

# Automated generation of planar geometry olympiad problems

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# Overall goal

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Design and implement software that is able to **generate planar geometry olympiad problems** by extending an initial configuration with new geometrical objects and subsequently finding non-trivial theorems in the generated configurations.



[www.skmo.sk](http://www.skmo.sk)

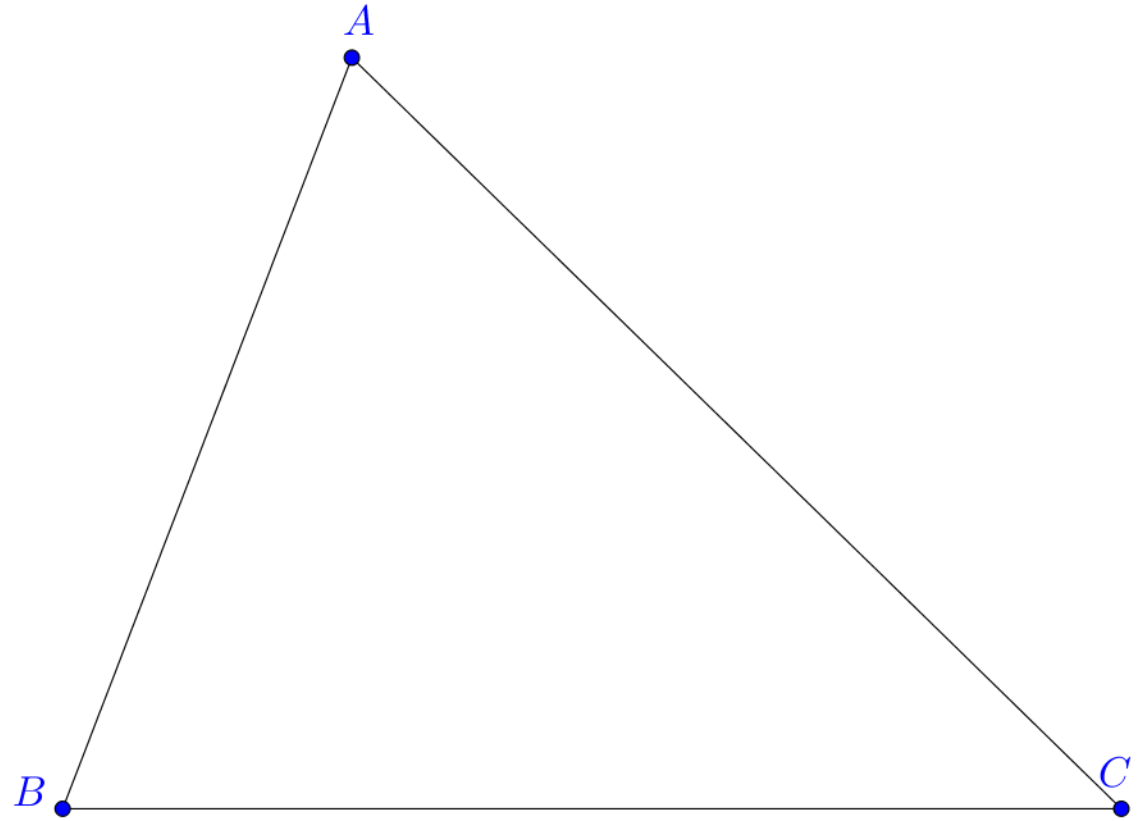


[imo-official.org](http://imo-official.org)

# Geometry problem

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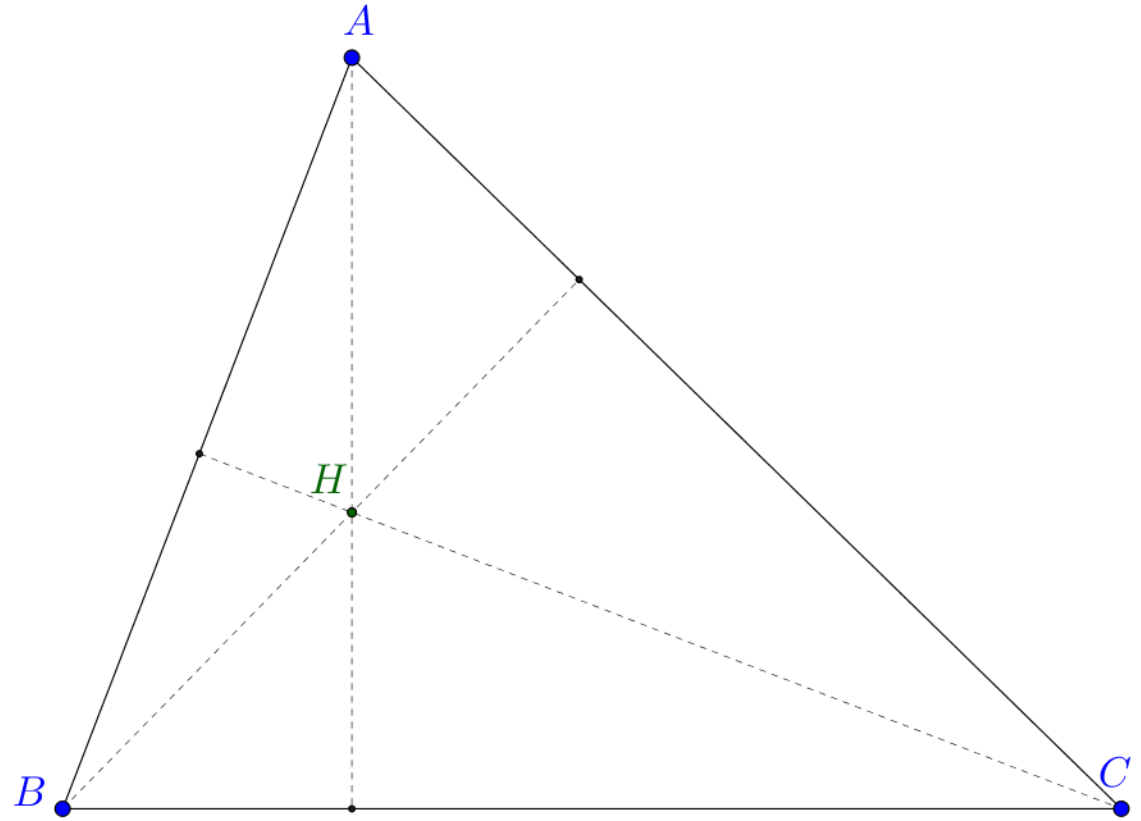
- Acute triangle  $ABC$



# Geometry problem

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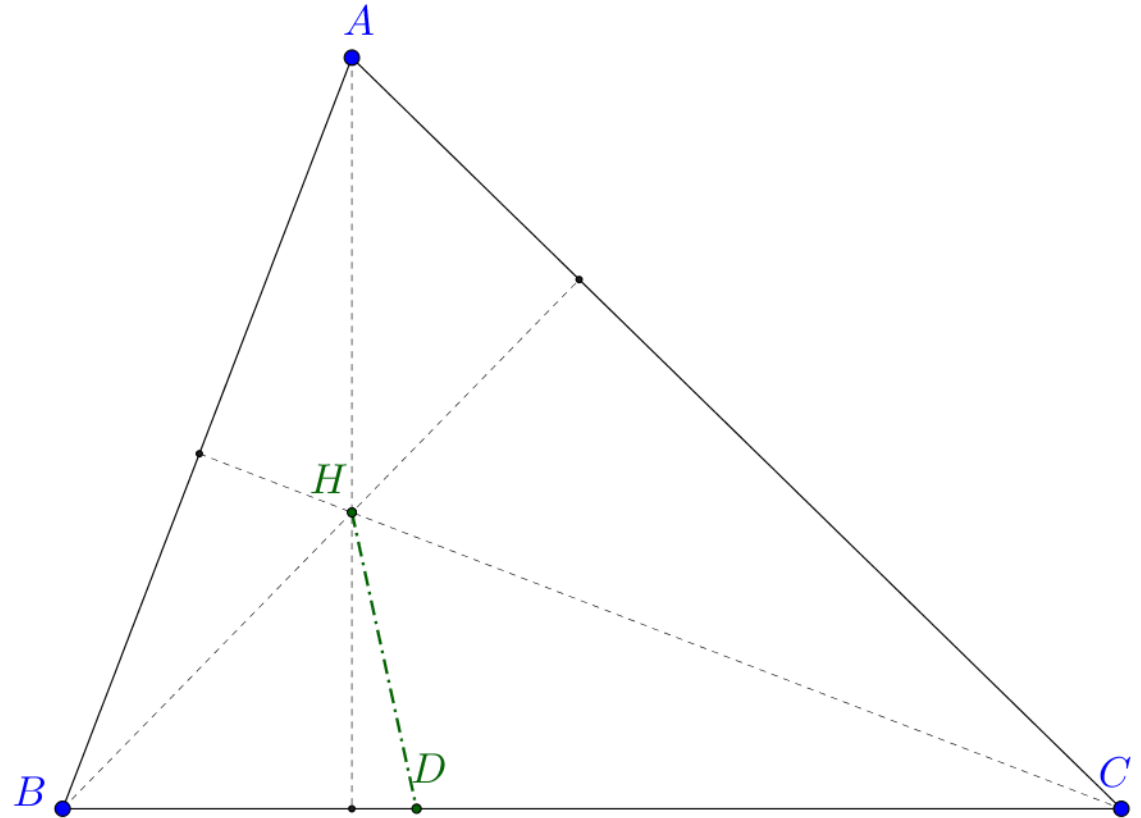
- Acute triangle  $ABC$
- $H$  is the orthocenter of  $ABC$



# Geometry problem

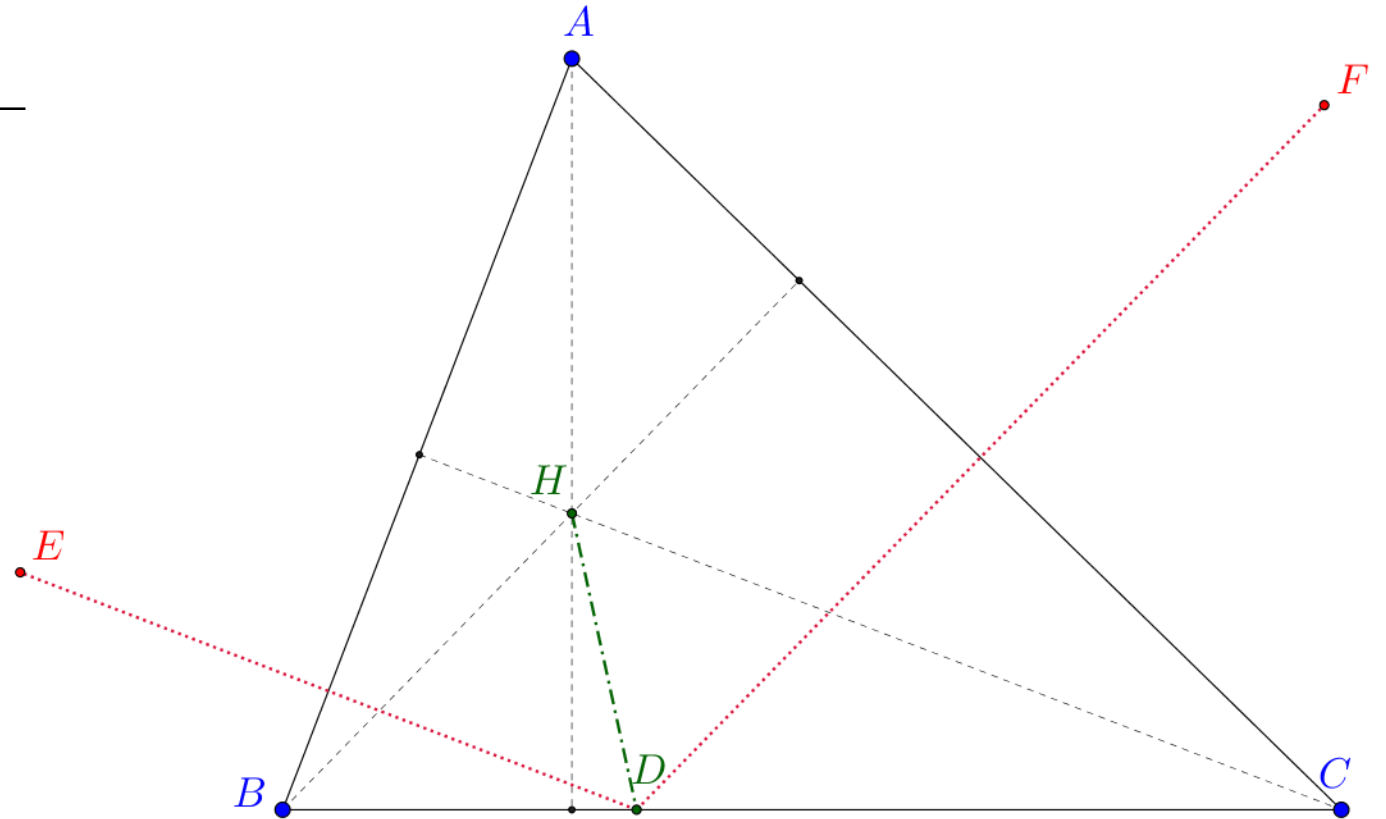
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- Acute triangle  $ABC$
- $H$  is the orthocenter of  $ABC$
- $D$  is the intersection of  $BC$  and the angle bisector of angle  $BHC$



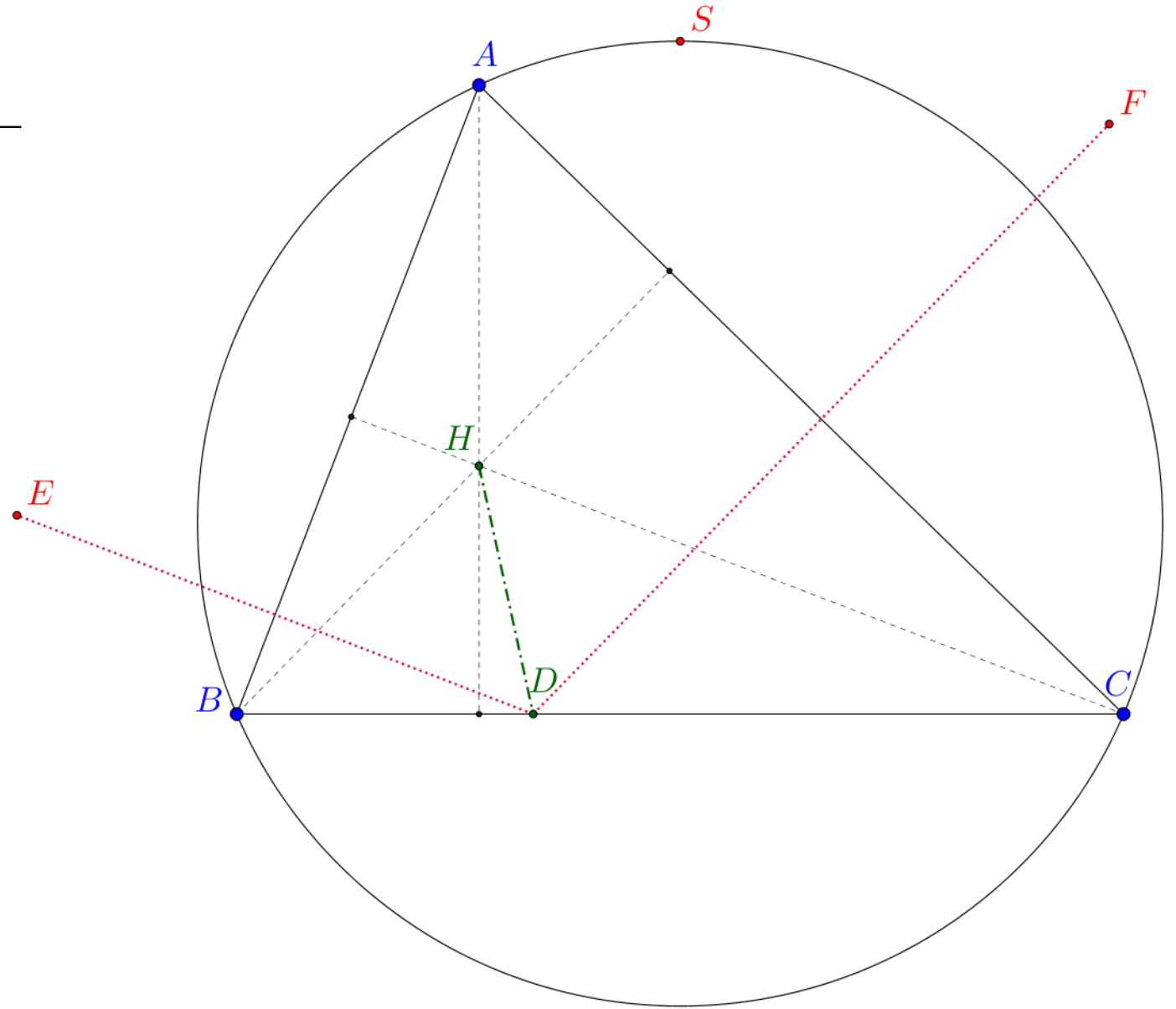
# Geometry problem

- Acute triangle  $ABC$
- $H$  is the orthocenter of  $ABC$
- $D$  is the intersection of  $BC$  and the angle bisector of angle  $BHC$
- $E, F$  are the reflections of  $D$  across  $AB$  and  $AC$ , respectively



# Geometry problem

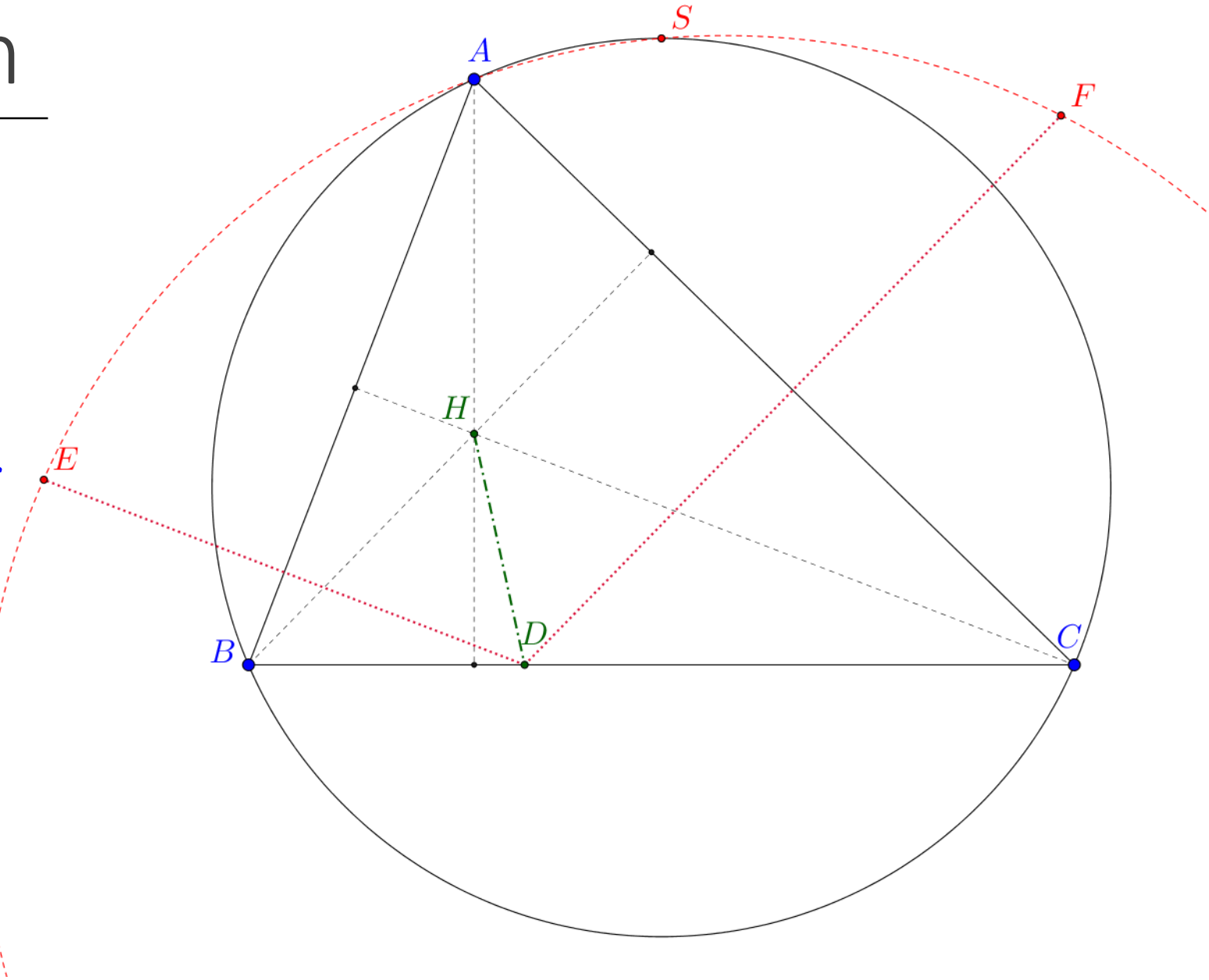
- Acute triangle  $ABC$
- $H$  is the orthocenter of  $ABC$
- $D$  is the intersection of  $BC$  and the angle bisector of angle  $BHC$
- $E, F$  are the reflections of  $D$  across  $AB$  and  $AC$ , respectively
- $S$  is the midpoint of arc  $BAC$



# Geometry problem

- Acute triangle  $ABC$
- $H$  is the orthocenter of  $ABC$
- $D$  is the intersection of  $BC$  and the angle bisector of angle  $BHC$
- $E, F$  are the reflections of  $D$  across  $AB$  and  $AC$ , respectively
- $S$  is the midpoint of arc  $BAC$

Prove:  $A, E, S, F$  are concyclic

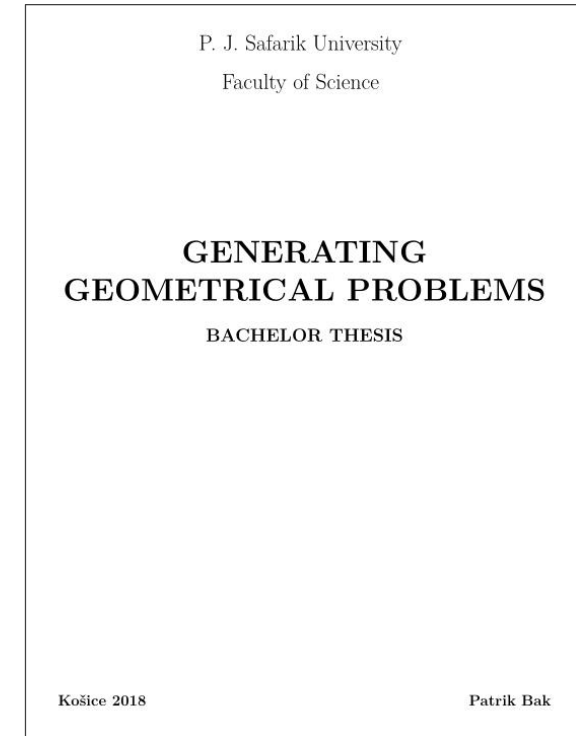




# What was done in the bachelor thesis?

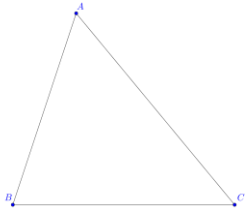
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- 1. Object **representation** of geometrical objects, configurations and theorems
- 2. Algorithms that **generate** geometrical objects and configurations
- 3. Algorithms that **find** all possible theorems relevant to a configuration
- 4. Algorithms that **verify** a theorem using numerical analytic geometry



# Example of an input and output

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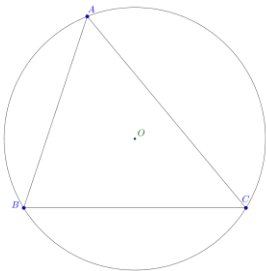
**Initial configuration:** triangle



**Constructor:** midpoint



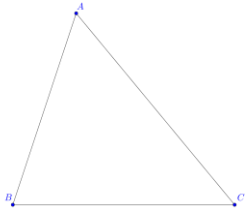
3 iterations



**Constructor:** circumcenter

# Example of an input and output

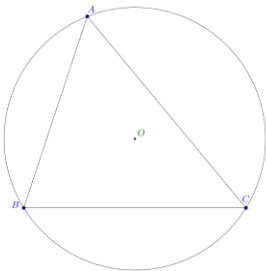
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**Initial configuration:** triangle



**Constructor:** midpoint



**Constructor:** circumcenter



3 iterations

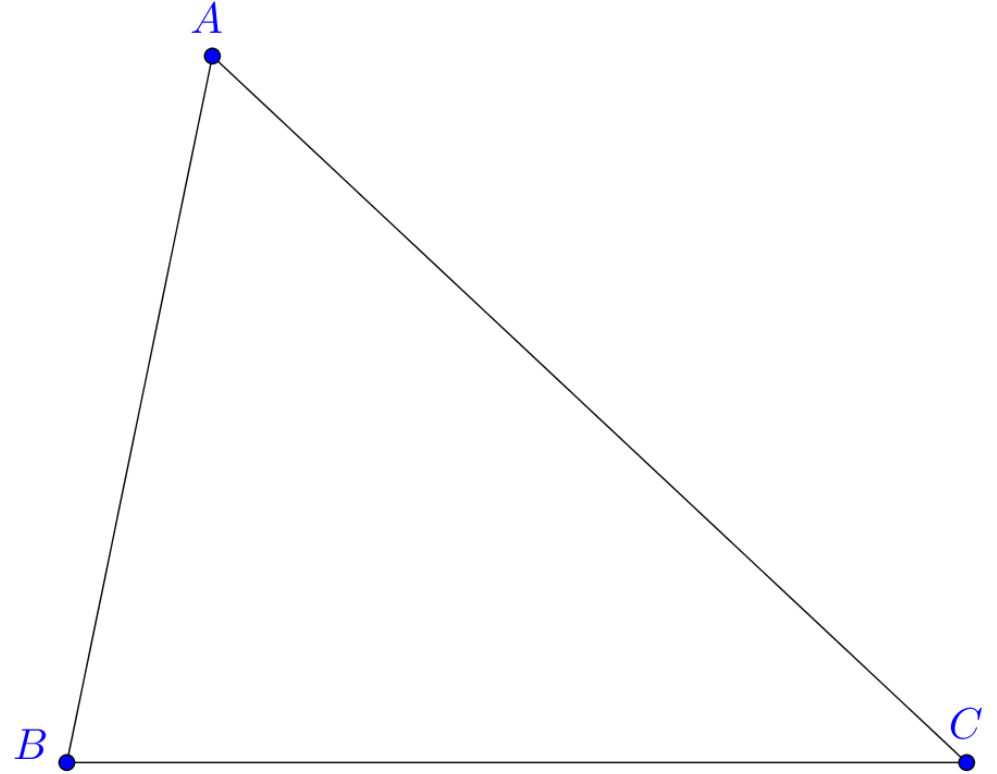
- Configurations: 73
- Configurations with theorems: 54
- Total number of theorems: 242
- Duration: 360 ms

Something  
interesting?

# Generated problem

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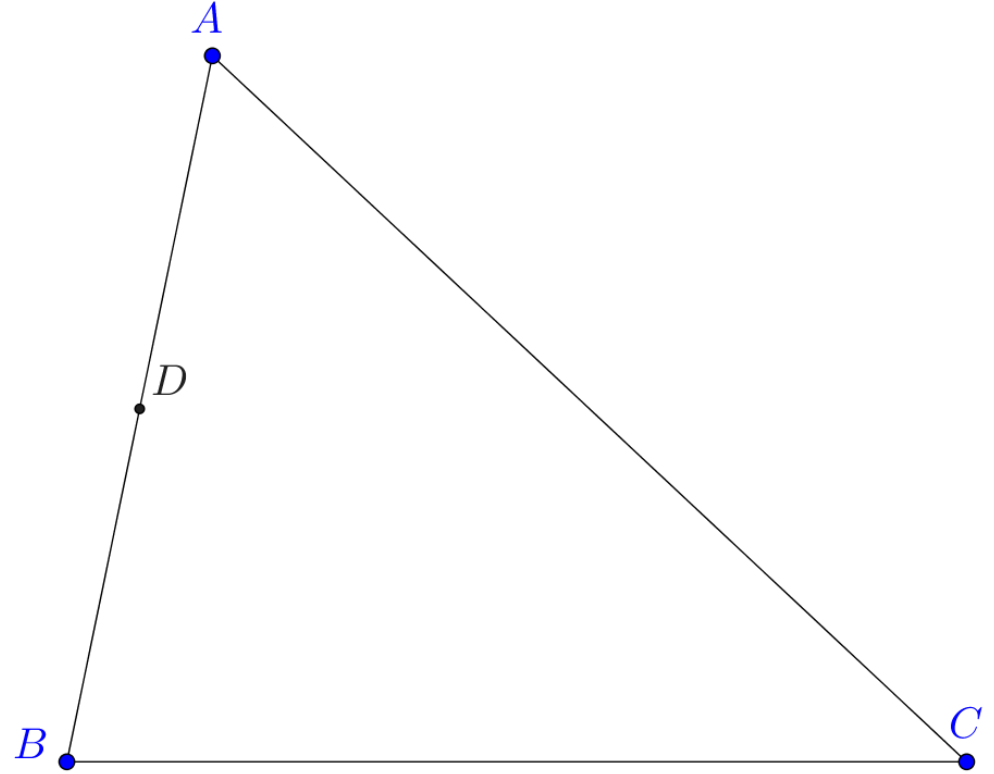
- Acute triangle  $ABC$



# Generated problem

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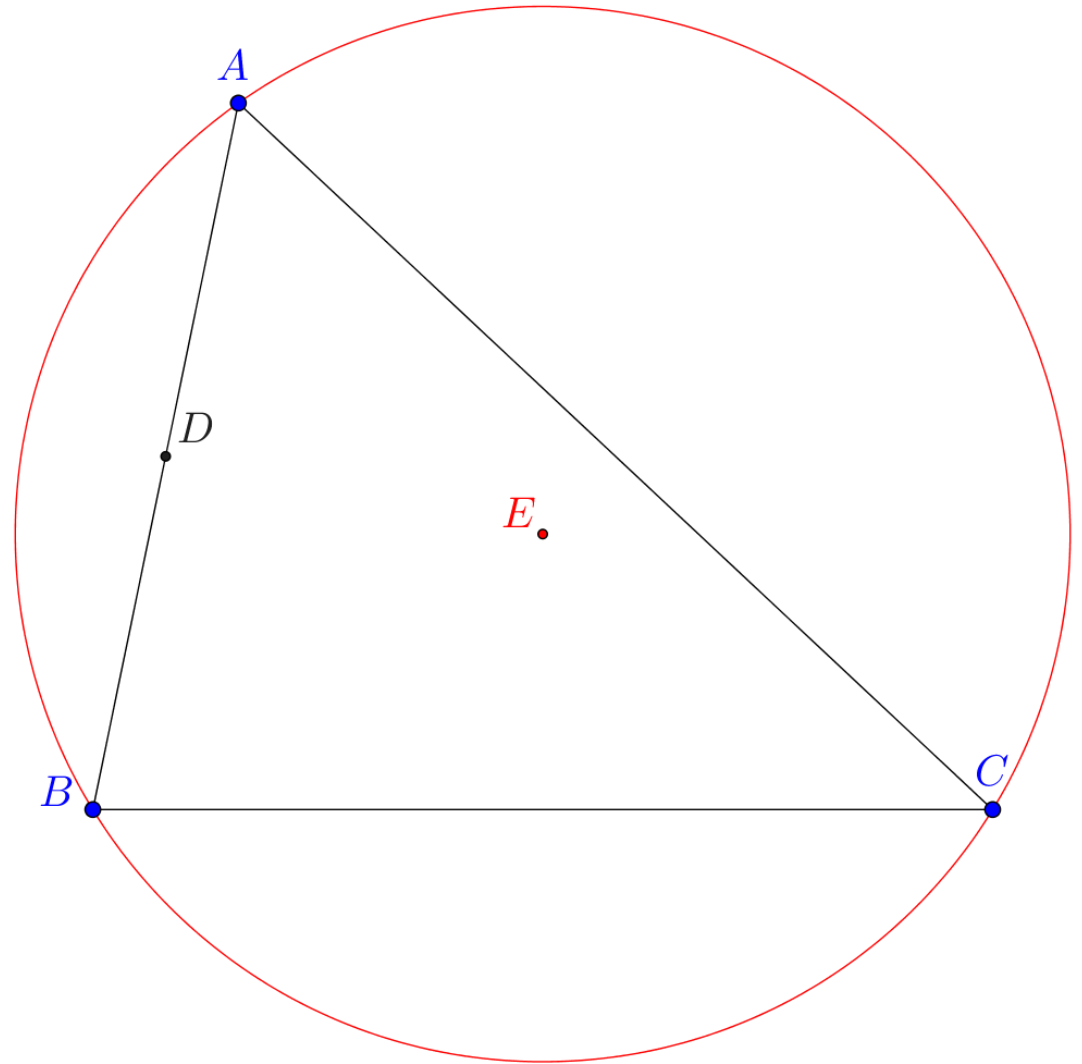
- Acute triangle  $ABC$
- $D$  is the midpoint of  $AB$



# Generated problem

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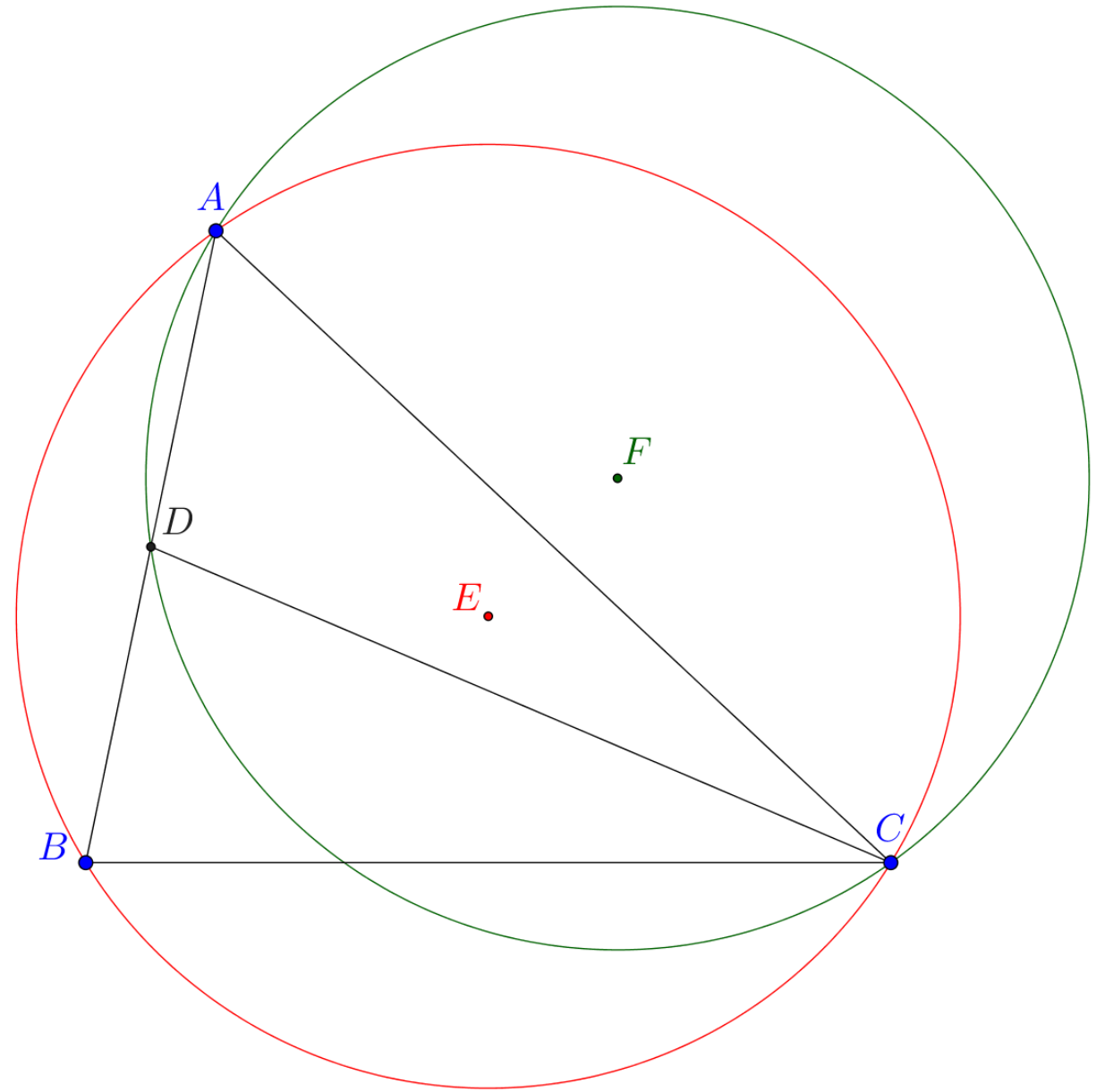
- Acute triangle  $ABC$
- $D$  is the midpoint of  $AB$
- $E$  is the circumcenter of  $ABC$



# Generated problem

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- Acute triangle  $ABC$
- $D$  is the midpoint of  $AB$
- $E$  is the circumcenter of  $ABC$
- $F$  is the circumcenter of  $ACD$

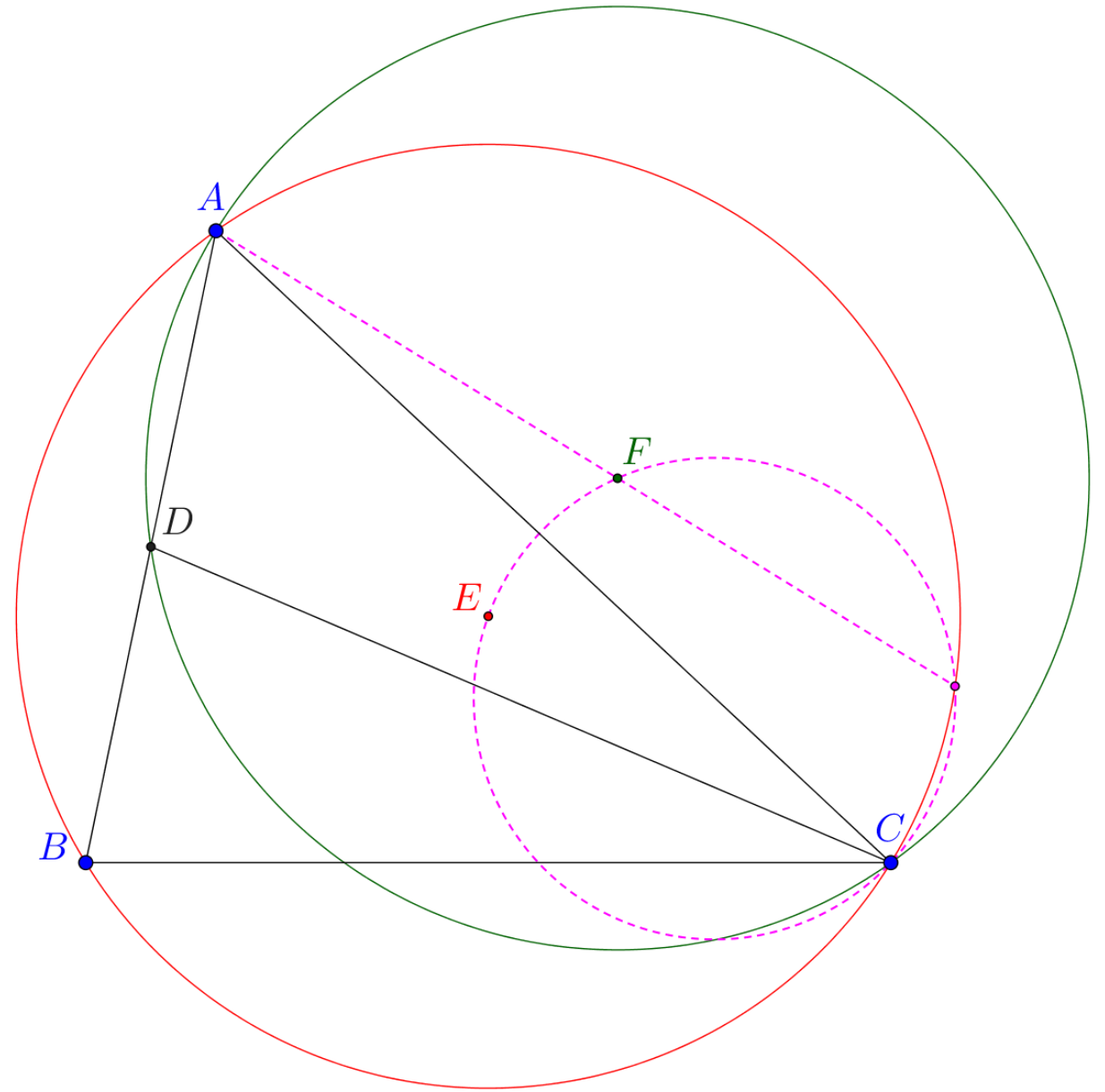


# Generated problem

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- Acute triangle  $ABC$
- $D$  is the midpoint of  $AB$
- $E$  is the circumcenter of  $ABC$
- $F$  is the circumcenter of  $ACD$

Prove:  $AF$ ,  $(CEF)$ ,  $(ABC)$  are concurrent

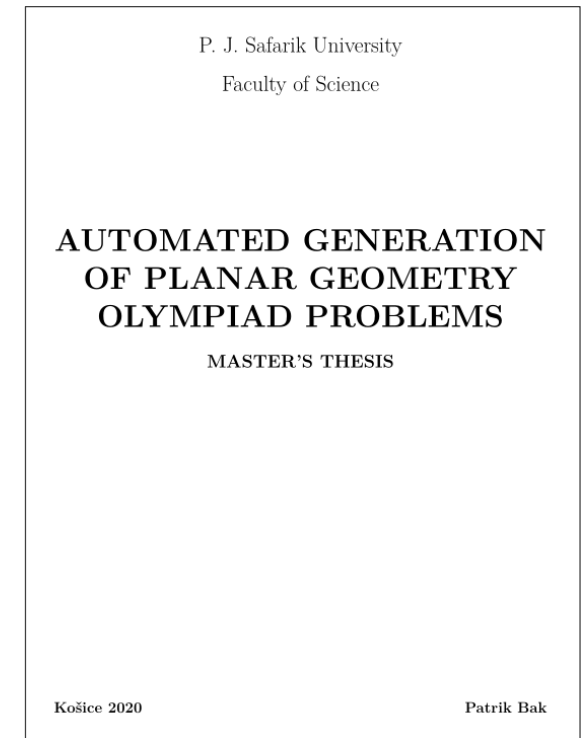




# What is left for the master thesis?

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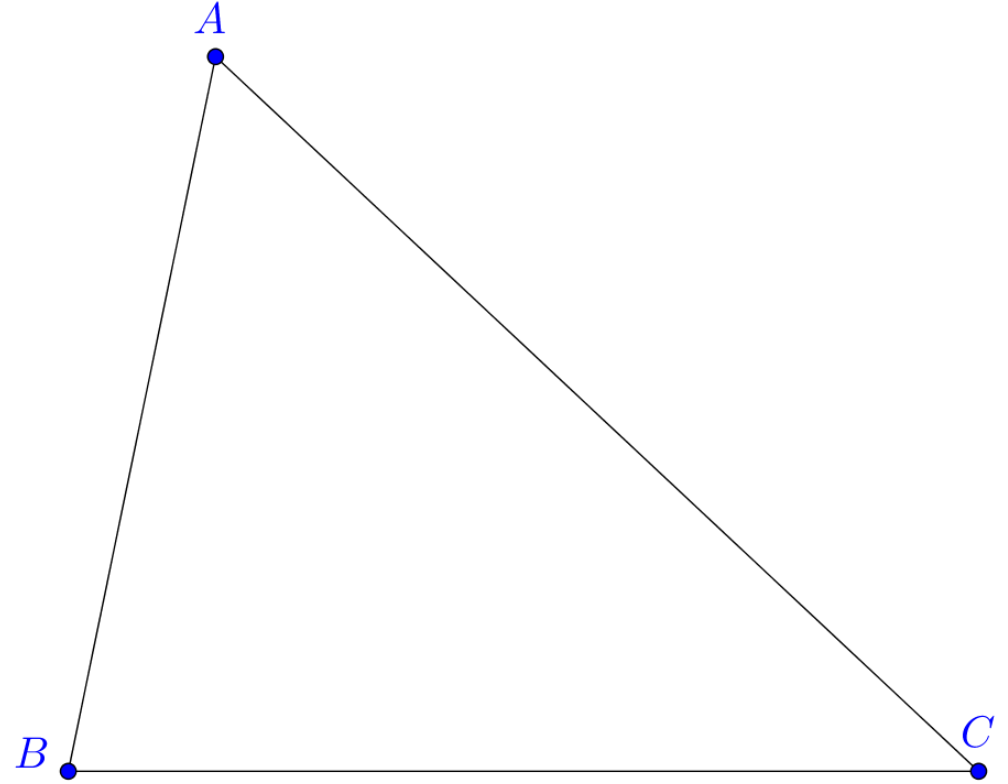
- 1. Automatic **detection** of trivial theorems



# Trivial problem

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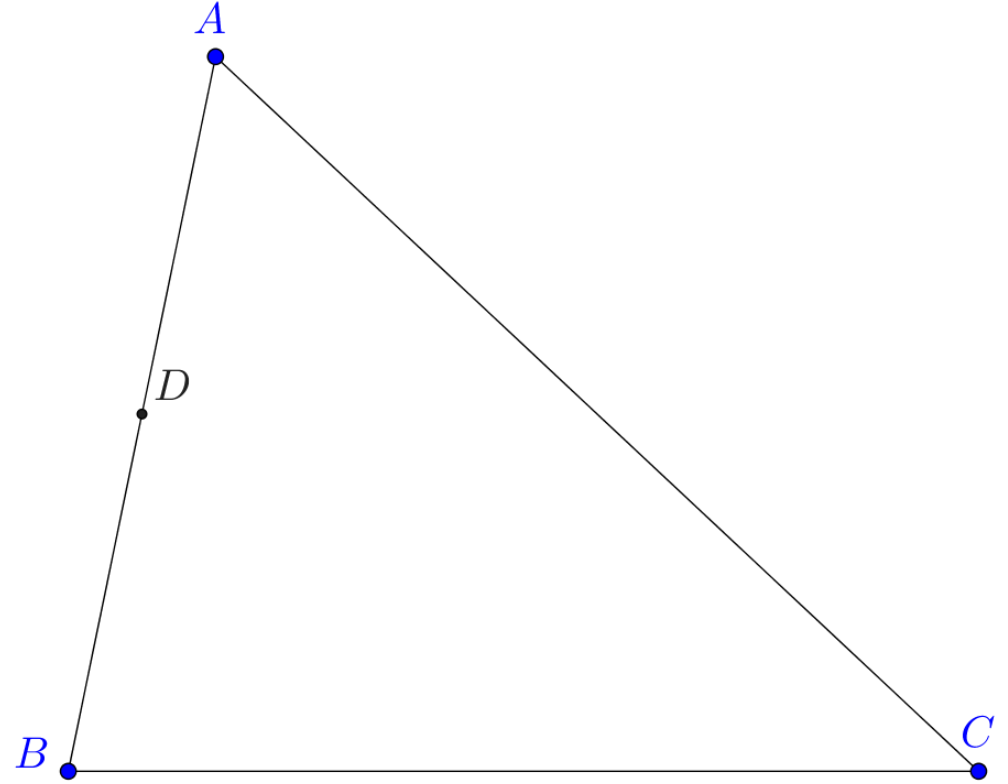
- Acute triangle  $ABC$



# Trivial problem

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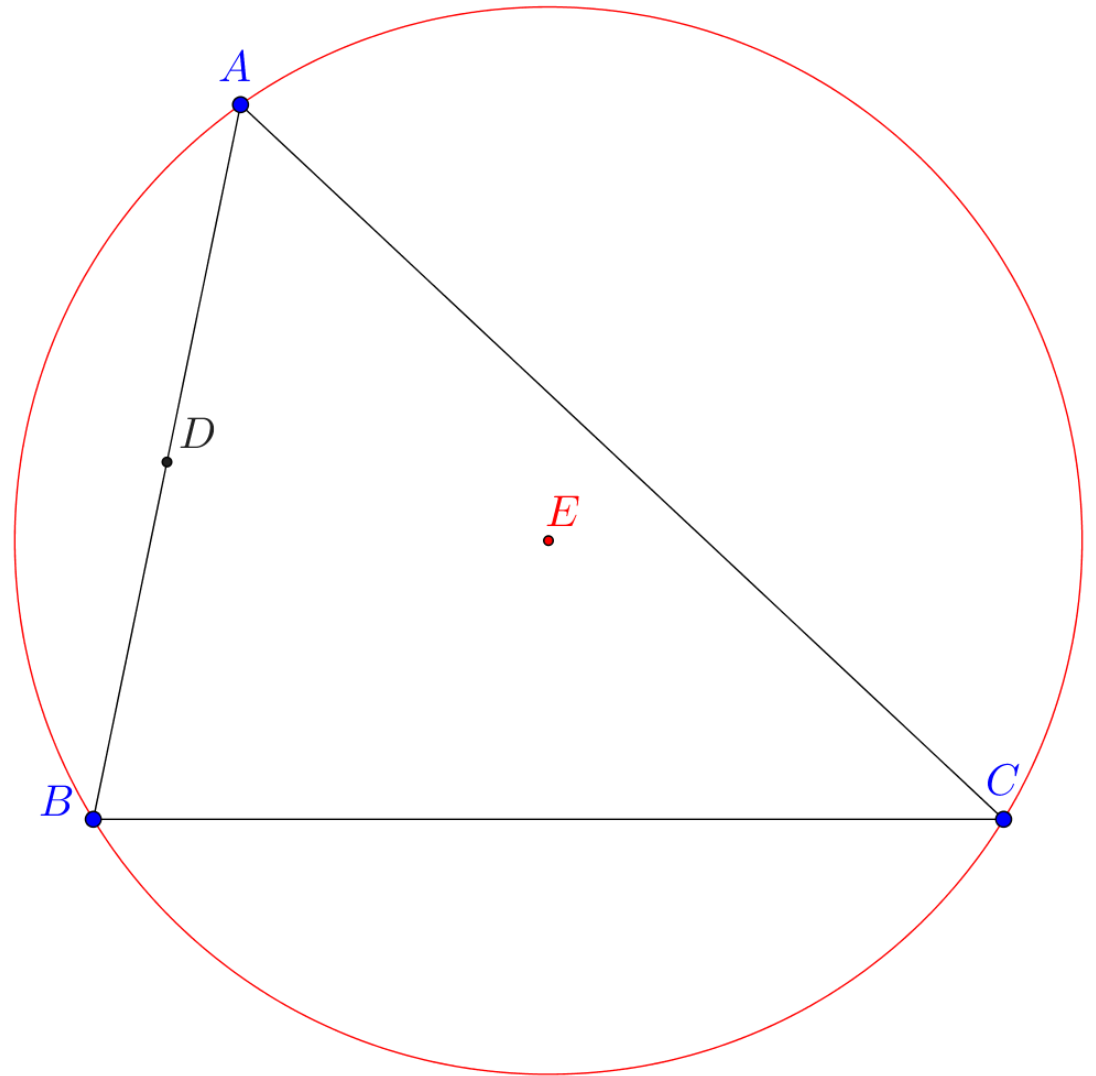
- Acute triangle  $ABC$
- $D$  is the midpoint of  $AB$



# Trivial problem

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- Acute triangle  $ABC$
- $D$  is the midpoint of  $AB$
- $E$  is the circumcenter of  $ABC$

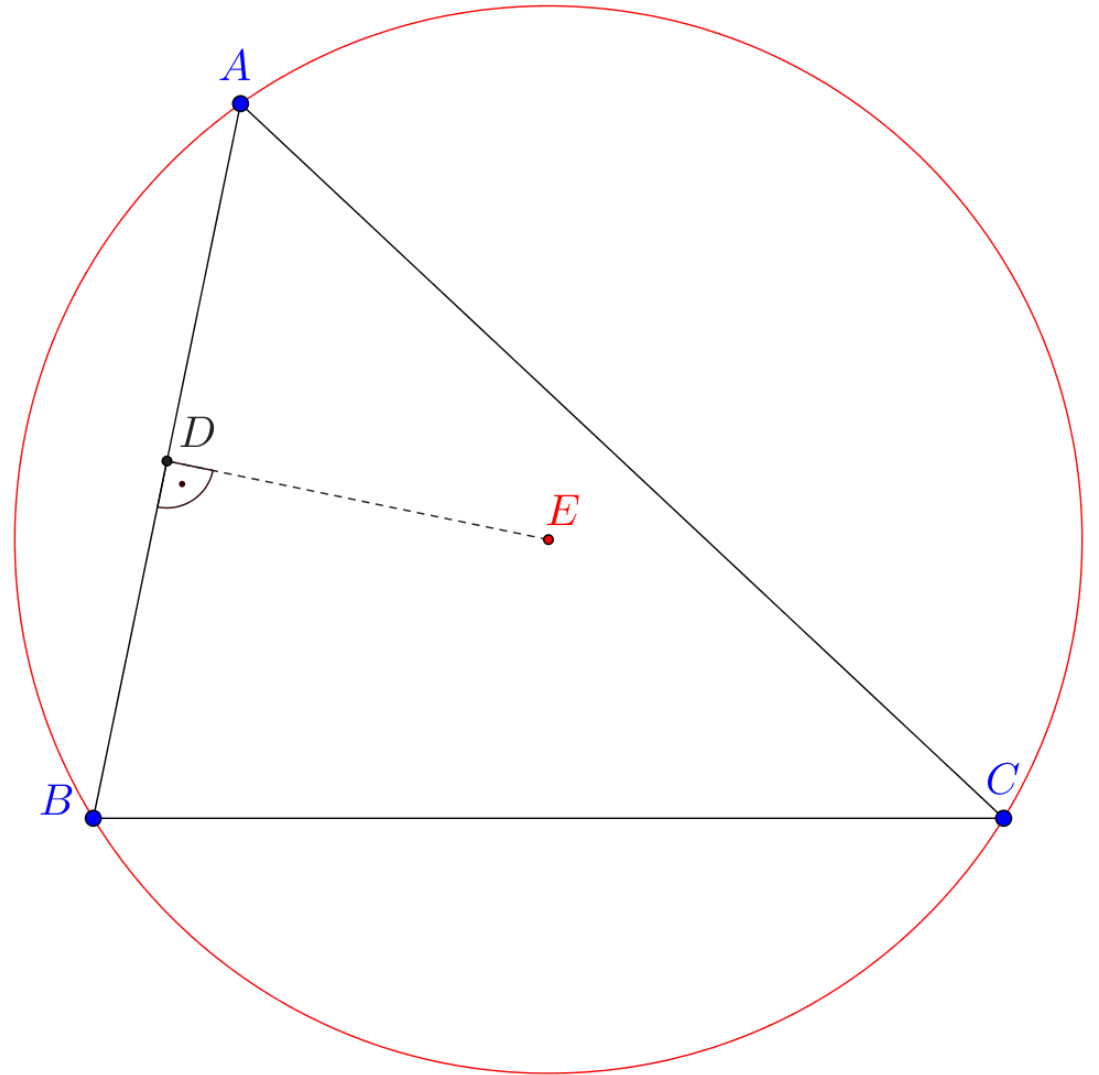


# Trivial problem

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- Acute triangle  $ABC$
- $D$  is the midpoint of  $AB$
- $E$  is the circumcenter of  $ABC$

Prove:  $ED$  and  $AB$  are perpendicular



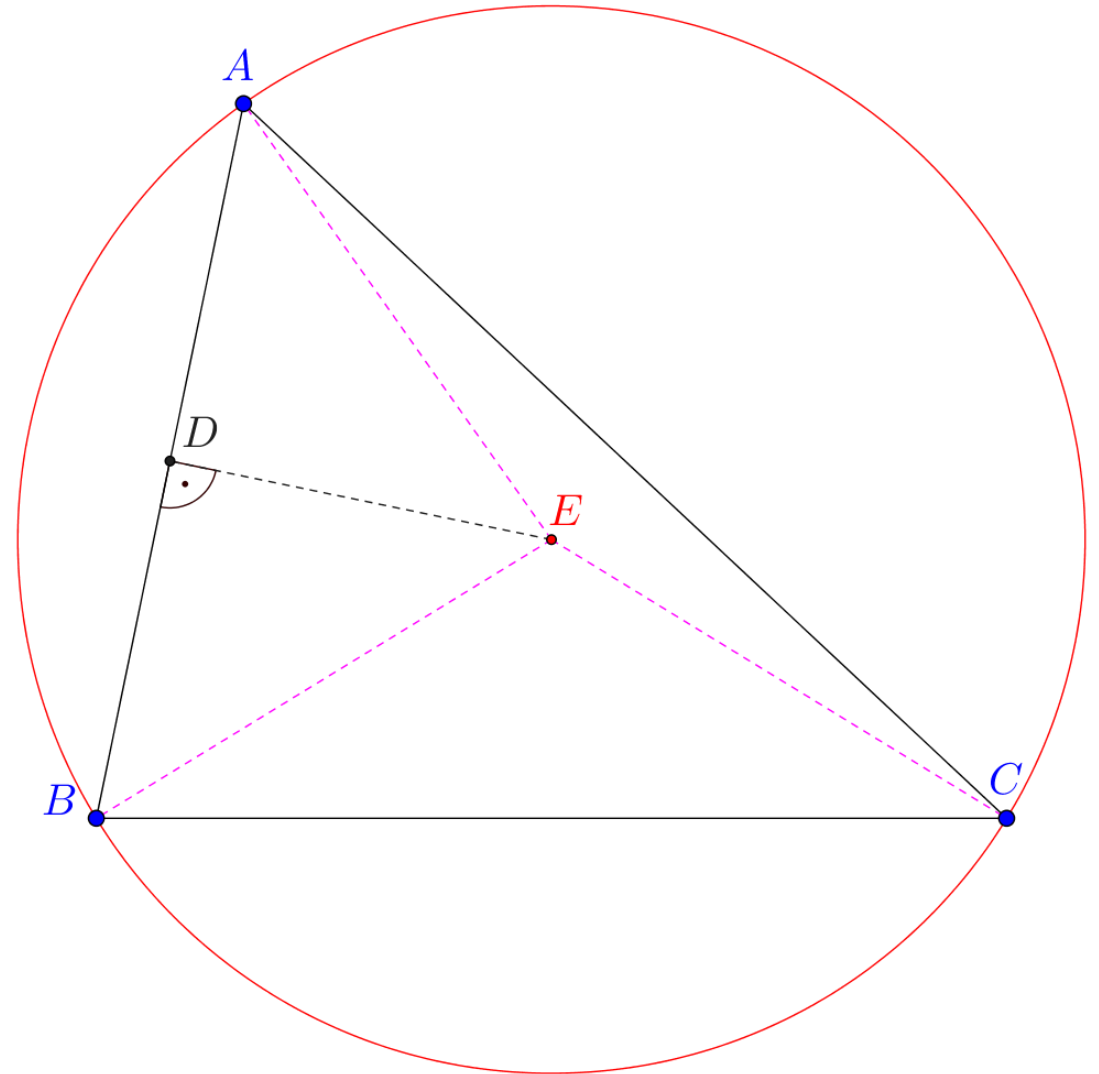
# Trivial problem

---

- Acute triangle  $ABC$
- $D$  is the midpoint of  $AB$
- $E$  is the circumcenter of  $ABC$

Prove:  $ED$  and  $AB$  are perpendicular

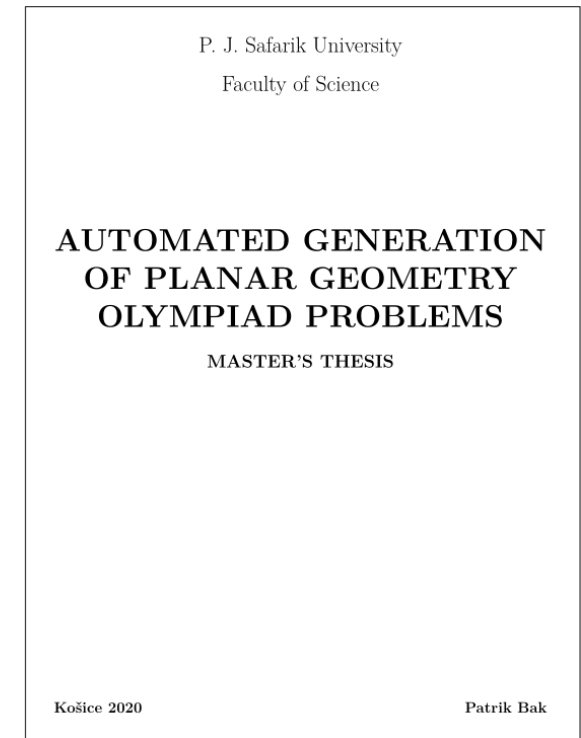
Prove:  $EA = EB = EC$



# What is left for the master thesis?

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- 1. Automatic **detection** of trivial theorems
- 2. Automatic **generalization** of theorems

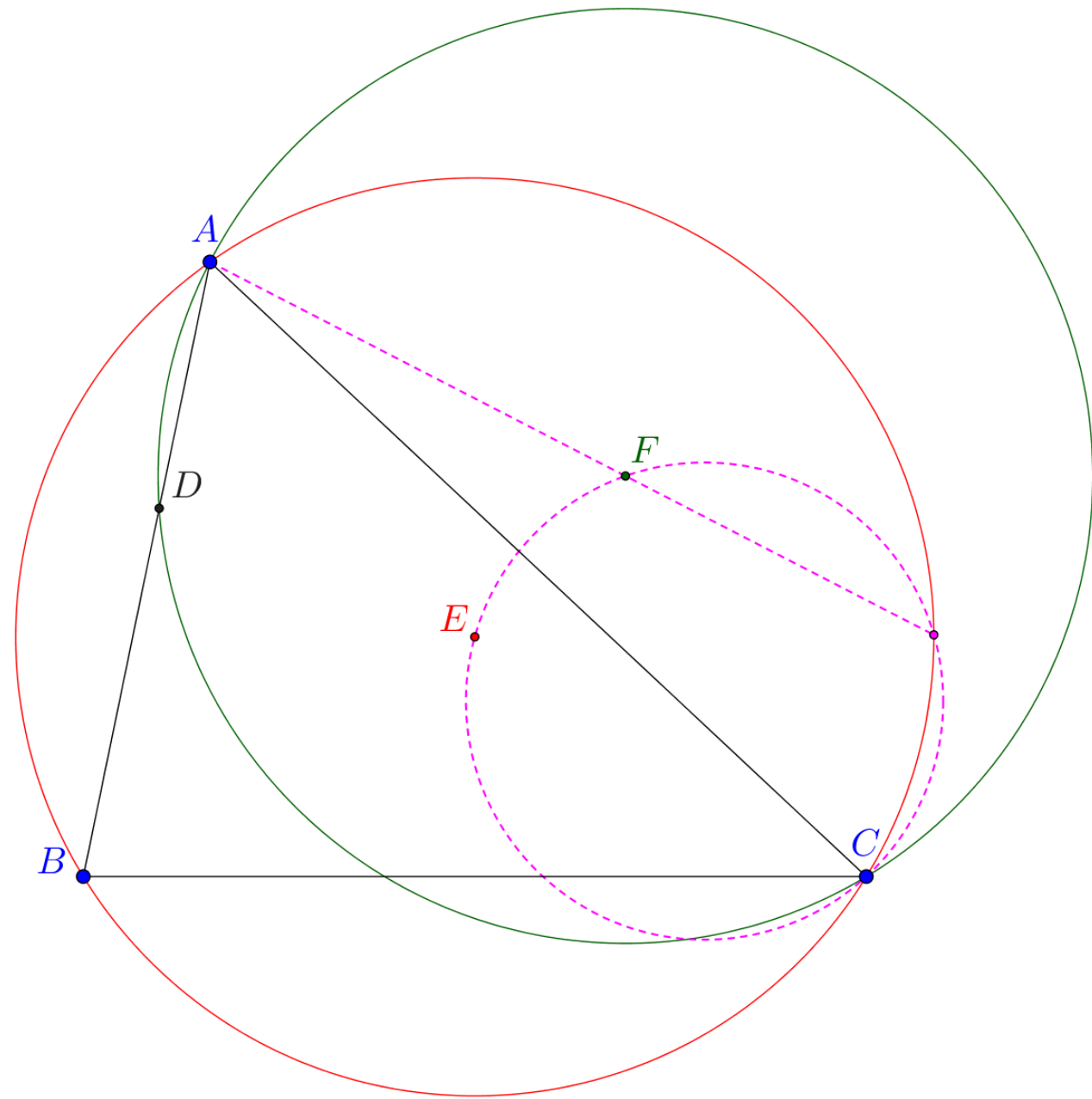


# Generalized problem

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- Acute triangle  $ABC$
- $D$  is ~~(the midpoint)~~ any point of  $AB$
- $E$  is the circumcenter of  $ABC$
- $F$  is the circumcenter of  $ACD$

Prove:  $AF$ ,  $(CEF)$ ,  $(ABC)$  are concurrent

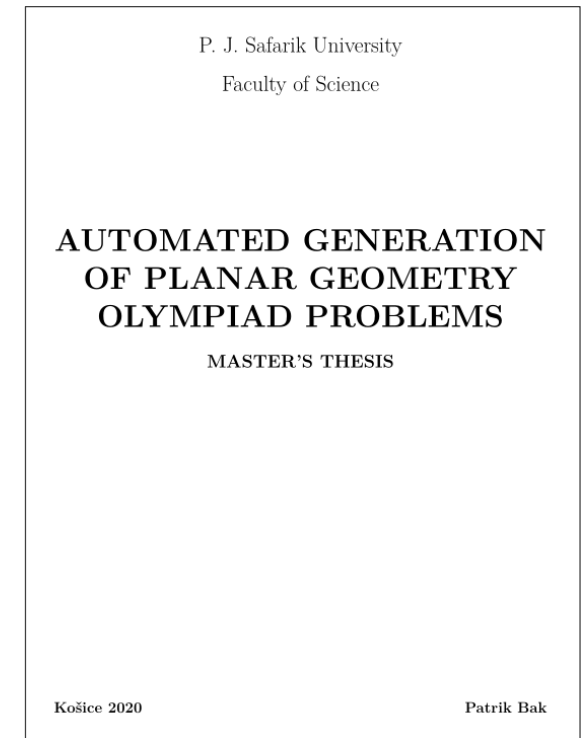




# What is left for the master thesis?

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- 1. Automatic **detection** of trivial theorems
- 2. Automatic **generalization** of theorems
- 3. Automatic **simplification** of configurations



# Simplification of configurations

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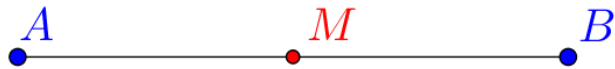
- Line segment  $AB$



# Simplification of configurations

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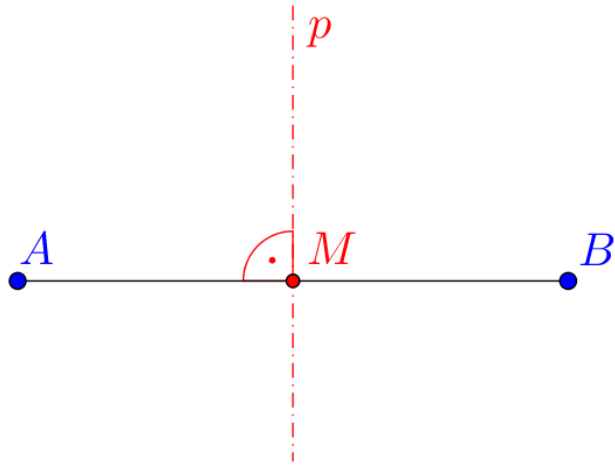
- Line segment  $AB$
- $M$  is the midpoint of  $AB$



# Simplification of configurations

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- Line segment  $AB$
- $M$  is the midpoint of  $AB$
- $p$  is the perpendicular line to  $AB$  at  $M$



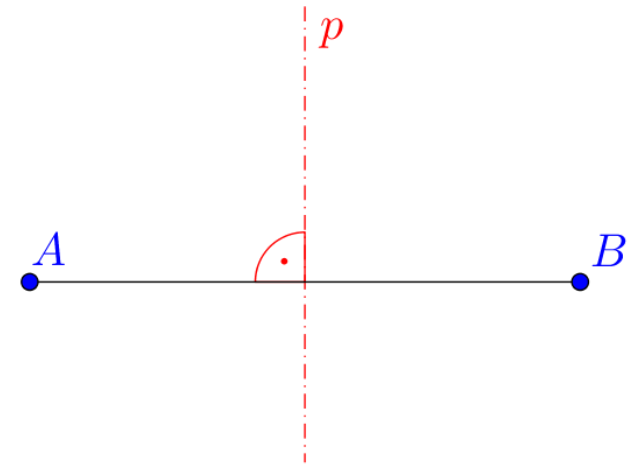
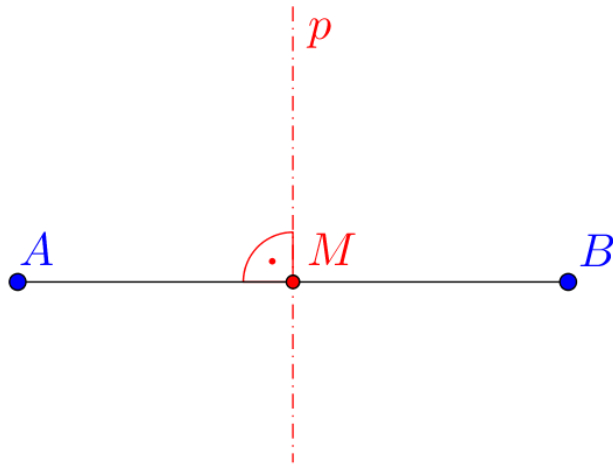
# Simplification of configurations

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- Line segment  $AB$
- $M$  is the midpoint of  $AB$
- $p$  is the perpendicular line to  $AB$  at  $M$



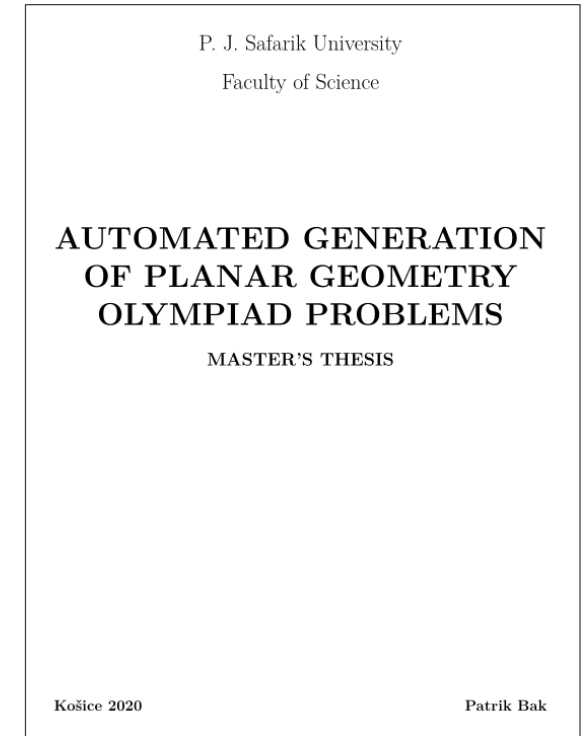
- Line segment  $AB$
- ~~$M$  is the midpoint of  $AB$~~
- $p$  is the perpendicular bisector of  $AB$



# What is left for the master thesis?

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- 1. Automatic **detection** of trivial theorems
- 2. Automatic **generalization** of theorems
- 3. Automatic **simplification** of configurations
- 4. Implementing a **database** for generated results

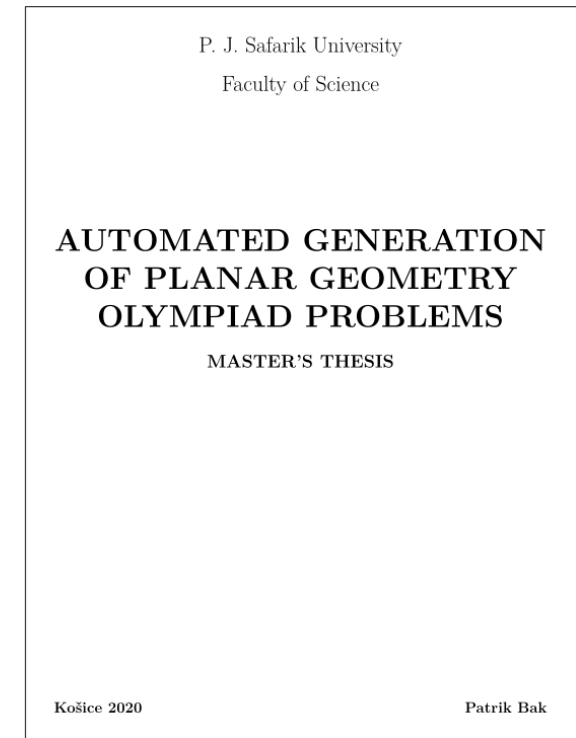


# What is left for the master thesis?

---

- 1. Automatic **detection** of trivial theorems
- 2. Automatic **generalization** of theorems
- 3. Automatic **simplification** of configurations
- 4. Implementing a **database** for generated results
- 5. Implementing a **user interface**

*(plus some other things listed in the bachelor thesis)*



# Implementation

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Entity Framework



<https://github.com/PatrikBak/GeoGen>





**KEEP  
CALM  
AND  
WRITE YOUR  
MASTER THESIS**