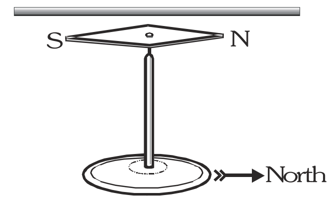
MAGNETIC EFFECT OF CURRENT

The branch of physics which deals with the magnetism due to electric current or moving charge (i.e. electric current is equivalent to the charges or electrons in motion) is called electromagnetism.

ORESTED'S DISCOVERY

The relation between electricity and magnetism was discovered by Orested in 1820. Orested showed that the electric current through the conducting wire deflects the magnetic needle held below the wire.



- When the direction of current in conductor is reversed then deflection of magnetic needle is also reversed
- On increasing the current in conductor or bringing the needle closer to the conductor the deflection of magnetic needle increases.

  Oersted discovered a magnetic field around a conductor carrying electric current. Other related facts are as follows:

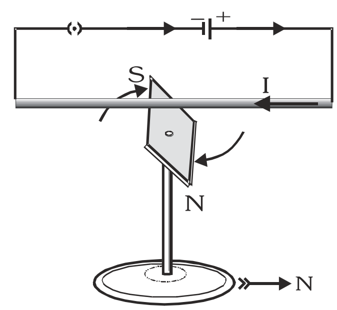
  (a) A magnet at rest produces a magnetic field around it while an electric charge at rest produce an electric field around it.

  (b) A current carrying conductor has a magnetic field and not an electric field around it. On the other hand, a charge moving with a uniform velocity has an electric as well as a magnetic field around it.

  (c) An electric field cannot be produced without a charge whereas a magnetic field can be produced without a magnet.

  (d) No poles are produced in a coil carrying current but such a coil shows north and south polarities.

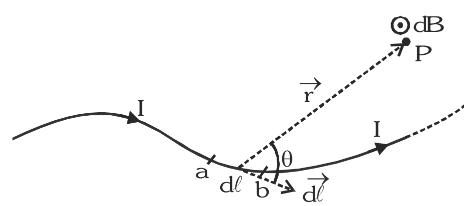
  (e) All oscillating or an accelerated charge produces E.M. waves also in additions to electric and magnetic fields.



Oersted's experiment. Current in the wire deflects the compass needle

- **Current Element**

  A very small element ab of length $d\ell$ of a thin conductor carrying current I called current element. Current element is a vector quantity whose magnitude is equal to the product of current and length of small element having the direction of the flow of current.



- **Biot – Savart's Law**

  With the help of experimental results, Biot and Savart arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the field. That expression is based on the following experimental observations for the magnetic field $d\vec{B}$ at a point P associated with a length element $d\vec{\ell}$ of a wire carrying a steady current I.

$$dB \propto I,\ \ dB \propto d\ell\ ,\ dB \propto \sin\theta \text{ and } dB \propto \frac{1}{r^2} \Rightarrow dB \propto \frac{Id\ell\sin\theta}{r^2} \Rightarrow dB = \frac{\mu_0}{4\pi}\frac{Id\ell\sin\theta}{r^2}$$

Vector form of Biot-Savart's law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\ell\sin\theta}{r^2}\tilde{n}$$

$\hat{n}$ =unit vector perpendicular to the plane of $(Id\vec{\ell})$ and $(\vec{r})$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\vec{r}}{r^3}$$

$[\because\ Id\vec{\ell}\ \ \ \vec{r} = (Id\ell)\ (r)\sin\theta\ \ \tilde{n}]$

GOLDEN KEY POINTS

- According to $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\vec{r}}{r^3}$, direction of magnetic field vector $(d\vec{B})$ is always perpendicular to the plane of vectors $(Id\vec{\ell})$ and ($\vec{r}$), where plane of $(Id\vec{\ell})$ and ($\vec{r}$) is the plane of wire.
- Magnetic field on the axis of current carrying conductor is always zero (θ=0 or θ = 180)
- Magnetic field on the perimeter of circular loop or coil is always minimum.

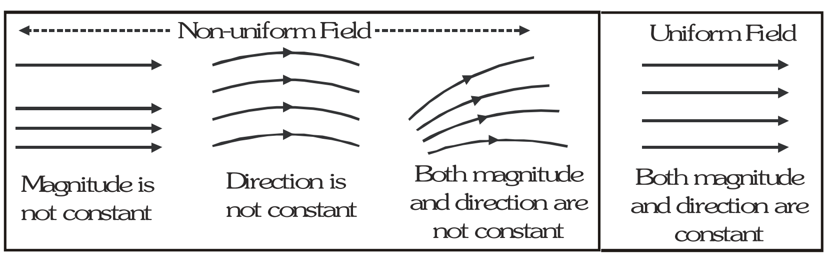
MAGNETIC FIELD LINES (By Michal Faraday)

In order to visualise a magnetic field graphically, Michal faraday introduced the concept of field lines.

Field lines of magnetic field are imaginary lines which represents direction of magnetic field continuously.

GOLDEN KEY POINTS

- Magnetic field lines are closed curves.
- Tangent drawn at any point on field line represents direction of the field at that point.
- Field lines never intersects to each other.
- At any place crowded lines represent stronger field while distant lines represents weaker field.
- In any region, if field lines are **equidistant and straight** the field is uniform otherwise not.



- Magnetic field lines emanate from or enters in the surface of a magnetic material at any angle.
- Magnetic field lines exist inside every magnetised material.
- Magnetic field lines can be mapped by using iron dust or using compass needle.

RIGHT HAND THUMB RULE

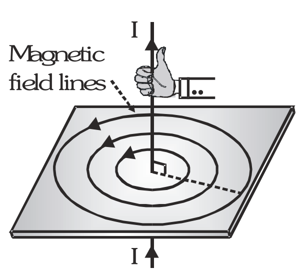
This rule gives the pattern of magnetic field lines due to current carrying wire.

(i) Straight current

Thumb $\rightarrow$ In the direction of current

Curling fingers $\rightarrow$ Gives field line pattern

Case I : wire in the plane of the paper

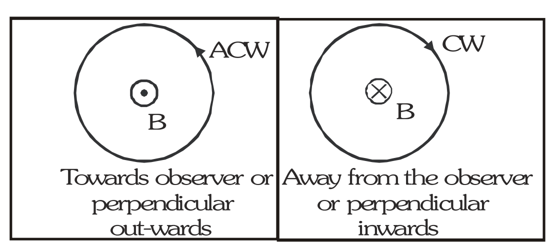


(ii) Circular current

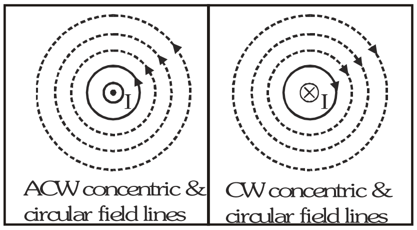
Curling fingers $\rightarrow$ In the direction of current,

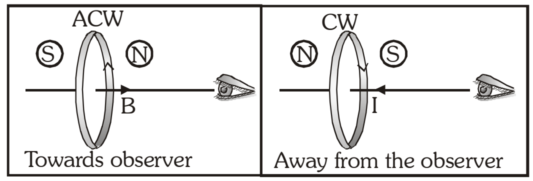
Thumb $\rightarrow$ Gives field line pattern

Case I : wire in the plane of the paper



Case II : Wire is $\perp$ to the plane of the paper. ***Case II :*** Wire is $\perp$ to the plane of the paper

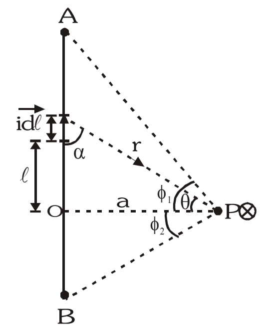




GOLDEN KEY POINTS

- When current is straight, field is circular
- When current is circular, field is straight (along axis)
- When wire is in the plane of paper, the field is perpendicular to the plane of the paper.
- When wire is perpendicular to the plane of paper, the field is in the plane of the paper.

APPLICATION OF BIOT-SAVART LAW :



- **Magnetic field surrounding a thin straight current carrying conductor**

  AB is a straight conductor carrying current i from B to A. At a point P, whose perpendicular distance from AB is OP =a, the direction of field is perpendicular to the plane of paper, inwards (represented by a cross)

  ℓ=a $\tan\theta \Rightarrow$dl=a $\sec^2\theta d\theta$...(i)

  α=90 $-\theta$ & r=a$\sec\theta$

- **By Biot-Savart's law**

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{id\ell\sin\alpha}{r^2}\otimes \text{ (due to a current element } id\ell \text{ at point P)}$$

$$\Rightarrow B=\int dB = \int\frac{\mu_0}{4\pi}\frac{id\ell\sin\alpha}{r^2} \quad \text{(due to wire AB)} \quad \therefore B=\frac{\mu_0 i}{4\pi}\int\cos\theta d\theta$$

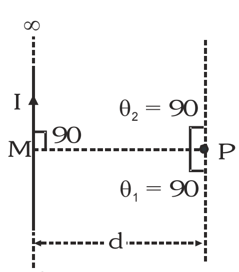
Taking limits of integration as $-\phi_2 -\phi_2$ to ϕ_1

$$B = \frac{\mu_0 i}{4\pi a}\int_{-\phi_2}^{\phi_1}\cos\theta d\theta = \frac{\mu_0 i}{4\pi a}\left[\sin\theta\right]_{-\phi_2}^{\phi_1} \frac{\mu_0 i}{4\pi a}\left[\sin\phi_1 + \sin\phi_2\right] \quad \text{(inwards)}$$

Example

Magnetic field due to infinite length wire at point 'P'

Solution

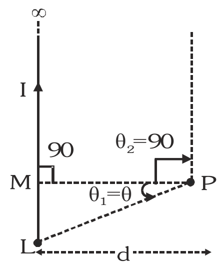


$$B_P = \frac{\mu_0 I}{4\pi d}[\sin 90 + \sin 90] \qquad B_P = \frac{\mu_0 I}{2\pi d}$$

Example

Magnetic field due to semi infinite length wire at point 'P'

Sol. $B_P = \dfrac{\mu_0 I}{4\pi d}[\sin\theta + \sin 90]$



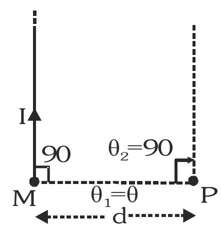
$B_P = \dfrac{\mu_0 I}{4\pi d}[\sin\theta + 1]$

Example

Magnetic field due to special semi infinite length wire at point 'P'

Solution

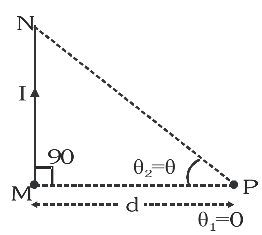
$B_P = \dfrac{\mu_0 I}{4\pi d}[\sin 0 + \sin 90]$



$B_P = \dfrac{\mu_0 I}{4\pi d}$

Example

Magnetic field due to special finite length wire at point 'P'

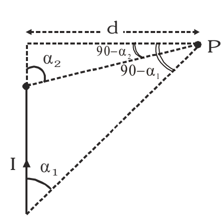


Solution

$B_P = \dfrac{\mu_0 I}{4\pi d}[\sin 0 + \sin\theta]$; $B_P = \dfrac{\mu_0 I}{4\pi d}\sin\theta$

Example

If point 'P' lies out side the line of wire then magnetic field at point 'P' :

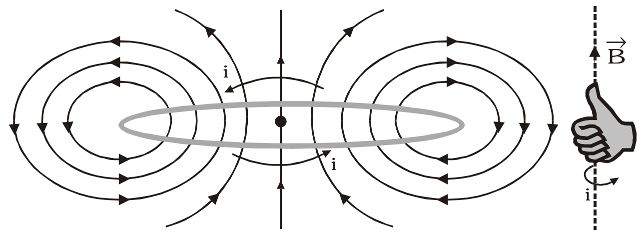


Solution

$$B_P = \frac{\mu_0 I}{4\pi d}\left[\sin(90-\alpha_1) - \sin(90-\alpha_2)\right] = \frac{\mu_0 I}{4\pi d}(\cos\alpha_1 - \cos\alpha_2)$$

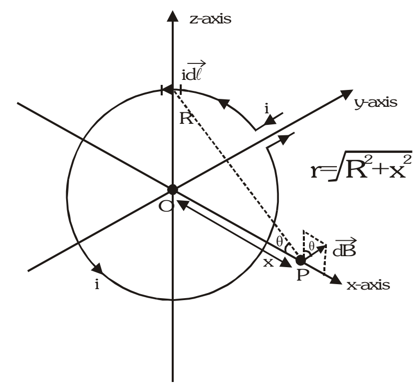
- **Magnetic field due to a loop of current**

Magnetic field lines due to a loop of wire are shown in the figure



The direction of magnetic field on the axis of current loop can be determined by right hand thumb rule. If fingers of right hand are curled in the direction of current, the stretched thumb is in the direction of magnetic field.

- **Calculation of magnetic field**

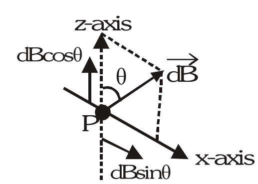


Consider a current loop placed in y-z plane carrying current i in anticlockwise sense as seen from positive x-axis. Due to a small current element $id\vec{\ell}$ shown in the figure, the magnetic field at P is given by $dB = \frac{\mu_0}{4\pi}\frac{id\ell\sin 90^0}{r^2}$.

The angle between $id\vec{\ell}$ and $\vec{r}$ is 90^0 because $id\vec{\ell}$ is along y-axis, while $\vec{r}$ lies in x-z plane. The direction of $\overrightarrow{dB}$ is perpendicular to $\vec{r}$ as shown. The vector $\overrightarrow{dB}$ can be resolved into two components, dB cosθ along z-axis and dB sinθ along x-axis.

For any two diametrically opposite current elements, the components along x-axis add up, while the other two components cancel out. Therefore, the field at P is due to x-component of field only. Hence, we have

$$B= \int dB\sin\theta = \int \frac{\mu_0}{4\pi}\frac{id\ell}{r^2}\sin\theta = \int \frac{\mu_0}{4\pi}\frac{id\ell}{r^2}\times\frac{R}{r} \quad \therefore B= \frac{\mu_0 iR}{4\pi r^3}\int d\ell \Rightarrow \int d\ell = 2\pi R$$



$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{i\times 2\pi R^2}{r^3} = \frac{\mu_0}{4\pi}\frac{i\times 2\pi R^2}{\left(R^2+x^2\right)^{3/2}} \quad \left(\because r=\sqrt{R^2+x^2}\right)$$

(a) At the centre, x=0, $B_{centre} = \frac{\mu_0 i}{2R}$

(b) At points very close to centre, $x<<R \Rightarrow B=\frac{\mu_0 i}{2}\left(1+\frac{x^2}{R^2}\right)^{-3/2} = \frac{\mu_0 i}{2}\left(1-\frac{3x^2}{2R^2}\right)$

(c) At points far off from the centre, $x>>R \Rightarrow B=\frac{\mu_0\ell}{4\pi}\frac{2\pi R^2}{x^3}$

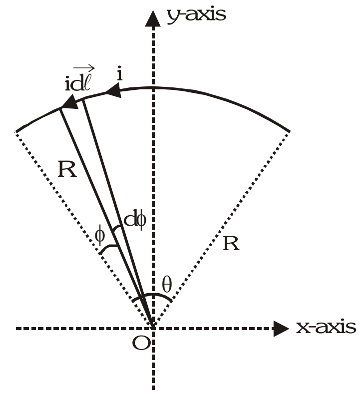
(d) The result in point (c) is also expressed as $B = \frac{\mu_0}{4\pi}\frac{2M}{x^3}$

where $M= \ell\times\pi R^2$, is called magnetic dipole moment.

Example

Find the magnetic field at the centre of a current carrying conductor bent in the form of an arc subtending angle θ at its centre. Radius of the arc is R.

Solution



Let the arc lie in x-y plane with its centre at the origin.

Consider a small current element $id\vec{\ell}$ as shown.

The field due to this element at the centre is

$$dB = \frac{\mu_0}{4\pi}\frac{id\ell\sin 90^0}{R^2} \quad \left(\because id\vec{\ell} \text{ and R are perpendicular}\right)$$

Now $d\ell = Rd\phi \quad \therefore dB = \dfrac{\mu_0}{4\pi}\dfrac{iRd\phi}{R^2} \Rightarrow dB = \dfrac{\mu_0}{4\pi}\dfrac{i}{R}d\phi$

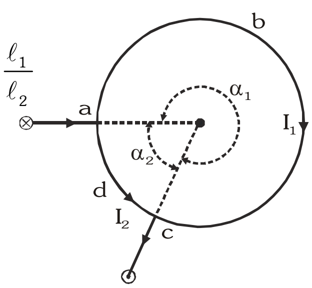
The direction of field is outward perpendicular to plane of paper

Total magnetic field $B = \int dB \therefore B = \dfrac{\mu_0 i}{4\pi R}\displaystyle\int_0^\theta d\phi = \dfrac{\mu_0 i}{4\pi R}\left[\phi\right]_0^\theta \quad \therefore B = \dfrac{\mu_0 i}{4\pi R}\theta$

Example

Find the magnetic field at the centre of a current carrying conductor bent in the form of an arc subtending angle α_1 and α_2 at the centre.

Solution



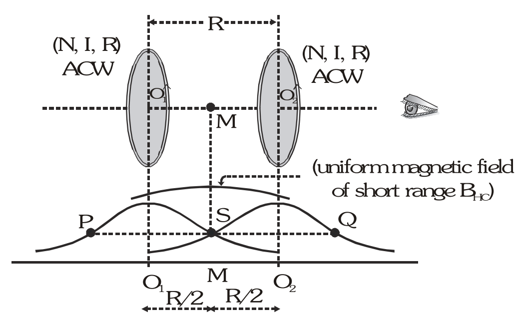
Magnetic field at the centre of arc abc and adc wire of circuit loop

$$B_{abc} = \frac{\mu_0 I_1\alpha_1}{4\pi r} \text{ and } B_{adc} = \frac{\mu_0 I_2\alpha_2}{4\pi r} \Rightarrow \frac{B_{abc}}{B_{adc}} = \frac{I_1\alpha_1}{I_2\alpha_2} \quad \because \text{angle} = \frac{\text{arc length}}{\text{radius}} \Rightarrow \frac{\alpha_1}{\alpha_2} = \frac{\ell_1}{\ell_2}$$

$$\therefore V = I_1R_1 = I_2R_2 \Rightarrow \frac{I_1}{I_2} = \frac{R_2}{R_1} \Rightarrow \frac{I_1}{I_2} = \frac{\ell_2}{\ell_1} \left(\because R = \frac{\rho\ell}{A} \Rightarrow R \propto \ell\right)$$

$$\therefore \frac{B_{abc}}{B_{adc}} = \left(\frac{\ell_2}{\ell_1}\right)\left(\frac{\ell_1}{\ell_2}\right) \Rightarrow \frac{B_{\alpha_1}}{B_{\alpha_2}} = \frac{1}{1}$$

HELMHOLT'Z COILS ARRANGEMENT

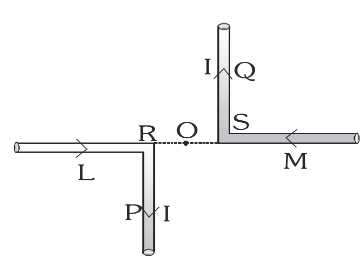


This arrangement is used to produce **uniform** magnetic field of **short range.** It consists :-

- Two identical co–axial coils (N, I, R same)
- Placed at distance (center to center) equal to radius ('R') of coils
- Planes of both coils are parallel to each other.
- Current direction is same in both coils (observed from same side) otherwise this arrangement is not called "Helmholtz coil arrangement".

Example

A pair of stationary and infinitely long bent wires are placed in the x-y plane as shown in fig. The wires carry currents of 10 ampere each as shown. The segments L and M are along the x-axis. The segments P and Q are parallel to the y-axis such that OS = OR = 0.02 m. Find the magnitude and direction of the magnetic induction at the origin O.



Solution

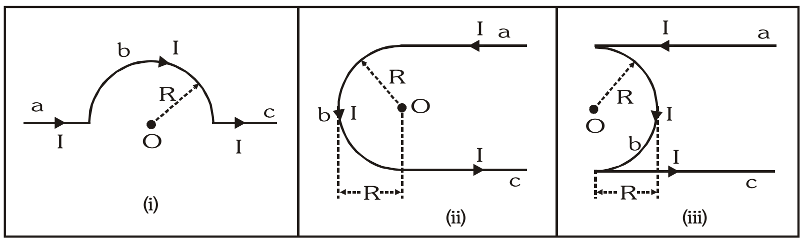
As point O is along the length of segments L and M so the field at O due to these segments will be zero. Further, as the point O is near one end of a long wire, $\vec{B}_R = \vec{B}_P + \vec{B}_Q = \frac{\mu_0}{4\pi}\frac{I}{d}(\tilde{k}) + \frac{\mu_0}{4\pi}\frac{I}{d}(\tilde{k})$ [as RO = SO = d]

so, $\vec{B}_R = \frac{\mu_0}{4\pi}\left(\frac{2I}{d}\right)(\tilde{k})$ Substituting the given data, $\vec{B}_R = 10^{-7}\ \frac{2\times 10}{0.02}(\tilde{k}) = 10^{-4}\frac{Wb}{m^2}(\tilde{k})$

B = 10^{-4} T and in (+z) direction.

Example

Calculate the field at the centre of a semi-circular wire of radius R in situations depicted in figure (i), (ii) and (iii) if the straight wire is of infinite length.



Solution

The magnetic field due to a straight current carring wire of infinite length, for a point at a distance R from one of its ends is zero if the point is along its length and $\frac{\mu_0 I}{4\pi R}$ if the point is on a line perpendicular to its length while at the centre of a semicircular coil is $\frac{\mu_0 I}{4R}$ so net magnetic field at the centre of semicircular wire is $\vec{B}_R = \vec{B}_a + \vec{B}_b + \vec{B}_c$

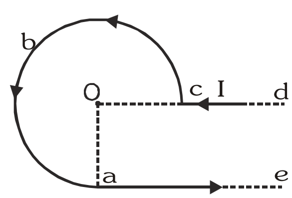
(i) $\vec{B}_R = 0 + \frac{\mu_0}{4}\frac{I}{R}\otimes + 0 = \frac{\mu_0 I}{4R}\otimes$(into the page)

(ii) $\vec{B}_R = \frac{\mu_0}{4\pi}\frac{I}{R}\odot + \frac{\mu_0}{4}\frac{I}{R}\odot + \frac{\mu_0}{4\pi}\frac{I}{R}\odot = \frac{\mu_0}{4\pi}\frac{I}{R}[\pi + 2]\odot$ (out of the page)

(iii) $\vec{B}_R = \frac{\mu_0}{4\pi}\frac{I}{R}\odot + \frac{\mu_0}{4}\frac{I}{R}\otimes + \frac{\mu_0}{4\pi}\frac{I}{R}\odot = \frac{\mu_0}{4\pi}\frac{I}{R}[\pi - 2]\otimes$ (in to the page)

Example

Calculate the magnetic induction at the point O, if the current carrying wire is in the shape shown in figure. The radius of the curved part of the wire is a and linear parts are assumed to be very long and parallel.



Solution

Magnetic induction at the point O due to circular portion of the wire

$B_1 = \frac{\mu_0 I\alpha}{4\pi R} = \frac{\mu_0 i}{4\pi a}\times\frac{3}{2}\pi\odot$ (out of the page) $(\because \alpha = \frac{3\pi}{2})$

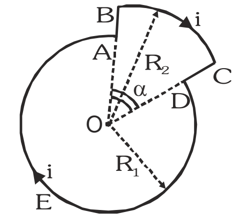
Magnetic induction at O due to wire cd will be zero since O lies on the line cd itself when extended backward. Magnetic induction at O due to infinitely long straight wire ae is

$\therefore\ B_2 = \frac{\mu_0 i}{4\pi r}[\sin\phi_1 + \sin\phi_2]$ where r = a, $\phi_1 = 0$, $\phi_2 = \frac{\pi}{2} \Rightarrow B_2 = \frac{\mu_0 i}{4\pi a}\left[\sin 0° + \sin\left(\frac{\pi}{2}\right)\right] = \frac{\mu_0 i}{4\pi a}$

Because both the fields are in same direction i.e. perpendicular to plane of paper and directed upwards, hence the resultant magnetic induction at O is $B = B_1 + B_2 = \frac{\mu_0 i}{4\pi a}\left(\frac{3\pi}{2}+1\right)\odot$

Example

In the frame work of wires shown in figure, a current i is allowed to flow. Calculate the magnetic induction at the centre O. If angle α is equal to 90 , then what will be the value of magnetic induction at O ?



Solution

Magnetic induction at O due to the segment BC is $B_1 = \dfrac{\mu_0 i}{4\pi R_2}\alpha \otimes$

Similarly, the magnetic induction at O due to circular segment AED is $B_2 = \dfrac{\mu_0 i}{4\pi R_1}(2\pi - \alpha)\otimes$

Magnetic field due to segments AB and CD is zero, because point 'O' lies on axis of these parts.

Hence resultant magnetic induction at O is $B = B_1 + B_2 = \dfrac{\mu_0 i}{4\pi}\left(\dfrac{\alpha}{R_2} + \dfrac{2\pi - \alpha}{R_1}\right), \otimes$

If α = 90 $= \dfrac{\pi}{2}$, then $B = \dfrac{\mu_0 i}{4\pi}\left(\dfrac{\pi}{2R_2} + \dfrac{3\pi}{2R_1}\right) = \dfrac{\mu_0 i}{8}\left(\dfrac{1}{R_2} + \dfrac{3}{R_1}\right)$

Example

Two concentric circular coils X and Y of radii 16 cm and 10 cm respectively lie in the same vertical plane containing the north-south direction. Coil X has 20 turns and carries a current of 16 A; coil Y has 25 turns and carries a current of 18 A. The sense of the current in X is anticlockwise, and in Y clockwise, for an observer looking at the coils facing the west.

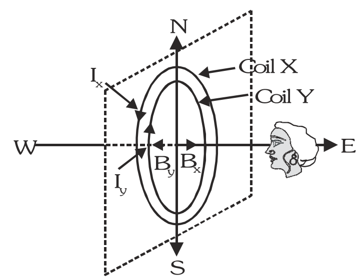
What is the magnitude and direction of the magnetic field at their common centre

(i) Due to coil X alone ? (ii) Due to coil Y alone ? (iii) Due to both the coils ?

Solution

According to the figure the magnitude of the magnetic field at the centre of coil X is

$$B_x = \frac{\mu_0}{2}\frac{I_x N_x}{r_x} = \frac{2\pi \times 10^{-7}}{2} \quad \frac{16\times 20}{0.16} = 4\pi \quad 10^{-4}\ \mathrm{T}$$



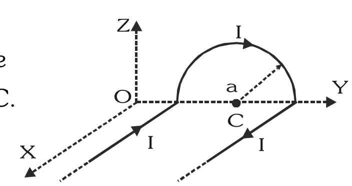
Since the current in coil X is anticlockwise, the direction of B_x is towards the east as shown in figure.

The magnitude of magnetic field at the centre of the coil Y is given by $B_Y = \dfrac{\mu_0}{2}\dfrac{I_Y N_Y}{r_Y} = \dfrac{4\pi\times 10^{-7}}{2} \quad \dfrac{18\times 25}{2} = 9\pi \quad 10^{-4}$ T

$\because$ since the current in coil Y is clockwise, the direction of field B_Y is towards the west (see fig.). Since the two fields are collinear and oppositely directed. The magnitude of the resultant field = difference between the two fields and its direction is that of the bigger field. Hence the net magnetic field at the common centre is 5 π 10^{-4} T and is directed towards the west.

Example

A long wire bent as shown in the figure carries current I. If the radius of the semi-circular portion is "a" then find the magnetic induction at the centre C.



Solution

Due to semi circular part $\vec{B}_1 = \dfrac{\mu_0 I}{4a}\left(-\tilde{i}\right)$

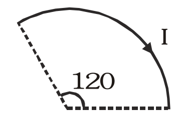
due to parallel parts of currents $\vec{B}_2 = 2\times\dfrac{\mu_0 I}{4\pi a}(-\tilde{k})$, $B_{net} = B_C = \vec{B}_1 + \vec{B}_2 = \dfrac{\mu_0 I}{4a}(-\tilde{i}) + \dfrac{\mu_0 I}{2\pi a}(-\tilde{k})$

magnitude of resultant field $B = \sqrt{B_1^2 + B_2^2} = \dfrac{\mu_0 I}{4\pi a}\sqrt{\pi^2 + 4}$

Example

A piece of wire carrying a current of 6 A is bent in the form of a circular arc of radius 10.0 cm, and it subtends an angle of 120 at the centre. Find the magnetic field due to this piece of wire at the centre.

Solution

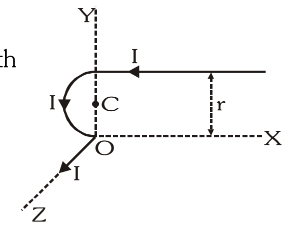


Magnetic field at centre of arc B = $\frac{\mu_0 I\alpha}{4\pi R}$, α = 120 = $\frac{2\pi}{3}$ rad

$$B = \frac{\mu_0 I}{4\pi R}\times\frac{2\pi}{3} = \frac{\mu_0 I}{6R} = \frac{4\pi\times10^{-7}\times6\times100}{6\times10}T = 12.57\ \mu T\otimes$$

Example

An infinitely long conductor as shown in fig. carrying a current I with a semicircular loop on X-Y plane and two straight parts, one parallel to x-axis and another coinciding with Z-axis. What is the magnetic field induction at the centre C of the semi-circular loop.



Solution

The magnetic field induction at C due to current through straight part of the conductor parallel to X-axis is

$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{(r/2)}\left[\sin\frac{\pi}{2}+\sin 0\right] = \frac{\mu_0}{2\pi}\frac{I}{r} \text{ acting along + Z direction. i.e. } \vec{B}_1 = \frac{\mu_0}{2\pi}\frac{I}{r}\hat{k}$$

The magnetic field induction at C due to current through the semi-circular loop in X-Y plane is

$$B_2 = \frac{\mu_0}{4\pi}\frac{I}{(r/2)}(\pi) = \frac{\mu_0}{2}\frac{I}{r} \text{ acting along + Z-direction i.e. } \vec{B}_2 = \frac{\mu_0 I}{2r}\hat{k}$$

The magnetic field induction at C due to current through the straight part of the conductor coinciding with Z-axis is

$$B_3 = \frac{\mu_0}{4\pi}\frac{I}{r/2}\left[\sin\frac{\pi}{2}+\sin 0\right] = \frac{\mu_0 I}{2\pi r} \text{ acting along (–X)-axis i.e. } \vec{B}_3 = \frac{\mu_0}{2\pi}\frac{I}{r}(-\hat{i})$$

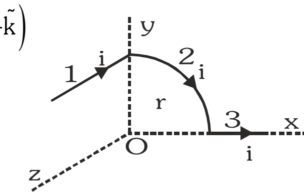
Total magnetic field induction at C is $\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \frac{\mu_0 I}{2\pi r}\hat{k} + \frac{\mu_0 I}{2\pi}\hat{k} - \frac{\mu_0 I}{2\pi r}\hat{i} = \frac{\mu_0 I}{2\pi r}\left[(1+\pi)\hat{k}-\hat{i}\right]$

Example

A conductor carrying a current i is bented as shown in figure. Find the magnitude of magnetic field at the origin

Solution

Field at O due to part 1 $\vec{B}_1 = \frac{\mu_0 i}{4\pi r}(-\hat{i})$ Field at O due to part 2 $\vec{B}_2 = \frac{1}{4}\left(\frac{\mu_0 i}{2r}\right)(-\hat{k})$

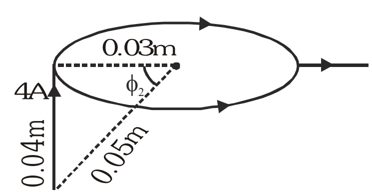


$\because$ Wire 3 passes through origin when it is extended backwards $\vec{B}_3 = 0$

$$\vec{B}_0 = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = -\frac{\mu_0 i}{4r}\left[\frac{\hat{i}}{\pi}+\frac{\hat{k}}{2}\right]$$

Example

A conductor of length 0.04 m is tangentially connected to a circular loop of radius 0.03 m perpendicular to its plane. Find the magnetic field induction at the centre of the loop if 4 ampere current is passed through the conductor as shown in fig.



Solution

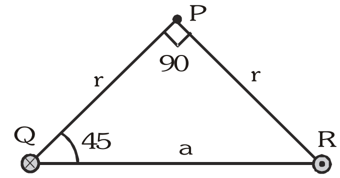
Magnetic field induction at the centre of the loop due to the straight current-carrying conductor,

$$B = \frac{\mu_0 I}{4\pi r}\left[\sin\phi_1 + \sin\phi_2\right] = \frac{4\pi\times10^{-7}\times4}{4\pi\times0.03}\left[\sin 0° + \frac{0.04}{0.05}\right] = 1.07\quad 10^{-5}T$$

Magnetic fields due to the two halves of the loop are equal in magnitude and opposite in direction. So, the magnetic field induction due to the loop at the centre of the loop is zero. So, the magnetic field induction at the centre of the loop is 1.07 10^{-5}T.

Example

Figure shows a right-angled isosceles triangle PQR having its base equal to a. A current of I ampere is passing downwards along a thin straight wire cutting the plane of the paper normally as shown at Q. Likewise a similar wire carries an equal current passing normally upwards at R. Find the magnitude and direction of the magnetic induction at P. Assume the wires to be infinitely long.



Solution

Let $r = PQ = PR$ and $a^2 = r^2 + r^2 = 2r^2$ or $r = \frac{a}{\sqrt{2}}$

Magnetic induction at P due to conductor at Q is $B_1 = \frac{\mu_0 I}{2\pi r} = \frac{\sqrt{2}\mu_0 I}{2\pi a} = \frac{\mu_0 I}{\sqrt{2}\pi a}$ (along PR)

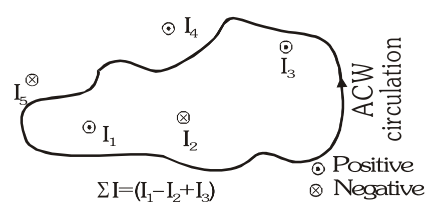
Magnetic induction at P due to conductor at R is $B_2 = \frac{\mu_0 I}{\sqrt{2}\pi a}$ (along PQ)

Now, resultant of these two is $B = \sqrt{B_1^2 + B_2^2} = \sqrt{\left(\frac{\mu_0 I}{\sqrt{2}\pi a}\right)^2 + \left(\frac{\mu_0 I}{\sqrt{2}\pi a}\right)^2} = \frac{\sqrt{2}\,\mu_0 I}{\sqrt{2}\pi a} = \frac{\mu_0 I}{\pi a}$

The direction of $\vec{B}$ is towards the mid-point of the line QR.

AMPERE'S CIRCUITAL LAW

Ampere's circuital law state that line integral of the magnetic field around any closed path in free space or vacuum is equal to μ_0 times of net current or total current which crossing through the area bounded by the closed path. Mathematically $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \Sigma I$



This law independent of size and shape of the closed path.

Any current outside the closed path is not included in writing the right hand side of law

Note :

- This law suitable for infinite long and symmetrical current distribution.
- Radius of cross section of thick cylinderical conductor and current density must be given to apply this law.

MAGNETOMOTIVE FORCE (M.M.F.)

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \Sigma I \text{, where } \vec{B} = \mu_0 \vec{H}\text{, } \oint \mu_0 \vec{H} \cdot d\vec{\ell} = \mu_0 \Sigma I \Rightarrow \oint \vec{H} \cdot d\vec{\ell} = \Sigma I$$

The line integral of magnetising field around any closed path is equal to net current crossing through the area bounded by the closed path, also called 'magnetomotive force'. Magnetomotive force (M.M.F.) $= \oint \vec{H} \cdot d\vec{\ell}$

APPLICATION OF AMPERE'S CIRCUITAL LAW

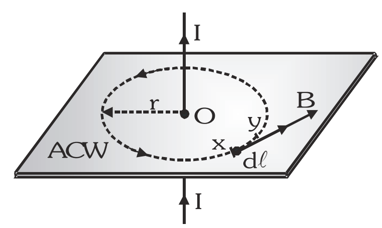
- **Magnetic field due to infinite long thin current carrying straight conductor**

Consider a circle of radius 'r'. Let XY be the small element of length $d\ell$. $\vec{B}$ and $d\vec{\ell}$ are in the same direction because direction of along the tangent of the circle. By A.C.L.

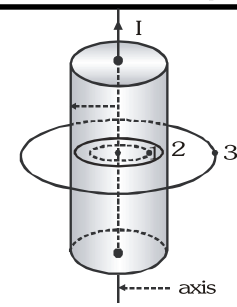
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \Sigma I \text{ , } \oint B\, d\ell \cos\theta = \mu_0 I \quad (\text{where } \theta = 0)$$

$$\oint B\, d\ell \cos 0° = \mu_0 I \Rightarrow B \oint d\ell = \mu_0 I \quad (\text{where } \oint d\ell = 2\pi r)$$

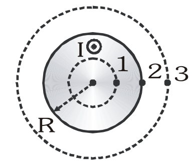
$$B(2\pi r) = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$



- **Magnetic field due to infinite long solid cylinderical conductor**



Cross-sectional view



  - For a point inside the cylinder $r < R$, Current from area πR^2 is $= I$

    so current from area πr^2 is $= \dfrac{I}{\pi R^2}(\pi r^2) = \dfrac{Ir^2}{R^2}$

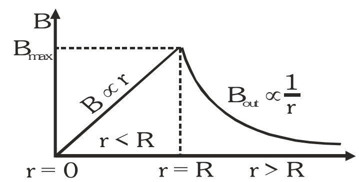
    By Ampere circuital law for circular path 1 of radius r

    $$B_{in}(2\pi r) = \mu_0 I' = \mu_0 \frac{Ir^2}{R^2} \Rightarrow B_{in} = \frac{\mu_0 Ir}{2\pi R^2} \Rightarrow B_{in} \propto r$$

  - For a point on the axis of the cylinder $(r = 0)$; $B_{axis} = 0$
  - For a point on the surface of cylinder $(r = R)$

    By Ampere circuital law for circular path 2 of radius R

    $$B_s(2\pi R) = \mu_0 I \Rightarrow B_s = \frac{\mu_0 I}{2\pi R} \quad \text{(it is maximum)}$$



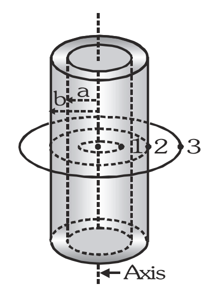
  - For a point outside the cylinder $(r > R)$:-

    By Ampere circuital law for circular path 3 of radius r

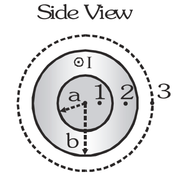
    $$B_{out}(2\pi r) = \mu_0 I \Rightarrow B_{out} = \frac{\mu_0 I}{2\pi r} \Rightarrow B_{out} \propto \frac{1}{r}$$

Magnetic field outside the cylinderical conductor does not depend upon nature (thick/thin or solid/hollow) of the conductor as well as its radius of cross section.

- **Magnetic field due to infinite long hollow cylinderical conductor**



Side View



Cross sectional view

  - For a point at a distance r such that $r < a < b$ $\quad B_1 = 0$
  - For a point at a distance r such that $a < r < b$

    $$B_2(2\pi r) = \mu_0 I' \Rightarrow \quad B_2(2\pi r) = \mu_0 I\left(\frac{r^2 - a^2}{b^2 - a^2}\right)$$

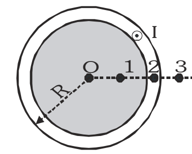
    $$B_2 = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - a^2}{b^2 - a^2}\right)$$

    → $r = a$ (inner surface) $\Rightarrow B_{IS} = 0$

    → $r = b$ (outer surface) $\Rightarrow B_{OS} = \dfrac{\mu_0 I}{2\pi b}$ (maximum)

  - For a point at a distance r such that $r > b > a$, $B_3(2\pi r) = \mu_0 I \Rightarrow B_3 = \dfrac{\mu_0 I}{2\pi r}$
  - For a point at the axis of cylinder $r = 0$ $\quad B_{axis} = 0$

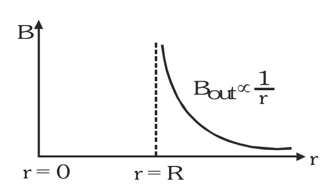
Magnetic field at specific positions for thin hollow cylinderical conductor



At point 1 $\quad B_1 = 0$

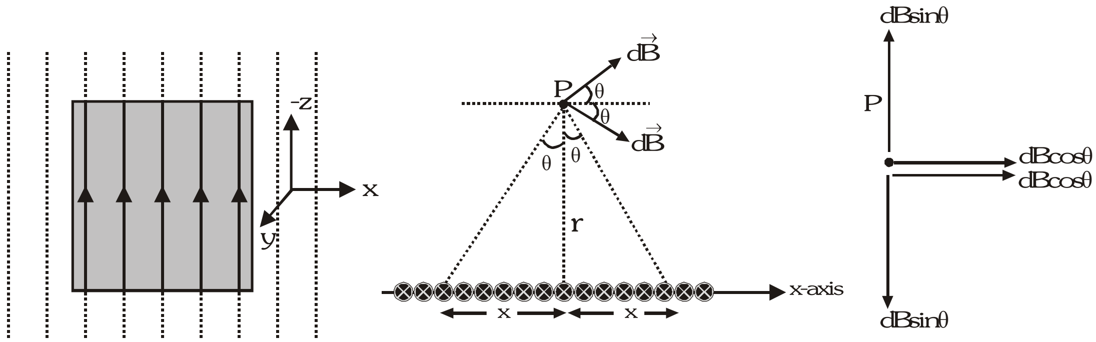
At point 2 $\quad B_2 = \dfrac{\mu_0 I}{2\pi R}$ (maximum) [outer surface] and

$\quad B_2 = 0$ (minimum) [inner surface]

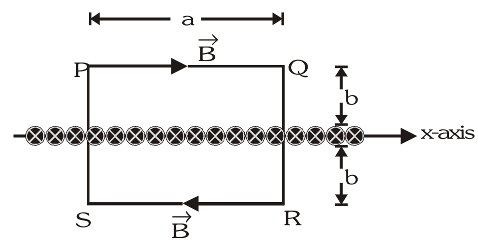


At point 3 $\quad B_3 = \dfrac{\mu_0 I}{2\pi r}$ (for the point on axis $B_{axis} = 0$)

Magnetic field due to an infinite plane sheet of current



An infinite sheet of current lies in x-z plane, carrying current along-z axis. The field at any point P on y is along a line parallel to x-z plane. We can take a rectangular amperian loop as shown. If you traverse the loop in clockwise direction, inward current will be positive.



By Ampere circuital law, $\oint\limits_{PQRS} \vec{B}.d\vec{\ell} = \mu_0 \ell_{enclosed}$ ....(i)

Let λ represents current per unit length.

The current enclosed is given by $\ell_{enclosed} = \lambda a$

Now, $\oint\limits_{PQRS} \vec{B}.d\vec{\ell} = \int\limits_{PQ} \vec{B}.d\vec{\ell} + \int\limits_{QR} \vec{B}.d\vec{\ell} + \int\limits_{RS} \vec{B}.d\vec{\ell} + \int\limits_{SR} \vec{B}.d\vec{\ell}$

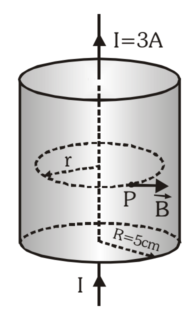
Now, $\int\limits_{QR} \vec{B}.d\vec{\ell} = \int\limits_{SP} \vec{B}.d\vec{\ell} = 0$ as $\vec{B} \perp \overrightarrow{d\ell}$

Also, $\int\limits_{PQ} \vec{B}.d\vec{\ell} + \int\limits_{RS} \vec{B}.d\vec{\ell} = 2 \times B \times a \left(\text{as } \vec{B} \parallel d\vec{\ell}\right)$ $\therefore$ $2B \times a = \mu_0 \lambda a \Rightarrow B = \dfrac{\mu_0 \lambda}{2}$

Example

A long straight solid conductor of radius 5 cm carries a current of 3A, which is uniformly distributed over its circular cross-section. Find the magnetic field induction at a distance 4 cm from the axis of the conductor. Relative permeability of the conductor = 1000.

Solution



Imagine a circular path of radius r whose centre lies on the axis of solid conductor such that the point P lies on it.

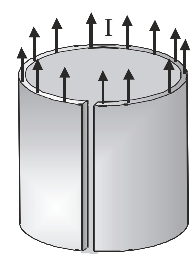
the current threading this closed path $I' = \dfrac{I}{\pi R^2} \times \pi r^2 = \dfrac{Ir^2}{R^2}$

Magnetic field B acts tangential to the amperian circular path at P and is same in magnitude at every point on circular path.

Using Ampere circuital law $\oint \vec{B}.d\vec{\ell} = \mu_0 \mu_r I' \Rightarrow$ $B\,(2\pi r) = \mu_0 \mu_r \left(\dfrac{Ir^2}{R^2}\right) \Rightarrow B = \dfrac{\mu_0 \mu_r I r}{2\pi R^2}$

$$\therefore\ B = \frac{4\pi \times 10^{-7} \times 1000 \times 3 \times 0.04}{2\pi \times (0.05)^2} = 9.6 \quad 10^{-3}\ \text{T.}$$

Example



A current I flows along a thin walled tube of radius R with a long longitudinal slit of width b (<<R). What is the magnetic field induction at a distance r (< R) ?

Solution

Using principle of superposition,

field due to strip in place of slit + field due to tube with slit B = 0

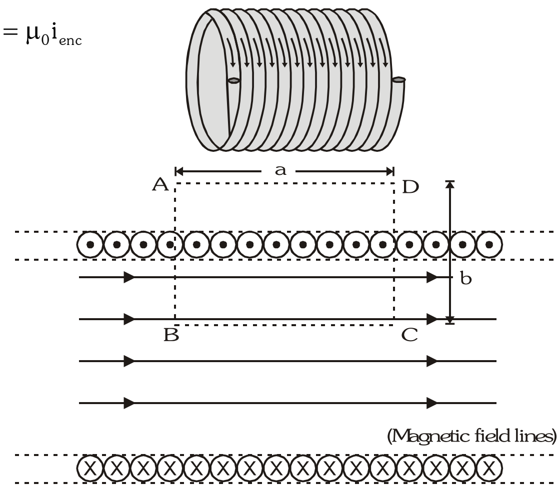
so $\quad B$ = field due to strip in place of slit= $\dfrac{\mu_0}{2\pi r} \times \dfrac{I}{(2\pi R - b)} \times b$

MAGNETIC FIELD DUE TO SOLENOID

It is a coil which has length and used to produce uniform magnetic field of long range. It consists a conducting wire which is tightly wound over a cylinderical frame in the form of helix. All the adjacent turns are electrically insulated to each other. The magnetic field at a point on the axis of a solenoid can be obtained by superposition of field due to large number of identical circular turns having their centres on the axis of solenoid.

Magnetic field due to a long solenoid

A solenoid is a tightly wound helical coil of wire. If length of solenoid is large, as compared to its radius, then in the central region of the solenoid, a reasonably uniform magnetic field is present. Figure shows a part of long solenoid with number of turns/length n.We can find the field by using Ampere circuital law.



Consider a rectangular loop ABCD. For this loop $\oint_{ABCD} \vec{B}.\vec{d\ell} = \mu_0 i_{enc}$

Now $\oint_{ABCD} \vec{B}.\vec{d\ell} = \oint_{AB} \vec{B}.\vec{d\ell} + \oint_{BC} \vec{B}.\vec{d\ell} + \oint_{CD} \vec{B}.\vec{d\ell} + \oint_{DA} \vec{B}.\vec{d\ell} = B \times a$

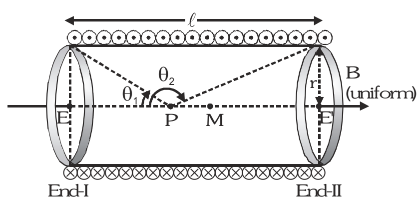
This is because $\oint_{AB} \vec{B}.\vec{d\ell} = \oint_{CD} \vec{B}.\vec{d\ell} = 0,\ \vec{B} \perp \vec{d\ell}$.

And, $\oint_{DA} \vec{B}.\vec{d\ell} = 0$ ($\because$ $\vec{B}$ outside the solenoid is negligible

Now, $i_{enc} = (n \times a) \times i$ s $\Rightarrow B \times a = \mu_0 (n \times a \times i) \Rightarrow B = \mu_0 ni$

Finite length solenoid :

Its length and diameter are comparable.



By the concept of BSL magnetic field at the axial point 'P' obtained as : $B_P = \frac{\mu_0 nI}{2}(\cos\theta_1 - \cos\theta_2)$

Angle θ_1 and θ_2 both measured in same sense from the axis of the solenoid to end vectors.

Infinite length solenoid :

Its length very large as compared to its diameter i.e. ends of solenoid tends to infinity.

(a) Magnetic field at axial point which is well inside the solenoid

$$\theta_1 \simeq 0 \text{ and } \theta_2 \simeq 180 \Rightarrow B \simeq \frac{\mu_0 nI}{2}[\cos 0 - \cos 180] \simeq \frac{\mu_0 nI}{2}[(1) - (-1)] \simeq \mu_0 nI$$

(b) Magnetic field at both axial end points of solenoid

$$\theta_1 = 90 \text{ and } \theta_2 \simeq 180 \Rightarrow B \simeq \frac{\mu_0 nI}{2}[\cos 90 - \cos 180] \simeq \frac{\mu_0 nI}{2}[(0) - (-1)] \simeq \frac{\mu_0 nI}{2}$$

Example

The length of solenoid is 0.1m. and its diameter is very small. A wire is wound over it in two layers. The number of turns in inner layer is 50 and that of outer layer is 40. The strength of current flowing in two layers in opposite direction is 3A. Then find magnetic induction at the middle of the solenoid.

Solution

Direction of magnetic field due to both layers is opposite, as direction of current is opposite so

$$B_{net} = B_1 - B_2 = \mu_0 n_1 I_1 - \mu_0 n_2 I_2 = \mu_0 \frac{N_1}{\ell} I - \mu_0 \frac{N_2}{\ell} I \quad (\because\ I_1 = I_2 = I)$$

$$= \frac{\mu_0 I}{\ell}(N_1 - N_2) = \frac{4\pi \times 10^{-7} \times 3}{0 \cdot 1}(50 - 40) = 12\pi \quad 10^{-5}\,T$$

Example

A closely wound, solenoid 80 cm. long has 5 layers of winding of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0A. Estimate the magnetic field

(a) Inside the solenoid (b) Axial end points of the solenoid

Solution

(a) Magnetic field inside the solenoid

$$B_{in} = \mu_0 nI = \mu_0 \frac{N}{\ell} I,\ (N=400 \quad 5= 2000) = \frac{4\pi \times 10^{-7} \times 2000 \times 8}{(80 \times 10^{-2})} = 8\pi \quad 10^{-3}\ T$$

(b) Magnetic field at axial end points of solenoid $B_{ends} = \frac{\mu_0 nI}{2} = \frac{8\pi \times 10^{-3}}{2} = 4\pi \quad 10^{-3}\ T$

Example

A straight long solenoid is produced magnetic field 'B' at its centre. If it cut into two equal parts and same number of turns wound on one part in double layer. Find magnetic field produced by new solenoid at its centre.

Solution

Magnetic field produced by a long solenoid is $B = \mu_0 nI$, where $n = N/\ell$

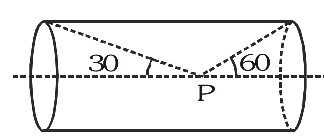
$\because$ Same number of turns wound over half length

$\therefore$ Magnetic field produced by new solenoid is $B' = \mu_0 \left(\frac{N}{\ell/2}\right) I = 2\left(\frac{\mu_0 NI}{\ell}\right) = 2B$

Example

Find out magnetic field at axial point 'P' of solenoid shown in figure

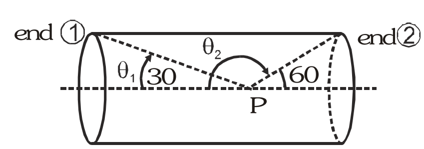
(where turn density 'n' and current through it is I)



Solution

Magnetic field at point 'P' due to finite length solenoid

$$B_P = \frac{\mu_0 nI}{2}[\cos\theta_1 - \cos\theta_2], \quad \text{where } \theta_1 = 30 \ (CW),$$

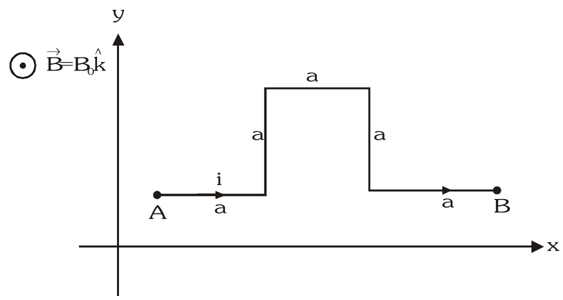


$$\theta_2 = (180 - 60) = 120 \ (CW) = \frac{\mu_0 nI}{2}[\cos 30 - \cos 120]$$

$$= \frac{\mu_0 nI}{2}\left[\frac{\sqrt{3}}{2} - \left(-\frac{1}{2}\right)\right] = \frac{\mu_0 nI}{4}(\sqrt{3} + 1)$$

Example

A uniform magnetic field $\vec{B} = B_0\hat{k}$ exists in a region. A current carrying wire is placed in x-y plane as shown. Find the force acting on part AB of the wire.



Solution

The conductor consists of 5 straight sections viz AC, CD, DE, EF and FB as shown.
As the field is uniform, force on the sections is given by

$$\vec{F} = i\left(\hat{i}\times\vec{B}\right).\ \text{Thus, } \vec{F}_{AC} = i\left(a\hat{i}\times B_0\hat{k}\right) = -B_0 ia\hat{j}$$

$$\vec{F}_{CD} = i\left(a\hat{j}\times B_0\hat{k}\right) = B_0 ia\hat{i},\ \vec{F}_{DE} = i\left(a\hat{i}\times B_0\hat{k}\right) = -B_0 ia\hat{j},\ \vec{F}_{EF} = i\left(-a\hat{j}\times B_0\hat{k}\right) = -B_0 ia\hat{i}$$

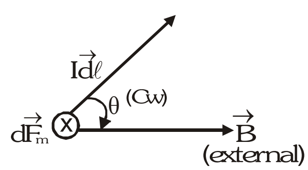
$$\vec{F}_{FB} = i\left(a\hat{j}\times B_0\hat{k}\right) = -B_0 ia\hat{j}\ \ \text{Net force, } \vec{F} = \vec{F}_{AC} + \vec{F}_{CD} + \vec{F}_{DE} + \vec{F}_{EF} + \vec{F}_{FB} = -3B_0 ia\hat{j}$$

CURRENT CARRYING CONDUCTOR IN MAGNETIC FIELD

When a current carrying conductor placed in magnetic field, a magnetic force exerts on each free electron which are present inside the conductor. The resultant of these forces on all the free electrons is called magnetic force on conductor.

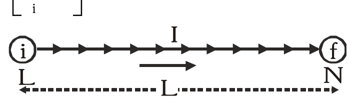
- **Magnetic force on current element**

Through experiments Ampere established that when current element $Id\vec{\ell}$ is placed in magnetic field $\vec{B}$, it experiences a magnetic force $d\vec{F}_m = I(d\vec{\ell}\times\vec{B})$



- Current element in a magnetic field does not experience any force if the current in it is parallel or anti–parallel with the field $\theta = 0$ or 180 $dF_m = 0$ (min.)
- Current element in a magnetic field experiences maximum force if the current in it is perpendicular with the field $\theta = 90$ $dF_m = BId\ell$ (max.)
- Magnetic force on current element is always perpendicular to the current element vector and magnetic field vector. $d\vec{F}_m \perp Id\vec{\ell}$ and $d\vec{F}_m \perp \vec{B}$ (always)
- Total magnetic force on straight current carrying conductor in uniform magnetic field given as

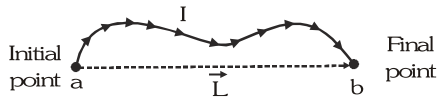
$$\vec{F}_m = \int_i^f d\vec{F}_m \left[\int_i^f d\vec{\ell}\right] = I \quad \vec{B},\ \vec{F}_m = I(\vec{L}\times\vec{B})$$



Where $\vec{L} = \int_i^f d\vec{\ell}$, vector sum of all length elements from initial to final point, which is in accordance with the law of vector addition and $|\vec{L}|$ = length of the condutor.

- Total magnetic force on arbitrary shape current carrying conductor in uniform magnetic field $\vec{B}$ is

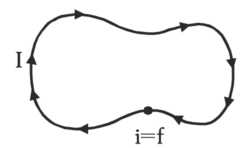
$$\int_i^f d\vec{F}_m = I\left[\int_i^f d\vec{\ell}\right] \vec{B}, \quad \vec{F}_m = I(\vec{L} \times \vec{B}) \quad (L = ab)$$



Where $\vec{L} = \int_i^f d\vec{\ell}$, vector sum of all length elements from initial to final point or displacement between free ends of an arbitrary conducter from initial to final point.

GOLDEN KEY POINT

- A current carrying closed loop (or coil) of any shape placed in uniform magnetic field then no net magnetic force act on it (Torque may or may not be zero)



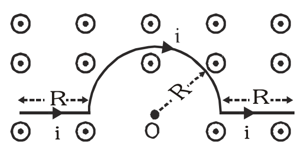
$$\vec{L} = \int_i^f d\vec{\ell} = 0 \quad \text{or} \quad \oint d\vec{\ell} = 0$$

So net magnetic force acting on a current carrying closed loop $\vec{F}_m = 0$ (always)

- When a current carrying closed loop (or coil) of any shape placed in non uniform magnetic field then net magnetic force is always acts on it (Torque may or may not be zero)

Example

A wire bent as shown in fig carries a current i and is placed in a uniform field of magnetic induction $\vec{B}$ that emerges from the plane of the figure. Calculate the force acting on the wire.

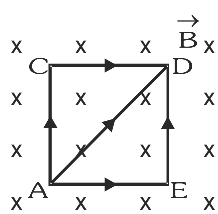


Solution

The total force on the whole wire is $F_m = I|\vec{L}|B = I(R + 2R + R)B = 4RIB$

Example

A square of side 2.0 m is placed in a uniform magnetic field $\vec{B} = 2.0$ T in a direction perpendicular to the plane of the square inwards. Equal current i = 3.0 A is flowing in the directions shown in figure. Find the magnitude of magnetic force on the loop.



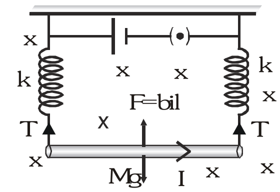
Solution

Net force on the loop = 3 ($\vec{F}_{AD}$) $\quad\because$ Force on wire ACD = Force on AD = Force on AED

$\Rightarrow F_{net}$ = 3(i) (AD) (B) = (3) (3.0) ($2\sqrt{2}$)(2.0) N = $36\sqrt{2}$ N. Direction of this force is towards EC.

Example

A metal rod of mass 10 gm and length 25 cm is suspended on two springs as shown in figure. The springs are extended by 4 cm. When a 20 ampere current passes through the rod it rises by 1 cm. Determine the magnetic field assuming acceleration due to gravity to be 10 m/s^2.



Sol. Let tension in each spring is = T_0

Initially the rod will be in equilibrium if $2T_0 = Mg$ then $T_0 = kx_0$ ...(i)

Now when the current I is passed through the rod it will experience a force

F = BIL vertically up; so in this situation for its equilibrium,

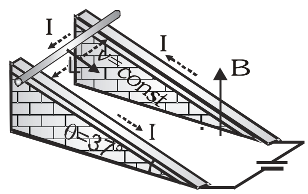
$2T + BIL = Mg$ with $T = kx$...(ii) $(x = 4 - 1 = 3\text{cm})$

So from eq. (i) and eq.(ii) $\frac{T}{T_0} = \frac{Mg - BIL}{Mg} \Rightarrow \frac{x}{x_0} = 1 - \frac{BIL}{Mg}$

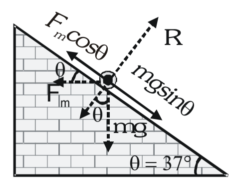
$$\Rightarrow B = \frac{Mg(x_0 - x)}{ILx_0} = \frac{10\times10^{-3}\times10\times3\times10^{-2}}{20\times25\times10^{-2}\times4\times10^{-2}} = 1.5 \quad 10^{-2}\text{T}$$

Example

Two conducting rails are connected to a source of e.m.f. and form an incline as shown in fig. A bar of mass 50 g slides without friction down the incline through a vertical magnetic field B. If the length of the bar is 50 cm and a current of 2.5 A is provided by the battery, for what value of B will the bar slide at a constant velocity ? [g = 10 m/s^2]



Solution



Force on current carrying wire F = BIL

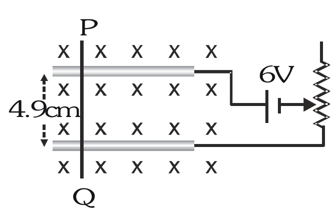
The rod will move down the plane with constant velocity only if

$F\cos\theta = mg\sin\theta \Rightarrow BIL\cos\theta = mg\sin\theta$

$$\text{or,} \quad B = \frac{mg}{IL}\tan\theta = \frac{50\times10^{-3}\times10}{2.5\times50\times10^{-2}}\times\frac{3}{4} = 0.3 \text{ T}$$

Example

A wire PQ of mass 10g is at rest on two parallel metal rails. The separation between the rails is 4.9 cm. A magnetic field of 0.80 tesla is applied perpendicular to the plane of the rails, directed downwards. The resistance of the circuit is slowly decreased. When the resistance decreases to below 20 ohm, the wire PQ begins to slide on the rails. Calculate the coefficient of friction between the wire and the rails.



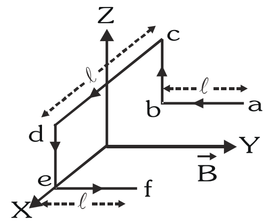
Solution

Wire PQ begins to slide when magnetic force is just equal to the force of friction, i.e.,

$\mu\ mg = i\ell\ B \sin\theta\ (\theta = 90)$ so $i = \dfrac{E}{R} = \dfrac{6}{20} = 0.3$ A so $\mu = \dfrac{i\ell B}{mg} = \dfrac{(0.3)(4.9\times10^{-2})(0.8)}{(10\times10^{-3})(9.8)} = 0.12$

Example

A wire abcdef with each side of length 'ℓ' bent as shown in figure and carrying a current I is placed in a uniform magnetic field B parallel to +y direction.What is the force experienced by the wire.



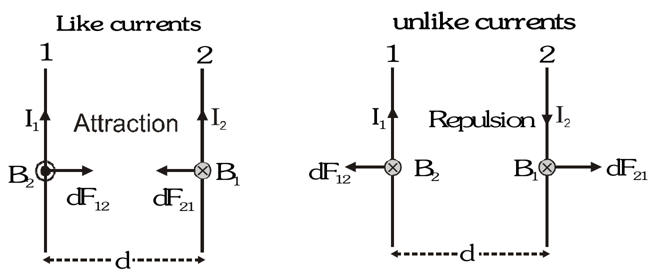
Solution

Magnetic force on wire abcdef in uniform magnetic field is $\vec{F}_m = I\ (\vec{L}\times\vec{B})$,

$\vec{L}$ is displacement between free ends of the conductor from initial to

final point. $\vec{L} = (\ell)\,\tilde{i}$ and $\vec{B} = (B)\,\tilde{j}$; $F_m = I(\vec{L}\times\vec{B}) = BIL\ (\tilde{i}\times\tilde{j}) = BI\ell\,(\tilde{k}) = BI\ell$, along +z direction

MAGNETIC FORCE BETWEEN TWO PARALLEL CURRENT CARRYING CONDUCTORS



The net magnetic force acts on a current carrying conductor due to its own field is zero. So consider two infinite long parallel conductors separated by distence 'd' carrying currents I_1 and I_2.

Magnetic field at each point on conductor (ii) due to current I_1 is $B_1 = \dfrac{\mu_0 I_1}{2\pi d}$ [uniform field for conductor (2)]

Magnetic field at each point on conductor (i) due to curent I_2 is $B_2 = \dfrac{\mu_0 I_2}{2\pi d}$ [Uniform field for conductor (1)]

consider a small element of length 'dℓ' on each conductor. These elements are right angle to the external magnetic field, so magnetic force experienced by elements of each conductor given as

$$dF_{12} = B_2\, I_1\, d\ell = \left(\frac{\mu_0 I_2}{2\pi d}\right) I_1\, d\ell \quad ...(i) \quad (\text{Where } I_1\, d\ell \perp B_2)$$

$$dF_{21} = B_1\, I_2\, d\ell = \left(\frac{\mu_0 I_1}{2\pi d}\right) I_2\, d\ell \quad ...(ii) \quad (\text{Where } I_2\, d\ell \perp B_1)$$

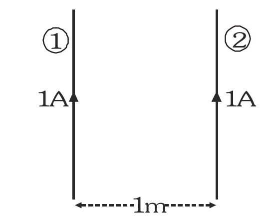
Where dF_{12} is magnetic force on element of conductor (i), due field of conductor (i) and dF_{21} is magnetic force on element of conductor (ii), due to field of conductor (i).

Magnetic force per unit length of each conductor is $\dfrac{dF_{12}}{d\ell} = \dfrac{dF_{21}}{d\ell} = \dfrac{\mu_0 I_1 I_2}{2\pi d}$

$$f = \frac{\mu_0 I_1 I_2}{2\pi d}\ \text{N/m (in S.I.)} \qquad f = \frac{2 I_1 I_2}{d}\ \text{dyne/cm (In C.G.S.)}$$

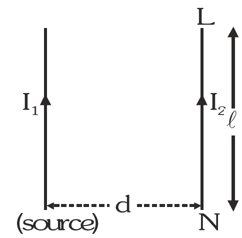
Definition of ampere :

Magnetic force/unit length for both infinite length conductor gives as



$$f = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{(4\pi\times10^{-7})(1)(1)}{2\pi(1)} = 2 \times 10^{-7}\ \text{N/m}$$

'Ampere' is the current which, when passed through each of two parallel infinite long straight conductors placed in free space at a distance of 1 m from each other, produces between them a force of 2×10^{-7} N/m

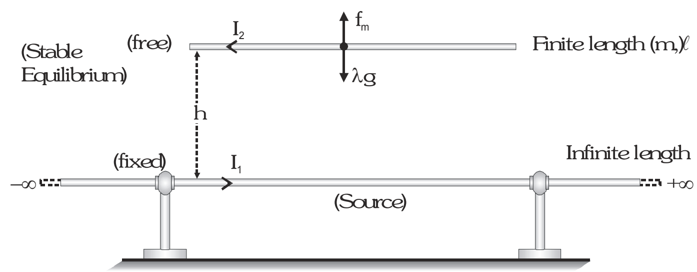


- Force scale $f = \dfrac{\mu_0 I_1 I_2}{2\pi d}$ is applicable when at least one conductor must be of infinite length so it behaves like source of uniform magnetic field for other conductor.

  Magnetic force on conductor 'LN' is $F_{LN} = f \ell \Rightarrow F_{LN} = \left(\dfrac{\mu_0 I_1 I_2}{2\pi d}\right)\ell$

- **Equillibrium of free wire**

  Case I : Upper wire is free : Consider a long horizontal wire which is rigidly fixed another wire is placed directly above and parallel to fixed wire.



Magnetic force per unit length of free wire $f_m = \dfrac{\mu_0 I_1 I_2}{2\pi h}$, and it is repulsive in nature because currents are unlike.

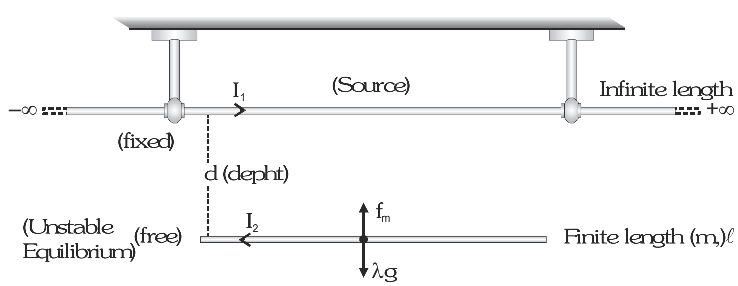
Free wire may remains suspended if the magnetic force per unit length is equal to weight of its unit length

At balanced condition f_m = W'. Weight per unit length of free wire $= \dfrac{\mu_0 I_1 I_2}{2\pi h} = \dfrac{m}{\ell} g$ (stable equillibrium condition)

If free wire is slightly pluked and released then it will executes S.H.M. in vertical plane.

The time period of motion is $T = 2\pi\sqrt{\dfrac{h}{g}}$

Case II : Lower wire is free : Consider a long horizontal wire which is rigidly fixed. Another wire is placed directly below and parallel to the fixed wire.



Magnetic force per unit length of free wire is $f_m = \dfrac{\mu_0 I_1 I_2}{2\pi d}$, and it is attractive in nature because currents are like.

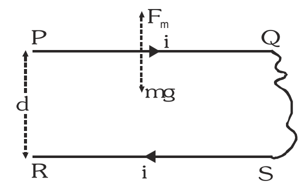
Free wire may remains suspended if the magnetic force per unit length is equal to weight of its unit length
At balanced condition f_m = W'

Weight per unit length of free wire $\dfrac{\mu_0 I_1 I_2}{2\pi d} = \dfrac{m}{\ell} g$ (unstable equillibrium condition)

Example

Two horizontal parallel straight conductors, each 20 cm long, are arranged one vertically above the other and carry equal currents in opposite directions. The lower conductor is fixed while the other is free to move in guides remaining parallel to the lower. If the upper conductor weights 1.20 g, what is the approximate current that will maintain the conductors at a distance 0.75 cm apart.

Solution



In equilibrium magnetic force F_m will balance weight mg.

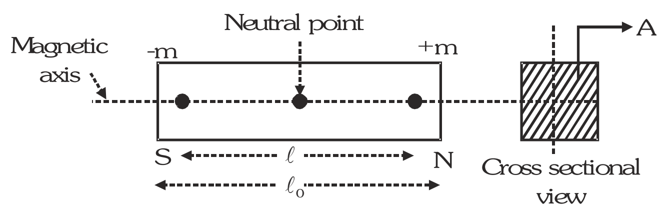
So $mg = F_m \Rightarrow mg = \dfrac{\mu_0 i^2 \ell}{2\pi d}$

$$\Rightarrow i = \sqrt{\frac{2\pi mgd}{\mu_0 \ell}} = \sqrt{\frac{2\pi\times1.2\times10^{-3}\times9.8\times0.75\times10^{-2}}{4\pi\times10^{-7}\times20\times10^{-2}}} = \sqrt{2205} = 47\text{ A}$$

MAGNETIC DIPOLE MOMENT

A magnetic dipole consists of a pair of magnetic poles of equal and opposite strength separated by small distance. Ex. Magnetic needle, bar magnet, solenoid, coil or loop.

- **Magnetic moment of Bar magnet**



The magnetic moment of a bar magnet is defined as a vector quantity having magnitude equal to the product of pole strength (m) with effective length (ℓ) and directed along the axis of the magnet from south pole to north pole.

$\vec{M} = m\vec{\ell}$ → It is an axial vector
→ S.I. unit : - $A.m^2$

GOLDEN KEY POINTS

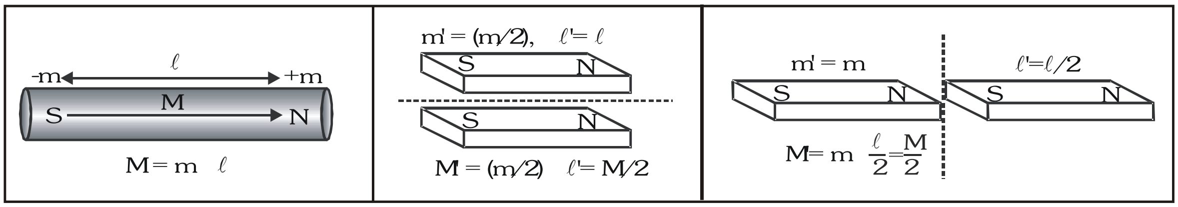
- Attractive property : A bar magnet attracts certain magnetic substances (eg. Iron dust). The attracting power of the bar magnet is maximum at two points near the ends called poles. So the attracting power of a bar magnet at its poles called 'pole strength'
- The 'pole strength' of north and south pole of a bar magnet is conventionally represented by +m and –m respectively.
- The 'pole strength' is a scalar quantity with S.I. unit A–m.
- The 'pole strength' of bar magnet is directly proportional to its area of cross section. $m \propto A$
- The attracting power of a bar magnet at its centre point is zero, so it is called 'neutral point'.
- Magnetic poles are always exists in pairs i.e. mono pole does not exist in magnetism. So Gauss law in magnetism given as $\oint \vec{B}\cdot d\vec{s} = 0$
- Effective length or magnetic length :– It is distance between two poles along the axis of a bar magnet. As pole are not exactly at the ends, the effective length (ℓ) is less than the geometrical length (ℓ_0) of the bar magnet. $\ell \simeq 0\cdot91\,\ell_0$
- **Inverse square law (Coulomb law) :** The magnetic force between two isolated magnetic poles of strength m_1 and m_2 lying at a distance 'r' is directly proportional to the product of pole strength and inversely proportional to the square of distance between their centres. The magnetic force between the poles can be attractive or repulsive according to the nature of the poles.

$$\left.\begin{array}{l} F_m \propto m_1 m_2 \\ F_m \propto \dfrac{1}{r^2} \end{array}\right] \quad F_m = k\,\frac{m_1 m_2}{r^2} \qquad \text{where } k < \begin{array}{l} \dfrac{\mu_0}{4\pi}\ (\text{S.I.}) \\ 1\ (\text{C.G.S.}) \end{array}$$

Inverse square law of Coulomb in magnetism is applicable only for two long bar magents becasue isolated poles cannot exist.

- If a magnet is cut into two equal parts along the length then pole strength is reduced to half and length remains unchanged. New magnetic dipole moment $M'=m'(\ell)=\frac{m}{2}\times\ell=\frac{M}{2}$.

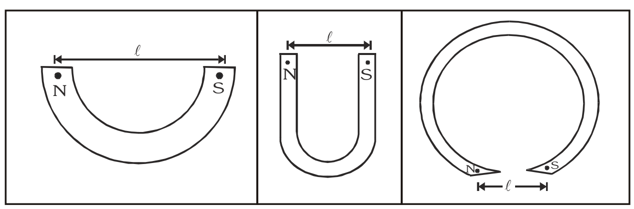
  The new magnetic dipole moment of each part becomes half of original value.



- If a magnet is cut into two equal parts transverse to the length then pole strength remains unchanged and length is reduced to half. New magnetic dipole moment $M' = m\left(\frac{\ell}{2}\right)=\frac{M}{2}$.

  The new magnetic dipole moment of each part becomes half of original value.

- The magnetic dipole moment of a magnet is equal to product of pole strength and distance between poles. $M=m\ell$



- As magnetic moment is a vector, in case of two magnets having magnetic moments M_1 and M_2 with angle θ between them, the resulting magnetic moment.

$$M = \left[M_1^2 + M_2^2 + 2M_1M_2\cos\theta\right]^{1/2} \quad \text{with} \qquad \tan\phi = \left[\frac{M_2\sin\theta}{M_1+M_2\cos\theta}\right]$$

Example

The force between two magnetic poles in air is 9.604 mN. If one pole is 10 times stronger than the other, calculate the pole strength of each if distance between two poles is 0.1 m?

Solution

Force between poles $F=\frac{\mu_0}{4\pi}\frac{m_1m_2}{r^2}$ or $9.604\times10^{-3}=\frac{10^{-7}\times m\times 10m}{0.1\times0.1}$ or $m^2 = 96.04\ N^2T^{-2} \Rightarrow m=9.8 N/T$

So strength of other pole is 9.8 10 = 98 N/T

Example

A steel wire of length L has a magnetic moment M. It is then bent into a semicircular arc. What is the new magnetic moment?

Solution

If m is the pole strength then M=m.L $\Rightarrow m = \dfrac{M}{L}$

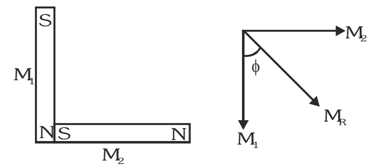
If it is bent into a semicircular arc then L=π r $\Rightarrow$ $r = \dfrac{L}{\pi}$

So new magnetic moment $M' = m \times 2r = \dfrac{M}{L} \times 2 \times \dfrac{L}{\pi} = \dfrac{2M}{\pi}$

Example

Two identical bar magnets each of length L and pole strength m are placed at right angles to each other with the north pole of one touching the south pole of other. Evaluate the magnetic moment of the system.

Solution

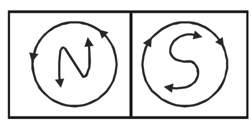


$M_1 = M_2 = mL \quad \therefore \quad M_R = \sqrt{M_1^2 + M_2^2 + 2M_1M_2\cos\dfrac{\pi}{2}} = \sqrt{2}\ mL$

and $\tan\phi = \dfrac{M\sin 90°}{M + M\cos 90°} = 1$ i.e. $\phi = \tan^{-1} 1 = 45$

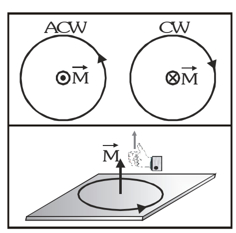
MAGNETIC MOMENT OF CURRENT CARRYING COIL (LOOP)

Current carrying coil (or loop) behaves like magnetic dipole. The face of coil in which current appears to flow anti clock wise acts as north pole while face of coil in which current appears to flow clock wise acts as south pole.



- A loop of geometrical area 'A', carries a current 'I' then magentic moment of coil M = I A
- A coil of turns 'N', geometrical area 'A', carries a current 'I' then magentic moment M = N I A

Magnetic moment of current carrying coil is an axial vector $\vec{M} = NI\vec{A}$ where $\vec{A}$ is a area vector perpendicular to the plane of the coil and along its axis. SI UNIT : A-m^2 or J/T



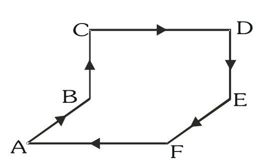
Direction of $\vec{M}$ find out by right hand thumb rule

• Curling fingers ⇒ In the direction of current • Thumb ⇒ Gives the direction of $\vec{M}$

For a current carrying coil, its magnetic moment and magnetic field vectors both are parallel axial vectors.

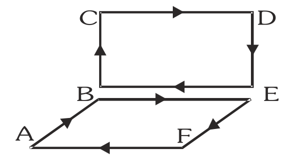
Example

Find the magnitude of magnetic moment of the current carrying loop ABCDEFA. Each side of the loop is 10 cm long and current in the loop is i = 2.0 A



Solution

By assuming two equal and opposite currents in BE, two current carrying loops (ABEFA and BCDEB) are formed. Their magnetic moments are equal in magnitude but perpendicular to each other.

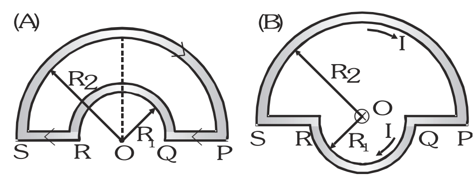


Hence, $M_{net} = \sqrt{M^2 + M^2} = \sqrt{2}M$ where $M = iA = (2.0)(0.1)(0.1) = 0.02$ A-m^2

$\Rightarrow M_{net} = (\sqrt{2})(0.02)$ A-m^2 = 0.028 A-m^2

Example

The wire loop PQRSP formed by joining two semicircular wires of radii R_1 and R_2 carries a current I as shown in fig. What is the magnetic induction at the centre O and magnetic moment of the loop in cases (A) and (B) ?



Solution

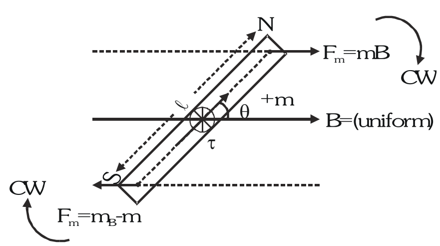
As the point O is along the length of the straight wires, so the field at O due to them will be zero and hence.

(A) $\vec{B} = \dfrac{\mu_0}{4\pi}\left[\dfrac{\pi I}{R_2}\otimes + \dfrac{\pi I}{R_1}\odot\right]$ i.e., $\vec{B} = \dfrac{\mu_0}{4\pi}\pi I\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]\odot$ & $\vec{M} = NI\vec{S} = 1 \quad I\left[\dfrac{1}{2}\pi R_2^2 \otimes + \dfrac{1}{2}\pi R_1^2 \odot\right] = \dfrac{1}{2}\pi I\left[R_2^2 - R_1^2\right]\otimes$

(B) Following as in case (A), in this situation, $\vec{B} = \dfrac{\mu_0}{4\pi}\pi I\left[\dfrac{1}{R_1} + \dfrac{1}{R_2}\right]\otimes$ and, $\vec{M} = \dfrac{1}{2}\pi I\left[R_2^2 + R_1^2\right]\otimes$

MAGNETIC DIPOLE IN MAGNETIC FIELD

Torque on magnetic dipole



(a) Bar magnet

τ = force perpendicular distance between force couple

$\tau = (mB)(\ell\sin\theta)$, where $M = m\ell$

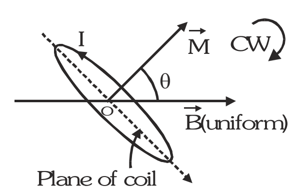
$\boxed{\tau = MB\sin\theta}$
- $\theta = 90 \Rightarrow \tau = MB$ (maximum)
- $\theta = 0$ or $180 \Rightarrow \tau = 0$ (minimum)

Vector form $\vec{\tau} = \vec{M} \times \vec{B}$

(b) Coil or Loop

$\vec{\tau} = \vec{M} \times \vec{B} \qquad \vec{\tau} = NI(\vec{A} \times \vec{B})$

$\boxed{\tau = BINA\sin\theta}$
- $\theta = 90 \Rightarrow \tau = BINA$ (maximum)
- $\theta = 0$ or $180 \Rightarrow \tau = 0$ (minimum)



GOLDEN KEY POINTS

- Torque on dipole is an axial vector and it is directed along axis of rotation of dipole.
- Tendency of torque on dipole is try to align the $\vec{M}$ in the direction of $\vec{B}$ or tries to makes the axis of dipole parallel to $\vec{B}$ or makes the plane of coil (or loop) perpendicular to $\vec{B}$.
- Dipole in uniform magnetic field
  - $F_{net}=0$ (no translatory motion)
  - τ may or may not be zero (decides by θ)
- Dipole in non uniform magnetic field
  - $F_{net}\neq 0$ (translatory motion)
  - τ may or may not be zero (decides by θ)
- When a current carrying coil (or loop) is placed in longitudenal magnetic field then maxmium torque acts on it. $\theta = 90$ $(\vec{M} \perp \vec{B}) \Rightarrow \tau_{max} = MB = BINA$
- When a current carrying coil (or loop) is placed in transverse magnetic field the no torque acts on it.

  $\theta = 0$ $(\vec{M} \| \vec{B})$ or $\theta = 180$ $(\vec{M}$ anti$\| \vec{B}) \Rightarrow \tau_{min} = 0$

WORK DONE IN ROTATING A MAGNETIC DIPOLE

Work done in rotating a dipole in a uniform magnetic field through small angle 'dθ'

$$dW = \tau.d\theta = MB\sin\theta d\theta$$

So work done in rotating a dipole from angular position θ_1 to θ_2 with respect to the Magnetic field direction

$$W = \int_{\theta_1}^{\theta_2} dW = \int_{\theta_1}^{\theta_2} MB\sin\theta d\theta = MB(\cos\theta_1 - \cos\theta_2)$$

- If magnetic dipole is rotated from field direction i.e. $\theta_1 = 0$ to position $\theta_2=\theta$

  then work done is $W_\theta = MB\,(1 - \cos\theta) = 2MB\,\sin^2\theta/2$

  in one rotation $\theta = 0$ or $360 \Rightarrow W = 0$ in 1/4 rotation $\theta = 90 \Rightarrow W = MB$

  in half rotation $\theta = 180 \Rightarrow W = 2MB$ in 3/4 rotation $\theta = 270 \Rightarrow W = MB$
- Work done to rotate a dipole in a magnetic field is stored in the form of potential energy of magnetic dipole.

POTENTIAL ENERGY OF MAGNETIC DIPOLE

The potential energy of dipole defined as work done in rotating the dipole from a direction perpendicular to the given direction. $U = W_\theta - W_{90} \Rightarrow U = MB\,(1 - \cos\theta) - MB = MB\cos\theta$, In vector form $U = -\vec{M}\cdot\vec{B}$

GOLDEN KEY POINTS

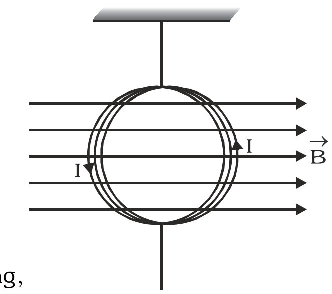
- When $\vec{M}$ and $\vec{B}$ are parallel ($\theta = 0$), the dipole has minimum potential energy and it is in stable equillibrium.

  $$U = -MB \text{ (minimum)}$$
- When $\vec{M}$ and $\vec{B}$ are anti parallel ($\theta = 180$), the dipole has maximum potential energy and it is in unstable equillibrium. $U = MB$ (maximum)
- When $\vec{M}$ and $\vec{B}$ are prpendicular to each other ($\theta = 90$), the dipole has potential energy U=0 and in this situation maximum torque acts on it hence no equillibrium.

Example

A circular coil of 25 turns and radius 6.0 cm, carrying a current of 10 A, is suspended vertically in a uniform magnetic field of magnitude 1.2 T. The field lines run horizontally in the plane of the coil. Calculate the force and torque on coil due to the magnetic field. In which direction should a balancing torque be applied to prevent the coil from turning ?

Solution



Magnetic force $F_m = I\left(\oint d\vec{\ell} \times \vec{B}\right)$

For coil or close loop $\oint d\vec{\ell} = 0$ so $\vec{F}_m = 0$

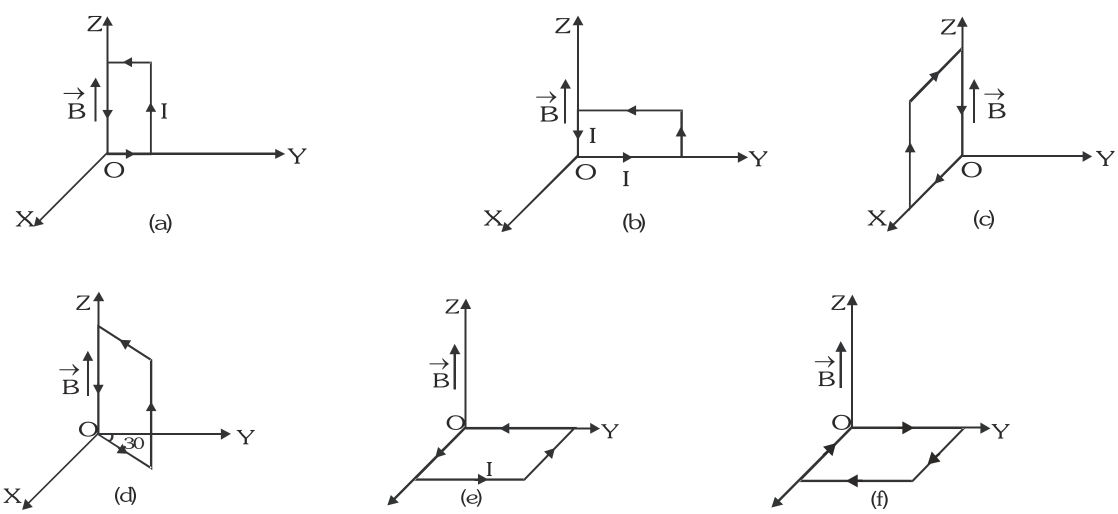
The torque $\vec{\tau}$ on a coil of any shape having N turns and current I in a magnetic field B is given by τ = NIABsinθ

τ = 25 10 π 6 6 10^{-4} 1.2 sin90 = 3.39 N

The direction of $\vec{\tau}$ is vertically upwards. To prevent the coil from turning, an equal and opposite torque must be applied.

Example

A uniform magnetic field of 5000 gauss is established along the positive z-direction. A rectangular loop of side 20 cm and 5 cm carries a current of 10 A is suspended in this magnetic field. What is the torque on the loop in the different cases shown in the following figures ? What is the force in each case ? Which case corresponds to stable equilibrium ?



Solution

(a) Torque on loop, τ=BIA sinθ

Here, θ=90 ; B=5000 gauss=5000 10^{-4} tesla = 0.5 tesla

I = 10 ampere, A=20 5cm^2=100 10^{-4}=10^{-2} m^2

Now, τ = 0.5 10 10^{-2}=5 10^{-2} Nm It is directed along –y-axis

(b) Same as (a).

(c) τ=5 10^{-2} Nm along –x-direction

(d) τ =5 10^{-2} N m at an angle of 240 with +x direction.

(e) τ is zero. [$\because$Angle between plane of loop and direction of magnetic field is 90]

(f) τ is zero.

Resultant force is zero in each case. Case (e) corresponds to stable equilibrium.

Example

A circular coil of 100 turns and having a radius of 0.05 m carries a current of 0.1 A. Calculate the work required to turn the coil in an external field of 1.5 T through 180 about an axis perpendicular to the magnetic field ? The plane of coil is initially at right angles to magnetic field.

Solution

Work done W = MB $(\cos\theta_1 - \cos\theta_2)$ =NIAB $(\cos\theta_1 - \cos\theta_2)$

$\Rightarrow$ W = NIπr^{2}B $(\cos\theta_1 - \cos\theta_2)$ = 100 0.1 3.14 $(0.05)^2$ 1.5 (cos 0 –cosπ) = 0.2355J

Example

A bar magnet of magnetic moment $1.5\ JT^{-1}$ lies aligned with the direction of a uniform magnetic field of 0.22 T.

(a) What is the amount of work required to turn the magnet so as to align its magnetic moment. (i) Normal to the field direction? (ii) Opposite to the field direction?

(b) What is the torque on the magnet in case (i) and (ii)?

Solution

Here, $M = 1.5\ JT^{-1}$, $B = 0.22$ T.

(a) P.E. with magnetic moment aligned to field = – MB
P.E. with magnetic moment normal to field = 0
P.E. with magnetic moment antiparallel to field = + MB

(i) Work done = increase in P.E. = 0 – (–MB) = MB = 1.5 0.22 = 0.33 J.

(ii) Work done = increase in P.E. = MB – (–MB) = 2MB = 2 1.5 0.22 = 0.66 J.

(b) We have $\tau = MB\sin\theta$

(i) $\tau = MB\sin\theta = 1.5\ \ 0.22\ \ 1 = 0.33$ J. ($\theta=90 \Rightarrow \sin\theta=1$)
This torque will tend to align M with B.

(ii) $\tau = MB\sin\theta = 1.5\ \ 0.22\ \ 0=0$ ($\theta=180 \Rightarrow \sin\theta=0$)

Example

A short bar magnet of magnetic moment 0.32 J/T is placed in uniform field of 0.15 T. If the bar is free to rotate in plane of field then which orientation would correspond to its (i) stable and (ii) unstable equilibrium? What is potential energy of magnet in each case?

Solution

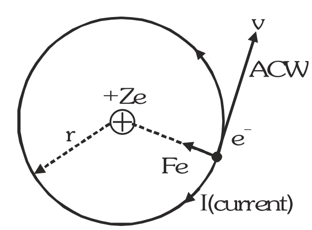
(i) If M is parallel to B then $\theta=0$. So potential energy $U = U_{min} = -MB$
$U_{min} = -MB = -0.32\ \ 0.15$ J$= -4.8\ \ 10^{-2}$ J (stable equilibrium)

(ii) If M is antiparallel to B then $\theta = \pi$ So potential energy
$U = U_{max} = +MB = +0.32\ \ 0.15 = 4.8\ \ 10^{-2}$ J (unstable equilibrium.)

ATOMIC MAGNETISM

An atomic orbital electron, which doing bounded uniform circular motion around nucleus. A current constitues with this orbital motion and hence orbit behaves like current carrying loop. Due to this magnetism produces at nucleus position. This phenomenon called as 'atomic magnetism.

Bohr's postulates :



(i) $\dfrac{mv^2}{r} = \dfrac{kze^2}{r^2}$ (ii) $L = mvr = n\left(\dfrac{h}{2\pi}\right)$, where n = 1, 2,3 ....

Basic elements of atomic magnetism :

(a) **Orbital current :-** $I = ef = \dfrac{e}{T} = \dfrac{ev}{2\pi r} = \dfrac{e\omega}{2\pi}$

(b) **Magnetic induction at nucleus position :-** As circular orbit behaves like current carrying loop, so magnetic induction at nucleus position $B_N = \dfrac{\mu_0 I}{2r}$

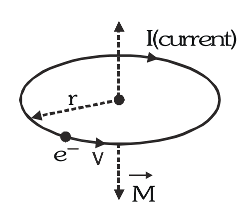
$$B_N = \frac{\mu_0 ef}{2r} = \frac{\mu_0 e}{2Tr} = \frac{\mu_0 ev}{4\pi r^2} = \frac{\mu_0 e\omega}{4\pi r}$$

(c) **Magnetic moment of circular orbit :-** Magnetic dipole moment of circular orbit

M = IA where A is area of circular orbit. $M = ef\left(\pi r^2\right) = \dfrac{\pi er^2}{T} = \dfrac{evr}{2} = \dfrac{e\omega r^2}{2}$

- **Relation between magnetic moment and angular momentum of orbital electron**

Magnetic moment $M = \dfrac{evr}{2}\times\dfrac{m}{m} = \dfrac{eL}{2m}$ ($\because$ angular momentum $L = mvr$)



Vector form $\vec{M} = \dfrac{-e\vec{L}}{2m}$

For orbital electron its $\vec{M}$ and $\vec{L}$ both are antiparallel axial vectors.

BOHR MAGNETON (μ_B)

According of Bohr's theory, angular momentum of orbital electron is given by

$L = \dfrac{nh}{2\pi}$, where n = 1, 2, 3 .... and h is plank's constant.

Magnetic moment of orbital electron is given by $M = \dfrac{eL}{2m} = n\dfrac{eh}{4\pi m}$

- If n = 1 then $M = \dfrac{eh}{4\pi m}$, which is Bohr magneton denoted by μ_B

- **Definition of μ_B :**
  Bohr magneton can be defined as the magnetic moment of orbital electron which revolves in first orbit of an atom.

- $\mu_B = \dfrac{eh}{4\pi m} = \dfrac{1.6\times10^{-19}\times6.6\times10^{-34}}{4\times3.14\times9.1\times10^{-31}} = 0.923 \quad 10^{-23}$ A.m.^{2}

- **Basic elements of atomic magnetism for first orbit of H-atom (n=1, z = 1)**

  (a) Accurate form :- (v = 2.18 $\quad 10^6$ m/sec, f = 6.6 $\quad 10^{15}$ cy/sec. r = 0.529Å)
  - Orbital current I = 0.96 mA
  - Magnetic induction at nucleus position B_N = 12.8 T
  - Magnetic moment of orbital electron M = 0.923 $\quad 10^{-23}$ A.m^2

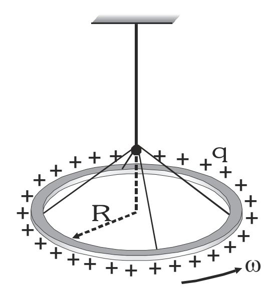
  (b) Simple form :- (v $\simeq$ 2 $\quad 10^6$ m/sec, f $\simeq$ 6 $\quad 10^{15}$cy/sec, r $\simeq$ 0.5Å)
  - Orbital current I $\simeq$ 1mA
  - Magnetic induction at nucleus position $B_N \simeq 4\pi$T
  - Magnetic moment of orbital electron M = μ_B A.m^2

A NONCONDUCTING CHARGED BODY IS ROTATED WITH SOME ANGULAR SPEED.

In this case the ratio of magnetic moment and angular momentum is constant which is equal to $\dfrac{q}{2m}$ here q = charge and m = the mass of the body.

Example

In case of a ring, of mass m, radius R and charge q distributed on it circumference.



Angular momentum $\quad L = I\omega = (mR^2)(\omega) \quad$ ... (i)

Magnetic moment $\quad M = iA = (qf)\,(\pi R^2)$

$$M = (q)\left(\frac{\omega}{2\pi}\right)(\pi R^2) = q\frac{\omega R^2}{2} \quad \text{...(ii)}$$

$\because f = \dfrac{\omega}{2\pi}$ From Eqs. (i) and (ii) $\quad \dfrac{M}{L} = \dfrac{q}{2m}$

Although this expression is derived for simple case of a ring, it holds good for other bodies also. For example, for a disc or a sphere. $M = \dfrac{qL}{2m} \Rightarrow M = \dfrac{q(I\omega)}{2m}$, where L = I ω

Rigid body	Ring	Disc	Solid sphere	Spherical shell
Moment of inertia (I)	mR^2	$\dfrac{mR^2}{2}$	$\dfrac{2}{5}mR^2$	$\dfrac{2}{3}mR^2$
Magnetic moment= $\dfrac{qI\omega}{2m}$	$\dfrac{q\omega R^2}{2}$	$\dfrac{q\omega R^2}{4}$	$\dfrac{q\omega R^2}{5}$	$\dfrac{q\omega R^2}{3}$

FORCE ON A CHARGED PARTICLE IN A MAGNETIC FIELD

Force experienced by a current element $I d\vec{\ell}$ in magnetic field $\vec{B}$ is given by $d\vec{F} = I d\vec{\ell}\ \ \vec{B}$ ....(i)

Now if the current element $I d\vec{\ell}$ is due to the motion of charge particles, each particle having a charge q moving with velocity $\vec{v}$ through a cross-section A, $I d\vec{\ell} = (nqAv)\, d\vec{\ell} = (nqdV)\,\vec{v}$ [with volume dV=A $d\ell$]

From eq^n (i) we can write $d\vec{F} = ndV\ q(\vec{v} \times \vec{B})$

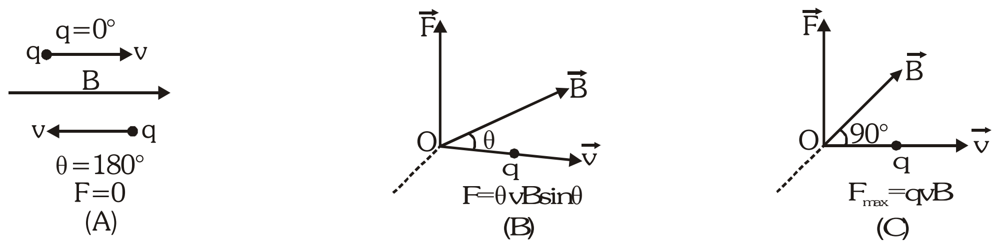
ndV=the total number of charged particles in volume dV

(n=number of charged particles per unit volume), force on a charged particle $\vec{F} = \frac{1}{n}\frac{d\vec{F}}{dV} = q(\vec{v} \times \vec{B})$

GOLDEN KEY POINT

- The force $\vec{F}$ is always perpendicular to both the velocity $\vec{v}$ and the field $\vec{B}$
- A charged particle at rest in a steady magnetic field does not experience any force.

  If the charged particle is at rest then $\vec{v} = \vec{0}$, so $\vec{v} \times \vec{B} = \vec{0}$
- A moving charged particle does not experience any force in a magnetic field if its motion is parallel or antiparallel to the field.



- If the particle is moving perpendicular to the field. In this situation all the three vectors $\vec{F}$, $\vec{v}$ and $\vec{B}$ are mutually perpendicular to each other. Then sin θ = max = 1, i.e., θ = 90 ,

  The force will be maximum F_{max} = q v B
- Work done by force due to magnetic field in motion of a charged particle is always zero.

  When a charged particle move in a magnetic field, then force acts on it is always perpendicular to displacement, so the work done,W = $\int \vec{F}.d\vec{s} = \int F ds\ \cos 90° = 0$ (as θ = 90),

  And as by work-energy theorem W = Δ KE, the kinetic energy $\left(=\frac{1}{2}mv^2\right)$ remains unchanged and hence speed of charged particle v remains constant.

  However, in this situation the force changes the direction of motion, so the direction of velocity $\vec{v}$ of the charged particle changes continuously.

MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

Motion of a charged particle when it is moving collinear with the field magnetic field is not affected by the field (i.e. if motion is just along or opposite to magnetic field) (∵ F = 0) Only the following two cases are possible :

- **Case I :**

  When the charged particle is moving perpendicular to the field.

  The angle between $\vec{B}$ and $\vec{v}$ is θ=90 . So the force will be maximum (= qvB) and always perpendicular to motion (and also field); Hence the charged particle will move along a circular path (with its plane perpendicular to the field). Centripetal force is provided by the force qvB, So $\frac{mv^2}{r} = qvB \Rightarrow r = \frac{mv}{qB}$

  Angular frequency of circular motion, called cyclotron or gyro-frequency. $\omega = \frac{v}{r} = \frac{qB}{m}$

and the time period, $T=\frac{2\pi}{\omega}=2\pi\frac{m}{qB}$ i.e., time period (or frequency) is independent of speed of particle and radius of the orbit. Time period depends only on the field B and the nature of the particle, i.e., specific charge (q/m) of the particle.

This principle has been used in a large number of devices such as cyclotron (a particle accelerator), bubble-chamber (a particle detector) or mass-spectrometer etc.

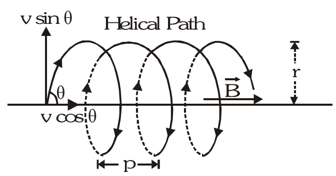
- **Case II :**

The charged particle is moving at an angle θ to the field : (θ ≠ 0 , 90 or 180). Resolving the velocity of the particle along and perpendicular to the field.The particle moves with constant velocity v cos θ along the field (∵ no force acts on a charged particle when it moves parallel to the field).

And at the same time it is also moving with velocity v sinθ perpendicular to the field due to which it will describe a circle (in a plane perpendicular to the field)

Radius of the circular path $r=\frac{m(v\sin\theta)}{qB}$ and Time period $T=\frac{2\pi r}{v\sin\theta}=\frac{2\pi m}{qB}$

So the resultant path will be a helix with its axis parallel to the field $\vec{B}$ as shown in fig.



The pitch p of the helix = linear distance travelled in one rotation $p=T\ (v\cos\theta)=\frac{2\pi m}{qB}(v\cos\theta)$

Example

An electron emitted by a heated cathode and accelerated through a potential difference of 2.0 kV enters a region with uniform magnetic field of 0.15 T. Determine the radius of the trajectory of the electron if the field is –

(a) Transverse to its initial velocity(b) Makes an angle of 30° with the initial velocity [Given : m_e = 9 10^{-31} kg]

Solution

$$\frac{1}{2}mv^2 = eV \Rightarrow v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times2\times10^{3}}{9\times10^{-31}}} = \frac{8}{3}\quad 10^7\ \text{m/s}$$

(a) Radius $r_1=\frac{mv}{qB}=\frac{9\times10^{-31}\times(8/3)\times10^{7}}{1.6\times10^{-19}\times0.15}=10^{-3}\ \text{m} = 1\text{mm}$

(b) Radius $r_2=\frac{mv\sin\theta}{qB}=r_1\sin\theta = 1\quad \sin30 = 1\quad \frac{1}{2} = 0.5\ \text{mm}$

MOTION OF CHARGED PARTICLE IN COMBINED ELECTRIC AND MAGNETIC FIELDS

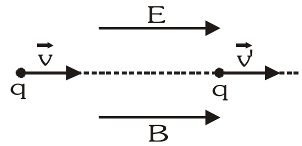
Let a moving charged particle is subjected simultaneously to both electric field $\vec{E}$ and magnetic field $\vec{B}$.

The moving charged particle will experience electric force $\vec{F}_e=q\vec{E}$ and magnetic force $\vec{F}_m=q(\vec{v}\times\vec{B})$.

Net force on the charge particle $\boxed{\vec{F}=q(\vec{E}+\vec{v}\times\vec{B})}$ **"Lorentz-force"**

Depending on the direction of $\vec{v},\vec{E}$ and $\vec{B}$ various situation are possible and the motion in general is quite complex.

Case I : $\vec{v}$, $\vec{E}$ and $\vec{B}$ all the three are collinear :

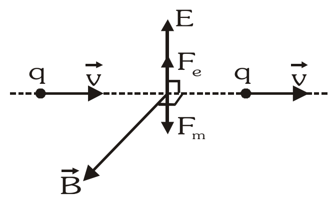


As the particle is moving parallel or antiparallel to the field. The magnetic force on it will be zero and only electric force will act So, acceleration of the particle $\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$

Hence, the particle will pass through the field following a straight line path (parallel to the field) with change in its speed.

In this situation speed, velocity, momentum and kinetic energy all will change without change in direction of motion as shown in figure above.

Case II : $\vec{v}$, $\vec{E}$ and $\vec{B}$ are mutually perpendicular :



If in this situation direction and magnitude of $\vec{E}$ and $\vec{B}$ are such that

Resultant force $\vec{F} = \vec{F}_e + \vec{F}_m = 0 \Rightarrow \vec{a} = \frac{\vec{F}}{m} = 0$

Then as shown in fig., the particle will pass through

the field with same velocity $\because$ $F_e = F_m$ i.e. $qE = qvB \Rightarrow v = \frac{E}{B}$

Example

A beam of protons is deflected sideways. Could this deflection be caused by

(i) a magnetic field (ii) an electric field? If either possible, what would be the difference?

Solution

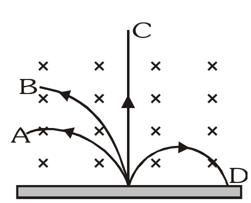
Yes, the moving charged particle (e.g. proton, α-particles etc.) may be deflected sideway either by an electric or by a magnetic field.

(i) The force exerted by a magnetic field on the moving charged particle is always perpendicular to direction of motion, so that no work is done on the particle by this magnetic force. That is the magnetic field simply deflects the particle and does not increase its kinetic energy.

(ii) The force exerted by electric field on the charged particle at rest or in motion is always along the direction of field and the kinetic energy of the particle changes.

Example

A neutron, a proton, an electron an α-particle enter a region of constant magnetic field with equal velocities. The magnetic field is along the inwards normal to the plane of the paper. The tracks of the particles are shown in fig. Relate the tracks to the particles.



Sol. Force on a charged particle in magnetic field $\vec{F} = q\ (\vec{v} \times \vec{B})$

For neutron q=0, F=0 hence it will pass undeflected i.e., tracks C corresponds to neutron.

If the particle is negatively charged, i.e. electron. $\vec{F} = -e(\vec{v} \times \vec{B})$

It will experience a force to the right; so track D corresponds to electron.

If the charge on particle is positive. It will experience a force to the left; so both tracks A and B corresponds to positively charged particles (i.e., protons and α-particles). When motion of charged particle perpendicular to the magnetic field the path is a circle with radius

$$r=\frac{mv}{qB} \quad \text{i.e. } r \propto \frac{m}{q} \text{ and as } \left(\frac{m}{q}\right)_\alpha = \left(\frac{4m}{2e}\right) \text{ while } \left(\frac{m}{q}\right)_p = \frac{m}{e} \Rightarrow \left(\frac{m}{q}\right)_\alpha > \left(\frac{m}{q}\right)_p$$

So $r_\alpha > r_p \Rightarrow$ track B to α-particle and A corresponds to proton.

Example

An electron does not suffer any deflection while passing through a region. Are you sure that there is no magnetic field? Is the reverse definite?

Solution

If electron passing through a certain region does not suffer any deflection, then we are not sure that there is no magnetic field in that region. This is due to that electron suffers no force when it moves parallel or antiparallel to magnetic field. Thus the magnetic field may exist parallel or antiparallel to the direction of motion of electron.

The reverse is not true since an electron can also be deflected by the electric field.

Example

In a chamber, a uniform magnetic field of 8 10^{-4} T is maintained. An electron with a speed of 4.0 10^6m/s enters the chamber in a direction normal to the field.

(a) Describe the path of the electron.

(b) What is the frequency of revolution of the electron?

(c) What happens to the path of the electron if it progressively loses its energy due to collisions with the atoms or molecules of the environment ?

Solution

(a) The path of the electron is a circle of radius $r=\frac{mv}{Be}=\frac{9.1\times10^{-31}\times4\times10^{6}}{1.6\times10^{-19}\times8\times10^{-4}}$ = 2.8 10^{-2}m

The sense of rotation of the electron in its orbit can be determined from the direction of the centripetal force. $\vec{F} = -e\ (\vec{v}\ \ \vec{B})$. So, if we look along the direction of $\vec{B}$, the electron revolves clockwise.

(b) the frequency of revolution of the electron in its circular orbit

$$f = \frac{eb}{2\pi m} = \frac{1.6\times10^{-19}\times8.0\times10^{-4}}{2\pi\times9.1\times10^{-31}} \text{ Hz} = 22.4 \text{ MHz}$$

(c) Due to collision with the atomic consistent of the environment, the electron progressively loses its speed. If the velocity vector of the electron remains in the same plane of the initial circular orbit after collisions, the radius of the circular orbit will decrease in proportion to the decreasing speed. However, in general, the velocity of the electron will not remain in the plane of the initial orbit after collision. In that case, the component of velocity normal to $\vec{B}$ will determine the radius of the orbit, while the component of velocity parallel to $\vec{B}$ remains constant. Thus, the path of the electron, between two collisions is, in general, helical. But an important fact must be noted: the frequency of orbital revolution remains the same, whatever be the speed of the electron.

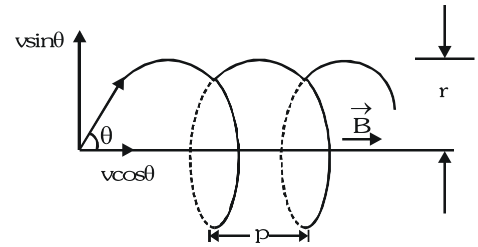
Example

A beam of protons with velocity 4 10^5 m/s enters a uniform magnetic field of 0.3 tesla at an angle of 60 to the magnetic field. Find the radius of the helical path taken by the proton beam. Also find the pitch of helix. Mass of proton=1.67 10^{-27}kg.

Solution

Radius of helix $r=\frac{mv\sin\theta}{qB}$ ($\therefore$ component of velocity $\perp$ to field is $v\sin\theta$)

$$=\frac{(1.67\times10^{-27})(4\times10^{5})\sqrt{\frac{3}{2}}}{(1.6\times10^{-19})0.3}=\frac{2}{\sqrt{3}}\quad 10^{-2}\text{m} = 1.2\text{cm}$$



Again, pitch $p=v\cos\theta \quad T$ (where $T=\frac{2\pi r}{v\sin\theta}$)

$$\therefore\ p=\frac{v\cos\theta\times2\pi r}{v\sin\theta}=\frac{\cos60^\circ\times2\pi\times(1.2\times10^{-2})}{\sin60^\circ}=4.35\quad 10^{-2}\text{m}=4.35\text{cm}$$

Example

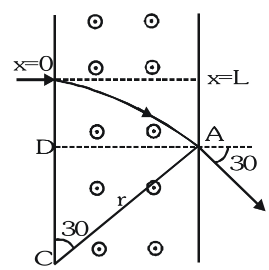
The region betwen x = 0 and x = L is filled with uniform, steady magnetic field $B_0\tilde{k}$. A particle of mass m, positive charge q and velocity $v_0\tilde{i}$ travels along X-axis and enters the region of magnetic field. Neglect the gravity throughout the question.

(a) Find the value of L if the particle emerges from the region of magnetic field with its final velocity at an angle 30 to its initial velocity.

(b) Find the final velocity of the particle and the time spent by it in the magnetic field, if the magnetic field now extends upto 2.1 L.

Solution

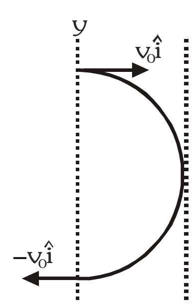
(a) The particle is moving with velocity $v_0\tilde{i}$, perpendicular to magnetic field $B_0\tilde{k}$. Hence the particle will move along a circular arc OA of radius $r=\frac{mv_0}{qB_0}$



Let the particle leave the magnetic field at A.

From ΔCDA, $\sin60=\frac{AD}{CA}=\frac{L}{r}\Rightarrow L = r\sin30=\frac{r}{2}\ \therefore L = \frac{mv_0}{2qB_0}$

(b) As the magnetic field extends upto 2.1 L i.e., L > 2r, so the particle completes half cycle before leaving the magnetic field, as shown in figure.

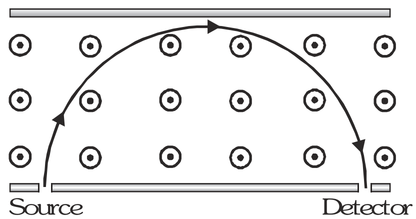


The magnetic field is always perpendicular to velocity vector, therefore the magnitude of velocity will remain the same.

$\therefore$Final velocity $= v_0(-\tilde{i})=-v_0\tilde{i}$ Time spent in magnetic field$=\frac{\pi r}{v_0}=\frac{\pi m}{qB_0}$

Example

A uniform magnetic field with a slit system as shown in fig. is to be used as a momentum filter for high energy charged particles. With a field of B tesla it is found that the filter transmits α-particle each of energy 5.3 MeV. The magnetic field is increased to 2.3 B tesla and deuterons are passed into the filter. What is the energy of each deuteron transmitted by the filter ?



Solution

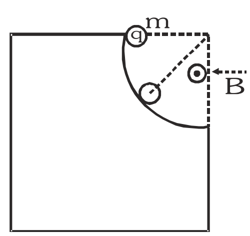
In case of circular motion of a charged particle in a magnetic field $\because E_k = \dfrac{r^2q^2B^2}{2m}$

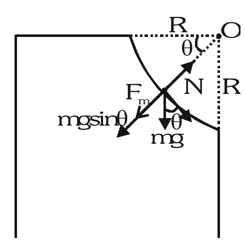
So according to the given problem $(E_k)_\alpha = \dfrac{r^2(2e)^2B^2}{2(4m)}$ and $(E_k)_D = \dfrac{r^2(e)^2(2.3B)^2}{2(2m)}$ i.e. $\dfrac{(E_k)_D}{(E_k)_\alpha} = \dfrac{(2.3)^2 \times 4}{2^2 \times 2}$

$\Rightarrow (E_k)_D = (5.3) \quad \dfrac{5.29}{2} = 14.02$ MeV

Example

A charged sphere of mass m and charge q starts sliding from rest on a vertical fixed circular track of radius R from the position as shown in figure . There exists a uniform and constant horizontal magnetic field of induction B. Find the maximum force exerted by the track on the sphere.





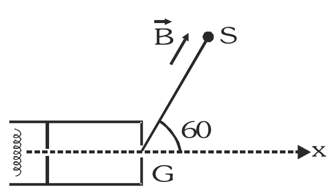
Sol. Magnetic force on sphere $F_m = qvB$ (directed radially outward)

$\because N - mg\sin\theta - qvB = \dfrac{mv^2}{R} \Rightarrow N = \dfrac{mv^2}{R} + mg\sin\theta + qvB$

Hence, at $\theta = \pi/2$ we get $N_{max} = \dfrac{2mgR}{R} + mg + qB\sqrt{2gR} = 3mg + qB\sqrt{2gR}$

Example

An electron gun G emits electrons of energy 2keV travelling in the positive x-direction. The electrons are required to hit the spot S, where GS = 0.1m, and the line GS makes an angle of 60 with the x-axis, as shown in the figure. A uniform magnetic field B parallel to GS exists in the region outside the electron gun. Find the minimum value of B needed to make the electron hit S.



Sol. The velocity of the electrons emitted by electron gun along x-axis, is $v = \sqrt{\frac{2E_K}{m}}$ $\because$ $E = \frac{1}{2}mv^2$

The velocity of the electron can be resolved into two components v cos θ and v sin θ, parallel and perpendicular to the magnetic field respectively. Due to component v cos θ electron will move in the direction of magnetic field with constant speed v cos θ but due to component v sin θ, it will move on a circular path in the plane ⊥ to magnetic field. Hence electron will move on a spiral path. As electrons are required to hit the spot S, hence distance travelled by electron in one time period along the direction of magnetic field must be just equal to GS. The electrons may also hit the spot S after two or more time periods but minimum value of B is required

As per above discussion,

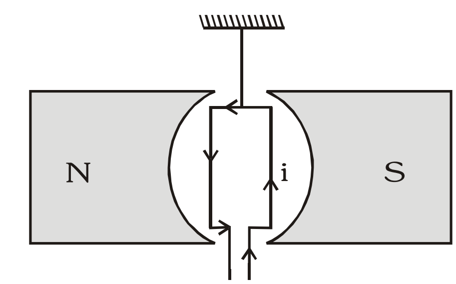
GS = Distance travelled along the direction of magnetic field in one time period

$$= (v\cos\theta)\quad T = v\cos\theta\quad \frac{2\pi m}{qB} = \sqrt{\frac{2E_K}{m}}\quad \cos\theta\quad \frac{2\pi m}{qB} \therefore B = \sqrt{\frac{2E}{m}}\quad \cos\theta\quad 2\pi m\quad \frac{1}{q\times(GS)}$$

$$= \sqrt{\frac{2\times2\times1.6\times10^{-16}}{9.1\times10^{-31}}}\quad \frac{\cos 60^\circ\times2\times3.14\times9.1\times10^{-31}}{1.6\times10^{-19}\times0.1} = 4.68\quad 10^{-3}\ T$$

MOVING COIL GALVANOMETER

A galvanometer is used to detect the current and has moderate resistance.



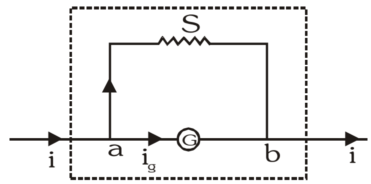
Principle. When a current carrying coil is placed in a magnetic field, it experiences a torque given by τ = NiAB sin θ where θ is the angle between normal to plane of coil and direction of magnetic field. In actual arrangement the coil is suspended between the cylindrical pole pieces of a strong magnet. The cylindrical pole pieces give the field radial such that sin θ=1 (always). So torque τ=NiAB If C is torsional rigidity (i.e., restoring couple per unit twist of the suspension wire), then for deflection θ of coil τ=Cθ. In equilibrium we have external couple = Restoring couple i.e.Cθ=NiAB or $\theta = \frac{NAB}{C}$ i i.e., $\theta \propto i$

In words the deflection produced is directly proportional to current in the coil.

The quantity $\frac{\theta}{i} = \frac{NAB}{C}$ is called the current sensitivity of the galvanometer. Obviously for greater sensitivity of galvanometer the number of turns N, area of coil A and magnetic field B produced by pole pieces should be larger and torsional rigidity C should be smaller. That is why the suspension wire is used of phosphor bronze for which torsional rigidity C is smaller.

CONVERSION OF GALVANOMETER INTO AMMETER

An ammeter is a low resistance galvanometer; used to measure current directly in amperes and is always connected in series with the circuit. To convert a galvanometer into ammeter, a low resistance, called shunt, is connected in parallel to the galvanometer as shown in figure.



Let i_g be the current in galvanometer for its full scale deflection and G the resistance of galvanometer. Let i is the range of ammeter and i_s the current in shunt S. Then potential difference across a and b is

$$V_{ab} = i_g G = i_S S. \quad \ldots(i)$$

At junction a, $i = i_S + i_g$ i.e., $i_S = i - i_g$

Therefore from (i) $i_g G = (i - i_g)S$ or $i_g(S+G) = iS$ i.e., $i_g = \frac{S}{S+G} i \ldots(ii)$

This is the working equation for conversion of galvanometer into ammeter. Here $i_g < i$.

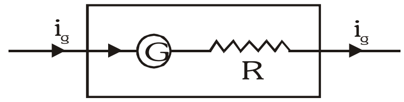
From (ii) shunt required $S = \frac{i_g G}{i - i_g}$. If $i_g << i$, $S = \left(\frac{i_g}{i}\right) G$

The resistance of ammeter R_A so formed is given by $\frac{1}{R_A} = \frac{1}{G} + \frac{1}{S} \Rightarrow R_A = \frac{SG}{S+G} \ldots.(iii)$

Note: Equation (ii) may also be used to increase the range of given ammeter. Here G will be resistance of given ammeter, S shunt applied, i_g its initial range and i the new range desired.

CONVERSION OF GALVANOMETER INTO VOLTMETER

A voltmeter is a high resistance galvanometer and is connected between two points across which potential difference, is to be measured i.e., voltmeter is connected in parallel with the circuit. To convert a galvanometer into voltmeter, a high resistance R in series is connected to the galvanometer.



If V is range of voltmeter, then $i_g = \frac{V}{R+G}$ or resistance in series $R = \frac{V}{i_g} - G \ldots(i)$

This is working equation for conversion of galvanometer into voltmeter.

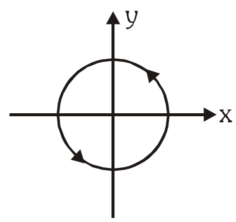
The resistance of voltmeter so formed is $R_V = R + G \ldots(ii)$

Note: Equation (i) may also be used to increase the range of voltmeter. If V_0 is initial range and V is new range of voltmeter, then $i_g = \frac{V_0}{G} = \frac{V}{R+G}$

SOME WORKED OUT EXAMPLES

Example#1

Current i = 2.5 A flows along the circle $x^2 + y^2 = 9\ cm^2$ (here x & y in cm) as shown. Magnetic field at point (0, 0, 4 cm) is



(A) $(36\pi \times 10^{-7}\,T)\tilde{k}$ (B) $(36\pi \times 10^{-7}\,T)(-\tilde{k})$ (C) $\left(\frac{9\pi}{5} \times 10^{-7}\,T\right)\tilde{k}$ (D) $\left(\frac{9\pi}{5} \times 10^{-7}\,T\right)(-\tilde{k})$

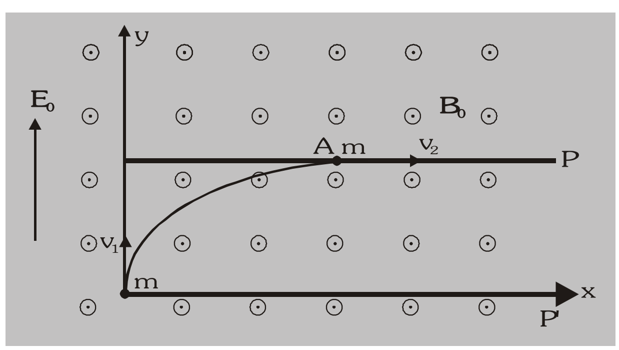
Solution **Ans. (A)**

Magnetic field on the axis of a circular loop

$$B = \left(\frac{\mu_0}{4\pi}\right) \times \frac{2\pi i R^2}{(R^2 + z^2)^{3/2}} = 10^{-7} \times \frac{2\pi \times 2.5 \times 3^2 \times 10^{-4}}{125 \times 10^{-6}}\tilde{k} = \left(\frac{9\pi}{25} \times 10^{-5}\,T\right)\tilde{k} = (36\pi \times 10^{-7}\,T)\tilde{k}$$

Example#2

There are constant electric field $E_0\tilde{j}$ & magnetic field $B\tilde{k}$ present between plates P and P'. A particle of mass m is projected from plate P' along y axis with velocity v_1. After moving on the curved path, it passes through point A just grazing the plate P with velocity v_2. The magnitude of impulse (i.e. $\vec{F}\Delta t = \Delta\vec{p}$) provided by magnetic force during the motion of particle from origin to point A is :–



(A) $m|v_2 - v_1|$ (B) $m\sqrt{v_1^2 + v_2^2}$ (C) mv_1 (D) mv_2

Solution **Ans. (D)**

Electric force is only responsible for the change in momentum along y–axis. Therefore impulse provide by magnetic force is $J_B = mv_2$.

Example#3

Three identical charge particles A, B and C are projected perpendicular to the uniform magnetic field with velocities v_1, v_2 and v_3 ($v_1 < v_2 < v_3$) respectively such that T_1, T_2 and T_3 are their respective time period of revolution and r_1, r_2 and r_3 are respective radii of circular path described. Then :-

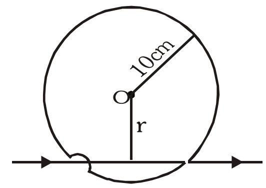
(A) $\frac{r_1}{T_1} > \frac{r_2}{T_2} > \frac{r_3}{T_3}$ (B) $T_1 < T_2 < T_3$ (C) $\frac{r_1}{T_1} < \frac{r_2}{T_2} < \frac{r_3}{T_3}$ (D) $r_1 = r_2 = r_3$

Solution **Ans. (C)**

$$T=\frac{2\pi m}{qB}\ \&\ r=\frac{mv}{qB}\Rightarrow\frac{r}{T}\propto v$$

Example#4

An infinitely long straight wire is bent as shown in figure. The circular portion has a radius of 10 cm with its center O at a distance r from the straight part. The value of r such that the magnetic field at the center O of the circular portion is zero will be :-



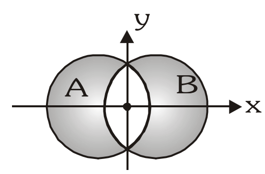
(A) $\frac{10}{\pi}$ cm (B) $\frac{20}{\pi}$ cm (C) $\frac{1}{5\pi}$ cm (D) $\frac{5}{\pi}$ cm

Solution **Ans. (A)**

$$B_{\text{circular loop}}=-B_{\text{wire}}\Rightarrow\frac{\mu_0 I}{2\times10}=\frac{\mu_0 I}{2\pi r}\Rightarrow r=\frac{10}{\pi}\text{cm}$$

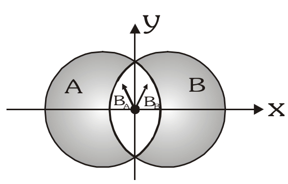
Example#5

Two cylindrical straight and very long non magnetic conductors A and B, insulated from each other, carry a current I in the positive and the negative z–direction respectively. The direction of magnetic field at origin is



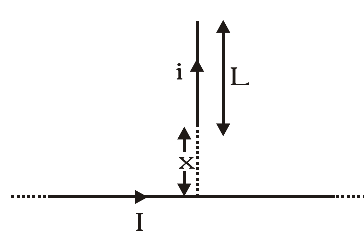
(A) $-\tilde{i}$ (B) $+\tilde{i}$ (C) $\tilde{j}$ (D) $-\tilde{j}$

Solution **Ans. (C)**



Example#6

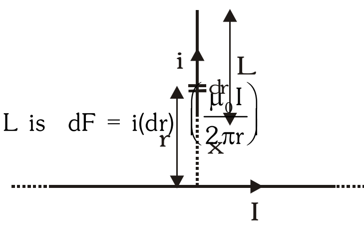
The magnetic force between wires as shown in figure is :-



(A) $\frac{\mu_0 iI^2}{2\pi}\ell n\left(\frac{x+L}{2x}\right)$ (B) $\frac{\mu_0 iI^2}{2\pi}\ell n\left(\frac{2x+L}{2x}\right)$ (C) $\frac{\mu_0 iI}{2\pi}\ell n\left(\frac{x+L}{x}\right)$ (D) None of these

Solution **Ans. (C)**

Magnetic field at dr, B = $\dfrac{\mu_0 I}{2\pi r}$

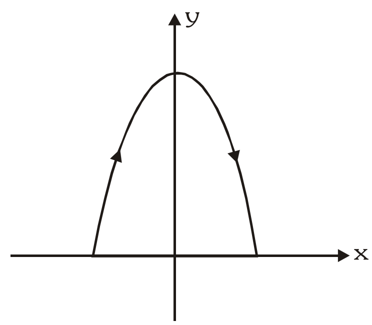


Force on small element at a distance r of wire of length L is dF = $i(dr)\left(\dfrac{\mu_0 I}{2\pi r}\right)$

$$F = \frac{\mu_0 i I}{2\pi}\int_x^{x+L}\frac{dr}{r} = \frac{\mu_0 i I}{2\pi}\ell n\left(\frac{x+L}{x}\right)$$

Example#7

A wire carrying a current of 4A is bent in the form of a parabola $x^2 + y = 16$ as shown in figure, where x and y are in meter. The wire is placed in a uniform magnetic field $\vec{B} = 5\hat{k}$ tesla. The force acting on the wire is



(A) $80\ \hat{j}$ N (B) $-\ 80\ \hat{j}$ N (C) $-160\ \hat{j}$ N (D) $160\ \hat{j}$ N

Solution **Ans. (C)**

$$\vec{F} = I(\vec{\ell}\times\vec{B}) = 4(8\hat{i}\times 5\hat{k}) = -160\ \hat{j}\ N$$

Example#8

A conducting coil is bent in the form of equilateral triangle of side 5 cm. Current flowing through it is 0.2 A. The magnetic moment of the triangle is :-

(A) $\sqrt{3}$ 10^{-2} A–m^2 (B) 2.2 10^{-4} A–m^2 (C) 2.2 10^{-2} A–m^2 (D) $\sqrt{3}$ 10^{-4} A–m^2

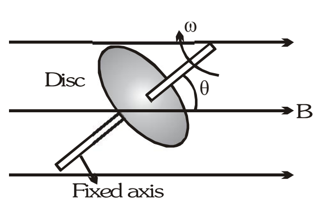
Solution **Ans. (B)**

Magnetic moment of current carrying triangular loop M = IA

$$M = 0.2\left(\frac{1}{2}\times 5\times 10^{-2}\times\frac{\sqrt{3}\times 5\times 10^{-2}}{2}\right) = 2.2\quad 10^{-4}\ \text{A-m}^2$$

Example#9

A disc of radius r and carrying positive charge q is rotating with angular speed ω in a uniform magnetic field B about a fixed axis as shown in figure, such that angle made by axis of disc with magnetic field is θ. Torque applied by axis on the disc is



(A) $\dfrac{q\omega r^2 B\sin\theta}{2}$, clockwise (B) $\dfrac{q\omega r^2 B\sin\theta}{4}$, anticlockwise

(C) $\dfrac{q\omega r^2 B\sin\theta}{2}$, anticlockwise (D) $\dfrac{q\omega r^2 B\sin\theta}{4}$, clockwise

Solution **Ans. (D)**

$$\frac{M}{L}=\frac{q}{2m}\Rightarrow M=\frac{q}{2m}\times\frac{mr^2}{2}\omega\Rightarrow\tau=\left|\vec{M}\times\vec{B}\right|=\frac{q\omega r^2B\sin\theta}{4}(\text{clockwise})$$

Example#10

A particle of mass m and charge q is thrown from origin at t = 0 with velocity $2\hat{i}+3\hat{j}+4\hat{k}$ units in a region with uniform magnetic field $2\hat{i}$ units. After time t = $\frac{\pi m}{qB}$, an electric field $\vec{E}$ is switched on, such that particle moves on a straight line with constant speed. $\vec{E}$ may be

(A) $-8\hat{j}+6\hat{k}$ units (B) $-6\hat{i}-9\hat{k}$ units (C) $-12\hat{j}+9\hat{k}$ units (D) $8\hat{j}-6\hat{k}$ units

Solution **Ans. (A)**

At t = $\frac{\pi n}{qB}$, $\vec{v}=2\hat{i}-3\hat{j}-4\hat{k}$; For net force to be zero $q\vec{v}\times\vec{B}+q\vec{E}=0\Rightarrow\vec{E}=-\vec{v}\times\vec{B}=-8\hat{j}+6\hat{k}$

Example#11

A particle of specific charge $\frac{q}{m}=\pi\times10^{10}\,C\,kg^{-1}$ is projected from the origin along the positive x–axis with a velocity of 10^5 ms^{-1} in a uniform magnetic field $\vec{B}=-2\times10^{-3}\hat{k}$ tesla. Choose correct alternative(s)

(A) The centre of the circle lies on the y–axis

(B) The time period of revolution is 10^{-7} s.

(C) The radius of the circular path is $\frac{5}{\pi}$ mm

(D) The velocity of the particle at t = $\frac{1}{4}\times10^{-7}$s is $10^5\hat{j}$ m / s

Solution **Ans. (A,B,C,D)**

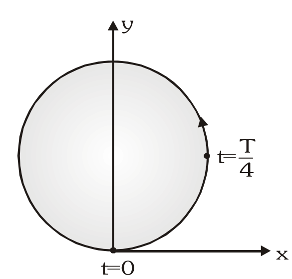
$\vec{F}=q\left(\vec{v}\times\vec{B}\right)$, $\vec{v}=10^5\hat{i}$, $\vec{B}=-2\times10^{-3}\hat{k}$ $\Rightarrow$ Force will be in y–direction

$\Rightarrow$ Motion of particle will be in xy plane

Time period $T=\frac{2\pi m}{qB}=\frac{2\pi}{\pi\times10^{10}\times2\times10^{-3}}=10^{-7}$s

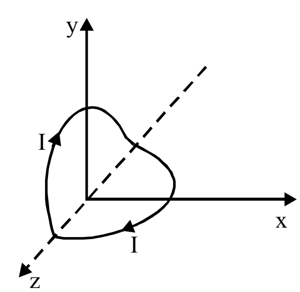
Radius of path $=\frac{mv}{qB}=\frac{10^5}{\pi\times10^{10}\times2\times10^{-3}}=\frac{5}{\pi}\times10^{-3}\,\text{m}=\frac{5}{\pi}\text{mm}$

At t = $\frac{T}{4}=\frac{10^{-7}}{4}$ s. Velocity of particle will be in +y direction.



Example#12

A circular current carrying loop of radius R is bent about its diameter by 90 and placed in a magnetic field $\vec{B}=B_0(\hat{i}+\hat{j})$ as shown in figure.



(A) The torque acting on the loop is zero

(B) The magnetic moment of the loop is $\frac{I\pi R^2}{2}\left(-\hat{i}-\hat{j}\right)$

(C) The angular acceleration of the loop is non zero.

(D) The magnetic moment of the loop is $\frac{I\pi R^2}{2}\left(-\hat{i}+\hat{j}\right)$

Solution **Ans. (A,B)**

$$\vec{M} = \frac{I\pi R^2}{2}\left(-\tilde{i} - \tilde{j}\right);\ \vec{\tau} = \vec{M} \times \vec{B} = 0$$

Example#13

A current–carrying ring is placed in a magnetic field. The direction of the field is perpendicular to the plane of the ring–

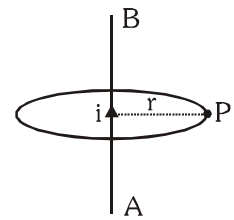
(A) There is no net force on the ring.
(B) The ring will tend to expand.
(C) The ring will tend to contract.
(D) Either (B) or (C) depending on the directions of the current in the ring and the magnetic field.

Solution **Ans. (A,D)**

Net force = 0 and ring will tend to expand/contracts depending on I & B.

Example#14

In the given figure, B is magnetic field at P due to shown segment AB of an infinite current carrying wire. A loop is taken as shown in figure. Which of the following statement(s) is/are correct.



(A) $\oint \vec{B} \cdot \vec{d\ell} = \mu_0 i$ (B) $B = \frac{\mu_0 i}{2\pi r}$

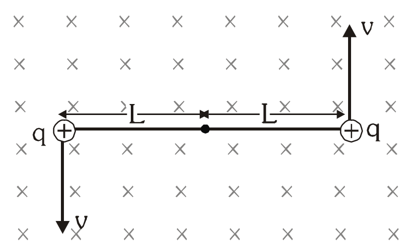
(C) Magnetic field at P will be tangential (D) None of these

Solution **Ans. (C)**

Magnetic field at P is tangential.

Example#15 to 17

Two charge particles each of mass 'm', carrying charge +q and connected with each other by a massless inextensible string of length 2L are describing circular path in the plane of paper, each with speed $v = \frac{qB_0L}{m}$ (where B_0 is constant) about their centre of mass in the region in which an uniform magnetic field $\vec{B}$ exists into the plane of paper as shown in figure. Neglect any effect of electrical & gravitational forces.



15. The magnitude of the magnetic field such that no tension is developed in the string will be

(A) $\frac{B_0}{2}$ (B) B_0 (C) $2B_0$ (D) 0

16. If the actual magnitude of magnetic field is half to that of calculated in part (i) then tension in the string will be

(A) $\frac{3}{4}\frac{q^2B_0^2L}{m}$ (B) zero (C) $\frac{q^2B_0^2L}{2m}$ (D) $\frac{2q^2B_0^2L}{m}$

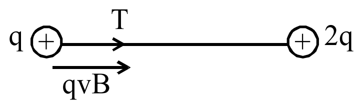
17. Given that the string breaks when the tension is $T = \frac{3}{4}\frac{q^2B_0^2L}{m}$. Now if the magnetic field is reduced to such a value that the string just breaks then find the maximum separation between the two particles during their motion

(A) 16 L (B) 4L (C) 14L (D) 2L

Solution

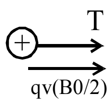
15. Ans. (B)

$T + qvB = \frac{mv^2}{L}$



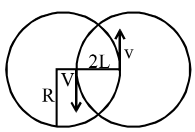
$T = 0; \quad \Rightarrow q\left(\frac{qB_0L}{m}\right)B = \frac{mv^2}{L} \Rightarrow B = B_0$

16. Ans. (C)



$T + \frac{qvB_0}{2} = \frac{mv^2}{R}$; $T = \frac{m(qB_0L)^2}{m^2R} - \frac{q(qB_0L)}{m} \times \frac{B_0}{2} = \frac{q^2B_0^2L}{2m}$

17. Ans. (C)

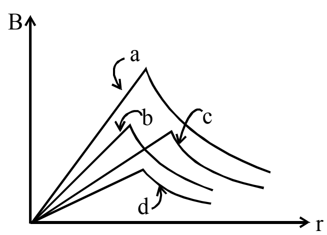


Maximum reperation = 2R + (2R – 2L)

$T + qvB = \frac{mv^2}{R} \Rightarrow B = B_0/4 \Rightarrow \quad R = 4L \Rightarrow$ maximum seperation =16L – 2L= 14L

Example#18 to 20

Curves in the graph shown give, as functions of radial distance r, the magnitude B of the magnetic field inside and outside four long wires a, b, c and d, carrying currents that are uniformly distributed across the cross sections of the wires. The wires are far from one another.



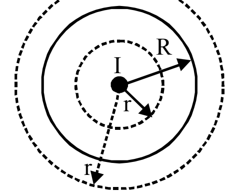
18. Which wire has the greatest radius?
(A) a (B) b (C) c (D) d

19. Which wire has the greatest magnitude of the magnetic field on the surface?
(A) a (B) b (C) c (D) d

20. The current density in wire a is
(A) greater than in wire c
(B) less than in wire c
(C) equal to that in wire c
(D) not comparable to that in wire c due to lack of information

Solution

18. Ans. (C)



Inside the cylinder : $B = \frac{\mu_0 I}{2\pi R^2} r$ ...(i)

Outside the cylinder $\therefore B = \frac{\mu_0 I}{2\pi r}$ ...(ii)

Inside cylinder B $\propto$ r and outside $B \propto \frac{1}{r}$

So from surface of cylinder nature of magnetic field changes.
Hence it is clear from the graph that wire 'c' has greatest radius.

19. Ans. (A)

Magnitude of magnetic field is maximum at the surface of wire 'a'.

20. Ans. (A)

Inside the wire $B(r) = \frac{\mu_0}{2\pi}\frac{I}{R^2}r$; $\frac{dB}{dr} = \frac{\mu_0}{2\pi}\frac{I}{R^2}$ i.e. slope $\propto \frac{I}{\pi R^2}$ $\propto$ current density.

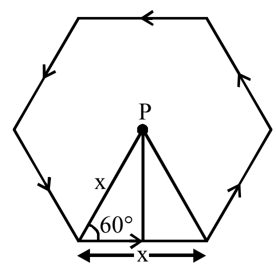
It can be seen that slope of curve for wire a is greater than wire c.

Example#21

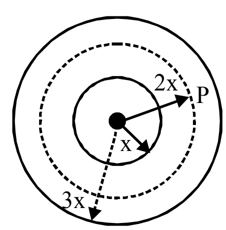
Column-I gives some current distributions and a point P in the space around these current distributions. Column-II gives some expressions of magnetic field strength. Match column-I to corresponding field strength at point P given in column-II

	Column – I		Column – II
(A)	A conducting loop shaped as regular hexagon of side x, carrying current i. P is the centroid of hexagon	(P)	$\frac{3\mu_0 i}{32\pi x}$
(B)	A cylinder of inner radius x and outer radius 3x, carrying current i. Point P is at a distance 2x from the axis of the cylinder	(Q)	$\frac{\sqrt{3}\mu_0 i}{\pi x}$
(C)	Two coaxial hollow cylinders of radii x and 2x, each carrying current i, but in opposite direction. P is a point at distance 1.5x from the axis of the cylinders	(R)	$\frac{\mu_0 i}{2x}$
(D)	Magnetic field at the centre of an n-sided regular polygon, of circum circle of radius x, carrying current i, $n \to \infty$, P is centroid of the polygon.	(S)	$\frac{\mu_0 i}{3\pi x}$
		(T)	Zero

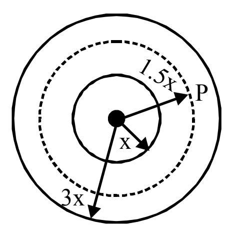
Solution Ans. (A) $\to$ (Q) ; (B) $\to$ (P) ; (C) $\to$ (S) ; (D) $\to$ (R)



For A : $B_P = 6 \times \frac{\mu_0 i}{4\pi(x \sin 60°)}[\sin 30° + \sin 30°] = \frac{\sqrt{3}\mu_0 i}{\pi x}$



For B: $B_P = \frac{\mu_0\left(\frac{3}{8}i\right)}{2\pi(2x)} = \frac{3\mu_0 i}{32\pi x}$

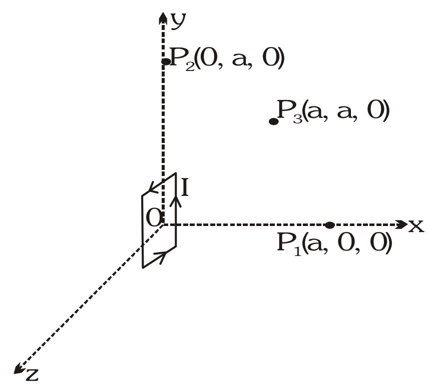


For C : $B_P = \frac{\mu_0 i}{2\pi(1.5x)} = \frac{\mu_0 i}{3\pi x}$

For D : If $n \to \infty$, n sided polygon $\to$ circle so $B = \frac{\mu_0 i}{2x}$

Example#22

A very small current carrying square loop (current I) of side 'L' is placed in y-z plane with centre at origin of the coordinate system (shown in figure). In column–I the coordinate of the points are given & in column–II magnitude of strength of magnetic field is given. Then



	Column I		Column II
(A)	At point O (0, 0, 0)	(P)	$\frac{\mu_0 I L^2}{2\pi a^3}$
(B)	At point P_1 (a, 0, 0) (here a > > L)	(Q)	$\frac{2\sqrt{2}\mu_0 I}{\pi L}$
(C)	At point P_2 (0, a, 0) (here (a > > L)	(R)	$\frac{\mu_0\sqrt{5}\, I L^2}{16\pi a^3}$
(D)	At point P_3 (a, a, 0) (here a > > L)	(S)	$\frac{\mu_0 I L^2}{4\pi a^3}$
		(T)	$\frac{\mu_0\sqrt{5} I L^2}{4\pi a^3}$

2. **Ans. (A) → (Q); (B) → (P); (C) → (S); (D) → (R)**

For A : $B_{P_1} = 4 \times \frac{\mu_0 I}{4\pi(L/2)}\left[\sin\frac{\pi}{4} + \sin\frac{\pi}{4}\right] = \frac{2\sqrt{2}\mu_0 I}{\pi L}$

For B: $B_{P_2} = \frac{2kM}{r^3} = \frac{2\left(\frac{\mu_0}{4\pi}\right)(IL^2)}{a^3} = \frac{\mu_0 IL^2}{2\pi a^3}$

For C : $B_{P_3} = \frac{kM}{r^3} = \frac{\left(\frac{\mu_0}{4\pi}\right)(IL^2)}{a^3} = \frac{\mu_0 IL^2}{4\pi a^3}$

For D : $B_{P_4} = \frac{kM}{r^3}\sqrt{1+3\cos^2\theta}$ where $\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow B_{P_4} = \frac{\left(\frac{\mu_0}{4\pi}\right)(IL^2)}{\left(a\sqrt{2}\right)^3}\sqrt{1+3\left(\frac{1}{2}\right)} = \frac{\mu_0\sqrt{5} IL^2}{16\pi a^3}$