A APPENDIX

A.1 Proof of Theorem 3.1

We begin by introducing some notation from prior work [1]. First, recall that a degree sequence for a relation R and variable V is a vector $\boldsymbol{f}_{R.V}$ such that $f_{R.V,i}$ is the frequency of the ith most frequent value, and $F_{R.V,i} = \sum_{j=1,i} f_{R.V,j}$ is the cumulative degree sequence (CDS). We represent relations R as tensors M_R with one dimension for each variable present in the relation. The value at a particular entry $M_{i_1,\ldots,i_{|\boldsymbol{V}_R|}}$ is equal to the frequency of that tuple, $(i_1, \ldots, i_{|\boldsymbol{V}_R|})$, in the relation R. Note that entries can be zero if that tuple does not appear in R. Additionally, the discrete derivative and integral of a tensor M on a variable V is defined as,

Definition A.1.

$$(\Delta_V M)_{v_i} = M_{v_i} - M_{v_i - 1} \tag{1}$$

$$(\Sigma_V M)_{v_i} = \sum_{j=1,v_i} M_j \tag{2}$$

A tensor M is *consistent* with a set of degree sequences $\boldsymbol{f}_R$ if the following is true,

$$(\sum_{V' \neq V} M)_i \leq f_{R.V,i} \qquad \forall\ i \in \mathbb{D}_V, V \in \boldsymbol{V}_R$$

Briefly, this means that if we contract the tensor down to a single dimension, then the resulting vector is less than the DF $F_{R.V}$ at all points. The set $\mathcal{M}_{\boldsymbol{f}_R}$ is the set of tensors consistent with $\boldsymbol{f}_R$.

Lastly, we define the *value tensor*, $E^{\boldsymbol{F}_R}$, and use it to explicitly define the *worst-case tensor*, $C^{\boldsymbol{F}_R}$.

Definition A.2. The *value* tensor, $\boldsymbol{E}^{\boldsymbol{F}_R} \in \mathbb{R}_+^{[\boldsymbol{n}]}$, is defined by the following linear optimization problem:

$$\forall \boldsymbol{m} \in [\boldsymbol{n}] : \qquad E_{\boldsymbol{m}}^{\boldsymbol{F}_R} \stackrel{\text{def}}{=} \text{ Maximize: } \sum_{\boldsymbol{s} \leq \boldsymbol{m}} M_{\boldsymbol{s}} \tag{3}$$
$$\text{Where: } \boldsymbol{M} \in \mathcal{M}_{\boldsymbol{f}_R}$$

The *worst-case* tensor, $\boldsymbol{C}^{\boldsymbol{F}_R} \in \mathbb{R}^{[\boldsymbol{n}]}$, is defined as:

$$\boldsymbol{C}^{\boldsymbol{f}_R} \stackrel{\text{def}}{=} \Delta_{V_1} \cdots \Delta_{V_d} \boldsymbol{E}^{\boldsymbol{F}_R} \tag{4}$$

Or, equivalently,

$$\Sigma_{V_1} \cdots \Sigma_{V_d} \boldsymbol{C}^{\boldsymbol{f}_R} = \boldsymbol{E}^{\boldsymbol{F}_R} \tag{5}$$

Note that this worst-case tensor is equivalent to the worst-case instance, $W(R)$ of a relation R, as depicted in Figure 2.

Getting back to Theorem 3.1 of this work, we start by proving that it holds for star queries before expanding to all Berge-acyclic queries. Consider the following query,

$$Q_{STAR} = R(V_1, \ldots, V_d) S_1(V_1) \ldots S_d(V_d)$$

If $\boldsymbol{M}$ is the count tensor of the relation R and $\boldsymbol{a}^{(V_i)}$ is the count tensor of S_i, in this case a simple non-increasing vector, then we can express the query size as follows,

$$|Q_{STAR}(D)| = \boldsymbol{M} \cdot \boldsymbol{a}^{(V_1)} \cdots \boldsymbol{a}^{(V_d)}$$

Given this notation, we consider part of Theorem 3.2 from [1].

Theorem A.3. ***[Thm. 3.2 from [1]]*** *Let $\boldsymbol{f}_R$ be the set of degree sequences as above, and let $\boldsymbol{V}, \boldsymbol{C}$ defined by* (3)-(5). *Then:*

(1) We can define the value tensor as follows,

$$\forall \boldsymbol{m} \in [\boldsymbol{n}] : \qquad E_{\boldsymbol{m}}^{\boldsymbol{F}_R} = \min\left(F_{R.V_1}(m_1), \ldots, F_{R.V_d}(m_d)\right) \tag{6}$$

(2) For any non-increasing vectors $\boldsymbol{a}^{(V_p)} \in \mathbb{R}_+^{[n_p]}$, $p = 2, d$, the vector $\boldsymbol{C}^{\boldsymbol{f}_R} \cdot \boldsymbol{a}^{(V_2)} \cdots \boldsymbol{a}^{(V_d)}$ is in $\mathbb{R}_+^{[n_1]}$ and non-increasing.

(3) For all count tensors $\boldsymbol{M}_R$, and all non-increasing vectors $\boldsymbol{a}^{(X_1)} \in \mathbb{R}_+^{[n_1]}, \ldots, \boldsymbol{a}^{(X_d)} \in \mathbb{R}_+^{[n_d]}$:

$$\boldsymbol{M}_R \cdot \boldsymbol{a}^{(V_1)} \cdots \boldsymbol{a}^{(V_d)} \leq \boldsymbol{C}^{\boldsymbol{f}_R} \cdot \boldsymbol{a}^{(V_1)} \cdots \boldsymbol{a}^{(V_d)} \tag{7}$$

Directly implying,

$$|Q_{STAR}(D)| \leq |Q_{STAR}(W(s(D)))| \tag{8}$$

Let $\hat{\boldsymbol{F}}_R$ be an upper bound of $\boldsymbol{F}_R$, i.e. $\hat{\boldsymbol{F}}_{R.V}(i) \geq \boldsymbol{F}_{R.V}(i)\ \forall\ V \in \boldsymbol{V}_R, i$, and define $\hat{f}_{R.V} = \Delta_V \hat{\boldsymbol{F}}_{R.V}$ and $\hat{f}_R$ as the set of these degree sequences which, as specified in Theorem 3.1, must be non-increasing. Further, note that item 1 and item 1 relies only on the properties of the worst-case instance's inherent structure, so it immediately applies to $\boldsymbol{C}^{\hat{F}_R}$.

Based on the above, we can prove the following lemma,

Lemma A.4. *For all non-increasing vectors $\boldsymbol{a}^{(V_1)} \in \mathbb{R}_+^{[n_1]}, \ldots, \boldsymbol{a}^{(V_d)} \in \mathbb{R}_+^{[n_d]}$:*

$$\boldsymbol{C}^{f_R} \cdot \boldsymbol{a}^{(V_1)} \cdots \boldsymbol{a}^{(V_d)} \leq \boldsymbol{C}^{\hat{F}_R} \cdot \boldsymbol{a}^{(V_1)} \cdots \boldsymbol{a}^{(V_d)} \tag{9}$$

Directly implying,

$$|Q_{STAR}(W(s(D)))| \leq |Q_{STAR}(W(\Delta\hat{S}))| \tag{10}$$

Proof. Following the original proof of item 2, we begin by simplifying the problem using 1-0 vectors. In particular, let $\boldsymbol{b}^{(m)} \in \mathcal{R}^n$ be the vector with m 1's followed by $n - m$ 0's. Because the $\boldsymbol{a}^{V_i}$ are non-increasing integral vectors, they can be represented as a sum of 1-0 vectors, so it suffices to consider the case where each of them is a 1-0 vector. In this case, the problem description becomes,

$$\boldsymbol{C}^{f_R} \cdot \boldsymbol{b}^{(m_1)} \cdots \boldsymbol{b}^{(m_d)} \leq \boldsymbol{C}^{\hat{F}_R} \cdot \boldsymbol{b}^{(m_1)} \cdots \boldsymbol{b}^{(m_d)}$$

Multiplying against $\boldsymbol{b}^{(m)}$ is the same as summing over the first m indices, so this can be alternatively expressed as,

$$\Sigma_{m_1} \ldots \Sigma_{m_d} \boldsymbol{C}^{f_R} \leq \Sigma_{m_1} \ldots \Sigma_{m_d} C^{\hat{f}_R}$$

Considering the definition of the value tensor $E_{\boldsymbol{m}}^{f_R}$, we can rephrase this as follows where $\boldsymbol{m} = (m_1, \ldots, m_d)$,

$$E_{\boldsymbol{m}}^{f_R} \ \leq\ E_{\boldsymbol{m}}^{\hat{f}_R}$$

Lastly, we insert the alternative definition of E^{f_R} provided in item 1 and the fact that each $\hat{\boldsymbol{F}}_{R.V_i}$ is an upper bound of $\boldsymbol{F}_{R.V_i}$ to prove the lemma,

$$\min(F_{R.V_1}(m_1), \ldots, F_{R.V_d}(m_d)) \ \leq\ \min(\hat{F}_{R.V_1}(m_1), \ldots, \hat{F}_{R.V_d}(m_d))$$

□

To prove that this can be extended to general queries, we rely on more theory from [1].

Theorem A.5. ***Implied by Thm. 4.2 of [1]*** *If the following is true for a set of database instances, $\mathcal{D}$,*

(1) For any non-increasing vectors $\boldsymbol{a}^{(V_p)} \in \mathbb{R}_+^{[n_p]}$, $p = 2, d$, the vector $\boldsymbol{C}^{f_R} \cdot \boldsymbol{a}^{(V_2)} \cdots \boldsymbol{a}^{(V_d)}$ is in $\mathbb{R}_+^{[n_1]}$ and non-increasing.
(2) For all relations $R \in \mathcal{D}$ with count tensor $\boldsymbol{M}_R$, and all non-increasing vectors $\boldsymbol{a}^{(X_1)} \in \mathbb{R}_+^{[n_1]}, \ldots, \boldsymbol{a}^{(X_d)} \in \mathbb{R}_+^{[n_d]}$:

$$\boldsymbol{M}_R \cdot \boldsymbol{a}^{(V_1)} \cdots \boldsymbol{a}^{(V_d)} \leq \boldsymbol{C}^{f_R} \cdot \boldsymbol{a}^{(V_1)} \cdots \boldsymbol{a}^{(V_d)} \tag{11}$$

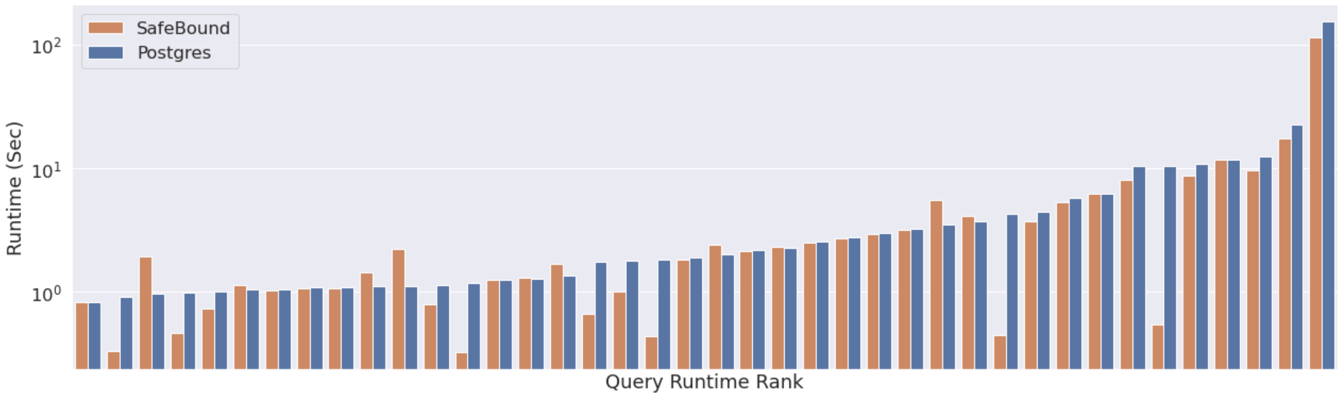
Then, for any Berge-acyclic query, Q,

$$|Q(D)| \leq |Q(W(s))| \quad \forall D \in \mathcal{D} \tag{12}$$

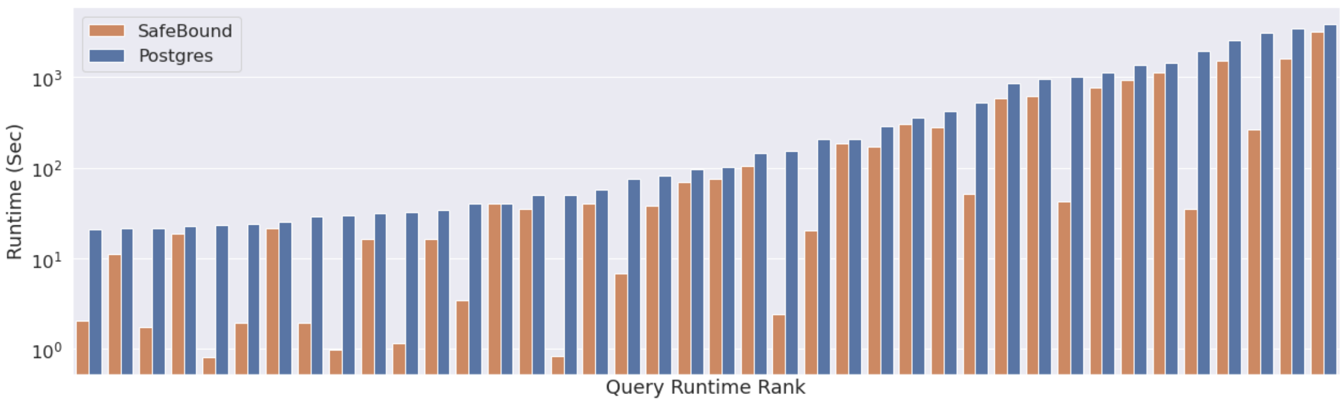
The first immediately holds for $\boldsymbol{C}^{\hat{F}_R}$ from Theorem A.3 while the latter holds for the set of all database instances, D, such that $D \models S$ due to Lemma A.4.

B DISAGGREGATED RESULTS

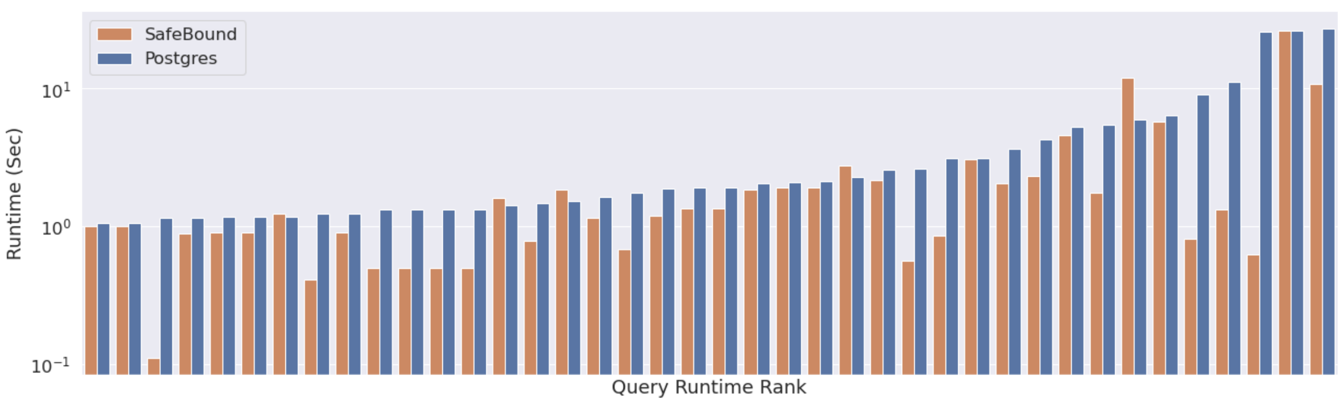
In this section, we provide alternate versions of Figures 6, 7, and 9.a which are disaggregated by benchmark.



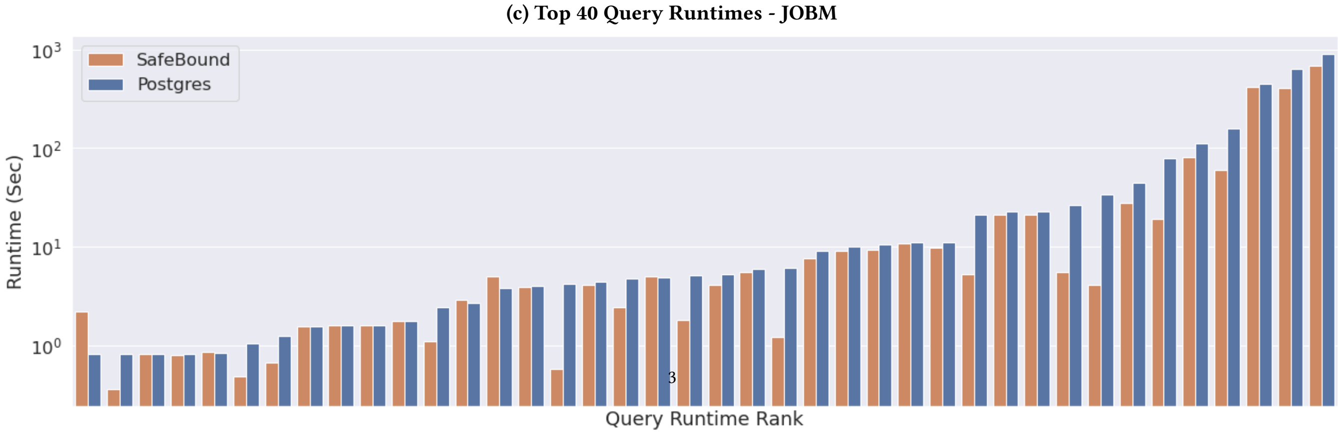
(a) Top 40 Query Runtimes - JOBLight



(b) Top 40 Query Runtimes - JOBLightRanges

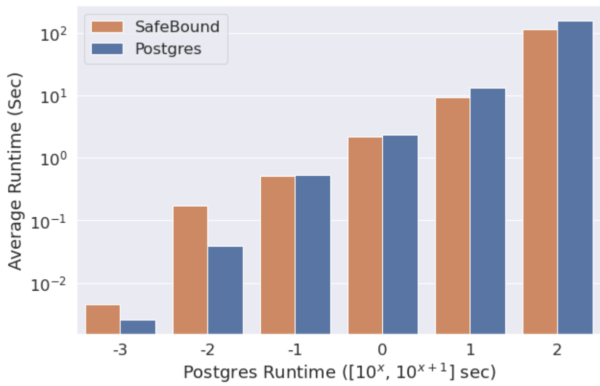


(c) Top 40 Query Runtimes - JOBM

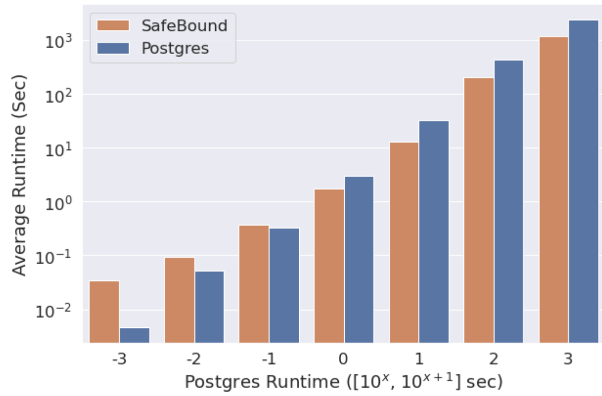


(d) Top 40 Query Runtimes - Stats

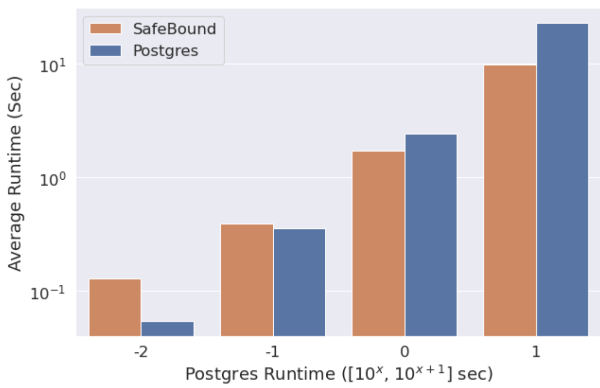
Figure 1: Top Query Runtime Graphs



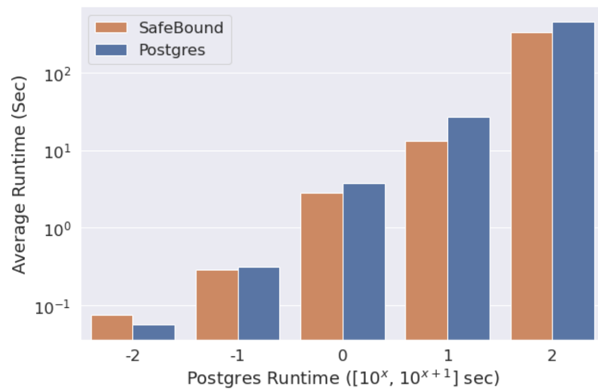
(a) Binned Runtimes - JOBLight



(b) Binned Runtimes - JOBLightRanges



(c) Binned Runtimes - JOBM

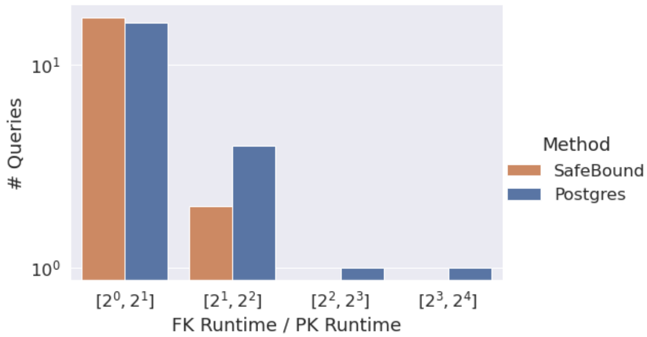


(d) Binned Runtimes - Stats

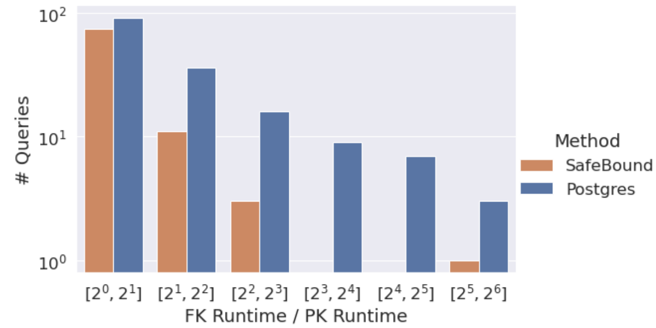
Figure 2: Binned Runtime Graphs

REFERENCES

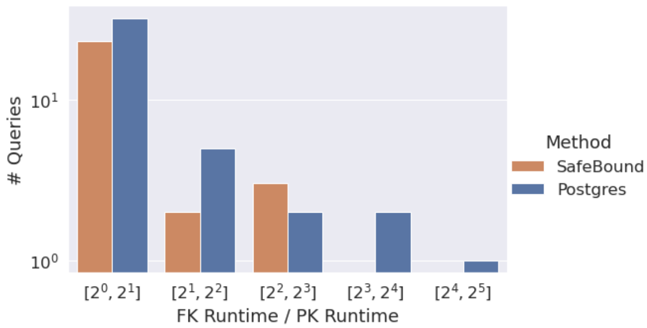
[1] Kyle Deeds, Dan Suciu, Magda Balazinska, and Walter Cai. 2022. Degree Sequence Bound For Join Cardinality Estimation. *CoRR* abs/2201.04166 (2022). arXiv:2201.04166 https://arxiv.org/abs/2201.04166



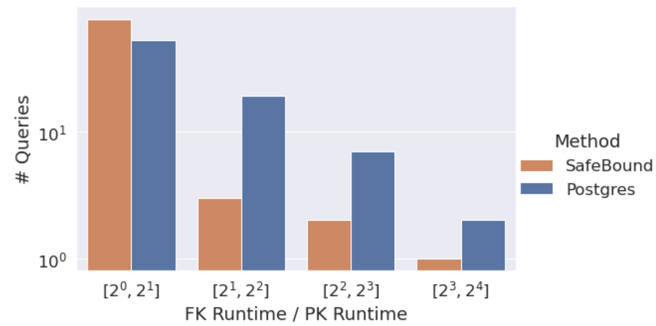
(a) FK Regressions - JOBLight



(b) FK Regressions - JOBLightRanges



(c) FK Regressions - JOBM



(d) FK Regressions - Stats

Figure 3: FK Regressions Graphs