Properties of fuzzy Set.

1) Commutativity :
$$\tilde{A} \cup \tilde{B} = \tilde{B} \cup \tilde{A}$$
$$\tilde{A} \cap \tilde{B} = \tilde{B} \cap \tilde{A}$$

2.) Associative:
i) $\tilde{A} \cup (\tilde{B} \cup \tilde{C}) = (\tilde{A} \cup \tilde{B}) \cup \tilde{C}$

ii) $\tilde{A} \cap (\tilde{B} \cap \tilde{C}) = (\tilde{A} \cap \tilde{B}) \cap \tilde{C}$

3.) Distributive:
i) $\tilde{A} \cup (\tilde{B} \cap \tilde{C}) = (\tilde{A} \cup \tilde{B}) \cap (\tilde{A} \cup \tilde{C})$

ii) $\tilde{A} \cap (\tilde{B} \cup \tilde{C}) = (\tilde{A} \cap \tilde{B}) \cup (\tilde{A} \cap \tilde{C})$

4) Idempotence:
$$\tilde{A} \cup \tilde{A} = \tilde{A}$$
$$\tilde{A} \cap \tilde{A} = \tilde{A}$$

5.) Identity :
i) $\tilde{A} \cup \phi = \tilde{A}$ iii) $\tilde{A} \cap \phi = \phi$

ii) $\tilde{A} \cap X = \tilde{A}$ iv) $\tilde{A} \cup X = X$

6.) Transitivity : If $\tilde{A} \subseteq \tilde{B} \subseteq \tilde{C}$, then $\tilde{A} \subseteq \tilde{C}$

7) Involution : $(\tilde{A}^c)^c = \tilde{A}$

8.) De-Morgan's law :
i) $(\tilde{A} \cap \tilde{B})^c = (\tilde{A}^c \cup \tilde{B}^c)$

ii) $(\tilde{A} \cup \tilde{B})^c = (\tilde{A}^c \cap \tilde{B}^c)$

Que: The task is to recognize Englisly alphabe-tical characters (F, E, X, Y, I, T) in an Image Processing system.

Define two fuzzy sets $\tilde{T}$ and $\tilde{F}$ to represent the identification of characters I and $\bar{F}$

$$\tilde{T} = \{ (F, 0.4), (E, 0.3), (X, 0.1), (I, 0.9), (T, 0.8) \}$$
$$\underline{(Y, 0)}$$

$$\tilde{F} = \{ (E, 0.19), (E, 0.8), (X, 0.1), (Y, 0.2), (I, 0.3), (T, 0.5) \}$$

Find the following :

a) $\tilde{T} \cup \tilde{F}$

b) $\tilde{T} - \tilde{F}$

c) $\tilde{F} \cup \tilde{F}^c$

d) Verify De-Morgan's Low

$$(\tilde{T} \cup \tilde{F})^c = \tilde{T}^c \cap \tilde{F}^c$$

Sol^n :

$$\tilde{T} = \{(F, 0.4), (E, 0.3), (X, 0.1), (1, 0.9), (T, 0.8)\}$$

$$\tilde{F} = \{(F, 0.19), (E, 0.8), (X, 0.1), (Y, 0.2), (1, 0.3), (T, 0.5)\}$$

i) $\tilde{T} \cup \tilde{F}$

$$\mu_{\tilde{T} \cup \tilde{F}} = \max(\mu_Y(x), \mu_{\tilde{T}}(x))$$

$$= \{(F, 0.4), (E, 0.8), (X, 0.1), (1, 0.9), (Y, 0.2), (T, 0.8)\}$$

ii) $\tilde{T} - \tilde{F}$

$$\mu_{\tilde{T} - \tilde{F}} = \mu(1 \cap F^c)$$

$$= \{(F, 0.4), (E, 0.2), (X, 0.1), (1, 0.7), (T, 0.5)\}$$

$$F^c = \{(F, 0.81), (E, 0.2), (X, 0.9), (Y, 0.8), (1, 0.7), (T, 0.5)\}$$

$$\boxed{\mu \cap F^c = \min(\mu_Y(x), \mu_{\tilde{F^c}}(x)}$$

c) $\tilde{F} \cup \tilde{F}^c$

$$= \{(F, 0.81), (E, 0.8), (X, 0.9),$$
$$(Y, 0.8), (1, 0.7), (T, 0.5)\}$$

d) $(T \cup \tilde{F})^c = T^c \cap \tilde{F}^c$

$\underline{(T \cup \tilde{F})^c}$

$$\{(F, 0.6), (E, 0.2), (X, 0.9),$$
$$(Y, 0.8), (1, 0.1), (T, 0.2)\}$$

$$\tilde{T}^c = \{(F, 0.6), (E, 0.7), (X, 0.9), (Y, 1),$$
$$(1, 0.1), (T, 0.2)\}$$

$$\tilde{F}^c = \{(F, 0.81), (E, 0.2), (X, 0.9), (Y, 0.8),$$
$$(1, 0.7), (T, 0.5)\}$$

$$\underline{\tilde{T}^c \cap \tilde{F}^c} = \{(F, 0.6), (E, 0.2), (X, 0.9),$$
$$(Y, 0.8), (1, 0.1), (T, 0.2)\}$$

Que: Consider the fuzzy sets $\tilde{A}$ and $\tilde{B}$ defined on the interval $X = [0, 5]$ of real numbers, by the membership grade functions

$$\mu_{\tilde{A}}(x) = \frac{x}{x+1} \quad, \quad \mu_{\tilde{B}}(x) = 2^{-x}$$

Determine the mathematical formulae and graphs of the membership grade function of each of the following sets

a) $\tilde{A}^C$, $\tilde{B}^C$

b) $\tilde{A} \cup \tilde{B}$

c) $\tilde{A} \cap \tilde{B}$

d) $(\tilde{A} \cup \tilde{B})^C$