MATHEMATICS FOR DATA SCIENCE II

WEEK 4

TOPICS COVERED IN WEEK 3

Vector Spaces

A vector space V over $\mathbb{R}$ is a set along with two functions

$$+ : V \times V \to V \qquad \text{and} \qquad \cdot : \mathbb{R} \times V \to V$$

(i.e. for each pair of elements v_1 and v_2 in V, there is a unique element $v_1 + v_2$ in V, and for each $c \in \mathbb{R}$ and $v \in V$ there is a unique element $c \cdot v$ in V)

that satisfies the following conditions:

Note: It is also represented as (V ; +; ·; R)

Vec. addition

i) $v_1 + v_2 = v_2 + v_1$ for all $v_1, v_2 \in V$

ii) $(v_1 + v_2) + v_3 = v_1 + (v_2 + v_3)$ for all $v_1, v_2, v_3 \in V$

iii) There exists an element in V denoted by 0 such that $v + 0 = v$ for all $v \in V$

iv) For each element $v \in V$ there exists an element $v' \in V$ such that $v + v' = 0$

Sc. mult

v) For each element $v \in V$, $1v = v$

vi) For each pair of elements $a, b \in \mathbb{R}$ and each element $v \in V$, $(ab)v = a(bv)$

distributive

vii) For each element $a \in \mathbb{R}$ and each pair of elements v_1 and v_2, $a(v_1 + v_2) = av_1 + av_2$

viii) For each pair of elements $a, b \in \mathbb{R}$ and each element $v \in V$, $(a + b)v = av + bv$

Note:

• To prove a set is a vector space, we need to verify additive and multiplicative closure and all the other axioms given above.

• If just one of the vector space axiom fails to hold, then V is not a vector space.

Subspaces

A non-empty subset W of a vector space V is called a subspace of V if W is a vector space under the operations addition and scalar multiplication defined in V.

To show that a non-empty set W is a vector subspace, one doesn't need to check all the vector space axioms.

<u>Conditions for a subspace:</u>

If W is a non-empty subset of a vector space V , then W is a subspace of V if and only if the following conditions hold:

(1) If w_1 and w_2 are in W , then $w_1 + w_2 \in W$.

(2) For all $c \in R$ and for all $w \in W$, $c \cdot w \in W$.

A subspace W of a vector space V is called a proper subspace if $W \subsetneq V$.

$\mathbb{R}^2 \longrightarrow$ any line passing thru origin $W = \{(x,y) : y = mx\}$

Note:

Every vector space V over R has two trivial subspaces:

- V itself is a subspace of V .
- The subset consisting of the zero vector $\{0_V\}$ of V is also a subspace of V.

Example:

Non-trivial subspaces of R^2:

- All lines passing through origin.

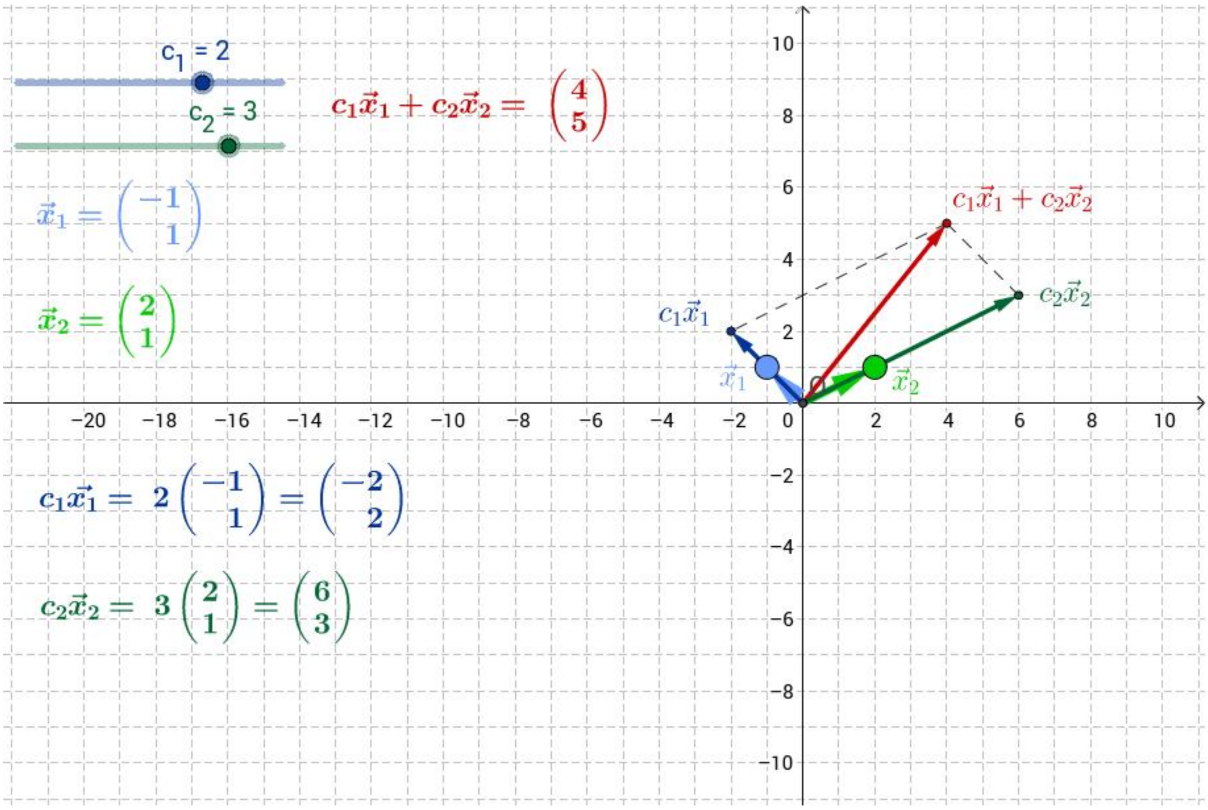
Non-trivial subspaces of R^3:

- All lines passing through origin
- All planes passing through origin

Linear Combination

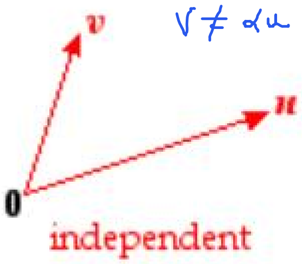
Let V be a vector space and $v_1, v_2, \ldots, v_n \in V$. Then $\sum_{i=1}^{n} \alpha_i v_i$ is said to be a linear combination of the vectors $v_1, v_2, \ldots, v_n$ with coefficients $\alpha_1, \alpha_2, \ldots, \alpha_n \in R$. Note that the linear combination of a set of vectors is another vector in V , since a vector space is closed under addition and scalar multiplication.

$2(-1, 1) + 3(2, 1) =$ $(4, 5)$



Linear dependence and independence

$v \neq \alpha u$

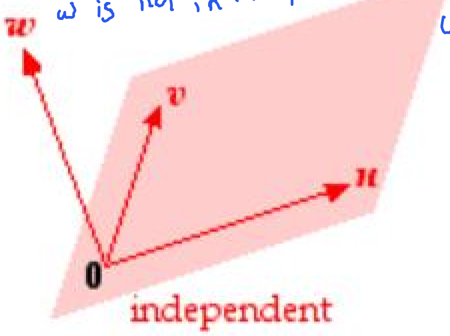


$v = \alpha u$

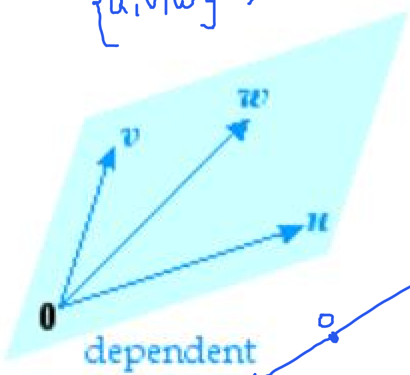
same line

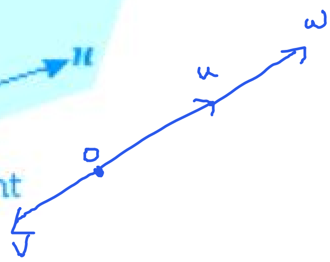
$u, v \rightarrow$ diff lines on a plane

w is not in the plane containing u & v



$\{u, v, w\} \rightarrow$ lie on same plane





Geometrically, any two vectors that lie on the same line, any three vectors that lie on the same plane are linearly dependent.

Linear Dependence: A set of vectors $\{v_1, v_2, ..., v_n\}$ is said to be linearly dependent, if there exist scalars $\alpha_1, \alpha_2, ..., \alpha_n \in R$ not all zero, such that $\alpha_1 v_1+\alpha_2 v_2+...+\alpha_n v_n = 0$.

Linear Independence: A set of vectors $\{v_1, v_2, ..., v_n\}$ is said to be linearly independent if they are not linearly dependent. In other words, if there exist $\alpha_1, \alpha_2, ..., \alpha_n \in R$ such that $\alpha_1 v_1+\alpha_2 v_2+...+\alpha_n v_n = 0$, then $\alpha_i = 0$ for all i.
If a set is linearly independent, then the only linear combination of these vectors that can yield the zero vector is when all the coefficients are zero.

Ways of checking linear independence

$V=0$ $\quad \alpha V = 0 \Rightarrow \alpha = 0$ the only solution? No

Let S be a subset of $\mathbf{R}^2$

zero vector

- If S={0}, then S is linearly dependent
- If S={v} where v is a non-zero vector, then S is linearly independent
- If S= $\{v_1, v_2\}$,

 Method I: One is a multiple of other

 Lin. dep.

 Method II: Determinant method

 $v_1 = (x_1, y_1) \quad v_2 = (x_2, y_2)$

 $\det \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix} \rightarrow \begin{cases} =0 & \text{lin. depen} \\ \neq 0 & \text{lin. indep} \end{cases}$

- If S is a set with three or more vectors, then S is linearly dep.

Let S be a subset of $\mathbf{R}^3$

- If S={0}, then S is linearly dep.
- If S={v} where v is a non-zero vector, then S is linearly indep
- If S= $\{v_1, v_2\}$, check if one vector is a multiple of other

 $v_2 = \alpha v_1 \Rightarrow$ lin. dep

 $v_2 \neq \alpha v_1 \Rightarrow$ lin. indep

- If S=$\{v_1, v_2, v_3\}$,

 Method I: Check if one vector is a multiple of other/linear combination of other vectors

 $\underset{v_1}{(1,0,0)}, \underset{v_2}{(0,1,0)}, \underset{v_3}{(1,1,0)} \quad v_3 = v_1 + v_2 \quad$ Lin. dep

 $\underset{v_1}{(1,2,1)} \quad \underset{v_2}{(2,4,2)} \quad \underset{v_3}{(1,0,0)} \quad v_2 = 2v_1 \quad$ Lin. dep.

Method II: Determinant method

$$\det\begin{bmatrix} | & | & | \\ v_1 & v_2 & v_3 \\ | & | & | \end{bmatrix} \begin{array}{ll} = 0 & L.D \\ \neq 0 & L.I. \end{array}$$

- If S is a set with four or more vectors in $\mathbb{R}^3$, then S is linearly _dep_________

More generally, if S be a subset of $\mathbf{R}^n$

- If S={0}, then S is linearly _dep_________
- If S={v} where v is a non-zero vector, then S is linearly _ind_________
- If S= $\{v_1, v_2\}$, check if one vector is a multiple of other.
- If S has n vectors use determinant method.
- If S is a set with k vectors in R^n, $k \geq n + 1$, then S is always linearly dependent.

- A set containing a zero vector is always _lin. dep_________

- A superset of a linearly dependent set is _lin. dep_________

$A \subseteq B$ B - superset of A $\{v_1, v_2, \ldots, v_k\} \to$ lin. dep $\Rightarrow \sum_{j=1}^{k} \alpha_j v_j = 0$, $\alpha_j \neq 0$

$\{v_1, v_2, \ldots, v_k, v_{k+1}, \ldots, v_r\}$

- A subset of a linearly independent set is _lin. ind_____________

$A \subseteq B$ B – L.I Suppose A is L.D $\Rightarrow$ B is L.D "contradiction"

So A is L.I

Checking linear independence using system of homogeneous equations

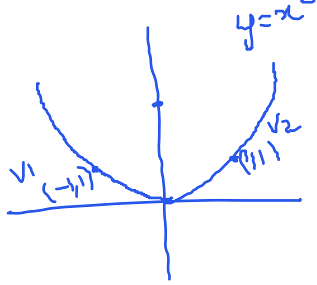
To check whether the set $\{v_1, v_2, \ldots, v_n\}$ is linearly independent or not, we need to verify whether the homogeneous system Vx = 0 has only the trivial solution or not, where V is the matrix whose j^{th} column is the vector v_j .

- ➢ If Vx=0 reduces to a system which has <u>only the trivial solution</u>, then the set is <u>linearly independent</u>.
- ➢ If we get a <u>non-zero solution</u>, then the set is <u>linearly dependent</u>.

Note that if the matrix V is square, then the only thing we need to verify is whether the determinant of V is non-zero or not.

v_1, v_2 $\quad -2v_1+3v_2$

TOPICS IN WEEK 4



$v_1+v_2=(0,2)$

$y=x^2$
↓
cannot be a subsp

- Span of a set
- Spanning set
- Basis "representative" of a v.s.
- Dimension
- Finding basis
- Rank of a matrix

- **Span of a set** $\quad S \subseteq V \quad V$ - v.s.

The span of a set S, denoted by span(S) is the set of all finite linear combinations of the elements of the set S. That is,

$$span(S) = \{\sum_{i=1}^{n} \alpha_i v_i : v_i \in S, \alpha_i \in \mathbb{R}\}$$

1) $w_1, w_2 \in span(S)$
$\Rightarrow w_1+w_2 = \sum_{i=1}^{k_1} \alpha_i v_i + \sum_{i=1}^{k_2} \beta_i v_i \in span(S)$

2) $\alpha w_1 = \sum_{i=1}^{n} (\alpha\alpha_i) v_i \in span(S)$

Note that **span(S) is a vector subspace**. of V.

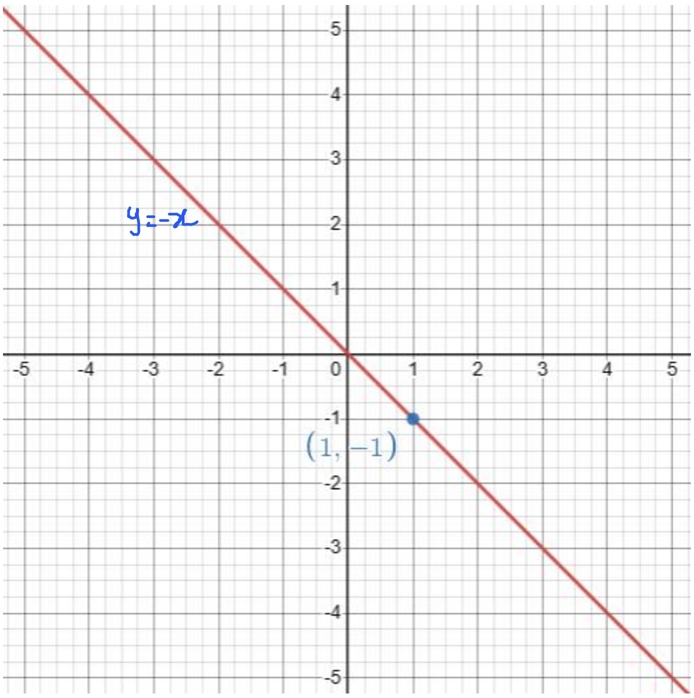
Example:

Let S = {(1, -1)} ⊂ R^2. The span of S will contain all possible finite linear combinations of the elements of S.

span(S) = $\{\alpha(1,-1) : \alpha \in \mathbb{R}\} = \{(\alpha,-\alpha) : \alpha \in \mathbb{R}\} = \{(x,y) : x+y=0\} = W$

Spanning set

Thus the span of {(1, -1)} is the line y = $-x$



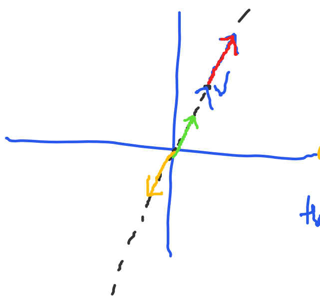
$span\{(1,-1)\} = span\{(-1,1)\}$
$= span\{(5,-5)\}$

$span\{\overset{v_1}{(1,-1)}, \overset{v_2}{(2,-2)}\} = \{\alpha_1 v_1 + \alpha_2 v_2 : \alpha_1, \alpha_2 \in \mathbb{R}\}$
$v_2 = 2v_1$
$= \{\alpha_1 v_1 + 2\alpha_2 v_1 : \alpha_1, \alpha_2 \in \mathbb{R}\}$
$= \{\underbrace{(\alpha_1+2\alpha_2)}_{\alpha} v_1 : \alpha \in \mathbb{R}\}$
$= span\{v_1\}$

$\alpha=1 \quad \alpha v = v$

$\alpha > 1 \quad \alpha > 1 \quad \alpha v \to$ stretches along v

$0<\alpha<1 \quad \alpha v \to$ shrinks but remains in the same dirn



$\alpha<0 \to \alpha v$ is along the same line in opp. dirn

Exercises: 1) What is the span of {(x, y) : (x, y) ≠ (0, 0)}? Line passing thru origin in $\mathbb{R}^2$

$\underbrace{(x,y)}_{v}$

$\text{span}\{v\} = \{\alpha v : \alpha \in \mathbb{R}\} = \{(\alpha x, \alpha y) : \alpha \in \mathbb{R}\}$

2) What is the span of {(a, b, c) : (a, b, c) ≠ (0, 0, 0)}? Line passing thru origin in $\mathbb{R}^3$

$\text{span}\{v\} = \{\alpha v : \alpha \in \mathbb{R}\} =$

3) Let S = {(1, 1, 0), (1, 0, 0)}. What is the span of S? plane passing thru origin in $\mathbb{R}^3$

$v_1 = (1,1,0)$, $v_2 = (1,0,0)$

$v_1 \,\&\, v_2$ lin. ind

Eqn of a plane in $\mathbb{R}^3$: $ax + by + cz = 0$
passing thru origin

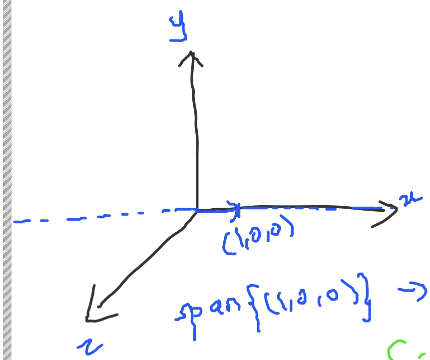
$\text{span}\{v_1, v_2\} = \{\alpha_1 v_1 + \alpha_2 v_2 : \alpha_1, \alpha_2 \in \mathbb{R}\}$

$= \{\alpha_1 (1,1,0) + \alpha_2 (1,0,0) : \alpha_1, \alpha_2 \in \mathbb{R}\}$

$= \{(\underbrace{\alpha_1 + \alpha_2}_{x}, \underbrace{\alpha_1}_{y}, \underbrace{0}_{z}) : \alpha_1, \alpha_2 \in \mathbb{R}\}$

$= \{(x, y, 0) : x, y \in \mathbb{R}\} = xy\text{-plane}$

$x = \alpha_1 + \alpha_2$
$y = \alpha_1$
$\boxed{z = 0}$
$x = y + \alpha_2$



$\text{span}\{(1,0,0)\} \to x\text{-axis}$

$v_1 = (1,0,0)$ $v_2 = (2,0,0)$ $v_1, v_2 \to$ lin. dep

$\text{span}\{v_1, v_2\}$ = line thru origin in $\mathbb{R}^3$

$v_1 = (1,0)$ $v_2 = (1,1)$ $v_1, v_2 \to$ lin. ind

$S_1 = \{v_1, v_2\}$

$\text{span}\{v_1, v_2\} = \{\alpha_1 v_1 + \alpha_2 v_2 : \alpha_1, \alpha_2 \in \mathbb{R}\} = \mathbb{R}^2$

$\text{span}(S_1) = \mathbb{R}^2$
$S_1 \to$ lin. ind

$(x, y) \in \mathbb{R}^2$ Does $(x, y) \in \text{span}\{v_1, v_2\}$? Yes

$(x, y) = \alpha_1 (1,0) + \alpha_2 (1,1) = (\alpha_1 + \alpha_2, \alpha_2)$

Spanning set:

Let V be a vector space and S be a subset of V. S is said to be a spanning set if span(S) = V .

$\alpha_2 = y$ $\alpha_1 + \alpha_2 = x$
$\alpha_1 = x - y$

$(x, y) = (x - y) v_1 + y v_2$

Examples:

For R^2

$S = \{\text{Any two lin. ind vectors}\}$ $\text{span}(S) = \mathbb{R}^2$

$v_3 = v_1 + v_2$

$\text{span}\{\underbrace{(1,0)}_{v_1}, \underbrace{(0,1)}_{v_2}, \underbrace{(1,1)}_{v_3}\} = \mathbb{R}^2$

S_2 - lin. dep
$\text{span}(S_2) = \mathbb{R}^2$

$= \{\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 \underset{(v_1 + v_2)}{v_3} : \alpha_i \in \mathbb{R}\}$

For R^3

$S = \{\text{any three lin. ind vectors}\}$ $\text{span}(S) = \mathbb{R}^3$

$= \{\underbrace{(\alpha_1 + \alpha_3)}_{\alpha} v_1 + \underbrace{(\alpha_2 + \alpha_3)}_{\beta} v_2 : \alpha, \beta \in \mathbb{R}\}$

$= \text{span}\{v_1, v_2\} = \mathbb{R}^2$

$\text{span}\{\underbrace{v_1, v_2, \ldots, v_k}_{S}\} = \text{span}\{\underbrace{v_{i_1}, v_{i_2}, \ldots, v_{i_r}}\}$

$\hookrightarrow$ max. lin. independent subset of S

Is the spanning set of a space unique? No

Let S = {(1, 0), (0, 1), (1, 1)}. Then S spans $\mathbb{R}^2$

S = {(1, 1), (1, 0)} spans $\mathbb{R}^2$

If S_1 and S_2 are two spanning set of a space V, does S_1 and S_2 contain the same number of vectors? No

Exercises : 1) Choose the set of correct options.

○ Option 1: If S is a spanning set of the vector space V , then S ∪ {v} must be a spanning set of V , for all v ∈ V .

○ Option 2: Span of an empty set is the zero vector space.

○ Option 3: If S is a spanning set of the vector space V , then S \ {v} must be a spanning set of V , for all v ∈ S.

$S = \{(1,0),(0,1)\}$ $\text{Span}(S) = \mathbb{R}^2$

$S \setminus \{(0,1)\} \to$ does not span $\mathbb{R}^2$

○ Option 4: If S is a spanning set of the vector space V , then S ∪ {v} may not be a spanning set of V , for all v ∈ V .

2) Let V be the subspace of R^3 defined as follows:

$x - y + z = 0$

V = {(x, y, z) | x = y − z, and x, y, z ∈ R} → plane in $\mathbb{R}^3$

Choose the set of correct options from the following.

○ Option 1: {(1, 1, 0), (1, 0, −1)} is a linearly independent set of V.

$x = y - z$ $\quad$ $x = y - z$

○ Option 2: {(1, 1, 0), (1, 0, −1), (0, 1, 1)} is a linearly independent set of V.

lin. ind

Option 3: {(0, 1, 1), (1, 0, −1)} is a spanning set of V.

$x=y-z$ $x=y-z$

Option 4: {(1, 1, 0)} is a spanning set of V.

need 2 vectors

$V=\{(x,y,z) : x=y-z\}$ How do you find a spanning set?

→ reduce the no. of variables

$=\{(y-z,y,z) : y,z\in\mathbb{R}\}$

$=\{y(1,1,0)+z(-1,0,1) : y,z\in\mathbb{R}\}$

$\{v_1, v_2\}$ → spanning set for V

Building spanning sets

non-zero vector

$v=(x,y,z)$ not origin

"add"

We may append vectors to a set to build spanning sets for a vector space. Consider R^3 .

empty

• Step 1: Start with $S_0 = \emptyset$. Span(S_0) = {(0, 0, 0)}.

some random non-zero vector in $\mathbb{R}^3$

• Step 2: Since span(S_0) ≠ R^3, append a vector, say (1, 1, 0) to S_0. Now $S_1 = S_0 \cup \{(1, 1, 0)\} = \{(1, 1, 0)\}$. span(S1) is a line in R^3 , which still doesn't cover the entire space R^3.

v_1

some nonzero vector lin. ind. to v_1

• Step 3: Now choose a vector outside span(S_1). Let $S_2 = S_1 \cup \{(1, 1, 1)\} = \{(1, 1, 0), (1, 1, 1)\}$. Span($S_2$) is the plane x = y, which still doesn't cover R^3 .

v_2 v_1, v_2 → lin. ind

v_1, v_2, v_3 → lin. ind

v_3

• Step 4: We now choose a vector outside span(S_2). Let $S_3 = S_2 \cup \{(1, 0, 0)\} = \{(1, 1, 0), (1, 1, 1), (1, 0, 0)\}$. Now, it is easy to verify that S_3 spans R^3. Notice that at each stage we added a vector which was not in the span of the previous vectors.

HW: Can you construct a spanning set for R^3 starting with the set S $\{(1, 2, 3)\}$?

$\begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} \rightarrow$ scalar matrices

αI

$\text{Span}\left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\} =$ all 2×2 scalar matrices

$= \left\{\alpha \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} : \alpha \in \mathbb{R}\right\} = \left\{\begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} : \alpha \in \mathbb{R}\right\}$

$\text{Span}(S) = \left\{\underbrace{\begin{bmatrix} \alpha_1 & 0 \\ 0 & 0 \end{bmatrix}}_{\alpha_1 A_1} + \underbrace{\begin{bmatrix} 0 & \alpha_2 \\ 0 & 0 \end{bmatrix}}_{\alpha_2 A_2} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & \alpha_3 \end{bmatrix}}_{\alpha_3 A_3} : \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}\right\} = \left\{\begin{bmatrix} \alpha_1 & \alpha_2 \\ 0 & \alpha_3 \end{bmatrix} : \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}\right\}$

$\Rightarrow$ all 2×2 upper triangular matrices

Exercise 3: Let S be the set of $S = \left\{\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}_{A_1}, \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A_2}, \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}_{A_3}\right\}$ matrices where

Choose the correct statement(s):

✗ • Option 1: Span(S) is the vector space consisting of only lower triangular square matrices of order 2.

✓ • Option 2: Span(S) is the vector space consisting of only upper triangular square matrices of order 2.

✗ • Option 3: Span(S) is the vector space consisting of all the square matrices of order 2.

$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \in \text{Span}(S)$?

✗ • Option 4: Span(S) is the vector space consisting of only scalar matrices of order 2.

$\begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix}$

Basis of a Vector Space

B — L.I, span(B) = V

A basis B of a vector space V is a linearly independent set that also spans V.

For R^2 $\{(1,0),(0,1)\}$ ✓ → standard basis

$\{(1,0),(0,1),(1,1)\}$ → not a basis → not L.I, spans $\mathbb{R}^2$

For R^3

$B = \{(1,0,0),(0,1,0),(0,0,1)\}$ → L.I

$\{(1,0)\}$ → not a basis → L.I, not spanning $\mathbb{R}^2$

$(x,y,z) \in \mathbb{R}^3$ $\quad (x,y,z) \in \text{span}(B)$

$(x,y,z) = x(1,0,0) + y(0,1,0) + z(0,0,1)$

For $M_n(R)$ → set of all $n \times n$ matrices with real entries

$M_2(\mathbb{R})$ $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ → basis

HW: $M_3(\mathbb{R})$

Is every spanning set of V a basis for V? No

Spanning set need not be L.I

But basis has to be L.I

Is every linearly independent set in V a basis for V? No

↓ need not span ✓

Equivalent conditions for basis

The following conditions are equivalent for a set B to be a basis of a vector space V :

1. B is linearly independent and span(B) = V .

2. B is a maximal linearly independent set.

3. B is a minimal spanning set.

Exercise 4: If $\{v_1, v_2, v_3\}$ forms a basis of R^3, then which of the following are true?

○ Option 1: $\{v_1, v_2, v_1 + v_3\}$ forms a basis of R^3.

○ Option 2: $\{v_1, v_1 + v_2, v_1 + v_3\}$ forms a basis of R^3.

○ Option 3: $\{v_1, v_1 + v_2, v_1 - v_3\}$ forms a basis of R^3.

○ Option 4: $\{v_1, v_1 - v_2, v_1 - v_3\}$ forms a basis of R^3.

Is the basis of a space unique?

Suppose B_1 and B_2 are two basis for V, then

number of vectors in B_1 ____ number of vectors in B_2

Exercise 5: If S be a subset of R^5 such that span(S) = R^5, then what is the minimum number of elements possible in S?

Exercise 6: Choose the set of correct options.

○ Option 1: If a subset S of R^2 contains only two elements then S is a basis of R_2.

○ Option 2: If a subset S of R^3 contains only one elements then S can never be a basis of R^3.

○ Option 3: If a subset S of R contains only one non zero element then S is a basis of R.

○ Option 4: If S_1 and S_2 are two bases of R^3, then $S_1 = S_2$

Dimension of a vector space

The dimension of a vector space V, is the size/cardinality of a basis B of V. It is denoted by dim(V).

Examples:

- Any basis for R^n has exactly __ elements and hence dim(R^n) = ___.
- Let W be the subspace of R^3 spanned by {(1, 0, 0), (0, 1, 0), (3, 5, 0)}. What is the dimension of W?

Steps to obtain basis from a spanning set: Method 1

- Step 1: Write down the vectors as rows of a matrix A.
- Step 2: Apply row reduction on A to obtain a row echelon form.
- Step 3: The non-zero rows of the reduced matrix form a basis for W.

The number of non-zero rows in the reduced matrix is the dimension of the subspace W.

Example: Let W =span {(1, 1, 2), (2, –1, 4), (3, 0, 6)}. Find a basis for W.

Note: In this method, the basis vectors may be different from the vectors in the spanning set.
Suppose we want a basis which contains vectors from the spanning set, then we can use the following method.

Steps to obtain basis from a spanning set: Method 2

Step 1: The vectors are arranged as columns of a matrix, say A.

Step 2: The matrix A is reduced to row echelon/reduced row echelon form, say R.

Step 3: The columns of A corresponding to the columns of R containing pivots form a basis for the column space of A.

Example: Let W =span {(1, 1, 2), (2, −1, 4), (3, 0, 6)}. Find a basis for W.

Choose the set of correct options.

: The dimension of the vector space $M_{1\times 2}(\mathbb{R})$ is 2.

The dimension of the vector space $M_{2\times 1}(\mathbb{R})$ is 1.

The dimension of the vector space $M_{3\times 3}(\mathbb{R})$ is 3.

: A basis of $M_{2\times 2}(\mathbb{R})$ is the set $\left\{\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\1 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}\right\}$.

Find a basis for the following sets

- $V_1 = \{A \mid A \in M_{2\times 2}(\mathbb{R})$ and A is a symmetric matrix, i.e., $A = A^T\}$
- $V_2 = \{A \mid A \in M_{2\times 2}(\mathbb{R})$ and A is a scalar matrix$\}$
- $V_3 = \{A \mid A \in M_{2\times 2}(\mathbb{R})$ and A is a diagonal matrix$\}$
- $V_4 = \{A \mid A \in M_{2\times 2}(\mathbb{R})$ and A is an upper triangular matrix$\}$
- $V_5 = \{A \mid A \in M_{2\times 2}(\mathbb{R})$ and A is a lower triangular matrix$\}$

All $V_i, i = 1, 2, 3, 4, 5$ are subspaces of the vector space $M_{2\times 2}(\mathbb{R})$.

Find the dimension of the vector space
$V = \{A \mid \text{sum of entries in each row is } 0, \text{ and } A \in M_{3\times 2}(R)\}$.

Let V be a vector space which is defined as follows:

$V = \{(x, y, z, w) \mid x + z = y + w\} \subseteq R^4$

with usual addition and scalar multiplication. Find a basis for V.

How to find basis for subspaces?

R^2

R^3