数学魔法师

对于$a>b\in R^{+}$,M为平均值，若满足:

①$a>M(a,b)>b$ ②$M(a,b)=M(b,a)$ ③$M(at,bt)=tM(a,b)\ (t\geqslant 0)$

算术、几何、调和、反调和、二次根次、指数、质心、对数、$Seiffert$、$Toader$、海伦

$$A(a,b)=\frac{a+b}{2},G(a,b)=\sqrt{ab},H(a,b)=\frac{2ab}{a+b},C(a,b)=\frac{a^2+b^2}{a+b}$$

$$Q(a,b)=\sqrt{\frac{a^2+b^2}{2}},I(a,b)=\frac{1}{e}\left(\frac{b^b}{a^a}\right)^{\frac{1}{b-a}},\bar{C}(a,b)=\frac{2(a^3-b^3)}{3(a^2-b^2)}$$

$$L(a,b)=\frac{a-b}{\ln a-\ln b},\hat{S}(a,b)=\frac{a-b}{2\arcsin\frac{a-b}{a+b}},S(a,b)=\frac{a-b}{2\arctan\frac{a-b}{a+b}}$$

$$T(a,b)=\frac{2}{\pi}\int_0^{\frac{\pi}{2}}\sqrt{a^2\cos^2\theta+b^2\sin^2\theta}\,d\theta,He(a,b)=\frac{a+\sqrt{ab}+b}{3}$$

$H<G<L<\hat{S}<I<A<S<\bar{C}<Q<C$ 幂平均$M_q(a,b)=\left(\frac{a^q+b^q}{2}\right)^{\frac{1}{q}}(q\uparrow M_q\uparrow)$

若两类$M_1(a,b)=a_1,M_2(a,b)=b_1$,再将$a_1,b_1$替换$a,b$,将得到新平均值,迭代

$M_1<M_2\rightarrow\lim a_n=\lim b_n=M_3(a,b)$,记做$M_1.M_2\rightarrow M_3$,即④$M_3(M_1,M_2)=M_3(a,b)$

$$eg:A.H\rightarrow G,\sqrt{\left(\frac{a+b}{2}\right)\left(\frac{2ab}{a+b}\right)}=\sqrt{ab},G(A(a,b),H(a,b))=G(a,b)$$

$$eg:A.G\rightarrow\frac{a\pi}{2}\frac{1}{K\left(\frac{\sqrt{a^2-b^2}}{a}\right)}(\text{第一类})=\frac{\pi}{2}\left(\int_0^{\frac{\pi}{2}}\frac{d\theta}{\sqrt{a^2\cos^2\theta+b^2\sin^2\theta}}\right)^{-1}$$

$$need:\int_0^{\frac{\pi}{2}}\frac{d\theta}{\sqrt{a^2\cos^2\theta+b^2\sin^2\theta}}=\int_0^{\frac{\pi}{2}}\frac{d\theta}{\sqrt{\left(\frac{a+b}{2}\right)^2\cos^2\theta+ab\sin^2\theta}}$$

$$eg:G.H\rightarrow\frac{2b}{\pi}K\left(\frac{\sqrt{a^2-b^2}}{a}\right)=\frac{2}{\pi}\int_0^{\frac{\pi}{2}}\frac{abd\theta}{\sqrt{a^2\cos^2\theta+b^2\sin^2\theta}}$$

$$need:\int_0^{\frac{\pi}{2}}\frac{\sqrt{ab}\frac{2ab}{a+b}d\theta}{\sqrt{ab\cos^2\theta+\left(\frac{2ab}{a+b}\right)^2\sin^2\theta}}=\int_0^{\frac{\pi}{2}}\frac{2abd\theta}{\sqrt{(a+b)^2\cos^2\theta+4ab\sin^2\theta}}$$

$$=\int_0^{\frac{\pi}{2}}\frac{abd\theta}{\sqrt{a^2\cos^2\theta+b^2\sin^2\theta}}$$