1 PROOF

THEOREM 1.1 (EQUISATISFIABILITY OF η-PROJECTION). *The projected constraints can be satisfied by the same set of parameter type combinations as the unprojected constraints. In other words, the projected constraints and unprojected constraints are equisatisfiable.*

This theorem is equivalent to the following theorem: Every parameter type combination that satisfy the projected constraints can be *extended* to a solution of the unprojected constraints by (1) adding back the pre-determined types of global variables and literals and (2) determine the types of local expressions and local variables. Meanwhile, every solution to the unprojected constraints can be *reduced* to a parameter type combination that satisfy the projected constraints by (1) remove the constraints about global variables and literals and (2) remove the types of local expressions and local variables.

The first part (extension) is proved in this paragraph. We note that every conjunction term in a the unprojected constraints as a conjunction. e.g., $1_1 = str \wedge 1_2 = str \wedge 1_3 = bool \wedge 0_1 = F_1 \wedge 3_1 = str$ and $1_1 = list \wedge 1_2 = list \wedge 1_3 = bool \wedge 0_1 = F_1 \wedge 3_1 = list$ are the conjunctions of the unprojected constraint of $sorted(p, c)$. The theorem can be proved by observing that all constraints about parameters in different conjunctions are mutually exclusive, i.e., a parameter type combination either does not satisfy any conjunction or satisify exactly one conjunction. The parameter type combination that does not satisfy any conjunction can not satisfy projected and unprojected constraints. While the parameter type combination does satisfy one conjunction satisfies the *same* conjunction in projected and unprojected constraints. Furthermore, the types of local expressions can be determined by the parameter type combination and the types of global variables and literals, since it would be the only free meta-variable by then. e.g.,$(1_1 : str, 1_2 : str, 1_3 : bool, 0_1 : F_1) \rightarrow (1_1 : str, 1_2 : str, 1_3 : bool, 0_1 : F_1, 3_1 : bool)$. Thus, the parameter type combinations that satisfy the projected constraints can also satisfy the unprojected constraints.

The second part (reduction) is proved in this paragraph. This part can be proved by observing the mutual exclusive. The reduced conjunction is still satisfied with the same parameter type combination.