1) Proof by contradiction:

Lets say $G(V, E)$ is an undirected graph. It has unique weights. Lets define T_1 and T_2 as two distinct MSTs for the graph G. E1 is the edge with the lowest weight in T_1 but not T_2. E2 is ~~the~~ edge with the lowest weight in T_2 but not T_1. Due to the edge weights being unique, without loss of generality we can say $w(e_1) < w(e_2)$. If we add e_1 into T_2 it creates a cycle. then if you remove e_2 the cycle would be broken. This now gives you a spanning tree: $T = T_2 \cup \{e_1\} \setminus \{e_2\}$. Now the spanning tree, T, has a lower weight than T_2 which is a contradiction. Because T_2 was already an MST, this is wrong. This proves, given an undirected graph with unique weights there exists a unique MST.

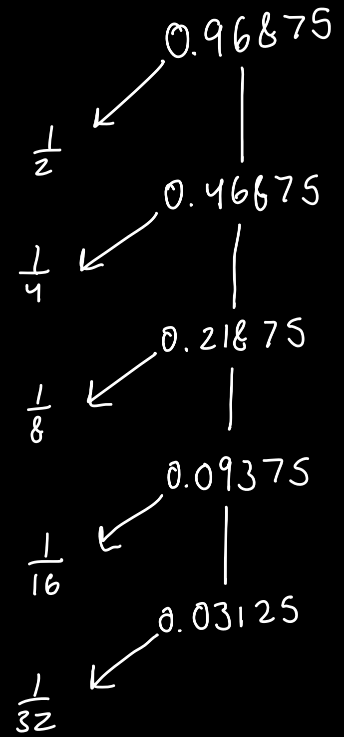
2) The method described is correct and called Kruskal's algorithm. It is correct because the algorithm will always choose the edge with the lowest weight that has 2 connections to components. Then, if the edge is not a part of the MST, it will be added to it and will not cause a cycle to be formed. On the other hand, if it is already part of the MST, it will be skipped. By doing this the algorithm creates a MST and has a runtime of $O(E \log E)$ where E is the number of edges in the graph.

3) a) show the frequencies of all chars sum to 1

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots = \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n = \frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

b) show what the huffman encoding is for each char

freq$\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}\right)$ $n=5$



c) What is the number of bits per char?

From the tree constructed above we find the expected number of bits to be $\log_2 n$