Curious-II: A Multi/Many-Objective Optimization Algorithm with Subpopulations based on Multi-novelty Search*

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Abstract. Novelty search's ability to explore efficiently the fitness space has been receiving increasing attention. However, different novelty metrics differ in how they explore the search space. In fact, here we show that novelty metrics are complementary and a multi-novelty approach improves the performance substantially. Specifically, we propose a multi-novelty search multi/many-objective algorithm that has both Euclidian distance and prediction-error novelty metrics (Curious II). In one hand, the Euclidian distance based novelty metric makes the subpopulation explore subspaces with low crowd density and avoids premature convergence. On the other hand, the prediction-error novelty metric guides a subpopulation to explore subspaces with unexpected objective fitness since the deceptive or less biased regions have a high-error (i.e., high novelty) with surrogate models. Experiments here reveal that using both novelty metrics in a multi-novelty algorithm has strong benefits. The proposed algorithm (Curious II) was compared with four state-of-the-art algorithms (NSGA-III, MOEA/D, BiGE and KnEA) and two novelty search-based algorithms on the WFG1-8 test problem with up to 10 objectives. Curious II outperforms all the others in 26 out of 32 tasks for HV index, 29 out of 32 tasks for GD index and 24 out of 32 tasks for IGD index.

Keywords: Multi-objective optimization · Multi-novelty search · Surrogate-assisted evolutionary algorithms.

1 Introduction

In the real world, there exist problems that require the optimization of multiple conflicting objective functions simultaneously [5], which are known as multi-objective or many-objective optimization problems (MOPs or MaOPs) [11]. Generally, an MOP/MaOP can be defined as follows:

$$\begin{aligned}&\text{Minimize} F(x) = (f_1(x), \cdots, f_M(x)),\\ &\text{s.t.} x \in \Omega \subset R^n,\end{aligned}$$

* Supported by organization x.

where x is an n-dimensional decision variable, $f_m(x), m = 1, 2, \ldots, M$ is the m-th objective function, and Ω defines the decision space. If M is 2 or 3, the problem is a MOP. Otherwise, it is a MaOP.

The multi-objective evolutionary algorithms (MOEAs) for solving MaOPs can be divided into three categories [16]: (1) modification of the classical dominance relationship; (2) application of a performance indicator; (3) decomposition of the MaOP into single-objective optimization problems [26]. MOEAs based on the Pareto dominance relationship or its variants belong to the first category [24, 8, 21]. MOEAs of the second category use indicators to evaluate solutions and guide the search process [3, 17, 13]. The decomposition-based MOEAs decompose an MaOP into several single-objective optimization problems (SOPs) or simpler MOPs for collaborative solutions [10, 27].

As for most MOEAs, individual evaluations are usually performed using metrics related to fitness functions. However, fitness functions do not always appreciate the step-by-step stepping stones that guide the search process to the optimal solution [1]. Therefore, novelty search-based metric is one kind of performance indicator to substitute the fitness function, which can save so-called novelty solutions to the next generation. The problem is how to combine performance-driven search and novelty search. In the latest literature, subpopulation algorithm based on novelty (SAN) [23], Curious I [22] and a decomposition-based interactive co-evolutionary algorithm (CIEMO/D) [2] are representatives.

In this paper we go beyond previous algorithms to propose a multi-novelty approach called "Curious II". It makes use of multi-novelty and multi-objective subpopulations, which consists of three parts: (1) the improved general subpopulation framework (IGSF) to encourage individual interactions between subpopulations. And self-adapting differential evolution algorithm [4] (jDE) is used for the genetic variation of generations; (2) multi-novelty search technique (MNS) with two novelty metrics keeps the search focus on novel solutions while the fitness functions focus on the fitted ones. The prediction error-based metric relies on the surrogate model–the radial basis function network (RBFN); (3) archive self-updated criterion preserve the elite/novel individuals of each generation while preventing degradation in the selection process by shared adaptive thresholds.

Our contributions are:

- **Pioneer multi-novelty search approach** - We propose the first method (Curious II) that uses two different novelty metrics as the subjective fitness. This approach enables each generation to retain innovative individuals to search different regions of the objective space, enhancing the diversity of optimal solutions.
- **Improved multi-population interaction framework** - We have proposed GSF to enable different subpopulations to share the fitness space for evolution. This time we came up with IGSF to make them memorizes the generation's adaptive evolution weight and speed up their evolution.
- **State of the art results** - Experiments demonstrate that the performance of the proposed MOEA significantly outperforms that of 6 state-of-the-art MOEAs on MOPs/MaOPs.

2 Preliminary Knowledge

2.1 Self-adapting Differential Evolution Algorithm

For the subpopulations, they repeat the mutation and crossover procedures for each individual in the genotype spaces. For SAN and Curious I, they both choose differential evolution (DE) to generate offspring. However, the IGSF uses jDE because the novelty fitness space is dynamic and various parameters are needed.

The DE algorithm is represented by real-valued vectors. For each individual x_i, where i is its index in the population, the mutation procedure creates a mutation vector v_i as below:

$$\mathrm{v}_i = \mathrm{x}_{r1} + F\left(\mathrm{x}_{r2} - \mathrm{x}_{r3}\right)$$

where $r1, r2$ and $r3$ are randomly selected individuals and F is a hyperparameter.

The trial vector chooses either the gene from the parent x_i or from the mutation vector v_i in the crossover process:

$$u_i = \begin{cases} x_{i,j} \text{ if } rand() > CR \text{ and } j \neq rnd_i \\ v_{i,j} \text{ if } rand() \leq CR \text{ or } j = rnd_i \end{cases}$$

where $rand()$ is a random number, CR is a hyperparameter, j is the variable index of x_i and rnd_i is a random index to ensure at least one variable will come from the mutation vector v_i.

The jDE uses self-adapting mechanism on the control parameters F and CR. For the solution represented by $\mathrm{x}_{i,G}$ (G denotes the current generation), new control parameters $F_{i,G+1}$ and $CR_{i,G+1}$ are obtained before next mutation is performed, which influence the mutation and crossover operations of the new vector $\mathrm{x}_{i,G+1}$. They are calculated as:

$$F_{i,G+1} = \begin{cases} F_l + \mathrm{rand}_1 * F_u, & \text{if } \mathrm{rand}_2 < \tau_1 \\ F_{i,G}, & \text{otherwise} \end{cases}$$

$$CR_{i,G+1} = \begin{cases} \mathrm{rand}_3, & \text{if } \mathrm{rand}_4 < \tau_2 \\ CR_{i,G}, & \text{otherwise} \end{cases}$$

where $rand_j$, $j \in \{1, 2, 3, 4\}$ are random values. τ_1 and τ_2 represent probabilities to adjust F and CR, respectively.

2.2 Novelty Search and Novelty Metrics

Novelty search is an exploration algorithm driven by the novelty of a behavior of individuals. The novelty metrics are the specific distance between them (real-valued vectors). Here presents two kinds of novelty metrics which combine the multi-novelty search.

Objective-Specific Distance-based Novelty The distance-based metric is applied in objective space and gauges the distance of the novelty solutions to their k-nearest neighbors in a search space. The metric is defined as follows:

$$novelty_1 = p(\mathrm{x}) = \frac{1}{k}\sum_{i=1}^{k} dist\left(f(\mathrm{x}), f(\mu_i)\right)$$

where $f()$ is the original objective function and μ_i is the i-th nearest neighbor of individual x according to the Euclidean distance-based measure $dist()$.

Error-Specific Learning-based Novelty Learning-based novelty metric, which was measured in behavior space, can be the prediction error (PE) of a surrogate model to approximate the optimization functions [7, 18], whihc is defined as:

$$novelty_2 = PredictionError = |\hat{f}(\mathrm{x}) - f(\mathrm{x})|$$

where $\hat{f}()$ is the objective function fitted by the surrogate model. Other variations of PE include the dispersion in predictions and predictive variance.

2.3 Radial Basis Function Networks

Many regression or classification techniques can be used as surrogate models in MOEAs, such as RBFNs [25], Kriging models [6] and polynomial regression (PR) models. RBFNs are distinguished from others due to their highly nonlinear, universal approximation and faster learning speed.

An RBFN is a feed forward neural network composed of three layers. The input layer connects the network to its environment. The hidden layer applies a nonlinear transformation to the inputs, with a radial basis function like the Gaussian function. The output layer is linear and serves as a summation unit.

The multivariate Gaussian density function is chosen for the hidden node transfer function and expressed as:

$$a_{hk} = \exp\left(-\frac{\|x_h - c_k\|^2}{\sigma_h^2}\right)$$

where a_{hk} is the activation of the h-th hidden unit with input x_k. c_k is the RBF unit center determined by clustering. σ_h is a distance scaling parameter, determining over what distance in the input space the unit will have a influence.

3 The Proposed Curious II

3.1 Algorithm Overview

The basic framework of the proposed Curious II with three stages is shown in Fig.1. First, both subpopulations and RBFN are initialized and then enter different stages of the algorithm. The stage 1 is the subpopulations parallel evolution and interaction stage under the IGSF. The stage 2 is the is the multi-objective and novelty metrics guidance stage based on MNS while the stage 3 is the archive self-updated stage.

Firstly, the initial population P with N individuals is randomly generated which are also used for initial training of the RBFN_1. After that, the population P is divided into $M+2$ subpopulations randomly with the same size, where M is the number of the objectives. With the $M+2$ subpopulations and RBFN_G, the three stages are performed in turn, which is illustrated in Sections 3.2-3.4.

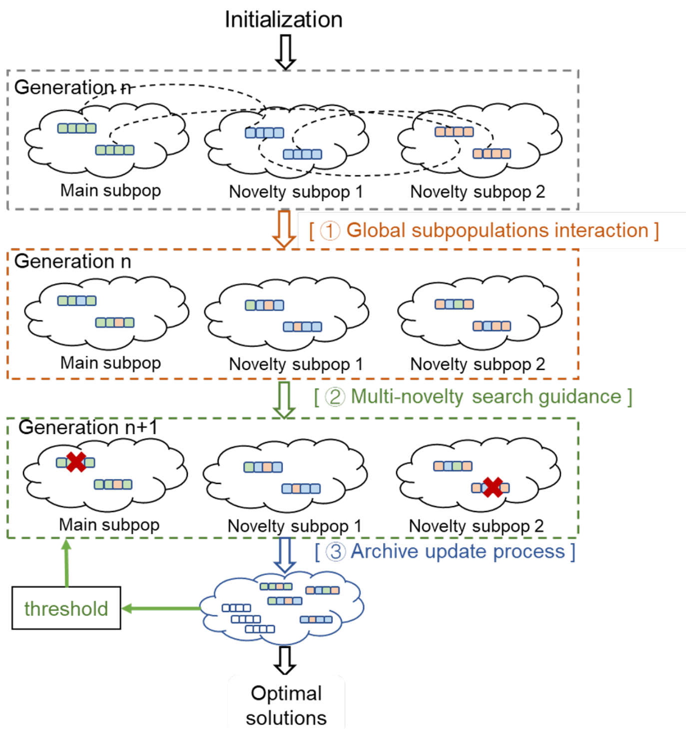


Fig. 1: General framework of Curious II.

3.2 Global Subpopulations Interaction Stage with IGSF

The global subpopulations interaction is uniform described by the GSF [23]. First, each subpopulation can use existing different structured evolutionary algorithms (EAs) in its formalization to evolve [14, 15, 12, 19]. Secondly, each subpopulation optimizes a unique objective function with all subpopulations evolve together to approximate the Pareto-front (PF). However, MOEAs with only the above characteristics are prone to generate solutions around the extreme value of each objective and thus lead to poor distribution of the PF. To enhance the performance of GSF, novel designs need to be considered further.

In this paper, we propose IGSF in Algorithm 1. The main improvement is that, there are main subpopulations exploring various single-objective space and novelty subpopulations searching solutions based on additional diversity-based fitness functions. In IGSF, two latter functions are posed to raise novelty subpopulations. They are the distance-novelty based subpopulation and the learning-novelty based subpopulation. IGSF are applied to merge subpopulations together. The interaction is such that, each individual of each subpopulation acts as a parent and another parent is randomly selected from other subpopulations. The mutation and crossover operators conform to jDE process in Section 2.1.

Algorithm 1: IGSF of global subpopulations interaction stage.

Input: $M+2$(the number of subpopulations), N_m(size of subpopulation), $P_1, \ldots, P_{M+2}$(subpopulation), $F, CR, F_l, F_u, \tau_1, \tau_2$(the hyperparameters of jDE)
Output: offspring $O_1, \ldots, O_{M+2}$

for m = *1 to* $M+2$ **do**
  $F, CR \leftarrow weight(optimal\_individual(P_m))$;
  for n = *1 to* N_m **do**
    `// choose parents`
    $x_{n1} \leftarrow Individual_{m,n}$;
    $x_{n2}, x_{n3} \leftarrow rand(Individual) \in rand(subpop)$;
    $x_{m,n}, weight(x_{m,n}) \leftarrow jDE(x_{n1}, x_{n2}, x_{n3})$;
  end
  offspring $O_m \leftarrow x_{m,n}, para(x_{m,n})$;
end

3.3 Multi-novelty Search (MNS) Guidance Stage

The second stage is used to guide the selection process in the subpopulations, i.e., the process of replacing the parent by the offspring in Algorithm 2. These new offsprings in each subpopulation will be exerted different selection forces on according to the different fitness functions as shown in Fig.2. For the convergence of the solution set, the selected metric for the established main subpopulations is the value of the decomposed subproblem of the multi-objectives. However, for diversity of solution sets, the selected metric for the established novelty subpopulations are novelty metrics in Section 2.2.

Algorithm 2: MNS based multi-novelty search guidance stage.

Input: $M+2$(the number of subpopulations), N_m(size of subpopulation), $P_1, \ldots, P_{M+2}$(subpopulation), $O_1, \ldots, O_{M+2}$(offspring), RBFN)
Output: New subpopulation $P_1, \ldots, P_{M+2}$

for m = *1 to* M **do**
  for n = *1 to* N_m **do**
    if $objective_m(O_{m,n}) > objective_m(Indi_{m,n})$ **then**
      $P_m \leftarrow Individual_{m,n} \leftarrow O_{m,n}$;
    end
  end
end
for i = *1 to 2* **do**
  `// multi-novelty subpopulation`
  for n = *1 to* N_m **do**
    if $novelty_i(O_{M+i,n}) > novelty_i(Indi_{M+i,n})$ **then**
      $P_{M+i} \leftarrow Individual_{M+i,n} \leftarrow O_{M+i,n}$;
    end
  end
end

As can be seen in Fig.2, If the PF of the problem is full of solutions, then the distance-based novelty fitness will prefer the opposite direction while the learning-based fitness do prefer where solutions are sparse than the PF.

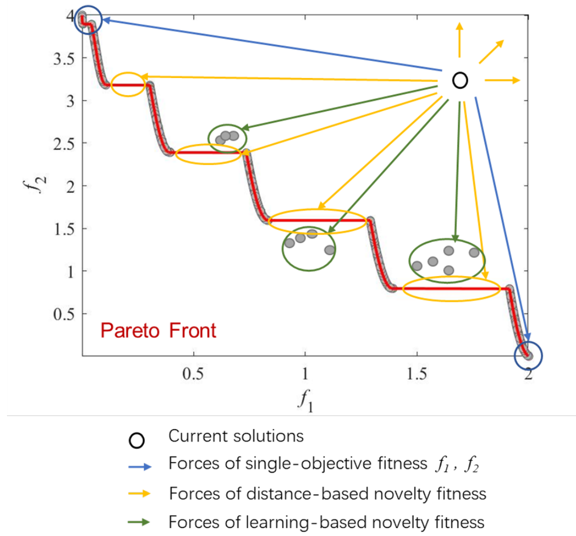


Fig. 2: Different selection forces according to the different fitness functions.

3.4 Archive Self-adapted Update Stage

In stage 3, archive update criterions are inherited from MNS, i.e., whether the offsprings can enter archive depends entirely on their novelty metric values. However, in this process, we propose an adaptive entry threshold, which is called the novelty threshold. This threshold is shared by two different metric calculations, i.e., the number of acceptances and rejections by the two novelty metrics jointly adjust the change in the size of this threshold during the evolutionary process.

The following are the dynamics adjustment used to update the threshold:

- if the size of archive $<= H_a$, the novelty threshold is H_0
- if the size of archive $> H_a$ and a new individual is accepted by the archive, multiply H_0 by H_{inc}
- if the size of archive $> H_a$ and a new individual is rejected by the archive, multiply H_0 by H_{dec}.

4 Experiments

4.1 The Experimental Setup

The typical WFG 1-8 test suite[9] are used for comparison on solving MOPs/MaOPs. The test suite has a series of complex problem properties. In the experiment, 20 distance-related variables and 4 position-related variables are chosen to be optimized. The 32 instances include each problem with 2, 3, 5, and 10 objectives.

As for the comparison algorithms, they are divided into two types. The first type is the state-of-the-art algorithms widely used for MOPs/MaOPs, including NSGA-III, MOEA/D, BiGE and KnEA. The second type is the predecessor algorithms of Curious II, namely Curious I and SAN. Among the second type, the difference between Curious II and Curious I is that, the former uses IGSF while the latter uses DE based GSF; the difference between Curious II and SAN is that SAN uses DE based GSF and only distance-based novelty search technique. By comparing Curious II with Curious I and SAN, we can investigate the effects of IGSF and MNS.

The indicator chosen for comparison is the widely used hypervolume (HV) [28] and inverted generational distance (IGD). HV calculates the volume of the area enclosed by the solution set and a reference point specified at the normalized maximum point $[1, \cdots, 1]$. The IGD indicator measures the average distance from each reference point (10000 uniformly distributed points sampled on the true PF of the MOP) to its nearest solution in the objective space. The two indicators verifies both the convergence and diversity of solutions.

The experiment is conducted on the Evolutionary Multi-Objective Optimization Platform 2.0 (PlatEMO 2.0) [20] in MATLAB. The parameters of the first type algorithms are basically the same as in their proposing papers. The parameters of Curious II are listed in Table 1.

Table 1: Parameters' value used in Curious II and other algorithms

Parameters		values	
Evaluation times	250000	K (in K-NN)	5
SBX probability / CR	0.6	F_l	0.1
Mutation probability / F	0.1	F_u	0.3
τ_1	0.5	τ_2	0.5
H_0	0.1	H_a	1
H_{inc}	0.99	H_{dec}	1.01
Subpopulation size	30	Number of runs	30
Number of objectives (M)		population size (P)	
2		120	
3		150	
5		210	
10		360	

4.2 Experimental Results and Discussion

To demonstrate the state of the art of the proposed method, we compare it with two types of comparison algorithms. The first-type algorithms includes NSGA-III, MOEA/D, BiGE, KnEA while the second-type methods includes Curious I and SAN. The results of a indicator are its mean and standard deviation value over 30 runs of the EMO algorithms. Fig.3 shows the set of optimal results for each run of the above methods on two-objective optimization problems.

Results and discussion on first type comparison algorithms Tables 2-3 show the HV and IGD results of the first-type algorithms and the bold numbers indicate the best (or similarly best) results. In general, the Curious II shows the best optimization performance on 32 MaOP instances in terms of the two metrics simultaneously.

Compared to the EMO algorithms in columns 2-5, Curious II has obtained the significantly better results on WFG 5-7 MaOP instances with 2-5 objectives and all instances in WFG 1-4 in terms of HV metric. For IGD indicator, Curious II performed slightly worse on the WFG 1-2, 8 instances with different number of objectives but better on others.

Among all the instances, the PFs of WFG4-WFG8 are regular, while those of WFG1-WFG3 are irregular. These results demonstrate that the Curious II algorithm exhibits advantages for at least one indicators, and even for all indicators, on WFG 1-7 MaOPs with different Pareto-Fronts.

We believe that the comparison with the first class of algorithms can show that Curious II, an algorithm inspired by performance-driven search, exhibits advantages different from previous EMO algorithms for solving MaOPs.

Table 2: HV results (mean and standard deviation) obtained by Curious II, NSGA-III, MOEA/D, BiGE and KnEA on WFG1-WFG8 problems with 2,3,5,10 objectives.

Problems	M	Curious II	NSGA-III	MOEA/D	BiGE	KnEA
WFG1	2	**0.698(0.000)**	0.599(0.044)	0.684(0.002)	0.666(0.062)	0.677(0.006)
	3	**0.954(0.002)**	0.916(0.038)	0.906(0.016)	0.364(0.015)	0.356(0.015)
	5	**0.993(0.004)**	0.930(0.032)	0.903(0.049)	**0.990(0.002)**	0.988(0.003)
	10	**1.000(0.000)**	0.780(0.080)	0.652(0.083)	0.998(0.001)	0.893(0.033)
WFG2	2	**0.634(0.000)**	0.609(0.010)	0.567(0.021)	0.630(0.001)	0.549(0.0024)
	3	**0.946(0.000)**	0.926(0.006)	0.836(0.030)	0.892(0.007)	0.881(0.006)
	5	**0.998(0.001)**	0.986(0.005)	0.790(0.035)	0.994(0.001)	0.992(0.001)
	10	**1.000(0.000)**	0.982(0.010)	0.690(0.060)	0.996(0.001)	0.991(0.002)
WFG3	2	**0.585(0.001)**	0.568(0.007)	0.547(0.016)	0.576(0.003)	0.581(0.000)
	3	**0.423(0.001)**	0.383(0.007)	0.316(0.024)	0.370(0.008)	0.339(0.014)
	5	**0.284(0.001)**	0.125(0.013)	0.010(0.018)	0.246(0.012)	0.144(0.021)
	10	**0.174(0.001)**	0.000(0.000)	0.000(0.000)	0.043(0.027)	0.000(0.000)
WFG4	2	**0.349(0.003)**	**0.348(0.000)**	0.346(0.018)	0.345(0.001)	0.345(0.001)
	3	**0.598(0.000)**	0.570(0.000)	0.554(0.003)	0.523(0.003)	0.515(0.003)
	5	**0.861(0.001)**	0.806(0.001)	0.700(0.031)	0.793(0.003)	0.785(0.003)
	10	**0.972(0.016)**	0.910(0.007)	0.343(0.056)	**0.969(0.001)**	0.955(0.002)
WFG5	2	**0.316(0.000)**	0.314(0.000)	0.308(0.000)	0.214(0.029)	0.311(0.002)
	3	**0.552(0.002)**	0.530(0.000)	0.518(0.005)	0.502(0.003)	0.496(0.004)
	5	**0.790(0.001)**	0.761(0.001)	0.681(0.012)	0.743(0.003)	0.743(0.003)
	10	0.857(0.004)	0.878(0.003)	0.425(0.028)	**0.907(0.001)**	0.902(0.001)
WFG6	2	**0.323(0.002)**	**0.322(0.004)**	0.321(0.006)	0.319(0.003)	0.265(0.014)
	3	**0.560(0.005)**	0.538(0.004)	0.526(0.008)	0.480(0.006)	0.475(0.007)
	5	**0.802(0.012)**	0.765(0.006)	0.636(0.010)	0.750(0.005)	0.744(0.008)
	10	0.839(0.014)	0.865(0.013)	0.251(0.051)	**0.905(0.007)**	0.897(0.010)
WFG7	2	**0.349(0.001)**	**0.348(0.000)**	0.344(0.001)	0.342(0.001)	0.327(0.011)
	3	**0.598(0.001)**	0.570(0.000)	0.530(0.013)	0.536(0.003)	0.536(0.003)
	5	**0.857(0.008)**	0.806(0.001)	0.663(0.018)	0.793(0.004)	0.793(0.003)
	10	0.937(0.010)	0.910(0.010)	0.339(0.032)	**0.973(0.001)**	0.966(0.001)
WFG8	2	**0.317(0.003)**	**0.320(0.003)**	0.309(0.006)	**0.318(0.000)**	0.175(0.048)
	3	0.490(0.002)	**0.527(0.002)**	0.512(0.005)	0.462(0.006)	0.460(0.005)
	5	0.722(0.005)	**0.739(0.002)**	0.569(0.073)	0.717(0.004)	0.704(0.003)
	10	0.812(0.014)	0.814(0.015)	0.054(0.031)	**0.905(0.003)**	0.870(0.005)

Results and discussion on second type comparison algorithms To validate the effects of IGSF and MNS in Curious II, empirical HV and IGD results are shown in Tables 3. This table is different from the previous ones. “>’ indicates that the algorithm of the left column has better results than the algorithm of the right column, and “=" indicates that the algorithms perform similarly. Curious II significantly outperforms Curious I 23 times on HV metric. Except for 6 instances of WFG 6-7, Curious II always wins Curious I on IGD indicator. However, Curious I with MNS but without IGSF wins SAN 26 times on both HV metric and IGD metric. HV and IGD indicators reflect that the diversity

Table 3: IGD results (mean and standard deviation) obtained by Curious II, NSGA-III, MOEA/D, BiGE and KnEA on WFG1-WFG8 problems with 2,3,5,10 objectives.

Problems	M	Curious II	NSGA-III	MOEA/D	BiGE	KnEA
WFG1	2	**0.006(0.001)**	0.182(0.083)	0.024(0.002)	0.102(0.226)	0.043(0.018)
	3	**0.080(0.001)**	0.144(0.043)	0.204(0.013)	1.272(0.040)	1.297(0.045)
	5	0.785(0.114)	**0.405(0.032)**	0.793(0.035)	0.542(0.022)	**0.402(0.009)**
	10	1.754(0.055)	1.214(0.125)	1.767(0.090)	1.290(0.060)	**0.940(0.043)**
WFG2	2	**0.004(0.001)**	0.043(0.015)	0.150(0.077)	0.021(0.002)	0.625(0.140)
	3	**0.058(0.001)**	0.126(0.002)	0.267(0.035)	0.213(0.013)	0.178(0.008)
	5	**0.383(0.036)**	**0.391(0.002)**	0.763(0.047)	0.599(0.012)	0.467(0.018)
	10	1.227(0.093)	1.398(0.327)	1.821(0.040)	1.459(0.028)	**1.095(0.033)**
WFG3	2	**0.003(0.001)**	0.034(0.014)	0.073(0.032)	0.024(0.007)	0.013(0.000)
	3	**0.005(0.005)**	0.103(0.012)	0.230(0.058)	0.110(0.017)	0.165(0.028)
	5	**0.032(0.006)**	0.534(0.045)	1.013(0.271)	0.228(0.037)	0.398(0.054)
	10	**0.102(0.012)**	1.280(0.335)	5.574(0.483)	0.471(0.105)	1.245(0.108)
WFG4	2	**0.006(0.007)**	**0.009(0.000)**	**0.012(0.001)**	0.018(0.002)	0.016(0.001)
	3	**0.040(0.002)**	0.166(0.000)	0.192(0.003)	0.234(0.007)	0.214(0.004)
	5	**0.405(0.004)**	0.967(0.001)	1.574(0.126)	1.123(0.023)	1.030(0.011)
	10	**2.986(0.447)**	4.028(0.074)	9.546(0.094)	4.057(0.047)	3.916(0.023)
WFG5	2	**0.062(0.000)**	0.063(0.000)	0.069(0.000)	0.349(0.110)	0.067(0.003)
	3	**0.094(0.003)**	0.180(0.000)	0.196(0.002)	0.245(0.006)	0.223(0.004)
	5	**0.478(0.005)**	0.959(0.000)	1.423(0.062)	1.151(0.026)	1.018(0.011)
	10	**3.727(0.023)**	3.934(0.024)	9.209(0.087)	4.127(0.056)	3.932(0.017)
WFG6	2	**0.049(0.004)**	**0.049(0.006)**	**0.051(0.011)**	0.053(0.006)	0.180(0.039)
	3	**0.093(0.014)**	0.175(0.002)	0.199(0.005)	0.265(0.008)	0.259(0.009)
	5	**0.486(0.040)**	0.960(0.001)	1.636(0.023)	1.105(0.021)	1.051(0.017)
	10	**3.917(0.220)**	**4.040(0.022)**	9.678(0.152)	**3.989(0.038)**	**4.013(0.041)**
WFG7	2	0.006(0.002)	0.009(0.000)	0.013(0.002)	0.025(0.003)	0.057(0.026)
	3	0.040(0.004)	0.166(0.000)	0.258(0.024)	0.237(0.009)	0.194(0.004)
	5	**0.434(0.046)**	0.967(0.001)	1.686(0.050)	1.123(0.026)	1.025(0.012)
	10	**3.604(0.173)**	4.037(0.211)	9.515(0.131)	4.046(0.057)	3.860(0.026)
WFG8	2	0.060(0.004)	**0.054(0.006)**	0.076(0.011)	**0.056(0.001)**	0.736(0.277)
	3	0.227(0.007)	**0.188(0.002)**	0.214(0.005)	0.286(0.009)	0.276(0.008)
	5	**0.723(0.021)**	0.948(0.002)	1.475(0.092)	1.124(0.020)	1.077(0.012)
	10	4.213(0.088)	4.068(0.155)	8.568(0.353)	**3.989(0.046)**	**4.013(0.036)**

and convergence of Curious II is better than its predecessor algorithms, while that of Curious I is better than SAN.

Comparing Curious II with Curious I, we can conclude that IGSF is better than GSF for subpopulation-based EAs. Moreover, by comparing Curious I with SAN, we find that MNS outperforms single-novelty search with distance-based novelty metric. It is mainly due to the addition of an uncommon distance calculation by MNS, which promotes the improvement of the diversity of solutions.

5 Conclusion

In this paper, we propose an multi-novelty search technique based many-objective optimization algorithm, termed Curious II. To tackle both MOPs and MaOPs with various shapes of PFs, there are 3 stages in Curious II. Each subpopulation first reproduces offspring in stage 1 by a novel information sharing mechanism. In stage 2, each main subpopulation optimizes one single objective, while the novel subpopulation optimizes the novelty fitness of the individuals. To further obtain a good scaling distribution along the PF, the archive update criterion screens the elite/innovative individuals in each round in stage 3.

Table 4: HV and IGD results (mean and standard deviation) obtained by Curious II, Curious I and SAN on WFG1-WFG8 problems with 2,3,5,10 objectives.

Problems	HV					IGD				
	CuriousII		Curious I		SAN	CuriousII		CuriousI		SAN
WFG1	0.698(0.000)	>	0.688(0.005)	>	0.672(0.014)	0.006(0.001)	<	0.023(0.008)	<	0.053(0.022)
	0.954(0.002)	>	0.888(0.021)	=	0.902(0.030)	0.080(0.001)	<	0.487(0.145)		0.388(0.182)
	0.993(0.004)	>	0.971(0.003)	>	0.961(0.009)	0.785(0.114)	<	1.254(0.056)	<	1.321(0.096)
	1.000(0.000)	>	0.997(0.001)	>	0.994(0.004)	1.754(0.055)	<	1.990(0.049)	<	2.095(0.146)
WFG2	0.634(0.000)	>	0.630(0.004)	>	0.626(0.006)	0.004(0.001)	<	0.007(0.005)	<	0.020(0.007)
	0.946(0.000)	>	0.944(0.001)	>	0.938(0.001)	0.058(0.001)	<	0.067(0.002)	<	0.132(0.096)
	0.998(0.001)	=	0.998(0.002)	>	0.995(0.002)	0.383(0.036)	<	0.431(0.052)	<	0.573(0.067)
	1.000(0.000)	=	1.000(0.000)	>	0.999(0.001)	1.227(0.093)	<	1.377(0.127)	<	1.626(0.230)
WFG3	0.585(0.001)	>	0.583(0.003)	>	0.584(0.001)	0.003(0.001)	<	0.007(0.006)	=	0.005(0.002)
	0.423(0.001)	>	0.416(0.004)	>	0.412(0.005)	0.005(0.005)	<	0.019(0.008)	<	0.033(0.014)
	0.284(0.001)	>	0.276(0.004)	>	0.264(0.012)	0.032(0.006)	<	0.052(0.011)	<	0.090(0.036)
	0.174(0.001)	>	0.169(0.008)	>	0.137(0.015)	0.102(0.012)	<	0.116(0.015)	<	0.199(0.030)
WFG4	0.349(0.003)	>	0.327(0.002)		0.331(0.003)	0.006(0.007)	<	0.056(0.008)		0.046(0.009)
	0.598(0.000)	>	0.548(0.004)	>	0.541(0.006)	0.040(0.002)	<	0.118(0.006)	<	0.152(0.008)
	0.861(0.001)	>	0.799(0.002)	>	0.762(0.012)	0.405(0.004)	<	0.458(0.005)	<	0.708(0.043)
	0.972(0.016)	>	0.866(0.009)	>	0.841(0.031)	2.986(0.447)	<	3.925(0.225)	<	4.260(0.310)
WFG5	0.316(0.000)	=	0.316(0.001)	=	0.315(0.000)	0.062(0.000)	<	0.063(0.002)	=	0.062(0.001)
	0.552(0.002)	>	0.547(0.005)	>	0.537(0.004)	0.094(0.003)	<	0.104(0.010)	<	0.143(0.010)
	0.790(0.001)	>	0.767(0.007)	>	0.745(0.015)	0.478(0.005)	<	0.510(0.011)	<	0.742(0.045)
	0.857(0.004)	>	0.820(0.010)	>	0.790(0.037)	3.727(0.023)	<	3.911(0.137)	<	4.301(0.350)
WFG6	0.323(0.002)	=	0.326(0.004)	=	0.326(0.004)	0.049(0.004)	=	0.045(0.007)	=	0.044(0.007)
	0.560(0.005)	=	0.559(0.011)	=	0.553(0.006)	0.093(0.014)	=	0.089(0.024)	<	0.119(0.016)
	0.802(0.012)	>	0.789(0.025)	>	0.759(0.023)	0.486(0.040)	<	0.539(0.072)	<	0.742(0.045)
	0.839(0.014)	=	0.837(0.019)	>	0.801(0.030)	3.917(0.220)	=	3.968(0.256)	<	4.540(0.331)
WFG7	0.349(0.001)	=	0.351(0.002)	=	0.350(0.000)	0.006(0.002)		0.001(0.001)	<	0.004(0.000)
	0.598(0.001)	=	0.599(0.001)	>	0.586(0.001)	0.040(0.004)		0.033(0.000)	<	0.095(0.004)
	0.857(0.008)		0.859(0.004)	>	0.817(0.011)	0.434(0.046)	=	0.428(0.023)	<	0.710(0.040)
	0.937(0.010)	>	0.925(0.013)	>	0.905(0.016)	3.604(0.173)	<	3.755(0.254)	=	3.951(0.296)
WFG8	0.317(0.002)	>	0.308(0.003)	>	0.304(0.003)	0.060(0.004)	<	0.075(0.005)	<	0.089(0.004)
	0.490(0.002)	>	0.479(0.006)	>	0.471(0.004)	0.227(0.007)	<	0.245(0.012)	<	0.277(0.009)
	0.722(0.005)	>	0.700(0.015)	>	0.659(0.018)	0.723(0.021)	<	0.799(0.051)	<	0.950(0.045)
	0.812(0.014)	>	0.783(0.023)	>	0.697(0.021)	4.213(0.088)	<	4.663(0.326)	<	5.040(0.279)

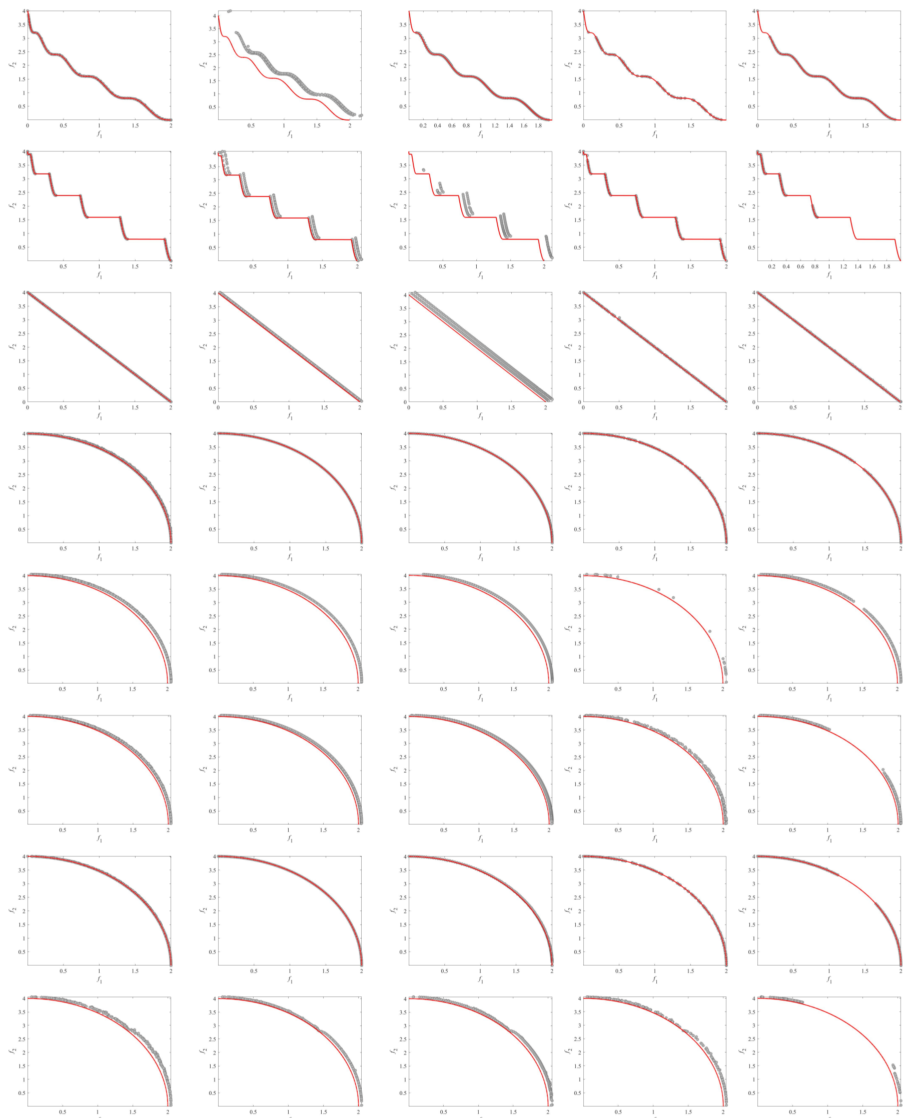
The performance of Curious II has been investigated on 32 test instances Empirical results show that Curious II outperforms 6 representative EMO algorithms (NSGA-III, MOEA/D, BiGE, KnEA, SAN and Curious I) on the indicators of HV and IGD. Curious II outperforms all the others in 26 out of 32 tasks for HV index and 24 out of 32 tasks for IGD index.

The distinct algorithm features is demonstrated by empirical comparisons:

(1) The IGSF of Curious II can be used as a general framework for a class of multi-objective optimization algorithms. Under IGSF, the cooperative interaction of multiple subpopulations can form an alternative to the evolution of a single population, so that the multi-objective function is decomposed. At the same time, in this framework, the jDE method, which has dominant individual memory and weighted dominance in the previous generation, works as a genetic variation algorithm. This framework improves the evolutionary efficiency of subpopulations and the developmental synchronization of dominant individuals.

(2) The MNS in stage 2 uses two kinds of novelty distance measurements. Since many points in search space collapse into the same point in behavior space, the distance based on Euclidean distance can preserve more sparse points

in the behavior space to prevent premature convergence. The distance based on prediction error can further dynamically reflect the occupied objective spaces by judging which solution is suspicious. Therefore, MNS more strongly guides each generation of individuals toward a more innovative (and more contributing) space in direction, significantly increasing the diversity of solutions.



(a) Curious II (b) NSGA-III (c) MOEA/D (d) BiGE (e) KnEA

Fig. 3: The performance of part of EMO algorithms on two-objective optimization WFG problems.

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