International Finance
Chapter 3: The Monetary Model

Zichao Yang

Zhongnan University of Economics & Law

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What is Monetary Model

The monetary model is central to international macroeconomic analysis and is a recurrent theme in this class.

The model identifies a set of underlying economic fundamentals that determine the nominal exchange rate in the long run.

The monetary approach assume that all prices are perfectly flexible and centers on conditions for stock equilibrium in the money market.

Notation Rules

The level of a variable will be denoted in upper case letters.

The natural logarithm of a variable will be denoted in lower case letters.

Exception: the level of the interest rate is denoted in lower case. Hence, i_t is the level value of the nominal interest rate.

Cassel's Approach

Purchasing-power parity (PPP) is a key building block of the monetary model.

Since the purchasing power of the home currency is $1/P$ and the purchasing power of the foreign currency is $1/P^*$, in equilibrium, the relative value of the two currencies should reflect their relative purchasing power $S = P/P^*$ (in American terms).

Q: then, what is the appropriate definition of the price level?

Cassel's Approach

Cassel suggested to use the general price level. And whether the general price level samples prices of non-traded goods or not is irrelevant. Hence, the **consumer price index (CPI)** is typically used in empirical implementations of this theory.

"Some people believe that Purchasing Power Parities should be calculated exclusively on price indices for such commodities are for the subject of trade between the two countries. This is a misinterpretation of the theory...The whole theory of purchasing power parity essentially refers to the internal value of the currencies concerned, and variations in this value can be measured only by general index figures representing as far as possible the whole mass of commodities marketed in the country."

Cassel's Approach

This theory implies that the log real exchange rate $q \equiv s + p^* - p$ is constant. However, the real world data does not support this theory.

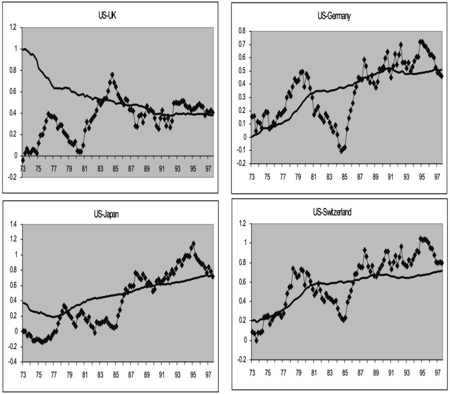

Figure 3.1: Log nominal exchange rates (boxes) and CPI-based PPPs (solid).

Cassel's Approach

However, we do see the nominal exchange rate and PPP tend to revert toward each other in the long run. Nowadays, people view Casselian PPP a theory of the long-run determination of the exchange rate: **PPP is a long-run attractor for the nominal exchange rate.**

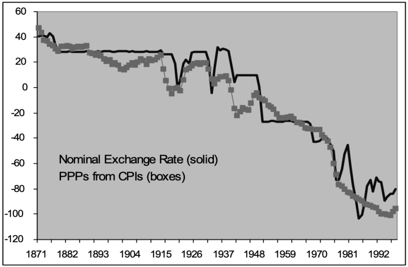

Figure 3.2: US–UK log nominal exchange rates and CPI-based PPPs multiplied by 100. 1871-1997.

The Commodity-Arbitrage Approach

The commodity-arbitrage view of PPP, proposed by Samuelson simply holds that the law-of-one price holds for all internationally traded goods.

People argue that the appropriate price index should cover only those goods that are traded internationally.

The **producer price index (PPI)** should be a better choice for studying PPP since it is more heavily weighted toward traded goods than the CPI.

PPP is clearly violated in the short run. However, there exists econometric evidence to support long-run PPP.

Money Supply

Consider a small open economy that maintains a fixed exchange rate $\bar{s}$.

B_t: the monetary base

R_t: the stock of foreign excahgne reserves held by the central bank

D_t: the domestic credit extended by the central bank

Then the money supply is: $M_t = \mu B_t = \mu(R_t + D_t)$, μ is the money multiplier.

A logarithmic expansion of the money supply at its mean value gives us:

$$m_t = \theta r_t + (1 - \theta) d_t \qquad \text{(How?)}$$

Money Demand

Based on the transaction motivation, we can write the money demand equation as:

$$m_t^d - p_t = \phi y_t - \lambda i_t + \epsilon_t$$

How to interpret this equation?

The log real money demand $m_t^d - p_t$ depends positively on y_t and negatively on the opportunity cost of holding money i_t.

$0 < \phi < 1$ is the income elasticity of money demand, $0 < \lambda$ is the interest semi-elasticity of money demand and $\epsilon_t \overset{iid}{\sim} (0, \sigma_\epsilon^2)$.

Equilibrium Condition

If we assume the exchange rate is fixed: $\bar{s}$, then:

(1) if the purchasing-power parity (PPP) holds: $p_t = \bar{s} + p_t^*$

(2) if the uncovered interest parity (UIP) holds:
$i_t - i_t^* = E_t s_{t+1} - s_t = \bar{s} - \bar{s} = 0$

Furthermore, assume that the money market is continuously in equilibrium by equating m_t^d to m_t, we can get:

$$\left.\begin{aligned} p_t &= \bar{s} + p_t^* \\ i_t &= i_t^* \\ m_t &= \theta r_t + (1-\theta) d_t \\ m_t^d - p_t &= \phi y_t - \lambda i_t + \epsilon_t \end{aligned}\right\} \Rightarrow \theta r_t + (1-\theta) d_t = \bar{s} + p_t^* + \phi y_t - \lambda i_t^* - (1-\theta) d_t + \epsilon_t$$

Equilibrium Condition

The equilibrium condition tells us:

- If the home country experiences any of the following: a high rate of income growth, declining interest rates, or rising prices, the demand for nominal money balances will grow.
- If money demand growth is not satisfied by a increase in domestic credit d_t, the public will accumulate international reserves r_t.
- If the central bank conducts domestic credit expansion that exceeds money demand growth, the public will eliminate the excess supply of money by running a balance of payments deficit.

Monetary Model Under Flexible Exchange Rates

Under flexible exchange rates, the equilibrium in the domestic and foreign money market are given by:

$$m_t - p_t = \phi y_t - \lambda i_t$$
$$m_t^* - p_t^* = \phi y_t^* - \lambda i_t^*$$

where $0 < \phi < 1$ is the income elasticity of money demand, and $\lambda > 0$ is the interest semi-elasticity of money demand.

International capital market equilibrium is described by uncovered interest parity:

$$i_t - i_t^* = E_t s_{t+1} - s_t$$

Price level and the exchange rate are related through purchasing-power parity:

$$s_t = p_t - p_t^*$$

Monetary Model Under Flexible Exchange Rates

Now we define a new concept, the economic **fundamentals**, which can be written as:

$$f_t \equiv (m_t - m_t^*) - \phi(y_t - y_t^*)$$

Eventually, we can get a equation describe the relationship between exchange rate and fundamentals:

$$\left.\begin{aligned} m_t - p_t &= \phi y_t - \lambda i_t \\ m_t^* - p_t^* &= \phi y_t^* - \lambda i_t^* \\ i_t - i_t^* &= E_t s_{t+1} - s_t \\ s_t &= p_t - p_t^* \\ f_t &\equiv (m_t - m_t^*) - \phi(y_t - y_t^*) \end{aligned}\right\} \Rightarrow s_t = \gamma f_t + \psi E_t s_{t+1}$$

where $\gamma \equiv 1/(1+\lambda)$ and $\psi \equiv \lambda\gamma = \lambda/(1+\lambda)$.

Monetary Model Under Flexible Exchange Rates

The above equation is the basic first-order stochastic equation of the monetary model. This equation tells us:

- The expectations of future values of the exchange rate are embodied in the current exchange rate.
- High relative money growth at home leads to a weakening of the home currency while high relative income growth leads to a strengthening of the home currency.

Monetary Model Under Flexible Exchange Rates

In the previous slids, we know that:

$$
\begin{aligned}
s_t &= \gamma f_t + \psi E_t s_{t+1} \\
s_{t+1} &= \gamma f_{t+1} + \psi E_t s_{t+2} \\
s_{t+2} &= \gamma f_{t+2} + \psi E_t s_{t+3} \\
&\dots
\end{aligned}
$$

Eventually, we can get:

$$s_t = \gamma \sum_{j=0}^{k} (\psi)^j E_t f_{t+j} + (\psi)^{k+1} E_t s_{t+k+1}$$

Next step, we want to discuss the behavior of the term of $(\psi)^k E_t s_{t+k}$ when $k \to \infty$.

Fundamentals (no bubbles) Solution

We know that $\psi = \lambda/(1+\lambda) < 1$, let us further impose the transversality condition that:

$$\lim_{k\to\infty} (\psi)^k E_t s_{t+k} = 0$$

Then the present-value formula can be written as:

$$s_t = \gamma \sum_{j=0}^{k} (\psi)^j E_t f_{t+j}$$

- Exchange rate is the discounted present value of expected future value of the fundamentals.
- Monetary approach is sometimes referred to as the "asset" approach to the exchange rate.
- Exchange rate is more volatile than the fundamentals, just as stock prices are more volatile than dividends.

Rational Bubble Solution

We add a bubble term into the original equation:

$$b_t = (1/\psi)b_{t-1} + \eta_t$$
$$\hat{s}_t = s_t + b_t$$

where $1/\psi > 1$ and $\eta_t \overset{iid}{\sim} (0, \sigma_\eta^2)$.

Now $\hat{s}_t$ violates the transversality condition because:

$$(\psi)^k E_t \hat{s}_{t+k} = \underbrace{\psi^k E_t s_{t+k}}_{0} + \psi^k E_t b_{t+k} = b_t$$

However, $\hat{s}_t$ is indeed another solution to $s_t = \gamma f_t + \psi E_t s_{t+1}$:

$$s_t + b_t = \gamma f_t + \psi[E_t s_{t=1} + (1/\psi)b_t]$$

Rational Bubble Solution

$\hat{s}_t$ is another viable solution, but in this solution the bubble will eventually dominate and will drive the exchagne rate arbitrarily far away from the fundamentals f_t.

The bubble arises in a model where people have rational expectations so it is referred to as a rational bubble.

The picture displays a realization of rational bubble.

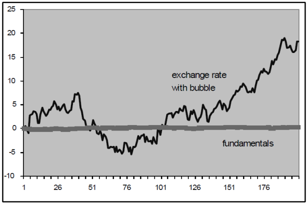

Figure 3.3: A realization of a rational bubble where $\psi = 0.99$, and the fundamentals follow a random walk. The stable line is the realization of the fundamentals.

Fundamentals and Exchanges Rate Volatility

Table 3.1: Descriptive statistics for exchange-rate and equity returns, and their fundamentals.

	Mean	Std.Dev.	Min.	Max.	Autocorrelations ρ_1	ρ_4	ρ_8	ρ_{16}
Returns								
S&P	2.75	5.92	-13.34	18.31	0.24	-0.10	0.15	0.09
UKP	0.41	5.50	-13.83	16.47	0.12	0.03	0.01	-0.29
DEM	0.46	6.35	-13.91	15.74	0.09	0.23	0.04	-0.07
YEN	0.73	6.08	-15.00	16.97	0.13	0.18	0.06	-0.29
Deviation from fundamentals								
Div.	1.31	0.30	0.49	1.82	1.01	1.03	1.05	0.94
UKP	0	0.18	-0.46	0.47	0.89	0.61	0.25	-0.12
DEM	0	0.31	-0.61	0.59	0.98	0.91	0.77	0.55
YEN	0	0.38	-0.85	0.50	0.98	0.88	0.76	0.68

Stylized Facts on Volatility and Dynamics

- The volatility of exchange rate returns is indistinguishable from stock return volatility.
- Returns for both stocks and exchange rates have low first-order serial correlation.
- The negative autocorrelations in exchange returns at 16 quarters suggest the possibility of mean reversion.
- The deviation from the fundamentals display substantial persistence, and much less volatile than returns.

Excess Volatility and the Monetary Model

The monetary model can be made consistent with the excess volatility in the exchange rate if the growth rate of the fundamentals is a persistent stationary process.

$$\Delta f_t = \rho \Delta f_{t-1} + \epsilon_t \quad \text{where } \epsilon_t \overset{iid}{\sim} (0, \sigma_\epsilon^2)$$

Then we can rewrite $s_t = \gamma \sum_{j=0}^{k} (\psi)^j E_t f_{t+j}$ as;

$$s_t = f_t + \frac{\rho\psi}{1-\rho\psi} \Delta f_t$$

$$Var(\Delta s_t) = \frac{(1-\rho\psi)^2 + 2\rho\psi(1-\rho)}{(1-\rho\psi)^2} Var(\Delta f_t) > Var(\Delta f_t)$$

However, this is not very encouraging since the levels of the fundamentals are explosive. (Why?)