Let $(x_1, x_2 \ldots x_n)$ be sample of size 'n' taken

Mean $\rightarrow \theta_1$ Variance $\rightarrow \theta_2$

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} \cdot e^{-(x_i - \theta_1)^2 / 2\theta_2}$$

take log

$$\log L(\theta_1, \theta_2) = -\frac{n}{2} \log(2\pi\theta_2) - \frac{1}{2\theta_1} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

for θ_1 diff $\log(L(\theta_1, \theta_2))$ w.r.t θ_1 & set it to zero

$$\frac{\partial \log(L)}{\partial \theta_1} = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1) = 0.$$

$$\theta_1 = \frac{1}{n} \sum_{i=1}^{n} x_i$$

MLE of S_1 is sample mean.

for θ_2 diff. w.r.t. θ_2 & put zero.

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2.$$

2) Binomial distribution.

$m \Rightarrow$ no. of trails.

$\theta = (0, 1)$ prob. of success

$$L_0 = \prod_{i=1}^{n} f(x_i, n, \theta)$$

PMF

$$f(x, n, \theta) = {}^{n}C_{x} \cdot \theta^{1} \cdot (1-\theta)^{m-n}$$

$$L(\theta) = \prod_{i=1}^{n} ({}^{n}C_{x_i}) \cdot \theta^{x_i} \cdot (1-\theta)^{m-x_i}$$

take log

$$\frac{\partial \log(L)}{\partial \theta} = \frac{1}{\theta} \sum_{i=1}^{n} x_i - \frac{1}{1-\theta} \sum_{i=1}^{m} (n - x_i) = 0.$$

$$\Rightarrow \frac{1}{\theta} \sum_{i=1}^{n} x_i = \frac{1}{1-\theta} \sum_{i=1}^{m} (m - x_i)$$

Multiply by $\theta(1-\theta)$

$$\Rightarrow (1-\theta) \sum_{i=1}^{n} x_i = \theta \sum_{i=1}^{m} (m - x_i).$$

$$\theta = \frac{\sum_{i=1}^{n} x_i}{m}.$$

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