A Neuro-Symbolic Computing Approach to Symbol Grounding for $\mathcal{ALC}$-Ontologies

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ABSTRACT

Neural-symbolic computing aims at integrating robust neural learning and sound symbolic reasoning into a single framework, to leverage the complementary strengths of both of these, seemingly unrelated (maybe even contradictory) AI paradigms. The central challenge in neural-symbolic computing is to unify the formulation of neural learning and symbolic reasoning into a single framework with common semantics, that is, to seek a joint representation between a neural model and a logical theory that can support the basic grounding learned by the neural model and also stick to the semantics of the logical theory. In this paper, we propose differentiable fuzzy $\mathcal{ALC}$ (DF-ALC) for this role, as a neural-symbolic approach with the desired semantics of $\mathcal{ALC}$. DF-ALC unifies the description logic $\mathcal{ALC}$ and neural models for symbol grounding; in particular, it infuses an $\mathcal{ALC}$ knowledge base into neural models through differentiable concept and role embeddings. We define a hierarchical loss to the constraint that the grounding learned by neural models must be semantically consistent with $\mathcal{ALC}$ knowledge bases, and we prove soundness of the semantics of DF-ALC under the open-world assumption and soundness of learning to ground for a DF-ALC ontology. We further define a rule-based loss for DF-ALC adapting to semantic image interpretation. The experiment results show that DF-ALC with rule-based loss can improve the performance of object detectors.

1 INTRODUCTION

For decades now, trends in the computational modeling of intelligent behavior have followed a recurring pattern, cycling between a primary focus on symbolic logic and automated reasoning, and on pattern recognition (neural networks). In recent years, neural network models, powered by ever-increasing amounts of data and computing resources, have been in the spotlight and driven much of the progress in AI — due in small part to the success of computational neuroscience and in large part to the success of deep learning in AI. Nevertheless, there are some inhibitors to making the most of their potential, such as the lack of transparency of some AI algorithms which are unable to fully explain the reasoning for their decisions (aka the "black box" nature), as well as the lack of high-quality training data, without a foundation of which, even the most performant models can be rendered useless — the current neural network models are flawed in its lack of model interpretability and the need for large amounts of data for learning. The classic symbolic AI [5, 15, 28, 39, 57], on the other hand, relies less heavily on data but more on knowledge representation using symbolic logic and automated reasoning, and thus bears full transparency by pinning down its internal working to a set of logical statements which have a well-defined syntax and semantics. In this sense, symbolic systems may have the potential to provide a good complement to neural systems, and these two fundamental AI paradigms can in principle be integrated in a way that neural systems' transparency can be promoted through the injection of symbolic systems. This fancier vision of AI has resulted in a relevant and promising research area — neural-symbolic computing [11, 12, 20, 21, 23, 31, 34, 38, 52].

Neural-symbolic computing aims at computing with both learning and reasoning abilities, to step towards more comprehensive intelligence. Current learning ability relies largely on differentiable programming to draw conclusions from observations and apply them, while current reasoning ability relies largely on logical programming to give conclusions inferred from premises and rules through deductive reasoning, give rules according to observations comprising premises and conclusions through inductive reasoning, and give premises that can interpret conclusions according to rules through abductive reasoning. So it comes with challenges in the integration and representation of these two kinds of programming paradigms. From the perspective of integration, research works differ in logical techniques that are mainly consumed. Neural-symbolic inductive logical programming works seek to learn probabilistic logical rules from observations. This requires learning model parameters in a continuous space and the structure in a discrete space [6, 19, 45, 56, 58]. To combine the ability of deductive reasoning, the first line of research learn to reason by modeling the inference procedure using neural networks or replacing logical computations with differentiable functions [13, 18, 26, 42, 43, 50]. But this neglects factual knowledge which bridges the physical world and the conceptual world, so the second line of research aim to find an interpretation that satisfies theories which can be a mapping between these two worlds[1] by encoding the satisfiability of theories in the loss function [41, 46, 49, 53, 55]. But these works cannot find explanations of observations according to theories, so abductive learning-based neural-symbolic works are proposed to use the explanations getting through abductive reasoning to promote the interpretability of the computing [7, 27, 51, 60]. From the view of the knowledge expressivity, some works are based on classical logic — propositional logic [7, 50, 51, 60], description logic [6, 17, 18], or first-order logic [19, 26, 42, 43, 45, 46, 56, 58], others are based on non-classical logic, such as fuzzy logic [13, 41, 53], or probabilistic logic [35, 56]. These works have well-verified effectiveness of neural-symbolic computing in knowledge acquisition [45], query

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[1]The satisfiability problem (SAT) involves determining whether there exists an interpretation, or assignment of truth values, that satisfies a given logical formula.

answering [1], semantic image interpretation [14, 29], and entity linking [30].

We concern about the symbol grounding problem [8, 10, 24]: ground conceptual symbols' meanings in perceptual instances. Neural networks may be one way to ground concrete instances in the capacity to categorize them [25] but fail to capture the inherent logical relations between symbols. Symbol grounding problem is the main challenge in neural-symbolic AI, and can be better resolved by neural-symbolic computing. Given a symbol set, The neural module in neural-symbolic computing maps input instances to these symbols through supervised (/semi-supervised) learning. And the symbolic module finds the interpretation that can satisfy the semantics of these symbols through reasoning. Both the learned mapping and the interpretation are grounding (meaning) of these symbols but are from different views. However, most neural-symbolic computing works fail to do symbol grounding, some works are due to the inability to interact with many-valued logic (non-classical logic), and other works such as SATNet [49] are due to the lack of modeling specific logical semantics. Logic Tensor Network (LTN) [4] is the state of art neural-symbolic work for symbol grounding based on real logic, the semantics of which is close to the semantics of fuzzy first-order logic, but differs in syntax, where real logic types functions and predicates. LTN transforms the symbol grounding problem into gradient-descent-based optimization through learning (i.e. searching the grounding of symbols in theories by maximizing the satisfiability of theories) and reasoning (i.e. querying the truth value of a formula from theories). The efficiency of LTN under open-world assumption (OWA)[2] is evaluated with incomplete grounding revision task in [54] showing that LTN cannot completely revise the grounding (with 88% F1 score). And LTN cannot get interpretable and safe symbol grounding, may meet reasoning shortcut problem[36, 37] (Though result is correct, the interpretation process is wrong). These are due to the missing properties in approximate reasoning of real logic analyzed in [53], the unreasonable semantics under OWA, and the mismatched way to combining neural network and logical semantics. Real logic as an extension to first-order logic is quite expressive and is undecidable, which means that we cannot find a sound, complete, and terminating decision algorithm for real logic [47]. Hence, we consider Differentiable Fuzzy Logic (DFL) based on description logics (DLs) [3], a family of knowledge representation languages that strike a balance between expressivity and decidability; most DLs are decidable fragments of first-order logic.

DLs are widely used in ontological modeling by providing logical semantics for Web Ontology Language (OWL) as an underpinning of logical reasoning [33]. Our work is based on a fundamental description logic — Attributive Concept Language with Complements ($\mathcal{ALC}$) [44], which is a decidable fragment of first-order logic and has strong expressive power.

In this study, we present the differentiable fuzzy $\mathcal{ALC}$ (DF-ALC) as a neuro-symbolic approach that can combine effective information from any $\mathcal{ALC}$ ontology O and any neural network for getting symbol grounding. For observed instances X, the neural network maps them into invariant features as their symbol groundings, and then can theoretically categorize them as symbols Y with a probably approximately correct distribution $p(Y|X)$, which is an incomplete grounding $\mathcal{I}'$ in practice. Based on perceptual grounding $\mathcal{I}'$, DF-ALC revises it into a grounding $\mathcal{I}''$ that satisfies O. While the grounding that satisfies O can be multiple, the direction of grounding should retain as much valid information as possible in $\mathcal{I}'$. So we target to find the grounding that meets :

$$\max \ l_1(\mathcal{I}'', \mathcal{I}') \ \text{ s.t. } \ l_2(\mathcal{I}'', O) = 1$$

where l_1 measures the constancy (keeping the reliable parts of perceptual grounding), l_2 measures the satisfiability.

A symbol grounding application of our work is semantic image interpretation (SII) [14, 29, 32, 40], which aims to generate a structured and human-readable description of the content of images. Current successful SII researches [2, 14] rely on background knowledge of the images. LTN [14] models predicates and functions as neural networks and learns the symbol grounding through maximizing the satisfiability in a supervised way. The main struggle of these neural-symbolic works in leveraging logical knowledge to adapt to the symbol grounding problem is that the revision signal cannot be properly conveyed. In our work, rather than maximizing satisfiability to generate grounding, we propose a rule-based loss to learn a symbol grounding problem — semantic image interpretation.

The key contributions of this work can be summarized as follows:

- We present a neural-symbolic approach DF-ALC which facilitates a sound and complete mechanism to revise the probabilistic semantics by a neural model according to a consistent $\mathcal{ALC}$ ontology. This makes us the first to combine differentiable fuzzy logic with fuzzy description logics.
- Experiments show that DF-ALC can keep the reliable component of the perceptual grounding. Meanwhile, unknown situations are few to affect grounding, this further demonstrates that the semantics of DF-ALC are solid in terms of crisp $\mathcal{ALC}$ under OWA.
- To get safe symbol grounding, rather than ground by maximizing satisfiability. We designed rule-based loss, which mitigates the reasoning shortcut problem, and helps fuzzy description logic adapt to interpret images, and improves the performance in image object classification.
- The source code, alongside the experimental settings, is publicly accessible at https://anonymous.4open.science/r/DF-ALC.

Except for semantic image interpretation, the application of DF-ALC is wide, e.g. in dialogue state tracker, and ontology-mediated query answering, which is further elaborated with related works in the appendix.

2 PRELIMINARIES

2.1 The Description Logic $\mathcal{ALC}$

Let N_C and N_R be pairwise disjoint and countably infinite sets of *concept names* and *role names*, respectively. $\mathcal{ALC}$*-concepts* are inductively constructed based on the following syntax rule:

$$C, D \to \top | \bot | A | \neg C | C \sqcap D | C \sqcup D | \exists r.C | \forall r.C,$$

where $A \in N_C$, $r \in N_R$, and C and D range over concepts. A concept of the form $A \in N_C$ is called *atomic*, otherwise it is *compound*. An

[2] it assumes that the truth value of any formula out of theories is unknown

ontology $\mathcal{O}$ consists of a TBox and an ABox. An *$\mathcal{ALC}$-TBox* $\mathcal{T}$ is a finite set of axioms of the form:

$C \sqsubseteq D$ *(concept inclusion)*, and $C \equiv D$ *(concept equivalence)*,

where C and D are concepts. The *disjointness* between C, D is $C \sqcap D \sqsubseteq \bot$. We use the axiom $C \equiv D$ as abbreviation for $C \sqsubseteq D$ and $D \sqsubseteq C$.

Let N_I be disjoint and countably infinite sets of *individual names*, while an *$\mathcal{ALC}$-ABox* $\mathcal{A}$ is a finite set of crisp assertions of the form:

$a : C$ *(concept assertion)*, and $(a, b) : r$ *(role assertion)*,

where C is a concept, r is a role name, and a, b are individuals from N_I.

An $\mathcal{ALC}$ ontology is comprised of an $\mathcal{T}$ and $\mathcal{A}$, denoted as $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$. The *signature* of $\mathcal{O}$ is $sig(\mathcal{O}) = N_C \cup N_R \cup N_I$.

The semantics of $\mathcal{O}$ is defined in terms of an *interpretation* $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$, where $\Delta^{\mathcal{I}}$ denotes the *domain of the interpretation* (a non-empty *crisp set*), and $\cdot^{\mathcal{I}}$ denotes the *interpretation function*, which assigns to every concept name $A \in N_C$ a set $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$, and to every role name $r \in N_R$ a binary relation $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$. The interpretation function $\cdot^{\mathcal{I}}$ is inductively extended to concepts as follows:

$$\top^{\mathcal{I}} = \Delta^{\mathcal{I}}, \bot^{\mathcal{I}} = \emptyset, (\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \backslash C^{\mathcal{I}},$$
$$(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}, (C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}},$$
$$(\exists r.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \exists b.(a, b) \in r^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}\},$$
$$(\forall r.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \forall b.(a, b) \in r^{\mathcal{I}} \rightarrow b \in C^{\mathcal{I}}\}.$$

Let $\mathcal{I}$ be an interpretation. A concept inclusion $C \sqsubseteq D$ is *true* in $\mathcal{I}$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$. A concept assertion $a : A$ is *true* in $\mathcal{I}$ iff $a^{\mathcal{I}} \in C^{\mathcal{I}}$. A role assertion $(a, b) : r^{\mathcal{I}}$ is *true* in $\mathcal{I}$ iff $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$. $\mathcal{I}$ is a *model* of an ontology $\mathcal{O}$, write $\mathcal{I} \models \mathcal{O}$, iff every axiom in $\mathcal{O}$ is *true* in $\mathcal{I}$. An axiom β is entailed by an ontology $\mathcal{O}$, write $\mathcal{O} \models \beta$, iff β is true in every model $\mathcal{I}$ of $\mathcal{O}$. An ontology $\mathcal{V}$ is entailed by another ontology $\mathcal{O}$, write $\mathcal{O} \models \mathcal{V}$, iff every model of $\mathcal{V}$ is also a model of $\mathcal{O}$. An ontology $\mathcal{O}$ is *consistent (true)* if there exists a model $\mathcal{I}$ of $\mathcal{O}$. A concept C is satisfiable w.r.t. $\mathcal{O}$ if there exists a model $\mathcal{I}$ of $\mathcal{O}$ and some $d \in \Delta^{\mathcal{I}}$ with $d \in C^{\mathcal{I}}$. A concept assertion $a : C$ is satisfiable in $\mathcal{I}$ iff $a^{\mathcal{I}} \in C^{\mathcal{I}}$. A role assertion $(a, b) : r$ is satisfiable in $\mathcal{I}$ iff $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$.

Other basic reasoning problems are polynomial-time reducible to the satisfiability problem. A concept inclusion $C \sqsubseteq D$ is true in $\mathcal{I}$ iff the concept $C \sqcap \neg D$ is unsatisfiable in $\mathcal{I}$. The retrieval problem of computing the instantiation of concept C is polynomial-time reducible to that of checking the satisfiability of $a : C$.

Under the interpretation $\mathcal{I}$, concepts and roles are mapped into crisp sets in $\Delta^{\mathcal{I}}$, so the vagueness cannot be modeled.

2.2 Zadeh-$\mathcal{ALC}$

Fuzzy set theory and fuzzy logic were proposed by Zadeh [59] to manage imprecise and vague knowledge. Based on fuzzy set theory [59], a *fuzzy set* X w.r.t. an universe is characterized by a *membership function* $\mu_X : U \rightarrow [0, 1]$. Each element $u \in U$ is assigned with an X-membership degree $\mu_X(u)$. In fuzzy logic, $\mu_X(u)$ is the *truth-value* of the statement 'u is X'.

Fuzzy $\mathcal{ALC}$ retains the same syntax with $\mathcal{ALC}$, only semantics changes. Here, we follow fuzzy $\mathcal{ALC}$ proposed in [48], which is based on Gödel logic [16], and call it Zadeh-$\mathcal{ALC}$.

A fuzzy interpretation (also called *grounding* here) $\mathcal{I}$ consists of a non-empty domain $\Delta^{\mathcal{I}}$ and an *interpretation function* $\cdot^{\mathcal{I}}$ defined as: (1) an individual a is interpreted by $\mathcal{I}$ as an element $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$, and; (2) a concept C is interpreted by $\mathcal{I}$ as a fuzzy set $C^{\mathcal{I}} : \Delta^{\mathcal{I}} \rightarrow [0, 1]$, and; (3) a role r is interpreted by $\mathcal{I}$ as a fuzzy set $r^{\mathcal{I}} : \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \rightarrow [0, 1]$.

The fuzzy interpretation function $\cdot^{\mathcal{I}}$ is inductively extended to concepts as follows, for all $a \in \Delta^{\mathcal{I}}$:

$$\top^{\mathcal{I}}(a) = 1, \bot^{\mathcal{I}}(a) = 0, (\neg C)^{\mathcal{I}}(a) = 1 - C^{\mathcal{I}}(a), \tag{1}$$
$$(C \sqcap D)^{\mathcal{I}}(a) = \min\{C^{\mathcal{I}}(a), D^{\mathcal{I}}(a)\}, \tag{2}$$
$$(C \sqcup D)^{\mathcal{I}}(a) = \max\{C^{\mathcal{I}}(a), D^{\mathcal{I}}(a)\}, \tag{3}$$
$$(\exists r.C)^{\mathcal{I}}(a) = \sup_{b \in \Delta^{\mathcal{I}}}\{\min\{r^{\mathcal{I}}(a, b), C^{\mathcal{I}}(b)\}\}, \tag{4}$$
$$(\forall r.C)^{\mathcal{I}}(a) = \inf_{b \in \Delta^{\mathcal{I}}}\{\max\{1 - r^{\mathcal{I}}(a, b), C^{\mathcal{I}}(b)\}\}. \tag{5}$$

A Zadeh-$\mathcal{ALC}$ TBox is a finite set of *fuzzy inclusion* of the form $C \sqsubseteq D$. $C \sqsubseteq D$ is true (i.e., truth-value is 1) in $\mathcal{I}$ (or we say $\mathcal{I}$ satisfies $C \sqsubseteq D$) iff,

$$\forall a \in \Delta^{\mathcal{I}},\ C^{\mathcal{I}}(a) \leq D^{\mathcal{I}}(a) \tag{6}$$

We say that two concepts C and D are *fuzzy equivalent* ($C \cong D$) when $C^{\mathcal{I}}(a) = D^{\mathcal{I}}(a)$ for all $a \in \Delta^{I}$.

A Zadeh-$\mathcal{ALC}$ is a finite set of *fuzzy assertion* of the form $(a : C) \bowtie n$ or $(\langle a, b \rangle : r) \bowtie n$, where $\bowtie$ stands for $\geq, >, \leq, <$, and $n \in [0, 1]$ is the *truth value*. Formally, a fuzzy interpretation $\mathcal{I}$ satisfies a fuzzy assertion $(a : C) \bowtie n$ (resp. $(\langle a, b \rangle : r) \bowtie n$) iff $C^{\mathcal{I}}(a^{\mathcal{I}}) \bowtie n$ (resp. $r^{\mathcal{I}}(a^{\mathcal{I}}, b^{\mathcal{I}}) \bowtie n$). For simplicity, we write a fuzzy assertion as $\varphi = \{\phi \bowtie n\}$, where ϕ is $(a : C)$ or $(\langle a, b \rangle : r)$. Here, we set the $\bowtie$ be $=$ by considering two assertions of the from $\phi \geq n$ and $\phi \leq n$.

A fuzzy interpretation $\mathcal{I}$ is a *model* of a fuzzy ontology $\mathcal{O}$, write $\mathcal{I} \models \mathcal{O}$, iff $\mathcal{I}$ satisfies each axiom in $\mathcal{O}$. A fuzzy ontology $\mathcal{O}$ *fuzzy entails* a fuzzy assertion φ, write $\mathcal{O} \models \varphi$ iff every model of $\mathcal{O}$ also satisfies φ.

A *crisp* $\mathcal{ALC}$ ontology is a specialism of fuzzy $\mathcal{ALC}$ ontology, and can easily be extended to a fuzzy ontology by assigning truth value 1 to assertions.

2.3 Ontology-based Semantic Image Interpretation

Let $S = \{s_1, ..., s_n\}$ be a set of segments (a segment is a set of contiguous pixels) returned by a low-level analysis (e.g. object detection) of picture $\mathcal{P}$. Given an ontology $\mathcal{O}$, the semantic image interpretation task can be formed as labelling picture $\mathcal{P}$ with an interpretation $\mathcal{I}$ defined in the domain S, which maps each segment $s \in S$ to a set of values $\{C^{\mathcal{I}}(s) \mid C \text{ is any concept in } \mathcal{O}\}$.

3 DIFFERENTIABLE FUZZY $\mathcal{ALC}$

DF-ALC is an extension of fuzzy $\mathcal{ALC}$. The semantics of ontology represented in DF-ALC can be infused into symbol grounding in a continuous space. To solve a symbol grounding problem, such as the semantic image interpretation problem shown in Figure 1, where the concept symbols are about cat, bird and their components and the `isPartOf`(s_2, s_1) relation symbol denotes image segment s_2 is a part of s_1. When the neural model is poorly trained (e.g. in a low-resource situation), DF-ALC-represented knowledge helps to

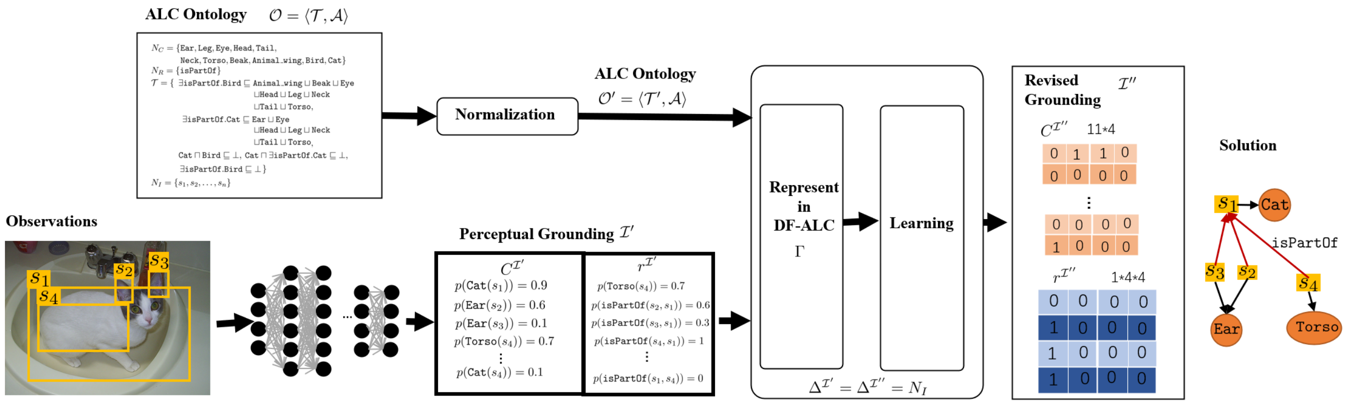


Figure 1: An ontology-based semantic image interpretation example utilizing DF-ALC. Ontology gives background knowledge about the cat and bird. The neural model does the low-level analysis for the image, which gives the wrong grounding for the image.

revise the perceptual grounding. The $\mathcal{ALC}$ ontology $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$ is one of the inputs of our model, which contains the inherent relations between symbols. Another input, the perceptual grounding $\mathcal{I}'$ contains valuable information for grounding but is not a model of fuzzy extended $\mathcal{O}$, so it is not reasonable. To get the grounding that can retain information of $\mathcal{I}'$ as much as possible, ontology should be interpretated in a continuous domain as the same as the perceptual grounding. So $\mathcal{T}$ is normalized to keep axioms in the standard format, and the normalized ontology $\mathcal{O}'$ is transformed into fuzzy $\mathcal{ALC}$ ontology. Revising perceptual grounding is realized by maximizing the satisfiability of the reformulated DF-ALC ontology.

3.1 Normalization

Given an $\mathcal{ALC}$ ontology $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$, concepts in $\mathcal{T}$ are transformed into negation normal forms using De Morgan's Laws until all concepts have no indirect negation. Then we recursively apply NF1-9 in Figure 2 until all the axioms are in the forms in Figure 3.

Any axioms in an $\mathcal{ALC}$ ontology can be transformed into the normal forms in Figure 3, using the rules in Figure 2. For the concept equivalence axiom $C \equiv D \Leftrightarrow C \sqsubseteq D, D \sqsubseteq C$, two inclusions should use the same set of introduced concept names. The introduced concepts should also not interfere with the semantics of the ontology, so a logical reasoner is used here to add assertions about introduced concept names to the ABox. Theorem 1 ensures that the interpretation of the normalized ontology is semantically equivalent to the interpretation of $\mathcal{O}$.

Theorem 1. *For any $\mathcal{ALC}$ ontology $\mathcal{O}$, one can construct in polynomial time a normalized $\mathcal{ALC}$-ontology $\mathcal{O}$' of polynomial size in $|\mathcal{O}|$ using the normalization described above such that (i) for every model $\mathcal{I}$ of $\mathcal{O}$, there exists a model $\mathcal{J}$ of $\mathcal{O}'$ such that $\mathcal{I}$ is semantically equivalent to $\mathcal{J}$ in $sig(\mathcal{O})$, denoted as $\mathcal{I} \sim_{sig(\mathcal{O})} \mathcal{J}$, and (ii) for every model $\mathcal{J}$ of $\mathcal{O}'$ there exists a model $\mathcal{I}$ of $\mathcal{O}$ such that $\mathcal{I} \sim_{sig(\mathcal{O})} \mathcal{J}$.*

After normalization, we can see that formula in the form of Figure 3 has at most one logical operation except the subclass operator, so any normalized $\mathcal{ALC}$ ontology in Figure 2 can be efficiently used as the input to a neural network.

NF 1: $\widehat{D} \sqsubseteq \widehat{E} \Rightarrow \widehat{D} \sqsubseteq A, A \sqsubseteq \widehat{E}$
NF 2: $\widehat{D} \sqcap C \sqsubseteq B \Rightarrow \widehat{D} \sqsubseteq A, A \sqcap C \sqsubseteq B$
NF 3: $C \sqcup \widehat{D} \sqsubseteq B \Leftrightarrow \widehat{D} \sqsubseteq B, C \sqsubseteq B$
NF 4: $\exists r.\widehat{D} \sqsubseteq B \Rightarrow \widehat{D} \sqsubseteq A, \exists r.A \sqsubseteq B$
NF 5: $\forall r.\widehat{D} \sqsubseteq B \Rightarrow \widehat{D} \sqsubseteq A, \forall r.A \sqsubseteq B$
NF 6:$B \sqsubseteq D \sqcap E \Leftrightarrow B \sqsubseteq D, B \sqsubseteq E$
NF 7: $B \sqsubseteq D \sqcup \widehat{E} \Rightarrow B \sqsubseteq D \sqcup A, A \sqsubseteq \widehat{E}$
NF 8: $B \sqsubseteq \exists r.\widehat{D} \Rightarrow A \sqsubseteq \widehat{D}, B \sqsubseteq \exists r.A$
NF 9: $B \sqsubseteq \forall r.\widehat{D} \Rightarrow A \sqsubseteq \widehat{D}, B \sqsubseteq \forall r.A$
NF 10 (De Morgan's Laws):

$$\neg(C \sqcap D) \Leftrightarrow \neg C \sqcup \neg D, \neg(C \sqcup D) \Leftrightarrow \neg C \sqcap \neg D,$$

$$\neg\exists r.C \Leftrightarrow \forall r.\neg C, \neg\forall r.C \Leftrightarrow \exists r.\neg C, \neg\neg C \Leftrightarrow C$$

$\widehat{D}, \widehat{E}$ are complex concepts. So they are neither $\top$, $\bot$ nor concept names.
A is a new introduced concept name.
B is a concept name or a concept name with negation.
C, D, E are arbitrary concepts.

Figure 2: Normalization rules for $\mathcal{ALC}$

Form 1: $C \sqsubseteq B$
Form 2: $C_1 \sqcap C_2 \sqsubseteq B$ **Form 3:** $B \sqsubseteq C_1 \sqcup C_2$
Form 4: $C \sqsubseteq \exists r.B$ **Form 5:** $C \sqsubseteq \forall r.B$
Form 6: $\exists r.B \sqsubseteq C$ **Form 7:** $\forall r.B \sqsubseteq C$

C, B, C_1, C_2 are concept names or concept names with negation.
r is a role name.

Figure 3: Normalized forms of $\mathcal{ALC}$ ontologies for DF-ALC

Table 1: Performance of hierarchical loss based on four kinds of axioms given the perceptual grounding in three examples. Unknown situations expect no revision. True situations expect revision executed in the implication.

Example	Description	Given an Ontology with Axiom	Expected	Performance of Hierarchical Loss
1	$(\mathsf{r}^{\mathcal{I}}(s_1,\ s_2) > \alpha) \wedge (\mathsf{B}^{\mathcal{I}}(s_1) > \alpha) \implies (\mathsf{A}^{\mathcal{I}}(s_2) > \alpha)$	$(\forall\mathsf{r.A} \sqsubseteq \mathsf{B})$ or $(\exists\mathsf{r.A} \sqsubseteq \mathsf{B})$	Unknown	do not revise, as expected
1	$(\mathsf{r}^{\mathcal{I}}(s_1,\ s_2) > \alpha) \wedge (\mathsf{B}^{\mathcal{I}}(s_1) > \alpha) \implies (\mathsf{A}^{\mathcal{I}}(s_2) > \alpha)$	$(\mathsf{B} \sqsubseteq \forall\mathsf{r.A})$ or $(\mathsf{B} \sqsubseteq \exists\mathsf{r.A})$	True	can revise, but not in an expected way
2	$(\mathsf{r}^{\mathcal{I}}(s_1,\ s_2) > \alpha) \wedge (\mathsf{A}^{\mathcal{I}}(s_1) > \alpha) \implies (\mathsf{B}^{\mathcal{I}}(s_2) > \alpha)$	$(\forall\mathsf{r.A} \sqsubseteq \mathsf{B})$ or $(\exists\mathsf{r.A} \sqsubseteq \mathsf{B})$	True	can revise, but not in an expected way
2	$(\mathsf{r}^{\mathcal{I}}(s_1,\ s_2) > \alpha) \wedge (\mathsf{A}^{\mathcal{I}}(s_1) > \alpha) \implies (\mathsf{B}^{\mathcal{I}}(s_2) > \alpha)$	$(\mathsf{B} \sqsubseteq \forall\mathsf{r.A})$ or $(\mathsf{B} \sqsubseteq \exists\mathsf{r.A})$	Unknown	can revise, but not in an expected way
3	$(\mathsf{A}^{\mathcal{I}}(s_2) > \alpha) \wedge (\mathsf{B}^{\mathcal{I}}(s_1) > \alpha) \implies (\mathsf{r}^{\mathcal{I}}(s_1,\ s_2) > \alpha)$	$\mathsf{B} \sqsubseteq \exists\mathsf{r.A}$	True	do not revise, as expected
3	$(\mathsf{A}^{\mathcal{I}}(s_2) > \alpha) \wedge (\mathsf{B}^{\mathcal{I}}(s_1) > \alpha) \implies (\mathsf{r}^{\mathcal{I}}(s_1,\ s_2) > \alpha)$	$(\exists\mathsf{r.A} \sqsubseteq \mathsf{B})$ or $(\forall\mathsf{r.A} \sqsubseteq \mathsf{B})$	Unknown	do not revise, not as expected

3.2 Learning to Ground Symbols

After normalizing ontology $\mathcal{O}$ into $\mathcal{O}'$, only TBox of $\mathcal{O}$ and the concept name set change. Assign $\mathcal{T}'$ to the TBox of Γ. Fuzzy extend $\mathcal{A}$ to be the ABox of of Γ (i.e. transform each assertion ϕ in $\mathcal{A}$ to be $\phi = 1$). The signature of Γ is defined as the signature of $\mathcal{O}'$, containing N_C, N_R, and N_I. To enable differentiable operators to transfer gradient information, we reform the fuzzy interpretation as differentiable fuzzy interpretation. The domain of the grounding $\Delta^{\mathcal{I}}$ is N_I. The interpretation function of $\mathcal{I}$ is reformed as embedding functions (symbol grounding), which embeds each concept name $C \in N_C$ into $|\Delta|$-dimensional vector, $C^{\mathcal{I}} = \mathbb{R}^{|\Delta|}$, and each role name $r \in N_R$ into a $(|\Delta|, |\Delta|)$-dimensional matrix, $r^{\mathcal{I}} = \mathbb{R}^{|\Delta|} \times \mathbb{R}^{|\Delta|}$. The ith item of $C^{\mathcal{I}}$ is the truth value of $\Delta_i : C$, and the (i, j)th item of $r^{\mathcal{I}}$ is the truth value of $(\Delta_i, \Delta_j) : r$. Then, reform the perceptual grounding $\mathcal{I}'$ as embedding functions, as the initialization of the grounding of Γ.

The semantics of Zadeh-$\mathcal{ALC}$ is *sound* w.r.t crisp semantics under the open-world assumption. This is an extension of the soundness of fuzzy $\mathcal{ALC}$. We define the crisp transformation $\sharp(\cdot)$ of DF-ALC assertion φ into three-valued $\mathcal{ALC}$ assertion.

$$\sharp\varphi = \sharp\{\phi = n\} \mapsto \left\{ \begin{array}{cc} \phi & \text{when } n > \alpha \\ unknown & \text{when } 1-\alpha \leq n \leq \alpha \\ \neg\phi & \text{when } n < 1-\alpha \end{array} \right\} \quad (7)$$

where $\alpha \in [0.5, 1]$ is predefined according to the application, and $\neg\phi$ is $a : \neg C$ or $(a, b) : \neg r$. For TBox axioms, $\sharp(\cdot)$ to fuzzy TBox axioms: $\sharp\{\psi \in \mathcal{T}\} = \{\psi \in \mathcal{T}\}$. So for $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$, $\sharp\mathcal{O} = \sharp\{\varphi \in \mathcal{A}\} \cup \{\psi \in \mathcal{T}\}$.

Proposition 1. *(Soundness of the semantics) Let $\sharp\mathcal{O}$ be an $\mathcal{ALC}$ ontology, and φ be a fuzzy assertion. $\mathcal{O} \models \varphi$ iff. $\sharp\mathcal{O} \models \sharp\varphi$ (i.e. fuzzy entailment is consistent with entailment in $\mathcal{ALC}$).*

Backpropagation on the grounding of Γ can learn a model $\mathcal{I}''$ of Γ, which is also a model of $\mathcal{O}$ in the signature of $\mathcal{O}$, according to Theorem 1. The forward process is to compute the truth values of axioms in $\mathcal{T}'$, by maximizing the satisfiability of $\mathcal{T}'$ (minimizing the *hierarchical loss* in Equation 8).

$$Loss(\mathcal{I}, \Gamma) = \frac{1}{|\mathcal{T}'|} \sum_{\{C \sqsubseteq D\} \in \mathcal{T}'} \sum_{a \in \Delta^{\mathcal{I}}} \max(0, C^{\mathcal{I}}(a) - D^{\mathcal{I}}(a)) \quad (8)$$

where C and D is any concept.

The main idea of Equation 8 is to ensure that the interpretation $\mathcal{I}$ should satisfy every $C \sqsubseteq D \in \mathcal{T}'$, denoting that $\forall a \in \Delta^{\mathcal{I}}, C^{\mathcal{I}}(a) \leq D^{\mathcal{I}}(a)$. Though $C \sqsubseteq D$ is also equivalent to $C \to D \geq n$ in fuzzy $\mathcal{ALC}$, where $(C \to D)^{\mathcal{I}} = \min_{d \in \Delta^{\mathcal{I}}}\{\max\{1 - C^{\mathcal{I}}(a), D^{\mathcal{I}}(a)\}\}$, according to [48], it is hard to assign n. Besides, using $C \to D \geq n$ as constraint will lead to $D^{\mathcal{I}}(a) \geq n$ or $C^{\mathcal{I}}(a) \leq 1 - n$, which is highly dependent on n rather than the reliable observations of the neural system. So we use $\forall a \in \Delta^{\mathcal{I}}, C^{\mathcal{I}}(a) \leq D^{\mathcal{I}}(a)$ to constraint concept inclusion in Γ.

Proposition 2. ***(Soundness of learning to ground in DF-ALC)*** *When the hierarchical loss converges to 0, the learned interpretation $\mathcal{I}''$ is the model of the given $\mathcal{ALC}$ ontology $\mathcal{O}$. For any model $\mathcal{J}$ of $\mathcal{O}$, $\mathcal{I}'' \sim_{sig(\mathcal{O})} \mathcal{J}$.*

Prop.1 and Prop.2 are proved to be true in the Appendix.A

3.3 Rule-based Learning

But learning grounding by maximizing the satisfiability captured by DF-ALC semantics can meet reasoning shortcut problem, because this restraint can only ensure the revised grounding is a model of the given ontology, but there are lots of different models. Besides, the hierarchical loss proposed in Equation. 8 can lead $\mathsf{A} \sqsubseteq \mathsf{B}$ to learn a grounding $\mathcal{I}$ that $\mathsf{A}^{\mathcal{I}} = \mathsf{B}^{\mathcal{I}}$, so cannot be distinguish between $\sqsubseteq$ and $\equiv$. And $\mathsf{B}^{\mathcal{I}}(s)$ can be revised as 0.5, which loses information.

A relaxed revision is to reduce $\mathsf{A}^{\mathcal{I}}(s)$ or improve $\mathsf{B}^{\mathcal{I}}(s)$ based on the truthness of $\mathsf{B}^{\mathcal{I}}(s)$ to satisfy $\mathsf{A}^{\mathcal{I}} \leq \mathsf{B}^{\mathcal{I}}$. Here is the rule-based loss for axioms in the NF 1-3 shown in Fig. 2:

$$Loss_{\mathsf{NF1-NF3}}(\mathsf{A}^{\mathcal{I}}, \mathsf{B}^{\mathcal{I}}; \mathcal{I}, \Gamma) = \sum_{\mathsf{A} \sqsubseteq \mathsf{B}} \sum_{s \in \Delta^{\mathcal{I}}} ((1 - \mathsf{B}^{\mathcal{I}}(s)) * G(\mathsf{A}^{\mathcal{I}}(s), \mathsf{B}^{\mathcal{I}}(s))), \quad (9)$$

where A and B is any concept, $\mathsf{A}^{\mathcal{I}}$ and $\mathsf{B}^{\mathcal{I}}$ are vectors in the representation of DF-ALC calculated according to the semantics of Zadeh-$\mathcal{ALC}$. $G(v,\ w) = ReLU(v - w)$, and $G(v,\ w)$ does not take part in the gradient descent.

For axioms in the normal form 4-7, the semantics of $\exists$ and $\forall$ in Zadeh-$\mathcal{ALC}$ can not be reasoned in a proper way with hierarchical loss. So different losses are designed for axioms in normal form 4-7 respectively.

Consider four ontologies $\mathcal{O}_1 = \{\exists\mathsf{r.A} \sqsubseteq \mathsf{B}\}$, $\mathcal{O}_2 = \{\forall\mathsf{r.A} \sqsubseteq \mathsf{B}\}$, $\mathcal{O}_3 = \{\mathsf{B} \sqsubseteq \exists\mathsf{r.A}\}$, $\mathcal{O}_4 = \{\mathsf{B} \sqsubseteq \forall\mathsf{r.A}\}$, in the following three examples, the performance of DF-ALC with hierarchical loss and the revision calculus are shown in Table 1[3]. Example1 in Table 1 is explained as follow:

[3]Detailed analysis for example 2 and 3 is in the appendix.B

EXAMPLE 1. *Given a perceptual grounding $\mathcal{I}$ in the domain $\{s_1, s_2\}$, $\mathsf{A}^{\mathcal{I}}(s_1) = 0, \mathsf{A}^{\mathcal{I}}(s_2) = 0, \mathsf{B}^{\mathcal{I}}(s_1) = 0.9, \mathsf{B}^{\mathcal{I}}(s_2) = 0, \mathsf{r}^{\mathcal{I}}(s_1, s_2) = 0.9, \mathsf{r}^{\mathcal{I}}(s_1, s_1) = \mathsf{r}^{\mathcal{I}}(s_2, s_1) = \mathsf{r}^{\mathcal{I}}(s_2, s_2) = 0$, notated as vectors $\mathsf{A}^{\mathcal{I}} = \langle 0, 0\rangle$ and $\mathsf{B}^{\mathcal{I}} = \langle 0.9, 0\rangle$.*

According to the semantics of Zadeh-$\mathcal{ALC}$, in O_1, $(\exists \mathsf{r}.\mathsf{A})^{\mathcal{I}} = \langle 0, 0\rangle$, which satisfies $(\exists \mathsf{r}.\mathsf{A})^{\mathcal{I}} \leq \mathsf{B}^{\mathcal{I}}$, so hierarchical loss is 0, and no revision is executed. But this is not what we want. As we know that s_1 is likely to be B, and $\mathsf{r}^{\mathcal{I}}(s1, s2)$ is likely to be true, so s_2 is likely to be a membership of A. In O_2, $(\forall \mathsf{r}.\mathsf{A})^{\mathcal{I}} = \langle 0.1, 1\rangle$, which does not satisfy $(\forall \mathsf{r}.\mathsf{A})^{\mathcal{I}} \leq \mathsf{B}^{\mathcal{I}}$, and hierarchical loss is 1.1. Through gradient decent, until loss becomes 0, $\mathsf{A}^{\mathcal{I}} = \langle 0.24, 0\rangle, \mathsf{B}^{\mathcal{I}} = \langle 0.4, 1\rangle$, $\mathsf{r}^{\mathcal{I}}(s_2, s_1)$ will be 0.7 and $\mathsf{r}^{\mathcal{I}}(s_1, s_2)$ will be 1. In O_3, with hierarchical loss, $\mathcal{A}^{\mathcal{I}}$ will be $\langle 0.38, 0\rangle$, $\mathcal{B}^{\mathcal{I}}$ will be $\langle 0.35, 0\rangle$and $\mathsf{r}^{\mathcal{I}}(s_1, s_1)$ will be $\langle 0, 36\rangle$. In O_4, with hierarchical loss, $\mathcal{B}^{\mathcal{I}} = \langle 0, 0\rangle$ and $\mathsf{r}^{\mathcal{I}}(s_1, s_2) = 0$.

These situations should be solved in an axiom-level view, which means that the loss rather than the fuzzy semantics should be advanced. So we introduce the following rule-based loss to solve the problems meet in Table 1:

$$\begin{aligned}
&Loss_{\mathsf{NF4}}(\mathsf{A}^{\mathcal{I}}, \mathsf{B}^{\mathcal{I}}, \mathsf{r}^{\mathcal{I}}; \mathcal{I}, \Gamma) = \\
&\sum_{\mathsf{B}\sqsubseteq\exists \mathsf{r}.\mathsf{A}} \sum_{s\in\Delta^{\mathcal{I}}} ((1-\mathsf{A}^{\mathcal{I}}(s)) * G(\alpha', \mathsf{A}^{\mathcal{I}}(s)) * \\
&G(\sum_{a\in\Delta^{\mathcal{I}}} (\mathsf{B}^{\mathcal{I}}(a) \otimes \mathsf{r}^{\mathcal{I}}(a,s), \alpha')) + \\
&(1-\mathsf{r}^{\mathcal{I}}(s, a)) * G(\mathsf{B}^{\mathcal{I}}(s) \otimes \mathsf{A}^{\mathcal{I}}(a), \mathsf{r}^{\mathcal{I}}(s, a)))
\end{aligned} \tag{10}$$

where $\otimes$ is the t-norm. With the rule-based loss, the parts of interpretation that we believed to be true are not determined by a threshold, but by the speciality of the task and dataset. In this paper, we use the product t-norm as $\otimes$ in the rule-based loss validated by the evaluation in the experiments. $\alpha' \in [0.5, 1]$ is the threshold for the truth-value.

$$\begin{aligned}
&Loss_{\mathsf{NF5}}(\mathsf{A}^{\mathcal{I}}, \mathsf{B}^{\mathcal{I}}; \mathcal{I}, \Gamma) = \\
&\sum_{\mathsf{B}\sqsubseteq\forall \mathsf{r}.\mathsf{A}} \sum_{s\in\Delta^{\mathcal{I}}} ((1-\mathsf{A}^{\mathcal{I}}(s)) * G(\alpha, \mathsf{A}^{\mathcal{I}}(s)) * \\
&G(\sum_{a\in\Delta^{\mathcal{I}}} (\mathsf{B}^{\mathcal{I}}(a) \otimes \mathsf{r}^{\mathcal{I}}(a,s), \alpha')))
\end{aligned} \tag{11}$$

$$\begin{aligned}
&Loss_{\mathsf{NF6}}(\mathsf{A}^{\mathcal{I}}, \mathsf{B}^{\mathcal{I}}; \mathcal{I}, \Gamma) = \\
&\sum_{\exists \mathsf{r}.\mathsf{A}\sqsubseteq \mathsf{B}} \sum_{s\in\Delta^{\mathcal{I}}} ((1-\mathsf{B}^{\mathcal{I}}(s)) * G(\alpha', \mathsf{B}^{\mathcal{I}}(s)) * \\
&G(\sum_{a\in\Delta^{\mathcal{I}}} (\mathsf{A}^{\mathcal{I}}(a) \otimes \mathsf{r}^{\mathcal{I}}(s, a), \alpha')))
\end{aligned} \tag{12}$$

$$\begin{aligned}
&Loss_{\mathsf{NF7}}(\mathsf{A}^{\mathcal{I}}, \mathsf{B}^{\mathcal{I}}; \mathcal{I}, \Gamma) = \\
&\sum_{\forall \mathsf{r}.\mathsf{A}\sqsubseteq \mathsf{B}} \sum_{s\in\Delta^{\mathcal{I}}} ((1-\mathsf{B}^{\mathcal{I}}(s)) * G(\alpha', \mathsf{B}^{\mathcal{I}}(s)) * \\
&G(\sum_{a\in\Delta^{\mathcal{I}}} (\mathsf{A}^{\mathcal{I}}(a) \otimes \mathsf{r}^{\mathcal{I}}(s, a), \alpha')) + \\
&(1-\mathsf{A}^{\mathcal{I}}(s)) * G(\alpha', \mathsf{A}^{\mathcal{I}}(s)) * G(\sum_{a\in\Delta^{\mathcal{I}}} (\mathsf{B}^{\mathcal{I}}(a) \otimes \mathsf{r}^{\mathcal{I}}(a, s), \alpha')))
\end{aligned} \tag{13}$$

We only consider the situations when an assertion is larger than α' here,w.l.o.g., the opposite situations (less than $1-\alpha'$) are dual and can be added to the loss according to the distribution of perceptual grounding, e.g. the opposite situations are more plausible.

4 EXPERIMENTS

4.1 Performance Evaluation

We design two experiments to verify the efficiency of DF-ALC, and answer the following questions: can loss always converge to zero? If not, what can the result be in these cases? How successful the learning in DF-ALC is in keeping reliable observations while revising erroneous ones? To answer the first two questions, learning grounding for DF-ALC ontologies should be evaluated in different neural networks under various situations. However the distribution of observations and the properties of a specific neural network give bias (in a way of having the same pattern of errors) to the perceptual grounding. Therefore, we design an experimental task — masked ABox revision, for evaluation in various observation distributions. This task is not oriented to tackle a concrete symbol grounding problem. Given an $\mathcal{ALC}$ ontology $O = \langle \mathcal{T}, \mathcal{A}\rangle$, where $\mathcal{A}$ is completed by a logical reasoner, then fuzzy extended in DF-ALC. We assign the idea grounding $\mathcal{I}$ with the processed $\mathcal{A}$. Mask the random part of grounding $\mathcal{I}$ into a random truth value in an unknown region, and reformulate it into a differentiable fuzzy interpretation $\mathcal{I}'$ as the imitation of a perceptual grounding. Then transform ontology O into DF-ALC ontology Γ. Use $\mathcal{I}'$ as the initialization to learn the revised grounding $\mathcal{I}''$ based on Γ. Using the crisp transformation defined in Formula 7 to transform the revised grounding into crisp grounding, and evaluate it with the satisfiability calculated in the crisp mode. However, this experiment cannot show the constancy (in keeping the reliable parts) between $\mathcal{I}'$ and $\mathcal{I}''$, which means the degree that the revised grounding shifts from an ideal grounding. So we design another task called conjunctive query answering, which is a kind of ontology-mediated query answering. The target is to retrieve individuals for complex concepts based on the revised grounding.

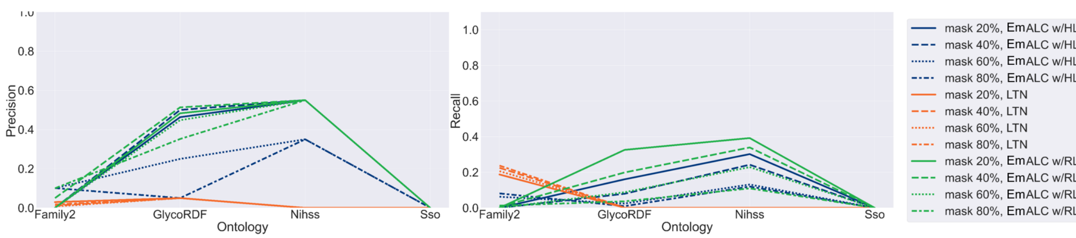
4.1.1 Settings. In the masked ABox revision task, we used 6 ontologies ("Ontodm" and "Nifdys" are not consistent), while in the conjunctive query answering task, we used 4 consistent ontologies. We used the Logical Tensor Network (LTN) as the comparison model. LTN, is a differentiable fuzzy logic model in product real logic based on first-order logic. We used normalized ontology and the same loss (hierarchical loss in Equation. 8) to train LTN.

The mask rate of ABox ranges from {20%, 40%, 60%, 80%}. We set the unknown region as [0.2, 0.8]. Meanwhile, the truth values greater (less) than $\alpha = 0.8$ ($1-\alpha = 0.2$) were assumed to be true (false). We used success rate (S.R.) to evaluate soundness. The success rate is the percentage of the TBox axiom in original ontology (without normalization) that is satisfied w.r.t. the crisp $\mathcal{I}''$. To be fair, we evaluated the results in the semantics of first-order logic. We tested conjunctive queries in the forms of $C\sqcap D$ and $C\sqcap\exists r.D$, where C and D are atomic concepts, and r is a role name. We generated 20 queries in each form, and the answer set of each query was not empty. Considering the time complexity of using a logical reasoner to get the true answer set, we only used two forms of conjunctive

	0.2							0.4							0.6							0.8						
	S.R.				KL			S.R.				KL			S.R.				KL			S.R.				KL		
	M	H	R	L	H	R	L	M	H	R	L	H	R	L	M	H	R	L	H	R	L	M	H	R	L	H	R	L
Family	0.0	**100.0**	93.1	**100.0**	6.0	**1.5**	10.3	0.0	**100.0**	93.1	**100.0**	7.1	**3.1**	10.3	0.0	**100.0**	93.1	**100.0**	8.2	**4.6**	10.3	0.0	**100.0**	93.1	**100.0**	9.0	**6.1**	10.3
Family2	0.0	**100.0**	73.8	91.8	5.1	**1.9**	9.5	0.0	**96.7**	73.8	88.5	6.2	**3.2**	9.9	0.0	**98.4**	73.8	91.8	7.1	**4.2**	10.4	0.0	**96.7**	73.8	91.8	8.5	**5.4**	11.2
GlycoRDF	4.1	**100.0**	90.9	96.4	3.7	**1.3**	2.2	4.1	**100.0**	90.9	96.4	4.2	**1.9**	2.8	4.1	**99.5**	90.9	95.9	4.7	**2.5**	3.1	4.1	**99.5**	90.9	95.9	5.0	**3.1**	3.4
Nifdys	7.3	**97.3**	89.2	95.0	1.0	**0.3**	0.7	2.7	**98.7**	84.5	94.7	1.1	**0.6**	0.8	0.4	**99.3**	82.7	94.8	**0.9**	**0.9**	**0.9**	0.1	**99.6**	82.7	94.7	**1.0**	1.2	**1.0**
Nihss	16.1	**100.0**	**100.0**	51.6	**0.4**	0.5	2.4	0.0	**100.0**	**100.0**	48.4	**0.6**	1.0	2.4	0.0	**100.0**	**100.0**	48.4	**1.0**	1.4	2.4	0.0	**100.0**	**100.0**	48.4	**1.1**	1.9	2.4
Ontodm	5.4	91.6	41.5	**98.5**	5.4	**1.2**	10.6	0.3	91.2	37.3	**98.2**	**0.6**	1.0	2.4	0.2	92.1	**97.6**	**1.0**	1.4	2.4	0.2	91.3	37.3	**97.4**	**1.1**	1.9	2.4	
Sso	0.0	**100.0**	**100.0**	**100.0**	**0.3**	0.5	0.5	0.0	**100.0**	**100.0**	**100.0**	0.7	1.0	**0.5**	0.0	**100.0**	**100.0**	**100.0**	1.0	1.5	**0.5**	0.0	**100.0**	**100.0**	**100.0**	1.3	2.0	**0.5**

Table 2: Success rate (S.R.) and KL divergence (KL) in four mask rate settings. M is the masked grounding. H is the revised grounding by DF-ALC with hierarchical loss. R is the revised grounding by DF-ALC with rule-based loss. L is the revised grounding by LTN.

(a) With queries in the form of $C \sqcap D$



(b) With queries in the form of $C \sqcap \exists r.D$

Figure 4: Conjunctive query answering results

queries in-depth 2 (the depth is determined by the conjunction amounts in the query). We chose all individuals with $Q^{\mathcal{I}}(a) \geq 0.8$ to be the answer for query Q. And use the answers generated by logical reasoner as ideal answers to evaluate the predicted answers with precision and recall as metrics. Detailed information (data, resources, training details, etc.) about the experiments is shown in the supplementary material.

4.1.2 Results. The revising masked grounding experiment uses a success rate to evaluate the ratio of TBox axiom that is interpreted as true, and a KL divergence to evaluate how the revised grounding is similar to the expected grounding, with results shown in Table 2. We can see that DF-ALC and LTN succeeded in most cases. Not surprisingly, the success rate is low for masked grounding, since any small fault in the grounding can dissatisfy an axiom in the ontology. DF-ALC does not perfectly predict "Ontodm" and "Nifdys", as these two ontologies are not consistent. In "Ontodm", DF-ALC predicts wrong grounding for disjoint concepts, and these concepts are incompletely asserted. The same problem occurs in "Nifdys". We further study the failures in "Family2", and find that the failures are caused by unknown cases. More specifically, we can see that an individual "F6M80" is asserted as a Male, but his parents are not asserted, therefore the values of "Son(F6M80)" and "Child(F6M80)" are unknown. In learned grounding of Γ, though they are all in unknown region $(0.2, 0.8)$, $\text{Son}^{\mathcal{I}''}(\text{F6M80}) = 0.5490 > \text{Child}^{\mathcal{I}''}(\text{F6M80}) = 0.5489$ can still lead to $\text{Son}^{\mathcal{I}''} \not\sqsubseteq \text{Child}^{\mathcal{I}''}$. For LTN, almost all of the axioms in the form of $\exists r.\top \sqsubseteq C$ are not learned well. For DF-ALC, several axioms fail in the complex forms (e.g. $\exists r_1.(\exists r_2.B) \sqsubseteq C$) when the masked rate gets higher. And it is worth noting that learning in "Family2", "GlycoRDF", "Nifdys", and "Ontodm" cannot get the hierarchical loss to converge to 0 in finite time in the four settings. But we still get the success rate of "Family2" and "GlycoRDF" being 100%, which is due to the crisp transformation for masked grounding. So if the given $\mathcal{ALC}$ ontology $\mathcal{O}$ is consistent, though the learning loss cannot converge to 0 in some cases, the crisp transformed grounding is the model of $\mathcal{O}$. DF-ALC with hierarchical loss does better than DF-ALC with rule-based loss in success rate, but worse in the KL divergence.

From the results shown in Figure 4, we find that both models cannot do well in this task as expected. Because the masked semantics loses much information, DF-ALC can revise the grounding

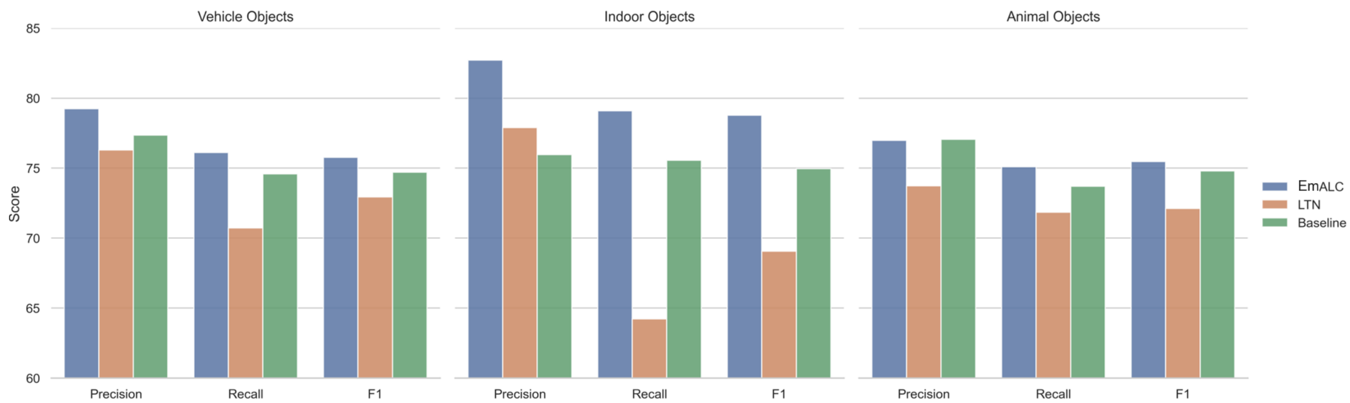


Figure 5: Object types classification results. The baseline model is the classification results of FRCNN. For evaluation, the performance are classified into three categories: vehicle, indoor, and animal. The metrics are macro averaged.

in a shifted direction. High precision (i.e. maintaining the reasoning properties well) and relatively high recall (i.e. being constancy with reliable part of observation) can be expected with DF-ALC compared to LTN when the mask rate is low (e.g. 0.2). While both DF-ALC and LTN have problems in revising role interpretation function.

Overall, DF-ALC outperforms LTN in most observation revision cases. The common and significant problem for both of them is to avoid the disturbance of unknown cases to satisfiability to knowledge.

4.2 Semantic Image Interpretation

In this experiment, we apply DF-ALC to solve the SII problem. We use PASCAL-PART dataset [9]. The dataset consists of images annotated with bounding boxes denoting distinct objects. The simple semantics between these objects like part-of relation can be detected by computing the pixel cover rate between bounding boxes, constructing the role interpretation function of perceptual interpretation $\mathcal{I}'$. Objects are then grounded by object detector Fast R-CNN (FRCNN) [22], which gives each object b the label C with $score(C, b)$, constructing the concept interpretation function of $\mathcal{I}'$. To revise $\mathcal{I}'$, we introduced an OWL ontology $\mathcal{O}_{partOf}$ with two kinds of axioms, which is similar to the ontology introduced in Figure. 1. The first kind of axiom depicts the part-of relation between types, e.g. $\exists \texttt{isPartOf.Chair} \sqsubseteq \texttt{Seat} \sqcup \texttt{Leg}$. The second kind of axiom asserts the disjointness between different types, e.g. $\texttt{Chair} \sqcap \exists \texttt{isPartOf.Chair} \sqsubseteq \bot$, $\texttt{Chair} \sqcap \texttt{Table} \sqsubseteq \bot$. Then we used the rule-based loss to revise $\mathcal{I}'$ according to ontology $\mathcal{O}_{partOf}$. LTN was trained with constraints following the settings proposed in [14].

The results are shown in Figure. 5. Indoor objects have the simplest relationships and animal objects have the most complex relationships, which interprets why DF-ALC performs the best in indoor objects. Animal objects can have many common types of objects, e.g. ear, head, and eye, but DF-ALC can still improve recall. LTN fails in improving the object types classification performance upon the baseline because the fuzzy logical operators cannot convey the proper information by maximizing the satisfiability. But LTN can do link prediction (e.g. revise the part-of-relation interpretation), which cannot be done by DF-ALC. On the whole, DF-ALC provides an unsupervised way to improve symbol grounding by utilizing logical knowledge. We further tested how DF-ALC perform when the FRCNN is not trained well, the result is shown in appendix.C.

5 CONCLUSION AND FUTURE WORK

This work presents DF-ALC, the first neural-symbolic approach that gets symbol grounding for $\mathcal{ALC}$ Ontologies. DF-ALC proposed two strategies to revise any neural networks' symbol grounding using fuzzy extended $\mathcal{ALC}$ semantics. One strategy is to maximize the satisfiability of $\mathcal{ALC}$ ontology $\mathcal{O}$ by minimizing hierarchical loss. When DF-ALC with hierarchical loss is converged to 0, the revised symbol grounding is proven to be a model of $\mathcal{O}$. But in some situations, the revision process can be wrong, so we proposed another strategy to minimize rule-based loss. DF-ALC with rule-based loss can mitigate the reasoning shortcut problem. Compared with the most related differentiable fuzzy logic model, LTN, we find that DF-ALC is better at retaining the reliable part of probability.

However, we have only tested how DF-ALC revises the output of the last layer of the neural network. While DF-ALC can also back-propagate the gradient to the neural network, and train DF-ALC with the neural system in an end-to-end way. Besides, the differentiable fuzzy existential quantifier operator in DF-ALC should further be investigated.

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A PROOF

The proof of Theorem 1 requires the introduction of Ackermann's Lemma:

$$\frac{O(\mathcal{S}^-), \alpha_1 \sqsubseteq \mathcal{S}, ..., \alpha_n \sqsubseteq \mathcal{S}\texttt{(premise)}}{O(\mathcal{S}^-)^{\mathcal{S}}_{\alpha_1 \sqcup ... \sqcup \alpha_n}\texttt{(conclusion)}} \tag{14}$$

$$\frac{O(\mathcal{S}^+), \mathcal{S} \sqsubseteq \alpha_1, ..., \mathcal{S} \sqsubseteq \alpha_n\texttt{(premise)}}{O(\mathcal{S}^+)^{\mathcal{S}}_{\alpha_1 \sqcap ... \sqcap \alpha_n}\texttt{(conclusion)}} \tag{15}$$

Theorem 1 Proof Here, $\mathcal{S} \in N_C$ ($\mathcal{S} \in N_R$) is the focused symbol, $\alpha_i (1 \leq i \leq n)$ are concepts (relations) that do not contain $\mathcal{S}$, $O(\mathcal{S}^-)$ and $O(\mathcal{S}^+)$ respectively indicate whether the ontology O is negative or positive with respect to $\mathcal{S}$, and $O^{\mathcal{S}}_{\alpha}$ denotes the ontology obtained by replacing every occurrence of $\mathcal{S}$ in O with α. In this context, α is called the definition of $\mathcal{S}$ in O. Rules 14 and 15 can be used to eliminate the symbol $\mathcal{S}$ only if either of the following conditions is met: (1) every $\mathcal{S}^+$-formula in O is of the form $\alpha \sqsubseteq \mathcal{S}$, and $\mathcal{S} \notin sig(\alpha)$; (2) every $\mathcal{S}^-$-formula in O is of the form $\mathcal{S} \sqsubseteq \alpha$, and $\mathcal{S} \notin sig(\alpha)$.

Given any $\mathcal{ALC}$ ontology O, let $|sig(O)|$ denote the size of O. Let the symbol set of the normalized ontology O' obtained via F1-F9 be $sig(O') \subseteq sig(O) \cup \mathrm{S}$, where S is the set of concept names introduced during normalization. Based on the aforementioned Ackermann's Lemma, the newly introduced concepts in F1, F2, F4, and F5 can be eliminated using Lemma 14. The newly introduced concepts in F7, F8, and F9 can be eliminated using Lemma 15. The semantics of the antecedents and consequents in F3, F6, and F10 are evidently equivalent. Therefore, it can be proven that for any newly introduced concept symbol $\mathcal{S} \in \mathrm{S}$, the ontology $O^{\mathcal{S}}_{\alpha}$ after elimination is semantically equivalent to O' on $sig(O)$. Consequently, after completely eliminating S, the models of O and O' are semantically equivalent on $sig(O)$.

Next, we prove that the normalization method in Em$\mathcal{ALC}$ has polynomial time and space complexity. Let the formula $\varphi = \hat{D} \sqsubseteq \hat{E}$, where $\hat{D}$ and $\hat{E}$ are complex concepts. Define the depth of a TBox formula as the number of concept and relation names it contains, denoted as $\texttt{depth}(\hat{D})$. For example, $\texttt{depth}(\exists r.\forall r_1.(A \sqcup B)) = 4$. Let the two formulas obtained after applying F1-F9 be φ_1 and φ_2. It is easy to see that $\texttt{depth}(\varphi_1) < \texttt{depth}(\varphi)$ and $\texttt{depth}(\varphi_2) < \texttt{depth}(\varphi)$ always hold. Therefore, each application of F1-F9 reduces the depth by at least one and increases the number of formulas by one, with the ontology size increasing by at most one. When finally transformed into NF1-NF7, the depth of the formulas is at least 2 and no longer applicable to F1-F9. Therefore, $|sig(O')| < \sum_{\varphi \in O} \texttt{depth}(\varphi)$, and the upper limit of applying F1-F9 is $\sum_{\varphi \in O} \texttt{depth}(\varphi)$, which is a polynomial multiple of $|sig(O)|$. Hence, the normalization method mentioned above has polynomial time and space complexity.

The properties that Zadeh-$\mathcal{ALC}$ holds are shown in Appendix Table 3. Only $C \sqcap \neg C \cong \bot$ and $C \sqcup \neg C \cong \top$ do not hold in DF-ALC and fuzzy $\mathcal{ALC}$. But for any C and $\mathcal{I}$, $(C \sqcup \neg C)^{\mathcal{I}} \geq 0.5$ and $(C \sqcap \neg C)^{\mathcal{I}} \leq 0.5$ hold.

Proposition 1 Proof. 1.$\Rightarrow$ Consider any crisp interpretation $\mathcal{I}$ that is a model of $\sharp O$. $\mathcal{I}$ can also be considered as a fuzzy interpretation that $C^{\mathcal{I}}(a) \in \{0, 0.5, 1\}$ and $r^{\mathcal{I}}(a,b) \in \{0, 0.5, 1\}$ hold. By induction on the structure of a concept C, $\mathcal{I}$ satisfies $a : C$ iff $C^{\mathcal{I}}(a) = 1$. $\mathcal{I}$ satisfies $a : C$ is unknown iff $C^{\mathcal{I}}(a) = 0.5$. And

Property	Fuzzy $\mathcal{ALC}$	Real Logic
$C \sqcap \neg C \cong \bot$		
$C \sqcup \neg C \cong \top$		
$C \sqcap C \cong C$	•	
$C \sqcup C \cong C$	•	
$\neg\neg C \cong C$	•	•
$\neg(C \sqcap D) \cong \neg C \sqcup \neg D$	•	
$\neg(C \sqcup D) \cong \neg C \sqcap \neg D$	•	
$C \sqcap (D \sqcup E) \cong (C \sqcap D) \sqcup (C \sqcap E)$	•	
$C \sqcup (D \sqcap E) \cong (C \sqcup D) \sqcap (C \sqcup E)$	•	
$\forall r.C \cong \neg\exists r.\neg C$	•	

Table 3: Properties of fuzzy $\mathcal{ALC}$ with equality assertion used in our model and product real logic [53] used in LTN [4]. Cell with • is the meaning of 'has'.

similarly for roles. Therefore, $\mathcal{I}$ is also a model of O. And for every model of $\sharp O$, $\mathcal{I}$ satisfies $\sharp\varphi$, so $\sharp O \models \sharp\varphi$ holds.

2. $\Leftarrow$ If $\sharp O \models \sharp\varphi$, consider the crisp interpretation $\mathcal{I}$ discussed above, it is similar to proof that each model $\mathcal{I}$ of $\sharp O$, is also the model of O, and satisfies φ, so we have $O \models \varphi$. To sum up, this proposition is proved to be true.

Proposition 2 Proof. when loss converges to 0, the learned $\mathcal{I}''$ satisfies any $C \sqsubseteq D$ in the normalized ontology O', so $\mathcal{I}''$ is the model of O'. And according to Theorem 1, any model of O' is semantically equivalent to the model of O. So this proposition is proved to be true.

B RULE-BASED LEARNING: EXAMPLE ANALYSE

Example 2. *Given a perceptual grounding $\mathcal{I}$ in the domain $\{s_1, s_2\}$, $\texttt{A}^{\mathcal{I}} = [0, 0.9]$, $\texttt{B}^{\mathcal{I}} = [0, 0]$, $\texttt{r}^{\mathcal{I}}$ is the same as in Example. 1.*

According to the semantics of Zadeh-$\mathcal{ALC}$, in O_1, $(\exists\texttt{r.A})^{\mathcal{I}} = [0.9, 0]$, which does not satisfy $(\exists\texttt{r.A})^{\mathcal{I}} \leq \texttt{B}^{\mathcal{I}}$, so hierarchical loss is 0.9. Through gradient decent, until loss becomes 0, $\texttt{A}^{\mathcal{I}}$ is decreased as $[0, 0]$, and $\texttt{B}^{\mathcal{I}}$ is increased as $[0.9, 0]$. $\texttt{A}^{\mathcal{I}}$ is not expected to be changed and $\texttt{B}^{\mathcal{I}}$ is expected to be increased as $[0.9, 0]$. In O_2, $(\forall\texttt{r.A})^{\mathcal{I}} = [0.9, 1]$, $\texttt{A}^{\mathcal{I}}$ will be revised as $[0, 0]$, $\texttt{B}^{\mathcal{I}}$ will be revised as $[0.5, 1]$, and $\texttt{r}^{\mathcal{I}}(s_1, s_2) = 1$. In O_3 and O_4, there is no revision.

Example 3. *Given a perceptual grounding $\mathcal{I}$ in the domain $\{s_1, s_2\}$, $\texttt{A}^{\mathcal{I}} = \langle 0, 0.9\rangle$ and $\texttt{B}^{\mathcal{I}} = \langle 0.9, 0\rangle$, $\texttt{r}^{\mathcal{I}}(s_1, s_2) = \texttt{r}^{\mathcal{I}}(s_1, s_1) = \texttt{r}^{\mathcal{I}}(s_2, s_1) = \texttt{r}^{\mathcal{I}}(s_2, s_2) = 0$.*

According to the semantics of Zadeh-$\mathcal{ALC}$, $(\exists\texttt{r.A})^{\mathcal{I}} = (\forall\texttt{r.A})^{\mathcal{I}} = [0, 0]$, which satisfies $(\exists\texttt{r.A})^{\mathcal{I}} \leq \texttt{B}^{\mathcal{I}}$, so no revision is executed. In O_3, $\texttt{B}^{\mathcal{I}}$ and $\texttt{r}^{\mathcal{I}}$ is revised if there are other s_n that $\texttt{A}^{\mathcal{I}}$ is not zero. In O_4, there is no revision.

C EXPERIMENT

In the masked ABox revision task, we used 6 ontologies ("Ontodm" and "Nifdys" are not consistent), while in the conjunctive query answering task, we used 4 consistent ontologies.

The ontologies used for the experiments are taken from Bioportal[4], which, currently, includes more than 700 biomedical ontologies

[4]http://bioportal.bioontology.org/ontologies

	Family	Family2	GlycoRDF	Nifdys	Nihss	Ontodm	Sso
# TBox axioms	2032	2054	1453	6435	318	3476	2050
# ABox axioms	224	224	518	2920	146	1113	366
# Concepts	19	19	113	2751	18	838	176
# Roles	4	4	91	68	16	78	22
# Individuals	202	202	219	102	106	187	158
Expressivity	$\neg$	$\neg, \sqcap, \exists$	$\neg, \sqcup, \exists$	$\neg, \sqcup, \exists$	$\neg$	$\neg, \sqcap, \sqcup, \exists, \forall$	/

Table 4: Ontology information

Table 5: Low-resource image object classification results. Mask rate denotes the rate of features extracted by FRCNN (baseline) that are randomly assigned.

		Vehicle			Indoor			Animal		
Mask Rate	Model	Precision	Recall	F1	Precision	Recall	F1	Precision	Recall	F1
0%	baseline	77.36	74.58	74.71	75.96	75.56	74.96	77.06	73.7	74.8
0%	DF-ALC (w/$O_{\mathcal{EL}}$)	**79.17** (↑ 1.81)	**76.37**(↑ 1.79)	**76.08** (↑ 1.37)	77.5 (↑ 1.54)	77.04 (↑ 1.68)	76.4 (↑ 1.44)	**77.24** (↑ 0.18)	**74.52** (↑ 0.82)	**75.34** (↑ 0.54)
0%	DF-ALC (w/$O_{\mathcal{ALC}}$)	78.35 (↑ 0.99)	75.83 (↑ 1.25)	75.97 (↑ 1.26)	**80.69** (↑ 4.73)	**80.74** (↑ 5.18)	**80.52** (↑ 5.56)	**77.24** (↑ 0.18)	73.92 (↑ 0.22)	75.05 (↑ 0.25)
3%	baseline	55.41	59.47	56.41	68.98	69.88	68.7	49.43	59.17	51.77
3%	DF-ALC (w/$O_{\mathcal{EL}}$)	56.5 (↑ 1.09)	60.68(↑ 1.21)	57.36 (↑ 0.95)	69.93 (↑ 0.95)	70.83 (↑ 0.95)	69.56 (↑ 0.86)	**50.22** (↑ 0.79)	**60.83** (↑ 1.66)	**52.76** (↑ 0.99)
3%	DF-ALC (w/$O_{\mathcal{ALC}}$)	**56.67** (↑ 1.26)	**60.87** (↑ 1.4)	**57.51** (↑ 1.1)	**72.67** (↑ 3.69)	**73.35** (↑ 3.47)	**71.84** (↑ 3.14)	50.2 (↑ 0.77)	60.79 (↑ 1.62)	52.73 (↑ 0.96)
5%	baseline	46.87	51.91	48.37	63.68	65.52	64.03	42.73	52.4	44.29
5%	DF-ALC (w/$O_{\mathcal{EL}}$)	48.4 (↑ 1.53)	53.83(↑ 1.92)	49.85 (↑ 1.48)	65.01 (↑ 1.33)	66.78 (↑ 1.26)	65.2 (↑ 1.17)	43.3 (↑ 0.57)	53.7 (↑ 1.3)	45.0 (↑ 0.71)
5%	DF-ALC (w/$O_{\mathcal{ALC}}$)	**48.45** (↑ 1.58)	**53.93** (↑ 2.02)	**49.91** (↑ 1.54)	**67.06** (↑ 3.38)	**68.56** (↑ 3.04)	**66.9** (↑ 2.87)	**43.45** (↑ 0.72)	**54.05** (↑ 1.65)	**45.18** (↑ 0.89)
10%	baseline	36.23	41.21	37.31	55.7	58.74	56.69	32.94	40.25	32.68
10%	DF-ALC (w/$O_{\mathcal{EL}}$)	37.28 (↑ 0.95)	42.61(↑ 1.4)	38.37 (↑ 1.06)	56.5 (↑ 0.8)	59.5 (↑ 0.76)	57.41 (↑ 0.72)	33.4 (↑ 0.46)	41.27 (↑ 1.02)	33.21 (↑ 0.53)
10%	DF-ALC (w/$O_{\mathcal{ALC}}$)	**37.35** (↑ 1.12)	**42.71** (↑ 1.5)	**38.45** (↑ 1.14)	**58.82** (↑ 3.12)	**61.71** (↑ 2.97)	**59.5** (↑ 2.81)	**33.53** (↑ 0.59)	**41.66** (↑ 1.41)	**33.35** (↑ 0.67)

from different sources. We require the ontologies to have at least the logical operator of negation, disjunction, or universal quantifier, as well as 100 ABox assertions. Five ontologies fall into this set, with two of them ("Ontodm" and "Nifdys") not consistent in some assertions; it remains to see whether DF-$\mathcal{ALC}$ would revise these errors. A taxonomy ontology (Sso) is also added for comparison. We also test a terseness ontology "Family", which contains multiple instantiated families but its knowledge is incomplete. Based on "Family", we augment it into "Family2" by adding some knowledge that can bridge with the instantiation. The information about these ontologies is shown in Table 4. Adam optimizer was used with a learning rate of 2e-4 to learn the grounding. Early stopping with 10 epochs tolerance was used to limit the running time.

In semantic image interpretation experiment, we introduced two ontologies with different expressive powers: $O_{\mathcal{ALC}}$ and $O_{\mathcal{EL}}$. The $O_{\mathcal{EL}}$ ontology consists of two types of statements. The first type declares concepts, such as screen $\sqsubseteq$ $\top$. The second type specifies isPartOf relationships between concepts, for example, pot $\sqsubseteq \exists$isPartOf.potted_plant.

The $O_{\mathcal{ALC}}$ ontology, in addition to the first type of statements, has more expressive statements for specifying isPartOf relationships between concepts, such as $\exists$isPartOf.Chair $\equiv$ Seat $\sqcup$ Leg and plant $\sqcup$ pot $\equiv \forall$isPartOf.potted_plant. It also includes a third type of statement, which specifies disjointness between different types, for example, Chair $\sqcap \exists$isPartOf.Chair $\sqsubseteq \bot$ and Chair $\sqcap$ Table $\sqsubseteq \bot$.

For fairness, Em$\mathcal{ALC}$ uses $O_{\mathcal{ALC}}$, while LTN uses $O_{\mathcal{EL}}$ (LTN was also tested with $O_{\mathcal{ALC}}$, but it did not perform well). The ontology content can be correspondingly divided into three categories for analysis: vehicles, indoor objects, and animals. Since these three categories of symbols do not overlap, during training, the ontology and image test sets are also divided into subsets according to these categories for separate correction. We masked random parts of FRCNN's perceptual grounding to simulate FRCNN is trained in low-resource situations, the revising results are shown in Table 5. Since LTN does not improve performance, the low-resource results using LTN are not shown in the table. When there are a few shallow errors in perceptual symbol grounding (mask rate $\leq$ 3%), the semantic constraints of $\sqcap$ and $\sqcup$ in the disjoint and equivalence axioms of $O_{\mathcal{ALC}}$ guide corrections, making convergence more difficult compared to $O_{\mathcal{EL}}$. However, as the errors become more complex, the performance improvement due to the semantics of $\mathcal{ALC}$ becomes more significant, validating the necessity of learning the semantics of negation ($\neg$). Since LTN does not improve performance, the results using LTN are not shown here. It can be seen that even in low-resource scenarios, Em$\mathcal{ALC}$ can improve the performance of the baseline model.