

Indeed, minimizing $||\frac{1}{N}\mathbf{H}^\top \mathbf{H} - \mathbf{I}||_F^2$ can alleviate collapse issues and enhance representation diversity, as proposed in CCA-SSG [1], known as *decorrelation-based* methods. Our IC is **parallel** to it.

[1] From Canonical Correlation Analysis to Self-supervised Graph Neural Networks.

In our study, we establish the connection between our Isotropic Constraints (IC) and this method.

Proposition 2 states that decorrelation loss can be regarded as a special case of our isotropic constraints under the conditions of *1) only considering 2-nd central moment* and *2) choosing eigenvectors of covariance matrix as projection vectors*.

Compared to decorrelation-based strategies, our method has the following advantages:

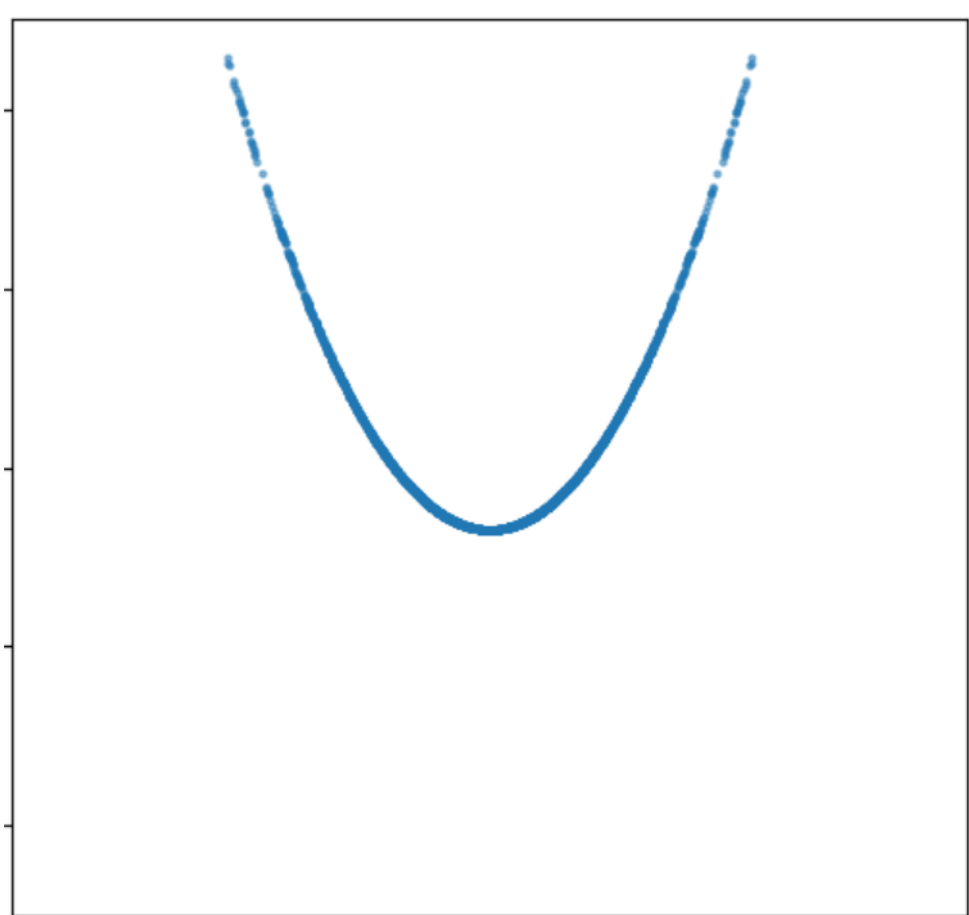
(1) Minimizing $||\frac{1}{N}\mathbf{H}^\top \mathbf{H} - \mathbf{I}||_F^2$ minimizes Pearson correlation coefficients between each pair of dimensions, potentially requiring BatchNorm, whereas our method is not limited to specific normalization techniques.

Table 7: Performance of isotropic constraints under different normalization schemes. (These experimental results have been added to our revision.)

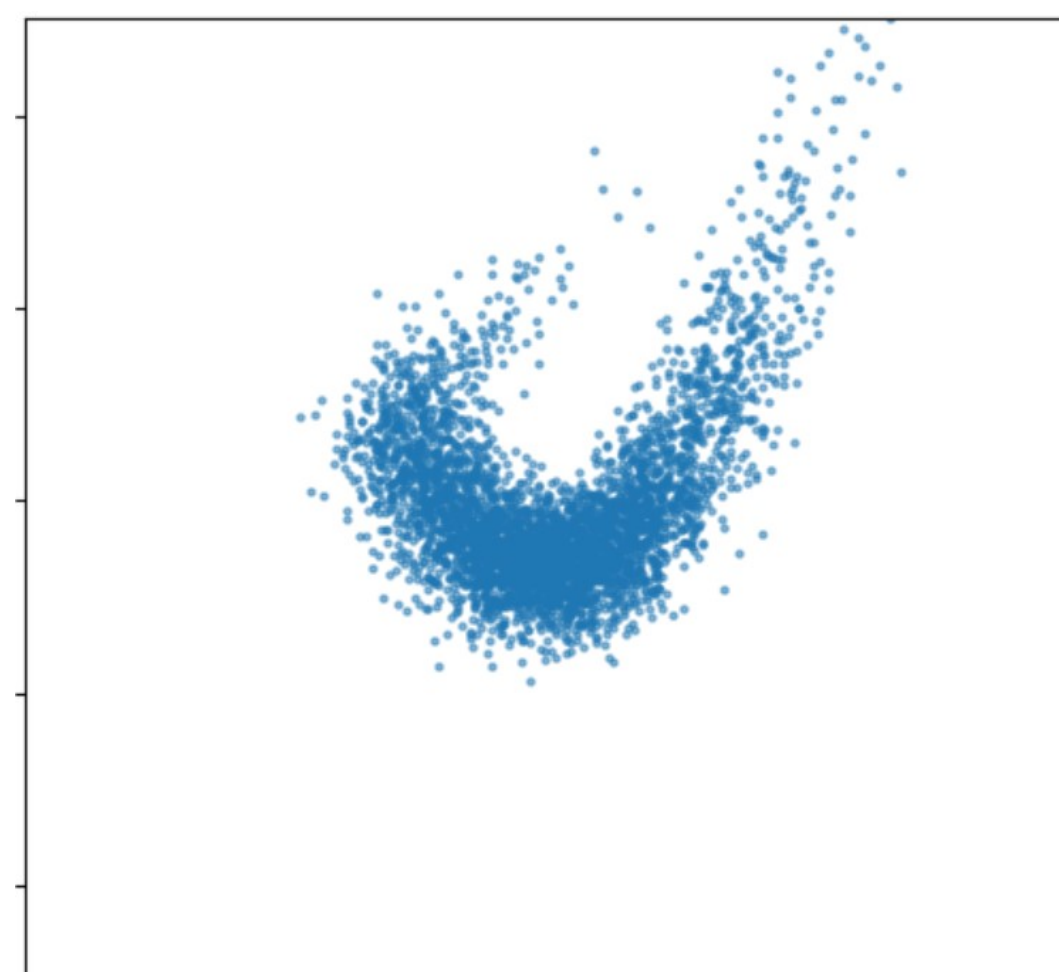
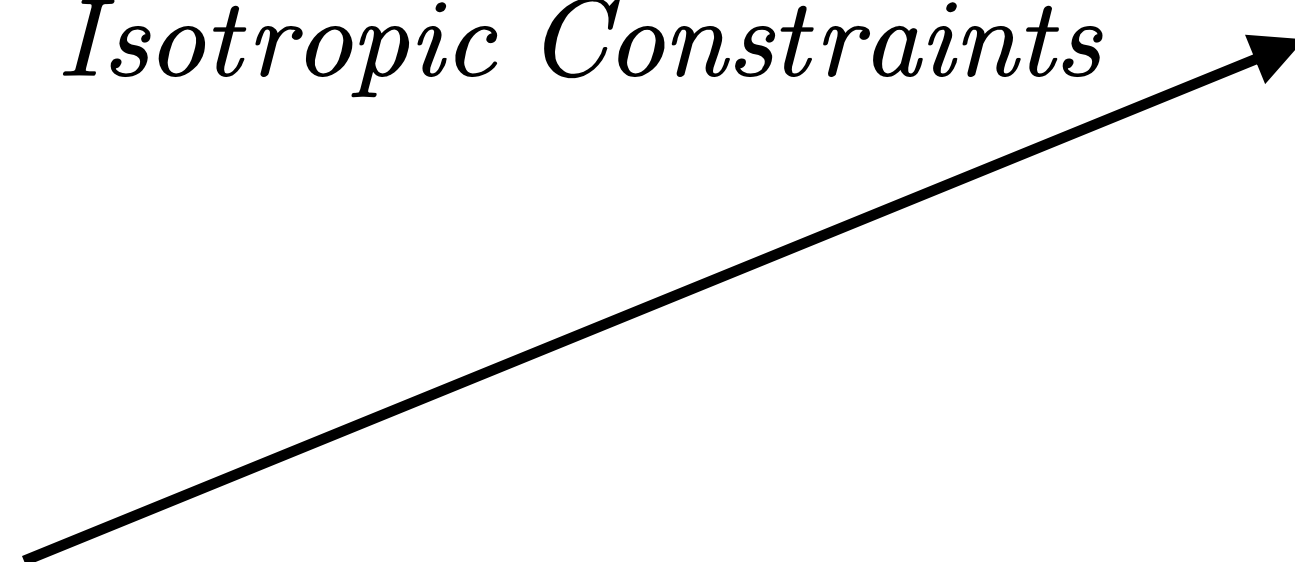
Dataset	Cora	Citeseer	Pubmed	Photo
Batch Norm	84.4 \pm 0.5	73.5 \pm 0.6	82.6 \pm 0.3	93.19 \pm 0.18
L2 Norm	84.3 \pm 0.4	73.6 \pm 0.5	82.4 \pm 0.4	93.15 \pm 0.20

(2) From a statistical perspective, $||\frac{1}{N}\mathbf{H}^\top \mathbf{H} - \mathbf{I}||_F^2$ can only remove Pearson linear correlations. It is powerless in the face of higher-order and complex correlations :

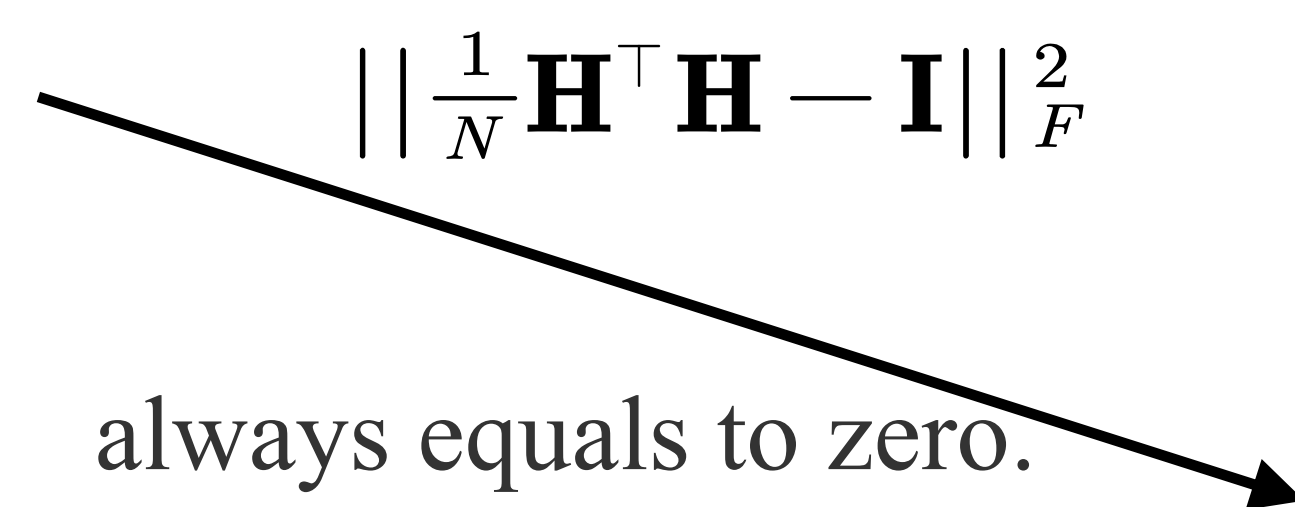
Initial distribution:



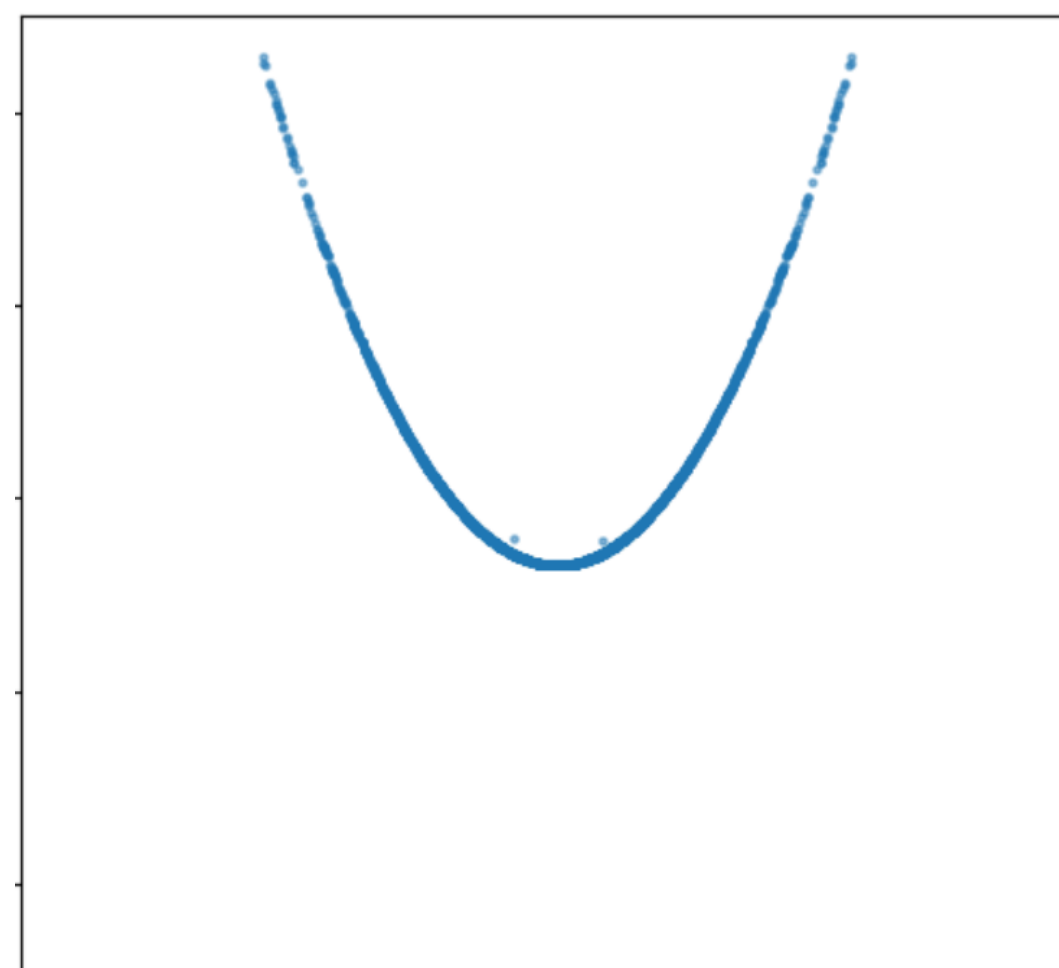
Isotropic Constraints



$||\frac{1}{N}\mathbf{H}^\top \mathbf{H} - \mathbf{I}||_F^2$



always equals to zero.



The training process did not converge significantly, and we only extracted the results to **highlight the differences**.

- For initial distribution, the covariance matrix of data is identity matrix \mathbf{I} .
- We set the sample points with ‘requires_grad=True’ and directly optimize them.
- From the results, it can be observed that our method exhibits a stronger capability in promoting representation diversity, because our method expects node representations to obey the same distribution along any space direction.
- This example also illustrates that eigenvalue distribution is not the optimal metric for measuring the utilization of the representation space.

In summary, our method and decorrelation-based approach investigate dimensional collapse issues and representation diversity from different perspectives. Researching the same problem from multiple angles and aspects undoubtedly contributes to the development of the self-supervised learning community.