When will the current yield equal the yield to maturity?

(1) The current yield better approximates the yield to maturity when the maturity of the bond is long.

(2) The current yield better approximates the yield to maturity when the bond's price is nearer to the bond's par value.

Current yield (i_c): $i_c = \frac{C}{P}$

Yield to maturity (i): $P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + ... + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$

For yield to maturity equation, we can simplify it as:

$$\begin{aligned} P &= \frac{C}{1+i} + \frac{C}{(1+i)^2} + ... + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n} \\ &= C \cdot \frac{1-(\frac{1}{1+i})^n}{i} + \frac{F}{(1+i)^n} \end{aligned}$$

(1) if $n \to \infty$, then

$$\begin{aligned} \lim_{n\to\infty} P &= \lim_{n\to\infty} C \cdot \frac{1-(\frac{1}{1+i})^n}{i} + \frac{F}{(1+i)^n} \\ &= \frac{C}{i} \end{aligned}$$

so $i = \frac{C}{P} = i_c$

(2) if the bond's price is the bond's par value, then $P = F$:

$$\begin{aligned} P = F = C \cdot \frac{1-(\frac{1}{1+i})^n}{i} + \frac{F}{(1+i)^n} \\ \Rightarrow F[1 - \frac{1}{(1+i)^n}] = \frac{C[1-\frac{1}{(1+i)^n}]}{i} \\ \Rightarrow F = \frac{C}{i} \Rightarrow i = \frac{C}{F} = \frac{C}{P} \end{aligned}$$

so $i = \frac{C}{P} = i_c$