ALTERNATING CURRENT

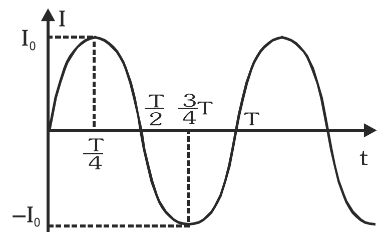
ALTERNATING CURRENT AND VOLTAGE

Voltage or current is said to be alternating if it is change continously in magnitude and perodically in direction with time. It can be represented by a sine curve or cosine curve

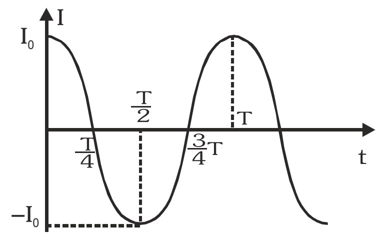
$I = I_0 \sin \omega t$ or $I = I_0 \cos \omega t$

where I = Instantaneous value of current at time t, I_0 = Amplitude or peak value

ω = Angular frequency $\omega = \dfrac{2\pi}{T} = 2\pi f$ T = time period f = frequency



I as a sine function of t



I as a cosine function of t

AMPLITUDE OF AC

The maximum value of current in either direction is called peak value or the amplitude of current. It is represented by I_0. Peak to peak value = $2I_0$

PERIODIC TIME

The time taken by alternating current to complete one cycle of variation is called periodic time or time period of the current.

FREQUENCY

The number of cycle completed by an alternating current in one second is called the frequency of the current.

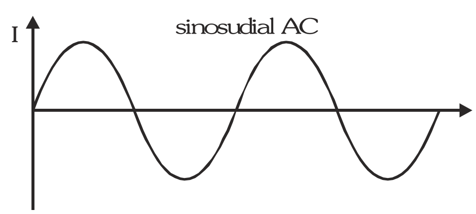
UNIT : cycle/s ; (Hz)

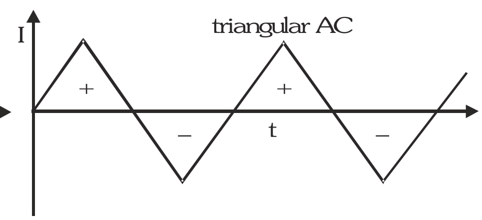
In India : f = 50 Hz , supply voltage = 220 volt **In USA** :f = 60 Hz ,supply voltage = 110 volt

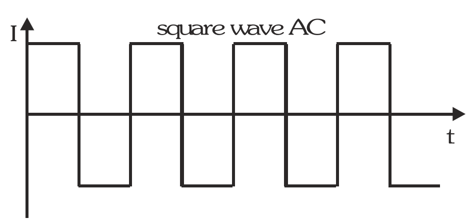
CONDITION REQUIRED FOR CURRENT/ VOLTAGE TO BE ALTERNATING

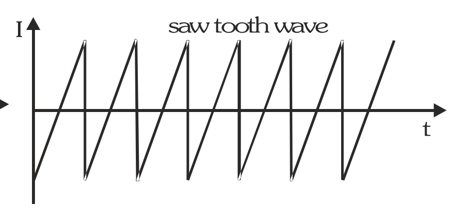
- Amplitude is constant
- Alternate half cycle is positive and half negative

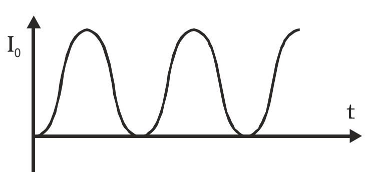
The alternating current continuously varies in magnitude and periodically reverses its direction.



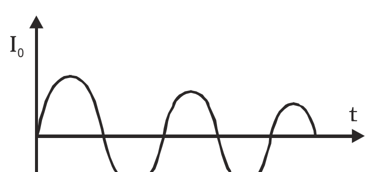




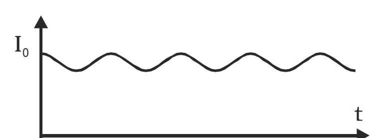




Not AC (direction not change)



Not AC (not periodic)



mixture of AC and DC

AVERAGE VALUE OR MEAN VALUE

The mean value of A.C over any half cycle (either positive or negative) is that value of DC which would send same amount of charge through a circuit as is sent by the AC through same circuit in the same time.

average value of current for half cycle $< I > = \dfrac{\int_0^{T/2} I dt}{\int_0^{T/2} dt}$

Average value of $I = I_0 \sin \omega t$ over the positive half cycle :

$$I_{av} = \frac{\int_0^{\frac{T}{2}} I_0 \sin\omega t\, dt}{\int_0^{\frac{T}{2}} dt} = \frac{2I_0}{\omega T}\left[-\cos\omega t\right]_0^{\frac{T}{2}} = \frac{2I_0}{\pi}$$

$< \sin\theta > = < \sin 2\theta > = 0$
$< \cos\theta > = < \cos 2\theta > = 0$
$< \sin\theta \cos\theta > = 0$
$< \sin^2\theta > = < \cos^2\theta > = \frac{1}{2}$

- For symmetric AC, average value over full cycle = 0,
  Average value of sinusoidal AC

Full cycle	(+ve) half cycle	(–ve) half cycle
0	$\frac{2I_0}{\pi}$	$\frac{-2I_0}{\pi}$

As the average value of AC over a complete cycle is zero, it is always defined over a half cycle which must be either positive or negative

MAXIMUM VALUE

- $I = a \sin\theta \Rightarrow I_{Max.} = a$
- $I = a + b \sin\theta \Rightarrow I_{Max.} = a + b$ (if a and b > 0)
- $I = a \sin\theta + b \cos\theta \Rightarrow I_{Max.} = \sqrt{a^2 + b^2}$
- $I = a \sin^2\theta \Rightarrow I_{Max.} = a$ (a > 0)

ROOT MEAN SQUARE (rms) VALUE

It is value of DC which would produce same heat in given resistance in given time as is done by the alternating current when passed through the same resistance for the same time.

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{\int_0^T dt}}$$ rms value = Virtual value = Apparent value

rms value of $I = I_0 \sin \omega t$:

$$I_{rms} = \sqrt{\frac{\int_0^T (I_0 \sin\omega t)^2 dt}{\int_0^T dt}} = \sqrt{\frac{I_0^2}{T}\int_0^T \sin^2\omega t\, dt} = I_0\sqrt{\frac{1}{T}\int_0^T \left[\frac{1-\cos 2\omega t}{2}\right] dt} = I_0\sqrt{\frac{1}{T}\left[\frac{t}{2} - \frac{\sin 2\omega t}{2\times 2\omega}\right]_0^T} = \frac{I_0}{\sqrt{2}}$$

- If nothing is mentioned then values printed in a.c circuit on electrical appliances, any given or unknown values, reading of AC meters are assumed to be RMS.

Current	Average	Peak	RMS	Angular fequency
$I_1 = I_0 \sin \omega t$	0	I_0	$\frac{I_0}{\sqrt{2}}$	ω
$I_2 = I_0 \sin \omega t \cos \omega t = \frac{I_0}{2}\sin 2\omega t$	0	$\frac{I_0}{2}$	$\frac{I_0}{2\sqrt{2}}$	2ω
$I_3 = I_0 \sin\omega t + I_0 \cos\omega t$	0	$\sqrt{2}\, I_0$	I_0	ω

- For above varieties of current rms $= \dfrac{\text{Peak value}}{\sqrt{2}}$

Example

If $I = 2\sqrt{t}$ ampere then calculate average and rms values over t = 2 to 4 s

Solution

$$<I> = \frac{\int_2^4 2\sqrt{t}.dt}{\int_2^4 dt} = \frac{4}{3}\frac{(t^{\frac{3}{2}})_2^4}{(t)_2^4} = \frac{2}{3}\left[8-2\sqrt{2}\right] \quad \text{and } I_{rms} = \sqrt{\frac{\int_2^4 (2\sqrt{t})^2 dt}{\int_2^4 dt}} = \sqrt{\frac{\int_2^4 4t\,dt}{2}} = \sqrt{2\left[\frac{t^2}{2}\right]_2^4} = 2\sqrt{3}\text{ A}$$

Example

Find the time required for 50Hz alternating current to change its value from zero to rms value.

Solution

$$\because I = I_0 \sin \omega t \therefore \frac{I_0}{\sqrt{2}} = I_0 \sin\omega t \Rightarrow \sin\omega t = \frac{1}{\sqrt{2}} \quad \omega t = \frac{\pi}{4} \Rightarrow \left(\frac{2\pi}{T}\right)t = \frac{\pi}{4} \quad \Rightarrow t = \frac{T}{8} = \frac{1}{8\times 50} = 2.5\text{ ms}$$

Example

If E = 20 sin (100π t) volt then calculate value of E at $t = \frac{1}{600}$ s

Solution

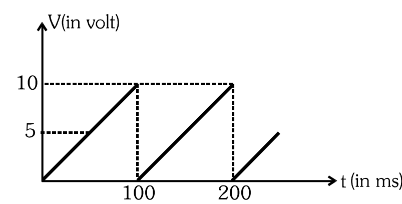
$$\text{At } t = \frac{1}{600}\text{ s} \quad E = 20 \text{ Sin}\left[100\pi\times\frac{1}{600}\right] = 20 \sin\left[\frac{\pi}{6}\right] = 20 \quad \frac{1}{2} = 10\text{V}$$

Example

A periodic voltage wave form has been shown in fig.

Determine. (a) Frequency of the wave form.

(b) Average value.



Solution

(a) After 100 ms wave is repeated so time period is

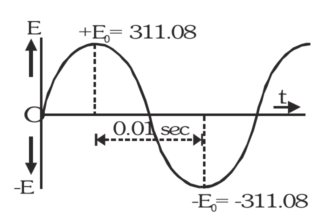
$T = 100$ ms. $\Rightarrow f = \frac{1}{T} = 10$ Hz

(b) Average value = Area/time period $= \frac{(1/2)\times 100\times 10}{(100)} = 5$ volt

Example

Explain why A.C. is more dangerous than D.C. ?

Solution



There are two reasons for it :

- A.C. attracts while D.C. repels.
- A.C. gives a huge and sudden shock.

for 220 V ac $V_{rms} = 220$ V

Hence, $V_0 = \sqrt{2}.V_{rms} = 12.414 \quad 220 = 311.08$ V

Voltage change from $+V_0$ (positive peak) to $-V_0$ (negative peak) = 311.08 – (–311.08) = 622.16 V

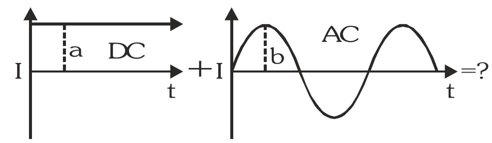
This change takes place in half cycle i.e., in $\frac{1}{100}$ s (for a 50 Hz A.C.)

A shock of 622.16 within 0.01 s is huge and sudden, hence fatal.

Example

If a direct current of value a ampere is superimposed on an alternating current I = b sinωt flowing through a wire, what is the

effective value of the resulting current in the circuit ?



Solution

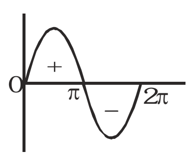
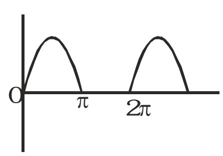
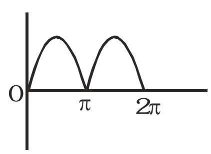
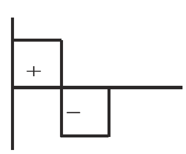
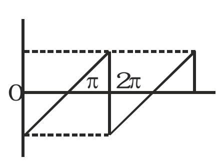
As current at any instant in the circuit will be,

$$I = I_{DC} + I_{AC} = a + b\sin\omega t$$

$$\therefore \quad I_{eff} = \sqrt{\frac{1}{T}\int_0^T I^2 dt} = \sqrt{\frac{1}{T}\int_0^T (a + b\sin\omega t)^2 dt} = \sqrt{\frac{1}{T}\int_0^T (a^2 + 2ab\sin\omega t + b^2\sin^2\omega t)dt}$$

but as $\frac{1}{T}\int_0^T \sin\omega t dt = 0$ and $\frac{1}{T}\int_0^T \sin^2\omega t dt = \frac{1}{2}$ $\quad \therefore \quad I_{eff} = \sqrt{a^2 + \frac{1}{2}b^2}$

SOME IMPORTANT WAVE FORMS AND THEIR RMS AND AVERAGE VALUE

Nature of wave form	Wave–form	RMS Value	Average or mean Value
Sinusoidal		$\frac{I_0}{\sqrt{2}}$ = 0.707 I_0	$\frac{2I_0}{\pi}$ = 0.637 I_0
Half wave rectifired		$\frac{I_0}{2}$ = 0.5 I_0	$\frac{I_0}{\pi}$ = 0.318 I_0
Full wave rectifired		$\frac{I_0}{\sqrt{2}}$ = 0.707 I_0	$\frac{2I_0}{\pi}$ 0.637 I_0
Square or Rectangular		I_0	I_0
Saw Tooth wave		$\frac{I_0}{\sqrt{3}}$	$\frac{I_0}{2}$

MEASUREMENT OF A.C.

Alternating current and voltages are measured by a.c. ammeter and a.c. voltmeter respectively. Working of these instruments is based on heating effect of current, hence they are also called hot wire instruments.

Terms	D.C. meter	A.C. meter
Name	moving coil	hot wire
Based on	magnetic effect of current	heating effect of current
Reads	average value	r.m.s. value
If used in	A.C. circuit then they reads zero $\because$ average value of A.C. = zero	A.C. or D.C. then meter works properly as it measures rms value
Deflection	deflection $\propto$ current $\phi \propto I$ (linear)	deflection $\propto$ heat $\phi \propto I^2_{rms}$ (non linear)
Scale	Uniform Seperation	Non uniform sepration
ϕ = Number of divisions	I - 1 2 3 4 5 ϕ - 1 2 3 4 5	I - 1 2 3 4 5 ϕ - 1 4 9 16 25

- D.C meter in AC circuit reads zero because < AC > = 0 (for complete cycle)
- AC meter works in both AC and DC

PHASE AND PHASE DIFFERENCE

(a) Phase

$I = I_0 \sin(\omega t \pm \phi)$

Initial phase = ϕ (it does not change with time)

Instantaneous phase = $\omega t \pm \phi$ (it changes with time)

- Phase decides both value and sign. ◎ UNIT: radian

(b) Phase difference

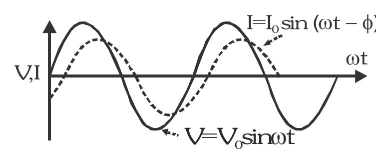
Voltage $V = V_0 \sin(\omega t + \phi_1)$ Current $I = I_0 \sin(\omega t + \phi_2)$

- Phase difference of I w.r.t. V $\phi = \phi_2 - \phi_1$
- Phase difference of V w.r.t. I $\phi = \phi_1 - \phi_2$

LAGGING AND LEADING CONCEPT

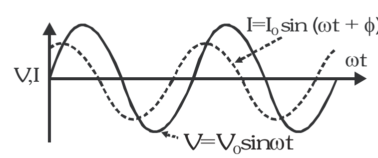
(a) V leads I or I lags V $\rightarrow$ It means, V reach maximum before I

Let if $V = V_0 \sin \omega t$ then $I = I_0 \sin(\omega t - \phi)$
and if $V = V_0 \sin(\omega t + \phi)$ then $I = I_0 \sin \omega t$



(b) V lags I or I leads V $\rightarrow$ It means V reach maximum after I

Let if $V = V_0 \sin \omega t$ then $I = I_0 \sin(\omega t + \phi)$
and if $V = V_0 \sin(\omega t - \phi)$ then $I = I_0 \sin \omega t$



PHASOR AND PHASOR DIAGRAM

A diagram representing alternating current and voltage (of same frequency) as vectors (phasor) with the phase angle between them is called phasor diagram.

Let $V = V_0 \sin \omega t$ and $I = I_0 \sin(\omega t + \phi)$

In figure (a) two arrows represents phasors. The length of phasors represents the maximum value of quantity. The projection of a phasor on y-axis represents the instantaneous value of quantity

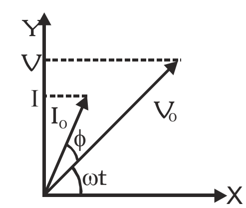


fig (a)

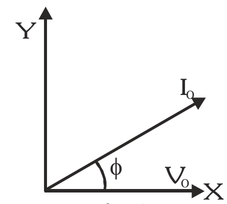


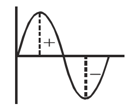
fig (b)

ADVANTAGES OF AC

- A.C. is cheaper than D.C
- It can be easily converted into D.C. (by rectifier)
- It can be controlled easily (choke coil)
- It can be transmitted over long distance at negligible power loss.
- It can be stepped up or stepped down with the help of transformer.

GOLDEN KEY POINTS

- AC can't be used in
  (a) Charging of battery or capacitor (as its average value = 0)
  (b) Electrolysis and electroplating (Due to large inertia, ions can not follow frequency of A.C)
- The rate of change of A.C. →
  → Minimum, at that instant when they are near their peak values
  → Maximum, at that instant when they change their direction.
- For alternating current $I_0 > I_{rms} > I_{av.}$
- Average value over half cycle is zero if one quarter is positive and the other quarter is negative.



- Average value of symmetrical AC for a cycle is zero thats why average potential difference on any element in A.C circuit is zero.
- The instrument based on heating effect of current are works on both A.C and D.C supply and also provides same heating for same value of A.C (rms) and D.C. that's why a bulb bright equally in D.C. and A.C. of same value.
- If the frequency of AC is f then it becomes zero 2f times in one second and the direction of current changes 2f times in one second. Also it become maximum 2f times in one second.

Example

The Equation of current in AC circuit is $I = 4\sin\left[100\pi t + \frac{\pi}{3}\right]$ A. Calculate.

(i) RMS Value (ii) Peak Value (iii) Frequency (iv) Initial phase (v) Current at t = 0

Solution

(i) $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$ A (ii) Peak value I_0 = 4A

(iii) $\because$ $\omega = 100\ \pi$ rad/s $\therefore$ frequency $f = \frac{100\pi}{2\pi} = 50$ Hz

(iv) Initial phase $= \frac{\pi}{3}$ (v) At t = 0, $I = 4\sin\left[100\pi \times 0 + \frac{\pi}{3}\right] = 4\ \frac{\sqrt{3}}{2} = 2\sqrt{3}$ A

Example

If $I = I_0 \sin \omega t$, $E = E_0 \cos\left[\omega t + \frac{\pi}{3}\right]$. Calculate phase difference between E and I

Solution

$I = I_0 \sin \omega t$ and $E = E_0 \sin\left[\frac{\pi}{2} + \omega t + \frac{\pi}{3}\right]$ $\therefore$ phase difference $= \frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$

Example

If E = 500 sin (100 πt) volt then calculate time taken to reach from zero to maximum.

Solution

$\because \omega = 100\ \pi \Rightarrow T = \frac{2\pi}{100\pi} = \frac{1}{50}$ s, time taken to reach from zero to maximum $= \frac{T}{4} = \frac{1}{200}$ s

Example

If Phase Difference between E and I is $\frac{\pi}{4}$ and f = 50 Hz then calculate time difference.

Solution

$\because$ $2\pi \equiv T \therefore \frac{\text{Phase difference}}{2\pi} = \frac{\text{time difference}}{T} \Rightarrow$ Time difference $= \frac{T}{2\pi}\ \frac{\pi}{4} = \frac{T}{8} = \frac{1}{50 \times 8} = 2.5$ms

Example

Show that average heat produced during a cycle of AC is same as produced by DC with $I = I_{rms}$.

Solution

For AC, $I = I_0 \sin\omega t$, the instantaneous value of heat produced (per second) in a resistance R is,

$H = I^2R = I_0^2\sin^2\omega t \quad R$ the average value of heat produced during a cycle is :

$$H_{av} = \frac{\int_0^T H\,dt}{\int_0^T dt} = \frac{\int_0^T (I_0^2 \sin^2 \omega t \times R)dt}{\int_0^T dt} = \frac{1}{2}I_0^2 R \quad \left[\because \int_0^T I_0^2 \sin^2 \omega t\,dt = \frac{1}{2}I_0^2 T\right] \Rightarrow H_{av} = \left(\frac{I_0}{\sqrt{2}}\right)^2 R = I_{rms}^2 R \text{ .....(i)}$$

However, in case of DC, $H_{DC} = I^2 R$...(ii) $\because I = I_{rms}$ so from equation (i) and (ii) $H_{DC} = H_{av}$

AC produces same heating effects as DC of value $I = I_{rms}$. This is also why AC instruments which are based on heating effect of current give rms value.

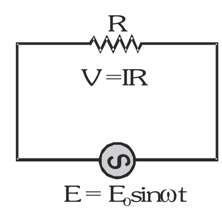
DIFFERENT TYPES OF AC CIRCUITS

In order to study the behaviour of A.C. circuits we classify them into two categories :

(a) Simple circuits containing only one basic element i.e. resistor (R) or inductor (L) or capacitor (C) only.

(b) Complicated circuit containing any two of the three circuit elements R, L and C or all of three elements.

AC CIRCUIT CONTAINING PURE RESISTANCE



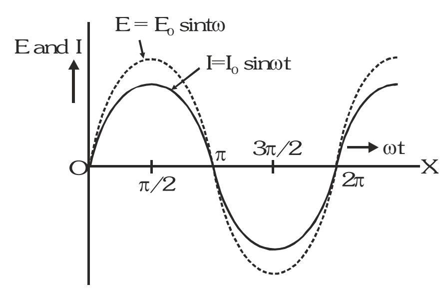
Let at any instant t the current in the circuit = I.

Potential difference across the resistance = I R.

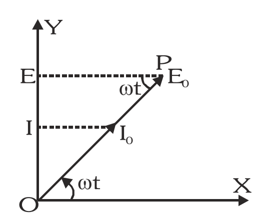
with the help of kirchoff's circuital law E – I R = 0

$$\Rightarrow E_0 \sin \omega t = I R \Rightarrow I = \frac{E_0}{R}\sin\omega t = I_0 \sin\omega t \left(I_0 = \frac{E_0}{R}\right.$$

= peak or maximum value of current)



Alternating current developed in a pure resistance is also of sinusoidal nature. In an a.c. circuits containing pure resistance, the voltage and current are in the same phase. The vector or phasor diagram which represents the phase relationship between alternating current and alternating e.m.f. as shown in figure.

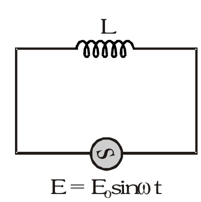


In the a.c. circuit having R only, as current and voltage are in the same phase, hence in fig. both phasors E_0 and I_0 are in the same direction, making an angle ωt with OX. Their projections on Y-axis represent the instantaneous values of alternating current and voltage.

i.e. $I = I_0 \sin\omega t$ and $E = E_0 \sin\omega t$.

Since $I_0 = \frac{E_0}{R}$, hence $\frac{I_0}{\sqrt{2}} = \frac{E_0}{R\sqrt{2}} \Rightarrow I_{rms} = \frac{E_{rms}}{R}$

AC CIRCUIT CONTAINING PURE INDUCTANCE



A circuit containing a pure inductance L (having zero ohmic resistance) connected with a source of alternating emf. Let the alternating e.m.f. $E = E_0 \sin \omega t$

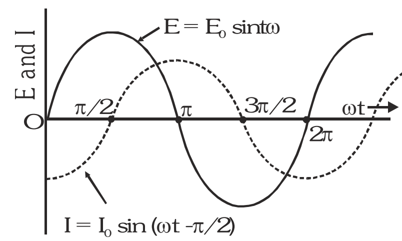
When a.c. flows through the circuit, emf induced across inductance $= -L\frac{dI}{dt}$

Negative sign indicates that induced emf acts in opposite direction to that of applied emf.

Because there is no other circuit element present in the circuit other then inductance so with the help of

Kirchoff's circuital law $E+\left(-L\frac{dI}{dt}\right)=0 \Rightarrow E=L\frac{dI}{dt}$ so we get $I=\frac{E_0}{\omega L}\sin\left(\omega t-\frac{\pi}{2}\right)$

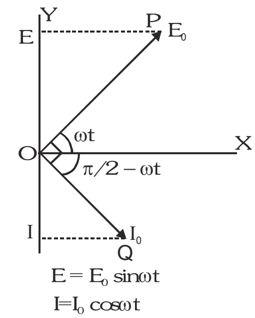
Maximum current $I_0=\frac{E_0}{\omega L}\times 1=\frac{E_0}{\omega L}$, Hence, $I=I_0\sin\left(\omega t-\frac{\pi}{2}\right)$



In a pure inductive circuit current always lags behind the emf by $\frac{\pi}{2}$.

or alternating emf leads the a. c. by a phase angle of $\frac{\pi}{2}$.

Expression $I_0=\frac{E_0}{\omega L}$ resembles the expression $\frac{E}{I}=R$.



This non-resistive opposition to the flow of A.C. in a circuit is called the inductive reactance (X_L) of the circiut.

$X_L=\omega L=2\pi f L$ where f = frequency of A.C.

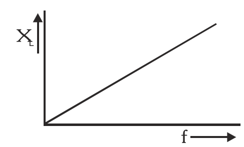
Unit of X_L : ohm

(ωL) = Unit of L Unit of ω = henry sec^{-1}

$$=\frac{\text{Volt}}{\text{Ampere / sec}}\times sec^{-1}=\frac{\text{Volt}}{\text{Ampere}}=\text{ohm}$$

Inductive reactance $X_L \propto f$

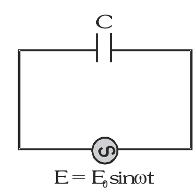
Higher the frequency of A.C. higher is the inductive reactance offered by an inductor in an A.C. circuit.



For d.c. circuit, f = 0 $\therefore X_L=\omega L=2\pi f L=0$

Hence, inductor offers no opposition to the flow of d.c. whereas a resistive path to a.c.

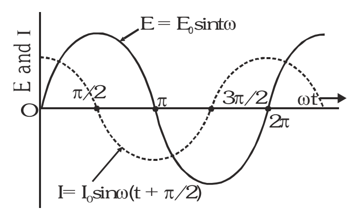
AC CIRCUIT CONTAINING PURE CAPACITANCE



A circuit containing an ideal capacitor of capacitance C connected with a source of alternating emf as shown in fig. The alternating e.m.f. in the circuit $E=E_0\sin\omega t$

When alternating e.m.f. is applied across the capacitor a similarly varying alternating current flows in the circuit.

The two plates of the capacitor become alternately positively and negatively charged and the magnitude of the charge on the plates of the capacitor varies sinusoidally with time. Also the electric field between the plates of the capacitor varies sinusoidally with time.Let at any instant t charge on the capacitor = q

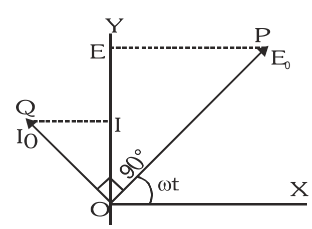


Instantaneous potential difference across the capacitor $E=\frac{q}{C}$

$\Rightarrow q = C\,E \Rightarrow q = CE_0\sin\omega t$

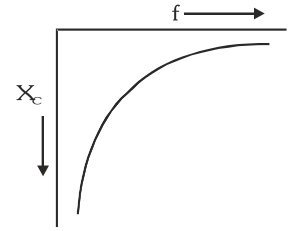
The instantaneous value of current $I=\frac{dq}{dt}=\frac{d}{dt}\left(CE_0\sin\omega t\right)=CE_0\omega\cos\omega t$

$\Rightarrow I=\frac{E_0}{(1/\omega C)}\sin\left(\omega t+\frac{\pi}{2}\right)=I_0\sin\left(\omega t+\frac{\pi}{2}\right)$ where $I_0=\omega CV_0$



In a pure capacitive circuit, the current always leads the e.m.f. by a phase angle of $\pi/2$. The alternating emf lags behinds the alternating current by a phase angle of $\pi/2$.

IMPORTANT POINTS



$\frac{E}{I}$ is the resistance R when both E and I are in phase, in present case they differ in phase by $\frac{\pi}{2}$, hence $\frac{1}{\omega C}$ is not the resistance of the capacitor, the capacitor offer opposition to the flow of A.C. This non-resistive opposition to the flow of A.C. in a pure capacitive circuit is known as capacitive reactance X_C. $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

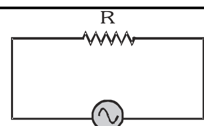
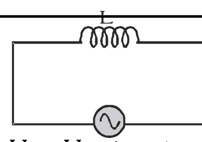
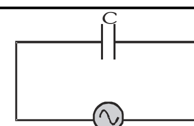
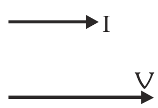
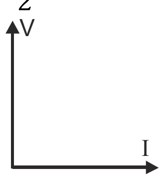
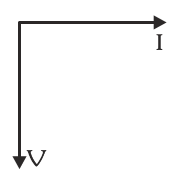
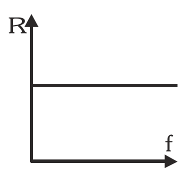
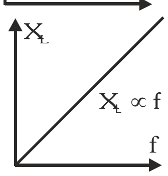
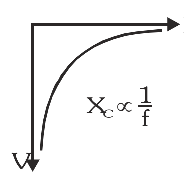
Unit of X_C : ohm

Capacitive reactance X_C is inversely proportional to frequence of A.C. X_C decreases as the frequency increases. This is because with an increase in frequency, the capacitor charges and discharges rapidly following the flow of current.

For d.c. circuit $f = 0$ $\quad \therefore X_C = \frac{1}{2\pi f C} = \infty$ but has a very small value for a.c.

This shows that capacitor blocks the flow of d.c. but provides an easy path for a.c.

INDIVIDUAL COMPONENTS (R or L or C)

TERM	R	L	C
Circuit			
Supply Voltage	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$
Current	$I = I_0 \sin \omega t$	$I = I_0 \sin (\omega t - \frac{\pi}{2})$	$I = I_0 \sin (\omega t + \frac{\pi}{2})$
Peak Current	$I_0 = \frac{V_0}{R}$	$I_0 = \frac{V_0}{\omega L}$	$I_0 = \frac{V_0}{1/\omega C} = V_0 \omega C$
Impedance (Ω)	$\frac{V_0}{I_0} = R$	$\frac{V_0}{I_0} = \omega L = X_L$	$\frac{V_0}{I_0} = \frac{1}{\omega C} = X_C$
$Z = \frac{V_0}{I_0} = \frac{V_{rms}}{I_{rms}}$	R = Resistance	X_L=Inductive reactance.	X_C=Capacitive reactance.
Phase difference	zero (in same phase)	$+\frac{\pi}{2}$ (V leads I)	$-\frac{\pi}{2}$ (V lags I)
Phasor diagram			
Variation of Z with f	 R does not depend on f		
G, S_L, S_C (mho, seiman)	G=1/R = conductance.	$S_L = 1/X_L$ Inductive susceptance	$S_C = 1/X_C$ Capacitive susceptance
Behaviour of device in D.C. and A.C	Same in A C and D C	L passes DC easily (because $X_L = 0$) while gives a high impedance for the A.C. of high frequency ($X_L \propto f$)	C - blocks DC (because $X_C = \infty$) while provides an easy path for the A.C. of high frequency $\left[X_C \propto \frac{1}{f}\right]$
Ohm's law	$V_R = IR$	$V_L = IX_L$	$V_C = IX_C$

GOLDEN KEY POINTS

- Phase diference between capacitive and inductive reactance is π
- Inductor called Low pass filter because it allow to pass low frequency signal.
- Capacitor is called high pass filter because it allow to pass high frequency signal.

Example

What is the inductive reactance of a coil if the current through it is 20 mA and voltage across it is 100 V.

Solution

$$\because \; V_L = IX_L \qquad \therefore X_L = \frac{V_L}{I} = \frac{100}{20\times10^{-3}} = 5 \text{ k}\Omega$$

Example

The reactance of capacitor is 20 ohm. What does it mean?

What will be its reactance if frequency of AC is doubled?

What will be its, reactance when connected in DC circuit? What is its consequence?

Solution

The reactance of capacitor is 20 ohm. It means that the hindrance offered by it to the flow of AC at a specific frequency is equivalent to a resistance of 20 ohm. The reactance of capacitance $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

Therefore by doubling frequency, the reactance is halved i.e., it becomes 10 ohm. In DC circuit f = 0. Therefore reactance of capacitor = ∞ (infinite). Hence the capacitor can not be used to control DC.

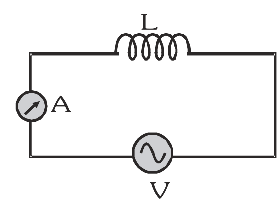
Example

A capacitor of 50 pF is connected to an a.c. source of frequency 1kHz Calculate its reactance.

Solution

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\times10^3\times50\times10^{-12}} = \frac{10^7}{\pi}\ \Omega$$

Example



In given circuit applied voltage $V = 50\sqrt{2}$ sin $100\pi t$ volt and ammeter reading is 2A then calculate value of L

Solution

$V_{rms} = I_{rms} X_L \qquad \because$ Reading of ammeter = I_{rms}

$$X_L = \frac{V_{rms}}{I_{rms}} = \frac{V_0}{\sqrt{2}\ I_{rms}} = \frac{50\sqrt{2}}{\sqrt{2}\times2} = 25\ \Omega \Rightarrow L = \frac{X_L}{\omega} = \frac{25}{100\pi} = \frac{1}{4\pi}\ \text{H}$$

Example

A 50 W, 100 V lamp is to be connected to an AC mains of 200 V, 50 Hz. What capacitance is essential to be put in series with the lamp ?

Solution

$\because$ resistance of the lamp $R = \frac{V_s^2}{W} = \frac{(100)^2}{50}$ = 200 Ω and the maximum curent $I = \frac{V}{R} = \frac{100}{200} = \frac{1}{2}$A

$\therefore$ when the lamp is put in series with a capacitance and run at 200 V AC, from V = IZ

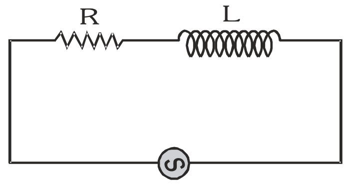
$Z = \frac{V}{I} = \frac{200}{\frac{1}{2}} = 400\Omega$ Now as in case of C–R circuit $Z = \sqrt{R^2 + \frac{1}{(\omega C)^2}}$,

$$\Rightarrow R^2 + \frac{1}{(\omega C)^2} = (400)^2 \Rightarrow \frac{1}{(\omega C)^2} = 16\times10^4 - (200)^2 = 12\times10^4 \Rightarrow \frac{1}{\omega C} = \sqrt{12}\times10^2$$

$$\Rightarrow C = \frac{1}{100\pi\times\sqrt{12}\times10^2}F = \frac{100}{\pi\sqrt{12}}\mu F = 9.2\mu F$$

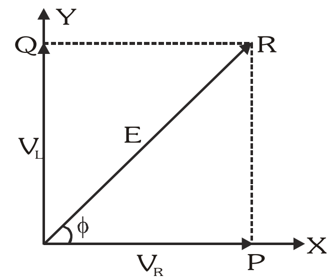
RESISTANCE AND INDUCTANCE IN SERIES (R-L CIRCUIT)

A circuit containing a series combination of a resistance R and an inductance L, connected with a source of alternating e.m.f. E as shown in figure.



$E = E_0 \sin\omega t$

PHASOR DIAGRAM FOR L-R CIRCUIT



Let in a L-R series circuit, applied alternating emf is $E = E_0 \sin\omega t$. As R and L are joined in series, hence current flowing through both will be same at each instant. Let I be the current in the circuit at any instant and V_L and V_R the potential differences across L and R respectively at that instant.

Then $V_L = IX_L$ and $V_R = IR$

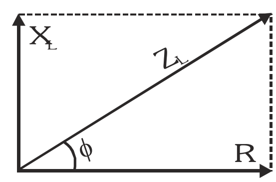
Now, V_R is in phase with the current while V_L leads the current by $\frac{\pi}{2}$.

So V_R and V_L are mutually perpendicular (Note : $E \neq V_R + V_L$)

The vector OP represents V_R (which is in phase with I), while OQ represents V_L (which leads I by 90).

The resultant of V_R and V_L = the magnitude of vector OR $E = \sqrt{V_R^2 + V_L^2}$

Thus $E = V_R + V_L = I (R + X_L) \Rightarrow I = \dfrac{E}{\sqrt{R^2 + X_L^2}}$



The phasor diagram shown in fig. also shows that in L-R circuit the applied emf E leads the current I or conversely the current I lags behind the e.m.f.

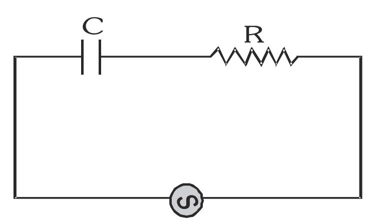
E. by a phase angle ϕ $\tan\phi = \dfrac{V_L}{V_R} = \dfrac{IX_L}{IR} = \dfrac{X_L}{R} = \dfrac{\omega L}{R}$ $\Rightarrow \phi = \tan^{-1}\left(\dfrac{\omega L}{R}\right)$

Inductive Impedance Z_L :

In L-R circuit the maximum value of current $I_0 = \dfrac{E_0}{\sqrt{R^2 + \omega^2 L^2}}$ Here $\sqrt{R^2 + \omega^2 L^2}$ represents the effective opposition offered by L-R circuit to the flow of a.c. through it. It is known as impedance of L-R circuit and is represented by Z_L. $Z_L = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + (2\pi f L)^2}$ The reciprocal of impedance is called admittance

$$Y_L = \frac{1}{Z_L} = \frac{1}{\sqrt{R^2 + \omega^2 L^2}}$$

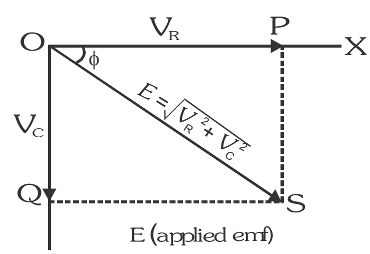
RESISTANCE AND CAPACITOR IN SERIES (R-C CIRCUIT)



$E = E_0 \sin\omega t$

A circuit containing a series combination of a resistance R and a capacitor C, connected with a source of e.m.f. of peak value E_0 as shown in fig.

PHASOR DIAGRAM FOR R-C CIRCUIT



Current through both the resistance and capacitor will be same at every instant and the instantaneous potential differences across C and R are

$V_C = I X_C$ and $V_R = I R$

where X_C = capacitive reactance and I = instantaneous current.

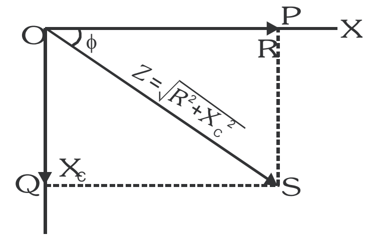
Now, V_R is in phase with I, while V_C lags behind I by 90 .

The phasor diagram is shown in fig.

The vector OP represents V_R (which is in phase with I)

and the vector OQ represents V_C (which lags behind I by $\frac{\pi}{2}$).

The vector OS represents the resultant of V_R and V_C = the applied e.m.f. E.



Hence $\quad V_R^2 + V_C^2 = E^2 \quad \Rightarrow E = \sqrt{V_R^2 + V_C^2}$

$\Rightarrow E = I\,(R + X_C) \Rightarrow I = \dfrac{E}{\sqrt{R^2 + X_C^2}}$

The term $\sqrt{(R^2 + X_C^2)}$ represents the effective resistance of the R-C circuit and called the capacitive impedance Z_C of the circuit.

Hence, in C-R circuit $\quad Z_C = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}$

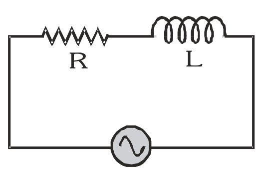
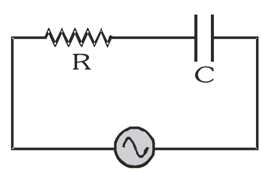
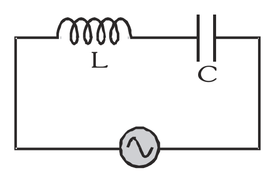
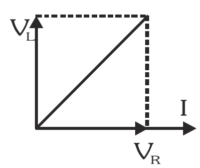
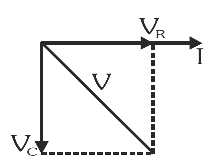
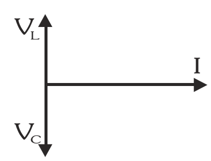
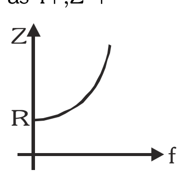
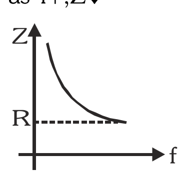
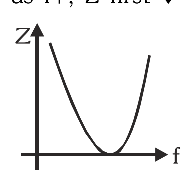
Capacitive Impedance Z_C :

In R-C circuit the term $\sqrt{R^2 + X_C^2}$ effective opposition offered by R-C circuit to the flow of a.c. through it. It is known as impedance of R-C circuit and is represented by Z_C

The phasor diagram also shows that in R-C circuit the applied e.m.f. lags behind the current I (or the current I leads the emf E) by a phase angle ϕ given by

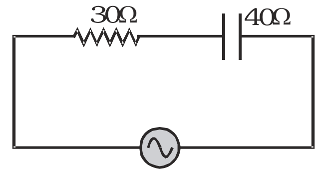
$$\tan\phi = \frac{V_C}{V_R} = \frac{X_C}{R} = \frac{1/\omega C}{R} = \frac{1}{\omega CR}, \quad \tan\ \phi = \frac{X_C}{R} = \frac{1}{\omega CR} \ \Rightarrow\ \phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

COMBINATION OF COMPONENTS (R-L or R-C or L-C)

TERM	R-L	R-C	L-C
Circuit			
	I is same in R & L	I is same in R & C	I is same in L & C
Phasor diagram			
	$V^2 = V_R^2 + V_L^2$	$V^2 = V_R^2 + V_C^2$	$V = V_L - V_C\ (V_L > V_C)$ $V = V_C - V_L\ (V_C > V_L)$
Phase difference	V leads I (ϕ = 0 to $\frac{\pi}{2}$)	V lags I ($\phi = -\frac{\pi}{2}$ to 0)	V lags I ($\phi = -\frac{\pi}{2}$, if $X_C > X_L$)
in between V & I		V leads I ($\phi = +\frac{\pi}{2}$, if $X_L > X_C$)	Impedance $\quad Z = \sqrt{R^2 + X_L^2}$
$Z = \sqrt{R^2 + (X_C)^2}$	$Z = \lvert X_L - X_c \rvert$		
Variation of Z	as f↑, Z ↑	as f↑, Z↓	as f↑, Z first ↓ then↑
with f			
At very low f	$Z \simeq R\ (X_L \to 0)$	$Z \simeq X_C$	$Z \simeq X_C$
At very high f	$Z \simeq X_L$	$Z \simeq R\ (X_C \to 0)$	$Z \simeq X_L$

Example

Calculate the impedance of the circuit shown in the figure.

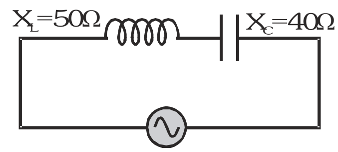


Solution

$$Z = \sqrt{R^2 + (X_c)^2} = \sqrt{(30)^2 + (40)^2} = \sqrt{2500} = 50\ \Omega$$

Example

If $X_L = 50\ \Omega$ and $X_C = 40\ \Omega$ Calculate effective value of current in given circuit.



V=40sin 100π volt

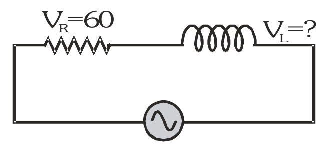
Solution

$$Z = X_L - X_C = 10\ \Omega$$

$$I_0 = \frac{V_0}{Z} = \frac{40}{10} = 4\text{ A} \Rightarrow I_{rms} = \frac{4}{\sqrt{2}} = 2\sqrt{2}\text{ A}$$

Example

In given circuit calculate, voltage across inductor



V=100√2 sinωt volt

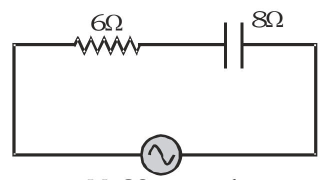
Solution

$$\because V^2 = V_R^2 + V_L^2 \qquad \therefore \qquad V_L^{\,2} = V^2 - V_R^{\,2}$$

$$V_L \quad \sqrt{V^2 - V_R^2} = \sqrt{(100)^2 - (60)^2} = \sqrt{6400} = 80\text{ V}$$

Example

In given circuit find out (i) impedance of circuit (ii) current in circuit



V=20sinωt volt

Solution

(i) $Z = \sqrt{R^2 + X_C^2} = \sqrt{(6)^2 + (8)^2} = 10\ \Omega$

(ii) $V = IZ \Rightarrow I = \frac{V_0}{Z} = \frac{20}{10} = 2\text{A}$ so $I_{rms} = \frac{2}{\sqrt{2}} = \sqrt{2}$ A

Example

When 10V, DC is applied across a coil current through it is 2.5 A, if 10V, 50 Hz A.C. is applied current reduces to 2 A. Calculate reactance of the coil.

Solution

For 10 V D.C. $\because V = IR$ $\therefore$ Resistance of coil $R = \frac{10}{2.5} = 4\Omega$ For 10 V A.C. $\nleftarrow$ $V = I Z \Rightarrow Z = \frac{V}{I} = \frac{20}{10} = 5\Omega$

$$\because Z = \sqrt{R^2 + X_L^2} = 5 \Rightarrow R^2 + X_L^2 = 25 \Rightarrow X_L^2 = 5^2 - 4^2 \Rightarrow X_L = 3\ \Omega$$

Example

When an alternating voltage of 220V is applied across a device X, a current of 0.5 A flows through the circuit and is in phase with the applied voltage. When the same voltage is applied across another device Y, the same current again flows through the circuit but it leads the applied voltage by π/2 radians.

(a) Name the devices X and Y.

(b) Calculate the current flowing in the circuit when same voltage is applied across the series combination of X and Y.

Solution

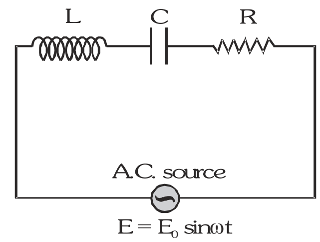
(a) X is resistor and Y is a capacitor

(b) Since the current in the two devices is the same (0.5A at 220 volt)
When R and C are in series across the same voltage then

$$R = X_C = \frac{220}{0.5} = 440\ \Omega \Rightarrow I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + X_C^2}} = \frac{220}{\sqrt{(440)^2 + (440)^2}} = \frac{220}{440\sqrt{2}} = 0.35\text{A}$$

INDUCTANCE, CAPACITANCE AND RESISTANCE IN SERIES

(L-C-R SERIES CIRCUIT)



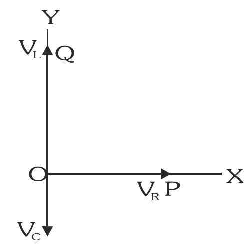
A circuit containing a series combination of an resistance R, a coil of inductance L and a capacitor of capacitance C, connected with a source of alternating e.m.f. of peak value of E_0, as shown in fig.

PHASOR DIAGRAM FOR SERIES L-C-R CIRCUIT

Let in series LCR circuit applied alternating emf is $E = E_0 \sin \omega t$.

As L,C and R are joined in series, therefore, current at any instant through the three elements has the same amplitude and phase.

However voltage across each element bears a different phase relationship with the current.



Let at any instant of time t the current in the circuit is I

Let at this time t the potential differences across L, C, and R

$V_L = I X_L$, $V_C = I X_C$ and $V_R = I R$

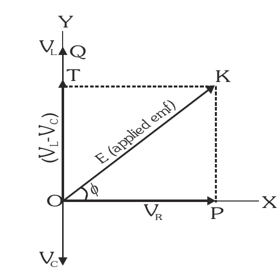
Now, V_R is in phase with current I but V_L leads I by 90

While V_C legs behind I by 90 .

The vector OP represents V_R (which is in phase with I) the vector

OQ represent VL (which leads I by 90)

and the vector OS represents V_C (which legs behind I by 90)

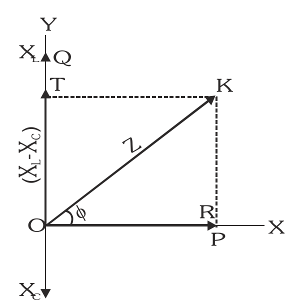


V_L and V_C are opposite to each other.

If $V_L > V_C$ (as shown in figure) the their resultant will be $(V_L - V_C)$ which is represented by OT.

Finally, the vector OK represents the resultant of V_R and $(V_L - V_C)$, that is, the resultant of all the three = applied e.m.f.

Thus $E = \sqrt{V_R^2 + (V_L - V_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2} \Rightarrow I = \dfrac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$

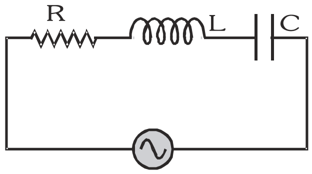
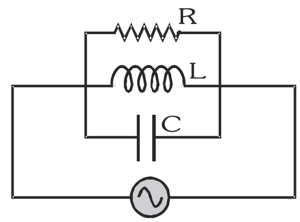
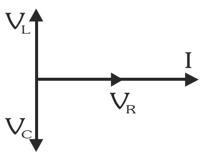
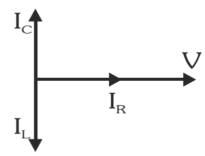
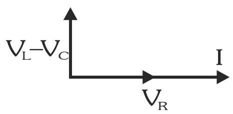
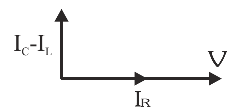
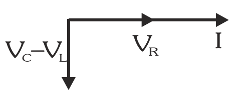
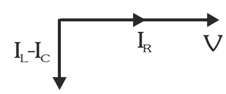
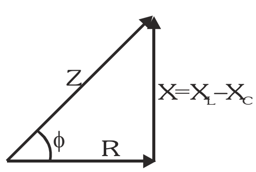
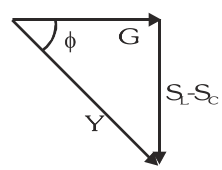


Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$

The phasor diagram also shown that in LCR circuit the applied e.m.f.

leads the current I by a phase angle ϕ $\tan\phi = \dfrac{X_L - X_C}{R}$

SERIES LCR AND PARALLEL LCR COMBINATION

SERIES L-C-R CIRCUIT	PARALLEL L-C-R CIRCUIT
1. Circuit diagram	
	 V same for R,L and C
I same for R, L & C	V same for R, L & C
2. Phasor diagram	
	
(i) If $V_L > V_C$ then	(i) if $I_C > I_L$ then
	
(ii) If $V_C > V_L$ then	(ii) if $I_L > I_C$ then
	
(iii) $V = \sqrt{V_R^2 + (V_L - V_C)^2}$	(iii) $I = \sqrt{I_R^2 + (I_L - I_C)^2}$
Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$	Admittance $Y = \sqrt{G^2 + (S_L - S_C)^2}$
$\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{V_L - V_C}{V_R}$	$\tan\phi = \dfrac{S_L - S_C}{G} = \dfrac{I_L - I_C}{I_R}$
(iv) Impedance triangle	(iv) Admittance triangle
	

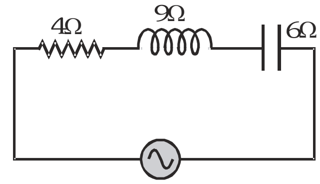
GOLDEN KEY POINTS

- **Series**
  - (a) if $X_L > X_C$ then V leads I,ϕ (positive) circuit nature inductive
  - (b) if $X_C > X_L$ then V lags I,ϕ (negative) circuit nature capacitive

  Parallel
  - (a) if $S_L > S_C$ ($X_L < X_C$) then V leads I,ϕ (positive) circuit nature inductive
  - (b) if $S_C > S_L$ ($X_C < X_L$) then V lags I, ϕ (negative) circuit nature capacitive
- In A.C. circuit voltage for L or C may be greater than source voltage or current but it happens only when circuit contains L and C both and on R it never greater than source voltage or current.
- In parallel A.C.circuit phase difference between I_L and I_C is π

Example

Find out the impedance of given circuit.



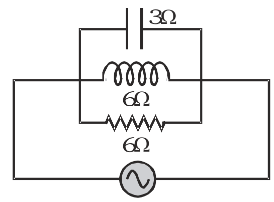
Solution

$$Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{4^2+(9-6)^2}=\sqrt{4^2+3^2}=\sqrt{25}=5\Omega$$

($\because$ $X_L > X_C$ $\therefore$ Inductive)

Example

Find out impedance of given circuit.



Solution

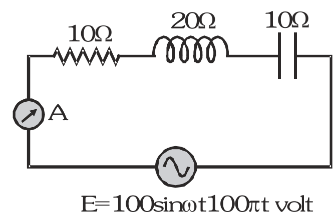
$$Y^2 = G^2 + (S_L - S_C)^2 \; \frac{1}{36}+\left[\frac{1}{6}-\frac{1}{3}\right]^2$$

$$Y = \frac{\sqrt{2}}{6}\Omega \quad \Rightarrow \quad Z = \frac{6}{\sqrt{2}}\ \Omega \quad \text{(capacitive, because } X_L > X_C)$$

Example

Find out reading of A C ammeter and also calculate the potential difference across, resistance and capacitor.

Solution



$$Z=\sqrt{R^2+(X_L-X_C)^2}=10\sqrt{2}\ \Omega \qquad \Rightarrow \qquad I_0=\frac{V_0}{Z}=\frac{100}{10\sqrt{2}}=\frac{10}{\sqrt{2}}A$$

$\because$ ammeter reads RMS value, so its reading = $\dfrac{10}{\sqrt{2}\sqrt{2}}$ = 5A

so V_R = 5 10 = 50 V and V_C = 5 10 = 50 V

Example

In LCR circuit with an AC source R = 300 Ω , C = 20 μF, L = 1.0 H, E_{rms} = 50V and f = 50/π Hz. Find RMS current in the circuit.

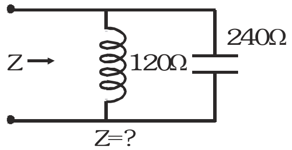
Solution

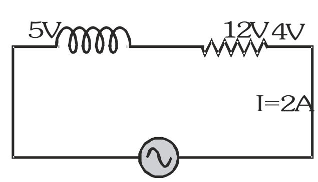
$$I_{rms}=\frac{E_{rms}}{Z}=\frac{E_{rms}}{\sqrt{R^2+\left[\omega L-\frac{1}{\omega C}\right]^2}}=\frac{50}{\sqrt{300^2+\left[2\pi\times\frac{50}{\pi}\times 1-\frac{1}{20\times10^{-6}\times 2\pi\times\frac{50}{\pi}}\right]^2}}$$

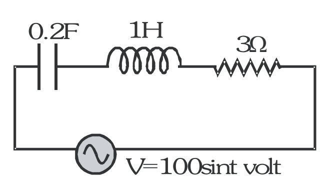
$$\Rightarrow \quad I_{rms}=\frac{50}{\sqrt{(300)^2+\left[100-\frac{10^3}{2}\right]^2}}=\frac{50}{100\sqrt{9+16}}=\frac{1}{10}=0.1A$$

Example

Calculate impedance of the given circuit :

(i)


(ii)


(iii)


Solution

(i) It is parallel circuit so Y is evaluated

$$Y = S_L - S_C = \frac{1}{120} - \frac{1}{240} = +\frac{1}{240} \quad \Rightarrow \quad Z = 240\ \Omega \text{ (inductive)}$$

(ii) $V_s^2 = 5^2 + 12^2 = 169 \quad \Rightarrow \quad V_s = 13$ volt therefore $Z = \frac{V_s}{I} = \frac{13}{2} = 6.5\Omega$

(iii) $R = 3\Omega$, $X_L = \omega L = 1$ as $(\omega = 1)$

$$X_C = \frac{1}{\omega C} = \frac{1}{(0.2).1} = 5\Omega \qquad \text{so} \qquad Z^2 = R^2 + (X_L - X_C)^2 = 3^2 + (1-5)^2 = 25 \Rightarrow Z = 5\ \Omega$$

RESONANCE

A circuit is said to be resonant when the natural frequency of circuit is equal to frequency of the applied voltage. For resonance both L and C must be present in circuit.

There are two types of resonance : (i) Series Resonance (ii) Parallel Resonance

SERIES RESONANCE

(a) At Resonance

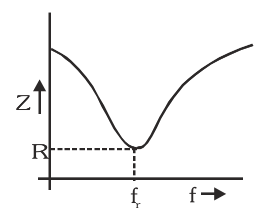
(i) $X_L = X_C$ (ii) $V_L = V_C$ (iii) $\phi = 0$ (V and I in same phase)

(iv) $Z_{min} = R$ (impedance minimum) (v) $I_{max} = \frac{V}{R}$ (current maximum)

(b) Resonance frequency

$$\because \quad X_L = X_C \Rightarrow \omega_r L = \frac{1}{\omega_r C} \Rightarrow \omega_r^2 = \frac{1}{LC} \Rightarrow \omega_r = \frac{1}{\sqrt{LC}} \Rightarrow f_r = \frac{1}{2\pi\sqrt{LC}}$$

(c) Variation of Z with f



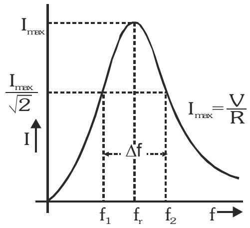
(i) If $f < f_r$ then $X_L < X_C$ circuit nature capacitive, ϕ (negative)

(ii) At $f = f_r$ then $X_L = X_C$ circuit nature, Resistive, ϕ = zero

(iii) If $f > f_r$ then $X_L > X_C$ circuit nature is inductive, ϕ (positive)

Variation of I with f as f increase, Z first decreases then increase

(d)



as f increase, I first increase then decreases

- At resonance impedance of the series resonant circuit is minimum so it is called 'acceptor circuit' as it most readily accepts that current out of many currents whose frequency is equal to its natural frequency. In radio or TV tuning we receive the desired station by making the frequency of the circuit equal to that of the desired station.

Half power frequencies

The frequencies at which, power become half of its maximum value called half power frequencies

Band width $= \Delta f = f_2 - f_1$

Quality factor Q : Q-factor of AC circuit basically gives an idea about stored energy & lost energy.

$$Q = 2\pi \frac{\text{maximum energy stored per cycle}}{\text{maximum energy loss per cycle}}$$

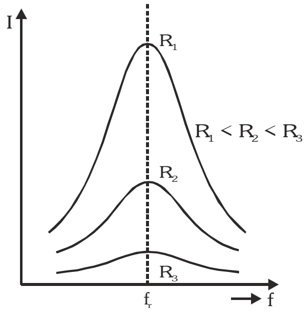
(i) It represents the sharpness of resonance. (ii) It is unit less and dimension less quantity

(iii) $Q = \frac{(X_L)_r}{R} = \frac{(X_C)_r}{R} = \frac{2\pi f_r L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{f_r}{\Delta f} = \frac{f_r}{\text{band width}}$

Magnification

At resonance V_L or V_C = QE (where E = supplied voltage)

So at resonance Magnification factor = Q-factor



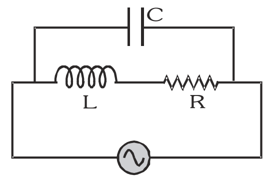
Sharpness

Sharpness ∝ Quality factor ∝ Magnification factor

R decrease ⇒ Q increases ⇒ Sharpness increases

PARALLEL RESONANCE

(a) At resonance

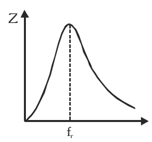


(i) $S_L = S_C$ (ii) $I_L = I_C$ (iii) $\phi = 0$

(iv) $Z_{max} = R$ (impedance maximum)

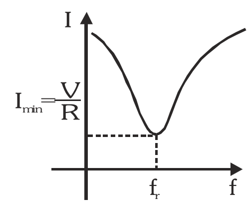
(v) $I_{min} = \dfrac{V}{R}$ (current minimum)

(b) Resonant frequency $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

(c) 

Variation of Z with f as f increases , Z first increases then decreases

- If $f < f_r$ then $S_L > S_C$, ϕ (positive), circuit nature is inductive
- If $f > f_r$ then $S_C > S_L$, ϕ (negative), circuit nature capacitive.



(d) Variation of I with f as f increases , I first decreases then increases

Note : For this circuit $f_r = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{L^2}} \Rightarrow Z_{max} = \dfrac{L}{RC}$ For resonance $\dfrac{1}{LC} > \dfrac{R^2}{L^2}$

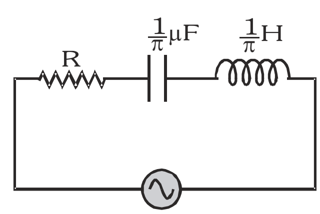
GOLDEN KEY POINTS

- Series resonance circuit gives voltage amplificaltion while parallel resonance circuit gives current amplification.
- At resonance current does not depend on L and C, it depends only on R and V.
- At half power frequencies : net reactance = net resistance.
- As R increases , bandwidth increases
- To obtain resonance in a circuit following parameter can be altered :

  (i) L (ii) C (iii) frequency of source.
- Two series LCR circuit of same resonance frequency f are joined in series then resonance frequency of series combination is also f
- The series resonance circuit called acceptor whereas parallel resonance circuit called rejector circuit.
- Unit of $\sqrt{LC}$ is second

Example

For what frequency the voltage across the resistance R will be maximum.



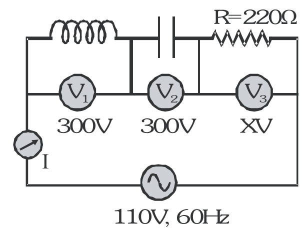
Solution

It happens at resonance

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\frac{1}{\pi}\times10^{-6}\times\frac{1}{\pi}}} = 500 \text{ Hz}$$

Example

A capacitor, a resistor and a 40 mH inductor are connected in series to an AC source of frequency 60Hz, calculate the capacitance of the capacitor, if the current is in phase with the voltage. Also calculate the value of X and I.



Solution

At resonance

$$\omega L = \frac{1}{\omega C},\quad C = \frac{1}{\omega^2 L} = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 \times (60)^2 \times 40 \times 10^{-3}} = 176\mu F$$

$$V = V_R \quad \Rightarrow \quad X = 110\ V \quad \text{and} \quad I = \frac{V}{R} = \frac{110}{220} = 0.5\ A$$

Example

A coil, a capacitor and an A.C. source of rms voltage 24 V are connected in series, By varying the frequency of the source, a maximum rms current 6 A is observed, If this coil is connected to a bettery of emf 12 V, and internal resistance 4Ω , then calculate the current through the coil.

Solution

At resonance current is maximum. $I = \frac{V}{R} \Rightarrow$ Resistance of coil $R = \frac{V}{I} = \frac{24}{6} = 4\ \Omega$

When coil is connected to battery, suppose I current flow through it then $I = \frac{E}{R+r} = \frac{12}{4+4} = 1.5\ A$

Example

Radio receiver recives a message at 300m band, If the available inductance is 1 mH, then calculate required capacitance

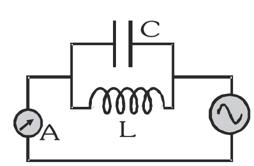
Solution

Radio recives EM waves. (velocity of EM waves c = 3 x 10^8 m/s)

$$\therefore \quad c = f\lambda \Rightarrow f = \frac{3\times 10^8}{300} = 10^6\ Hz \qquad \text{Now } f = \frac{1}{2\pi\sqrt{LC}} = 1 \quad 10^6 \Rightarrow C = \frac{1}{4\pi^2 L \times 10^{12}} = 25\ pF$$

Example

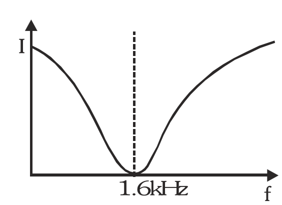
In a L–C circuit parallel combination of inductance of 0.01 H and a capacitor of 1 μF is connected to a variable frequency alternating current source as shown in figure. Draw a rough sketch of the current variation as the frequency is changed from 1kHz to 3kHz.



Solution

L and C are connected in parallel to the AC source,

so resonance frequency $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01\times 10^{-6}}} = \frac{10^4}{2\pi} \simeq 1.6\text{kHz}$



In case of parallel resonance, current in L–C circuit at resonance is zero, so the I-f curve will be as shown in figure.

POWER IN AC CIRCUIT

The average power dissipation in LCR AC circuit

Let $V = V_0 \sin\omega t$ and $I = I_0 \sin(\omega t - \phi)$

Instantaneous power $P = (V_0 \sin\omega t)(I_0 \sin(\omega t - \phi)) = V_0 I_0 \sin\omega t (\sin\omega t\cos\phi - \sin\phi\cos\omega t)$

Average power $<P> = \frac{1}{T}\int_0^T (V_0 I_0 \sin^2\omega t\cos\phi - V_0 I_0 \sin\omega t\cos\omega t\sin\phi)dt$

$$= V_0 I_0\left[\frac{1}{T}\int_0^T \sin^2\omega t\cos\phi dt - \frac{1}{T}\int_0^T \sin\omega t\cos\omega t\sin\phi dt\right] = V_0 I_0\left[\frac{1}{2}\cos\phi - 0\times\sin\phi\right]$$

$$\Rightarrow \quad <P> = \frac{V_0 I_0\cos\phi}{2} = V_{rms}\, I_{rm,s}\, \cos\phi$$

Instantaneous power	Average power/actual power/ dissipated power/power loss	Virtual power/ apparent Power/rms Power	Peak power
$P = VI$	$P = V_{rms}\, I_{rms} \cos\phi$	$P = V_{rms}\, I_{rms}$	$P = V_0\, I_0$

- $I_{rms} \cos\phi$ is known as active part of current or wattfull current, workfull current. It is in phase with voltage.
- $I_{rms} \sin\phi$ is known as inactive part of current, wattless current, workless current. It is in quadrature (90°) with voltage.

Power factor :

Average power $\overline{P} = E_{rms} I_{rms}\cos\phi = \text{r m s power}\times\cos\phi$

Power factor $(\cos\phi) = \frac{\text{Average power}}{\text{r m s Power}}$ and $\cos\phi = \frac{R}{Z}$

Power factor : (i) is leading if I leads V (ii) is lagging if I lags V

GOLDEN KEY POINTS

- $P_{av} \le P_{rms}$.
- Power factor varies from 0 to 1
-

Pure/Ideal	ϕ	V	Power factor = $\cos\phi$	Average power
R	0	V, I same Phase	1 (maximum)	$V_{rms}.\ I_{rms}$
L	$+\frac{\pi}{2}$	V leads I	0	0
C	$-\frac{\pi}{2}$	V lags I	0	0
Choke coil	$+\frac{\pi}{2}$	V leads I	0	0

- At resonance power factor is maximum ($\phi = 0$ so $\cos\phi = 1$) and $P_{av} = V_{rms}\ I_{rms}$

Example

A voltage of 10 V and frequency 10^3 Hz is applied to $\frac{1}{\pi}\mu F$ capacitor in series with a resistor of 500Ω. Find the power factor of the circuit and the power dissipated.

Solution

$$\because \quad X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi\times 10^3\times\frac{10^{-6}}{\pi}} = 500\Omega \quad \therefore Z = \sqrt{R^2 + X_C^2} = \sqrt{(500)^2 + (500)^2} = 500\sqrt{2}\,\Omega$$

Power factor $\cos\phi = \frac{R}{Z} = \frac{500}{500\sqrt{2}} = \frac{1}{\sqrt{2}}$, Power dissipated $= V_{rms}\, I_{rms}\cos\phi = \frac{V_{rms}^2}{Z}\cos\phi = \frac{(10)^2}{500\sqrt{2}}\times\frac{1}{\sqrt{2}} = \frac{1}{10}$ W

Example

If V = 100 sin 100 t volt and I = 100 sin (100 t + $\frac{\pi}{3}$) mA for an A.C. circuit then find out

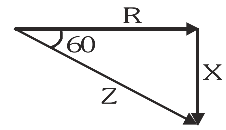
(a) phase difference between V and I (b) total impedance, reactance, resistance
(c) power factor and power dissipated (d) components contains by circuits

Solution

(a) Phase difference $\phi = -\frac{\pi}{3}$ (I leads V)

(b) Total impedance $Z = \frac{V_0}{I_0} = \frac{100}{100 \times 10^{-3}} = 1k\Omega$ Now resistance $R = Z\cos 60° = 1000 \times \frac{1}{2} = 500\Omega$

reactance $X = Z\sin 60° = 1000 \times \frac{\sqrt{3}}{2} = \frac{500}{\sqrt{3}}\Omega$



(c) ϕ = – 60 $\Rightarrow$ Power factor = $\cos\phi$ = cos (–60) = 0.5 (leading)

Power dissipated $P = V_{rms} I_{rms} \cos\phi = \frac{100}{\sqrt{2}} \times \frac{0.1}{\sqrt{2}} \times \frac{1}{2} = 2.5$ W

(d) Circuit must contains R as $\phi \neq \frac{\pi}{2}$ and as ϕ is negative so C must be their, (L may exist but $X_C > X_L$)

Example

If power factor of a R-L series circuit is $\frac{1}{2}$ when applied voltage is V = 100 sin 100πt volt and resistance of circuit is 200Ω then calculate the inductance of the circuit.

Solution

$\cos\phi = \frac{R}{Z} \Rightarrow \frac{1}{2} = \frac{R}{Z} \Rightarrow Z = 2R \Rightarrow \sqrt{R^2 + X_L^2} = 2R \Rightarrow X_L = \sqrt{3}\ R$

$\omega L = \sqrt{3}\ R \Rightarrow L = \frac{\sqrt{3}R}{\omega} = \frac{\sqrt{3} \times 200}{100\pi} = \frac{2\sqrt{3}}{\pi}H$

Example

A circuit consisting of an inductance and a resistacne joined to a 200 volt supply (A.C.). It draws a current of 10 ampere. If the power used in the circuit is 1500 watt. Calculate the wattless current.

Solution

Apparent power = 200   10 = 2000 W

$\therefore$ Power factor $\cos\phi = \frac{\text{True power}}{\text{Apparent power}} = \frac{1500}{2000} = \frac{3}{4}$

Wattless current = $I_{rms} \sin\phi = 10\sqrt{1 - \left(\frac{3}{4}\right)^2} = \frac{10\sqrt{7}}{4}A$

Example

A coil has a power factor of 0.866 at 60 Hz. What will be power factor at 180 Hz.

Solution

Given that cos φ = 0.866, ω = 2πf = 2π   60 = 120π rad/s, ω' = 2πf' = 2π   180 = 360π rad/s

Now, cos φ = R/Z $\Rightarrow$ R = Z cos φ = 0.866 Z

But $Z = \sqrt{R^2 + (\omega L)^2} \Rightarrow \omega L = \sqrt{Z^2 - R^2} = \sqrt{Z^2 - (0.866\ \ Z)^2} = 0.5\ Z \therefore L = \frac{0.5Z}{\omega} = \frac{0.5Z}{120\pi}$

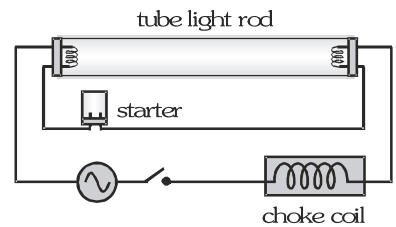
When the frequency is changed to ω' = 2π   180 = 3   120π = 300 rad/s, then inductive reactance ω' L = 3 ωL = 3   0.5 Z = 1.5 Z

$\therefore$ New impedence $Z' = \sqrt{[R' + (\omega' L)^2]} = \sqrt{(0.866\ \ Z)^2 + (1.5\ \ Z)^2} = Z\sqrt{[(0.866)^2 + (1.5)^2]} = 1.732Z$

$\therefore$ New power factor $= \frac{R}{Z'} = \frac{0.866\ \ Z}{1.732\ \ Z} = 0.5$

CHOKE COIL

In a direct current circuit, current is reduced with the help of a resistance. Hence there is a loss of electrical energy I^2 R per sec in the form of heat in the resistance. But in an AC circuit the current can be reduced by choke coil which involves very small amount of loss of energy. Choke coil is a copper coil wound over a soft iron laminated core. This coil is put in series with the circuit in which current is to be reduced. It also known as ballast.



Circuit with a choke coil is a series L-R circuit. If resistance of choke coil = r (very small)

The current in the circuit $I = \frac{E}{Z}$ with $Z = \sqrt{(R+r)^2 + (\omega L)^2}$ So due to large inductance L of the coil, the current in the circuit is decreased appreciably. However, due to small resistance of the coil r,

The power loss in the choke $P_{av} = V_{rms} I_{rms} \cos\phi \rightarrow 0 \because \cos\phi = \frac{r}{Z} = \frac{r}{\sqrt{r^2 + \omega^2 L^2}} \approx \frac{r}{\omega L} \rightarrow 0$

GOLDEN KEY POINT

- Choke coil is a high inductance and negligible resistance coil.
- Choke coil is used to control current in A.C. circuit at negligible power loss
- Choke coil used only in A.C. and not in D.C. circuit
- Choke coil is based on the principle of wattless current.
- Iron cored choke coil is used generally at low frequency and air cored at high frequency.
- Resistance of ideal choke coil is zero

Example

A choke coil and a resistance are connected in series in an a.c circuit and a potential of 130 volt is applied to the circuit. If the potential across the resistance is 50 V. What would be the potential difference across the choke coil.

Solution

$$V = \sqrt{V_R^2 + V_L^2} \Rightarrow V_L = \sqrt{V^2 - V_R^2} = \sqrt{(130)^2 - (50)^2} = 120 \text{ V}$$

Example

An electric lamp which runs at 80V DC consumes 10 A current. The lamp is connected to 100 V – 50 Hz ac source compute the inductance of the choke required.

Solution

Resistance of lamp $R = \frac{V}{I} = \frac{80}{10} = 8\Omega$

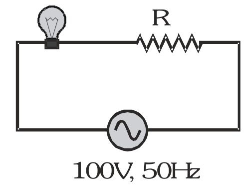
Let Z be the impedance which would maintain a current of 10 A through the Lamp when it is run on 100 Volt a.c. then. $Z = \frac{V}{I} = \frac{100}{10} = 10\ \Omega$ but $Z = \sqrt{R^2 + (\omega L)^2}$

$$\Rightarrow (\omega L)^2 = Z^2 - R^2 = (10)^2 - (8)^2 = 36 \Rightarrow \omega L = 6 \Rightarrow L = \frac{6}{\omega} = \frac{6}{2\pi \times 50} = 0.02\text{H}$$

Example

Calculate the resistance or inductance required to operate a lamp (60V, 10W) from a source of (100 V, 50 Hz)

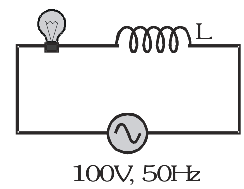
Solution



(a) Maximum voltage across lamp = 60V

$\because$ $V_{Lamp} + V_R = 100$ $\therefore$ $V_R = 40V$

Now current throgth Lamp is = $\dfrac{\text{Wattage}}{\text{voltage}} = \dfrac{10}{60} = \dfrac{1}{6}A$



But $V_R = IR$ $\Rightarrow$ $40 = \dfrac{1}{6}(R)$ $\Rightarrow$ $R = 240\ \Omega$

(b) Now in this case $(V_{Lamp})^2 + (V_L)^2 = (V)^2$

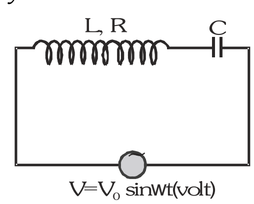
$(60)^2 + (V_L)^2 = (100)^2 \Rightarrow V_L = 80\ V$

Also $V_L = IX_L = \dfrac{1}{6}X_L$ so $X_L = 80\ \ 6 = 480\ \Omega = L\ (2\pi f) \Rightarrow L = 1.5\ H$

A capacitor of suitable capacitance replace a choke coil in an AC circuit, the average power consumed in a capacitor is also zero. Hence, like a choke coil, a capacitor can reduce current in AC circuit without power dissipation.

Cost of capacitor is much more than the cost of inductance of same reactance that's why choke coil is used.

Example A choke coil of resistance R and inductance L is connected in series with a capacitor C and complete combination is connected to a.c. voltage, Circuit resonates when angular frequency of supply is $\omega = \omega_0$.



(a) Find out relation betwen ω_0, L and C

(b) What is phase difference between V and I at resonance, is it changes when resistance of choke coil is zero.

Solution (a) At resonance condition $X_L = X_C \Rightarrow \omega_0 L = \dfrac{1}{\omega_0 C}$ $\Rightarrow$ $\omega_0 = \dfrac{1}{\sqrt{LC}}$

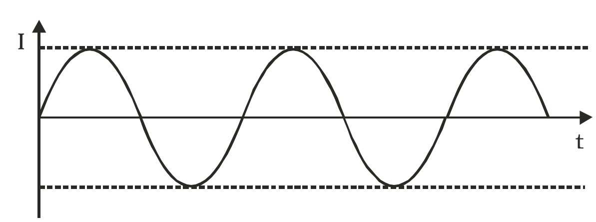
(b) $\because$ $\cos\phi = \dfrac{R}{Z} = \dfrac{R}{R} = 1$ $\therefore$ $\phi = 0$ No, It is always zero.

LC OSCILLATION

The oscillation of energy between capacitor (electric field energy) and inductor (magnetic field energy) is called LC Oscillation.

UNDAMPED OSCILLATION

When the circuit has no resistance, the energy taken once from the source and given to capacitor keeps on oscillating between C and L then the oscillation produced will be of constant amplitude. These are called undamped oscillation.



After switch is closed

$$\frac{Q}{C} + L\frac{di}{dt} = 0 \quad \Rightarrow \quad \frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0 \quad \Rightarrow \quad \frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$$

By comparing with standard equation of free oscillation $\left[\frac{d^2x}{dt^2} + \omega^2 x = 0\right]$

$\omega^2 = \frac{1}{LC}$ Frequency of oscillation $f = \frac{1}{2\pi\sqrt{LC}}$

Charge varies sinusoidally with time $q = q_m \cos \omega t$

current also varies periodically with t $I = \frac{dq}{dt} = q_m \omega \cos\left(\omega t + \frac{\pi}{2}\right)$

If initial charge on capacitor is q_m then electrical energy strored in capacitor is $U_E = \frac{1}{2}\frac{q_m^2}{C}$

At t = 0 switch is closed, capacitor is starts to discharge.

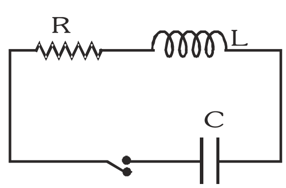
As the capacitor is fully discharged, the total electrical energy is stored in the inductor in the form of magnetic energy.

$U_B = \frac{1}{2}LI_m^2$ where I_m = max. current

$(U_{max})_{EPE} = (U_{max})_{MPE} \Rightarrow \frac{1}{2}\frac{q_m^2}{C} = \frac{1}{2}LI_m^2$

DAMPED OSCILLATION

Practically, a circuit can not be entirely resistanceless, so some part of energy is lost in resistance and amplitude of oscillation goes on decreasing. These are called damped oscillation.

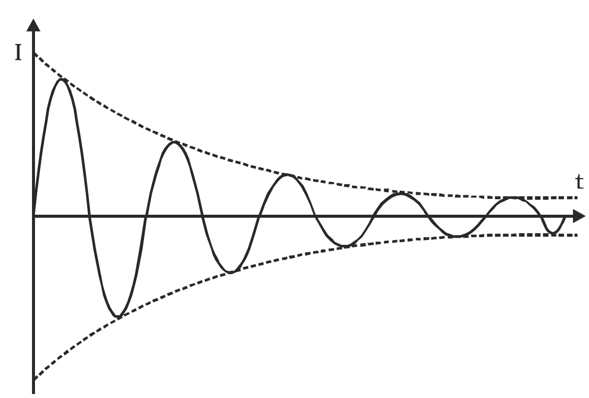


Angular frequency of oscillation $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$

frequency of oscillation $f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$

oscillation to be real if $\frac{1}{LC} - \frac{R^2}{4L^2} > 0$

Hence for oscilation to be real $\frac{1}{LC} > \frac{R^2}{4L^2}$



GOLDEN KEY POINTS

- In damped oscillation amplitude of oscillation decreases exponentially with time.
- At $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}.....$ energy stored is completely magnetic.
- At $t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}.....$ energy is shared equally between L and C
- Phase difference between charge and current is $\frac{\pi}{2}$ $\begin{bmatrix} \text{when charge is maximum, current minimum} \\ \text{when charge is minimum, current maximum} \end{bmatrix}$

Example

An LC circuit contains a 20mH inductor and a 50μF capacitor with an initial charge of 10mC. The resistance of the circuit is negligible. Let the instant the circuit is closed to be t = 0.

(a) What is the total energy stored initially.

(b) What is the natural frequency of the circuit.

(c) At what time is the energy stored is completely magnetic.

(d) At what times is the total energy shared equally between inductor and the capacitor.

Solution

(a) $U_E = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2} \times \frac{(10 \times 10^{-3})^2}{50 \times 10^{-6}} = 1.0\text{J}$

(b) $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20 \times 10^{-3} \times 50 \times 10^{-6}}} = 10^3$ rad/sec $\Rightarrow$ f = 159 Hz

(c) $\because$ $q = q_0 \cos \omega t$

Energy stored is completely magnetic (i.e. electrical energy is zero, q = 0)

at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}.........$ where $T = \frac{1}{f}$ = 6.3 ms

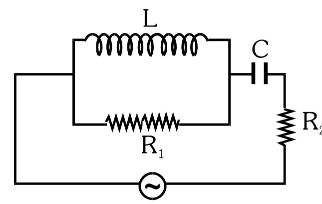
(d) Energy is shared equally between L and C when charge on capacitor become $\frac{q_0}{\sqrt{2}}$

so, at $t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}......$ energy is shared equally between L and C

SOME WORKED OUT EXAMPLES

Example#1

For the given circuit



(A) The phase difference between I_L & I_{R_1} is 0

(B) The phase difference between V_C & V_{R_2} is 90

(C) The phase difference between I_L & I_{R_1} is 180

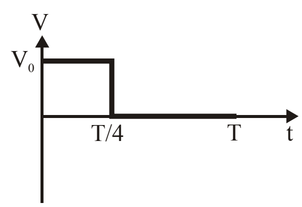
(D) The phase difference between V_C & V_{R_2} is 180

Solution **Ans. (B)**

The phase difference between V_C and V_{R_2} is $\pi/2$ rad or 90

Example#2

A periodic voltage V varies with time t as shown in the figure. T is the time period. The r.m.s. value of the voltage is :-



(A) $\dfrac{V_0}{8}$ (B) $\dfrac{V_0}{2}$ (C) V_0 (D) $\dfrac{V_0}{4}$

Solution **Ans. (B)**

Root mean square value <V> $= \sqrt{\dfrac{\int_0^{T/4} V_0^2 dt}{\int_0^T dt}} = \sqrt{\dfrac{V_0^2\left(\dfrac{T}{4}\right)}{T}} == \sqrt{\dfrac{V_0^2\left(\dfrac{T}{4}\right)}{T}} = \sqrt{\dfrac{V_0^2}{4}} = \dfrac{V_0}{2}$

Example#3

The potential difference V and current I flowing through the AC circuit is given by V = 5 cos(ωt − π/6) volt and I = 10sinωt ampere. The average power dissipated in the circuit is

(A) $\dfrac{25\sqrt{3}}{2}$ W (B) 12.5 W (C) 25 W (D) 50 W

Solution **Ans. (B)**

$V = 5\cos(\omega t - \pi/6)$; $i = 10\sin\omega t = 10\cos(\omega t - \pi/2)$

$$\phi = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3};\ P = \frac{VI}{2}\cos\phi = \frac{5\times10}{2}\times\frac{1}{2} = 12.5\text{W}$$

Example#4

The radius of a coil decreases steadily at the rate of 10^{-2} m/s. A constant and uniform magnetic field of induction 10^{-3} Wb/m^2 acts perpendicular to the plane of the coil. The radius of the coil when the induced e.m.f. in the coil is 1μV, is :-

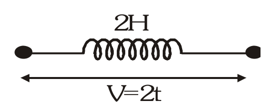
(A) $\frac{2}{\pi}$ cm (B) $\frac{3}{\pi}$ cm (C) $\frac{4}{\pi}$ cm (D) $\frac{5}{\pi}$ cm

Solution **Ans. (D)**

$$e = \frac{d\phi}{dt} = \frac{d}{dt}(\pi r^2 B) = 2\pi rB\frac{dr}{dt} \Rightarrow r = \frac{e}{\left(2\pi B\frac{dr}{dt}\right)} \Rightarrow r = \frac{10^{-6}}{2\pi\times10^{-3}\times10^{-2}} = \frac{5}{\pi}\text{cm}$$

Example#5

A time varying voltage V = 2t volt is applied across an ideal inductor of inductance L = 2H as shown in figure. Then select incorrect statement



(A) current versus time graph is a parabola

(B) energy stored in magnetic field at t = 2 s is 4J

(C) potential energy at time t = 1 s in magnetic field is increasing at a rate of 1 J/s

(D) energy stored in magnetic field is zero all the time

Solution **Ans. (D)**

$$V = 2t \Rightarrow L\frac{di}{dt} = 2t \Rightarrow 2\quad\frac{di}{dt} = 2t \Rightarrow \frac{di}{dt} \Rightarrow i = \frac{t^2}{2} \Rightarrow i - t \text{ graph parabola}$$

$$U = \frac{1}{2}Li^2 = \frac{1}{2}\quad 2\quad 4 = 4\text{J and } \frac{dU}{dt} = Li\frac{di}{dt} = 2\quad\frac{t^2}{2}\quad t = t^3 = 1 \text{ J/s}$$

Example#6

A circular coil of 500 turns encloses an area of 0.04 m^2. A uniform magnetic field of induction 0.25 Wb/m^2 is applied perpendicular to the plane of the coil. The coil is rotated by 90 in 0.1 second at a constant angular velocity about one of its diameters. A galvanometer of resistance 25Ω was connected in series with the the coil. The total charge that will pass through the galvanometer is -

(A) 0.4 C (B) 1 C (C) 0.2 C (D) Zero

Solution **Ans. (C)**

Induced current $I = \frac{e}{R} \Rightarrow \frac{dq}{dt} = \frac{e}{R} = \left(\frac{d\phi}{dt}\right)\frac{1}{R} \Rightarrow q = \frac{\Delta\phi}{R} = \frac{NBA}{R}$

Total charge $q = \frac{500\times0.25\times0.04}{25} = 0.2$ C

Example#7

A condenser of capacity 6 μF is fully charged using a 6-volt battery. The battery is removed and a resistanceless 0.2 mH inductor is connected across the condenser. The current which is flowing through the inductor when one-third of the total energy is in the magnetic field of the inductor is :-

(A) 0.1 A (B) 0.2 A (C) 0.4 A (D) 0.6 A

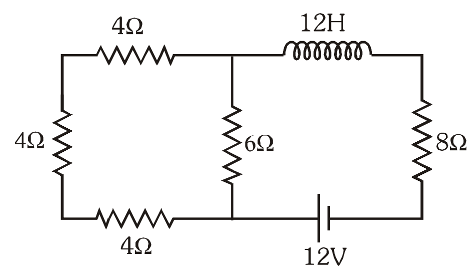
Solution **Ans. (D)**

Total energy = Intial energy on capacitor = $\frac{1}{2}CV^2$, Magnetic field energy = $\frac{1}{2}LI^2$

$$\therefore \quad \frac{1}{2}LI^2 = \frac{1}{3}\times\frac{1}{2}\ CV^2 \Rightarrow I = \sqrt{\frac{CV^2}{3L}} = \sqrt{\frac{6\times10^{-6}\times6\times6}{3\times2.0\times10^{-3}}} = 0.6\ A$$

Example#8

For the circuit shown, which of the following statement(s) is(are) correct?



(A) Its time constants is 2 second.

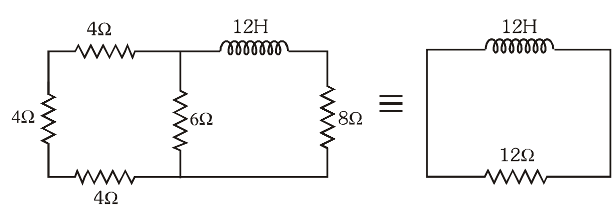
(B) In steady state, current through inductance will be 1A.

(C) In steady state, current through 4Ω resistance will be 2/3 A.

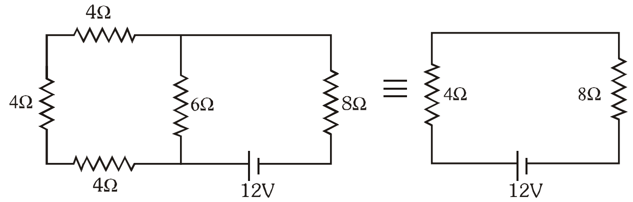
(D) In steady state, current throgh 8Ω resistance will be zero.

Solution **Ans. (B)**

Time constant $\tau = \frac{L}{R} = \frac{12}{12} = 1s$



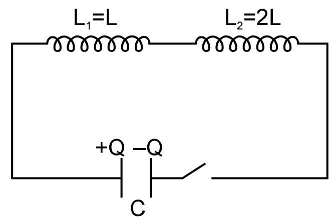
In steady state



current through 4Ω resistance = $\frac{6}{6+12}\times1 = \frac{1}{3}A$

Example#9

In the circuit shown the capacitor has charge Q. At t = 0 sec the key is closed. The charge on the capacitor at the instant potential difference across the inductor L_1 is zero, is



(A) Q (B) $\frac{Q}{3}$ (C) $\frac{2Q}{3}$ (D) 0

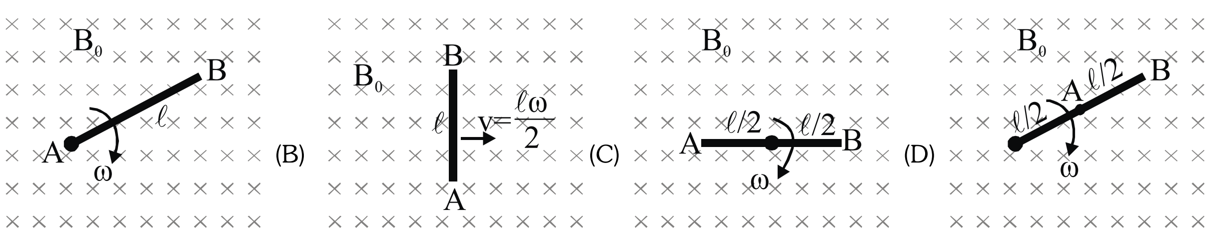
Solution **Ans.** (D)

When $V_{\text{across L}} = 0 \Rightarrow i = i_{max} \Rightarrow q_c = 0$

Example#10

The figure shows a rod of length ℓ with points A and B on it. The rod is moved in a uniform magnetic field (B_0) in different ways as shown. In which case potential difference ($V_A - V_B$) between A & B is minimum?

(A) (B) (C) (D)



Solution **Ans.** (C)

For (A): $V_B - V_A = \frac{1}{2}B\omega\ell^2 \Rightarrow V_A - V_B = -\frac{1}{2}B\omega\ell^2$

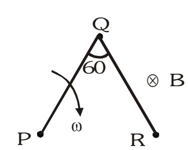
For (B) : $V_B - V_A = Bv\ell = B\left(\frac{\omega\ell}{2}\right)\ell = \frac{1}{2}B\omega\ell^2 \Rightarrow V_A - V_B = -\frac{1}{2}B\omega\ell^2$

For (C) : $V_A - V_B = 0$

For (D) : $V_B - V_A = \frac{1}{2}B\omega\ell^2 - \frac{1}{2}B\omega\left(\frac{\ell}{2}\right)^2 = \frac{3}{8}B\omega\ell^2 \Rightarrow V_A - V_B = -\frac{3}{8}B\omega\ell^2$

Example#11

A bent rod PQR (PQ = QR = ℓ) shown here is rotating about its end P with angular speed ω in a region of transverse magnetic field of strength B.



(A) e.m.f. induced across the rod is $B\omega\ell^2$

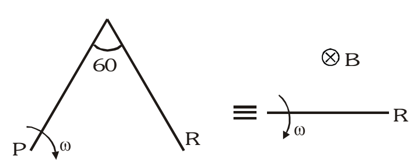
(B) e.m.f. induced across the rod is $B\omega\ell^2/2$

(C) Potential difference between points Q and R on the rod is $B\omega\ell^2/2$

(D) Potential difference between points Q and R on the rod is zero

Solution Ans. (B,D)

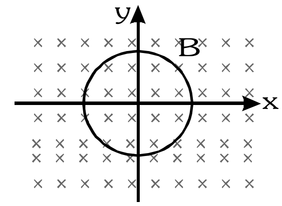
The rod in equivalent to a rod joining the ends P and R of the rod rotating is the same sense.



$V_R - V_P = \frac{B\omega\ell^2}{2}$; $V_Q - V_P = \frac{B\omega\ell^2}{2}$; $V_R - V_P = \frac{B\omega\ell^2}{2}$; $V_Q - V_R = 0$

Example#12

A conducting loop is kept so that its center lies at the origin of the coordinate system. A magnetic field has the induction B pointing along Z-axis as shown in the figure



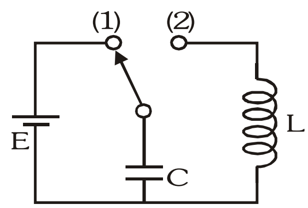
(A) No emf and current will be induced in the loop if it rotates about Z-axis
(B) emf is induced but no current flows if the loop is a fiber when it rotates about y-axis.
(C) emf is induced and induced current flows in the loop if the loop is made of copper & is rotated about y-axis.
(D) If the loop moves along Z-axis with constant velocity, no current flows in it.

Solution **Ans. (A,C,D)**

If the loop rotates about Z axis, the variation of flux linkage will be zero. Therefore no emf is induced. Consequently no current flows in the loop. When it rotates about y axis, its flux linkage changes. However, in insulators there can not be motional emf. If the loop is made of copper, it is conductive therefore induced current is set up. If the loop moves along the Z axis variation of flux linkage is zero. Therefore the emf and current will be equal to zero.n

Example#13

Initially key was placed on (1) till the capacitor got fully charged. Key is placed on (2) at t=0. The time when the energy in both capacitor and inductor will be same-



(A) $\dfrac{\pi\sqrt{LC}}{4}$ (B) $\dfrac{\pi\sqrt{LC}}{2}$ (C) $\dfrac{5\pi\sqrt{LC}}{4}$ (D) $\dfrac{5\pi\sqrt{LC}}{2}$

Solution. **Ans. (A,C)**

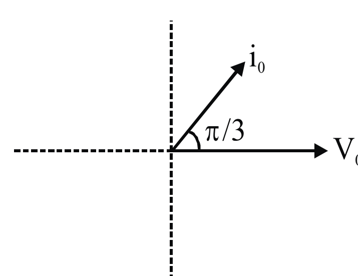
For given situation $\dfrac{q}{C}+L\dfrac{di}{dt}=0 \Rightarrow \dfrac{d^2q}{dt^2}+\dfrac{q}{LC}=0 \Rightarrow \dfrac{d^2q}{dt^2}+\omega^2q=0 \Rightarrow q = q_0 \cos \omega t$ & $i = -q_0\omega\sin\omega t$

According to given conditions $\dfrac{q^2}{2C}=\dfrac{1}{2}Li^2 \Rightarrow \dfrac{q_0 \cos^2 \omega t}{2C}=\dfrac{1}{2}Lq_0^2\omega^2 \sin^2 \omega t$

$\Rightarrow \cot^2\omega t = 1 \Rightarrow \omega t = \dfrac{\pi}{4},\dfrac{3\pi}{4},\dfrac{5\pi}{4},\dfrac{7\pi}{4}$ .... $\Rightarrow t = \dfrac{\pi\sqrt{LC}}{4},\dfrac{3\pi\sqrt{LC}}{4},\dfrac{5\pi\sqrt{LC}}{4},\dfrac{7\pi\sqrt{LC}}{4}$ .....

Example#14

For an LCR series circuit, phasors of current i and applied voltage $V = V_0 \sin\omega t$ are shown in diagram at t =0. Which of the following is/are **CORRECT**?



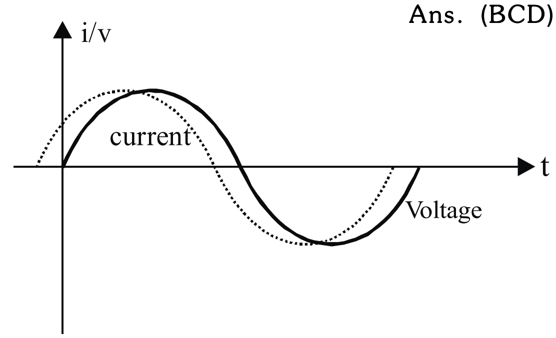
(A) At $t = \dfrac{\pi}{2\omega}$, instantaneous power supplied by source is negative.

(B) From $0 < t < \dfrac{2\pi}{3\omega}$, average power supplied by source is positive.

(C) At $t = \dfrac{5\pi}{6\omega}$, instantaneous power supplied by source is negative.

(D) If ω is increased slightly, angle between the two phasors decreases.

Solution **Ans. (BCD)**



The graph shows V & I as function of time.

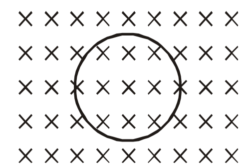
Current leads the voltage by $\pi/3$

Power is positive if V & I are of same sign.

Power is negative if V & I are of opposite sign

If $\omega\uparrow \Rightarrow \frac{1}{\omega C}\downarrow$ thus angle decreases.

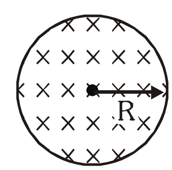
Example#15 to 17

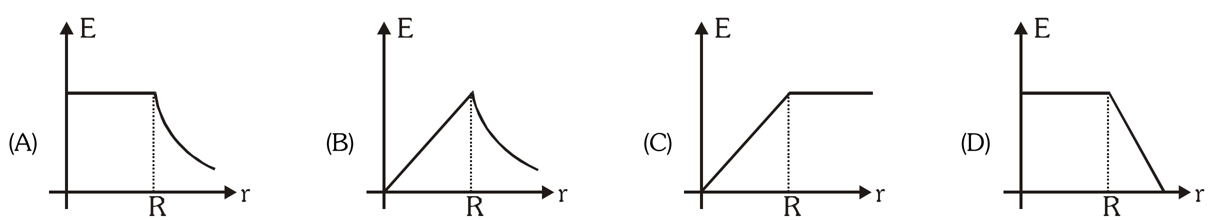


Consider a conducting circular loop placed in a magnetic field as shown in figure. When magnetic field changes with time, magnetic flux also changes and emf $e = -\frac{d\phi}{dt}$ is induced.

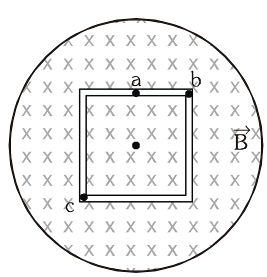
If resistance of loop is R then induced current is $i = \frac{e}{R}$. For current, charges must have non–zero average velocity. Magnetic force cannot make the stationary charges to move. Actually there is an induced electric field in the conductor caused by changing magnetic flux, which makes the charges to move, $\oint \vec{E}\cdot\vec{d\ell} = -\frac{d\phi}{dt}$. This induced electric field is non conservative by nature.

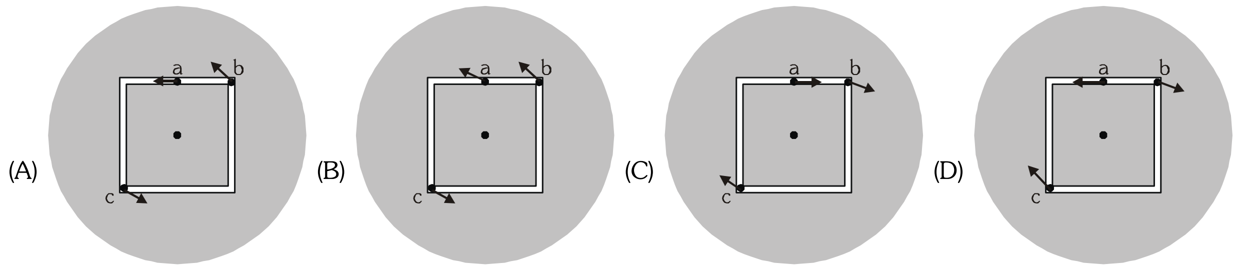
15. A cylindrical space of radius R is filled with a uniform magnetic induction B parallel to the axis of the cylinder. If $\frac{dB}{dt}$ = constant, the graph, showing the variation of induced electric field with distance r from the axis of cylinder, is



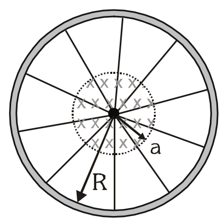


16. A square conducting loop is placed in the time varying magnetic field $\left(\frac{dB}{dt} = +\text{ve constant}\right)$. The centre of square coincides with axis of cylindrical region of magnetic field. The directions of induced electric field at point a, b and c.





17. A line charge λ per unit length is pasted uniformly onto the rim of a wheel of mass m and radius R. The wheel has light non–conducting spokes and is free to rotate about a vertical axis as shown in figure.A uniform magnetic field B exist as shown in figure. What is the angular velocity of the wheel when the field is suddenly switched off?



(A) $\dfrac{2\pi\lambda a^2 B}{mR}$ (B) $\dfrac{\pi\lambda a^2 B}{mR}$ (C) $\dfrac{3\pi\lambda a^2 B}{mR}$ (D) $\dfrac{\pi\lambda a^2 B}{2mR}$

Solution

15. Ans. (B)

For $r \le R$; $E(2\pi r) = \pi r^2 \dfrac{dB}{dt} \Rightarrow E \propto r$ For $r \ge R$; $E(2\pi r) = \pi R^2 \dfrac{dB}{dt} \Rightarrow E \propto \dfrac{1}{r}$

16. Ans. (A)

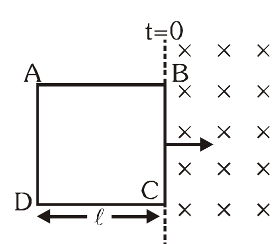
Induced electric field is circular.

17. Ans. (B)

$$\tau = \frac{\Delta L}{\Delta t} = \frac{I\omega}{\Delta t} = \frac{mR^2\omega}{\Delta t} \text{ but } \tau = \lambda(2\pi R)ER = \lambda\left(\pi a^2 \frac{\Delta B}{\Delta t}\right)R = \frac{\lambda\pi a^2 BR}{\Delta t} \Rightarrow \frac{mR^2\omega}{\Delta t} = \frac{\lambda\pi a^2 BR}{\Delta t} \Rightarrow \omega = \frac{\lambda\pi a^2 B}{mR}$$

Example#18 to 20

A conducting square wire frame ABCD of side ℓ is pulled by horizontal force so that it moves with constant velocity v. A uniform magnetic field of strength B is existing perpendicular to the plane of wire. The resistance per unit length of wire is λ and negligible self inductance. If at t= 0, frame is just at the boundary of magnetic field. Then



18. The emf across AB at $t = \dfrac{\ell}{2v}$ is

(A) $\dfrac{Bv\ell}{4}$ (B) zero (C) $Bv\ell$ (D) None of these

19. Potential difference across BC at time $t = \dfrac{\ell}{2v}$ is

(A) $\dfrac{Bv\ell}{4}$ (B) $Bv\ell$ (C) $\dfrac{3Bv\ell}{4}$ (D) None of these

20. Find the applied horizontal force on BC, (F) as a function of time 't' ($t < \dfrac{\ell}{v}$)

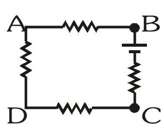
(A) $\dfrac{B^2v\ell}{\lambda}t$ (B) $\dfrac{B^2v\ell}{2\lambda}$ (C) $\dfrac{B^2v\ell}{4\lambda}$ (D) None of these

Solution

18. Ans. (B)

$e = \vec{B}\cdot(\vec{\ell}\times\vec{v})$ as $\vec{e} \,||\, \vec{v}$ $\because \theta = 0 \Rightarrow e = 0$

19. Ans. (C)



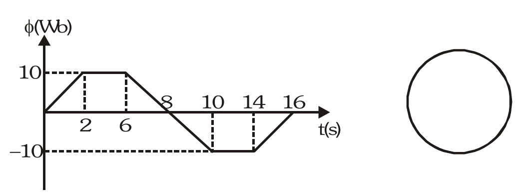
$V = E - ir = B\ell v - \dfrac{B\ell v}{4\lambda(\ell)}\lambda[\ell] = \dfrac{3B\ell v}{4}$

20. Ans. (C)

$$F = Bi\ell = \frac{B\ell(B\ell v)}{4\lambda(\ell)} = \frac{B^2\ell^2 v}{4\lambda(\ell)} = \frac{B^2\ell v}{4\lambda}$$

Example#21

Magnetic flux in a circular coil of resistance 10Ω changes with time as shown in figure. $\otimes$ direction indicates a direction perpendicular to paper inwards.



	Column–I		Column–II
(A)	At 1 second induced current is	(P)	Clockwise
(B)	At 5 second induced current is	(Q)	Anticlockwise
(C)	At 9 second induced current is	(R)	0.5 A
(D)	At 15 second induced current is	(S)	5 A
		(T)	None of these

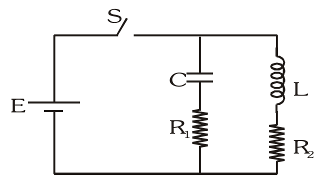
Solution **Ans.** (A) Q,R (B) T (C) P,R (D) Q,R

For (A) $\frac{d\phi}{dt} = 5 \Rightarrow i = \frac{5}{10} = \frac{1}{2}$ A, Anticlockwise   For (B) $\frac{d\phi}{dt} = 0 \Rightarrow$ i=0 = zero

For (C) $\frac{d\phi}{dt} = -5 \Rightarrow i = -\frac{5}{10} = -\frac{1}{2}$ A, Clockwise   For (D) $\frac{d\phi}{dt} = 5 \Rightarrow i = \frac{5}{10} = \frac{1}{2}$ A, Anticlockwise

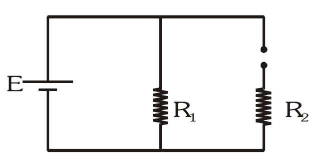
Example#22

In the circuit shown in figure E=25V, L=2H, C=60 μF, R_1 = 5Ω and R_2 = 10Ω. Switch S is closed at t = 0.



	Column-I		Column-II
(A)	Current through R_1 at t = 0	(P)	0
(B)	Current through R_2 at t = 0	(Q)	5A
(C)	Current through R_1 at t = ∞	(R)	2.5 A
(D)	Current through R_2 at t = ∞	(S)	7.5 A
		(T)	None of these

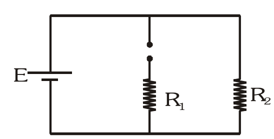
Solution **Ans.** (A)$\rightarrow$(Q), (B)$\rightarrow$(P), (C)$\rightarrow$(P), (D)$\rightarrow$(R)



At t = 0

$I_1 = \frac{E}{R_1} = \frac{25}{5} = 5A$

$I_2 = 0$



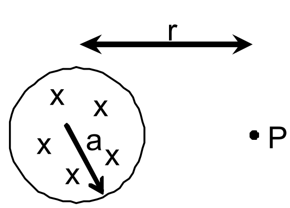
At t = ∞

$I_1 = 0$

$I_2 = \frac{E}{R_2} = \frac{25}{10} = 2.5$ A

Example#23

A uniform but time-varying magnetic field B(t) exists in a circular region of radius a and is directed into the plane of the paper, as shown. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region is proportional to $1/r^n$. Find the value of n.

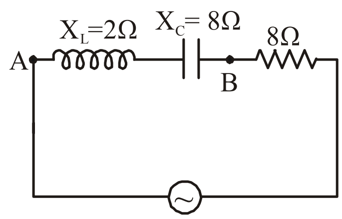


Solution **Ans.** 1

$$\frac{d\phi}{dt} = E(2\pi r) \Rightarrow E \propto \frac{1}{r} \Rightarrow n = 1$$

Example#24

An inductor (X_L = 2Ω) a capacitor (X_C = 8Ω) and a resistance (8Ω) is connected in series with an ac source. The voltage output of A.C source is given by V = 10 cos 2π 50t. Find the instantaneous p.d. between A and B when the voltage output from source is half of its maximum.



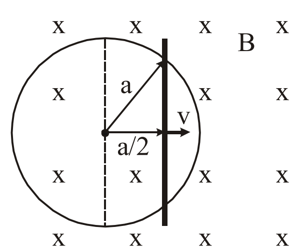
Solution **Ans.** 3

$$V_{AB} = \frac{6}{10} \times \left(\frac{10}{2}\right) = 3 \qquad \because R_{AB} = X_C - X_C \text{ and } Z = \sqrt{(X_C - X_L)^2 + R^2}$$

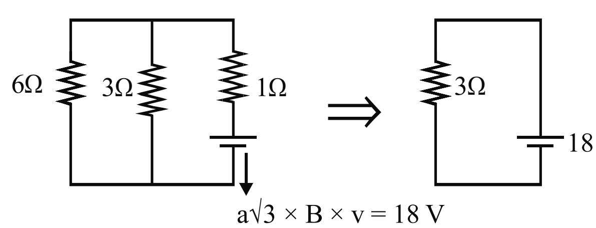
Example#25

Figure shows a uniform circular loop of radius 'a' having specific resistance ρ placed in a uniform magnetic field B perpendicular to plane of figure. A uniform rod of length 2a & resistance R moves with a velocity v as shown. Find the current in the rod when it has moved a distance $\frac{a}{2}$ from the centre of circular loop.

[Given B = √3, a =2, v =3, $\rho = \frac{9}{4\pi}$, R = 2/√3 all in SI units]



Solution **Ans.** 6



$$\Rightarrow i = \frac{18}{3} = 6A$$