WAVE THEORY, SOUND WAVES & DOPPLER'S EFFECTS

INTRODUCTION OF WAVES

What is wave motion ?

- When a particle moves through space, it carries KE with itself. Wherever the particle goes, the energy goes with it. (One way of transport energy from one place to another place)
- There is another way (wave motion) to transport energy from one part of space to other without any bulk motion of material together with it. Sound is transmitted in air in this manner.

Ex. You (Kota) want to communicate your friend (Delhi)

Write a letter   Use telephone

1st option involves the concept of particle & the second choice involves the concept of wave.

Ex. When you say "Namaste" to your friend no material particle is ejected from your lips to fall on your friends ear. Basically you create some disturbance in the part of the air close to your lips. Energy is transferred to these air particles either by pushing them ahead or pulling them back. The density of the air in this part temporarily increases or decreases. These disturbed particles exert force on the next layer of air, transferring the disturbance to that layer. In this way, the disturbance proceeds in air and finally the air near the ear of the listener gets disturbed.

Note :- In the above example air itself does not move.

> A **wave** is a disturbance that propagates in space, transports energy and momentum from one point to another without the transport of matter.

Few examples of waves :

The ripples on a pond (water waves), the sound we hear, visible light, radio and TV signals etc.

CLASSIFICATION OF WAVES

Wave classification According to

- Necessity of medium
  - (i) Elastic or mechanical wave
  - (ii) Electro magnetic wave (E.M. wave) or non-mech.
- Propagation of energy
  - (i) Progressive
  - (ii) Sationary
- Dimension
  - (i) One dimensional
  - (ii) Two dimensional
  - (iii) Three dimentional
- Vibration of particle
  - (i) Transverse
  - (ii) Longitudinal

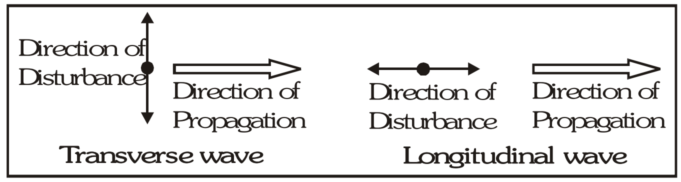
1. ***Based on medium necessity*** :- A wave may or may not require a medium for its propagation. The waves which do not require medium for their propagation are called non-mechanical, e.g. light, heat (infrared), radio waves etc. On the other hand the waves which require medium for their propagation are called mechanical waves. In the propagation of mechanical waves elasticity and density of the medium play an important role therefore mechanical waves are also known as **elastic waves**.

Example : Sound waves in water, seismic waves in earth's crust.

2. **Based on energy propagation :-** Waves can be divided into two parts on the basis of energy propagation (i) Progressive wave (ii) Stationary waves. The progressive wave propagates with constant velocity in a medium. In stationary waves particles of the medium vibrate with different amplitude but energy does not propagate.

3. *Based on direction of propagation* :- Waves can be one, two or three dimensional according to the number of dimensions in which they propagate energy. Waves moving along strings are one-dimensional. Surface waves or ripples on water are two dimensional , while sound or light waves from a point source are three dimensional.

4. *Based on the motion of particles of medium* :



Waves are of two types on the basis of motion of particles of the medium.

(i) Longitudinal waves

(ii) Transverse waves

In the transverse wave the direction associated with the disturbance (i.e. motion of particles of the medium) is at right angle to the direction of propagation of wave while in the longitudinal wave the direction of disturbance is along the direction of propagation.

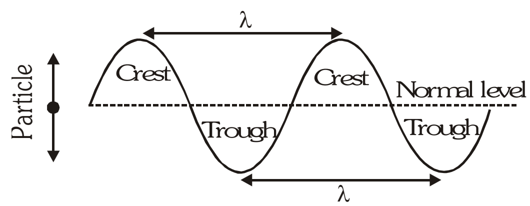
TRANSVERSE WAVE MOTION

Mechanical transverse waves produce in such type of medium which have shearing property, so they are known as shear wave or S-wave

Note :- Shearing is the property of a body by which it changes its shape on application of force.

⇒ Mechanical transverse waves are generated only in solids & surface of liquid.

In this individual particles of the medium execute SHM about their mean position in direction $\perp^r$ to the direction of propagation of wave motion.

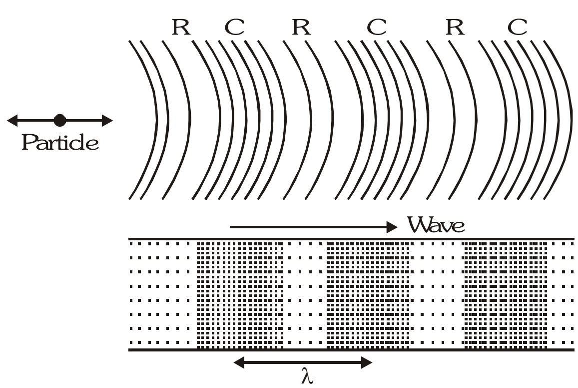


A **crest** is a portion of the medium, which is raised temporarily above the normal position of rest of particles of the medium , when a transverse wave passes.

A **trough** is a portion of the medium, which is depressed temporarily below the normal position of rest of particles of the medium , when a transverse wave passes.

LONGITUDINAL WAVE MOTION

In this type of waves, oscillatory motion of the medium particles produces regions of compression (high pressure) and rarefaction (low pressure) which propagated in space with time (see figure).

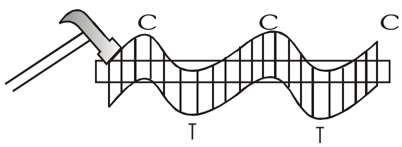


Note : The regions of high particle density are called compressions and regions of low particle density are called rarefactions.

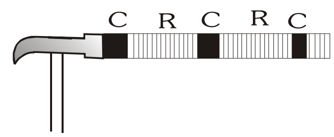
The propagation of sound waves in air is visualized as the propagation of pressure or density fluctuations. The pressure fluctuations are of the order of 1 Pa, whereas atmospheric pressure is 10^5 Pa.

Mechanical Waves in Different Media

- A mechanical wave will be transverse or longitudinal depends on the nature of medium and mode of excitation.
- In strings mechanical waves are always transverse when string is under a tension. In gases and liquids mechanical waves are always longitudinal e.g. sound waves in air or water. This is because fluids cannot sustain shear.
- In solids, mechanical waves (may be sound) can be either transverse or longitudinal depending on the mode of excitation. The speed of the two waves in the same solid are different. (Longitudinal waves travels faster than transverse waves). e.g., if we struck a rod at an angle as shown in fig. (A) the waves in the rod will be transverse while if the rod is struck at the side as shown in fig. (B) or is rubbed with a cloth the waves in the rod will be longitudinal. In case of vibrating tuning fork waves in the prongs are transverse while in the stem are longitudinal.



Transverse-wave
(A)

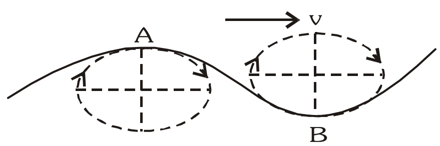


Longitudinal-wave
(B)

Further more in case of seismic waves produced by Earthquakes both S (shear) and P (pressure) waves are produced simultaneously which travel through the rock in the crust at different speeds

[$v_S \cong 5$ km/s while $v_P \cong 9$ km/s] S–waves are transverse while P–waves longitudinal.

Some waves in nature are neither transverse nor longitudinal but a combination of the two. These waves are called 'ripple' and waves on the surface of a liquid are of this type. In these waves particles of the medium vibrate up and down and back and forth simultaneously describing ellipses in a vertical plane.



Ripple

CHARACTERISTICS OF WAVE MOTION

Some of the important characteristics of wave motion are as follows :

- In a wave motion, the disturbance travels through the medium due to repeated periodic oscillations of the particles of the medium about their mean positions.
- The energy is transferred from place to another without any actual transfer of the particles of the medium.
- Each particle receives disturbance a little later than its preceding particle i.e., there is a regular phase difference between one particle and the next.
- The velocity with which a wave travels is different from the velocity of the particles with which they vibrate about their mean positions.
- The wave velocity remains constant in a given medium while the particle velocity changes continuously during its vibration about the mean position. It is maximum at the mean position and zero at the extreme position.
- For the propagation of a mechanical wave, the medium must possess the properties of inertia, elasticity and minimum friction amongst its particles.

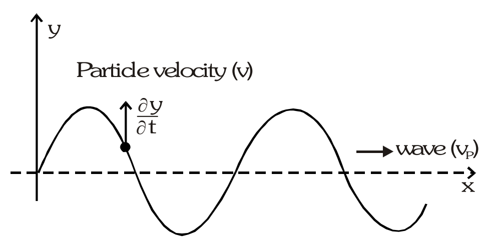
SOME IMPORTANT TERMS CONNECTED WITH WAVE MOTION

- **Wavelength (λ) [length of one wave]**

  Distance travelled by the wave during the time, any one particle of the medium completes one vibration about its mean position. We may also define wavelength as the distance between any two nearest particles of the medium, vibrating in the same phase.
- **Frequency (n)** :Number of vibrations (Number of complete wavelengths) complete by a particle in one second.

- **Time period (T)** : Time taken by wave to travel a distance equal to one wavelength.
- **Amplitude (A)** : Maximum displacement of vibrating particle from its equilibrium position.
- **Angular frequency (ω)** : It is defined as $\omega = \frac{2\pi}{T} = 2\pi n$
- **Phase** : Phase is a quantity which contains all information related to any vibrating particle in a wave. For equation $y = A \sin(\omega t - kx)$; $(\omega t - kx)$ = phase.
- **Angular wave number (k)** : It is defined as $k = \frac{2\pi}{\lambda}$
- **Wave number ($\vec{\nu}$)** : It is defined as $\vec{\nu} = \frac{1}{\lambda} = \frac{k}{2\pi}$ = number of waves in a unit length of the wave pattern.
- **Particle velocity, wave velocity and particle's acceleration** : In plane progressive harmonic wave particles of the medium oscillate simple harmonically about their mean position. Therefore, all the formulae what we have read in SHM apply to the particles here also. For example, maximum particle velocity is $\pm A\omega$ at mean position and it is zero at extreme positions etc. Similarly maximum particle acceleration is $\pm\omega^2 A$ at extreme positions and zero at mean position. However the wave velocity is different from the particle velocity. This depends on certain characteristics of the medium. Unlike the particle velocity which oscillates simple harmonically (between $+A\omega$ and $-A\omega$) the wave velocity is constant for given characteristics of the medium.
- **Particle velocity in wave motion :**

The individual particles which make up the medium do not travel through the medium with the waves. They simply oscillate about their equilibrium positions. The instantaneous velocity of an oscillating particle of the medium, through which a wave is travelling, is known as "Particle velocity".



- **Wave velocity** : The velocity with which the disturbance, or planes of equal phase (wave front), travel through the medium is called wave (or phase) velocity.

- **Relation between particle velocity and wave velocity :**

Wave equation :- $y = A \sin(\omega t - kx)$, Particle velocity $v = \frac{\partial y}{\partial t} = A\omega \cos(\omega t - kx)$.

Wave velocity $= v_p = \frac{\lambda}{T} = \lambda\frac{\omega}{2\pi} = \frac{\omega}{k}$, $\frac{\partial y}{\partial x} = -Ak\cos(\omega t - kx) = -\frac{A}{\omega}\omega k \cos(\omega t - kx) = -\frac{1}{v_p}\frac{\partial y}{\partial t} \Rightarrow \frac{\partial y}{\partial x} = -\frac{1}{v_p}\frac{\partial y}{\partial t}$

Note : $\frac{\partial y}{\partial x}$ represent the slope of the string (wave) at the point x.

Particle velocity at a given position and time is equal to negative of the product of wave velocity with slope of the wave at that point at that instant.

- **Differential equation of harmonic progressive waves :**

$$\frac{\partial^2 y}{\partial t^2} = -A\omega^2 \sin(\omega t - kx) \Rightarrow \frac{\partial^2 y}{\partial x^2} = -Ak^2 \sin(\omega t - kx) \Rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{1}{v_P^2}\frac{\partial^2 y}{\partial t^2}$$

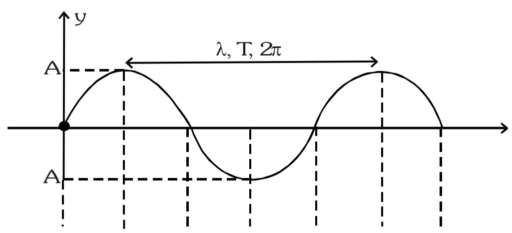
- **Particle velocity (v_p) and acceleration (a_p) in a sinusoidal wave :**

  The acceleration of the particle is the second particle is the second partial derivative of y (x, t) with respect to t,

$$\therefore\ a_P = \frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 A \sin(kx - \omega t) = -\omega^2 y(x,t)$$

  i.e., the acceleration of the particle equals $-\omega^2$ times its displacement, which is the result we obtained for SHM. Thus, $a_p = -\omega^2$ (displacement)

- **Relation between Phase difference, Path difference & Time difference**



Phase (ϕ)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π
Wave length (λ)	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$\frac{3\lambda}{4}$	λ	$\frac{5\lambda}{4}$	$\frac{3}{2}\lambda$
Time-period (T)	0	$\frac{T}{4}$	$\frac{T}{2}$	$\frac{3T}{4}$	T	$\frac{5T}{4}$	$\frac{3T}{2}$

$$\Rightarrow \frac{\Delta\phi}{2\pi} = \frac{\Delta\lambda}{\lambda} = \frac{\Delta T}{T} \Rightarrow \text{Path difference} = \left(\frac{\lambda}{2\pi}\right) \text{Phase difference}$$

Example

A progressive wave of frequency 500 Hz is travelling with a velocity of 360 m/s. How far apart are two points 60° out of phase.

Solution

We know that for a wave v = f λ So $\lambda = \frac{v}{f} = \frac{360}{500} = 0.72$ m

Phase difference $\Delta\phi = 60° = (\pi/180) \times 60 = (\pi/3)$ rad, so path difference $\Delta x = \frac{\lambda}{2\pi}(\Delta\phi) = \frac{0.72}{2\pi} \times \frac{\pi}{3} = 0.12$ m

THE GENERAL EQUATION OF WAVE MOTION

Some physical quantity (say y) is made to oscillate at one place and these oscillations of y propagate to other places. The y may be,

(i) displacement of particles from their mean position in case of transverse wave in a rope or longitudinal sound wave in a gas.

(ii) pressure difference (dP) or density difference (dρ) in case of sound wave or

(iii) electric and magnetic fields in case of electromagnetic waves.

The oscillations of y may or may not be simple harmonic in nature. Consider one-dimensional wave travelling along x-axis. In this case y is a function of x and t. i.e. y = f(x, t) But only those function of x & t, represent a wave motion which satisfy the differential equation. $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$ ...(i)

The general solution of this equation is of the form y (x, t) = f (ax ± bt) ...(ii)

Thus, any function of x and t and which satisfies equation (i) or which can be written as equation (ii) represents a wave. The only condition is that it should be finite everywhere and at all times, Further, if these conditions are satisfied, then speed of wave (v) is given by $v = \frac{\text{coefficient of t}}{\text{coefficient of x}} = \frac{b}{a}$

Example

Which of the following functions represent a travelling wave ?

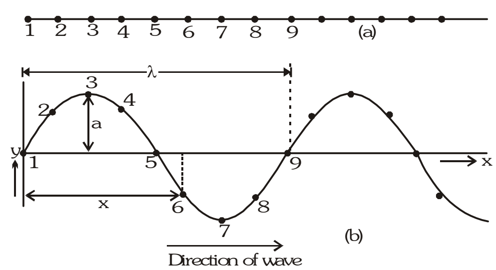
(a) $(x - vt)^2$ (b) $\ell n(x + vt)$ (c) $e^{-(x-vt)^2}$ (d) $\frac{1}{x + vt}$

Solution

Although all the four functions are written in the form f (ax $\pm$ bt), only (c) among the four functions is finite everywhere at all times. Hence only (c) represents a travelling wave.

Equation of a Plane Progressive Wave

If, on the propagation of wave in a medium, the particles of the medium perform simple harmonic motion then the wave is called a 'simple harmonic progressive wave'. Suppose, a simple harmonic progressive wave is propagating in a medium along the positive direction of the x-axis (from left to right). In fig. (a) are shown the equilibrium positions of the particles 1, 2, 3 ....



When the wave propagates, these particles oscillate about their equilibrium positions. In Fig. (b) are shown the instantaneous positions of these particles at a particular instant. The curve joining these positions represents the wave. Let the time be counted from the instant when the particle 1 situated at the origin starts oscillating. If y be the displacement of this particle after t seconds, then $y = a \sin \omega t$...(i)

where a is the amplitude of oscillation and $\omega = 2\pi$ n, where n is the frequency. As the wave reaches the particles beyond the particle 1, the particles start oscillating. If the speed of the wave be v, then it will reach particle 6, distant x from the particle 1, in x/v sec. Therefore, the particle 6 will start oscillating x/v sec after the particle 1. It means that the displacement of the particle 6 at a time t will be the same as that of the particle 1 at a time x/v sec earlier i.e. at time t – (x/v). The displacement of particle 1 at time t – (x/v) can be the particle 6, distant x from the origin (particle 1), at time t is given by

$$y = a \sin \omega \left(t - \frac{x}{v}\right) \qquad \text{But } \omega = 2\pi n,\ y = a \sin(\omega t - kx) \qquad \left(k = \frac{\omega}{v}\right) \quad ...(ii)$$

$$y = a \sin \left[\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right] \qquad \text{Also } k = \frac{2\pi}{\lambda} \ ...(iii) \qquad y = a \sin 2\pi \left[\frac{t}{T} - \frac{x}{\lambda}\right] \quad ...(iv)$$

This is the equation of a simple harmonic wave travelling along +x direction. If the wave is travelling along the –x direction then inside the brackets in the above equations, instead of minus sign there will be plus sign. For example, equation (iv) will be of the following form : $y = a \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda}\right)$. If ϕ be the phase difference between the above wave travelling along the +x direction and an other wave, then the equation of that wave will be

$$y = a \sin \left\{2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \pm \phi\right\}$$

Example

The equation of a wave is, $y(x, t) = 0.05 \sin\left[\frac{\pi}{2}(10x - 40t) - \frac{\pi}{4}\right]$m

Find : (a) The wavelength, the frequency and the wave velocity

(b) The particle velocity and acceleration at x=0.5 m and t=0.05 s.

Solution

(a) The equation may be rewritten as, $y(x,t) = 0.05\sin\left(5\pi x - 20\pi t - \frac{\pi}{4}\right)$m

Comparing this with equation of plane progressive harmonic wave,

$y(x,t) = A\sin(kx - \omega t + \phi)$ we have, wave number $k = \frac{2\pi}{\lambda} = 5\pi\,\text{rad/m}$ $\therefore \lambda = 0.4\text{m}$

The angular frequency is, $\omega = 2\pi f = 20\pi\,\text{rad/s}$ $\therefore f = 10\text{Hz}$

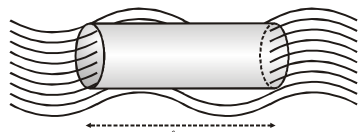
The wave velocity is, $v = f\lambda = \frac{\omega}{k} = 4\text{ms}^{-1}$ in $+x$ direction

(b) The particle velocity and acceleration are, $v_p = \frac{\partial y}{\partial t} = -(20\pi)(0.05)\cos\left(\frac{5\pi}{2} - \pi - \frac{\pi}{4}\right)$ =2.22m/s

$a_p = \frac{\partial^2 y}{\partial t^2} = -(20\pi)^2(0.05)\sin\left(\frac{5\pi}{2} - \pi - \frac{\pi}{4}\right)$ =140 m/s^2

INTENSITY OF WAVE

The amount of energy flowing per unit area and per unit time is called the intensity of wave. It is represented by I. Its units are J/m^{2}s or watt/metre^2. $I = 2\pi^2 f^2 A^2 \rho v$ i.e. $I \propto A^2$ and $I \propto A^2$.



If P is the power of an isotropic point source, then intensity at a distance r is given by,

$$I = \frac{P}{4\pi r^2} \text{ or } I \propto \frac{1}{r^2} \text{ (for a point source)}$$

If P is the power of a line source, then intensity at a distance r is given by,

$$I = \frac{P}{2\pi r\ell} \text{ or } I \propto \frac{1}{r} \text{ (for a line source) As, } I \propto A^2$$

Therefore, $A \propto \frac{1}{r}$ (for a point source) and $A \propto \frac{1}{\sqrt{r}}$ (for a line source)

SUPERPOSITION PRINCIPLE

Two or more waves can propagate in the same medium without affecting the motion of one another. If several waves propagate in a medium simultaneously, then the resultant displacement of any particle of the medium at any instant is equal to the vector sum of the displacements produced by individual wave. The phenomenon of intermixing of two or more waves to produce a new wave is called Superposition of waves. Therefore according to superposition principle.

> The resultant displacement of a particle at any point of the medium, at any instant of time is the vector sum of the displacements caused to the particle by the individual waves.

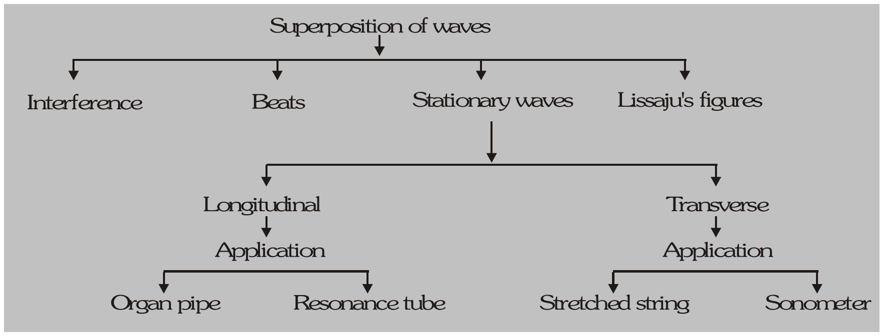
If $\vec{y}_1, \vec{y}_2, \vec{y}_3, \ldots$ are the displacement of particle at a particular time due to individual waves, then the resultant displacement is given by $\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \ldots$

Principle of superposition holds for all types of waves, i.e., mechanical as well as electromagnetic waves. But this principle is not applicable to the waves of very large amplitude.

Due to superposition of waves the following phenomenon can be seen

- **Interference** : Superposition of two waves having equal frequency and nearly equal amplitude.
- **Beats** : Superposition of two waves of nearly equal frequency in same direction.

- **Stationary waves :** Superposition of equal wave from opposite direction.
- **Lissajous' figure :** Superposition of perpendicular waves.



INTERFERENCE OF WAVES :

When two waves of equal frequency and nearly equal amplitude travelling in same direction having same state of polarisation in medium superimpose, then intensity is different at different points. At some points intensity is large, whereas at other points it is nearly zero.

Consider two waves $y_1 = A_1 \sin(\omega t - kx)$ and $y_2 = A_2 \sin(\omega t - kx + \phi)$

By principle of superposition $y = y_1 + y_2 = A \sin(\omega t - kx + \delta)$

where $A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$ and $\tan\delta = \dfrac{A_2 \sin\phi}{A_1 + A_2\cos\phi}$

As intensity $I \propto A^2$ so $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

- **Constructive interference (maximum intensity) :**

  Phase difference $\phi = 2n\pi$ or path difference $= n\lambda$ where $n = 0, 1, 2, 3, \ldots$

  $$\Rightarrow A_{max} = A_1 + A_2 \quad \text{and} \quad I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

- **Destructive interference (minimum intensity) :**

  Phase difference $\phi = (2n+1)\pi$, or path difference $= (2n-1)\dfrac{\lambda}{2}$ where $n = 0, 1, 2, 3, \ldots$

  $$\Rightarrow A_{min} = A_1 - A_2 \quad \text{and} \quad I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

GOLDEN KEY POINTS

- Maximum and minimum intensities in any interference wave form. $\dfrac{I_{Max}}{I_{Min}} = \left(\dfrac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\dfrac{A_1 + A_2}{A_1 - A_2}\right)^2$

- Average intensity of interference wave form :— < I >or $I_{av} = \dfrac{I_{max} + I_{min}}{2} = I_1 + I_2$

  if $A = A_1 = A_2$ and $I_1 = I_2 = I$ then $I_{max} = 4I$, $I_{min} = 0$ and $I_{AV} = 2I$

- Degree of interference Pattern (f) : Degree of hearing (Sound Wave) or

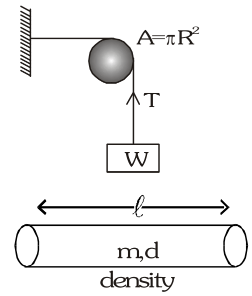
  Degree of visibility (Light Wave) $f = \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}$ 100

  In condition of perfect interference degree of interference pattern is maximum $f_{max} = 1$ or 100%

- Condition of maximum contrast in interference wave form $a_1 = a_2$ and $I_1 = I_2$ then $I_{max} = 4I$ and $I_{min} = 0$

  For perfect destructive interference we have a maximum contrast in interference wave form.

VELOCITY OF TRANSVERSE WAVE



Mass of per unit length $m = \dfrac{\pi r^2 \ell \times d}{\ell}$, $m = \pi r^2 d$, where d = Density of matter

Velocity of transverse wave in any wire $v = \sqrt{\dfrac{T}{m}}$ or $\sqrt{\dfrac{T}{\pi r^2 d}} = \sqrt{\dfrac{T}{Ad}}$ $\because \pi r^2 = A$

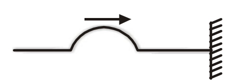
- If m is constant then, $v \propto \sqrt{T}$ it is called tension law.
- If tension is constant then $v \propto \sqrt{\dfrac{1}{m}}$ ← it is called law of mass
- If T is constant & take wire of different radius for same material then $v \propto \dfrac{1}{r}$ ← it is called law of radius
- If T is constant & take wire of same radius for different material. Then $v \propto \sqrt{\dfrac{1}{d}}$ ← law of density

REFLECTION FROM RIGID END

When the pulse reaches the right end which is clamped at the wall, the element at the right end exerts a force on the clamp and the clamp exerts equal and opposite force on the element. The element at the right end is thus acted upon by the force from the clamp. As this end remains fixed, the two forces are opposite to each other. The force from the left part of the string transmits the forward wave pulse and hence, the force exerted by the clamp sends a return pulse on the string whose shape is similar to a return pulse but is inverted. The original pulse tries to pull the element at the fixed end up and the return pulse sent by the clamp tries to pull it down, so the resultant displacement is zero. Thus, the wave is reflected from the fixed end and the reflected wave is inverted with respect to the original wave. The shape of the string at any time, while the pulse is being reflected, can be found by adding an inverted image pulse to the incident pulse.

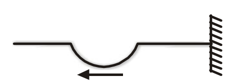
Equation of wave propagating in +ve x-axis

Incident wave $y_1 = a \sin(\omega t - kx)$



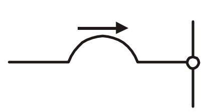
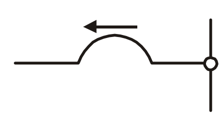
Reflected wave $y_2 = a \sin(\omega t + kx + \pi)$

or $y_2 = -a \sin(\omega t + kx)$



REFLECTION FROM FREE END

The right end of the string is attached to a light frictionless ring which can freely move on a vertical rod. A wave pulse is sent on the string from left. When the wave reaches the right end, the element at this end is acted on by the force from the left to go up. However, there is no corresponding restoring force from the right as the rod does not exert a vertical force on the ring. As a result, the right end is displaced in upward direction more than the height of the pulse i.e., it overshoots the normal maximum displacement. The lack of restoring force from right can be equivalent described in the following way. An extra force acts from right which sends a wave from right to left with its shape identical to the original one. The element at the end is acted upon by both the incident and the reflected wave and the displacements add. Thus, a wave is reflected by the free end without inversion.

Incident wave $y_1 = a \sin(\omega t - kx)$  Reflected wave $y_2 = a \sin(\omega t + kx)$ 

STATIONARY WAVES

* **Definition :** The wave propagating in such a medium will be reflected at the boundary and produce a wave of the same kind travelling in the opposite direction. The superposition of the two waves will give rise to a stationary wave. Formation of stationary wave is possible only and only in bounded medium.

ANALYTICAL METHOD FOR STATIONARY WAVES

- **From rigid end** : We know equation for progressive wave in positive x-direction $y_1 = a \sin(\omega t - kx)$
  After reflection from rigid end $y_2 = a \sin(\omega t + kx + \pi) = - a \sin(\omega t + kx)$
  By principle of super position. $y = y_1 + y_2 = a \sin(\omega t - kx) - a \sin(\omega t + kx) = - 2a \sin kx \cos \omega t$
  This is equation of stationary wave reflected from rigid end

  Amplitude $=2a \sin kx$ **Velocity of particle** $v_{pa} = \frac{dy}{dt} = 2a\,\omega \sin kx \sin \omega t$

  Strain $\frac{dy}{dx} = -2ak \cos kx \cos \omega t$ **Elasticity** $E = \frac{\text{stress}}{\text{strain}} = \frac{dp}{\frac{dy}{dx}}$ **Change in pressure** $dp = E\frac{dy}{dx}$

  - **Node** $x = 0, \frac{\lambda}{2}, \lambda$ .... $A = 0$, $V_{pa} = 0$, strain $\rightarrow$ max. **Change in pressure** $\rightarrow$ max
  - **Antinode** $x = \frac{\lambda}{4}, \frac{3\lambda}{4}$ .... $A \rightarrow$ max, $-V_{pa} \rightarrow$ max. strain $= 0$ **Change in pressure** $= 0$

- **From free end** : we know equation for progressive wave in positive x-direction $y_1 = a \sin(\omega t - kx)$
  After reflection from free end $y_2 = a \sin(\omega t + kx)$
  By Principle of superposition $y = y_1 + y_2 = a \sin(\omega t - kx) + a \sin(\omega t + kx) = 2a \sin \omega t \cos kx$

  Amplitude $= 2a \cos kx$,   **Velocity of particle** $= v_{Pa} = \frac{dy}{dt} = 2a\,\omega \cos \omega t \cos kx$

  Strain $\frac{dy}{dx} = -2ak \sin \omega t \sin kx$ **Change in pressure** $dp = E\frac{dy}{dx}$

  - **Antinode** : $x = 0, \frac{\lambda}{2}, \lambda$ .... $A \rightarrow$ Max, $V_{pa} = \frac{dy}{dt} \rightarrow$ max. Strain $= 0$, $dp = 0$
  - **Node** : $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ .... $A = 0$, $V_{pa} = \frac{dy}{dt} = 0$, strain $\rightarrow$ max , $dp \rightarrow$ max

PROPERTIES OF STATIONARY WAVES

The stationary waves are formed due to the superposition of two identical simple harmonic waves travelling in opposite direction with the same speed.

Important characteristics of stationary waves are:–

(i) Stationary waves are produced in the bounded medium and the boundaries of bounded medium may be rigid or free.

(ii) In stationary waves nodes and antinodes are formed alternately. Nodes are the points which are always in rest having maximum strain. Antinodes are the points where the particles vibrate with maximum amplitude having minimum strain.

(iii) All the particles except at the nodes vibrate simple harmonically with the same period.

(iv) The distance between any two successive nodes or antinodes is $\lambda/2$.

(v) The amplitude of vibration gradually increases from zero to maximum value from node to antinode.

(vi) All the particles in one particular segment vibrate in the same phase, but the particle of two adjacent segments differ in phase by 180

(vii) All points of the medium pass through their mean position simultaneously twice in each period.

(viii) Velocity of the particles while crossing mean position varies from maximum at antinodes to zero at nodes.

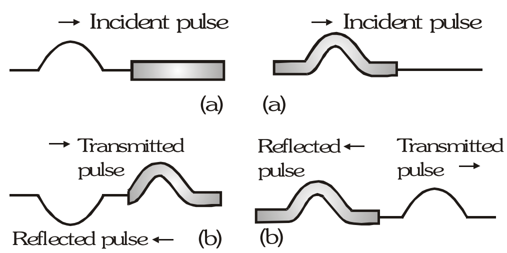
(ix) In a stationary wave the medium is splited into segments and each segment is vibrating up and down as a whole.

(x) In longitudinal stationary waves, condensation (compression) and refraction do not travel forward as in progressive waves but they appear and disappear alternately at the same place.

(xi) These waves do not transfer energy in the medium. Transmission of energy is not possible in a stationary wave.

TRANSMISSION OF WAVES

We may have a situation in which the boundary is intermediate between these two extreme cases, that is, one in which the boundary is neither rigid nor free. In this case, part of the incident energy is transmitted and part is reflected. For instance, suppose a light string is attached to a heavier string as in (figure). When a pulse travelling on the light reaches the knot, same part of it is reflected and inverted and same part of it is transmitted to the heavier string.



As one would expect, the reflected pulse has a smaller amplitude than the incident pulse, since part of the incident energy is transferred to the pulse in the heavier string. The inversion in the reflected wave is similar to the behaviour of a pulse meeting a rigid boundary, when it is totally reflected. When a pulse travelling on a heavy string strikes the boundary of a lighter string, as in (figure), again part is reflected and part is transmitted. However, in this case the reflected pulse is not inverted. In either case, the relative height of the reflected and transmitted pulses depend on the relative densities of the two string. In the previous section, we found that the speed of a wave on a string increases as the density of the string decreases. That is, a pulse travels more slowly on a heavy string than on a light string, if both are under the same tension. The following general rules apply to reflected waves. When a wave pulse travels from medium A to medium B and $v_A > v_B$ (that is, when B is denser than A), the pulse will be inverted upon reflection. When a wave pulse travels from medium A to medium B and $v_A < v_B$ (A is denser than B), it will not be inverted upon reflection.

GOLDEN KEY POINTS

Phenomenon of reflection and transmission of waves obeys the laws of reflection and refraction. The frequency of these wave remains constant i.e. does not change. $\omega_i = \omega_r = \omega_t = \omega$

From rarer to denser medium $y_i = a_i \sin(\omega t - k_1 x)$ $y_r = -a_i \sin(\omega t + k_1 x)$ $y_t = a_t \sin(\omega t - k_2 x)$

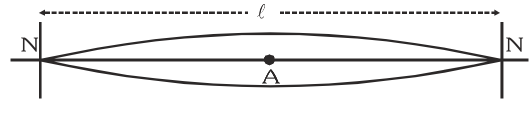
From denser to rarer medium $y_i = a_i \sin(\omega t - k_1 x)$ $y_r = a_i \sin(\omega t + k_1 x)$ $y_t = a_t \sin(\omega t - k_2 x)$

STATIONARY WAVE ARE OF TWO TYPES :

(i) Transverse st. wave (stretched string) **(ii)** Longitudinal st. wave (organ pipes)

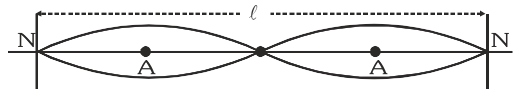
(i) Transverse Stationary wave (Fixed at Both ends)

Fundamental Harmonic



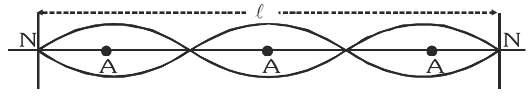
$$\frac{\lambda}{2} = \ell \Rightarrow \lambda = 2\ell \Rightarrow f = \frac{v}{2\ell}$$

Second Harmonic



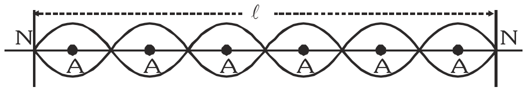
$$\frac{2\lambda}{2} = \ell \Rightarrow \lambda = \ell \Rightarrow f = \frac{2v}{2\ell}$$

Third Harmonic



$$\frac{3\lambda}{2} = \ell \Rightarrow \lambda = \frac{2\ell}{3} \Rightarrow f = \frac{3v}{2\ell}$$

p th harmonic

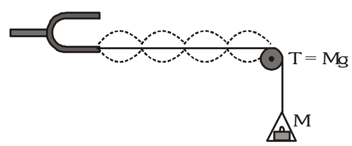


$$\frac{p\lambda}{2} = \ell \Rightarrow \lambda = \frac{2\ell}{p} \Rightarrow f = \frac{pv}{2\ell}$$

- **Law of length :** For a given string, under a given tension, the fundamental frequency of vibration is inversely proportional to the length of the string, i.e, $n \propto \frac{1}{\ell}$ (T and m are constant)
- **Law of tension :** The fundamental frequency of vibration of stretched string is directly proportional to the square root of the tension in the string, provided that length and mass per unit length of the string are kept constant. $n \propto \sqrt{T}$ (ℓ and m are constant)
- **Law of mass :** The fundamental frequency of vibration of a stretched string is inversely proportional to the square root of its mass per unit length provided that length of the string and tension in the string are kept constant, i.e., $n \propto \frac{1}{\sqrt{m}}$ (ℓ and T are constant)
- **Melde's experiment :** In Melde's experiment, one end of a flexible piece of thread is tied to the end of a tuning fork. The other end passed over a smooth pulley carries a pan which can be loaded. There are two arrangements to vibrate the tied fork with thread.

Transverse arrangement :

Case 1. In a vibrating string of fixed length, the product of number of loops and square root of tension are constant or $p\sqrt{T}$ = constant.

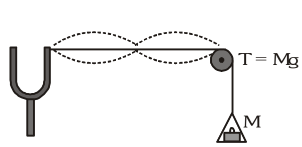


Case 2. When the tuning fork is set vibrating as shown in fig. then the prong vibrates at right angles to the thread. As a result the thread is set into motion. The frequency of vibration of the thread (string) is equal to the frequency of the tuning fork. If length and tension are properly adjusted then, standing waves are formed in the string. (This happens when frequency of one of the normal modes of the string matched with the frequency of the tuning fork). Then, if p loops are formed in the thread, then the frequency of the tuning fork is given by $n = \frac{p}{2\ell}\sqrt{\frac{T}{m}}$

Case 3. If the tuning fork is turned through a right angle, so that the prong vibrates along the length of the thread, then the string performs only a half oscillation for each complete vibrations of the prong. This is because the thread only makes node at the midpoint when the prong moves towards the pulley i.e. only once in a vibration.

Longitudinal arrangement :

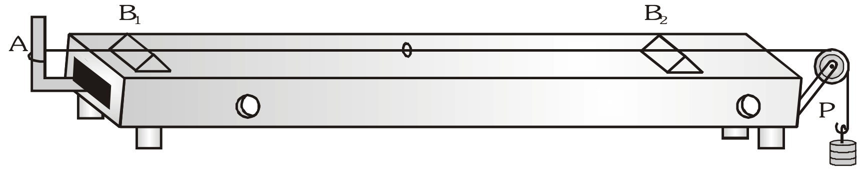
The thread performs sustained oscillations when the natural frequency of the given length of the thread under tension is half that of the fork.



Thus if p loops are formed in the thread, then the frequency of the tuning fork is $n = \frac{2p}{2\ell}\sqrt{\frac{T}{m}}$

SONOMETER :

Sonometer consists of a hollow rectangular box of light wood. One end of the experimental wire is fastened to one end of the box. The wire passes over a frictionless pulley P at the other end of the box. The wire is stretched by a tension T.



The box serves the purpose of increasing the loudness of the sound produced by the vibrating wire. If the length the wire between the two bridges is ℓ, then the frequency of vibration is $n = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$

To test the tension of a tuning fork and string, a small paper rider is placed on the string. When a vibrating tuning fork is placed on the box, and if the length between the bridges is properly adjusted, then when the two frequencies are exactly equal, the string quickly picks up the vibrations of the fork and the rider is thrown off the wire.

COMPARISON OF PROGRESSIVE AND STATIONARY WAVES

Progressive waves	Stationary waves
1. These waves travels in a medium with definite velocity.	These waves do not travel and remain confined between two boundaries in the medium.
2. These waves transmit energy in the medium.	These waves do not transmit energy in the medium.
3. The phase of vibration varies continuously from particle to particle.	The phase of all the particles in between two nodes is always same. But particles of two Adjacent nodes differ in phase by 180
4. No particle of medium is Permanently at rest.	Particles at nodes are permanently at rest.
5. All particles of the medium vibrate and amplitude of vibration is same.	The amplitude of vibration changes from particle to particle. The amplitude is zero for all at nodes and maximum at antinodes.
6. All the particles do not attain the maximum displacement position simultaneously.	All the particles attain the maximum displacement

SPEED OF LONGITUDINAL (SOUND) WAVES

Newton Formula $v_{medium} = \sqrt{\frac{E}{\rho}}$ (Use for every medium)

Where E = Elasticity coefficient of medium & ρ = Density of medium

- **For solid medium** $v_{solid} = \sqrt{\frac{Y}{\rho}}$ Where E = Y = Young's modulas
- **For liquid Medium** $v_{liquid} = \sqrt{\frac{B}{\rho}}$ Where E = B, where B = volume elasticity coefficient of liquid

- **For gas medium**

The formula for velocity of sound in air was first obtained by Newton. He assumed that sound propagates through air and temperature remains constant. (i.e. the process is isothermal) so Isothermal Elasticity = P $\therefore$ $v_{air} = \sqrt{(P/\rho}$

At NTP for air $P = 1.01 \times 10^5$ N/m^2 and $\rho = 1.3$ kg/m^3 so $v_{air} = \sqrt{\dfrac{1.01\times10^5}{1.3}} = 279$ m/s

However, the experimental value of sound in air is 332 m/s which is much higher than given by Newton's formula.

- **Laplace Correction**

In order to remove the discrepancy between theoretical and experimental values of velocity of sound, Laplace modified Newton's formula assuming that propagation of sound in air is adiabatic process, i.e.

Adiabatic Elasticity = γp so that $v = \sqrt{\dfrac{\gamma P}{\rho}}$ i.e. $v = \sqrt{1.41} \times 279 = 331.3$ m/s [as $\gamma_{air} = 1.41$]

Which is in good agreement with the experimental value (332 m/s). This in turn establishes that sound propagates adiabatically through gases.

The velocity of sound in air at NTP is 332 m/s which is much lesser than that of light and radio–waves (= 3×10^8 m/s). This implies that –

(a) If we set our watch by the sound of a distant siren it will be slow.

(b) If we record the time in a race by hearing sound from starting point it will be lesser than actual.

(c) In a cloud–lightening, though light and sound are produced simultaneously but as c > v, light proceeds thunder. An in case of gases –

$$v_s = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma PV}{\text{mass}}}\left[\text{as } \rho = \frac{\text{mass}}{\text{volume}} = \frac{M}{V}\right] \text{ or } v_s = \sqrt{\frac{\gamma\,\mu RT}{M}} \text{ [as PV = } \mu\text{RT] or } v_s = \sqrt{\frac{\gamma\, RT}{M_w}}$$

$$\left[\text{as} \quad \mu = \frac{\text{mass}}{M_w} = \frac{M}{M_w} \text{where } M_w = \text{Molecular weight}\right]$$

And from kinetic-theory of gases $v_{rms} = \sqrt{(3RT/M_w)}$ So $\dfrac{v_s}{v_{rms}} = \sqrt{\dfrac{\gamma}{3}}$

EFFECT OF VARIOUS QUANTITIES

(1) Effect of temperature

For a gas γ & M_W is constant $v \propto \sqrt{T} \Rightarrow \dfrac{v_2}{v_1} = \sqrt{\dfrac{T_2}{T_1}} = \sqrt{\dfrac{t+273}{273}} \Rightarrow v_t = v_0\left[1+\dfrac{t}{273}\right]^{\frac{1}{2}}$

By applying Binomial theorem.

(i) For any gas medium $v_t = v_0\left[1+\dfrac{t}{546}\right]$ **(ii)** For air : $v_t = v_0 + 0.61\,t$ m/sec. ($v_0 = 332$ m/sec.)

(2) Effect of Relative Humidity

With increase in humidity, density decreases so in the light of $v = \sqrt{\gamma P/\rho)}$ We conclude that with rise in humidity velocity of sound increases. This is why sound travels faster in humid air (rainy season) than in dry air (summer) at same temperature. Due to this in rainy season the sound of factories siren and whistle of train can be heard more than summer.

(3) Effect of Pressure

As velocity of sound $v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

So pressure has no effect on velocity of sound in a gas as long as temperature remain constant. This is why in going up in the atmosphere, though both pressure and density decreases, velocity of sound remains constant as long as temperature remains constant. Further more it has also been established that all other factors such as amplitude, frequency, phase, loudness pitch, quality etc. has partially no effect on velocity of sound.

Velocity of sound in air is measured by resonance tube or Hebb's method while in gases by Quinke's tube. Kundt's tube is used to determine velocity of sound in any medium solid, liquid or gas.

(4) Effect of Motion of Air

If air is blowing then the speed of sound changes. If the actual speed of sound is v and the speed of air is w, then the speed of sound in the direction in which air is blowing will be (v+ w), and in the opposite direction it will be (v – w).

(5) Effect of Frequency

There is no effect of frequency on the speed of sound. Sound waves of different frequencies travel with the same speed in air although their wavelength in air are different. If the speed of sound were dependent on the frequency, then we could not have enjoyed orchestra.

Example

A piezo electric quartz plate of thickness 0.005 m is vibrating in resonant conditions. Calculate its fundamental frequency if for quartz $Y = 8 \times 10^{10}$ N/m^2 and $\rho = 2.65 \times 10^3$ kg/m^3

Solution

We known that for longitudinal waves in solids $v = \sqrt{\frac{Y}{\rho}}$, So $v = \sqrt{\frac{8 \times 10^{10}}{2.65 \times 10^3}} = 5.5 \times 10^3$ m/s

Further more for fundamental mode of plate – $(\lambda/2) = L$ So $\lambda = 2 \times 5 \times 10^{-3} = 10^{-2}$ m

But as $v = f\lambda$, i.e., $f = (v/\lambda)$ so $f = [5.5 \times 10^3/10^{-2}] = 5.5 \times 10^5$ Hz = 550 kHz

Example

Determine the change in volume of 6 liters of alcohol if the pressure is decreased from 200 cm of Hg to 75 cm. [velocity of sound in alcohol is 1280 m/s, density of alcohol = 0.81 gm/cc, density of Hg = 13.6 gm/cc and g = 9.81 m/s^2]

Solution

For propagation of sound in liquid $v = \sqrt{(B/\rho)}$ i.e., $B = v^2\rho$

But by definition $B = -V\frac{\Delta P}{\Delta V}$ So $-V\frac{\Delta P}{\Delta V} = v^2\rho$, i.e. $\Delta V = \frac{V(-\Delta P)}{\rho v^2}$

Here $\Delta P = H_2\rho g - H_1\rho g = (75 - 200) \times 13.6 \times 981 = -1.667 \times 10^6$ dynes/cm^2

So $\Delta V = \frac{(6 \times 10^3)(1.667 \times 10^6)}{0.81 \times (1.280 \times 10^5)^2} = 0.75$ cc

Example

(a) Speed of sound in air is 332 m/s at NTP. What will the speed of sound in hydrogen at NTP if the density of hydrogen at NTP is (1/16) that of air.

(b) Calculate the ratio of the speed of sound in neon to that in water vapour at any temperature. [Molecular weight of neon = 2.02×10^{-2} kg/mol and for water vapours = 1.8×10^{-2} kg/mol]

Solution

The velocity of sound in air is given by $v = \sqrt{\dfrac{E}{\rho}} = \sqrt{\dfrac{\gamma P}{\rho}} = \sqrt{\dfrac{\gamma RT}{M}}$

(a) In terms of density and pressure $\dfrac{v_H}{v_{air}} = \sqrt{\dfrac{P_H}{\rho_H} \times \dfrac{\rho_{air}}{P_{air}}} = \sqrt{\dfrac{\rho_{air}}{\rho_H}}$ [as $P_{air} = P_H$]

$$\Rightarrow \quad v_H = v_{air} \times \sqrt{\frac{\rho_{air}}{\rho_H}} = 332 \times \sqrt{\frac{16}{1}} = 1328 \text{ m/s}$$

(b) In terms of temperature and molecular weight $\dfrac{v_{Ne}}{v_W} = \sqrt{\dfrac{\gamma_{Ne}}{M_{Ne}} \times \dfrac{M_W}{\gamma_W}}$ [as $T_N = T_W$]

Now as neon is mono atomic ($\gamma = 5/3$) while water vapours poly atomic ($\gamma = 4/3$) so

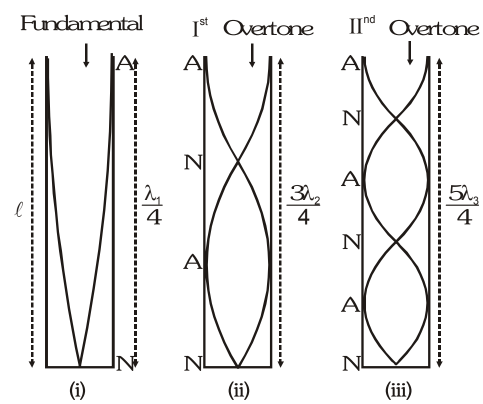
$$\frac{v_{Ne}}{v_W} = \sqrt{\frac{(5/3) \times 1.8 \times 10^{-2}}{(4/3) \times 2.02 \times 10^{-2}}} = \sqrt{\frac{5}{4} \times \frac{1.8}{2.02}} = 1.055$$

VIBRATION IN ORGAN PIPES

When two longitudinal waves of same frequency and amplitude travel in a medium in opposite directions then by superposition, standing waves are produced. These waves are produced in air columns in cylindrical tube of uniform diameter. These sound producing tubes are called organ pipes.

1. Vibration of air column in closed organ pipe :

The tube which is closed at one end and open at the other end is called close organ pipe. On blowing air at the open end, a wave travels towards closed end from where it is reflected towards open end. As the wave reaches open end, it is reflected again. So two longitudinal waves travel in opposite directions to superpose and produce stationary waves. At the closed end there is a node since particles does not have freedom to vibrate whereas at open end there is an antinode because particles have greatest freedom to vibrate.



Hence on blowing air at the open end, the column vibrates forming antinode at free end and node at closed end. If ℓ is length of pipe and λ be the wavelength and v be the velocity of sound in organ pipe then

Case (i) $\ell = \dfrac{\lambda}{4} \quad \Rightarrow \quad \lambda = 4\ell \quad \Rightarrow \quad n_1 = \dfrac{v}{\lambda} = \dfrac{v}{4\ell}$ fundamental frequency.

Case (ii) $\ell = \dfrac{3\lambda}{4} \quad \Rightarrow \quad \lambda = \dfrac{4\ell}{3} \quad \Rightarrow \quad n_2 = \dfrac{v}{\lambda} = \dfrac{3v}{4\ell}$ First overtone / IIIrd Harmonic

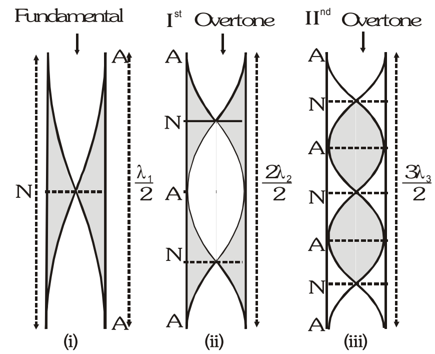
Case (iii) $\ell = \dfrac{5\lambda}{4} \quad \Rightarrow \quad \lambda = \dfrac{4\ell}{5} \quad \Rightarrow \quad n_3 = \dfrac{v}{\lambda} = \dfrac{5v}{4\ell}$ Second overtone / Vth Harmonic

When closed organ pipe vibrate in mth overtone then $\ell = (2m+1)\dfrac{\lambda}{4}$

$$\text{So} \qquad \lambda = \frac{4\ell}{(2m+1)} \Rightarrow n = (2m+1)\frac{v}{4\ell}$$

Hence frequency of overtones is given by $n_1 : n_2 : n_3$ .... = 1 : 3 : 5 ....

2. Vibration of air columns in open organ pipe :



The tube which is open at both ends is called an open organ pipe. On blowing air at the open end, a wave travel towards the other end after reflection from open end waves travel in opposite direction to superpose and produce stationary wave. Now the pipe is open at both ends by which an antinode is formed at open end. Hence on blowing air at the open end antinodes are formed at each end and nodes in the middle. If ℓ is length of the pipe and λ be the wavelength and v is velocity of sound in organ pipe.

Case (i) $\ell = \frac{\lambda}{2} \Rightarrow \lambda = 2\ell \Rightarrow n_1 = \frac{v}{\lambda} = \frac{v}{2\ell}$ Fundamental frequency.

Case (ii) $\ell = \frac{2\lambda}{2} \Rightarrow \lambda = \frac{2\ell}{2} \Rightarrow n_2 = \frac{v}{\lambda} = \frac{2v}{2\ell}$ First overtone frequency.

Case (iii) $\ell = \frac{3\lambda}{2} \Rightarrow \lambda = \frac{2\ell}{3} \Rightarrow n_3 = \frac{v}{\lambda} = \frac{3v}{2\ell}$ Second overtone frequency.

Hence frequency of overtones are given by the relation $n_1 : n_2 : n_3$ .... = 1 : 2 : 3 ....

When open organ pipe vibrate in m[th] overtone then $\ell = (m+1)\frac{\lambda}{4}$ so $\lambda = \frac{4\ell}{m+1} \Rightarrow n = (m+1)\frac{v}{2\ell}$

GOLDEN KEY POINTS

- A rod clamped at one end or a string fixed at one end is similar to vibration of closed end organ pipe.
- A rod clamped in the middle is similar to the vibration of open end organ pipe.
- If an open pipe is half submerged in water, it becomes a closed organ pipe of length half that of open pipe i.e. frequency remains same.
- Due to finite motion of air molecular in organ pipes reflection takes place not exactly at open end but some what above it so in an organ pipe antinode is not formed exactly at free–end but above it at a distance e = 0.6r (called end correction or Rayleigh's correction) with r being the radius of pipe. So for closed organ pipe $L \to L + 0.6r$ while for open $L \to L + 2 \times 0.6r$ (as both ends are open)

  so that – $f_C = \frac{v}{4(L+0.6r)}$ while $f_0 = \frac{v}{2(L+1.2r)}$

  This is why for a given v and L narrower the pipe higher will the frequency or pitch and shriller will be the sound.
- For an organ pipe (closed or open) if v = constant. $f \propto (1/L)$

  So with decrease in length of vibrating air column, i.e., wavelength ($\lambda \propto L$), frequency or pitch will increase and vice–versa. This is why the pitch increases gradually as an empty vessel fills slowly.
- For an organ pipe if f = constant. $v \propto \lambda$ or $v \propto L$, $f = \frac{v}{\lambda}$ = constant i.e. the frequency of an organ pipe will remain unchanged if the ratio of speed of sound in to its wave length remains constant.
- As for a given length of organ pipe ℓ = constant. $f \propto v$ So
  - **(a)** With rise in temperature as velocity will increase ($v \propto \sqrt{T}$), the pitch will increase.

    (Change in length with temperature is not considered unless stated)
  - **(b)** With change in gas in the pipe as v will change and so f will change ($v \propto \sqrt{\gamma / M}$)
  - **(c)** With increase in moisture as v will increase and so the pitch will also.

Example

For a certain organ pipe, three successive resonant frequencies are observed at 425, 595 and 765 Hz respectively. Taking the speed for sound in air to be 340 m/s (a) Explain whether the pipe is closed at one end or open at both ends (b) determine the fundamental frequency and length of the pipe.

Solution

(a) The given frequencies are in the ratio

425 : 595 : 765, i.e., 5 : 7 : 9

And as clearly these are odd integers so the given pipe is closed pipe.

(b) From part (b) it is clear that the frequency of 5th harmonic (which is third overtone) is 425 Hz

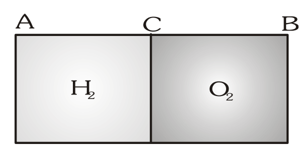
so $425 = 5f_C \Rightarrow f_C = 85$ Hz Further as $f_C = \frac{v}{4L}$, $L = \frac{v}{4f_C} = \frac{340}{4\times85} = 1$ m

Example

AB is a cylinder of length 1 m fitted with a thin flexible diaphram C at middle and two other thin flexible diaphram A and B at the ends. The portions AC and BC contain hydrogen and oxygen gases respectively. The diaphragms A and B are set into vibrations of the same frequency. What is the minimum frequency of these vibrations for which diaphram C is a node ? Under the condition of the experiment the velocity of sound in hydrogen is 1100 m/s and oxygen 300 m/s.

Solution

As diaphragm C is a node, A and B will be antinode (as in a organ pipe either both ends are antinode or one end node and the other antinode), i.e., each part will behave as closed end organ pipe so that



$f_H = \frac{v_H}{4L_H} = \frac{1100}{4\times0.5} = 550$ Hz And $f_0 = \frac{v_0}{4L_0} = \frac{300}{4\times0.5} = 150$Hz

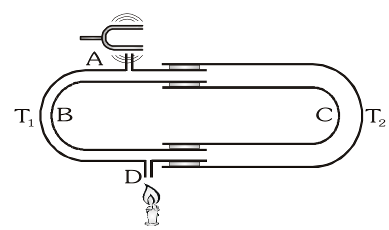
As the two fundamental frequencies are different, the system will vibrate with a common frequency f such that $f = n_H f_H = n_0 f_0$ i.e. $\frac{n_H}{n_0} = \frac{f_0}{f_H} = \frac{150}{550} = \frac{3}{11}$

i.e., the third harmonic of hydrogen and 11th harmonic of oxygen or 6th harmonic of hydrogen and 22nd harmonic of oxygen will have same frequency. So the minimum common frequency

$f = 3 \times 550$ or $11 \times 150 = 1650$ Hz

APPARATUS FOR DETERMINING SPEED OF SOUND

1. **Quinck's Tube :**

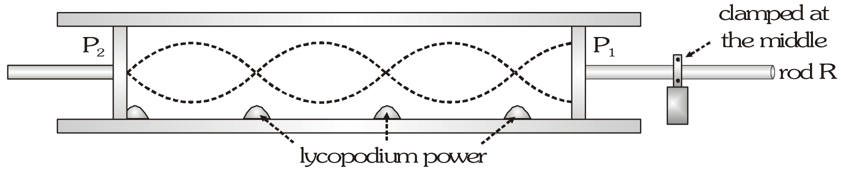


It consists of two U shaped metal tubes. Sound waves with the help of tuning fork are produced at A which travel through B & C and comes out at D where a sensitive flame is present. Now the two waves coming through different path interfere and flame flares up. But if they are not in phase, destructive interference occurs and flame remains undisturbed.

Suppose destructive interference occurs at D for some position of C. If now the tube C is moved so that interference condition is disturbed and again by moving a distance x, destructive interference occurs so that $2x = \lambda$. Similarly if the distance moved between successive constructive and destructive interference is x then $2x = \frac{\lambda}{2}$ Now by having value of x, speed of sound is given by $v = n\lambda$.

2. **Kundt's tube :** It is the used to determine speed of sound in different gases. It consists of a glass tube in which a small quantity of lycopodium powder is spread. The tube is rotated so that powder starts slipping. Now rod CD is rubbed at end D so that stationary waves form. The disc C vibrates so that air column also vibrates with the frequency of the rod. The piston P is adjusted so that frequency of air column become same as that of rod.

So resonance occurs and column is thrown into stationary waves. The powder sets into oscillations at antinodes while heaps of powder are formed at nodes.



Let n is the frequency of vibration of the rod then, this is also the frequency of sound wave in the air column in the tube.

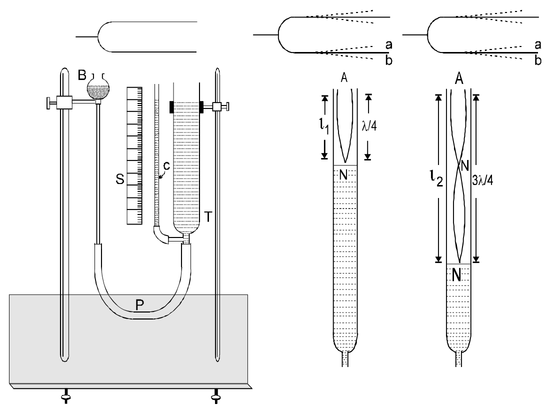
For rod : $\frac{\lambda_{rod}}{2} = \ell_{rod}$ For air : $\frac{\lambda_{air}}{2} = \ell_{air}$

Where ℓ_{air} is the distance between two heaps of powder in the tube (i.e. distance between two nodes). If v_{air} and v_{rod} are velocity of sound waves in the air and rod respectively, then $n = \frac{v_{air}}{\lambda_{air}} = \frac{v_{rod}}{\lambda_{rod}}$

Therefore, $\frac{v_{air}}{v_{rod}} = \frac{\lambda_{air}}{\lambda_{rod}} = \frac{\ell_{air}}{\ell_{rod}}$ Thus knowledge of v_{rod} determines v_{air}.

(ii) RESONANCE TUBE

Construction : The resonance tube is a tube T (figure) made of brass or glass, about 1 meter long and 5 cm in diameter and fixed on a vertical stand. Its lower end is connected to a water reservoir B by means of a flexible rubber tube. The rubber tube carries a pinch-cock P. The level of water in T can be raised or lowered by water adjusting the height of the reservoir B and controlling the flow of water from B to T or from T to B by means of the pinch-cock P. Thus the length of the air–column in T can be changed. The position of the water level in T can be read by means of a side tube C and a scale S.



Determination of the speed of sound in air by resonance tube

First of all the water reservoir B is raised until the water level in the tube T rises almost to the top of the tube. Then the pinch–cock P is tightened and the reservoir B is lowered. The water level in T stays at the top. Now a tuning fork is sounded and held over the mouth of tube .The pinch–cock P is opened slowly so that the water level in T falls and the length of the air–column increases. At a particular length of air–column in T, a loud sound is heard. This is the first state of resonance. In this position the following phenomenon takes place inside the tube.

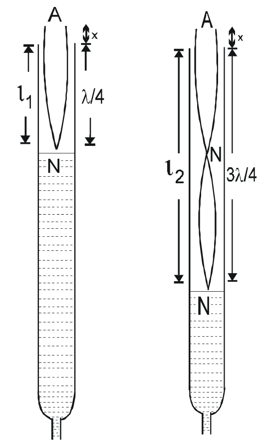
(i) For first resonance $\ell_1 = \lambda/4$

(ii) For second resonance $\ell_2 = 3\lambda/4 \Rightarrow \ell_2 - \ell_1 = \lambda/2 \Rightarrow \lambda = 2(\ell_2 - \ell_1)$

If the frequency of the fork be n and the temperature of the air-column be t°C, then the speed of sound at t°C is given by $v_t = n\lambda = 2n\ (\ell_2 - \ell_1)$

The speed of sound wave at 0°C $v_0 = (v_t - 0.61\ t)$ m/s.

End Correction : In the resonance tube, the antinode is not formed exactly at the open but slightly outside at a distance x. Hence the length of the air -column in the first and second states of resonance are $(l_1 + x)$ and $(\ell_2 + x)$ then



(i) For first resonance $\ell_1 + x = \lambda/4$ ....(i)

(ii) For second resonance $\ell_2 + x = 3\lambda/4$ ....(ii))

Subtract Equation (ii) from Equation (i)

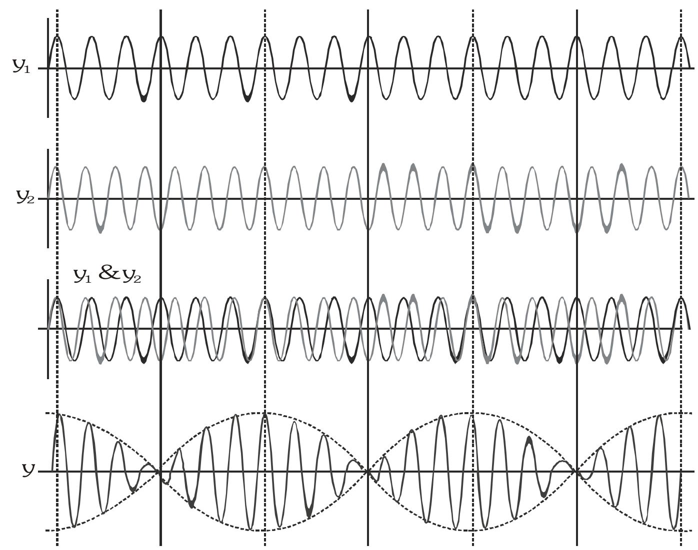
$$\ell_2 - \ell_1 = \lambda/2$$
$$\lambda = 2\,(\ell_2 - \ell_1)$$

Put the value of λ in Equation (i) $\ell_1 + x = \dfrac{2(\ell_2 - \ell_1)}{4}$

$$\Rightarrow \ell_1 + x = \frac{\ell_2 - \ell_1}{2} \quad \Rightarrow x = \frac{\ell_2 - 3\ell_1}{2}$$

BEATS

When two sound waves of same amplitude travelling in same direction with slightly different frequency superimpose, then intensity varies periodically with time. This effect is called Beats. Suppose two waves of frequencies f_1 and f_2 ($<f_1$) are meeting at some point in space. The corresponding periods are T_1 and T_2 ($>T_1$). If the two waves are in phase at t=0, they will again be in phase when the first wave has gone through exactly one more cycle than the second. This will happen at a time t=T, the period of the beat. Let *n* be the number of cycles of the first wave in time T, then the number of cycles of the second wave in the same time is (n–1). Hence, $T = nT_1 = (n-1)\,T_2$



Eliminating n we have $T = \dfrac{T_1T_2}{T_2 - T_1} = \dfrac{1}{\dfrac{1}{T_1} - \dfrac{1}{T_2}} = \dfrac{1}{f_1 - f_2}$

The reciprocal of the beat period is the beat frequency $f = \dfrac{1}{T} = f_1 - f_2$

Waves Interference On The Bases Of Beats :

Conditions : Two equal frequency waves travel in same direction.

Mathematical analysis

If displacement of first wave $y_1 = a \sin \omega_1 t \longrightarrow (N_1\,,\, a)$ $\quad I \propto N^2a^2$

Displacement of second wave $y_2 = a \sin \omega_2 t \longrightarrow (N_2\,,\, a)$

By superposition $y = y_1 + y_2$

Equation of resulting wave $y = a\,\{\sin 2\pi N_1 t + \sin 2\pi\, N_2 t\}$

$$y = a\left\{2\sin 2\pi\, t\frac{(N_1+N_2)}{2}\cos 2\pi t\frac{(N_1-N_2)}{2}\right\} = \left\{2a\cos 2\pi t\ \frac{(N_1-N_2)}{2}\right\}\ \sin 2\pi t\ \frac{(N_1+N_2)}{2} = A\sin 2\pi N't$$

Amplitude $A = 2a\ \text{Cos}\ 2\pi t\left(\frac{N_1-N_2}{2}\right) = 2a\cos \pi t\ (N_1 - N_2)$ **Frequency** $N' = \frac{N_1+N_2}{2}$

- **For max Intensity (A = ± 2a): –**

If $\cos \pi (N_1 - N_2)\, t = \pm 1 \Rightarrow \cos \pi (N_1 - N_2)\, t = \cos n\pi$, n = 0, 1, 2, ...

$$\Rightarrow \pi (N_1 - N_2)t = n\pi \Rightarrow t = \frac{n}{N_1-N_2} = 0, \frac{1}{\Delta N}, \frac{2}{\Delta N}, \frac{3}{\Delta N} \ \ldots\ldots\ldots\ldots$$

- **For Minimum Intensity (A = 0) :**

$$\Rightarrow \cos \pi (N_1 - N_2)\, t = 0 \Rightarrow \cos \pi (N_1 - N_2)\, t = \cos (2n + 1)\frac{\pi}{2} \quad n = 0, 1, 2 \ \ldots\ldots\ldots\ldots$$

$$\Rightarrow \pi (N_1 - N_2)\, t = (2n + 1)\frac{\pi}{2} \Rightarrow t = \frac{2n+1}{2(N_1-N_2)} = \frac{1}{2\Delta N}, \frac{3}{2\Delta N}, \frac{5}{2\Delta N} \ \ldots\ldots\ldots\ldots$$

GOLDEN KEY POINTS

- When we added wax on tuning fork then the frequency of fork decreases.
- When we file the tuning fork then the frequency of fork increases.

Example

A tuning fork having n = 300 Hz produces 5 beats/s with another tuning fork. If impurity (wax) is added on the arm of known tuning fork, the number of beats decreases then calculate the frequency of unknown tuning fork.

Solution

The frequency of unknown tuning fork should be 300 ± 5 = 295 Hz or 305 Hz.

When wax is added, if it would be 305 Hz, beats would have increases but with 295 Hz beats is decreases so frequency of unknown tuning fork is 295 Hz.

Example

A tuning fork having n = 158 Hz, produce 3 beats/s with another. As we file the arm of unknown, beats become 7 then calculate the frequency of unknown.

Solution

The frequency of unknown tuning fork should be 158 ± 3 = 155 Hz or 161 Hz.

After filling the number of beats = 7 so frequency of unknown tuning fork should be

158 ± 7 = 165 Hz or 151 Hz.

As both above frequency can be obtain by filing so frequency of unknown = 155/161 Hz.

SPECIAL POINTS

1. Displacement and pressure waves

A sound wave (i.e. longitudinal mechanical wave) can be described either in terms of the longitudinal displacement suffered by the particles of the medium (called displacement-wave) or in terms of the excess pressure generated due to compression and rarefaction (called pressure-wave). Consider a sound wave travelling in the x-direction in a medium. Suppose at time t, the particle at the undisturbed position x suffers a displacement y in the x-direction. The displacement wave then will be described by $y = A\sin(\omega t - kx)$ ...(i)

Now consider the element of medium which is confined with in x and x+Δx in the undisturbed state. If S is the cross-section, the volume element in undisturbed state will be V = S Δx. As the wave passes the ends at x and x +Δx are displaced by amount y and y + Δy so that increase in volume of the element will be $\Delta V = S\ \Delta y$.

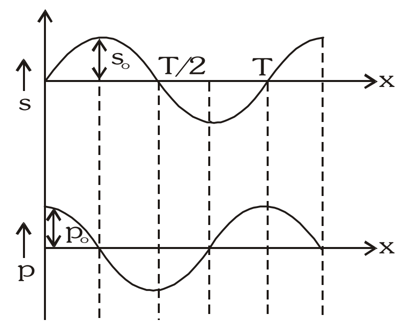
This in turn implies that volume strain for the element under consideration $\frac{\Delta V}{V} = \frac{S\ \Delta y}{S\ \Delta x} = \frac{\Delta y}{\Delta x}$ ... (ii)

So corresponding stress, i.e. excess pressure

$$P = B\left[\frac{-\Delta V}{V}\right]\left[\text{as } B = -V\frac{\Delta P}{\Delta V} = -V\frac{P}{\Delta V}\right] \text{ or } P = -B\frac{\Delta y}{\Delta x} \text{ [From equation (ii)) ... (iii)}$$

Note: For a harmonic -progressive - wave from $\frac{dy}{dx} = \frac{v_{pa}}{v}$, $P = -B\frac{dy}{dx} = -B\left(\frac{v_{pa}}{v}\right)$

i.e. pressure in a sound wave is equal to the product of elasticity of gas with the ratio of particle speed to wave speed.



But from equation (i) $\frac{\Delta y}{\Delta x} = -Ak\cos(\omega t - kx)$

So $P = AkB\cos(\omega t - kx)$

i.e. $P = P_0\cos(\omega t - kx)$ ...(iv)

with $P_0 = ABk$ ...(iv)

From equation (i) and (iv) it is clear that

- A sound wave may be considered as either a displacement wave
  $y = A\sin(\omega t - kx)$ or a pressure wave $P = P_0\cos(\omega t - kx)$.
- The pressure wave is 90^0 out of phase with respect to displacement wave, i.e, displacement will be maximum when pressure is minimum and vice-versa. This is shown in figure
- The amplitude of pressure wave:– $P_0 = ABk = Ak\rho v^2 = \rho vA\omega$ [as $v = \sqrt{B/\rho}$, $v = \omega/k$] ...(v)
- As sound-sensors (e.g. ear or mic) detects pressure changes, description of sound as pressure-wave is preferred over displacement wave.

2. Ultrasonic, Infrasonic and Audible (sonic) Sound :

Sound waves can be classified in three groups according to their range of frequencies.

- **Infrasonic Waves**

  Longitudinal waves having frequencies below 20 Hz are called infrasonic waves. They cannot be heard by human beings. They are produced during earthquakes. Infrasonic waves can be heard by snakes.
- **Audible Waves**

  Longitudinal waves having frequencies lying between 20-20,000 Hz are called audible waves.
- **Ultrasonic Waves**

  Longitudinal waves having frequencies above 20,000 Hz are called ultrasonic waves. They are produced and heard by bats. They have a large energy content.
- **Applications of Ultrasonic Waves**

  Ultrasonic waves have a large range of application. Some of them are as follows:

  (i) The fine internal cracks in metal can be detected by ultrasonic waves.
  (ii) Ultrasonic waves can be used for determining the depth of the sea, lakes etc.
  (iii) Ultrasonic waves can be used to detect submarines, icebergs etc.
  (iv) Ultrasonic waves can be used to clean clothes, fine machinery parts etc.
  (v) Ultrasonic waves can be used to kill smaller animals like rates, fish and frogs etc.

- **Shock Waves**

  If the speed of the body in air is greater than the speed of the sound, then it is called supersonic speed. Such a body leaves behind a conical region of disturbance which spreads continuously. Such a disturbance is called a **'Shock Wave'**. This wave carries huge energy. If it strikes a building, then the building may be damaged.

3. Sound intensity in decibels

The physiological sensation of loudness is closely related to the intensity of wave producing the sound. At a frequency of 1 kHz people are able to detect sounds with intensities as low as 10^{-12}W/m^2. On the other hand an intensity of 1 W/m^2 can cause pain, and prolonged exposure to sound at this level will damage a person's ears. Because the range in intensities over which people hear is so large, it is convenient to use a logarithmic scale to specify intensities. If the intensity of sound in watts per square meter is I, then the intensity level β in decibels (dB) is given by $\beta = 10 \log \frac{I}{I_0}$

where the base of the logarithm is 10, and I_0 =10^{-12} W/m^2 (roughly the minimum intensity that can be heard).

On the decibel scale, the pain threshold of 1 W/m^2 is then $\beta = 10 \log \frac{1}{10^{-12}} = 120\text{dB}$

Example

Calculate the change in intensity level when the intensity of sound increases by 10^6 times its original intensity.

Solution

Here $\frac{I}{I_0} = \frac{\text{Final intensity}}{\text{Initial intensity}} = 10^6$

Increase in intensity level, L = 10 $\log_{10}\left(\frac{I}{I_0}\right)$ = 10 $\log_{10}$ (10^6) = 10 $\log_{10}10$ = 10   6   1 = 60 decibels.

ECHO

Multiple reflection of sound is called an echo. If the distance of reflector from the source is d then, $2d = vt$

Hence, v = speed of sound and t, the time of echo. $\therefore d = \frac{vt}{2}$

Since, the effect of ordinary sound remains on our ear for 1\10 second, therefore, if the sound returns to the starting point before 1/10 second, then it will not be distinguished from the original sound and no echo will be heard. Therefore, the minimum distance of the reflector is, $d_{min} = \frac{v \times t}{2} = \left(\frac{330}{2}\right)\left(\frac{1}{10}\right) = 16.5\text{m}$

ACOUSTIC DOPPLER EFFECT (DOPPLER EFFECT FOR SOUND WAVES)

The apparent change in the frequency of sound when the source of sound, the observer and the medium are in relative motion is called Doppler effect. While deriving these expressions, we make the following assumptions

(i) The velocity of the source, the observer and the medium are along the line joining the positions of the source and the observer.

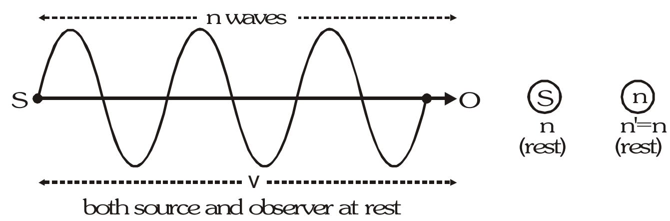
(ii) The velocity of the source and the observer is less than velocity of sound.

Doppler effect takes place both in sound and light. In sound it depends on whether the source or observer or both are in motion while in light it depends on whether the distance between source and observer is increasing or decreasing.

NOTATIONS

$n \rightarrow$ actual frequency	$n' \rightarrow$ observed frequency (apparent frequency)
$\lambda \rightarrow$ actual wave length	$\lambda' \rightarrow$ observed (apparent) wave length
$v \rightarrow$ velocity of sound	$v_s \rightarrow$ velocity of source
$v_0 \rightarrow$ velocity of observer	$v_w \rightarrow$ wind velocity

Case I : Source in motion, observer at rest, medium at rest :

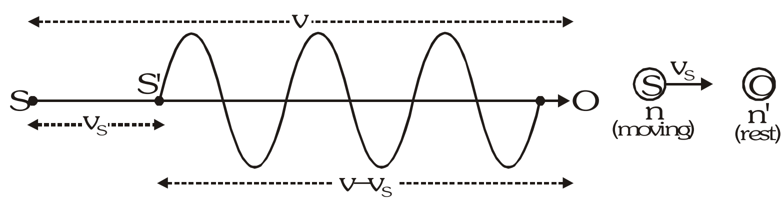


Suppose the source S and observer O are separated by distance v. Where v is the velocity of sound. Let n be the frequency of sound emitted by the source. Then n waves will be emitted by the source in one second. These n waves will be accommodated in distance v.

So, wave length $\lambda = \dfrac{\text{total distance}}{\text{total number of waves}} = \dfrac{v}{n}$

(i) Source moving towards stationary observer :

Let the sources start moving towards the observer with velocity v_s. After one second, the n waves will be crowded in distance $(v - v_s)$. Now the observer shall feel that he is listening to sound of wavelength λ' and frequency n'



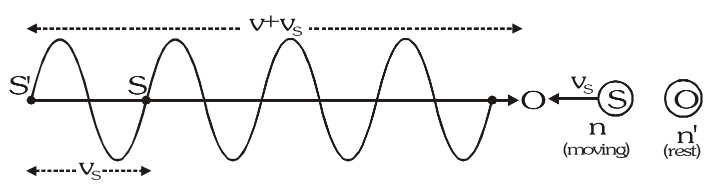
Now apparent wavelength $\lambda' = \dfrac{\text{total distance}}{\text{total number of waves}} = \dfrac{v - v_s}{n}$

and changed frequency, $n' = \dfrac{v}{\lambda'} = \dfrac{v}{\left(\dfrac{v - v_s}{n}\right)} = n\left(\dfrac{v}{v - v_s}\right)$

So, as the source of sound approaches the observer the apparent frequency n' becomes greater than the true frequency n.

(ii) When source move away from stationary observer :-

For this situation n waves will be crowded in distance $v + v_s$.



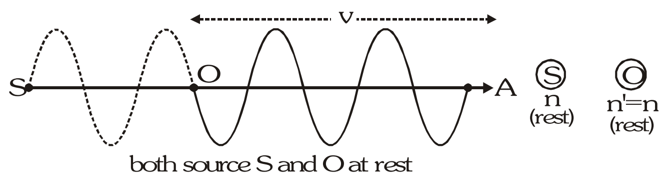
So, apparent wavelength $\lambda' = \dfrac{v + v_s}{n}$

and

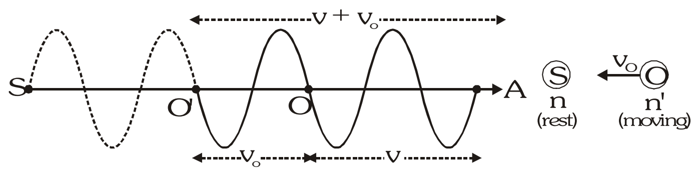
Apparent frequency $n' = \dfrac{v}{\lambda'} = \dfrac{v}{\left(\dfrac{v + v_s}{n}\right)} = n\left(\dfrac{v_s}{v + v_s}\right)$ So $n' < n$

Case II : Observer in motion, source at rest, medium at rest

Let the source (S) and observer (O) are in rest at their respective places. Then n waves given by source 'S' would be crossing observer 'O' in one second and fill the space OA (=v)



(i) Observer moves towards stationary source



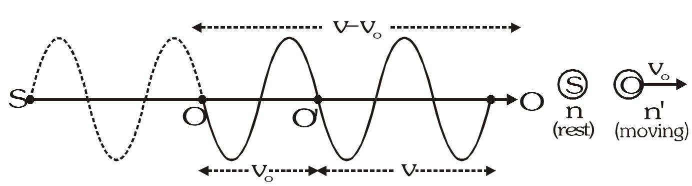
When observer 'O' moves towards 'S' with velocity v_o, it will cover v_o distance in one second. So the observer has received not only the n waves occupying OA but also received additional number of Δn waves occupying the distance OO' (= v_o).

So, total waves received by observer in one second i.e., apparent frequency (n') = Actual waves (n) + Additional waves (Δn)

$$n' = \frac{v}{\lambda} + \frac{v_o}{\lambda} = \frac{v + v_o}{(v/n)} = n\left(\frac{v + v_o}{v}\right)\left(\because \lambda = \frac{v}{n}\right) \quad (\text{so, } n' > n)$$

(ii) Observer moves away from stationary source :-

For this situation n waves will be crowded in distance $v - v_o$.



When observer move away from source with v_o velocity then he will get Δn waves less than real number of waves. So, total number of waves received by observer i.e.

Apparent frequency (n') = Actual waves (n) – reduction in number of waves (Δn)

$$n' = \frac{v}{\lambda} - \frac{v_o}{\lambda} = \frac{v - v_o}{\lambda} = \frac{v - v_o}{(v/n)} = \left(\frac{v - v_o}{v}\right)n \quad \left(\because \lambda = \frac{v}{n}\right) \quad (\text{so } n' < n)$$

GOLDEN KEY POINT

- If medium (air) is also moving with v_m velocity in direction of source to observer. Then velocity of sound relative to observer will be $v \pm v_m$ (–ve sign, if v_m is opposite to sound velocity). So, $n' = n\left(\frac{v \pm v_m \pm v_o}{v \pm v_m \mp v_s}\right)$
- If medium moves in a direction opposite to the direction of propagation of sound, then $n' = \left(\frac{v - v_m - v_o}{v - v_m - v_S}\right)n$
- Source in motion towards the observer. Both medium and observer are at rest. $n' = \left(\frac{v}{v - v_S}\right)n$

  So, when a source of sound approaches a stationary observer, the apparent frequency is more than the actual frequency.
- Source in motion away from the observer. Both medium and observer are at rest. $n' = \left(\frac{v}{v + v_S}\right)n$. So, when a source of sound moves away from a stationary observer, the apparent frequency is less than actual frequency.
- Observer in motion towards the source. Both medium and source are at rest. $n' = \left(\frac{v + v_o}{v}\right)n$. So, when observer is in motion towards the source, the apparent frequency is more than the actual frequency.
- Observer in motion away from the source. Both medium and source are at rest. $n' = \left(\frac{v - v_o}{v}\right)n$. So, when observer is in motion away from the source, the apparent frequency is less than the actual frequency.
- Both source and observer are moving away from each other. Medium at rest. $n' = \left(\frac{v - v_o}{v + v_S}\right)n$

DOPPLER'S EFFECT IN REFLECTION OF SOUND (ECHO)

When the sound is reflected from the reflector the observer receives two notes one directly from the source and other from the reflector. If the two frequencies are different then superposition of these waves result in beats and by the beat frequency we can calculate speed of the source.

If the source is at rest and reflector is moving towards the source with speed u,

then apparent frequency heard by reflector $n_1 = \left(\frac{v + u}{v}\right)n$

Now this frequency n_1 acts as a source so that apparent frequency received by observer is

$$n_2 = \left(\frac{v}{v - u}\right)n_1 = \left(\frac{v}{v - u}\right) \times \left(\frac{v + u}{v}\right)n = \left(\frac{v + u}{v - u}\right)n$$

If $u << v$ then $n_2 = n\left(1 + \frac{u}{v}\right)\left(1 - \frac{u}{v}\right)^{-1} \approx n\left(1 + \frac{u}{v}\right)^2 \approx n\left(1 + \frac{2u}{v}\right)$

Beat frequency $\Delta n = n_2 - n = \left(\frac{2u}{v}\right)n$ So speed of the source $u = \frac{v}{2}\left(\frac{\Delta n}{n}\right)$

CONDITIONS WHEN DOPPLER'S EFFECT IS NOT OBSERVED FOR SOUND WAVES

- When the source of sound and observer both are at rest then Doppler effect is not observed.
- When the source and observer both are moving with same velocity in same direction.
- When the source and observer are moving mutually in perpendicular directions.
- When the medium only is moving.
- When the distance between the source and observer is constant.

Example

When both source and observer approach each other with a speed equal to the half the speed of sound, then determine the percentage change in frequency of sound as detected by the listener.

Solution

Source $\bullet\!\xrightarrow{\frac{v}{2}} \quad \xleftarrow{\frac{v}{2}}\!\bullet$ Observer

$$n' = \left(\frac{v + \frac{v}{2}}{v - \frac{v}{2}}\right) n = \left(\frac{\frac{3}{2}v}{\frac{1}{2}v}\right) n = 3n$$

$$\%\ \text{change} = \frac{n' - n}{n} \times 100 = \frac{3n - n}{n} \times 100 = \frac{2n}{n} \times 100 = 200\ \%$$

Example

Two trains travelling in opposite directions at 126 km/hr each, cross each other while one of them is whistling. If the frequency of the node is 2.22 kHz find the apparent frequency as heard by an observer in the other train :

(a) Before the trains cross each other

(b) After the trains have crossed each other. (v_{sound} = 335 m/sec)

Solution

Here $v_1 = 126 \quad \frac{5}{18} = 35$ m/s

(i) In this situation $\circ\!\longrightarrow v_1 \qquad v_1 \longleftarrow\!\circ$

Observed frequency $n' = \left(\frac{v + v_1}{v - v_1}\right) \times n = \left(\frac{335 + 35}{335 - 35}\right) \times 2220 = 2738$ Hz

(ii) In this situation $v_1 \longleftarrow\!\circ \qquad \circ\!\longrightarrow v_1$

Observed frequency $n' = \left(\frac{v - v_1}{v + v_1}\right) \times n = \left(\frac{335 - 35}{335 + 35}\right) \times 2220 = 1800$ Hz

Example

A stationary source emits sound of frequency 1200 Hz. If wind blows at the speed of 0.1v, deduce

(a) The change in the frequency for a stationary observer on the wind side of the source.

(b) Report the calculations for the case when there is no wind but the observer moves at 0.1v speed towards the source. (Given : velocity of sound = v)

Solution

(a) Medium moves in the direction of sound propagation i.e. from source to observer

so effective velocity of sound $v_{eff} = v + v_m$

since both source and observer are at rest $n' = \left(\frac{v + v_m + 0}{v + v_m + 0}\right) n = \left(\frac{v + 0.1v}{v + 0.1v}\right) n = n$

so there is no change in frequency

(b) When observer move towards source $n' = \left(\frac{v + v_0}{v}\right) n = \left(\frac{v + 0.1\,v}{v}\right) n$

$= 1.1\ n = 1.1 \quad 1200\ \text{Hz} = 1320\ \text{Hz}$

Example

A bat is flitting about in a cave, navigating via ultrasonic beeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall ?

Solution

The apparent frequency heard by the bat of reflected sound

$$n' = \left(\frac{v + v_0}{v - v_s}\right) n = \left(\frac{v + 0.03v}{v - 0.03v}\right) \times 40 = \frac{1.03v}{0.97v} \times 40 = 42.47\ \text{kHz}$$

SOME WORKED OUT EXAMPLES

Example#1

A particle of mass 50 g participates in two simple harmonic oscillations, simultaneously as given by x_1 = 10(cm) cos[80π(s^{-1}) t] and x_2 = 5(cm) sin[(80π(s^{-1}) t + π/6]. The amplitude of particle's oscillations is given by 'A'. Find the value of A^2 (in cm^2).

(A) 175 (B) 165 (C) 275 (D) 375

Solution **Ans. (A)**

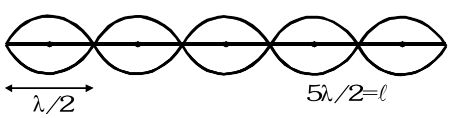
$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi} = \sqrt{10^2 + 5^2 + 2\times5\times10\times\frac{1}{2}} = \sqrt{175} \Rightarrow A^2 = 175$$

Example#2

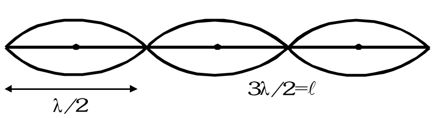
A sonometer wire resonates with a given tuning fork forming a standing wave with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire . When this mass is replaced by a mass 'M' kg, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. Find the value of M.

(A) 25 (B) 20 (C) 15 (D) 10

Solution **Ans. (A)**



$$f = \frac{\sqrt{\frac{9g}{\mu}}}{\frac{2\ell}{5}}$$



$$f = \frac{\sqrt{\frac{Mg}{\mu}}}{\frac{2\ell}{3}} \Rightarrow f = \frac{\sqrt{\frac{9g}{\mu}}}{\frac{2\ell}{5}} = \frac{\sqrt{\frac{Mg}{\mu}}}{\frac{2\ell}{3}} \Rightarrow \sqrt{M} = 5$$

Example#3

A steel wire of length 1 m and mass 0.1 kg and having a uniform cross-sectional area of 10^{-6} m^2 is rigidly fixed at both ends. The temperature of the wire is lowered by 20 C. If the wire is vibrating in fundamental mode, find the frequency (in Hz).(Y_{steel} = 2 10^{11} N/m^2, α_{steel} = 1.21 10^{-5}/ C)

(A) 11 (B) 20 (C) 15 (D) 10

Solution **Ans. (A)**

$$\Delta\ell = \alpha\ell\Delta\theta \Rightarrow Y = \frac{T/A}{\Delta\ell/\ell} \Rightarrow T = YA\frac{\Delta\ell}{\ell} \Rightarrow T = \alpha YA\Delta\theta = 48.4N \;;\; v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{48.4}{\left(\frac{0.1}{1}\right)}} = 22m/s$$

$\because$ for fundamental note $\ell = \frac{\lambda}{2} \Rightarrow \lambda = 2m \Rightarrow f = \frac{v}{\lambda} = \frac{22}{2} = 11Hz$

Example#4

A progressive wave on a string having linear mass density ρ is represented by $y = A\sin\left(\frac{2\pi}{\lambda}x - \omega t\right)$ where y is in mm. Find the total energy (in μJ) passing through origin from t = 0 to t = $\frac{\pi}{2\omega}$.

[Take : ρ = 3 10^{-2} kg/m; A = 1mm; ω = 100 rad/sec; λ = 16 cm]

(A) 6 (B) 7 (C) 8 (D) 9

Solution **Ans. (A)**

Total energy $\frac{1}{2}\rho A^2\omega^2 \times \frac{\lambda}{4}$

Example#5

Two tuning forks A and B lying on opposite sides of observer 'O' and of natural frequency 85 Hz move with velocity 10 m/s relative to stationary observer O. Fork A moves away from the observer while the fork B moves towards him. A wind with a speed 10 m/s is blowing in the direction of motion of fork A. Find the beat frequency measured by the observer in Hz. [Take speed of sound in air as 340 m/s]

(A) 5 (B) 6 (C) 7 (D) 8

Solution **Ans. (A)**

$$f_{\text{observer for source 'A'}} = f_0\left[\frac{v_{\text{sound}} - v_{\text{medium}}}{v_{\text{sound}} - v_{\text{medium}} + v_{\text{source}}}\right] = \frac{33}{34}f_0 \; ; \; f_{\text{observer for source 'B'}} = f_0\left[\frac{v_{\text{sound}} + v_{\text{medium}}}{v_{\text{sound}} + v_{\text{medium}} - v_{\text{source}}}\right] = \frac{35}{34}f_0$$

$\therefore$ Beat frequency $= f_1 - f_2 = \left(\frac{35-33}{34}\right)f_0 = 5$

Example#6

If y_1 = 5 (mm) sinπt is equation of oscillation of source S_1 and y_2 = 5 (mm) sin(πt + π/6) be that of S_2 and it takes 1 sec and ½ sec for the transverse waves to reach point A from sources S_1 and S_2 respectively then the resulting amplitude at point A, is

(A) $5\sqrt{2+\sqrt{3}}$ mm (B) $5\sqrt{3}$ mm (C) 5 mm (D) $5\sqrt{2}$ mm

Solution **Ans. (C)**

Wave originating at t =0 from S_1 reaches point A at t = 1.

Wave originating at t $=\frac{1}{2}$ from S_2 reaches point A at t = 1.

So phase difference in these waves = $\frac{\pi}{2} + \frac{\pi}{6}$; A = $\sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi} = 5$

Example#7

A transverse wave, travelling along the positive x-axis, given by y=Asin(kx−ωt) is superposed with another wave travelling along the negative x-axis given by y=−Asin(kx+ωt). The point x=0 is

(A) a node (B) an anitnode

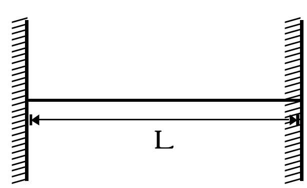
(C) neither a node nor an antinode (D) a node or antinode depending on t.

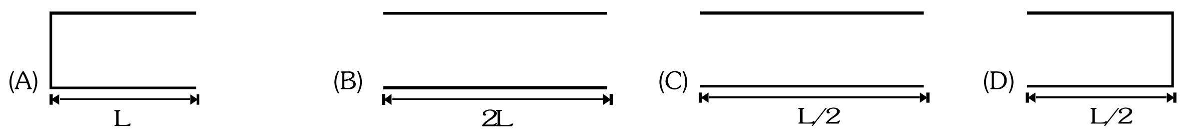
Solution **Ans. (B)**

At x =0, y_1 = Asin (−ωt) and y_2 = −Asinωt; $y_1 + y_2 = -2A\sin\omega t$ (antinode)

Example#8

Figure shows a streched string of length L and pipes of length L, 2L, L/2 and L/2 in options (A), (B), (C) and (D) respectively. The string's tension is adjusted until the speed of waves on the string equals the speed of sound waves in air. The fundamental mode of oscillation is then set up on the string. In which pipe will the sound produced by the string cause resonance?





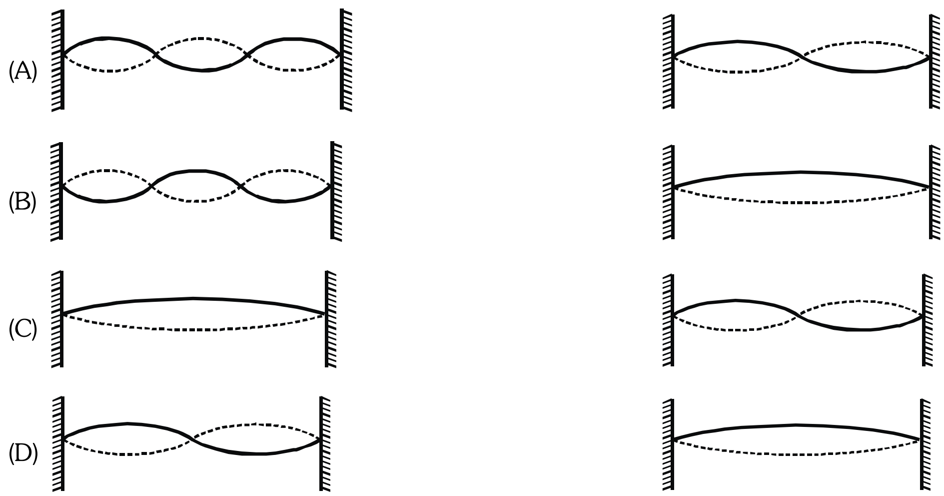
Solution

Ans. (B)

Example#9

String I and II have identical lengths and linear mass densities, but string I is under greater tension than string II. The accompanying figure shows four different situations, A to D, in which standing wave patterns exist on the two strings. In which situation it is possible that strings I and II are oscillating at the same resonant frequency?

String I **String II**



Solution **Ans. (C)**

Since tension in I > tension in II $\Rightarrow V_I > V_{II}$ Thus, for same frequency, $\lambda_I > \lambda_{II}$

Example#10

A standing wave is created on a string of length 120 m and it is vibrating in 6th harmonic. Maximum possible amplititude of any particle is 10 cm and maximum possible velocity will be 10 cm/s. Choose the correct statement.

(A) Angular wave number of two waves will be $\frac{\pi}{20}$.

(B) Time period of any particle's SHM will be 4π sec.

(C) Any particle will have same kinetic energy as potential energy.

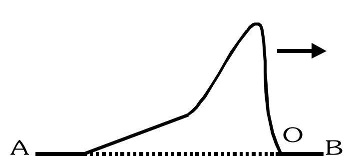
(D) Amplitude of interfering waves are 10 cm each.

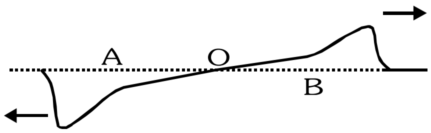
Solution **Ans. (A)**

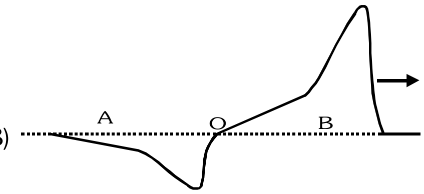
$$6\left(\frac{\lambda}{2}\right) = 120 \Rightarrow \lambda = 40 \Rightarrow k = \frac{\pi}{20} \Rightarrow A\omega = v_{max} \Rightarrow \omega = 1 \Rightarrow T = 2\pi$$

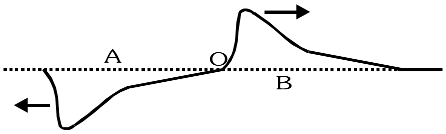
Example#11

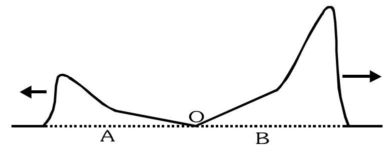
Two strings, A and B, of lengths 4L and L respectively and same mass M each, are tied together to form a knot 'O' and stretched under the same tension. A transverse wave pulse is sent along the composite string from the side A, as shown to the right. Which of the following diagrams correctly shows the reflected and transmitted wave pulses near the knot 'O'?



(A) 

(B) 

(C) 

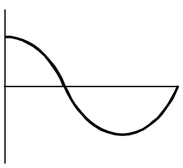
(D) 

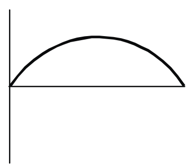
Solution **Ans. (A)**

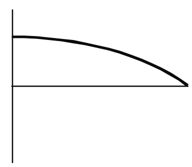
The wave suffers a phase difference of π when reflected by denser medium.

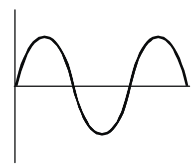
Example#12

Which of the figures, shows the pressure difference from regular atmospheric pressure for an organ pipe of length L closed at one end, corresponds to the 1st overtone for the pipe?

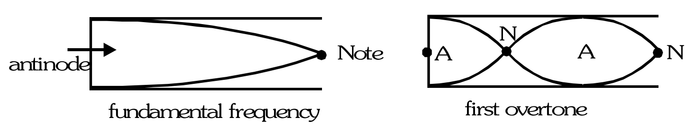
(A) 

(B) 

(C) 

(D) 

Solution **Ans. (A)**

For pressure standing wave 

Example#13

A man generates a symmetrical pulse in a string by moving his hand up and down. At $t = 0$ the point in his hand moves downward. The pulse travels with speed of 3 m/s on the string & his hands passes 6 times in each second from the mean position. Then the point on the string at a distance 3m will reach its upper extreme first time at time t =

(A) 1.25 sec. (B) 1 sec (C) $\frac{13}{12}$ sec (D) none

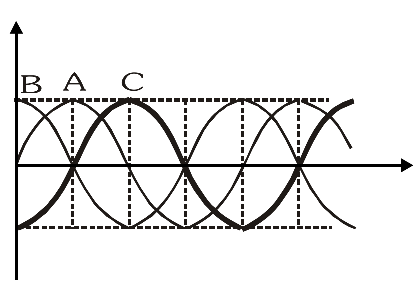
Solution **Ans. (A)**

Frequency of wave $= \frac{6}{2} = 3 \Rightarrow T = \frac{1}{3}s$; $\lambda = vT = (3)\left(\frac{1}{3}\right) = 1m$

Total time taken $= \frac{3}{3} + \frac{3T}{4} = 1.25$ sec

Example#14

Three progressive waves A, B and C are shown in figure.



With respect to wave A

(A) The wave C lags behind in phase by $\pi/2$ and B leads by $\pi/2$.

(B) The wave C leads in phase by π and B lags behind by π.

(C) The wave C leads in phase by $\pi/2$ and B lags behind by $\pi/2$.

(D) The wave C lags behind in phase by π and B leads by π.

Ans. (A)

Example#15

Following are equations of four waves :

(i) $y_1 = a \sin \omega\left(t - \frac{x}{v}\right)$ (ii) $y_2 = a \cos \omega\left(t + \frac{x}{v}\right)$

(iii) $z_1 = a \sin \omega\left(t - \frac{x}{v}\right)$ (iv) $z_2 = a \cos \omega\left(t + \frac{x}{v}\right)$

Which of the following statements is/are **CORRECT**?

(A) On superposition of waves (i) and (iii), a travelling wave having amplitude $a\sqrt{2}$ will be formed.

(B) Superposition of waves (ii) and (iii) is not possible.

(C) On superposition of waves (i) and (ii), a transverse stationary wave having maximum amplitude $a\sqrt{2}$ will be formed.

(D) On superposition of waves (iii) and (iv), a transverse stationary wave will be formed.

Solution **Ans. (AD)**

Superposition of waves (i) & (iii) will give travelling wave having amplitude of $a\sqrt{2}$

{waves are along x-axis but particle displacements are along y & z-axis respectively}

$$z_1 + z_2 = a\left[\sin \omega\left(t - \frac{x}{v}\right) + \sin\left\{\omega\left(t + \frac{x}{v}\right) + \frac{\pi}{2}\right\}\right]$$

Example#16

Two mechanical waves, $y_1 = 2 \sin 2\pi (50\, t - 2x)$ & $y_2 = 4 \sin 2\pi (ax + 100\, t)$ propagate in a medium with same speed.

(A) The ratio of their intensities is 1: 16 (B) The ratio of their intensities is 1: 4

(C) The value of 'a' is 4 units (D) The value of 'a' is 2 units

Solution **Ans. (AC)**

$I = \frac{1}{2}\rho v \omega^2 A^2$ and velocity $= \frac{\omega}{k}$

Example#17

Three simple harmonic waves, identical in frequency n and amplitude A moving in the same direction are superimposed in air in such a way, that the first, second and the third wave have the phase angles $\phi, \phi + \frac{\pi}{2}$ and $(\phi+\pi)$ respectively at a given point P in the superpositon.

Then as the waves progress, the superposition will result in

(A) a periodic, non-simple harmonic wave of amplitude 3A

(B) a stationary simple harmonic wave of amplitude 3A

(C) a simple harmonic progressive wave of amplitude A

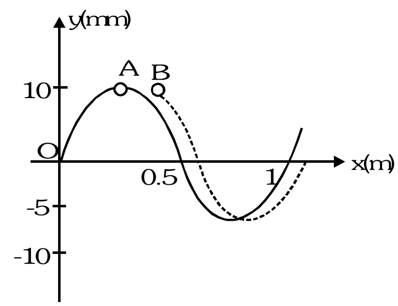
(D) the velocity of the superposed resultant wave will be the same as the velocity of each wave

Solution **Ans. (CD)**

Since the first wave and the third wave moving in the same direction have the phase angles ϕ and $(\phi+\pi)$, they superpose with opposite phase at every point of the vibrating medium and thus cancel out each other, in displacement, velocity and acceleration. They, in effect, destroy each other out. Hence we are left with only the second wave which progresses as a simple harmonic wave of amplitude A. The velocity of this wave is the same as if it were moving alone.

Example#18

Two idetncial waves A and B are produced from the origin at different instants t_A and t_B along the positive x-axis, as shown in the figure. If the speed of wave is 5m/s then



(A) the wavelength of the waves is 1m

(B) the amplitude of the waves is 10 mm

(C) the wave A leads B by 0.0167 s

(D) the wave B leads A by 1.67 s

Solution **Ans. (AB)**

Wavelength of the waves = 1m; Amplitude of the waves = 10 mm

Example#19

A progressive wave having amplitude 5 m and wavelength 3 m. If the maximum average velocity of particle in half time period is 5 m/s and wave is moving in the positive x-direction then find which may be the **correct** equation(s) of the wave? [where x in meter]

(A) $5\sin\left(\frac{2\pi}{5}t-\frac{2\pi}{3}x\right)$

(B) $4\sin\left(\frac{\pi t}{2}-\frac{2\pi}{3}x\right)+3\cos\left(\frac{\pi t}{2}-\frac{2\pi}{3}x\right)$

(C) $5\sin\left(\frac{\pi t}{2}-\frac{2\pi}{3}x\right)$

(D) $3\cos\left(\frac{2\pi}{5}t-\frac{2\pi}{3}x\right)-4\sin\left(\frac{2\pi}{5}t-\frac{2\pi}{3}x\right)$

Solution **Ans. (BC)**

$\because \lambda = 3\text{m} \quad \therefore k=\frac{2\pi}{\lambda}=\frac{2\pi}{3}$

Maximum displacement in half time period = 2a = 10 m

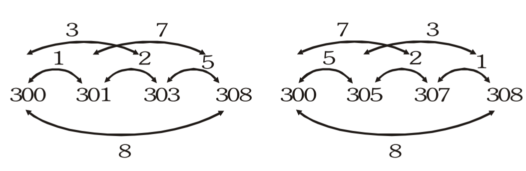
So maximum average velocity $=\dfrac{10}{T/2}=5 \Rightarrow T=4\text{ s} \Rightarrow \omega=\frac{2\pi}{T}=\frac{2\pi}{4}=\frac{\pi}{2}$

Example#20

You are given four tuning forks, the lowest frequency of the forks is 300 Hz. By striking two tuning forks at a time any of 1, 2, 3, 5, 7 & 8 Hz beat frequencies are heard. The possible frequencies of the other three forks are-

(A) 301,302 & 307 (B) 300,304 & 307 (C) 301, 303 & 308 (D) 305, 307 & 308

Solution **Ans. (CD)**



Example#21

A standing wave of time period T is set up in a string clamped between two rigid supports. At $t = 0$ antinode is at its maximum displacement 2A.

(A) The energy density of a node is equal to energy density of an antinode for the first time at $t = T/4$.

(B) The energy density of node and antinode becomes equal after T/2 second.

(C) The displacement of the particle at antinode at $t = \frac{T}{8}$ is $\sqrt{2}A$

(D) The displacement of the particle at node is zero

Solution **Ans. (CD)**

Equation of SHM of particle who is at antinode is $y = 2A\sin\left(\frac{2\pi}{T}\right)t$ at time $t = \frac{T}{8}$

$y = 2A\sin\frac{\pi}{4} = \sqrt{2}A$; Displacement of particle at note is always zero.

Example#22

Two notes A and B, sounded together, produce 2 beats per sec. Notes B and C sounded together produce 3 beats per sec. The notes A and C separately produce the same number of beats with a standard tuning fork of 456 Hz. The possible frequency of the note B is

(A) 453.5 Hz (B) 455.5 Hz (C) 456.5 Hz (D) 458.5 Hz

Solution **Ans. (ABCD)**

Let frequency of note B be n then according to question

$n_A = n-2$ or $n+2$

$n_C = n-3$ or $n+3$

As A & C produce same number of beats with T.F. of frequency 456 Hz so

$(n-2) - 456 = 456 - (n-3) \Rightarrow n = 458.5$ Hz

$(n+3) - 456 = 456 - (n-2) \Rightarrow n = 455.5$ Hz

$(n+2) - 456 = 456 - (n-3) \Rightarrow n = 456.5$ Hz

$(n+3) - 456 = 456 - (n+2) \Rightarrow n = 453.5$ Hz

Example#23 to 25

A metallic rod of length 1m has one end free and other end rigidly clamped. Longitudinal stationary waves are set up in the rod in such a way that there are total six antinodes present along the rod. The amplitude of an antinode is $4 \quad 10^{-6}$ m. Young's modulus and density of the rod are $6.4 \quad 10^{10}$ N/m^2 and $4 \quad 10^3$ Kg/m^3 respectively. Consider the free end to be at origin and at t=0 particles at free end are at positive extreme.

23. The equation describing displacements of particles about their mean positions is

(A) $s = 4\times10^{-6}\cos\left(\frac{11\pi}{2}x\right)\cos\left(22\pi\times10^3 t\right)$

(B) $s = 4\times10^{-6}\cos\left(\frac{11\pi}{2}x\right)\sin\left(22\pi\times10^3 t\right)$

(C) $s = 4\times10^{-6}\cos(5\pi x)\cos\left(20\pi\times10^3 t\right)$

(D) $s = 4\times10^{-6}\cos(5\pi x)\sin\left(20\pi\times10^3 t\right)$

24. The equation describing stress developed in the rod is

(A) $140.8\pi\times10^4\cos\left(\frac{11}{2}\pi x+\pi\right)\cos\left(22\pi\times10^3 t\right)$

(B) $140.8\pi\times10^4\sin\left(\frac{11}{2}\pi x+\pi\right)\cos\left(22\pi\times10^3 t\right)$

(C) $128\pi\times10^4\cos(5\pi x+\pi)\cos\left(20\pi\times10^3 t\right)$

(D) $128\pi\times10^4\sin(5\pi x+\pi)\cos\left(20\pi\times10^3 t\right)$

25. The magnitude of strain at midpoint of the rod at t= 1 sec is

(A) $11\sqrt{3}\pi\times10^{-6}$ (B) $11\sqrt{2}\pi\times10^{-6}$ (C) $10\sqrt{3}\pi\times10^{-6}$ (D) $10\sqrt{2}\pi\times10^{-6}$

Solution

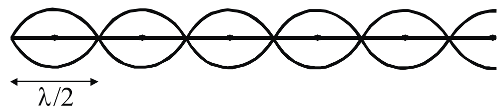
23. Ans. (A)

24. Ans. (B)

25. Ans. (B)

Solution (23 to 25)

Speed of wave $v = \sqrt{\frac{y}{\rho}} = 4 \times 10^{3}$



$\lambda = \frac{5\lambda}{2} + \frac{\lambda}{4} \Rightarrow \lambda = \frac{4\ell}{11}$

Frequency $\nu = \frac{v}{\lambda} = \frac{4 \times 10^{3}}{\frac{4}{11} \times 1} = 11 \times 10^{3}\,\text{Hz}$; Wave Number $K = \frac{2\pi}{\lambda} = \frac{11\pi}{2}$

(i) Equation of standing wave in the rod S = A coskx sin(ωt +ϕ) where A = 4 10^{-6} m

$\because$ at x =0, t =0 $\Rightarrow$S=A $\Rightarrow$ A =A $\Rightarrow$ cosk(0) sinϕ $\Rightarrow$ sinϕ =1 $\Rightarrow$ ϕ= $\frac{\pi}{2}$

$$S = 4 \times 10^{-6} \cos\left(\frac{11\pi}{2}x\right)\cos\left(22\pi \times 10^{3} t\right)$$

(ii) Strain $= \frac{ds}{dx} = -22\pi \times 10^{-6} \sin\left(\frac{11\pi}{2}x\right)\cos\left(22\pi \times 10^{3} t\right)$ $\because$stress = Y strain

$$\Rightarrow \text{stress} = 140.8 \times 10^{4} \cos\left(22\pi \times 10^{3} t\right)\sin\left(\frac{11\pi}{2}x + \pi\right)$$

(iii) Strain at t = 1s and $x = \frac{\ell}{2} = \frac{1}{2}\text{m}$; $\left|\frac{ds}{dx}\right|_{x=\frac{\ell}{2}}^{t=1} = 22\pi \times 10^{-6} \times \sin\left(\frac{11\pi}{4}\right) = 11\sqrt{2}\pi \times 10^{-6}$

Example#26 to 28

A detector at x =0 receives waves from three sources each of amplitude A and frequencies f +2, f and f–2.

26. The equations of waves are ; y_1=Asin[2π(f+2)t], y_2=Asin2πft and y_3=Asin[2π(f–2)t]. The time at which intensity is minimum, is

(A) t=0, 1/4, 1/2, 3/4,... sec
(B) t=1/6, 1/3, 2/3, 5/6, ...sec
(C) t=0, 1/2, 3/2, 5/2, ...sec
(D) t=1/2, 1/4, 1/6, 1/8, ...sec

27. The time at which intensity is maximum, is

(A) t=0, 1/4, 1/2, 3/4, ...sec
(B) t=1/6, 1/3, 2/3, 5/6...sec
(C) t=0, 1/2, 3/2, 5/2 ...sec
(D) t=1/2, 1/4, 1/6, 1/8...sec

28. If $I_0 \propto A^2$, then the value of maximum intensity, is

(A) $2I_0$
(B) $3I_0$
(C) $4I_0$
(D) $9I_0$

Solution

26. Ans. (B)

$$y = y_1 + y_2 + y_3 = A\sin 2\pi ft + A\sin[2\pi(f-2)t] + A\sin[2\pi(f+2)t] = A\sin 2\pi ft + 2A\sin 2\pi ft\cos 4\pi t$$

$$= A[1+2\cos 4\pi t]\sin 3\pi ft = A_0 \sin 2\pi ft$$

[where A_0 = Amplitude of the resultant oscillation = A [1+2cos4πt]

Intensity $\propto A_0^2 \ \therefore I \propto (1+2\cos 4\pi t)^2$

For maxima or minima of the intensity.

$$\frac{dI}{dt} = 0 \Rightarrow 2(1+2\cos 4\pi t)(2)(-\sin 4\pi t)4\pi = 0 \Rightarrow 1+2\cos 4\pi t = 0 \text{ or } \sin 4\pi t = 0$$

$$\therefore \cos 4\pi t = -\frac{1}{2} \Rightarrow 4\pi t = 2\pi n \pm \frac{2\pi}{3} \ \therefore t = \frac{n}{2}+\frac{1}{6} \Rightarrow t = \frac{1}{6},\frac{1}{3},\frac{2}{5},\frac{5}{6}\ldots \text{ (point of minimum intensity)}$$

27. Ans. (A)

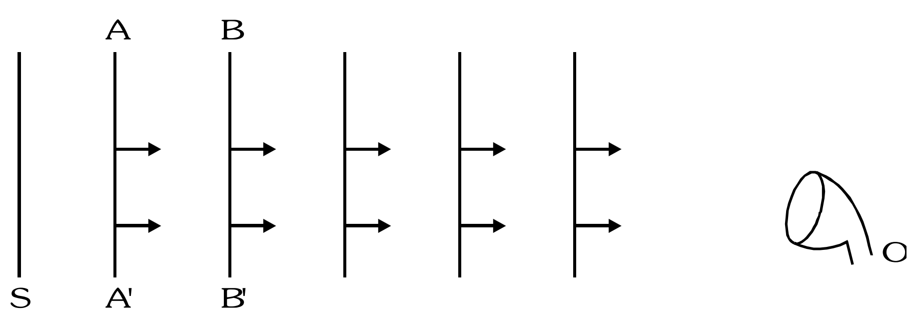
$$\sin 4\pi t = 0 \Rightarrow t = \frac{n}{4} \quad \therefore t = 0,\frac{1}{4},\frac{1}{2},\frac{3}{4}\ldots \text{ (point of maximum intensity)}$$

28. Ans. (D)

At t=0, $I_{max} \propto (1+2)^2A^2 = 9A^2 \ \therefore I_{max} = 9I_0$

Example#29

Consider a large plane diaphragm 'S' emitting sound and a detector 'O'. The diagram shows plane wavefronts for the sound wave travelling in air towards right when source, observer and medium are at rest. AA' and BB' are fixed imaginary planes. Column-I describes about the motion of source, observer or medium and column-II describes various effects. Match them correctly.



Column I	Column II
(A) Source starts moving towards right	(P) Distance between any two wavefronts will increase.
(B) Air starts moving towards right	(Q) Distance between any two wavefronts will decrease.
(C) Observer and source both move towards left with same speed.	(R) The time needed by sound to move from plane AA' to BB' will increase.
(D) Source and medium (air) both move towards right with same speed.	(S) The time needed by sound to move from plane AA' to BB' will decrease.
	(T) Frequency received by observer increases.

Solution Ans. (A) →(Q,T); (B) →(P,S); (C) →(P); (D) →(S,T)

Velocity of sound in a medium is always given in the reference frame of medium.

Example#30

Column I represents the standing waves in air columns and string. Column II represents frequency of the note. Match the column-I with column-II. [v = velocity of the sound in the medium]

Column -I		Column-II	
(A)	Second harmonic for the tube open at both ends	(P)	$\frac{v}{4\ell}$
(B)	Fundamental frequency for the tube closed at one end	(Q)	$\frac{v}{2\ell}$
(C)	First overtone for the tube closed at one end	(R)	$\frac{3v}{4\ell}$
(D)	Fundamental frequency for the string fixed at both ends	(S)	$\frac{v}{\ell}$
		(T)	$\frac{5v}{4\ell}$

Solution Ans. (A) S (B) P (C) R (D) Q

For (A) : For open organ pipe 2nd harmonic = $2\left(\frac{v}{2\ell}\right)$

For (B) : For closed organ pipe fundamental frequency = $\frac{v}{4\ell}$

For (C) : For closed organ pipe, first overtone frequency = $\frac{3v}{4\ell}$

For (D) : For string fixed at both ends, fundamental frequency = $\frac{v}{2\ell}$

Example#31

Two vibrating tuning forks produce progressive waves given by y_1= 4 sin(500πt) and y_2= 2 sin(506πt). These tuning forks are held near the ear of a person. The person will hear α beats/s with intensity ratio between maxima and minima equal to β. Find the value of β–α.

Solution Ans. 6

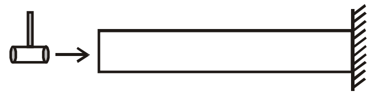
$y_1 = 4\sin(500\ \pi t)$ $\qquad y_2 = 2\ \sin(506\ \pi t)$

Number of beats $= \frac{n_1 - n_2}{2} = \frac{506-500}{2} = 3$ beat/sec.

As $I_1 \propto (16)$ and $I_2 \propto 4 \Rightarrow \frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2} \Rightarrow \left(\frac{4+2}{4-2}\right)^2 = \left(\frac{6}{2}\right)^2 = 9$

Example#32

A 1000 m long rod of density 10.0 10^4 kg/m^3 and having young's modulus Y = 10^{11} Pa, is clamped at one end. It is hammered at the other free end as shown in the figure. The longitudinal pulse goes to right end, gets reflected and again returns to the left end. How much time (in sec) the pulse take to go back to initial point?



Solution **Ans. 2**

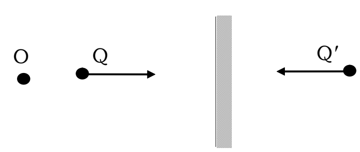
Velocity of longitudinal $\quad u = \sqrt{\dfrac{Y}{\rho}} = \sqrt{\dfrac{10^{11}}{10 \times 10^4}} = 10^3 \text{ ms}^{-1}$

Required time $\quad \dfrac{2\ell}{v} = \dfrac{2 \times 1000}{10^3} = 2 \text{ s}$

Example#33

A tuning fork P of unknown frequency gives 7 beats in 2 seconds with another tuning fork Q. When Q is moved towards a wall with a speed of 5 m/s, it gives 5 beats per second for an observer located left to it. On filing, P gives 6 beats per second with Q. The frequency (in Hz) of P is given by (80 α) + β. (α, $\beta \in I$, $0 \le \alpha, \beta \le 9$) then find the value of $\alpha + \beta$. Assume speed of sound = 332 m/s.

Solution **Ans. 9**



Let f_1 and f_2 be the frequencies of tuning forks P and Q,Then $| f_1 - f_2 | = 7/2$

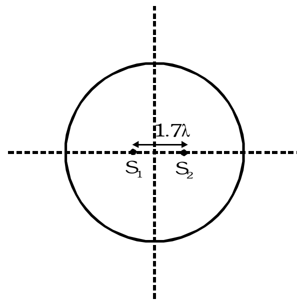
Apparent frequency for O corresponding to signal directly coming from $Q = f_2 \left(\dfrac{v}{v + v_q}\right)$

Apparent frequency of the echo $= f_2 \left(\dfrac{v}{v - v_q}\right) \quad \therefore \Delta f_2 = f_2 \left[\dfrac{2v_q v}{v^2 - v_q^2}\right]$

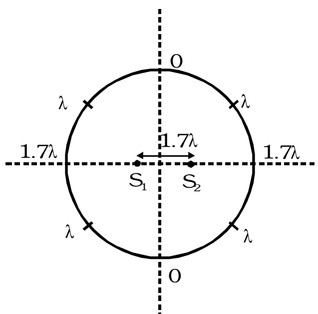
Since, $\Delta f_2 = 5$ (given) $\therefore f_2 = 163.5$ Hz. Now, $f_1 = 163.5 \pm 3.5 = 167$ or 160 Hz, when P is filed, its frequency will increase, since it is given that filed P gives greater number of beats with Q. It implies that f_1 must be 167 Hz.

Example#34

Find the number of maxima attend on circular perimeter as shown in the figure. Assume radius of circle >>>λ.



Solution **Ans. 6**



1 in each quadrant, 1 top point, 1 bottom point