

**Problem Chosen**

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Summary Sheet**

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# Modeling and Predicting the Time-To-Empty of Lithium-ion Batteries in Smartphones

## Summary

Smartphones, a kind of indispensable tool in modern life, have a battery performance which usually appears somewhat unpredictable, but this is not the case in reality. In order to predict the time-to-empty of lithium-ion batteries in smartphones under different conditions, we developed a continuous-time mathematical model for smartphone batteries, and successfully transformed our research results into practical recommendations for mobile phone users.

Initially, we started with the simplest and most reasonable description of battery power consumption, and then expanded it to incorporate factors such as screen usage, processor load, network connection, background applications, GPS usage, environmental factors, and others. We developed a model that returns the state of charge (SOC) as a function of time under actual usage conditions.

**Keywords:** smartphone, lithium battery,

# **Contents**

# 1 Introduction

## 1.1 Problem Background

Smartphones have become integral to contemporary living, yet their battery performance remains highly inconsistent. A device might operate seamlessly for a full day under moderate use, but in other instances, it may deplete rapidly even before midday. While many users attribute such variations to "frequent usage", the underlying causes of battery drain are far more complex. Key determinants include screen dimensions and brightness settings, computational load imposed by the processor, network-related activities, and background processes from applications that continue to consume power even when the phone appears idle. External conditions also exert considerable influence: certain batteries exhibit reduced efficiency in colder climates and are susceptible to overheating during sustained high-demand tasks. Moreover, the battery's historical usage patterns and charging behaviors significantly shape its long-term performance and reliability.



Figure 1: Diagram of mobile phone lithium battery

### 1.1.1 Usage Pattern Factors

Individual usage habits significantly impact battery performance:

- Frequency and intensity of usage (moderate vs. heavy use)
- Historical battery usage patterns over time[3]
- Charging behaviors and methods employed

### 1.1.2 Hardware, Software and Configuration Factors

Device-specific characteristics and settings play a crucial role:

- Screen dimensions and physical size
- Brightness level settings configured by the user
- Computational load imposed by the processor during operation

Background processes and network activities contribute to power drain:

- Network-related activities and data transmission
- Background processes from applications running during idle states
- Persistent background application consumption even when phone appears inactive

### 1.1.3 Environmental Conditions

External physical conditions exert a direct and sometimes significant influence on battery efficiency[1]. On one hand, lithium-ion batteries commonly exhibit reduced operational efficiency and effective capacity when exposed to colder climates and low temperatures. At low temperatures, the internal resistance of the battery increases, the battery capacity significantly decreases, and the voltage may drop below the lowest allowable voltage, making it impossible for the battery to function properly[2]. On the other hand, during periods of sustained high-demand processing tasks, these batteries are particularly susceptible to overheating, which can further degrade performance and long-term health. Finally, broader external temperature variations encountered during normal device operation consistently impact the rate of power drain and overall battery behavior.

## 1.2 Restatement of the Problem

Considering the background information and restricted conditions identified in the problem statement, the following problems we need to solve are listed as follows:

- **Problem 1:** Develop a model that uses continuous-time equations or systems of equations to represent the battery's state of charge, that is, establish a differential equation system that describes the change of state of charge (SOC) over time.
- **Problem 2:** Use our model to calculate or approximate the emptying time for various initial battery charge levels and usage scenarios. Compare the predicted results with observed or reasonable behaviors, quantify the uncertainty, and determine in which aspects the model performs well or poorly.
- **Problem 3:** Test the robustness of the model. This involves systematically altering the modeling assumptions, adjusting key parameters, and simulating different variations in usage patterns to analyze the extent to which these changes affect the final prediction results of the model.
- **Problem 4:** Translate the research findings into practical operational suggestions for mobile users, and explore how the operating system can implement more proactive and effective dynamic power management strategies based on the core insights of the model. At the same time, analyze the feasibility and potential value of extending this modeling framework to other portable electronic devices.

### 1.3 Literature Review

This issue focuses on the complex and seemingly unpredictable nature of smartphone battery drain. In recent years, research on power consumption models for portable devices has focused on both the underlying physical principles of lithium-ion batteries and the development of suitable continuous-time mathematical frameworks.

- During the use or storage of lithium batteries, certain degrees of failure may occur, including capacity reduction, short cycle life, increased internal resistance, abnormal voltage, and lithium precipitation, which significantly reduce the performance, consistency, reliability, and safety of lithium batteries[4]. Some researchers also studied the changes in capacity and temperature rise of NCM pouch-type lithium-ion batteries under different operating temperature conditions[5]. Based on previous studies, it was found that at low temperatures, lithium-ion batteries would exhibit phenomena such as lithium precipitation and increased impedance, which significantly affected the performance of lithium-ion batteries.
- Some simple continuous-time models, such as Enhanced Coulomb Counting, were used to analyse similar problems. However, its weakness will be exposed when we introduce nonlinear factors and consider the influence of internal resistance and temperature on the capacity of lithium-ion batteries, which is what Problem A specifically requires us to do[6]. Deterministic scenario simulation is often used to predict the time when power runs out. However, in reality, user behavior is not constant, and this kind of prediction is overly idealized. Local sensitivity analysis is often used to observe how much the "time-to-empty" changes when only one parameter is altered. Unfortunately, factors that affect the performance of lithium batteries in smartphones may interact with each other, and this method is unable to study such interactions[7].

The advantages and disadvantages of the three basic models are shown below in 2.

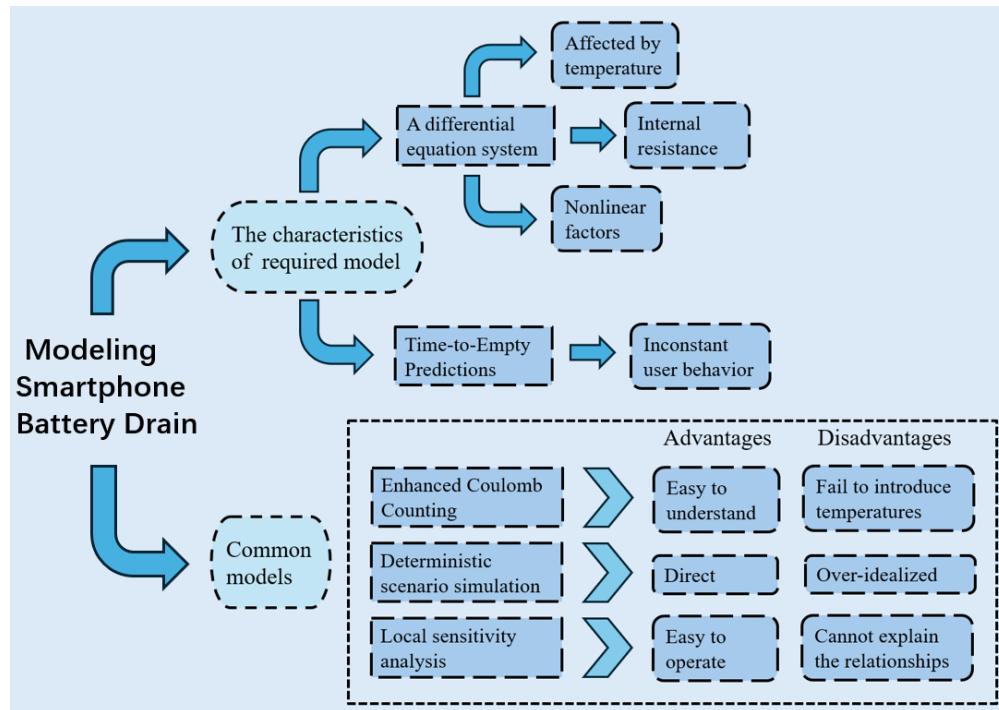


Figure 2: Literature Review

## 1.4 Our work

# 2 Model Preparation

## 2.1 Assumptions and Justifications

In addressing real-world problems that inherently involve numerous complex variables, our initial step involved formulating reasonable assumptions to simplify the modeling process, with each assumption accompanied by its corresponding justification.

- **Assumption:** The smartphones use lithium-ion batteries.
- **Justification:** The problem requires us to assume that smartphones use lithium-ion batteries.
- **Assumption:** The data we use are accurate and valid.
- **Justification:** Our data is collected from official technical specifications from manufacturers like Apple, some other official web sites and research papers, so it's reasonable to assume the high quality of their data.
- **Assumption:** Differential equations can be used to describe the process of how the battery's state changes over time.
- **Justification:** Differential equations are ideal for modeling continuous battery dynamics, as they capture real-time rate changes based on physical laws like energy conservation, aligning with the problem's requirement for a continuous-time approach.
- **Assumption:** The battery's internal resistance and effective capacity vary linearly with temperature within typical operating ranges (e.g., 10°C to 25°C).
- **Justification:** Empirical studies on lithium-ion batteries show that temperature effects on internal resistance and capacity can be approximated linearly for short-term predictions, balancing model accuracy and simplicity.

- **Assumption:** Power consumption from individual components (e.g., screen, CPU) is additive and can be modeled as a linear function of their operating states (e.g., brightness, load).
- **Justification:** This simplifies the integration of multiple power sources into a total system power consumption, supported by the superposition principle in circuit theory, and aligns with the continuous-time equation framework required by the problem.
- **Assumption:** Background application power consumption remains constant during idle periods unless explicitly changed by user activity.
- **Justification:** This reflects typical smartphone behavior where background tasks (e.g., push notifications) maintain a steady low-level energy draw, as noted in Document 1's discussion of "background applications that continue drawing energy."
- **Assumption:** Battery aging effects are negligible over the short-term simulations (e.g., a single day) but follow an exponential decay model for long-term analysis.
- **Justification:** Aging primarily impacts capacity over many charge cycles; ignoring it for daily use simplifies the model without significant loss of accuracy, while incorporating it via an exponential function ensures scalability for extended studies.

## 2.2 Notations

The key mathematical notations used in this paper are listed in Table 1.

Table 1: Notations

Symbol	Unit	Definition
$Q_{ref}$	mAh	Typical capacity of modern flagship smartphones
$Q_{eff}$	mAh	The effective available capacity of the battery
$V_{nom}$	V	Standard operating voltage platform for lithium-ion batteries
$V_{min}$	V	Voltage at which the protection circuit cuts off the power
$V_{max}$	V	Voltage at 100% State of Charge
$R_{int}$		Internal resistance, approximately 50m, increases sharply with decreasing temperature
$m_{bat}$	kg	Battery mass, approximately 0.05 kg (50 grams), used for thermal capacity calculation
$C_p$	J/(kg°C)	Average specific heat capacity of lithium-ion battery materials
$h_{eff}$	W/K	Effective heat transfer coefficient representing the phone's overall heat dissipation capability
$\eta$	dimensionless	Coulombic efficiency, typically close to 1, but decreases with aging or at low temperatures

\* There are some variables that are not listed here and will be discussed in detail in each section.

Table 3 lists the corresponding abbreviations for the basic concepts, which will frequently appear in this article.

Table 2: Abbreviations (Supplement)

Full Name	Abbreviation
Open Circuit Voltage	OCV
Time to Empty	TTE
State of Charge	SOC
Equivalent Circuit Model	ECM
Power Management Integrated Circuit	PMIC
Graphics Processing Unit	GPU
Image Signal Processor	ISP
Radio Resource Control (State)	RRC
Dynamic Power Management	DPM
State of Health	SOH
Central Processing Unit	CPU
Global Positioning System	GPS
Thermal Management System	TMS

\* This list extends the core notations. Additional model-specific abbreviations will be defined upon their first appearance in the respective sections.

## 2.3 Data Collection and Visualization

It has been discovered that it is difficult to inversely determine the coefficients of a continuous differential equation from discrete data through parameter estimation, because the data obtained is usually discrete.

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Table 3: Notations

Database Names	Database Websites
Los Angeles	<a href="https://geohub.lacity.org/">https://geohub.lacity.org/</a>
National Risk Index	<a href="https://www.fema.gov/">https://www.fema.gov/</a>
Bureau of the Census	<a href="https://data.census.gov/">https://data.census.gov/</a>
South Carolina	<a href="https://www.sc.edu/">https://www.sc.edu/</a>
World Bank	<a href="https://data.worldbank.org/">https://data.worldbank.org/</a>

## 3 Model I:A differential equation system

### 3.1 Construct an equivalent circuit model ECM

Based on the relevant literature, we firstly defined the battery state State of Charge, SOC as the percentage of the current remaining capacity relative to the rated capacity:

$$SOC(t) = \frac{Q(t)}{C} \times 100\%$$

Here,  $Q(t)$  represents the remaining battery power at time  $t$ , and  $C$  is the battery capacity measured in mAh. This provides a standardized starting point for modeling battery discharge under various conditions, ensuring consistency in time-to-empty predictions and simplifying initial value problems in differential equations.

3.2

In order to describe our process of calculating the value of  $\theta$  more clearly. We draw the flowchart Figure 3 below.

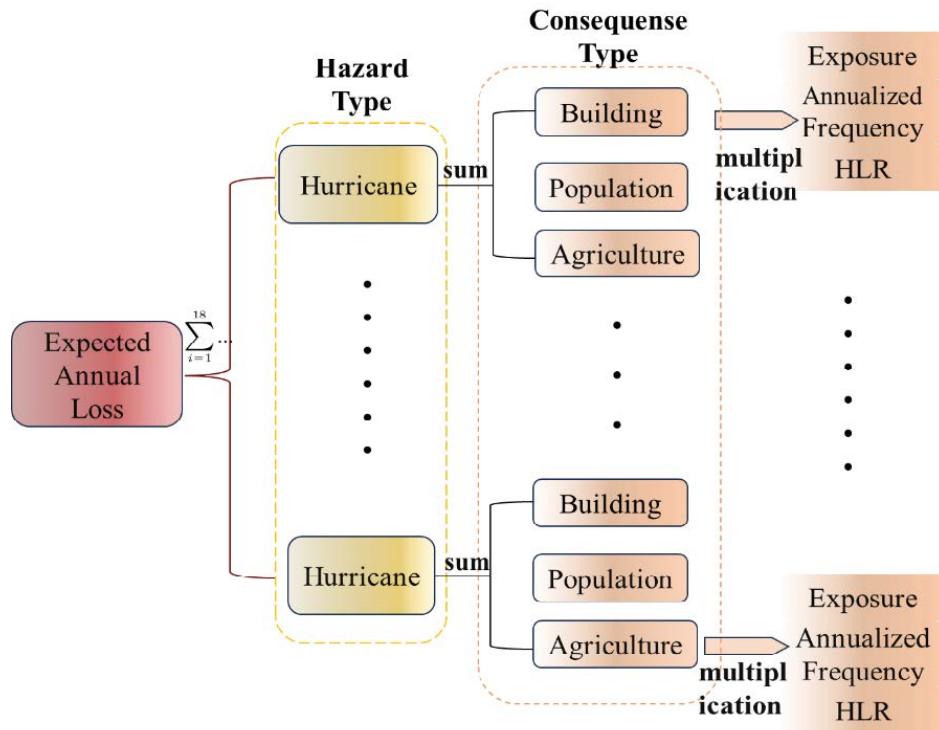


Figure 3: Calculation procedure for EAL

### 3.2.1

where  $R$  represents *Risk*.

This risk equation of  $R$  includes three components: a natural hazards risk component, a consequence enhancing component, and a consequence reduction component.  $EAL$  is the natural hazards risk component, measuring the expected loss of building value, population, and/or agriculture value each year due to natural hazards.  $CRF$  incorporates both the consequence enhancing component, denoted as  $SV$ , and the consequence reduction component, represented by  $CR$ .

For Equation (??), values for Risk and *EAL* are in units of dollars, representing the community average economic loss from natural hazards each year. For *SV* and *CR*, values are the index values for the community provided by the source data sets.

For Equation (??), each component is represented by a score that represents a community percentile ranking relative to all other communities at the same level. The composite risk score is calculated to measure a community's risk to all 18 hazard types. And it is also a community

percentile ranking in risk compared to all other communities at the same level. The risk score and *EAL* score are provided as both composite scores from the summation of all 18 hazard types.

We used the K-algorithm to categorize the disaster levels into five classes, as shown in Figure ??.

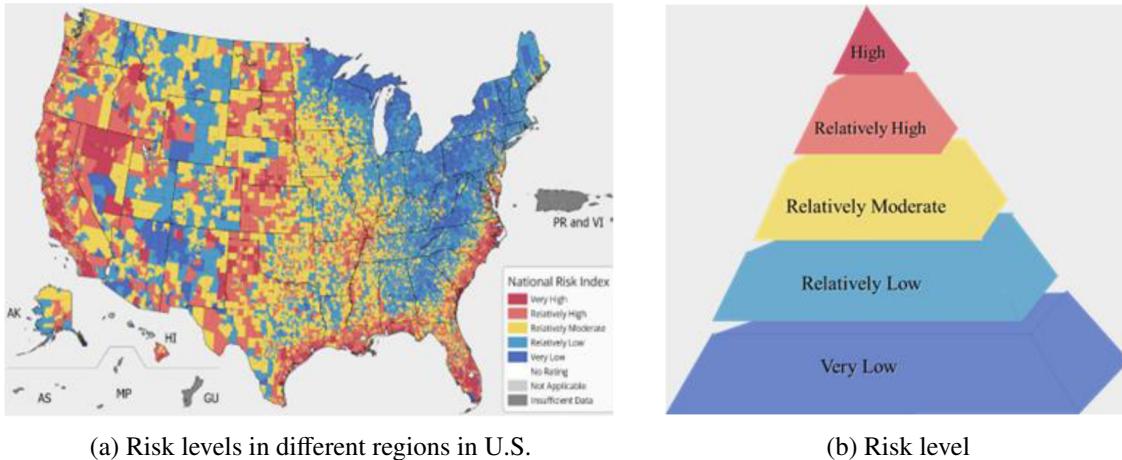


Figure 4: Risk rating and map presentation

### 3.3 Insurance Premium

Generally, the companies adopt the principle of Level Premium to determine the price of insurance. The company calculate the pure premium by using a pre-determined claim rate and a desired return on investment. The total premium is then obtained by adding surcharges at a certain expense ratio. Then, the total premium is obtained by adding surcharges at a certain expense ratio. Insurance companies increase their profitability by increasing expense ratios and reducing expected returns on investment. In addition, they utilize the Law of Large Numbers to set the overall payout ratio of the product so that it basically matches the statistical data, thus reducing the company's exposure to the risk of fluctuating payout ratios. For catastrophe insurance pricing, a natural disaster can cause huge property damage when it occurs. The insurance company may have a risk of becoming insolvent.

In finance, the capital asset pricing model (CAPM) is a model used to determine a theoretically appropriate required rate of return of an asset, to make decisions about adding assets to a well-diversified portfolio [2].

The reward-to-risk ratio for any individual security in the market is equal to the market reward-to-risk ratio, thus

$$\frac{E(r_i) - r_f}{\beta_i} = E(r_m) - r_f \quad (1)$$

$$\beta_i = \frac{\text{Cov}(R_i, R_m)}{\text{Var}(R_m)} = \rho(i, m) \frac{\sigma_i}{\sigma_m} \quad (2)$$

where

- $E(r_i)$  is the expected return on the capital asset,

- $E(r_m)$  is the expected return of the market,
- $r_f$  is the risk-free rate of interest such as interest arising from government bonds,
- $\beta_i$  is the sensitivity of the expected excess asset returns to the expected excess market returns,
- $\rho(i, m)$  denotes the correlation coefficient between the investment  $i$  and the market  $m$ ,
- $\sigma_i$  is the standard deviation for the investment  $i$ ,
- $\sigma_m$  is the standard deviation for the investment  $m$ .

Expected return on investment (ROI):

$$ROI = \int \frac{-A \times l(p) + (1+x)A \times R_{value} - I}{I} f(p) dp \quad (3)$$

Variance of ROI:

$$\text{Var}(ROI) = \int \left[ \frac{-A \times l(p) + A(1+x) \times R_{value} - l - (Ax \times R_{value} - l)}{I} \right]^2 f(p) dp \quad (4)$$

where

- $A$  is the sum insured,
- $I$  is the invested capital,
- $l(p)$  is the loss function,
- $x$  is the surcharge rate.

The surcharge rate  $x$  is a multiple of the average value of the loss:

$$x = \frac{\left(1 + r_f + \rho \times \frac{(A/I) \times \sigma}{\sigma_M} \times (r_M - r_f)\right) \times I}{A \times R_{value}} \quad (5)$$

where  $\sigma$  is the Standard deviation of  $R_{value}$ ,  $y$  is Pure premium per \$10,000:

$$y = 10000(1+x) \times P \quad (6)$$

where  $P$  is the probability of a disaster causing damage.

Typically, it is more reasonable to spend 3 – 10 percent of each person's annual income on insurance. We assume that each person is willing to spend 5% of his or her annual income each year to purchase catastrophe insurance with a one-year term. Insurance companies can make decisions from two perspectives based on the above formula:

- Introducing bankruptcy theory, after calculating the lowest order price,  $y$ , in the case where the probability of the firm's future bankruptcy is less than 10%, and then comparing it to the local per capita annual disposable income (GNI), it is expected that people in the locality will not be able to afford to consume catastrophe insurance and will not invest in it if the ratio of premiums per 10,000 to GNI is greater than 5%.
- We use 5% of the local national GNI per capita as the subscription price per \$10,000 of premium. If this price makes the likelihood of future insolvency of the company higher than 10%, no investment is made in that location.

The price of insurance also affects people's desire to buy to some extent, and an increase in the price of insurance may lead to a decrease in their desire to buy.

$$N = (1 - \omega y)N_A \quad (7)$$

$$\text{Total Revenue} = y \times N \quad (8)$$

where

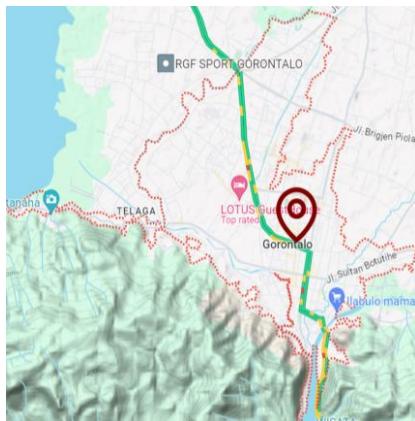
- $N_A$  is the total local population,
- $N$  is the number of local people with a strong desire to buy,
- $\omega$  is the factor that influences the price of insurance on the willingness of locals to buy, and is related to the average disposable income of locals as well as the gap between the rich and the poor,
- $\text{Total Revenue}$  is the projected total local insurance revenue.

The company first determines the area in which it wants to invest money to build the insurance and then determines the price of local insurance. We would like to maximize the company's total revenue:

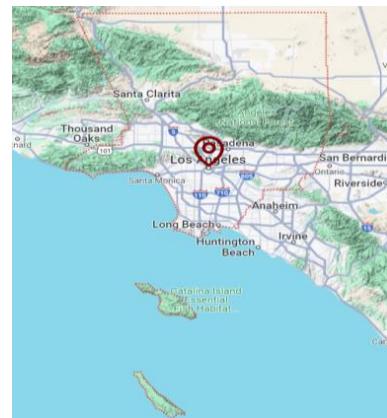
$$\begin{aligned} & \max \quad \text{Total Revenue} \\ \text{s.t. } & \begin{cases} 0 < \omega y < 1 \\ y \geq y_{10} \\ y < 0.05 \times GNI \end{cases} \end{aligned} \quad (9)$$

where  $y_{10}$  is the price of insurance when the firm's insolvency rate is 10 percent.

Our model is implemented in Gorontalo, Indonesia and Los Angeles, California. This is because both locations have similar and high risk indices, with Los Angeles having the highest disaster risk index in the United States.



(a) Gorontalo



(b) Los Angeles

Figure 5: Location of the two areas on the map

After searching for relevant data, we calculated that in order to ensure that the probability of the company's bankruptcy after investing in catastrophe insurance in Gorontalo is less than 10%, we need to charge a premium of \$342.745 for every \$10,000 of coverage, which is calculated in equation (??), of which \$283.465 is the pure premium and \$59.28 is the additional premium. Searching for relevant information we find that 5% of the per capita disposable income (GNI) of Gorontalo is only \$137.25, so the likelihood of residents being willing to purchase catastrophe insurance is low and the company should not invest in catastrophe insurance in the area.

In order to ensure that the probability of insolvency of the company after investing in catastrophe insurance in Los Angeles is less than 10%, through the formula (??) calculated that for every \$10,000 of coverage need to charge a premium of \$295.09 of which the pure premium is \$200 and the additional premium is \$95.09 (because of the higher return on investment in the market in the U.S.). The per capita disposable income in Los Angeles (5% of GNI is \$3,162.65), which is much higher than the cost of catastrophe insurance. In order to determine the most appropriate cost of insurance to earn a greater benefit, we plotted the trend of total premium income as a function of premiums.

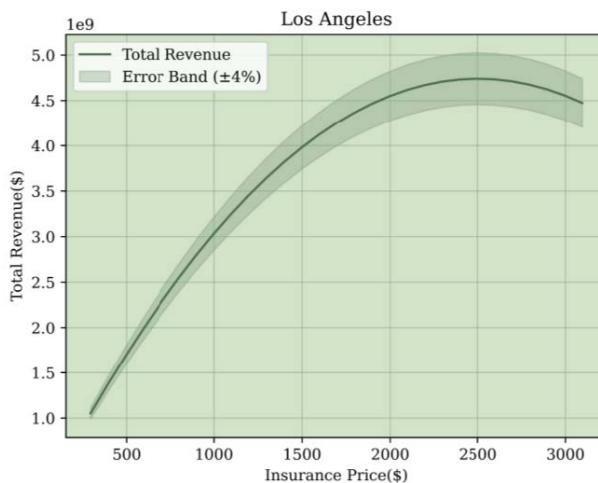


Figure 6: Relationship between company revenue and insurance price in Los Angeles

From the Figure ??, it can be seen that with the increase of premiums, the total income of insurance companies tends to increase first and then decrease. This is because when the premium is too low, although the number of insured people is high, the amount of single transaction is small and the number of guarantees is too high, which leads to a higher risk of bankruptcy of the insurance company; whereas too high a premium will reduce the consumer's expectations of catastrophe insurance, and the volume of insurance orders will be small.

In summary, for Los Angeles, a premium of about 2,500 per \$10,000 of coverage can be used, and 1.92 million people are expected to purchase the company's catastrophe insurance (the total population of Los Angeles is about 3.79 million). At this point, the insurance company's theoretical revenue would be around \$4.5 billion. Although Los Angeles has a high risk index, the profits are equally attractive, so the insurance company could take the risk of launching its catastrophe insurance business here.

## 3.4 Measures

### 3.4.1 Insurance Securitization

Catastrophe bonds are risk-linked securities that transfer a specified set of risks from a sponsor to investors. Catastrophe bonds emerged from a need by insurance companies to alleviate some of the risks they would face if a major catastrophe occurred, which would incur damages that they could not cover by the invested premiums [3, 4]. An insurance company issues bonds through an investment bank, which are then sold to investors. These bonds are inherently risky, and usually have maturities less than 3 years. If no catastrophe occurred, the insurance company would pay a coupon to the investors. But if a catastrophe did occur, then the principal would be forgiven and the insurance company would use this money to pay their claim-holders.

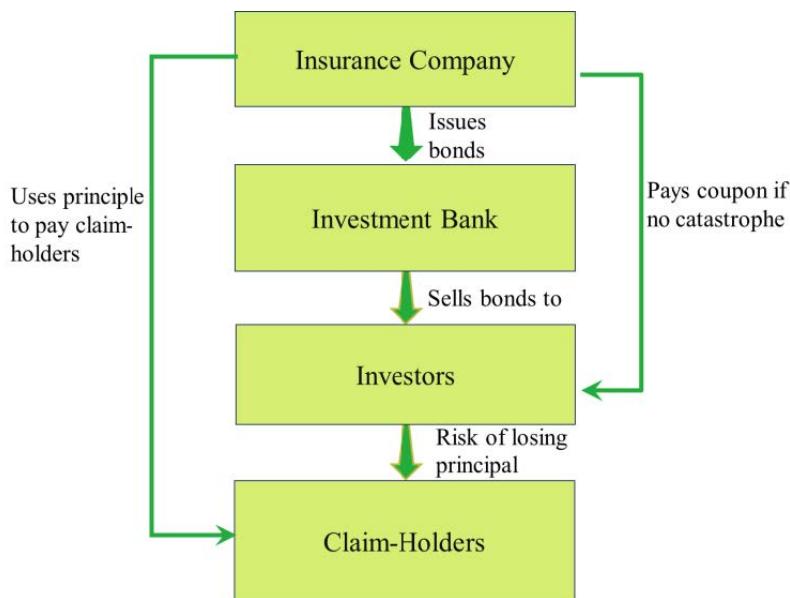


Figure 7: Insurance securitization schema

From an economic perspective, the securitization of insurance, particularly through instruments like catastrophe bonds, represents a significant innovation in the capital markets. This innovation not only diversifies investment opportunities but also plays a crucial role in enhancing the resilience of the insurance industry against catastrophic events. Catastrophe bonds allow insurance companies to transfer the risk of extreme events, such as natural disasters, to the capital markets, thereby reducing their potential liability and improving their solvency. This mechanism enables insurance firms to manage their risk exposure more effectively and to maintain stability in the face of potentially ruinous events. By doing so, it also ensures that insurance companies can continue to offer coverage for risks that might otherwise be uninsurable due to their catastrophic potential.

### 3.4.2 Co-operation with the Government

The government plays an important role in the country. The government can make some appealing policies to stimulate people to buy insurance and cooperate with insurance companies to undertake part of the risk. When people buy insurance, individuals are only required to bear part of the premium. The remainder is subsidized by the various levels of government. If necessary, special groups of people may be fully covered by government finances [5]. When a catastrophe

occurs, the government can act as a reinsurer and bear part of the amount of compensation. If the amount of compensation is small, the insurance company will pay directly. Otherwise, it can be covered or partially paid by the government. In this way, a multi-layered diversification of risk is constructed. It not only brings benefit protection to the people, but also drives the development of the insurance industry [6].

### 3.4.3 The Implementation of Measures

Through the two scenarios described above, the insurance company's market return on investment in the Gorontalo region  $r_m$  increased. When a natural disaster occurs, the amount of compensation paid by the insurance company is shared by the insurance company, the investors in the insurance securities, and the local government. In addition, the government subsidizes residents for catastrophe insurance, which increases the willingness of residents to purchase catastrophe insurance and reduces the actual cost paid by individuals. The insurance company can set premiums at the lowest premium (\$295.09) that can be assumed under the risk of insolvency. We plot the trend of total premium income as a function of premium at this point in time.

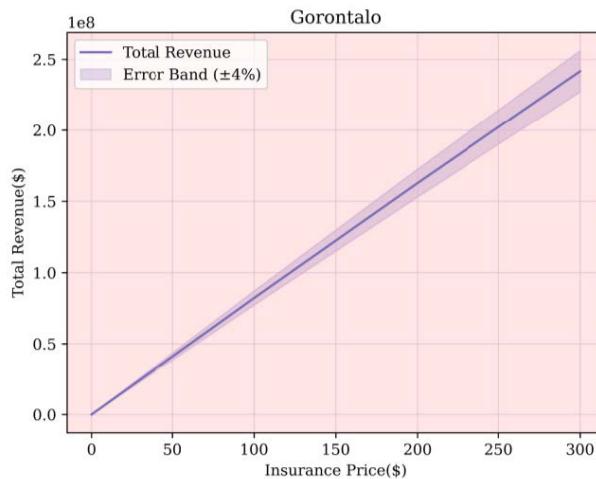


Figure 8: The Analog trend of total premium income in Gorontalo

As we can see from the picture. Under the company's affordable insolvency risk, the insurer expects maximum revenues of \$245 million. The company expects maximum revenue is \$245 million. At this time ,each \$10,000 of insurance amount charges \$295.09 of insurance premiums. We expect 830,255 people (about 73.26% of the total population) to have catastrophic insurance. Gorontalo residents pay only \$137.25 individually, and the remainder is subsidized by the Gorontalo government. The government guarantees the legal rights of Gorontalo residents as well as their social welfare.

### 3.5 Real Estate Decision Making

Our insurance model has a significant impact on the development decisions of real estate developers. Based on the above model derivation, it can be learned that for areas with high natural disasters and low per capita income, if the insurance company is willing to underwrite policies, it will result in the high bankruptcy rate of the company not being able to realize profitability [7]. Similarly, real estate developers will not choose the area for investment and development due to

high risk and lack of demand. That is, any area with high R value and income (GNI) below a certain value is not recommended for real estate developers to invest in. In addition to this, such areas have the following risk factors:



Figure 9: Risk factors

Further applying our model, we can calculate the insurance rate, which is the insurance premium divided by insurance amount. If the area has a high insurance rate by calculating, the property developer would have to bear a higher insurance cost during the construction of the building as well as during the unsold period. Therefore, property developers need to carefully consider and weigh the future profit and loss before making decisions.

Similarly, in other areas, we can calculate local insurance rates based on our model. According to this indicator, property developer can further determine the cost of developing land in local area and buying insurance. In this way we provide a reference for the property developers decision making.

Additionally, our model can also provide guidance about how property developers build construction. For each of the 18 hazard types, we can calculate the value of EAL and Rvalue (in dollars) for each hazard type. We find a positive correlation between EAL and Rvalue to some extent. Thus, property developers can determine the different major hazard type for each area based on Rvalue and thus build different types of homes. For example, in the city of New Orleans, USA, flooding ranks high on the list of 18 natural disasters in terms of Rvalue. Accordingly, many property companies, such as American Restorators LLC, are building houses with high foundations locally to minimize damage and achieve business profitability. Our model solves the problem about how to build on certain site. This approach not only maintains the interests of real property developers, but also protects the lives of people in the community.

## 4 Building Preservation Model

### 4.1 Building Value Quantification

Building value is measured in terms of the building's cultural value and community influence, economic value, and historical value. Therefore, we take these three main aspects as primary indicators.

#### 4.1.1 Indicators Determination

For the cultural value and community influence, we synthesized various factors, such as geography and network, and finally selected the three most representative secondary indicators to construct our model. Similarly, for the economic and historical value, we selected two secondary indicators each to improve the model. The specific description and indicators selected are shown in Table ??.

Table 4: Indicators

Object	Indicators	Description
Cultural Values and Community influence	NG	Number of Google search terms
	P	Participation in events held around the building
	ANV	Annual number of visitors
Economy	LV	Land value
	CC	Construction cost
History	NH	Number of historical research documents
	DP	Degree of preservation

- Cultural Value and Community Influence

  - Global Visibility

The cultural value of a building depends to a large extent on its global visibility. So we quantify its global visibility through two metrics, “Number of Google search terms” (NG) and “Annual number of visitors” (ANV). This approach balances online and offline, making the measurement of cultural values more quantifiable and accurate.

  - Impact on the Community

Buildings have a strong connection with local communities. When measuring the value of a building, we take into account its impact on the local community. Research has shown that the more influence a building has on the local community, the more the value of the landmark itself will increase. Besides, it will further promote the increase of influence, realizing a positive feedback loop. Therefore, we choose “Participation in events held around the building” (P) to quantify the building’s influence on the community. We calculate P as follows:

$$P = \frac{1}{N} \times \sum_{i=1}^N \frac{NCMP_i}{NTC_i} \quad (10)$$

where  $NCMP_i$  represents “Number of community members participating in activities” at the  $i$ th activity,  $NTC_i$  means the total number of people in the community at the time of the  $i$ th activity and N means the total number of activities conducted around the building.

- Economy Value

For economic value, we mainly consider the value of the building in terms of its construction. Therefore, we considered the value of the land it occupies. And it is measured by the indicator “Land value” (LV).

$$LV = P_c \times Area \quad (11)$$

where  $P_c$  represents the current price of the land and Area represents the area occupied by the building. Meanwhile, for the value created during the construction of the building itself, we use “Construction cost” (CC) for quantitative assessment. Taking inflation into account, we define Construction cost as all costs involved in the implementation of that construction project under this year’s Engineering News-Record (ENR) benchmark for the region. Both the  $LV$  and  $CC$  metrics are expressed in U.S. dollars.

- Historic Value

- Historical Research Value

The historical value of a building is largely dependent on its place in historical research. So we quantify its visibility and importance in the academic world through NH. NH refers to the number of historical research documents related to the building, including but not limited to books, papers, reports, etc. This indicator reflects the building’s attention and depth of research in the historical community. The higher NH value means the building has a higher historical research value.

- The Preservation Condition

The historic value of a building is also affected by its state of preservation. We use “Degree of preservation” (DP) to measure the extent to which a building has been preserved from its original state. It includes aspects such as structural integrity, exterior preservation, and interior decoration. The assessment of DP can be based on expert review, preservation grade, and comparative analysis with the original state. Highly preserved buildings not only better transmit history and culture, but also provide rich materials for future research.

#### 4.1.2 Weight Calculation

CRITIC is an objective assignment method based on data volatility. The idea of this method was based on two indicators, contrast intensity and correlation indicators. When calculating the weights, we need to multiply the contrast intensity with the correlation indicator and then normalize to get the final weights.

- Contrast intensity refers to the magnitude of the difference in values between evaluation programs for the same indicator, expressed as a standard deviation. The larger the standard deviation, the greater the fluctuation . That is, the larger the difference in the values taken between the programs, the higher the weight will be.
- The Sperman correlation coefficient is used to express the correlation between indicators. If there is a strong positive correlation between two indicators, it means that the less conflicting they are, the lower the weight will be.

1) There are  $n$  samples to be evaluated and  $p$  evaluation indicators to form the raw indicator data matrix.

$$X = \begin{pmatrix} x_{11} & \cdots & x_{1p} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{np} \end{pmatrix}$$

where  $x_{ij}$  represents the value of the  $j$ th evaluation indicator for the  $i$ th sample.

2) In order to remove the effect of the scale each indicator is normalized. The indicators we selected are of benefit attributes type, so the normalization formula:

$$X_{ij} = \frac{x_{ij} - \min(x_j)}{\max(x_j) - \min(x_j)}$$

where  $X_{ij}$  is normalized to obtain a numerical matrix.

3) Then we calculate the contrast intensity of the indicator:

$$\begin{cases} \bar{x}_j = \frac{1}{n} \sum_{i=1}^n x_{ij} \\ S_j = \sqrt{\frac{\sum_{i=1}^n (x_{ij} - \bar{x}_j)^2}{n-1}} \end{cases}$$

where  $S_j$  represents the strength of comparison of the  $j$ th indicator.

The larger the  $S_j$ , the greater the difference in values for that indicator. The more information the indicator reflects, the stronger the evaluation strength of the indicator itself and the more weight should be assigned to it.

4) Calculation of the conflicting nature of the indicators

$$\begin{cases} d_i = \text{rank}(x_{ij}) - \text{rank}(x_{ik}) \\ r_{jk} = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)} \\ R_j = \sum_{k=1, k \neq j}^p (1 - r_{jk}) \end{cases}$$

where  $R_{jk}$  denotes the Spearman correlation coefficient between evaluation indicators  $j$  and  $k$ .  $R_j$  denotes the conflictual of the  $j$ th indicator.

The Spearman correlation coefficient is used to express the correlation between indicators. The stronger the correlation between two indicators, the less they conflict, the more they reflect the same information, and the more repetitive the content of the evaluation is. To a certain extent, the evaluation strength of the indicator is weakened and the weight assigned to it should be reduced.

5) Calculation of the amount of information:

$$C_j = S_j \times R_j$$

6) Based on the amount of information, we calculate the weights of each indicator defined  $w_j$ :

$$w_j = \frac{C_j}{\sum_{j=1}^p C_j}$$

7) The score for each indicator is:

$$Score_{object} = \sum_{j=1}^p w_j s_j$$

where  $object$  represents “Cultural values and community influence”, “Economy”, “History”,  $s_j$  denotes the value of the  $j$ th secondary indicator.

Applying the CRITIC weighting method for each level 1 indicator separately, the objective weights for each level 2 indicator were obtained as shown in the following Table 1.

#### 4.1.3 Quantitative Results of Building Values

In order to assign weights to these three level 1 indicators to get the final building value, we use hierarchical analysis to construct a judgment matrix to get the weights of the three level 1 indicators:

$$\theta = (0.4432, 0.3873, 0.1694)$$

where the consistency ratio of the judgment matrix = 0.017591, and the consistency is acceptable. Ultimately, our building impact score is calculated as follows:

$$V_{score} = \sum_{i=1}^3 \theta_i Score_{object} \quad (12)$$

Table 5: The weight of indicators

Object	Weight	Indicators	Weight
Cultural Values and Community influence	0.4432	NG	0.1698
		P	0.4429
		ANV	0.3873
Economy	0.3873	LV	0.4272
		CC	0.5728
History	0.1694	NH	0.6286
		DP	0.3714

## 4.2 Determination of protection measures

### 4.2.1 Measure Score

In Model 1, we obtained a composite risk score  $R_{score}$  of each region by analyzing 18 natural hazards. Next, in the above section, we quantified the value of the building to get the score  $V_{score}$ . By multiplying the risk score and the value score,  $M_{score}$  is obtained, which is used to assess the conservation priority of the building and the extent and scale of conservation measures that need to be taken.

$$M_{score} = V_{score} \times R_{score} \quad (13)$$

A higher  $M_{score}$  indicates a higher value of the building, along with a higher risk of exposure to natural hazards. Therefore, more urgent and comprehensive protection measures are needed. Based on the statistical distribution of  $M_{score}$ , we set reasonable thresholds to recognize low, medium, and high grades, and the values of the specific thresholds need to be set based on expert recommendations and industry standards.

#### 4.2.2 Score of Protection Measures

- **Low:** For low-grade  $M_{score}$  buildings, basic conservation measures, such as routine maintenance and inspections and, where necessary, minor repairs, are undertaken. The risk or value of these buildings is low, so the measures taken are mainly preventive and low-cost.
- **Medium:** For medium-grade  $M_{score}$ , moderate protection measures are implemented, including enhanced structural inspections, improved safety features, and disaster preparedness programs. These measures aim to increase the resistance and resilience of buildings and require moderate investment.
- **High:** For high-grade  $M_{score}$ , implement comprehensive and high-intensity protection measures. This may include comprehensive structural reinforcement, installation of advanced security systems, and customized risk management plans in cooperation with external experts. Given the high risk or value of these buildings, the goal of the measures is to minimize potential losses, even if this means higher initial costs.

**Note that:** The “one-size-fits-all” approach to disaster protection measures ignores the impact of regional differences, architectural characteristics and socio-economic factors, and can lead to poor protection and resource utilization. So specific protection measures still need to be derived from a thoughtful local analysis by natural disaster experts.

#### 4.2.3 Mentoring for Community Leaders

Our model provides a quantitative and systematic framework for community leaders to help them determine the extent and priority of preservation measures based on a building’s risk

and value scores. The model makes the decision-making process more scientific and accurate by combining risk scores (which consider threats such as natural hazards) and value scores (which include historical, cultural, economic, and community importance). It promotes optimal allocation of resources and ensures that high-value or high-risk buildings are adequately protected, while also taking into account economic benefits. In addition, the model encourages community involvement and support, improves disaster response capacity, and supports sustainable community development. Through this approach, community leaders are able to make more informed decisions to protect and maintain important buildings in their neighborhoods, contributing to the overall well-being and development of the community.

## 5 Landmark Case Analysis

We select Tokyo Tower in Japan as the landmark for evaluation and analysis.



Figure 10: Location of Tokyo Tower

## 5.1 Insurance Pricing for Tokyo Tower

Designed by Japanese architect Tachu Naito, the Tokyo Tower cost \$8.4 million to build at the time and was constructed to send broadcast signals in Tokyo. Currently, Tokyo's GNI per capita is \$36,964.96, and the Tokyo capital market is currently functioning well. The risk score for Tokyo is calculated to be 86. According to the insurance pricing model, the optimal insurance rate for an insurance company to issue catastrophe insurance in Tokyo is 3.8%. In view of the special historical value of the Tokyo Tower, the insurance company may appropriately increase the insurance rate. As a result, the Tokyo government spends approximately US \$319,200 per year on catastrophe insurance for the Tokyo Tower.

## 5.2 Architectural Value of Tokyo Tower

In 2011, a major earthquake struck northeastern Japan, and this earthquake caused some damage to Tokyo Tower, bending the antenna at the tip of the tower by 2 degrees, resulting in the end of the experimental broadcasting of terrestrial wave digital sound broadcasting, and the interruption of the transmission of the 24/7 terrestrial analog television signals. Tokyo Tower entered a maintenance period during which analog signals and FM broadcasting-related services also began to be transferred to Tokyo Skytree, so the proportion of Tokyo Tower's actual use gradually decreased. However, Tokyo Tower has attracted more than 3 million visitors as a tourist attraction and has accumulated more than 150 million visitors. It is also a symbol of Japan's post-war prosperity and has a remarkable historical significance for the Japanese people.

According to the calculations of the building conservation model, the Mscore of Tokyo Tower is located in a high conservation level area. Therefore, the government should consider purchasing catastrophe insurance for Tokyo Tower and strengthening its daily supervision and maintenance to ensure that it can withstand natural disasters, such as earthquakes, and that its seismic treatment measures also need to be strengthened.

# 6 Sensitivity and Robustness Analysis

## 6.1 Sensitivity

In section 4.3, factor is introduced to estimate the parameters of the expected return of the market. Therefore, change the size of this parameter, that is, the capital market environment has changed. Below we analyze the sensitivity of this parameter. Gradual reduction of the parameter

by 5%. The reason for considering only a decrease in  $r$  and not an increase in  $r$  is to reflect the worst-case scenario, i.e. a gradual decrease in capital market returns, and to see if our model is sensitive to the parameter.

Therefore, Re-simulate the calculation results and obtain 3 sets of curves as shown in Figure ??.

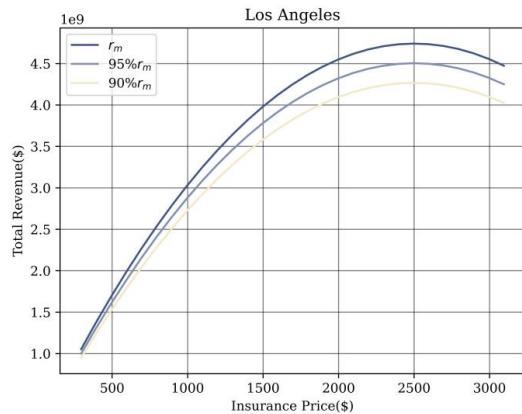


Figure 11: Sensitivity analysis of  $r_m$

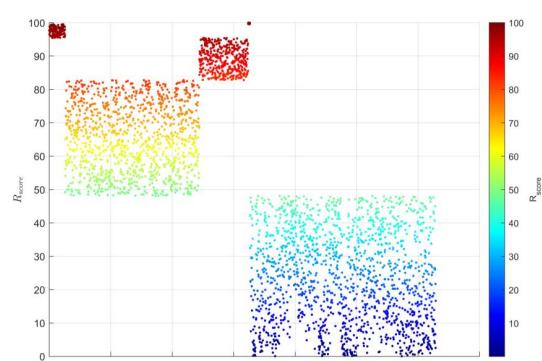


Figure 12: Robustness analysis of  $R_{value}$

The results show that as  $r$  decreases, the profit gained by the insurance company if it sells the insurance company at the same price tends to decrease. This makes sense because lower capital market interest rates result in lower profits for insurance companies. The trend of the curve obtained by sensitivity test is consistent with the actual situation.

## 6.2 Robustness

We verify the robustness of the model. Given the uncertainty of natural hazards, we may have errors in the calculation of Rvalue, which affects our grading of the risk indicators for each region. Randomly selecting some of the more than 3,000 counties in the United States and deviating its Rvalue by 5%, and again grading these areas, the results obtained are shown in Figure ??.

As can be seen from the figure, the grading of the original regions changes only slightly after randomly selecting regions to bias their measurements. This indicates that the small error in  $R$  does not cause large changes in the model results and our model is stable.

# 7 Model Evaluation

## 7.1 Strengths

**Robustness and Flexibility:** Our model demonstrates strong adaptability to various parameter variations through sensitivity analysis and robustness testing, and is able to provide reliable predictions under different scenarios.

**Comprehensive consideration:** Our model integrates social and economic factors, natural disaster risks, and provides a way to price insurance.

## 7.2 Weaknesses

**Data Dependency:** Although the model has a high demand for data quality, it reflects our scientific attitude of pursuing accuracy and real-time performance. By working with data providers,

we can continuously optimize the data collection and processing process.

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## A Letter to Community Members

Dear Esteemed Members of Tokyo Tower Community,

We are researchers dedicated to the study of architectural preservation. In order to realize the preservation of Tokyo Tower at a lower cost, please allow me to introduce our proposal on behalf of our team.

We have recently concluded a comprehensive analysis aimed at ensuring the future sustainability and preservation of our cherished Tokyo Tower. Our findings offer a strategic blueprint that prioritizes not only the physical well-being of this iconic structure but also its cultural and historical essence.

### **Recommendation Plan:**

Our proposed plan encompasses innovative preservation techniques, structural enhancements, and community-centric initiatives designed to safeguard and celebrate the tower's legacy. We aim to implement cutting-edge solutions that address both current vulnerabilities and future challenges.

### **Implementation Timeline:**

This ambitious project is scheduled over a five-year period, beginning with immediate preliminary assessments and followed by successive phases of structural upgrades and community engagement activities. This phased approach ensures meticulous attention to detail and the successful realization of our objectives.

### **Budget Overview:**

The financial blueprint for this venture is estimated at \$320,000 per annum. This budget encompasses costs associated with structural reinforcements, preservation technology, and community enrichment programs. It represents a balanced investment in tower's longevity and its significance to our community.

We stand at a pivotal moment in the history of Tokyo Tower, poised to embark on a journey that not only secures its future but also reinvigorates its role within our community. We invite you to support this noble cause, ensuring that Tokyo Tower remains a symbol of resilience, heritage, and communal unity for countless years to come.

Sincerely yours,  
**Team #2616863**

# Appendices

## Appendix A First Appendix

### Input Python source:

---

```
import numpy as np

def fft(x):
    n = len(x)
    if n == 1:
        return x
    even = fft(x[0::2])
    odd = fft(x[1::2])
    T = [np.exp(-2j * np.pi * k / n) * odd[k] for k in range(n // 2)]
    return [even[k] + T[k] for k in range(n // 2)] + [even[k] - T[k] for k in
        range(n // 2)]
```

---

### Input Matlab source:

---

```
function [t,seat,aisle]=OI6Sim(n,target,seated)
pab=rand(1,n);
for i=1:n
    if pab(i)<0.4
        aisleTime(i)=0;
    else
        aisleTime(i)=trirnd(3.2,7.1,38.7);
    end
end
```

---

## **Report on Use of AI**

In preparing for the COMAP mathematical modeling competition, our team has thoughtfully integrated artificial intelligence as a strategic tool to augment our research capabilities, deepen our analytical insights, and refine our solution development. We recognise that in the current AI era, such technologies serve best as assistants that enhance human expertise, not as shortcuts that replace critical thinking or intellectual rigor.

Accordingly, this report provides clear documentation of each instance of AI utilization, detailing both its functional contributions and the ethical framework guiding its application. By maintaining this transparency, we intend to uphold academic integrity, promote fairness in competition, and contribute to a culture of accountability within the scientific community.

We believe this forthright approach not only clarifies the supportive role AI played in our project but also reflects our teams commitment to principled and conscientious scholarship, where innovation is balanced with responsibility, and technological advantage is aligned with virtuous conduct.

### 1. OpenAI ChatGPT [8] (Jan 25, 2024 version, ChatGPT-4)

Query1: Give me some brainstorming ideas for innovative data visualization techniques.

Output: Here are some innovative data visualization technique ideas:

- (1) Interactive Dashboards: Develop dashboards that allow users to interact with the data, such as filtering, zooming, and selecting specific data points to see detailed information.
- (2) Animated Visualizations: Use animations to show changes in data over time, helping to illustrate trends, cycles, or progressions.
- (3) Augmented Reality (AR) Visualizations: Implement AR to overlay data on real-world objects or environments, enhancing understanding of spatial and contextual relationships.
- (4) 3D Visualizations and Models: Employ 3D models to represent complex data structures, allowing exploration from different angles and perspectives.
- (5) Narrative Visualizations: Combine data visualization with storytelling, guiding viewers through a narrative that explains the significance of the data in a compelling way.
- (6) Real-time Data Streaming: Visualize data in real-time, updating visuals as new data comes in, ideal for monitoring and tracking systems.
- (7) Data Sonification: Convert data points into audio signals, making it possible to “listen” to the data for patterns, trends, or anomalies.

These ideas can help present data in more engaging, insightful, and accessible ways.

### 2. OpenAI ChatGPT (Jan 25, 2024 version, ChatGPT-4)

Query1: What is Engineering News-Record? Give me a brief introduction.

Output: Engineering News-Record (ENR) is a prominent publication within the engineering and construction industry, providing news, analysis, commentary, and data vital for professionals in the field. The publication's roots can be traced back to two separate publications from the late 19th century, eventually merging into what we know today as ENR in 1917. It has

evolved to offer a wide range of content including top design firms, construction companies, projects in the U.S. and around the world, and industry trends affecting business management, design, construction methods, technology, and more. ENR is recognized for its comprehensive rankings of the largest construction and engineering firms, based on gross revenues, and for its ENR Award of Excellence, which honors individuals who have made significant contributions to the industry. Today, ENR is owned by BNP Media and continues to serve as an essential resource for contractors, project owners, engineers, architects, and other industry stakeholders with its print and digital magazine, website, newsletters, webinars, and events.

3. OpenAI ChatGPT (Jan 25, 2024 version, ChatGPT-4)

Query1: What's the ENR for?

Output: The Engineering News-Record (ENR) serves several vital functions in the engineering and construction industry. It is recognized for:

(1) News and Analysis: ENR provides comprehensive news coverage, analysis, and commentary on various topics relevant to the construction industry, including major projects, industry trends, technological advancements, and issues affecting the sector such as safety, law, and labor.

(2) Rankings and Data: It publishes annual rankings of the largest construction and engineering firms based on gross revenues, including lists like the Top 400 U.S. General Contractors, Top 500 U.S. Design Firms, and Top 600 Specialty Contractors, among others. These rankings are widely respected and used as benchmarks within the industry.

(3) Awards and Recognition: ENR recognizes industry excellence through various awards, including the ENR Award of Excellence, which honors individuals and projects that have made significant contributions to the industry.

(4) Industry Insights: The publication offers insights on business management, project delivery, technology, workforce development, and other critical areas impacting the engineering and construction fields.

ENR serves as a key resource for professionals across the construction industry, including contractors, project owners, engineers, architects, and suppliers, connecting diverse sectors with coverage that addresses common challenges and opportunities.