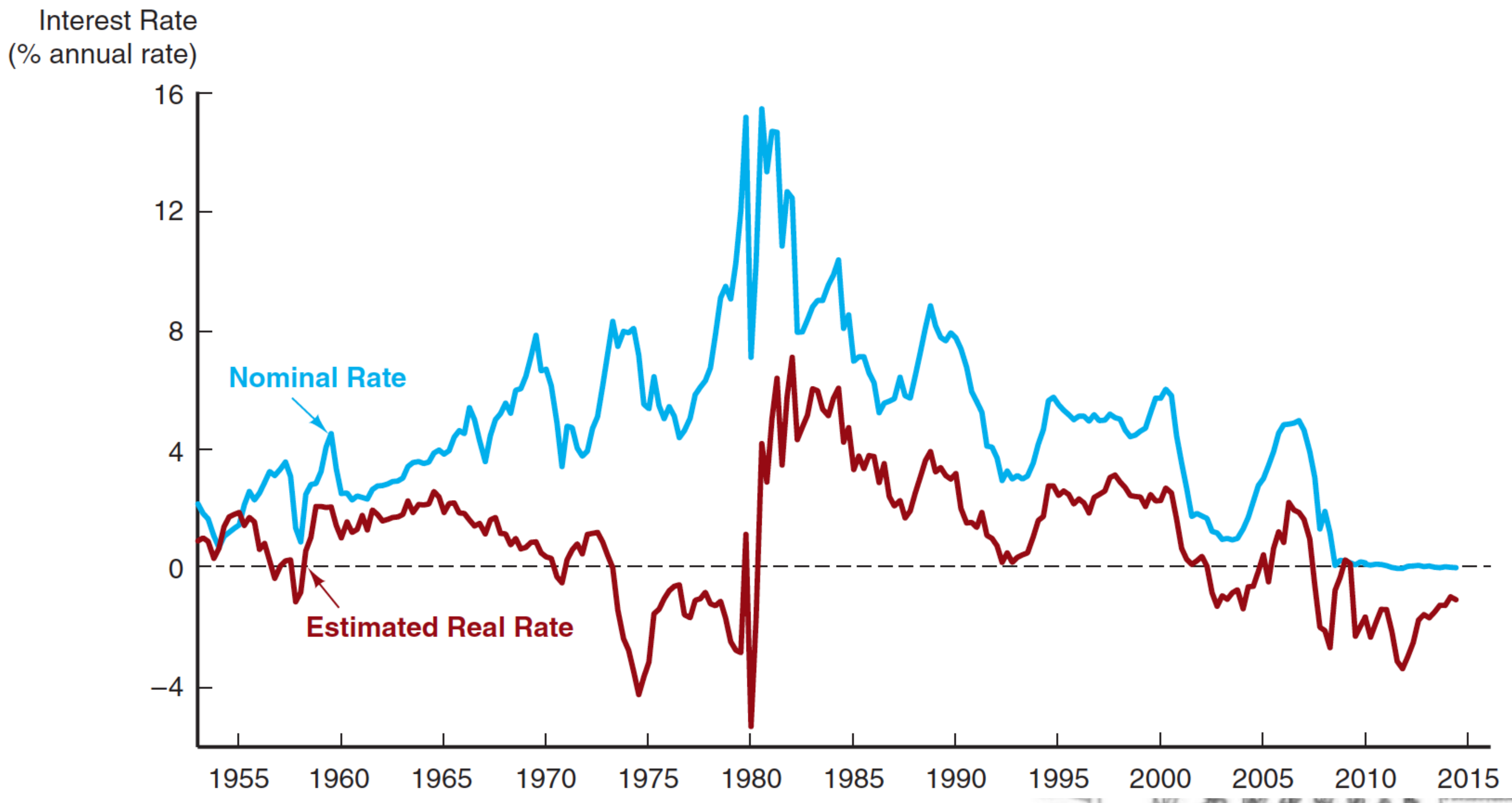
i= nominal interest rate

r= real interest rate

π^e= expected inflation rate

1. When the real interest rate is low, there are greater incentives to borrow and fewer incentives to lend.
2. The real interest rate is a better indicator of the incentives to borrow and lend.

Figure 1 Real and Nominal Interest Rates (Three-Month Treasury Bill), 1953–2014



Chapter Summary

1. Measuring interest rates: we examined several techniques for measuring the interest rate required on debt instruments.
2. The distinction between interest rates and returns: we examined what each means and how they should be viewed for asset valuation.
3. The concept of Duration.
4. The distinction between real and nominal interest rates: we examined the meaning of interest in the context of price inflation.

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