```python
import argparse
import os
from datetime import datetime
from collections import Counter, defaultdict
import json
import pickle

import numpy as np
import torch
from torch import nn
from torchvision import datasets, transforms, models
import torchvision
import matplotlib.pyplot as plt
from torchsummary import summary
from tqdm import tqdm
import pandas as pd
from scipy.stats import pearsonr, spearmanr
import torch.nn.functional as F
from torch.nn.functional import softmax
import numpy

# ============================================================
# Parsing the arguments
# ============================================================
parser = argparse.ArgumentParser()
parser.add_argument("--test_run", type=int, required=True, help="Experiment run index")
parser.add_argument("--resnet_ckpt_path", type=str, required=True,
                    help="Path to saved ResNet checkpoint (.pth)")
args = parser.parse_args()

test_run = args.test_run
resnet_ckpt_path = args.resnet_ckpt_path

# ============================================================
# Device configuration
# ============================================================
device = torch.device('cuda' if torch.cuda.is_available() else 'cpu')
print("Device:", device)
print("test_run", test_run)
print("resnet_ckpt_path:", resnet_ckpt_path)

# ============================================================
# DATA LOADING
# ============================================================
transformations = transforms.Compose([
    transforms.Resize(255),
    transforms.CenterCrop(224),
    transforms.ToTensor(),
    transforms.Normalize(mean=[0.485, 0.456, 0.406],
                         std=[0.229, 0.224, 0.225])
])

base_data_dir = f"/leonardo_work/IscrC_ArtLLMs/alitariqnagi_work/training_data_split_run_{test_run}"

train_set_mix = datasets.ImageFolder(
    os.path.join(base_data_dir, "train"),
    transform=transformations
)
```

```
val_set_mix = datasets.ImageFolder(
    os.path.join(base_data_dir, "val"),
    transform=transformations
)
test_set_mix = datasets.ImageFolder(
    os.path.join(base_data_dir, "test"),
    transform=transformations
)

print("Train size:", len(train_set_mix))
print("Val size:", len(val_set_mix))
print("Test size:", len(test_set_mix))

class_names = train_set_mix.classes
print("Classes:", class_names)

train_loader_mix = torch.utils.data.DataLoader(
    train_set_mix, batch_size=32, shuffle=True, num_workers=8
)
val_loader_mix = torch.utils.data.DataLoader(
    val_set_mix, batch_size=32, shuffle=False, num_workers=8
)
test_loader_mix = torch.utils.data.DataLoader(
    test_set_mix, batch_size=32, shuffle=False, num_workers=8
)

# Optional: quick visualization
dataiter = iter(train_loader_mix)
images, labels = next(dataiter)


def imshow(img):
    img = img / 2 + 0.5  # unnormalize
    npimg = img.numpy()
    plt.imshow(np.transpose(npimg, (1, 2, 0)))
    plt.axis("off")


imshow(torchvision.utils.make_grid(images))
plt.show()

model = models.resnet50(weights='ResNet50_Weights.IMAGENET1K_V1')

for param in model.parameters():
    param.requires_grad = True

model.fc = nn.Sequential(
    nn.Linear(2048, 512),
    nn.ReLU(),
    nn.Dropout(0.2),
    nn.Linear(512, len(class_names))
)

if torch.cuda.device_count() > 1:
    print(f"Using DataParallel Mode with cuda count {torch.cuda.device_count()}")
    model = torch.nn.DataParallel(model)

model.to(device)
print(summary(model, input_size=(3, 224, 224)))

```

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# ============================================================
# LOAD FINAL CHECKPOINT (NO TRAINING HERE)
# ============================================================
final_ckpt_path = resnet_ckpt_path
print("Loading final model checkpoint:", final_ckpt_path)

# Load on CPU first (safer across GPU setups)
state_dict = torch.load(final_ckpt_path, map_location="cpu")

# If checkpoint was saved using DataParallel, keys will start with "module."
if any(k.startswith("module.") for k in state_dict.keys()):
    print("[INFO] Detected DataParallel checkpoint. Stripping 'module.' prefix...")
    state_dict = {k.replace("module.", "", 1): v for k, v in state_dict.items()}

model.load_state_dict(state_dict, strict=True)

model.to(device)
model.eval()
print("Model loaded.\n")

# ============================================================
# CONFORMAL FUNCTIONS
# ============================================================

# different values will be overwritten in the loop
alpha = 0.1


##########################----------------------------------
# Calculation of Calibration Scores
# ----------------------------------------------------------------
def calibration_scores(model, data_loader, device):
    model.eval()

    scores_calibration_set = []

    with torch.no_grad():
        for X, y in tqdm(data_loader,
                         desc="##################Proceeding with the Calculation of Calibration Scores##################"):
            X = X.to(device)
            y = y.to(device)

            y = y.cpu().numpy()

            pred_logits = model(X)

            cal_smx = softmax(pred_logits, dim=1).cpu().numpy()
            scores_per_batch = 1 - cal_smx[np.arange(y.shape[0]), y]

            scores_calibration_set.extend(scores_per_batch)

    scores_calibration_set = numpy.array(scores_calibration_set)

    return scores_calibration_set


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##########################----------------------------------
# Calculation of Prediction Sets
# ----------------------------------------------------------------
def predict_with_conformal_sets(model, data_loader, calibration_scores_array, alpha, device):
    model.eval()

    prediction_sets = []
    true_labels = []

    n = len(calibration_scores_array)

    q_level = np.ceil((n + 1) * (1 - alpha)) / n
    qhat = np.quantile(calibration_scores_array, q_level, method='higher')

    with torch.no_grad():
        for X, y in tqdm(data_loader,
                         desc="###################Proceeding with the Calculation of Testing Scores###################"):
            X = X.to(device)
            y = y.cpu().numpy()
            pred_logits = model(X)

            val_smx = softmax(pred_logits, dim=1)
            val_smx = val_smx.cpu().numpy()

            masks = val_smx >= (1 - qhat)

            for _ in masks:
                prediction_sets.append(np.where(_)[0].tolist())

            true_labels.extend(y.tolist())

    return prediction_sets, true_labels


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##########################----------------------------------
# EVALUATION
# ----------------------------------------------------------------
def evaluate(prediction_sets, labels):
    number_of_correct_predictions = 0
    for pred_set, true_label in zip(prediction_sets, labels):
        if true_label in pred_set:
            number_of_correct_predictions += 1
    empirical_test_coverage = number_of_correct_predictions / len(labels)
    average_prediction_set_size = np.mean([len(s) for s in prediction_sets])
    return empirical_test_coverage, average_prediction_set_size


################################################################################################################


def feature_stratified_coverage_by_class(prediction_sets, true_labels, alpha=0.1, class_names=None):
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    class_to_indices = defaultdict(list)
    for idx, label in enumerate(true_labels):
        class_to_indices[label].append(idx)

    class_coverages = {}
    for cls, indices in class_to_indices.items():
        covered = sum(true_labels[i] in prediction_sets[i] for i in indices)
        coverage = covered / len(indices)
        class_coverages[cls] = coverage

    print("\n Feature-Stratified Coverage (by True Class Label):")
    for cls, cov in sorted(class_coverages.items()):
        name = class_names[cls] if class_names else str(cls)
        print(f"  Class {name:<20}: Coverage = {cov:.3f} (Target ≥ {1 - alpha:.2f})")

    fsc_metric = min(class_coverages.values())
    print(f"\n FSC Metric (minimum class-wise coverage): {fsc_metric:.3f}")

    return fsc_metric, class_coverages


def plot_class_wise_coverage(class_coverages, alpha=0.1, class_names=None,
title="Class-wise Coverage (FSC)"):
    classes = list(class_coverages.keys())
    coverages = [class_coverages[c] for c in classes]

    labels = [class_names[c] if class_names else str(c) for c in classes]

    plt.figure(figsize=(12, 6))
    bars = plt.bar(labels, coverages, color="pink", edgecolor="black")

    target = 1 - alpha
    plt.axhline(target, color='red', linestyle='--', label=f"Target Coverage
({target:.2f})")

    for bar, cov in zip(bars, coverages):
        if cov < target:
            bar.set_color('salmon')

    plt.xticks(rotation=45, ha="right")
    plt.ylim(0, 1.05)
    plt.ylabel("Coverage")
    plt.title(title)
    plt.legend()
    plt.tight_layout()
    filename = f"Class_wise_Coverage_(FSC)_{test_run}_{timestamp}.png"
    plt.savefig(os.path.join(results_directory, filename))
    plt.show()


def plot_coverage_vs_class_frequency(class_coverages, true_labels, alpha=0.1,
class_names=None,
                                     title="Coverage vs Class Frequency"):
    class_indices = sorted(class_coverages.keys())
    coverages = [class_coverages[c] for c in class_indices]

    total = len(true_labels)
    counts = Counter(true_labels)
    frequencies = [counts[c] / total for c in class_indices]

```

```
    labels = [class_names[c] if class_names else str(c) for c in class_indices]
    x = np.arange(len(class_indices))

    fig, ax1 = plt.subplots(figsize=(12, 6))

    bars1 = ax1.bar(x - 0.2, coverages, width=0.4, label="Coverage", color='pink', edgecolor='black')
    ax1.axhline(1 - alpha, color='red', linestyle='--', label=f"Target Coverage ({1 - alpha:.2f})")
    ax1.set_ylim(0, 1.05)
    ax1.set_ylabel("Coverage", color='pink')
    ax1.tick_params(axis='y', labelcolor='pink')

    for bar, cov in zip(bars1, coverages):
        if cov < (1 - alpha):
            bar.set_color('salmon')

    ax2 = ax1.twinx()
    bars2 = ax2.bar(x + 0.2, frequencies, width=0.4, label="Class Frequency", color='gray', alpha=0.5)
    ax2.set_ylabel("Class Frequency", color='gray')
    ax2.tick_params(axis='y', labelcolor='gray')

    plt.xticks(x, labels, rotation=45, ha="right")
    plt.title(title)
    fig.legend(loc='upper right', bbox_to_anchor=(0.85, 0.85))
    plt.tight_layout()
    filename = f"plot_coverage_vs_class_frequency_{test_run}_{timestamp}.png"
    plt.savefig(os.path.join(results_directory, filename))
    plt.show()


def plot_coverage_vs_class_frequency_with_correlation(class_coverages, true_labels, alpha=0.1, class_names=None,
                                                      title="Coverage vs Class Frequency"):
    class_indices = sorted(class_coverages.keys())
    coverages = [class_coverages[c] for c in class_indices]

    total = len(true_labels)
    counts = Counter(true_labels)
    frequencies = [counts[c] / total for c in class_indices]

    labels = [class_names[c] if class_names else str(c) for c in class_indices]
    x = np.arange(len(class_indices))

    pearson_corr, pearson_p = pearsonr(frequencies, coverages)
    spearman_corr, spearman_p = spearmanr(frequencies, coverages)

    print(f"\n Correlation between class frequency and coverage:")
    print(f"  - Pearson  r = {pearson_corr:.3f}, p = {pearson_p:.3e}")
    print(f"  - Spearman ρ = {spearman_corr:.3f}, p = {spearman_p:.3e}")

    fig, ax1 = plt.subplots(figsize=(12, 6))

    bars1 = ax1.bar(x - 0.2, coverages, width=0.4, label="Coverage", color='pink', edgecolor='black')
    ax1.axhline(1 - alpha, color='red', linestyle='--', label=f"Target Coverage ({1 - alpha:.2f})")
    ax1.set_ylim(0, 1.05)
```

```
    ax1.set_ylabel("Coverage", color='pink')
    ax1.tick_params(axis='y', labelcolor='pink')

    for bar, cov in zip(bars1, coverages):
        if cov < (1 - alpha):
            bar.set_color('salmon')

    ax2 = ax1.twinx()
    bars2 = ax2.bar(x + 0.2, frequencies, width=0.4, label="Class Frequency",
color='gray', alpha=0.5)
    ax2.set_ylabel("Class Frequency", color='gray')
    ax2.tick_params(axis='y', labelcolor='gray')

    plt.xticks(x, labels, rotation=45, ha="right")
    plt.title(title)
    fig.legend(loc='upper right', bbox_to_anchor=(0.85, 0.85))
    plt.tight_layout()
    filename =
f"plot_coverage_vs_class_frequency_with_correlation_{test_run}_{timestamp}.png"
    plt.savefig(os.path.join(results_directory, filename))
    plt.show()


def size_stratified_coverage(prediction_sets, true_labels, alpha=0.1,
                             bins=[[1], [2], [3, 4, 5], list(range(6, 100))]):
    bin_groups = defaultdict(list)

    for i, pred_set in enumerate(prediction_sets):
        set_size = len(pred_set)
        for bin_range in bins:
            if set_size in bin_range:
                bin_groups[str(bin_range)].append(i)
                break

    bin_coverages = {}
    for bin_key, indices in bin_groups.items():
        covered = sum(true_labels[i] in prediction_sets[i] for i in indices)
        coverage = covered / len(indices) if indices else 0
        bin_coverages[bin_key] = coverage

    print("\n Size-Stratified Coverage (SSC):")
    for bin_key, cov in bin_coverages.items():
        print(f"  Set Size {bin_key:10s}: Coverage = {cov:.3f} (Target ≥ {1 -
alpha:.2f})")

    ssc = min(bin_coverages.values())
    print(f"\n SSC Metric (min bin-wise coverage): {ssc:.3f}")
    return ssc, bin_coverages


def plot_ssc_coverage(bin_coverages, alpha=0.1, title="Size-Stratified Coverage"):
    bin_labels = list(bin_coverages.keys())
    coverages = [bin_coverages[k] for k in bin_labels]

    plt.figure(figsize=(10, 5))
    bars = plt.bar(bin_labels, coverages, color='pink', edgecolor='black')

    target = 1 - alpha
    plt.axhline(target, color='red', linestyle='--', label=f"Target Coverage
({target:.2f})")
```

```
398
399     for bar, cov in zip(bars, coverages):
400         if cov < target:
401             bar.set_color('grey')
402
403     plt.xticks(rotation=45, ha='right')
404     plt.ylim(0, 1.05)
405     plt.ylabel("Coverage")
406     plt.title(title)
407     plt.legend()
408     plt.tight_layout()
409     filename = f"plot_ssc_coverage_{test_run}__{timestamp}.png"
410     plt.savefig(os.path.join(results_directory, filename))
411     plt.show()
412
413
414 def plot_prediction_set_size_by_class(prediction_sets, true_labels, class_names=None,
    results_dir="", test_run="",
415                                       timestamp=""):
416     size_per_class = defaultdict(list)
417     for pred_set, true_label in zip(prediction_sets, true_labels):
418         size_per_class[true_label].append(len(pred_set))
419
420     sorted_classes = sorted(size_per_class.keys())
421     means = [np.mean(size_per_class[c]) for c in sorted_classes]
422     stds = [np.std(size_per_class[c]) for c in sorted_classes]
423     labels = [class_names[c] if class_names else str(c) for c in sorted_classes]
424
425     plt.figure(figsize=(12, 6))
426     plt.bar(labels, means, yerr=stds, capsize=5, color="lightgreen",
    edgecolor="black")
427     plt.ylabel("Avg Prediction Set Size")
428     plt.xlabel("Class")
429     plt.title("Average Prediction Set Size by Class")
430     plt.xticks(rotation=45, ha="right")
431     plt.grid(axis="y", linestyle="--", alpha=0.5)
432     plt.tight_layout()
433
434     filename = f"prediction_set_size_by_class_{test_run}__{timestamp}.png"
435     plt.savefig(os.path.join(results_directory, filename))
436     plt.show()
437
438
439 def plot_calibration_score_distribution(calibration_scores, results_dir="",
    test_run="", timestamp=""):
440     plt.figure(figsize=(10, 5))
441     plt.hist(calibration_scores, bins=30, color="pink", edgecolor="black", alpha=0.8)
442     plt.xlabel("Nonconformity Score (1 - P(True Class))")
443     plt.ylabel("Frequency")
444     plt.title("Calibration Score Distribution")
445     plt.grid(axis='y', linestyle='--', alpha=0.6)
446     plt.tight_layout()
447
448     filename = f"calibration_score_distribution_{test_run}__{timestamp}.png"
449     plt.savefig(os.path.join(results_directory, filename))
450
451     plt.show()
452
453
454 # ============================================================
```

```
# MAIN CONFORMAL EVALUATION FOR MULTIPLE ALPHA VALUES
# ============================================================

coverage_levels = np.arange(0.65, 0.95 + 1e-9, 0.05)

timestamp = datetime.now().strftime("%Y-%m-%d_%H-%M-%S")

base_results_root =
f"Results_normal_Conformal_Prediction_experiment_run_{test_run}_{timestamp}"
os.makedirs(base_results_root, exist_ok=True)
print("Base results directory:", base_results_root)

print("\n====== Computing calibration scores on validation set ======\n")
results_directory = base_results_root
scores_calibration = calibration_scores(model=model, data_loader=val_loader_mix,
device=device)

np.save(os.path.join(base_results_root,

f"calibration_scores_experiment_run_{test_run}_{timestamp}.npy"), scores_calibration)
with open(os.path.join(base_results_root,
                       f"scores_calibration_run_{test_run}__{timestamp}.pkl"), "wb")
as f:
    pickle.dump(scores_calibration, f)

plot_calibration_score_distribution(
    calibration_scores=scores_calibration,
    results_dir=base_results_root,
    test_run=test_run,
    timestamp=timestamp
)

all_alpha_metrics = []

for coverage_target in coverage_levels:
    alpha = 1.0 - coverage_target

    print("\n====================================================")
    print(f" Target coverage: {coverage_target:.2f}  |  alpha = {alpha:.2f}")
    print("====================================================\n")

    cov_tag = int(round(coverage_target * 100))
    results_directory = os.path.join(base_results_root, f"coverage_{cov_tag}")
    os.makedirs(results_directory, exist_ok=True)

    prediction_sets, true_labels = predict_with_conformal_sets(
        model=model,
        data_loader=test_loader_mix,
        calibration_scores_array=scores_calibration,
        alpha=alpha,
        device=device
    )
    test_set_coverage, avg_size = evaluate(prediction_sets=prediction_sets,
labels=true_labels)

    log_path = os.path.join(results_directory,

 f"logs_experiment_run_{test_run}_alpha_{alpha:.2f}_{timestamp}.txt")
    with open(log_path, "w") as f:
```

```
        f.write("########################## Results
##################################\n")
        f.write(f"test_run {test_run}\n")
        f.write(f"Target coverage: {coverage_target:.3f}\n")
        f.write(f"Alpha: {alpha:.3f}\n")
        f.write(f"Coverage: {test_set_coverage:.3f}\n")
        f.write(f"Average Prediction Set Size for Test Set: {avg_size:.3f}\n")

 f.write("#########################################################################\n")

    print("##########################Results:##################################")
    print(f"test_run {test_run}")
    print(f"Target coverage: {coverage_target:.3f}")
    print(f"Alpha: {alpha:.3f}")
    print(f"Coverage: {test_set_coverage:.3f}")
    print(f"Average Prediction Set Size for Test Set: {avg_size:.3f}")
    print("#########################################################################")

    set_sizes = [len(prediction_set) for prediction_set in prediction_sets]
    counts = Counter(set_sizes)
    total = sum(counts.values())
    sizes = sorted(counts.keys())
    frequencies = [counts[size] for size in sizes]
    percentages = [100 * freq / total for freq in frequencies]

    data = [{'size': len(prediction_set), 'correct': int(true in prediction_set)}
            for prediction_set, true in zip(prediction_sets, true_labels)]
    df = pd.DataFrame(data)
    coverage_stats = df.groupby('size').agg({'correct': ['mean', 'sum', 'count']})
    coverage_stats.columns = ['coverage', 'correct_count', 'total_count']
    coverage_stats = coverage_stats.reindex(sizes, fill_value=0)

    cumulative_correct = coverage_stats['correct_count'].cumsum()
    cumulative_coverage = 100 * cumulative_correct / len(true_labels)
    max_freq = max(frequencies)
    cumulative_scaled = (cumulative_coverage / 100) * max_freq

    fig, ax = plt.subplots(figsize=(13, 6))
    bars = ax.bar(sizes, frequencies, color='pink', edgecolor='black',
label='Frequency', alpha=0.7)
    ax.plot(sizes, cumulative_scaled, color='red', marker='o', linestyle='-',
            alpha=0.4, linewidth=2, label='Cumulative Coverage')

    for bar, size, pct in zip(bars, sizes, percentages):
        height = bar.get_height()
        count = counts[size]
        coverage_val = coverage_stats.loc[size, 'coverage'] * 100
        ax.annotate(f'{count} ({pct:.1f}%)\nCov: {coverage_val:.1f}%',
                    xy=(bar.get_x() + bar.get_width() / 2, height / 2),
                    ha='center', va='center', fontsize=9)

    for bar, cum_cov in zip(bars, cumulative_coverage):
        height = bar.get_height()
        ax.annotate(f'CumCov: {cum_cov:.1f}%',
                    xy=(bar.get_x() + bar.get_width() / 2, height),
                    xytext=(0, 8),
                    textcoords="offset points",
                    ha='center', va='bottom', fontsize=9, color='darkorange')

    mean_size = np.mean(set_sizes)
```

```
    median_size = np.median(set_sizes)
    ax.axvline(mean_size, color='gray', linestyle='--', label=f'Mean = {mean_size:.2f}')
    ax.axvline(median_size, color='orange', linestyle='--', label=f'Median = {median_size:.2f}')

    ax.set_xlabel("Prediction Set Size")
    ax.set_ylabel("Frequency")
    ax.set_title(f"Prediction Set Size Distribution with Coverage (Test Set), alpha={alpha:.2f}")
    ax.set_xticks(sizes)
    ax.grid(axis='y', linestyle='--', alpha=0.6)
    ax.legend(loc='upper right')

    plt.tight_layout()
    plt.savefig(os.path.join(
        results_directory,
 f"Prediction_Set_Size_Distribution_with_Coverage_(Test_Set)_{test_run}_alpha_{alpha:.2f}_{timestamp}.png"
    ))
    plt.show()

    fsc_metric, per_class_cov = feature_stratified_coverage_by_class(
        prediction_sets,
        true_labels,
        alpha=alpha,
        class_names=class_names
    )

    fsc_metric, class_coverages = feature_stratified_coverage_by_class(
        prediction_sets,
        true_labels,
        alpha=alpha,
        class_names=class_names
    )
    plot_class_wise_coverage(class_coverages, alpha=alpha, class_names=class_names)

    fsc_metric, class_coverages = feature_stratified_coverage_by_class(
        prediction_sets,
        true_labels,
        alpha=alpha,
        class_names=class_names
    )
    plot_coverage_vs_class_frequency(class_coverages, true_labels,
                                     alpha=alpha, class_names=class_names)

    plot_coverage_vs_class_frequency_with_correlation(
        class_coverages=class_coverages,
        true_labels=true_labels,
        alpha=alpha,
        class_names=class_names
    )

    ssc_metric, bin_coverages = size_stratified_coverage(
        prediction_sets,
        true_labels,
        alpha=alpha,
        bins=[[1], [2], [3], list(range(4, 100))]
    )
```

```
620
621     ssc_metric, bin_coverages = size_stratified_coverage(
622         prediction_sets,
623         true_labels,
624         alpha=alpha,
625         bins=[[1], [2], [3], list(range(4, 100))]
626     )
627     plot_ssc_coverage(bin_coverages, alpha=alpha)
628
629     plot_prediction_set_size_by_class(
630         prediction_sets=prediction_sets,
631         true_labels=true_labels,
632         class_names=class_names,
633         results_dir=results_directory,
634         test_run=test_run,
635         timestamp=timestamp
636     )
637
638     metrics = {
639         "test_run": test_run,
640         "timestamp": timestamp,
641         "alpha": alpha,
642         "target_coverage": coverage_target,
643         "test_set_coverage": test_set_coverage,
644         "average_prediction_set_size": avg_size,
645         "FSC": fsc_metric,
646         "SSC": ssc_metric,
647         "class_coverages": {str(k): float(v) for k, v in class_coverages.items()},
648         "bin_coverages": bin_coverages
649     }
650
651     with open(os.path.join(results_directory,
652
f"metrics_summary_{test_run}_alpha_{alpha:.2f}_{timestamp}.json"), "w") as f:
653         json.dump(metrics, f, indent=4)
654
655     all_alpha_metrics.append(metrics)
656
657 with open(os.path.join(base_results_root,
658                        f"metrics_summary_all_alphas_{test_run}_{timestamp}.json"),
"w") as f:
659     json.dump(all_alpha_metrics, f, indent=4)
660
661 print("\nFinished multi-alpha conformal evaluation for alphas corresponding to
coverage 0.65-0.95.")
662 print("Results saved under:", base_results_root)
663
```