

Algo HW 9  
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7.3) If we let  $m_1$ ,  $m_2$ , and  $m_3$  represent the given volumes of the materials given, then the linear program is:

$$\begin{aligned} &\text{Maximize } 1000m_1 + 1200m_2 + 1200m_3 \\ &2m_1 + m_2 + 3m_3 \leq 100 \\ &m_1 + m_2 + m_3 \leq 60 \\ &m_1 \leq 40 \\ &m_2 \leq 30 \\ &m_3 \leq 20 \\ &m_1, m_2, m_3 \geq 0 \end{aligned}$$

7.7)

- a) This linear program will always be feasible unless you add an additional constraint because at (0,0) it is always feasible
- b) If and only if  $a \leq 0$  or  $b \leq 0$  then it will be unbounded
- c) A finite and unique optimal solution only exists when  $a \neq b$  and both  $a > 0$  and  $b > 0$

7.12)

$$\begin{aligned} &\text{Maximize } x_1 - 2x_3 \text{ with bounds} \\ &x_1 - x_2 \leq 1 \\ &2x_2 - x_3 \leq 1 \\ &x_1, x_2, x_3 \geq 0 \end{aligned}$$

To maximize,  $x_3$  would have to have a value of 0 which is its minimum. This means  $2x_2 \leq 1$ . We maximize  $x_1$  so  $x_1 - x_2 \leq 1$ , this means  $x_2$  must be  $\frac{1}{2}$  and  $x_3$  must be  $\frac{3}{2}$ . Because all of these values are given in the problem, we have proven that  $(\frac{3}{2}, \frac{1}{2}, 0)$  is the optimal solution.

7.13)

- a) The payoff matrix:

H	-1	1
T	1	-1

- b) The value of the game is equal to 0

$$\begin{aligned} &-y_H + y_T \leq v \\ &y_T - y_H \leq v \\ &y_T + y_H = 1 \\ &y_T, y_H \geq 0 \end{aligned}$$

It is optimal at  $y_T = y_H = 0.5$ . Because of this both of the players should play with a probability of 0.5 for both heads and tails. This makes sure that the expected payoff for both players is 0 and there is no way to increase the odds by using another strategy.