Topic

[Euler's Path & Circuit]

CSE - 4th Semester

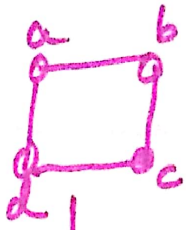
Eulerian path and Circuit

In Euler's path and Euler's Circuit each and every edge must be traversed (visit) exactly once but the basic difference between these two will be:

- In Euler's path initial and final vertex will be different because path is an open figure which starts and terminates at different vertices like 

(starts from vertex a and end with vertex e So both are different)

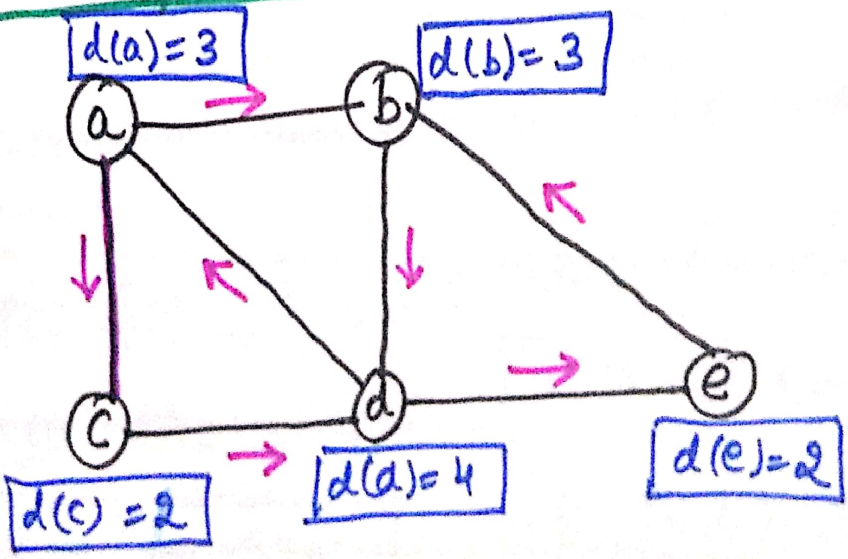
- In Euler's circuit initial and final vertex will be same because circuit is a closed figure and in closed figure (loop) initial and final vertex is same like



[This starts from a vertex and end to vertex a So initial and final vertex is same].

// Some examples of Euler's Path

*1st example of Euler's Path



graph-(1)

There is a condition to check any graph is having Euler's Path → "Two vertices will be of degree odd".

So in graph-(1)

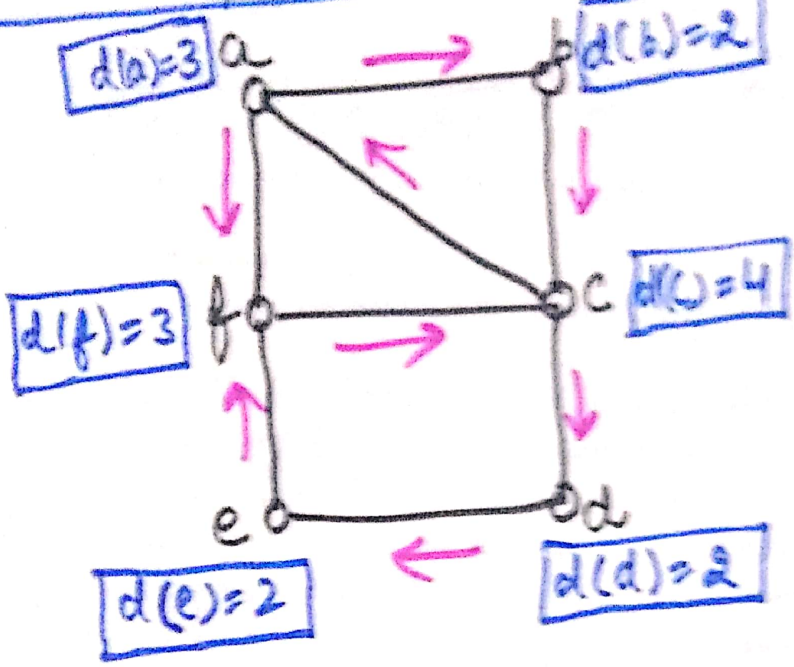
Two vertices a and b are of degree odd so this graph contains "Euler's Path"

Now Euler's Path will be

a → c → d → a → b → d → e → b

As you can see in this Euler's Path all the edges traversed exactly once (vertex can be repeated) and very important** Initial(a) and final vertex(b) are different.

2nd Example of Euler's Path



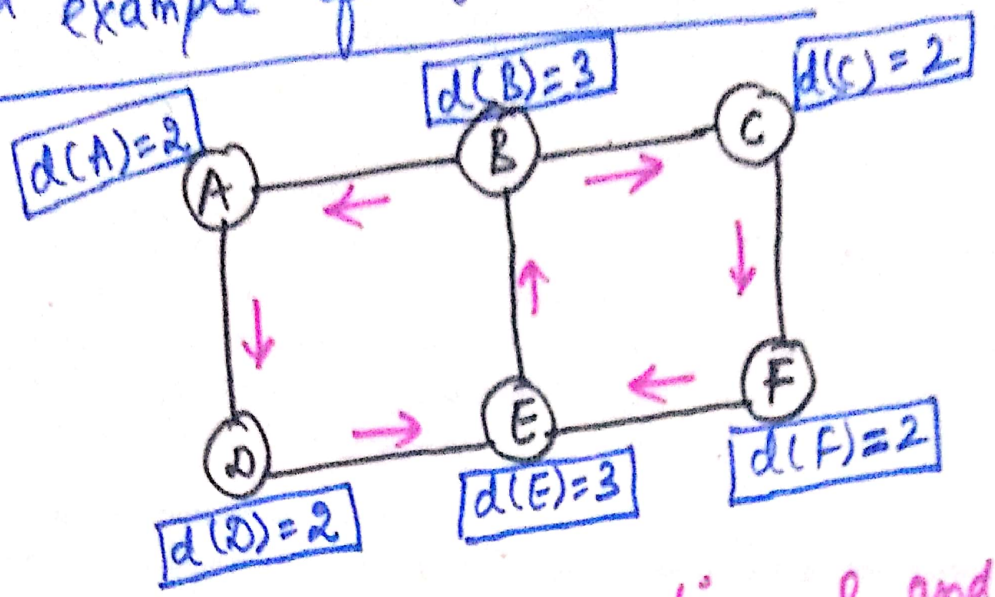
Now two vertices a and f are of degree odd So this graph contains "Euler's path"

Euler's Path of this graph will be

a → b → c → d → e → f → c → a → f

- all edges are traversed exactly once
- vertex can be repeated
- initial vertex (a) and final vertex (f) are different. [According to Path it must be different]

3rd example of Euler's Path



In this graph two vertices B and E are of degree odd So this graph contains "Euler's Path".

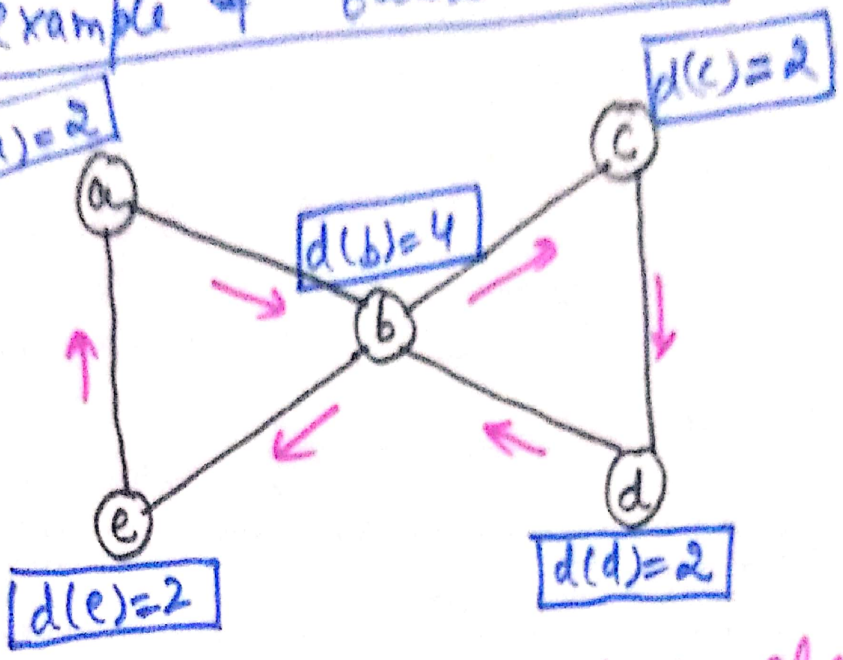
Euler's Path of this graph will be

B → C → F → E → B → A → D → E

- all edges are traversed exactly once
- vertex can be repeated
- initial vertex (B) and final vertex (E) are different [According to path it must be different]

// Some examples of Euler's Circuit

1st example of Euler's Circuit



There is a "condition" to check graph contains "Euler's circuit"

"all vertices" will be of degree "even"

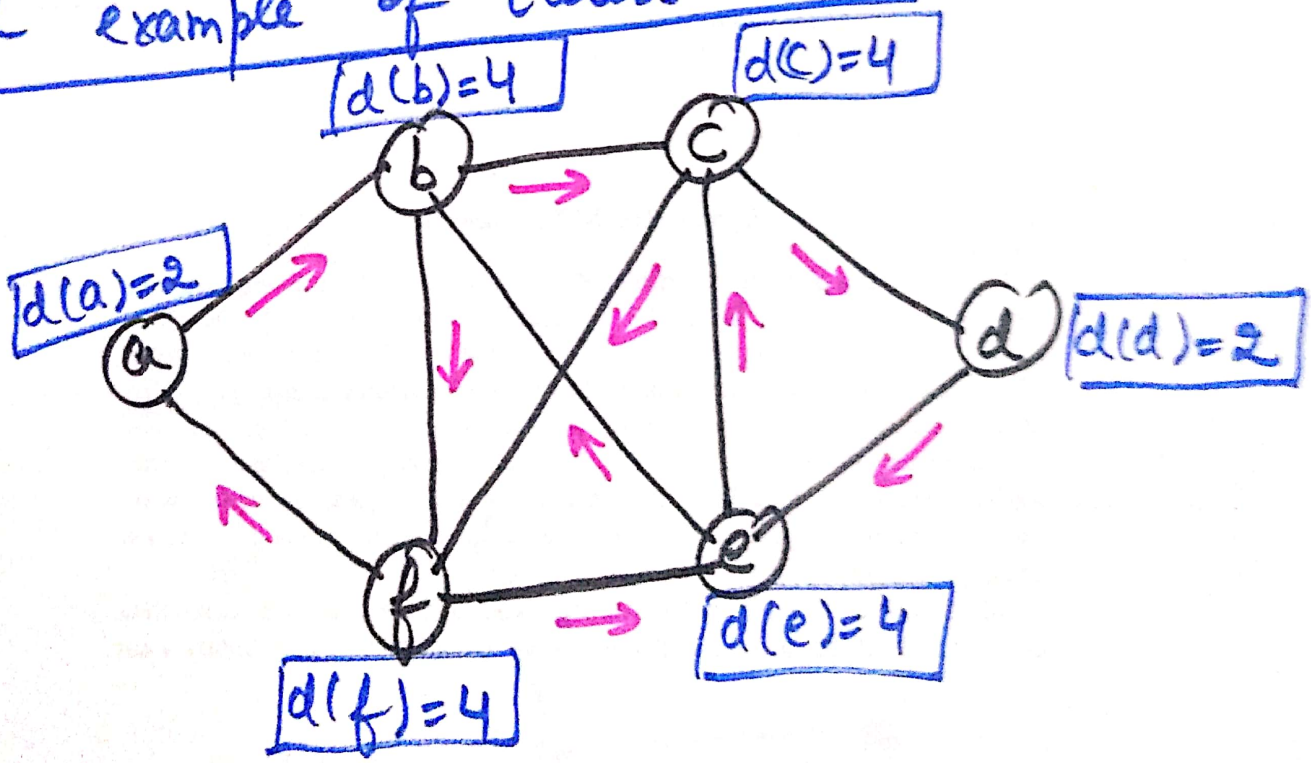
Now in above graph all vertices are of even degree so this graph contains "Euler's circuit"

Euler's Circuit will be

$a \to b \to c \to d \to b \to e \to a$

- All edges traversed exactly once.
- vertex can be repeated
- Initial and final vertex is same and (a) (a) according to circuit it must be same

* 2nd example of Euler's Circuit



So, All vertices are having even degree
So this graph contains Euler's Circuit

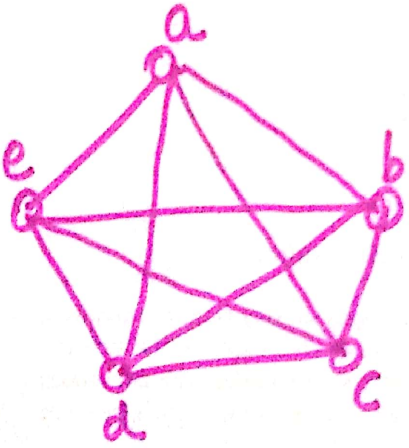
Eulers circuit of this graph will be

a → b → c → d → e → c → f → e → b → a

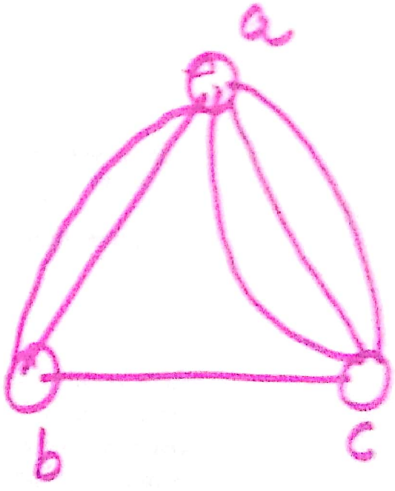
- all edges traversed exactly once.
- vertex can be repeated
- Initial vertex (a) and final vertex (a) is same and according to circuit it must be same.

Exercise

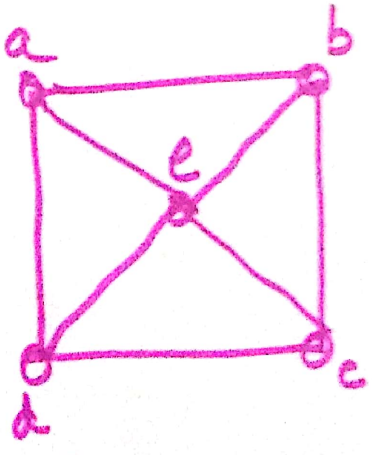
(a)



(b)



(c)



***

Any graph G which contains a Euler's circuit is called Euler's graph