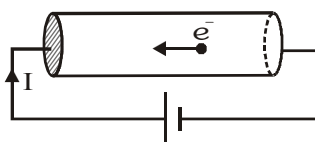


# CURRENT ELECTRICITY

In previous chapters we deal largely with electrostatics that is, with charges at rest. With this chapter we begin to focus on electric currents, that is, charges in motion.

## ELECTRIC CURRENT

Electric charges in motion constitute an electric current. Any medium having practically free electric charges, free to migrate is a conductor of electricity. The electric charge flows from higher potential energy state to lower potential energy state.



Positive charge flows from higher to lower potential and negative charge flows from lower to higher. Metals such as gold, silver, copper, aluminium etc. are good conductors. When charge flows in a conductor from one place to the other, then the rate of flow of charge is called electric current ( $I$ ). When there is a transfer of charge from one point to other point in a conductor, we say that there is an electric current through the area. If the moving charges are positive, the current is in the direction of motion of charge. If they are negative the current is opposite to the direction of motion. If a charge  $\Delta Q$  crosses an area in time  $\Delta t$  then the average electric current through the area, during this time as

• Average current  $I_{av} = \frac{\Delta Q}{\Delta t}$

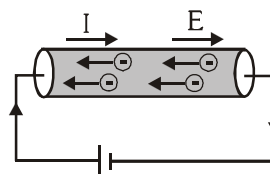
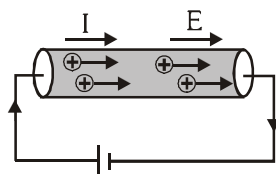
• Instantaneous current  $I = \lim_{\Delta t \rightarrow 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$

### GOLDEN KEY POINTS

- Current is a fundamental quantity with dimension  $[M^0 L^0 T^0 A]$
- Current is a scalar quantity with its SI unit **ampere**.

**Ampere** : The current through a conductor is said to be one ampere if one coulomb of charge is flowing per second through a cross-section of wire.

- The conventional direction of current is the direction of flow of positive charge or applied field. It is opposite to direction of flow of negatively charged electrons.



- The conductor remains uncharged when current flows through it because the charge entering at one end per second is equal to charge leaving the other end per second.
- For a given conductor current does not change with change in its cross-section because current is simply rate of flow of charge.

If  $n$  particles each having a charge  $q$  pass per second per unit area then current associated with cross-sectional area  $A$  is  $I = \frac{\Delta q}{\Delta t} = nqA$ .

If there are  $n$  particles per unit volume each having a charge  $q$  and moving with velocity  $v$  then current through cross-sectional area  $A$  is  $I = \frac{\Delta q}{\Delta t} = nqvA$

If a charge  $q$  is moving in a circle of radius  $r$  with speed  $v$  then its time period is  $T = 2\pi r/v$ . The equivalent current  $I = \frac{q}{T} = \frac{qv}{2\pi r}$ .

## CLASSIFICATION OF MATERIALS ACCORDING TO CONDUCTIVITY

### (i) Conductor

In some materials, the outer electrons of each atoms or molecules are only weakly bound to it. These electrons are almost free to move throughout the body of the material and are called free electrons. They are also known as conduction electrons. When such a material is placed in an electric field, the free electrons move in a direction opposite to the field. Such materials are called conductors.

### (ii) Insulator

Another class of materials is called insulators in which all the electrons are tightly bound to their respective atoms or molecules. Effectively, there are no free electrons. When such a material is placed in an electric field, the electrons may slightly shift opposite to the field but they can't leave their parent atoms or molecules and hence can't move through long distances. Such materials are also called dielectrics.

### (iii) Semiconductor

In semiconductors, the behaviour is like an insulator at low levels of temperature. But at higher temperatures, a small number of electrons are able to free themselves and they respond to the applied electric field. As the number of free electrons in a semiconductor is much smaller than that in a conductor, its behaviour is in between a conductor and an insulator and hence, the name semiconductor. A freed electron in a semiconductor leaves a vacancy in its normal bound position. These vacancies also help in conduction.

### Behavior of conductor in absence of applied potential difference :

In absence of applied potential difference electrons have random motion. The average displacement and average velocity is zero. There is no flow of current due to thermal motion of free electrons in a conductor.

The free electrons present in a conductor gain energy from temperature of surrounding and move randomly in the conductor.

The speed gained by virtue of temperature is called as thermal speed of an electron  $\frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}kT$

So thermal speed  $v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$  where  $m$  is mass of electron

At room temperature  $T = 300 \text{ K}$ ,  $v_{\text{rms}} = 10^5 \text{ m/s}$

- **Mean free path  $\lambda$**  : ( $\lambda \sim 10\text{\AA}$ )  $\lambda = \frac{\text{total distance travelled}}{\text{number of collisions}}$
- **Relaxation time** : The time taken by an electron between two successive collisions is called as relaxation time  $\tau$  : ( $\tau \sim 10^{-14}\text{s}$ ), Relaxation time :  $\tau = \frac{\text{total time taken}}{\text{number of collisions}}$

### Behavior of conductor in presence of applied potential difference :

When two ends of a conductors are joined to a battery then one end is at higher potential and another at lower potential. This produces an electric field inside the conductor from point of higher to lower potential

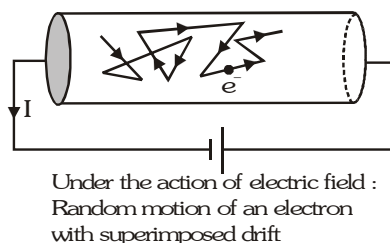
$E = \frac{V}{L}$  where  $V$  = emf of the battery,  $L$  = length of the conductor.

The field exerts an electric force on free electrons causing acceleration of each electron.

Acceleration of electron  $\vec{a} = \frac{\vec{F}}{m} = \frac{-e\vec{E}}{m}$

## DRIFT VELOCITY

Drift velocity is defined as the velocity with which the free electrons get drifted towards the positive terminal under the effect of the applied external electric field. In addition to its thermal velocity, due to acceleration given by applied electric field, the electron acquires a velocity component in a direction opposite to the direction of the electric field. The gain in velocity due to the applied field is very small and is lost in the next collision.



At any given time, an electron has a velocity  $\vec{v}_1 = \vec{u}_1 + \vec{a}\tau_1$ , where  $\vec{u}_1$  = the thermal velocity and

$\vec{a}\tau_1$  = the velocity acquired by the electron under the influence of the applied electric field.

$\tau_1$  = the time that has elapsed since the last collision. Similarly, the velocities of the other electrons are

$$\vec{v}_2 = \vec{u}_2 + \vec{a}\tau_2, \vec{v}_3 = \vec{u}_3 + \vec{a}\tau_3, \dots, \vec{v}_N = \vec{u}_N + \vec{a}\tau_N.$$

The average velocity of all the free electrons in the conductor is equal to the drift velocity  $\vec{v}_d$  of the free electrons

$$\vec{v}_d = \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \dots + \vec{v}_N}{N} = \frac{(\vec{u}_1 + \vec{a}\tau_1) + (\vec{u}_2 + \vec{a}\tau_2) + \dots + (\vec{u}_N + \vec{a}\tau_N)}{N} = \frac{(\vec{u}_1 + \vec{u}_2 + \dots + \vec{u}_N)}{N} + \vec{a} \left( \frac{\tau_1 + \tau_2 + \dots + \tau_N}{N} \right)$$

$$\therefore \frac{\vec{u}_1 + \vec{u}_2 + \dots + \vec{u}_N}{N} = 0 \quad \therefore \vec{v}_d = \vec{a} \left( \frac{\tau_1 + \tau_2 + \dots + \tau_N}{N} \right) \Rightarrow \vec{v}_d = \vec{a}\tau = -\frac{e\vec{E}}{m}\tau$$

**Note :** Order of drift velocity is  $10^{-4}$  m/s.

### Relation between current and drift velocity :

Let  $n$  = number density of free electrons and  $A$  = area of cross-section of conductor.

Number of free electrons in conductor of length  $L$  =  $nAL$ , Total charge on these free electrons  $\Delta q = neAL$

Time taken by drifting electrons to cross conductor  $\Delta t = \frac{L}{v_d}$   $\therefore$  current  $I = \frac{\Delta q}{\Delta t} = neAL \left( \frac{v_d}{L} \right) = neAv_d$

### Example

Find free electrons per unit volume in a metallic wire of density  $10^4$  kg/m<sup>3</sup>, atomic mass number 100 and number of free electron per atom is one.

### Solution

Number of free charge particle per unit volume ( $n$ ) =  $\frac{\text{total free charge particle}}{\text{total volume}}$

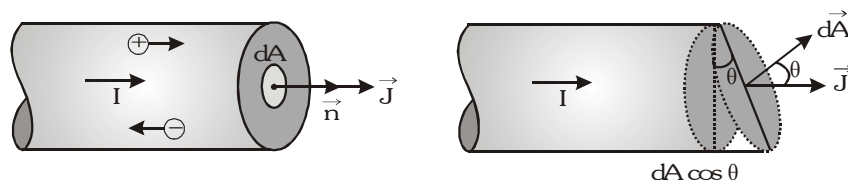
$\therefore$  No. of free electron per atom means total free electrons = total number of atoms =  $\frac{N_A}{M_w} \times M$

$$\text{So } n = \frac{\frac{N_A}{M_w} \times M}{V} = \frac{N_A}{M_w} \times d = \frac{6.023 \times 10^{23} \times 10^4}{100 \times 10^{-3}} = 6.023 \times 10^{28}$$

**CURRENT DENSITY (J)**

Current is a macroscopic quantity and deals with the overall rate of flow of charge through a section. To specify the current with direction in the microscopic level at a point, the term current density is introduced. Current density at any point inside a conductor is defined as a vector having magnitude equal to current per unit area surrounding that point. Remember area is normal to the direction of charge flow (or current passes) through that point.

- Current density at point P is given by  $\vec{J} = \frac{dI}{dA} \vec{n}$



- If the cross-sectional area is not normal to the current, but makes an angle  $\theta$  with the direction of current then  $J = \frac{dI}{dA \cos \theta} \Rightarrow dI = J dA \cos \theta = \vec{J} \cdot d\vec{A} \Rightarrow I = \int \vec{J} \cdot d\vec{A}$
- Current density  $\vec{J}$  is a vector quantity. It's direction is same as that of  $\vec{E}$ . It's S.I. unit is ampere/m<sup>2</sup> and dimension [L<sup>-2</sup>A].

**Example**

The current density at a point is  $\vec{J} = (2 \times 10^4 \hat{j}) \text{ Jm}^{-2}$ .

Find the rate of charge flow through a cross sectional area  $\vec{S} = (2\hat{i} + 3\hat{j}) \text{ cm}^2$

**Solution**

The rate of flow of charge = current =  $I = \int \vec{J} \cdot d\vec{S} \Rightarrow I = \vec{J} \cdot \vec{S} = (2 \times 10^4) [\hat{j} \cdot (2\hat{i} + 3\hat{j})] \times 10^{-4} \text{ A} = 6 \text{ A}$

**Example**

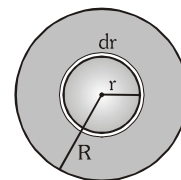
A potential difference applied to the ends of a wire made up of an alloy drives a current through it. The current density varies as  $J = 3 + 2r$ , where  $r$  is the distance of the point from the axis. If  $R$  be the radius of the wire, then the total current through any cross section of the wire.

**Solution**

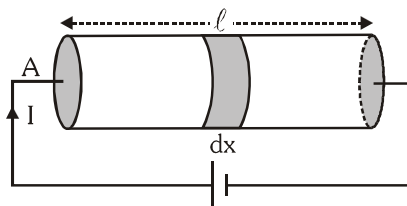
Consider a circular strip of radius  $r$  and thickness  $dr$

$$dI = \vec{J} \cdot d\vec{S} = (3 + 2r)(2\pi r dr) \cos 0^\circ = 2\pi(3r + 2r^2) dr$$

$$I = \int_0^R 2\pi(3r + 2r^2) dr = 2\pi \left( \frac{3r^2}{2} + \frac{2}{3}r^3 \right)_0^R = 2\pi \left( \frac{3R^2}{2} + \frac{2R^3}{3} \right) \text{ units}$$

**RELATION BETWEEN CURRENT DENSITY, CONDUCTIVITY AND ELECTRIC FIELD**

Let the number of free electrons per unit volume in a conductor =  $n$



Total number of electrons in  $dx$  distance =  $n (A dx)$

Total charge  $dQ = n (Adx)e$

Current  $I = \frac{dQ}{dt} = nAe \frac{dx}{dt} = neAv_d$ , Current density  $J = \frac{I}{A} = nev_d$

$$= ne \left( \frac{eE}{m} \right) \tau \quad \because v_d = \left( \frac{eE}{m} \right) \tau \Rightarrow J = \left( \frac{ne^2 \tau}{m} \right) E \Rightarrow J = \sigma E, \quad \text{where conductivity } \sigma = \frac{ne^2 \tau}{m}$$

$\sigma$  depends only on the material of the conductor and its temperature.

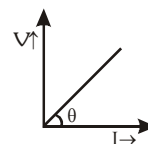
In vector form  $\vec{J} = \sigma \vec{E}$  Ohm's law (at microscopic level)

## RELATION BETWEEN POTENTIAL DIFFERENCE AND CURRENT (Ohm's Law)

If the physical conditions of the conductor (length, temperature, mechanical strain etc.) remains same, then the current flowing through the conductor is directly proportional to the potential difference across its two ends i.e.  $I \propto V \Rightarrow V = IR$  where  $R$  is a proportionality constant, known as electric resistance. Ohm's law (at macroscopic level)

- Ohm's law is not a universal law. The substances, which obey ohm's law are known as ohmic.
- Graph between  $V$  and  $I$  for a metallic conductor is a straight line as shown.

Slope of the line  $= \tan \theta = \frac{V}{I} = R$



### GOLDEN KEY POINTS

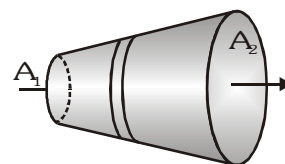
- 1 ampere of current means the flow of  $6.25 \times 10^{18}$  electrons per second through any cross section of conductor.
- Current is a scalar quantity but current density is a vector quantity.
- Order of free electron density in conductors  $= 10^{28}$  electrons/ $m^3$

Terms	Thermal speed	Mean free path	Relaxation time	Drift speed
	$v_T$	$\lambda$	$\tau$	$v_d$
Order	$10^5$ m/s	$10 \text{ \AA}$	$10^{-14}$ s	$10^{-4}$ m/s

- If a steady current flows in a metallic conductor of non uniform cross section.

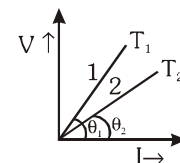
- Along the wire  $I$  is same.
- Current density and drift velocity depends on area

$$I_1 = I_2, A_1 < A_2 \Rightarrow J_1 > J_2, E_1 > E_2, v_{d1} > v_{d2}$$



- If the temperature of the conductor increases, the amplitude of the vibrations of the positive ions in the conductor also increase. Due to this, the free electrons collide more frequently with the vibrating ions and as a result, the average relaxation time decreases.
- At different temperatures  $V$ - $I$  curves are different.

Here  $\tan \theta_1 > \tan \theta_2$  So  $R_1 > R_2$  i.e.  $T_1 > T_2$



**Example**

What will be the number of electron passing through a heater wire in one minute, if it carries a current of 8 A.

**Solution**

$$I = \frac{Ne}{t} \Rightarrow N = \frac{It}{e} = \frac{8 \times 60}{1.6 \times 10^{-19}} = 3 \times 10^{21} \text{ electrons}$$

**Example**

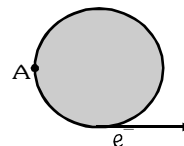
An electron moves in a circle of radius 10 cm with a constant speed of  $4 \times 10^6$  m/s. Find the electric current at a point on the circle.

**Solution**

Consider a point A on the circle. The electron crosses this point once in every revolution. The number of

$$\text{revolutions made by electron in one second is } n = \frac{v}{2\pi r} = \frac{4 \times 10^6}{2\pi \times 10 \times 10^{-2}} = \frac{2}{\pi} \times 10^7 \text{ rot/s.}$$

$$\therefore \text{Current } I = \frac{ne}{t} = \frac{2}{\pi} \times 10^7 \times 1.6 \times 10^{-19} \quad (\because t = 1 \text{ s.}) = \frac{3.2}{\pi} \times 10^{-12} \cong 1 \times 10^{-12} \text{ A}$$

**Example**

A current of 1.34 A exists in a copper wire of cross-section  $1.0 \text{ mm}^2$ . Assuming each copper atom contributes one free electron. Calculate the drift speed of the free electrons in the wire. The density of copper is  $8990 \text{ kg/m}^3$  and atomic mass = 63.50.

**Solution**

Mass of  $1 \text{ m}^3$  volume of the copper is =  $8990 \text{ kg} = 8990 \times 10^3 \text{ g}$

$$\text{Number of moles in } 1 \text{ m}^3 = \frac{8990 \times 10^3}{63.5} = 1.4 \times 10^5$$

Since each mole contains  $6 \times 10^{23}$  atoms therefore number of atoms in  $1 \text{ m}^3$

$$n = (1.4 \times 10^5) (6 \times 10^{23}) = 8.4 \times 10^{28}$$

$$\therefore I = neAv_d \quad \therefore v_d = \frac{I}{neA} = \frac{1.34}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-6}} = 10^{-4} \text{ m/s} = 0.1 \text{ mm/s} \quad (\because 1 \text{ mm}^2 = 10^{-6} \text{ m}^2)$$

**Example**

The current through a wire depends on time as  $i = (2 + 3t) \text{ A}$ . Calculate the charge crossed through a cross section of the wire in 10 s.

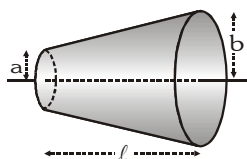
**Solution**

$$I = \frac{dq}{dt} \Rightarrow dq = (2 + 3t)dt \Rightarrow \int_0^{10} dq = \int_0^{10} (2 + 3t) dt \Rightarrow q = \left( 2t + \frac{3t^2}{2} \right)_0^{10}$$

$$q = 2t + \frac{3}{2} \times 100 = 20 + 150 = 170 \text{ C}$$

**Example**

Figure shows a conductor of length  $\ell$  carrying current  $I$  and having a circular cross-section. The radius of cross section varies linearly from  $a$  to  $b$ . Assuming that  $(b - a) \ll \ell$ . Calculate current density at distance  $x$  from left end.



### Solution

Since radius at left end is  $a$  and that of right end is  $b$ , therefore increase in radius over length  $\ell$  is  $(b - a)$ .

$$\text{Hence rate of increase of radius per unit length} = \left( \frac{b-a}{\ell} \right) \text{Increase in radius over length } x = \left( \frac{b-a}{\ell} \right) x$$

$$\text{Since radius at left end is } a \text{ so radius at distance } x, r = a + \left( \frac{b-a}{\ell} \right) x$$

$$\text{Area at this particular section } A = \pi r^2 = \pi \left[ a + \left( \frac{b-a}{\ell} \right) x \right]^2$$

$$\text{Hence current density } J = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{I}{\pi \left[ a + \frac{x(b-a)}{\ell} \right]^2}$$

### RESISTANCE

The resistance of a conductor is the opposition which the conductor offers to the flow of charge. When a potential difference is applied across a conductor, free electrons get accelerated and collide with positive ions and their motion is thus opposed. This opposition offered by the ions is called resistance of the conductor.

Resistance is the property of a conductor by virtue of which it opposes the flow of current in it.

**Unit :** ohm, volt/ampere,

**Dimension** =  $M L^2 T^{-3} A^{-2}$

**Resistance depends on :**

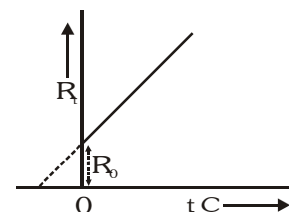
- Length of the conductor ( $R \propto \ell$ )
- Area of cross-section of the conductor  $R \propto \frac{1}{A}$
- Nature of material of the conductor  $R = \frac{\rho \ell}{A}$
- Temperature  $R_t = R_0 (1 + \alpha \Delta t)$

Where  $R_t$  = Resistance at  $t$  C,  $R_0$  = Resistance at 0 C

$\Delta t$  = Change in temperature,  $\alpha$  = Temperature coefficient of resistance

*\*[For metals :  $\alpha$  positive for semiconductors and insulators :  $\alpha$  negative]*

- Resistance of the conductor decreases linearly with decrease in temperature and becomes zero at a specific temperature. This temperature is called critical temperature. At this temperature conductor becomes a superconductor.

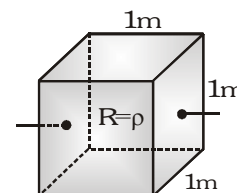


### RESISTIVITY

**Resistivity** :  $\rho = RA/\ell$  if  $\ell = 1\text{m}$ ,  $A = 1\text{m}^2$  then  $\rho = R$

The specific resistance of a material is equal to the resistance of the wire of that material with unit cross - section area and unit length.

**Resistivity depends on** (i) Nature of material (ii) Temperature of material  $\rho$  does not depend on the size and shape of the material because it is the characteristic property of the conductor material.



**Specific use of conducting materials :**

- The **heating element** of devices like heater, geyser, press etc are made of **microhm** because it has high resistivity and high melting point. It does not react with air and acquires steady state when red hot at 800 C.
- Fuse wire** is made of **tin lead alloy** because it has low melting point and low resistivity. The fuse is used in series, and melts to produce open circuit when current exceeds the safety limit.
- Resistances** of resistance box are made of **manganin** or **constantan** because they have moderate resistivity and very small temperature coefficient of resistance. The resistivity is nearly independent of temperature.
- The **filament of bulb** is made up of **tungsten** because it has low resistivity, high melting point of 3300 K and gives light at 2400 K. The bulb is filled with inert gas because at high temperature it reacts with air forming oxide.
- The **connection wires** are made of **copper** because it has low resistance and resistivity.

**COLOUR CODE FOR CARBON RESISTORS**

Colour	Strip A	Strip B	Strip C	Strip D (Tolerance)
Black	0	0	$10^0$	
Brown	1	1	$10^1$	
Red	2	2	$10^2$	
Orange	3	3	$10^3$	
Yellow	4	4	$10^4$	
Green	5	5	$10^5$	
Blue	6	6	$10^6$	
Violet	7	7	$10^7$	
Grey	8	8	$10^8$	
White	9	9	$10^9$	
Gold	-	-	$10^{-1}$	$\pm 5\%$
Silver	-	-	$10^{-2}$	$\pm 10\%$
No colour	-	-	-	$\pm 20\%$

May be remembered as  
**BBROY**  
 Great Britain  
 Very Good Wife.

**Example**

Draw a colour code for  $42 \text{ k } \Omega \pm 10\%$  carbon resistance.

**Solution**

According to colour code colour for digit 4 is yellow, for digit 2 it is red, for 3 colour is orange and 10% tolerance is represented by silver colour. So colour code should be yellow, red, orange and silver.

**Example**

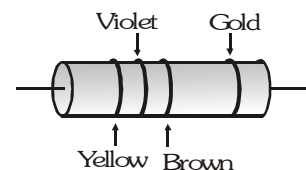
What is resistance of following resistor.

**Solution**

Number for yellow is 4, Number of violet is 7

Brown colour gives multiplier  $10^1$ , Gold gives a tolerance of  $\pm 5\%$

So resistance of resistor is  $47 \times 10^1 \Omega \pm 5\% = 470 \pm 5\% \Omega$ .





## COMBINATION OF RESISTORS

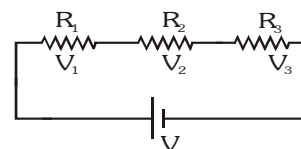
### Series Combination

- Same current passes through each resistance
- Voltage across each resistance is directly proportional to it's value

$$V_1 = IR_1, V_2 = IR_2, V_3 = IR_3$$

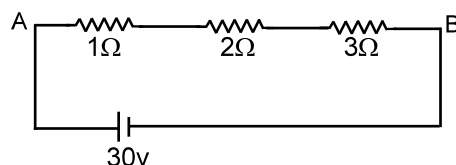
- Sum of the voltage across resistance is equal to the voltage applied across the circuit.

$$V = V_1 + V_2 + V_3 \Rightarrow IR = IR_1 + IR_2 + IR_3 \Rightarrow R = R_1 + R_2 + R_3 \quad \text{Where } R = \text{equivalent resistance}$$



### Example

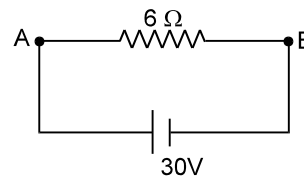
Find the current in the circuit



### Solution

$$R_{eq} = 1 + 2 + 3 = 6 \Omega \quad \text{the given circuit is equivalent to}$$

$$\text{current } i = \frac{V}{R_{eq}} = \frac{30}{6} = 5A$$



### Example

The resistance  $4R, 16R, 64R \dots \infty$  are connected in series. Find their equivalent resistance.

### Solution

$$\text{Resultant of the given combination } R_{eq} = 4R + 16R + 64R + \dots \infty = \infty$$

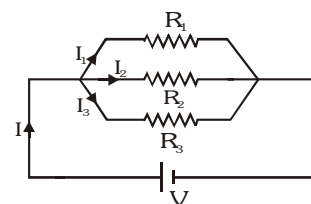
### Parallel Combination

- There is same drop of potential across each resistance.
- Current in each resistance is inversely proportional to the

$$\text{value of resistance. } I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3}$$

- Current flowing in the circuit is sum of the currents in individual resistance.

$$I = I_1 + I_2 + I_3 \Rightarrow \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \Rightarrow \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



### Example

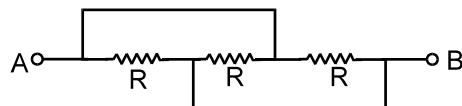
Resistance  $R, 2R, 4R, 8R \dots \infty$  are connected in parallel. What is their resultant resistance ?

### Solution

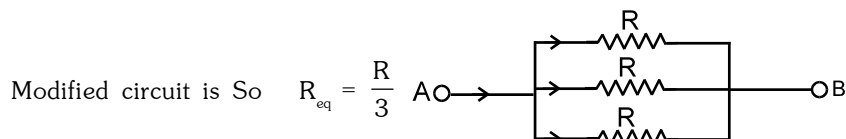
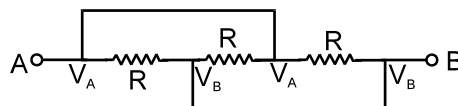
$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{4R} + \frac{1}{8R} \dots \infty = \frac{1}{R} \left[ 1 + \frac{1}{2} + \frac{1}{4} + \dots \infty \right] = \frac{1}{R} \left[ \frac{1}{1 - \frac{1}{2}} \right] = \frac{2}{R} \Rightarrow R_{eq} = \frac{R}{2}$$

**Example**

Find equivalent Resistance

**Solution**

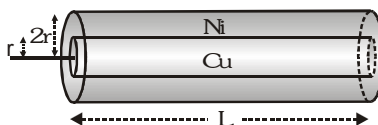
Here all the Resistance are connected between the terminals A and B

**Example**

A copper wire of length ' $\ell$ ' and radius ' $r$ ' is nickel plated till its final radius is  $2r$ . If the resistivity of the copper and nickel are  $\rho_{Cu}$  and  $\rho_{Ni}$ , then find the equivalent resistance of wire?

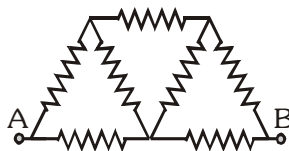
**Solution**

$$R = \rho \frac{\ell}{A}; \text{ Resistance of copper wire } R_{Cu} = \rho_{Cu} \frac{\ell}{\pi r^2} \quad (\because A = \pi r^2)$$

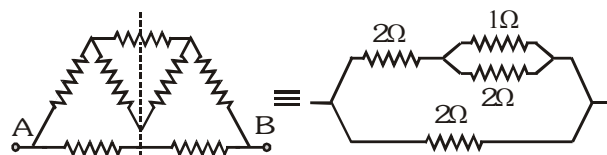


$$\therefore A_{Ni} = \pi(2r)^2 - \pi r^2 = 3\pi r^2 \Rightarrow \text{Resistance of Nickel wire } R_{Ni} = \rho_{Ni} \frac{\ell}{3\pi r^2}$$

Both wire are connected in parallel. So equivalent resistance  $R = \frac{R_{Cu} R_{Ni}}{R_{Cu} + R_{Ni}} = \left( \frac{\rho_{Cu} \rho_{Ni}}{3\rho_{Cu} + \rho_{Ni}} \right) \frac{\ell}{\pi r^2}$

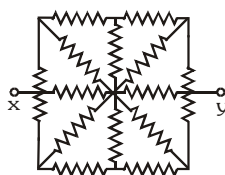
**Example**Each resistance is of  $1\ \Omega$  in the circuit diagram shown in figure. Find out equivalent resistance between A and B**Solution**

By symmetric line method  $R_{AB} = (2 + 1 \parallel 2) \parallel 2 = \frac{8}{7}\ \Omega$



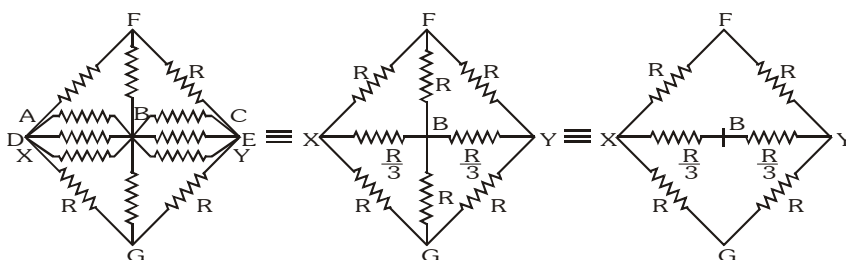
### Example

Identical resistance of resistance  $R$  are connected as in figure then find out net resistance between  $x$  and  $y$ .



### Solution

Given circuit can be modified according to following figures



$$\frac{1}{R_{xy}} = \frac{1}{2R} + \frac{3}{2R} + \frac{1}{2R} = \frac{5}{2R} \Rightarrow R_{xy} = \frac{2R}{5}$$

### KIRCHHOFF'S LAW

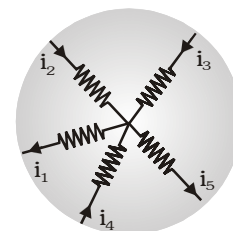
There are two laws given by Kirchhoff for determination of potential difference and current in different branches of any complicated network. **Law of conservation of charge is a consequence of continuity equation**

- First law (Junction Law or Current Law)**

In an electric circuit, the algebraic sum of the current meeting at any junction in the circuit is zero or Sum of the currents entering the Junction is equal to sum of the current leaving the Junction.  $\Sigma i = 0$

$$i_1 - i_2 - i_3 - i_4 + i_5 = 0 \Rightarrow i_1 + i_5 = i_2 + i_3 + i_4$$

This is based on law of conservation of charge.



- Second law (loop rule or potential law)**

In any closed circuit the algebraic sum of all potential differences and e.m.f. is zero.  $\Sigma E - \Sigma IR = 0$

while moving from negative to positive terminal inside the cell, e.m.f. is taken as positive while moving in the direction of current in a circuit the potential drop (i.e.  $IR$ ) across resistance is taken as positive.

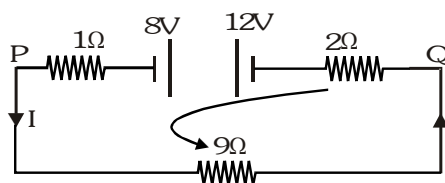
This law is based on law of conservation of energy.

## GOLDEN KEY POINTS

- If a wire is stretched to  $n$  times of its original length, its new resistance will be  $n^2$  times.
- If a wire is stretched such that its radius is reduced to  $\frac{1}{n}$ th of its original values, then resistance will increase  $n^4$  times similarly resistance will decrease  $n^4$  time if radius is increased  $n$  times by contraction.
- To get maximum resistance, resistance must be connected in series and in series the resultant is greater than largest individual.
- To get minimum resistance, resistance must be connected in parallel and the equivalent resistance of parallel combination is lower than the value of lowest resistance in the combination.
- Ohm's law is not a fundamental law of nature. As it is possible that for an element :-
  - $V$  depends on  $I$  non linearly (e.g. vacuum tubes)
  - Relation between  $V$  and  $I$  depends on the sign of  $V$  for the same value  
[Forward and reverse Bias in diode]
  - The relation between  $V$  and  $I$  is non unique. That is for the same  $I$  there is more than one value of  $V$ .
- In general :**
  - Resistivity of alloys is greater than their metals.
  - Temperature coefficient of alloys is lower than pure metals.
  - Resistance of most of non metals decreases with increase in temperature. (e.g. carbon)
  - The resistivity of an insulator (e.g. amber) is greater than the metal by a factor of  $10^{22}$
- Temperature coefficient ( $\alpha$ ) of semi conductor including carbon (graphite), insulator and electrolytes is negative.

## Example

In the given circuit calculate potential difference between the points P and Q.



## Solution

Applying Kirchhoff's voltage law (KVL)  $12 - 8 = (1)I + (9)I + (2)I \Rightarrow I = \frac{1}{3} \text{ A}$

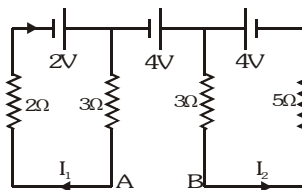
Potential difference between the points P and Q,  $V_P - V_Q = 9 \times \frac{1}{3} = 3\text{V}$

**Example**

In the given circuit calculate potential difference between A and B.

**Solution**

First applying KVL on left mesh  $2 - 3 I_1 - 2 I_1 = 0 \Rightarrow I_1 = 0.4$  amp.



Now applying KVL on right mesh.  $4 - 5 I_2 - 3 I_2 = 0 \Rightarrow I_2 = 0.5$  amp.

Potential difference between points A and B

$$V_A - V_B = -3 \cdot 0.4 - 4 + 3 \cdot 0.5 = -3.7 \text{ volt.}$$

**Example**

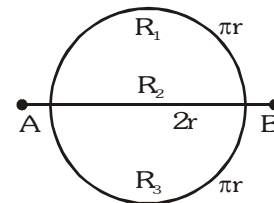
A wire of  $\rho_L = 10^{-6} \Omega/\text{m}$  is turned in the form of a circle of diameter 2 m. A piece of same material is connected in diameter AB. Then find resistance between A and B.

**Solution**

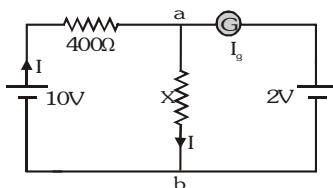
$$\because R = \rho_L \times \text{length}$$

$$\therefore R_1 = \pi \cdot 10^{-6} \Omega, R_2 = 2 \cdot 10^{-6} \Omega, R_3 = \pi \cdot 10^{-6} \Omega$$

$$\frac{1}{R_{AB}} = \frac{1}{\pi \times 10^{-6}} + \frac{1}{2 \times 10^{-6}} + \frac{1}{\pi \times 10^{-6}} ; \quad R_{AB} = 0.88 \cdot 10^{-6} \text{ ohm.}$$

**Example**

In the following circuit diagram, the galvanometer reading is zero. If the internal resistance of cells are negligible then what is the value of X ?

**Solution**

$$\because I_g = 0 \quad \therefore I = \frac{10}{400 + X} \text{ also potential difference across X is } 2V \Rightarrow I X = 2$$

$$\therefore I = \frac{10}{400 + X} \quad \frac{10X}{400 + X} = 2 \Rightarrow X = 100\Omega$$

**CELL**

Cell convert chemical energy into electrical energy.

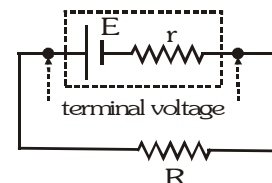
**ELECTRO MOTIVE FORCE (E. M. F.)**

The potential difference across the terminals of a cell when it is not giving any current is called emf of the cell. The energy given by the cell in the flow of unit charge in the whole circuit (including the cell) is called the emf of the cell.

- emf depends on : (i) nature of electrolyte (ii) metal of electrodes
- emf does not depend on : (i) area of plates (ii) distance between the electrodes  
(iii) quantity of electrolyte (iv) size of cell

**TERMINAL VOLTAGE (V)**

- When current is drawn through the cell or current is supplied to cell then, the potential difference across its terminals called terminal voltage.
- When I current is drawn from cell, then terminal voltage is less than it's e.m.f.  $V = E - Ir$

**INTERNAL RESISTANCE**

Offered by the electrolyte of the cell when the electric current flows through it is known as internal resistance.

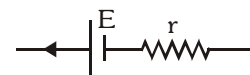
Distance between two electrodes increases  $\Rightarrow r$  increases

Area dipped in electrolyte increases  $\Rightarrow r$  decreases

Concentration of electrolyte increases  $\Rightarrow r$  increases

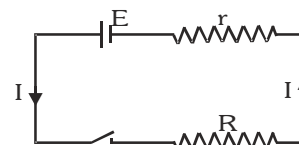
Temperature increases  $\Rightarrow r$  decreases

- **Terminal Potential Difference** : The potential difference between the two electrodes of a cell in a closed circuit i.e. when current is being drawn from the cell is called terminal potential difference.

**(a) When cell is discharging :**

Current inside the cell is from cathode to anode.

$$\text{Current } I = \frac{E}{r + R} \Rightarrow E = IR + Ir = V + Ir \Rightarrow V = E - Ir$$



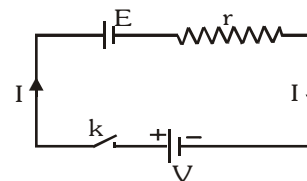
When current is drawn from the cell potential difference is less than emf of cell. Greater is the current drawn from the cell smaller is the terminal voltage. When a large current is drawn from a cell its terminal voltage is reduced.

**(b) When cell is charging :**

Current inside the cell is from anode to cathode.

$$\text{Current } I = \frac{V - E}{r} \Rightarrow V = E + Ir$$

During charging terminal potential difference is greater than emf of cell.

**(c) When cell is in open circuit :**

$$\text{In open circuit } R = \infty \quad \therefore I = \frac{E}{R + r} = 0 \Rightarrow V = E$$

In open circuit terminal potential difference is equal to emf and is the maximum potential difference which a cell can provide.

**(d) When cell is short circuited :**

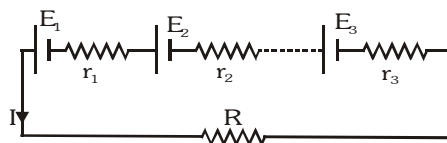
$$\text{In short circuit } R = 0 \quad \Rightarrow I = \frac{E}{R + r} = \frac{E}{r} \text{ and } V = IR = 0$$

In short circuit current from cell is maximum and terminal potential difference is zero.

## COMBINATION OF CELLS

### • Series combination

When the cells are connected in series the total e.m.f. of the series combination is equal to the sum of the e.m.f.'s of the individual cells and internal resistance of the cells also come in series.



Equivalent internal resistance  $r = r_1 + r_2 + r_3 + \dots$       Equivalent emf  $= E = E_1 + E_2 + E_3 + \dots$

Current  $I = \frac{E_{\text{net}}}{r_{\text{net}} + R}$ ,      If all  $n$  cell are identical then  $I = \frac{nE}{nr + R}$

• If  $nr \gg R$ ,  $I = \frac{E}{r} \approx$  current from one cell      • If  $nr \ll R$ ,  $I = \frac{nE}{R} \approx n$  current from one cell

### • Parallel combination

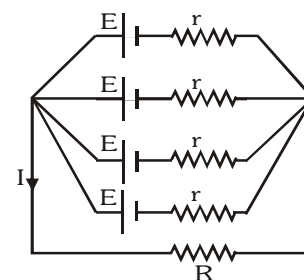
When the cells are connected in parallel, the total e.m.f. of the parallel combination remains equal to the e.m.f. of a single cell and internal resistance of the cell also come in parallel. If  $m$  identical cell connected in parallel

then total internal resistance of this combination  $r_{\text{net}} = \frac{r}{m}$ . Total e.m.f. of this combination  $= E$

Current in the circuit  $I = \frac{E}{R + \frac{r}{m}} = \frac{mE}{mR + r}$

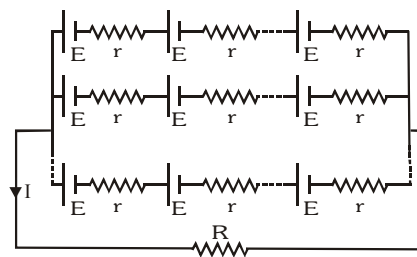
If  $r \ll mR$        $I = E/R =$  Current from one cell

If  $r \gg mR$        $I = \frac{mE}{r} = m$  current from one cell



### • Mixed combination

If  $n$  cells connected in series and their are  $m$  such branches in the circuit then total number of identical cell in this circuit is  $nm$ . The internal resistance of the cells connected in a row  $= nr$ . Since there are such  $m$  rows,



Total internal resistance of the circuit  $r_{\text{net}} = \frac{nr}{m}$

Total e.m.f. of the circuit  $=$  total e.m.f. of the cells connected in a row  $E_{\text{net}} = nE$

Current in the circuit  $I = \frac{E_{\text{net}}}{R + r_{\text{net}}} = \frac{nE}{R + \frac{nr}{m}}$

Current in the circuit is maximum when external resistance in the circuit is equal to the total internal resistance

of the cells  $R = \frac{nr}{m}$

## GOLDEN KEY POINTS

- At the time of charging a cell when current is supplied to the cell, the terminal voltage is greater than the e.m.f.  $E$ ,  $V = E + Ir$
- Series combination is useful when internal resistance is less than external resistance of the cell.
- Parallel combination is useful when internal resistance is greater than external resistance of the cell.
- Power in  $R$  (given resistance) is maximum, if its value is equal to net resistance of remaining circuit.
- Internal resistance of ideal cell = 0
- if external resistance is zero than current given by circuit is maximum.

## Example

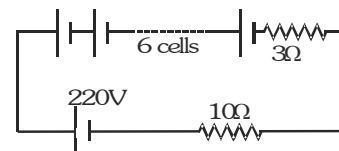
A battery of six cells each of e.m.f.  $2\text{ V}$  and internal resistance  $0.5\ \Omega$  is being charged by D. C. mains of e.m.f.  $220\text{ V}$  by using an external resistance of  $10\ \Omega$ . What will be the charging current.

## Solution

Net e.m.f of the battery =  $12\text{ V}$  and total internal resistance =  $3\ \Omega$

Total resistance of the circuit =  $3 + 10 = 13\ \Omega$

$$\text{Charging current } I = \frac{\text{Net e.m.f.}}{\text{total resistance}} = \frac{220 - 12}{13} = 16\text{ A}$$



## Example

A battery of six cells each of e.m.f.  $2\text{ V}$  and internal resistance  $0.5\ \Omega$  is being charged by D. C. mains of e.m.f.  $220\text{ V}$  by using an external resistance of  $10\ \Omega$ . What is the potential difference across the battery?

## Solution

In case of charging of battery, terminal potential  $V = E + Ir = 12 + 16 \times 3 = 60\text{ volt}$ .

## Example

Four identical cells each of e.m.f.  $2\text{ V}$  are joined in parallel providing supply of current to external circuit consisting of two  $15\ \Omega$  resistors joined in parallel. The terminal voltage of the equivalent cell as read by an ideal voltmeter is  $1.6\text{ V}$  calculate the internal resistance of each cell.

## Solution

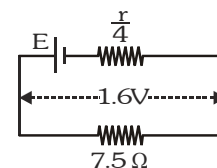
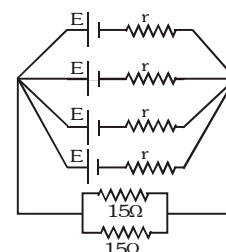
$$\text{Total internal resistance of the combination } r_{\text{eq}} = \frac{r}{4}$$

$$\text{Total e.m.f. } E_{\text{eq}} = 2\text{ V}$$

$$\text{Total external resistance } R = \frac{15 \times 15}{15 + 15} = \frac{15}{2} = 7.5\ \Omega$$

$$\text{Current drawn from equivalent cell } I = \frac{\text{terminal potential}}{\text{external resistance}} = \frac{1.6}{7.5}\text{ A}$$

$$\therefore E - I\left(\frac{r}{4}\right) = 1.6 \quad \therefore 2 - \frac{1.6}{7.5}\left(\frac{r}{4}\right) = 1.6 \Rightarrow r = 7.5\ \Omega$$





**Example**

The e.m.f. of a primary cell is 2 V, when it is shorted then it gives a current of 4 A. Calculate internal resistance of primary cell.

**Solution**

$$I = \frac{E}{r + R}, \text{ If cell is shorted then } R = 0, I = \frac{E}{r} \quad \therefore r = \frac{E}{I} = \frac{2}{4} = 0.5 \, \Omega$$

**Example**

$n$  rows each containing  $m$  cells in series, are joined in parallel. Maximum current is taken from this combination in a  $3 \, \Omega$  resistance. If the total number of cells used is 24 and internal resistance of each cell is  $0.5 \, \Omega$ , find the value of  $m$  and  $n$ .

**Solution**

$$\text{Total number of cell } mn = 24, \text{ For maximum current } \frac{mr}{n} = R \Rightarrow 0.5 m = 3 n, m = \frac{3n}{0.5} = 6n$$

$$\therefore 6n \cdot n = 24 \Rightarrow n = 2 \text{ and } m \cdot 2 = 24 \Rightarrow m = 12$$

**GALVANOMETER**

The instrument used to measure strength of current, by measuring the deflection of the coil due to torque produced by a magnetic field, is known as galvanometer.

**SHUNT**

The small resistance connected in parallel to galvanometer coil, in order to control current flowing through the galvanometer, is known as shunt.

- Merits of shunt**

- To protect the galvanometer coil from burning.
- Any galvanometer can be converted into ammeter of desired range with the help of shunt.
- The range an ammeter can be changed by using shunt resistance of different values.

- Demerits of shunt**

Shunt resistance decreases the sensitivity of galvanometer.

**CONVERSION OF GALVANOMETER INTO AMMETER**

A galvanometer can be converted into an ammeter by connecting low resistance in parallel to its coil.

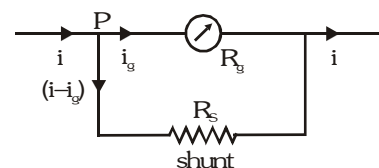
- The value of shunt resistance to be connected in parallel to galvanometer

$$\text{coil is given by : } R_s = \frac{R_g i_g}{i - i_g}$$

Where  $i$  = Range of ammeter

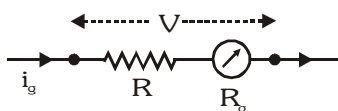
$i_g$  = Current required for full scale deflection of galvanometer.

$R_g$  = Resistance of galvanometer coil.



## CONVERSION OF GALVANOMETER INTO VOLTMETER

- The galvanometer can be converted into voltmeter by connecting high resistance in series with its coil.
- The high resistance to be connected in series with galvanometer coil is given by  $R = \frac{V}{i_g} - R_g$



### GOLDEN KEY POINT

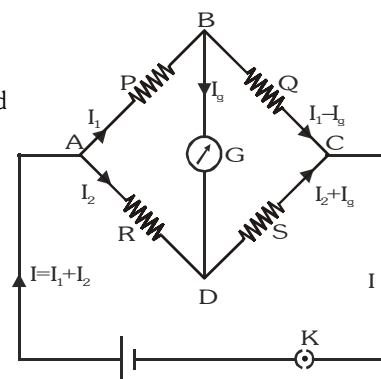
- The rate of variation of deflection depends upon the magnitude of deflection itself and so the accuracy of the instrument.
- A suspended coil galvanometer can measure currents of the order of  $10^{-9}$  ampere.
- $I_g$  is the current for full scale deflection. If the current for a deflection, of one division on the galvanometer scale is  $k$  and  $N$  is the total number of divisions on one side of the zero of galvanometer scale, then  $I_g = k N$ .
- A ballistic galvanometer is a specially designed moving coil galvanometer, used to measure charge flowing through the circuit for small time intervals.

## WHEAT STONE BRIDGE

- The configuration in the adjacent figure is called Wheat Stone Bridge.
- If current in galvanometer is zero ( $I_g = 0$ ) then bridge is said to be balanced

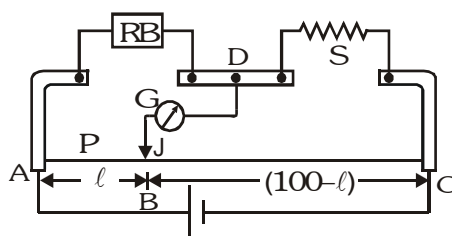
$$V_D = V_B \Rightarrow I_1 P = I_2 R \text{ \& } I_1 Q = I_2 S \Rightarrow \frac{P}{Q} = \frac{R}{S}$$

- If  $\frac{P}{Q} < \frac{R}{S}$  then  $V_B > V_D$  and current will flow from B to D.
- If  $\frac{P}{Q} > \frac{R}{S}$  then  $V_B < V_D$  and current will flow from D to B.



## METRE BRIDGE

It is based on principle of whetstone bridge. It is used to find out unknown resistance of wire. AC is 1 m long uniform wire R.B. is known resistance and S is unknown resistance. A cell is connected across 1 m long wire and Galvanometer is connected between Jockey and midpoint D. To find out unknown resistance we touch jockey from A to C and find balance condition. Let balance is at B point on wire.



$$AB = l \text{ cm}$$

$$BC = (100 - l) \text{ cm}$$

$$P = r l$$

$$Q = r(100 - l) \quad \text{where } r = \text{resistance per unit length on wire.}$$

$$\text{At balance condition : } \frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{r l}{r(100 - l)} = \frac{R}{S} \Rightarrow S = \frac{(100 - l)}{l} R$$

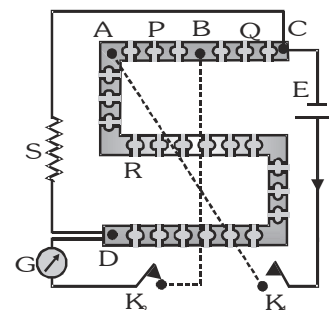
## POST OFFICE BOX

It is also based on wheat stone bridge. The resistance of  $10\Omega$ ,  $100\Omega$ , and  $1000\Omega$  are often connected between AB and BC. These are known as ratio arms. Resistance from  $1\Omega$  to  $5000\Omega$  are connected between A and D, this is known arm. Unknown resistance is connected between C and D.

A cell is connected between A and C with key  $K_1$  and Galvanometer is connected between B and D with key  $K_2$ .

First we select ratio of resistance Q and P. For given value of S we will take value of resistance from known arm in such a way that Galvanometer

show null deflection  $S = \frac{Q}{P} R$ . On decreasing the value of  $\frac{Q}{P}$  the sensitivity of the box increases. It is used to find out the breakage in telegraph line in post and telegraph offices.

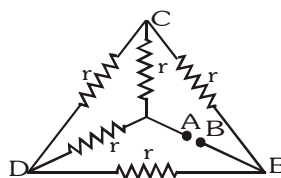


## GOLDEN KEY POINT

- To increase the range of an ammeter a shunt is connected in parallel with the galvanometer.
- To convert an ammeter of range  $I$  ampere and resistance  $R_g \Omega$  into an ammeter of range  $nI$  ampere, the value of resistance to be connected in parallel will be  $R_g(n - 1)$
- To increase the range of a voltmeter a high resistance is connected in series with it.
- To convert a voltmeter of resistance  $R_g \Omega$  and range  $V$  volt into a voltmeter of range  $nV$  volt, the value of resistance to be connected in series will be  $(n - 1)R_g$ .
- Resistance of ideal ammeter is zero & resistance of ideal voltmeter is infinite.
- The bridge is most sensitive when the resistance in all the four branches of the bridge is of same order.

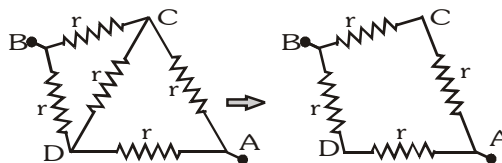
## Example

In the adjoining network of resistors each is of resistance  $r \Omega$ . Find the equivalent resistance between point A and B



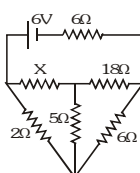
## Solution

Given circuit is balanced Wheat stone Bridge  $\therefore \frac{1}{R_{AB}} = \frac{1}{2r} + \frac{1}{2r} = \frac{1}{r}$   $R_{AB} = r$



## Example

Calculate magnitude of resistance  $X$  in the circuit shown in figure when no current flows through the  $5\Omega$  resistor?



**Solution**

Since wheat stone bridge is balanced so  $\frac{x}{18} = \frac{2}{6}$  or  $x = \frac{18 \times 2}{6} = 6\Omega$

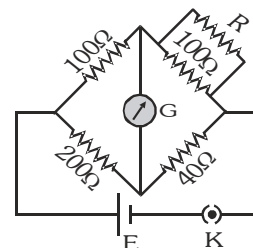
**Example**

For the following diagram the galvanometer shows zero deflection then what is the value of R ?

**Solution**

For balanced Wheat stone bridge  $\frac{100}{100R} = \frac{200}{(100 + R)}$

$$\Rightarrow \frac{100 + R}{R} = 5 \Rightarrow 100 + R = 5R \Rightarrow R = \frac{100}{4} = 25\Omega$$

**Example**

A 100 volt voltmeter whose resistance is  $20\text{ k}\Omega$  is connected in series to a very high resistance R. When it is joined in a line of 110 volt, it reads 5 volt. What is the magnitude of resistance R ?

**Solution**

When voltmeter connected in 110 volt line, Current through the voltmeter  $I = \frac{110}{(20 \times 10^3 + R)}$

The potential difference across the voltmeter  $V = IR_V \Rightarrow 5 = \frac{110 \times 20 \times 10^3}{(20 \times 10^3 + R)}$

$$\Rightarrow 20 \times 10^3 + R = 440 \times 10^3 \Rightarrow R = 420 \times 10^3 \Omega$$

**Example**

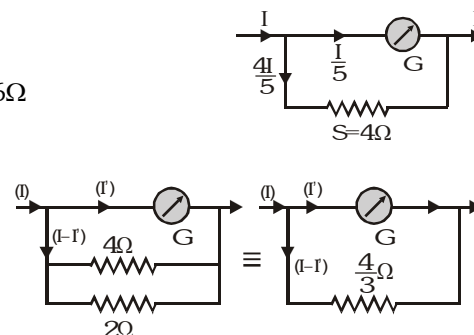
When a shunt of  $4\Omega$  is attached to a galvanometer, the deflection reduces to  $1/5^{\text{th}}$ . If an additional shunt of  $2\Omega$  is attached what will be the deflection ?

**Solution**

Initial condition : When shunt of  $4\Omega$  used  $\frac{I}{5} \times G = \frac{4}{5} I \times 4 \Rightarrow G = 16\Omega$

When additional shunt of  $2\Omega$  used  $I' \times 16 = (I - I') \times \frac{4}{3} \Rightarrow I' = \frac{I}{13}$

$\therefore$  it will reduce to  $\frac{I}{13}$  of the initial deflection

**Example**

A galvanometer having 30 divisions has current sensitivity of  $20\mu\text{A}/\text{division}$ . It has a resistance of  $25\Omega$ .

- How will you convert it into an ammeter measuring upto 1 ampere.
- How will you convert this ammeter into a voltmeter upto 1 volt.

**Solution**

The current required for full scale deflection  $I_g = 20\mu\text{A}$   $30 = 600\mu\text{A} = 6 \times 10^{-4}\text{A}$

- To convert it into ammeter, a shunt is required in parallel with it shunt resistance

$$R'_s = \frac{I_g R_g}{(I - I_g)} = \left( \frac{6 \times 10^{-4}}{1 - 6 \times 10^{-4}} \right) 25 = 0.015\Omega$$

- To convert galvanometer into voltmeter, a high resistance in series with it is required

$$\text{series resistance } R = \frac{V}{I_g} - R_g = \frac{1}{6 \times 10^{-4}} - 25 = 1666.67 - 25 = 1641.67 \Omega$$

## POTENTIOMETER

### • Necessity of potentiometer

Practically voltmeter has a finite resistance. (ideally it should be  $\infty$ ) in other words it draws some current from the circuit. To overcome this problem potentiometer is used because at the instant of measurement, it draws no current from the circuit. It means its effective resistance is infinite.

### • Working principle of potentiometer

Any unknown potential difference is balanced on a known potential difference which is uniformly distributed over entire length of potentiometer wire. This process is named as zero deflection or null deflection method.

### • Potentiometer wire

Made up of alloys of magnin, constantan, Eureka. Specific properties of these alloys are high specific resistance, negligible temperature co-efficient of resistance ( $\alpha$ ). Invariability of resistance of potentiometer wire over a long period.

## CIRCUITS OF POTENTIOMETER

- **Primary circuit** contains constant source of voltage rheostat or Resistance Box

- **Secondary, Unknown or galvanometer circuit**

Let  $\rho$  = Resistance per unit length of potentiometer wire

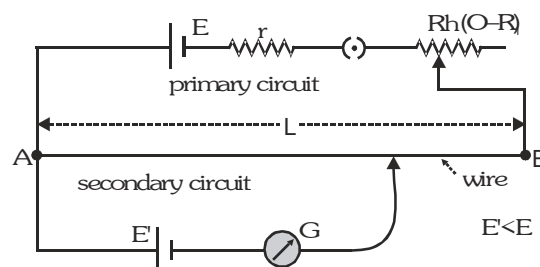
- **Potential gradient (x) (V/m)**

The fall of potential per unit length of potentiometer wire is called potential gradient.

$$x = \frac{V}{L} = \frac{\text{current} \times \text{resistance of potentiometer wire}}{\text{length of potentiometer wire}} = I \left( \frac{R}{L} \right)$$

The potential gradient depends only on primary circuit and is independent of secondary circuit.

- **Applications of potentiometer**



- To measure potential difference across a resistance.
- To find out emf of a cell
- Comparison of two emfs  $E_1/E_2$
- To find out internal resistance of a primary cell
- Comparison of two resistance.
- To find out an unknown resistance which is

- connected in series with the given resistance.
- To find out current in a given circuit
- Calibration of an ammeter or to have a check on reading of (A)
- Calibration of a voltmeter or to have a check on reading of (V)
- To find out thermocouple emf (e) (mV or mV)

### *difference between potentiometer and voltmeter*

Potentiometer	Voltmeter
<ul style="list-style-type: none"> <li>• It measures the unknown emf very accurately</li> <li>• While measuring emf it does not draw any current from the driving source of known emf.</li> <li>• While measuring unknown potential difference the resistance of potentiometer becomes infinite.</li> <li>• It is based on zero deflection method.</li> <li>• It has a high sensitivity.</li> <li>• it is used for various applications like measurement of internal resistance of cell, calibration of ammeter and voltmeter, measurement of thermo emf, comparison of emf's etc.</li> </ul>	<p>It measures the unknown emf approximately. While measuring emf it draws some current from the source of emf. While measuring unknown potential difference the resistance of voltmeter is high but finite. It is based on deflection method. Its sensitivity is low. It is only used to measure emf or unknown potential difference.</p>

**Example**

There is a definite potential difference between the two ends of a potentiometer. Two cells are connected in such a way that first time help each other, and second time they oppose each other. They are balanced on the potentiometer wire at 120 cm and 60 cm length respectively. Compare the electromotive force of the cells.

**Solution**

Suppose the potential gradient along the potentiometer wire =  $x$  and the emf's of the two cells are  $E_1$  and  $E_2$ .

When the cells help each other, the resultant emf =  $(E_1 + E_2)$   $E_1 + E_2 = x \cdot 120 \text{ cm} \dots(i)$

When the cells oppose each other, the resultant emf =  $(E_1 - E_2)$   $E_1 - E_2 = x \cdot 60 \text{ cm} \dots(ii)$

$$\text{From equation (i) and (ii)} \quad \frac{E_1 + E_2}{E_1 - E_2} = \frac{120 \text{ cm}}{60 \text{ cm}} = \frac{2}{1} \Rightarrow E_1 + E_2 = 2(E_1 - E_2) \Rightarrow 3E_2 = E_1 \Rightarrow \frac{E_1}{E_2} = \frac{3}{1}$$

**HEATING EFFECT OF CURRENT****CAUSE OF HEATING**

The potential difference applied across the two ends of conductor sets up electric field. Under the effect of electric field, electrons accelerate and as they move, they collide against the ions and atoms in the conductor, the energy of electrons transferred to the atoms and ions appears as heat.

- Joules's Law of Heating**

When a current  $I$  is made to flow through a passive or ohmic resistance  $R$  for time  $t$ , heat  $Q$  is produced such that

$$Q = I^2 R t = P t = V I t = \frac{V^2}{R} t$$

Heat produced in conductor does not depend upon the direction of current.

- SI unit** : joule ;

**Practical Units** : 1 kilowatt hour (kWh)

$$1 \text{ kWh} = 3.6 \times 10^6 \text{ joule} = 1 \text{ unit}$$

$$1 \text{ BTU (British Thermal Unit)} = 1055 \text{ J}$$

- Power** :  $P = V I = \frac{V^2}{R} = I^2 R$

- SI unit** : Watt

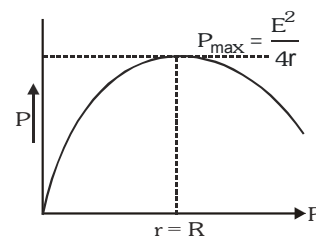
The watt-hour meter placed on the premises of every consumer records the electrical energy consumed.

- Power transferred to load by cell :**

$$P = I^2 R = \frac{E^2 R}{(r + R)^2} \Rightarrow P = P_{\max} \quad \text{if} \quad \frac{dP}{dR} = 0 \Rightarrow r = R$$

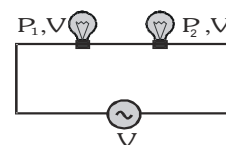
Power transferred by cell to load is maximum when

$$r = R \text{ and } P_{\max} = \frac{E^2}{4r} = \frac{E^2}{4R}$$



- Series combination of resistors (bulbs)**

$$\text{Total power consumed } P_{\text{total}} = \frac{P_1 P_2}{P_1 + P_2} \quad \text{If } n \text{ bulbs are identical } P_{\text{total}} = \frac{P}{n}$$



$$\text{In series combination of bulbs Brightness} \propto \text{Power consumed by bulb} \propto V \propto R \propto \frac{1}{P_{\text{rated}}}$$

Bulb of lesser wattage will shine more. For same current  $P = I^2 R$   $P \propto R$   $R \uparrow \Rightarrow P \uparrow$

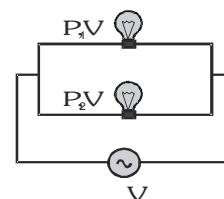
• **Parallel combination of resistors (bulbs)**

Total power consumed  $P_{\text{total}} = P_1 + P_2$

If  $n$  bulbs are identical  $P_{\text{total}} = nP$

In parallel combination of bulbs

Brightness  $\propto$  Power consumed by bulb  $\propto I \propto \frac{1}{R}$  Bulb of greater wattage will shine more.



For same  $V$  more power will be consumed in smaller resistance  $P \propto \frac{1}{R}$

- Two identical heater coils gives total heat  $H_s$  when connected in series and  $H_p$  when connected in parallel than

$\frac{H_p}{H_s} = 4$  [In this, it is assumed that supply voltage is same]

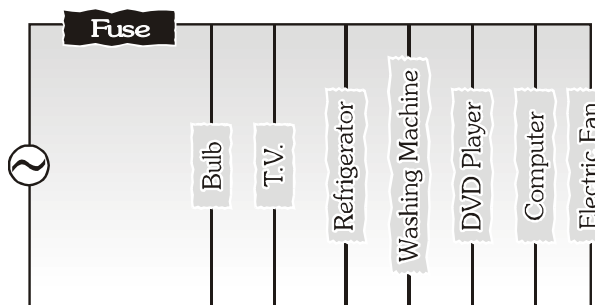
- If a heater boils  $m$  kg water in time  $T_1$  and another heater boils the same water in  $T_2$ , then both connected in

series will boil the same water in time  $T_s = T_1 + T_2$  and in parallel  $T_p = \frac{T_1 T_2}{T_1 + T_2}$  [Use time taken  $\propto$  Resistance]

- Instruments based on heating effect of current, works on both A.C. and D.C. Equal value of A.C. (RMS) and D.C. produces, equal heating effect. That why brightness of bulb is same whether it is operated by A.C. or same value D.C.

**FUSE WIRE**

The fuse wire for an electric circuit is chosen keeping in view the value of safe current through the circuit.



- The fuse wire should have high resistance per unit length and low melting point.
- However the melting point of the material of fuse wire should be above the temperature that will be reached on the passage of the current through the circuit
- A fuse wire is made of alloys of lead (Pb) and tin (Sn).
- Length of fuse wire is immaterial.
- The material of the filament of a heater should have high resistivity and high melting point.
- The temperature of the wire increases to such a value at which, the heat produced per second equals heat lost

per second due to radiation from the surface of wire  $I^2 \left( \frac{\rho \ell}{\pi r^2} \right) = H \times 2\pi r \ell$   $I^2 \propto r^3$

$H$  = heat lost per second per unit area due to radiation.

**Example**

An electric heater and an electric bulb are rated 500 W, 220 V and 100 W, 220 V respectively. Both are connected in series to a 220 V a.c. mains. Calculate power consumed by (i) heater (ii) bulb.

**Solution**

$$P = \frac{V^2}{R} \text{ or } R = \frac{V^2}{P}, \text{ For heater. Resistance } R_h = \frac{(220)^2}{500} = 96.8 \, \Omega, \text{ For bulb resistance } R_L = \frac{(220)^2}{100} = 484 \, \Omega$$

$$\text{Current in the circuit when both are connected in series } I = \frac{V}{R_L + R_h} = \frac{220}{484 + 96.8} = 0.38 \, \text{A}$$

$$(i) \text{ Power consumed by heater } = I^2 R_h = (0.38)^2 \times 96.8 = 13.98 \, \text{W}$$

$$(ii) \text{ Power consumed by bulb } = I^2 R_L = (0.38)^2 \times 484 = 69.89 \, \text{W}$$

**Example**

A heater coil is rated 100 W, 200 V. It is cut into two identical parts. Both parts are connected together in parallel, to the same source of 200 V. Calculate the energy liberated per second in the new combination.

**Solution**

$$\therefore P = \frac{V^2}{R} \quad \therefore R = \frac{V^2}{P} = \frac{(200)^2}{100} = 400 \, \Omega$$

$$\text{Resistance of half piece} = \frac{400}{2} = 200 \, \Omega$$

$$\text{Resistance of pieces connected in parallel} = \frac{200}{2} = 100 \, \Omega$$

$$\text{Energy liberated/second } P = \frac{V^2}{R} = \frac{200 \times 200}{100} = 400 \, \text{W}$$

**Example**

The power of a heater is 500W at 800 C. What will be its power at 200 C. If  $\alpha = 4 \times 10^{-4}$  per C ?

$$\text{Solution } P = \frac{V^2}{R} \quad \therefore \frac{P_{200}}{P_{800}} = \frac{R_{800}}{R_{200}} = \frac{R_0(1 + 4 \times 10^{-4} \times 800)}{R_0(1 + 4 \times 10^{-4} \times 200)} \Rightarrow P_{200} = \frac{500 \times 1.32}{1.08} = 611 \, \text{W}$$

**Example**

When a battery sends current through a resistance  $R_1$  for time  $t$ , the heat produced in the resistor is  $Q$ . When the same battery sends current through another resistance  $R_2$  for time  $t$ , the heat produced in  $R_2$  is again  $Q$ . Determine the internal resistance of battery.

**Solution**

$$\left[ \frac{E}{R_1 + r} \right]^2 R_1 = \left[ \frac{E}{R_2 + r} \right]^2 R_2 \Rightarrow r = \sqrt{R_1 R_2}$$

**Example**

A fuse with a circular cross-sectional radius of 0.15 mm blows at 15A. What is the radius of a fuse, made of the same material which will blow at 120 A?

**Solution**

$$\text{For fuse wire } I \propto r^{3/2}$$

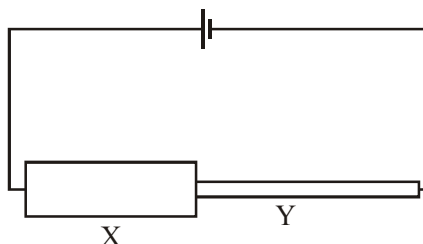
$$\text{so } \frac{r_2}{r_1} = \left( \frac{I_2}{I_1} \right)^{2/3} = \left( \frac{120}{15} \right)^{2/3} = (8)^{2/3} = 4 \Rightarrow r_2 = 4r_1 = 0.60 \, \text{mm}$$



## SOME WORKED OUT EXAMPLES

### Example#1

Figure shows a thick copper rod X and a thin copper wire Y, joined in series. They carry a current which is sufficient to make Y much hotter than X. Which one of the following is correct?



#### Density of conduction electrons

- (A) Same in X and Y
- (B) Same in X and Y
- (C) Same in X and Y
- (D) More in X than Y

#### Mean time between collisions of the electrons

- Less in X than Y
- Same in X and Y
- More in X than Y
- less in X than Y

### Solution

**Ans. (C)**

The number density  $n$  of conduction electrons in the copper is a characteristic of the copper and is about  $10^{29}$  at room temperature for both the copper rod X and the thin copper wire Y.

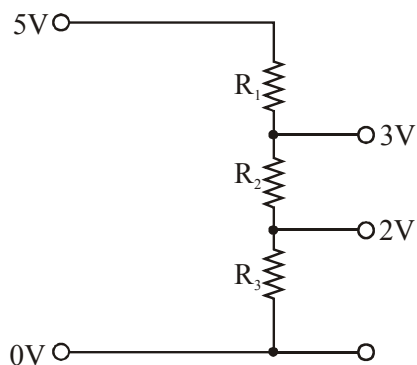
Both X and Y carry the same current  $I$  since they are joined in series.

From  $I = neAv_d$

We may conclude that rod X has a lower drift velocity of electrons compared to wire Y since rod X has larger cross-sectional area. This is so because the electrons in X collide more often with one another and with the copper ions when drifting towards the positive end. Thus, the mean time between collisions of the electrons is more in X and than in Y.

### Example#2

A potential divider is used to give outputs of 2V and 3V from a 5V source, as shown in figure. Which combination of resistances,  $R_1$ ,  $R_2$  and  $R_3$  gives the correct voltages?



- |     | $R_1$        | $R_2$        | $R_3$        |
|-----|--------------|--------------|--------------|
| (A) | 1 k $\Omega$ | 1 k $\Omega$ | 2 k $\Omega$ |
| (B) | 2 k $\Omega$ | 1 k $\Omega$ | 2 k $\Omega$ |
| (C) | 3 k $\Omega$ | 2 k $\Omega$ | 2 k $\Omega$ |
| (D) | 3 k $\Omega$ | 2 k $\Omega$ | 3 k $\Omega$ |

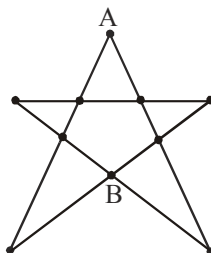
**Solution****Ans. (B)**

For resistors in series connection, current ( $I$ ) is the same through the resistors. In other words, ratio of the voltage drop across each resistor with its resistance is the same.

That is  $I = \frac{5-3}{R_1} = \frac{3-2}{R_2} = \frac{2}{R_3}$  i.e.,  $R_1 : R_2 : R_3 = 2 : 1 : 2$ .

**Example#3**

The resistance of all the wires between any two adjacent dots is  $R$ . The equivalent resistance between A and B as shown in figure is

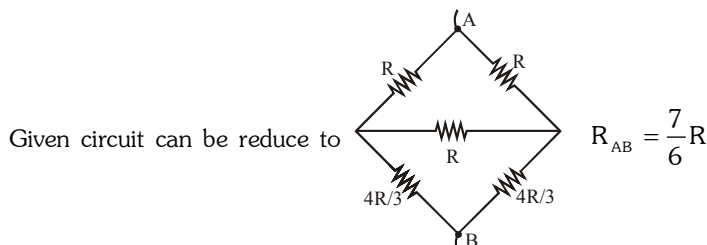


(A)  $\frac{7}{3} R$

(B)  $\frac{7}{6} R$

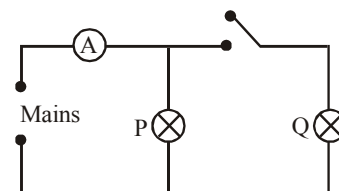
(C)  $\frac{14}{8} R$

(D) None of these

**Solution****Ans. (B)****Example#4**

How will the reading in the ammeter A of figure be affected if another identical bulb Q is connected in parallel to P as shown. The voltage in the mains is maintained at a constant value.

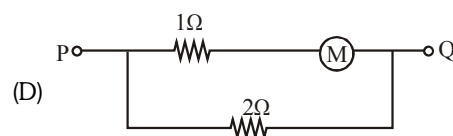
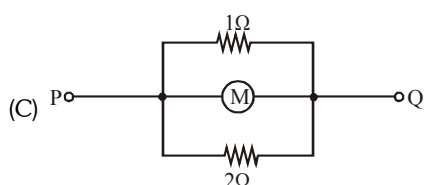
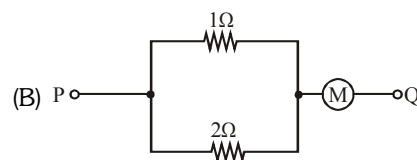
- (A) The reading will be reduced to one-half  
 (B) The reading will not be affected  
 (C) The reading will be doubled of the previous one  
 (D) The reading will be increased four-fold

**Solution****Ans. (C)**

Resistance is halved . Current is doubled.

**Example#5**

In which one of the following arrangements of resistors does the ammeter M, which has a resistance of  $2\Omega$ , give the largest reading when the same potential difference is applied between points P and Q?



**Solution**

Ans. (C)

Let  $V_{PQ} = E$

For (A) :  $I = \frac{E}{4}$

For (B) :  $I = \frac{E}{\frac{2}{3} + 2} = \frac{E}{\frac{8}{3}}$

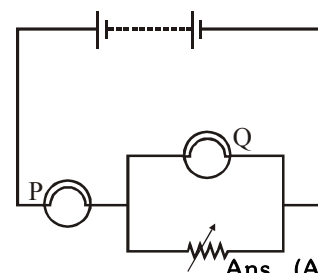
For (C) :  $I = \frac{E}{2}$

For (D) :  $I = \frac{E}{3}$

**Example#6**

The circuit shown in figure, contains a battery, a rheostat and two identical lamps. What will happen to the brightness of the lamps if the resistance of the rheostat is increased?

Lamp P	Lamp Q
(A) Less brighter	Brighter
(B) Less brighter	Less brighter
(C) Brighter	Less brighter
(D) No change	Brighter



Ans. (A)

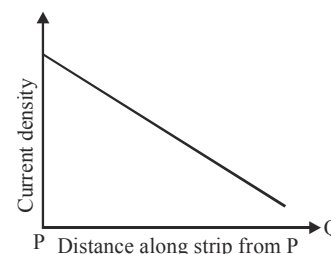
**Solution**

Consider two extreme cases. (i) When the resistance of the rheostat is zero, the current through Q is zero since Q is short-circuited. The circuit is then essentially a battery in series with lamp P. (ii) When the resistance of the rheostat is very large, almost no current flows through it. So, the currents through P and Q are almost equal. The circuit is essentially a battery in series with lamps P and Q.

**Example#7**

An electric current flows along an insulated strip PQ of a metallic conductor. The current density in the strip varies as shown in the graph. Which one of the following statements could explain this variation?

- (A) The strip is narrower at P than at Q
- (B) The strip is narrower at Q than at P
- (C) The potential gradient along the strip is uniform
- (D) The resistance per unit length of the strip is constant



Ans. (A)

**Solution**

The current density at P is higher than at Q. For the same current flowing through the metallic conductor PQ,

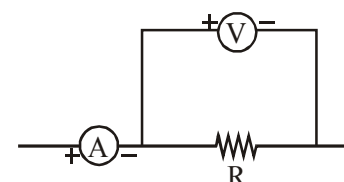
the cross-sectional area at P is narrower than at Q. The resistance per unit length  $r$  is given by  $r = \frac{\rho}{A}$

where  $\rho$  is the resistivity and  $A$  is the cross-sectional area of the conductor PQ. Thus,  $r$  is inversely proportional to the cross-sectional area  $A$  of the conductor.

**Example#8**

A candidate connects a moving coil voltmeter V, a moving coil ammeter A and a resistance R as shown in figure. If the voltmeter reads 20 V and the ammeter reads 4A, R is

- (A) equal to  $5\Omega$
- (B) greater than  $5\Omega$
- (C) less than  $5\Omega$
- (D) greater or less than  $5\Omega$  depending upon its material



Ans. (B)

**Solution**

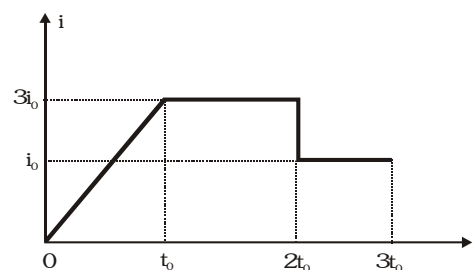
Let a current of  $x$  ampere passes through the voltmeter, then  $(4-x)$  ampere passes through the resistance R.

Therefore, voltmeter reading  $20 = (40 - x) R \therefore R = \frac{20}{4-x}$ , i.e.,  $R > 5\Omega$

**Example#9**

A time varying current  $i$  is passed through a resistance  $R$  as shown in figure. The total heat generated in the resistance is

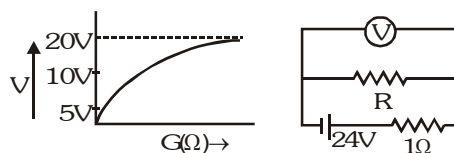
- (A)  $11i_0^2Rt_0$   
 (B)  $13i_0^2Rt_0$   
 (C)  $17i_0^2Rt_0$   
 (D)  $15i_0^2Rt_0$

**Solution****Ans. (B)**

$$\text{Total heat produced} = \int_0^{t_0} \left(\frac{3i_0}{t_0}t\right)^2 R dt + (3i_0)^2 R (2t_0 - t_0) + i_0^2 R (3t_0 - 2t_0) = 3i_0^2 Rt_0 + 9i_0^2 Rt_0 + i_0^2 Rt_0 = 13i_0^2 Rt_0$$

**Example#10**

A cell of internal resistance  $1\Omega$  is connected across a resistor. A voltmeter having variable resistance is used to measure potential difference across resistor. The plot of voltmeter reading  $V$  against  $G$  is shown. What is value of external resistor  $R$ ? ( $G$  = Resistance of galvanometer)



- (A)  $5\Omega$  (B)  $4\Omega$  (C)  $3\Omega$  (D) can't be determined

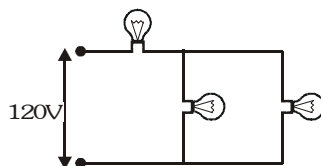
**Solution****Ans. (A)**

When galvanometer resistance tends to infinity  $G \rightarrow \infty$ ,

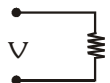
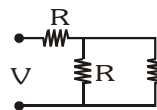
Potential difference across  $R$  is  $20V \Rightarrow 20 = 24 - i \cdot 1 \Rightarrow i = 4A$  also  $20 = 4R \Rightarrow R = 5\Omega$ .

**Example#11**

Three  $60W$ ,  $120V$  light bulbs are connected across a  $120V$  power source. If resistance of each bulb does not change with current then find out total power delivered to the three bulbs.



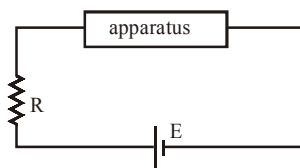
- (A)  $180W$  (B)  $20W$  (C)  $40W$  (D)  $60W$

**Solution****Ans. (C)**

Here  $R = \frac{V^2}{P}$ , Total power supplied  $= \frac{V^2}{3/2R} = \left(\frac{2}{3}\right)\left(\frac{V^2}{R}\right) = \frac{2}{3} \times 60 = 40W$

**Example#12**

An apparatus is connected to an ideal battery as shown in figure. For what value of current, power delivered to the apparatus will be maximum?



- (A)  $\frac{E}{R}$  (B)  $\frac{E}{2R}$  (C)  $\frac{E}{4R}$  (D) information insufficient

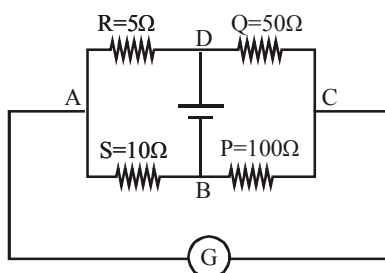
**Solution**

**Ans. (B)**

For maximum power  $R_{\text{ext}} = R_{\text{int}} = R \therefore \text{current} = \frac{E}{2R}$

**Example#13**

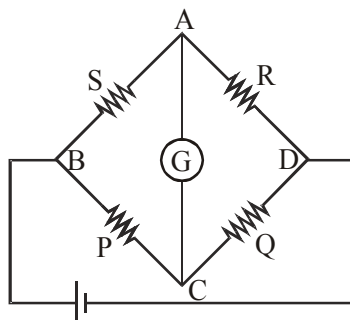
Figure shows a balanced Wheatstone's bridge



- (A) If P is slightly increased, the current in the galvanometer flows from C to A
- (B) If P is slightly increased, the current in the galvanometer flows from A to C
- (C) If Q is slightly increased, the current in the galvanometer flows from C to A
- (D) If Q is slightly increased, the current in the galvanometer flows from A to C

**Solution**

**Ans. (BC)**

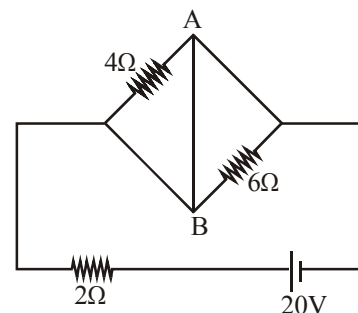


If P is slightly increased, potential of C will decrease. Hence current will flow from A to C. If Q is slightly increased, potential of C will increase, hence current will flow from C to A.

**Example#14**

In the circuit shown in figure.

- (A) power supplied by the battery is 200W
- (B) current flowing in the circuit is 5A
- (C) potential difference across the  $4\Omega$  resistance is equal to the potential difference across the  $6\Omega$  resistance
- (D) current in wire AB is zero.



**Ans. (ACD)**

**Solution**

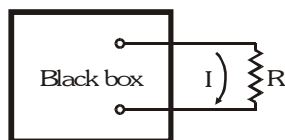
$4\Omega$  and  $6\Omega$  resistor are short-circuited. Therefore, no current will flow through these resistances. Current passing through the battery is  $I = (20/2) = 10\text{A}$ . This is also the current passing in wire AB from B to A.

Power supplied by the battery  $P = EI = (20)(10) = 200\text{W}$

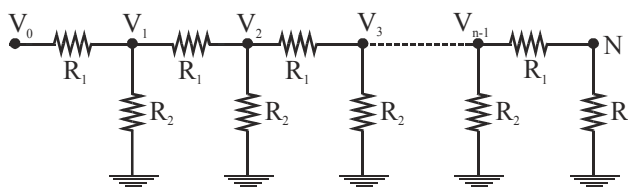
Potential difference across the  $4\Omega$  resistance = potential difference across the  $6\Omega$  resistance.

**Example#15**

In the given black box unknown emf sources and unknown resistances are connected by an unknown method such that (i) when terminals of 10 ohm resistances are connected to box then 1 ampere current flows and (ii) when 18 ohm resistances are connected then 0.6 A current flows then for what value of resistance does 0.1 A current flow?

(A) 118  $\Omega$ (B) 98  $\Omega$ (C) 18  $\Omega$ (D) 58  $\Omega$ **Solution****Ans. (A,B)****Example#16 to 18**

A network of resistance is constructed with  $R_1$  and  $R_2$  as shown in figure. The potential at the points 1, 2, 3.... N are  $V_1, V_2, V_3, \dots, V_N$ , respectively, each having a potential  $k$  times smaller than the previous one.



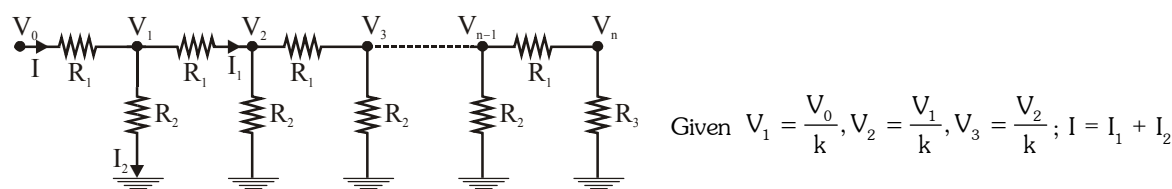
16. The ratio  $\frac{R_1}{R_2}$  is

(A)  $k^2 - \frac{1}{k}$ (B)  $\frac{k}{k-1}$ (C)  $k - \frac{1}{k^2}$ (D)  $\frac{(k-1)^2}{k}$ 

17. The ratio  $\frac{R_2}{R_3}$  is

(A)  $\frac{(k-1)^2}{k}$ (B)  $k^2 - \frac{1}{k}$ (C)  $\frac{k}{k-1}$ (D)  $k - \frac{1}{k^2}$ 

18. The current that passes through the resistance  $R_2$  nearest to the  $V_0$  is

(A)  $\frac{(k-1)^2}{k} \frac{V_0}{R_3}$ (B)  $\frac{(k+1)^2}{k} \frac{V_0}{R_3}$ (C)  $\left(k + \frac{1}{k^2}\right) \frac{V_0}{R_3}$ (D)  $\left(k - \frac{1}{k^2}\right) \frac{V_0}{R_3}$ **Solution**16. **Ans. (D)**

$$\frac{V_0 - V_1}{R_1} = \frac{V_1 - V_2}{R_1} + \frac{V_1 - 0}{R_2} \Rightarrow \frac{V_0 - V_1/k}{R_1} = \frac{V_0/k - V_0/k^2}{R_1} + \frac{V_0/k}{R_2} \Rightarrow \frac{R_1}{R_2} = \frac{(k-1)^2}{k}$$

17. Ans. (C)

Current in  $R_1$  and  $R_3$  will be same :  $\frac{V_{n-1} - V_n}{R_1} = \frac{V_n}{R_3} \Rightarrow \frac{V_{n-1} - \frac{V_{n-1}}{k}}{R_1} = \frac{V_{n-1}}{kR_3} \Rightarrow R_1 = R_3 (k-1)$

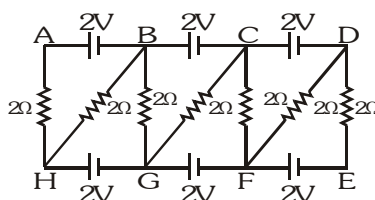
Put the value of  $R_1$  in (i)  $\frac{R_2}{R_3} = \frac{k}{k-1}$

18. Ans. (D)

Current in  $R_2$  nearest to  $V_0$  :  $I_2 = \frac{V_1}{R_2} = \frac{V_0/k}{R_3 \left( \frac{k}{k-1} \right)} = \left( \frac{k-1}{k^2} \right) \frac{V_0}{R_3}$

### Example#19 to 21

In given circuit, 7 resistors of resistance  $2\Omega$  each and 6 batteries of  $2V$  each, are joined together.



19. The potential difference  $V_D - V_E$  is-

(A)  $\frac{5}{6}V$  (B)  $\frac{6}{5}V$  (C)  $-\frac{5}{6}V$  (D)  $-\frac{22}{9}V$

20. The current through branch BG is-

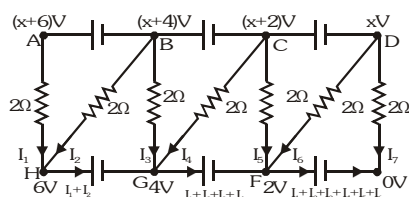
(A) 1A (B) 0.2A (C) 0.4A (D) 0.6A

21. The current through battery between A & B is-

(A) 0.6 A (B) 0.8 A (C) 0.4 A (D) 1 A

### Solution

19. Ans. (B)



$$I_1 + I_2 + I_3 + I_4 + I_5 + I_6 = I_7$$

$$\Rightarrow \frac{x}{2} + \frac{x-2}{2} + \frac{x}{2} + \frac{x-2}{2} + \frac{x}{2} + \frac{x-2}{2} = \frac{x}{2} \Rightarrow x = \frac{6}{5}V$$

20. Ans. (D)

Current through branch BG :  $I_3 = \frac{x}{2} = \frac{6}{10} = 0.6A$

21. Ans. (A)

Current through branch AB =  $I_1 = \frac{x}{2} = 0.6 A$

**Example#22 to 24**

An ammeter and a voltmeter are connected in series to a battery with an emf of 10V. When a certain resistance is connected in parallel with the voltmeter, the reading of the voltmeter decreases three times, whereas the reading of the ammeter increases the two times.

22. Find the voltmeter reading after the connection of the resistance.

- (A) 1V (B) 2V (C) 3V (D) 4V

23. If resistance of the ammeter is  $2\Omega$ , then resistance of the voltmeter is :-

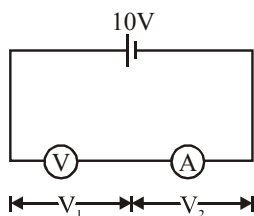
- (A)  $1\Omega$  (B)  $2\Omega$  (C)  $3\Omega$  (D)  $4\Omega$

24. If resistance of the ammeter is  $2\Omega$ , then resistance of the resistor which is added in parallel to the voltmeter is :-

- (A)  $\frac{3}{5}\Omega$  (B)  $\frac{2}{7}\Omega$  (C)  $\frac{3}{7}\Omega$  (D) None of these

**Solution**

22. Ans. (B)

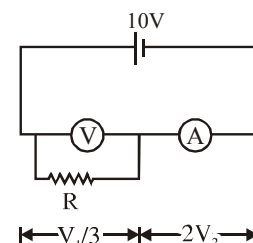


$$\text{Initially } V_1 + V_2 = 10V \dots(i)$$

$$\text{Finally } \frac{V_1}{3} + 2V_2 = 10V \dots(ii)$$

$$\text{From equation (i) \& (ii) We get } V_1 = 6 \text{ volt, } V_2 = 4 \text{ volt}$$

$$\therefore \text{ Final reading} = \frac{V_1}{3} = 2 \text{ volt}$$



23. Ans. (C)

$$\frac{V_1}{V_2} = \frac{R_A}{R_V} = \frac{4}{6} \text{ \& } R_A = 2\Omega \Rightarrow R_V = 3\Omega$$

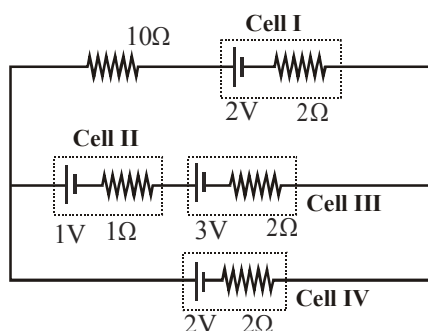
24. Ans. (A)

$$\frac{\frac{R_V R}{R_V + R}}{R_A} = \frac{\frac{V_1}{3}}{2V_2} = \frac{1}{4} \Rightarrow R = \frac{3}{5}\Omega$$



### Example#25

For the circuit shown in figure, 4 cells are arranged. In column I, the cell number is given while in column II, some statements related to cells are given. Match the entries of column I with the entries of column II.



#### Column I

- (A) Cell I
- (B) Cell II
- (C) Cell III
- (D) Cell IV

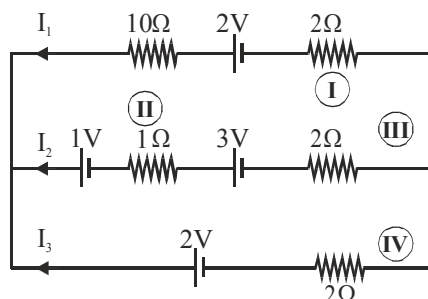
#### Column II

- (P) Chemical energy of cell is decreasing.
- (Q) Chemical energy of cell is increasing.
- (R) Work done by cell is positive.
- (S) Thermal energy developed in cell is positive.
- (T) None of these

**Solution**

**Ans. (A) Q,S (B) P, R, S (C) P,R,S (D) Q, S**

We have,  $I_1 = -\frac{2}{33}A, I_2 = \frac{14}{33}A, I_3 = -\frac{12}{33}A$

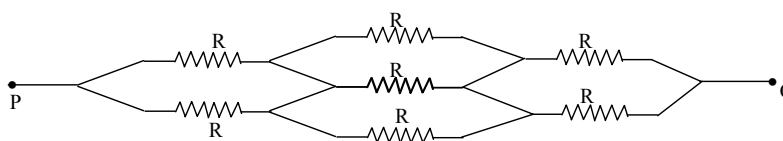


In each cell thermal energy will be dissipated due to internal resistance whether the chemical energy of the cell is increasing or decreasing.

- (i) Cell I is getting charged, hence its chemical energy increases.
- (ii) Cell II and III both are getting discharged, hence their chemical energy is decreasing. So, work done by both of them is positive.
- (iv) Cell IV is getting charged, hence its chemical energy increases.

### Example#26

Equivalent resistance for the given figure between P and Q is  $NR/3$ . Find value of N.



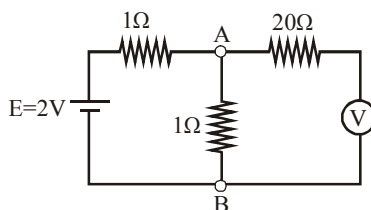
**Solution**

**Ans. 4**

$$\frac{R}{2} \text{ --- } \frac{R}{3} \text{ --- } \frac{R}{2} \quad R_{\text{net}} = \frac{4R}{3} \Rightarrow N = 4$$

**Example#27**

In the given circuit, the voltmeter and the electric cell are ideal. Find the reading of the voltmeter (in volt)

**Solution****Ans. 1**

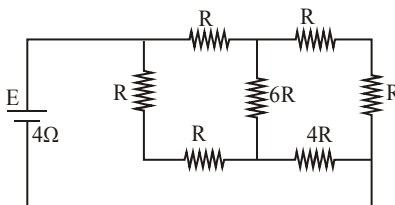
The electric current through ideal voltmeter is zero.

$$\text{According to loop rule, } E - 1 \cdot I - 1 \cdot I = 0 \Rightarrow I = \frac{E}{2} = \frac{2}{2} = 1 \text{ A}$$

$$\text{Reading of the voltmeter} = V_A - V_B = [1 \cdot 1] = [1 \cdot 1] = 1 \text{ V}$$

**Example#28**

A battery of internal resistance  $4\Omega$  is connected to the network of resistances as shown in figure. In order that the maximum power can be delivered to the network, the value of  $R$  in  $\Omega$  should be

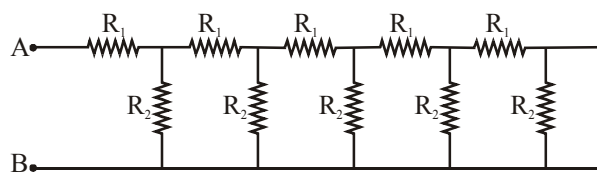
**Solution****Ans. 2**

For maximum power, external resistance is equal to internal resistance. Therefore,  $2R = 4$  or  $R = 2$

**Example#29**

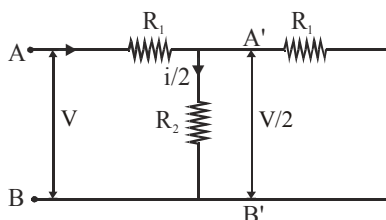
Consider an infinite ladder of network shown in figure. A voltage is applied between points A and B. If the

voltage is halved after each section, find the ratio of  $\frac{R_2}{R_1}$ .

**Solution****Ans. 2**

$$\text{Voltage across AB} = V, \text{ Voltage across A'B'} = \frac{V}{2} \text{ i.e., Voltage across } R_2 = \frac{V}{2}$$

$$\text{Now from Kirchhoff's law it is obvious that voltage across } R_1 = V - \frac{V}{2} = \frac{V}{2}$$



When the voltage is halved, current is also halved, i.e., current in  $R_2$  is half of that in  $R_1$ .

$$\text{So } R_1 i = R_2 \frac{i}{2} \Rightarrow \frac{R_2}{R_1} = 2$$

**Example#30**

A wire of length  $L$  and 3 identical cells of negligible internal resistances are connected in series. Due to the current, the temperature of the wire is raised by  $\Delta T$  in a time  $t$ . A number  $N$  of similar cells is now connected in series with a wire of the same material and cross-section but of length  $2L$ . The temperature of the wire is raised by the same amount  $\Delta T$  in the same time  $t$ , the value of  $N$  is

**Solution**

**Ans. 6**

Let  $R$  be the resistance of wire. Let  $R'$  be the resistance of another wire so  $R' = 2R$  ( $\because$  Length is twice)

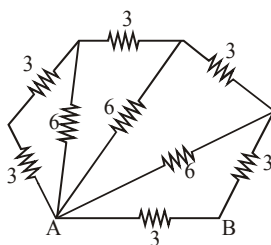
$$\text{In case (i) Energy released in } t \text{ second} = \frac{(3V)^2}{R} \times t \quad \text{In case (ii) } \therefore \text{Energy released in } t\text{-seconds} = \frac{(N^2 V^2)}{2R} \times t$$

$$\text{But } Q = mc\Delta T \quad \therefore mc\Delta T = \frac{(9V^2)}{R} \times t \dots (i) \quad \text{Applying } Q' = m'c\Delta T \Rightarrow 2mc\Delta T = \frac{(N^2 V^2)}{2R} \quad t \dots (ii)$$

$$\text{Dividing equation (ii) by equation (i)} \quad \frac{mc\Delta T}{2mc\Delta T} = \frac{9V^2 \times \frac{t}{R}}{N^2 V^2 \times \frac{t}{2R}} \quad \therefore \frac{1}{2} = \frac{9 \times 2}{N^2} \Rightarrow N^2 = 18 \times 2 \quad \therefore N = 6$$

**Example#31**

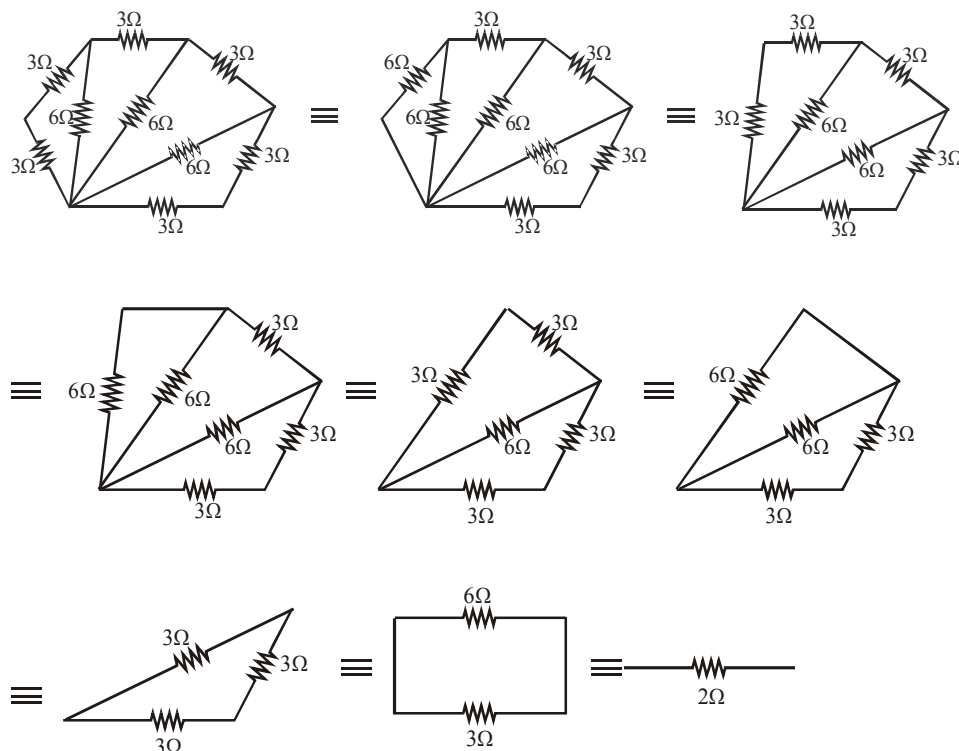
All resistances in the diagram below are in ohms. Find the effective resistance between the point A and B (in  $\Omega$ ).



**Solution**

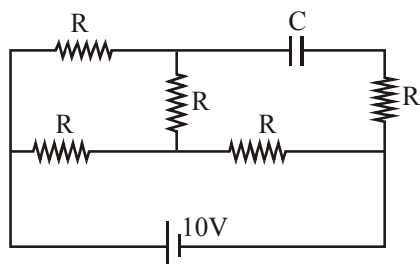
**Ans. 2**

The given system can be reduced as shown in figure.



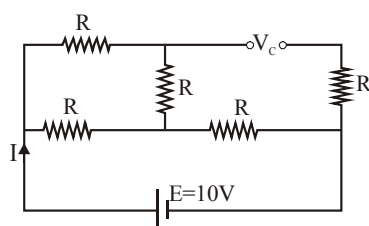
**Example#32**

Find the potential difference across the capacitor in volts.



**Solution**

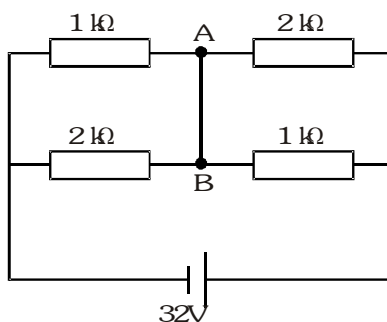
**Ans. 8**



In steady state  $I = \frac{E}{2R/3 + R} = \frac{3E}{5R}$ ;  $V_c = E - \frac{IR}{3} = E - \frac{E}{5} = \frac{4E}{5} = 8 \text{ Volts}$

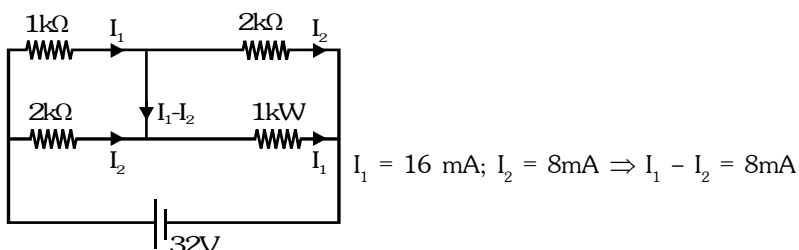
**Example#33**

In the given circuit, find the current (in mA) in the wire between points A and B.



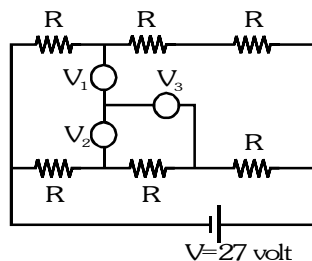
**Solution**

**Ans. 8**



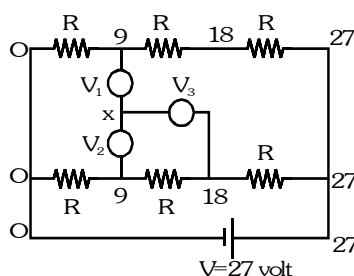
**Example#34**

In the circuit shown below, all the voltmeter identical and have very high resistance. Each resistor has the same resistance. The voltage of the ideal battery shown is 27 V. Find the reading of voltmeter  $V_3$  ( in volts).



**Solution**

**Ans. 6**



$$\frac{x-9}{R_v} + \frac{x-9}{R_v} + \frac{x-18}{R_v} = 0 \Rightarrow x = 12 \therefore V_3 = 6 \text{ volt}$$

**Example#35**

How much time heater will take to increase the temperature of 100 g water by 50 C if resistance of heating coil is  $484\Omega$  and supply voltage is 220V a.c.

**Solution**

$$\text{Heat given by heater} = \text{heat taken by water} \Rightarrow \frac{V^2}{R} t = ms J \Delta\theta \Rightarrow \frac{220 \times 220}{484}$$

$$t = (100 \times 10^{-3}) (4.2 \times 10^3) (50) \Rightarrow t = 210 \text{ s}$$