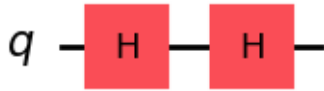





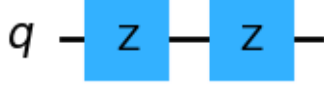


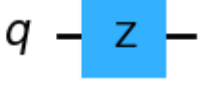
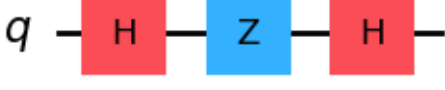



Catalogue of Equivalent Quantum Circuits



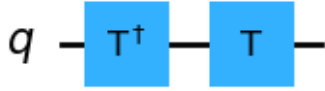

This document contains a list of equivalent quantum circuits used by QSimplify as simplification rules, compiled from various sources [1] [2] [3].

1, Single-qubit equivalences

Tabla 1: Single-qubit circuits and their simplified versions.

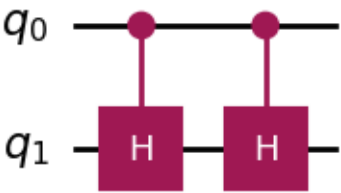
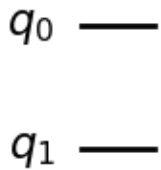
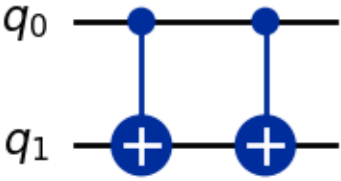
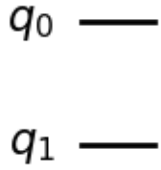
Circuit	Equivalent version (simplified)
	
	
	
	
	
	

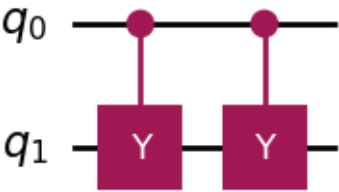
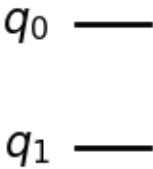
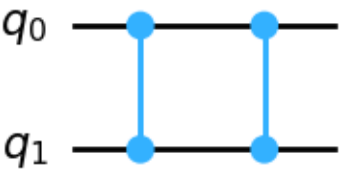
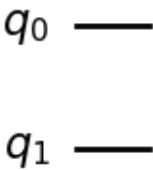
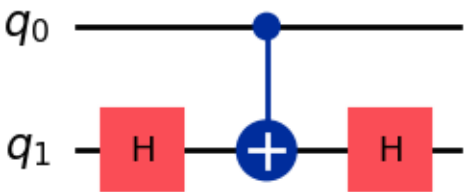
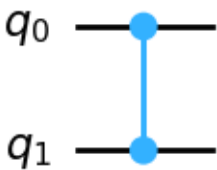
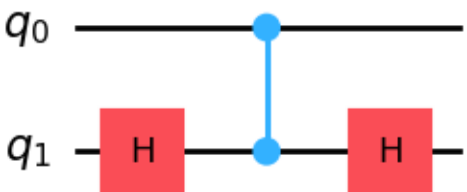
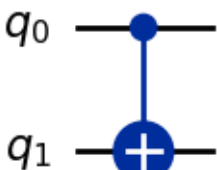
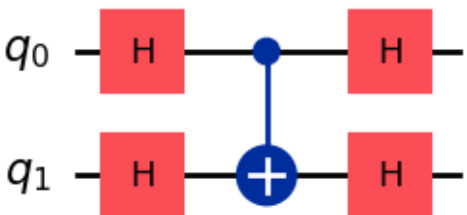
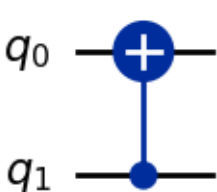
Circuit	Equivalent version (simplified)
$q - \boxed{s} - \boxed{s} -$	$q - \boxed{z} -$
$q - \boxed{\gamma} - \boxed{s} - \boxed{x} -$	$q - \boxed{s} -$
$q - \boxed{s} - \boxed{s^\dagger} -$	$q -$
$q - \boxed{s^\dagger} - \boxed{s} -$	$q -$
$q - \boxed{s} - \boxed{s} - \boxed{s} -$	$q - \boxed{s^\dagger} -$
$q - \boxed{s^\dagger} - \boxed{s^\dagger} - \boxed{s^\dagger} -$	$q - \boxed{s} -$
$q - \boxed{\tau} - \boxed{\tau} -$	$q - \boxed{s} -$

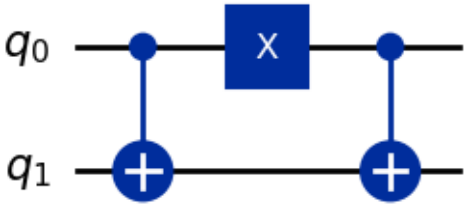
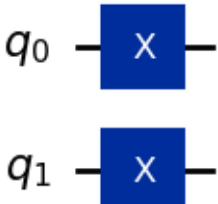
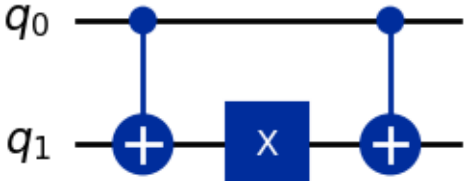
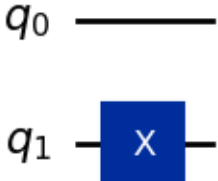
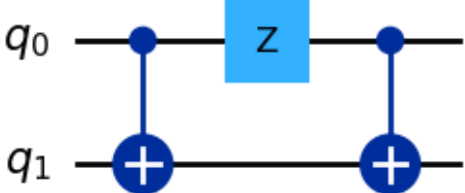
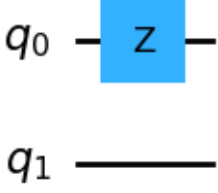
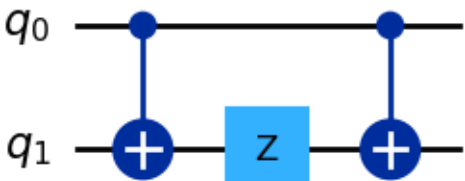
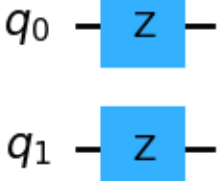
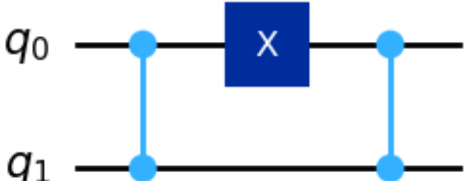
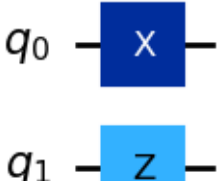
Circuit	Equivalent version (simplified)
	
	

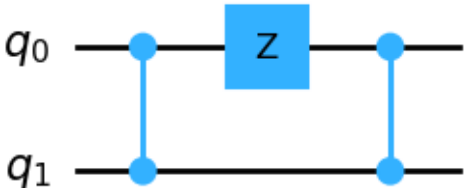
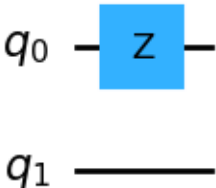
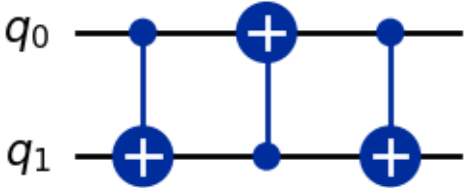
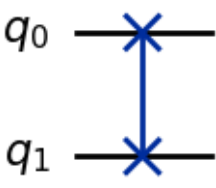
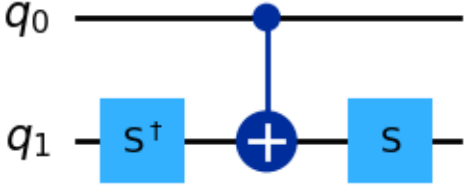
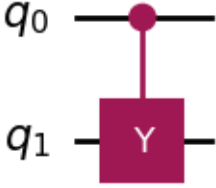
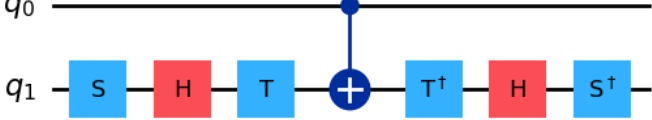
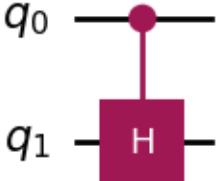
2. Two-qubit equivalences

Tabla 2: Two-qubit circuits and their simplified versions.

Circuit	Equivalent version (simplified)
	
	

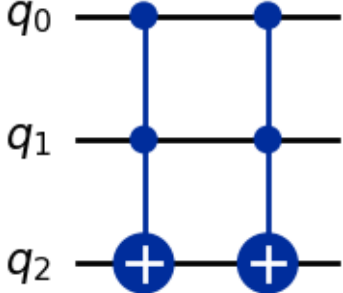
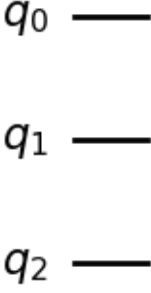
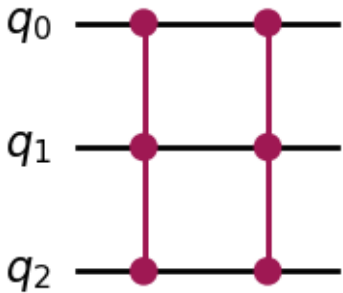
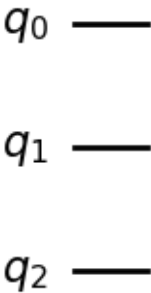
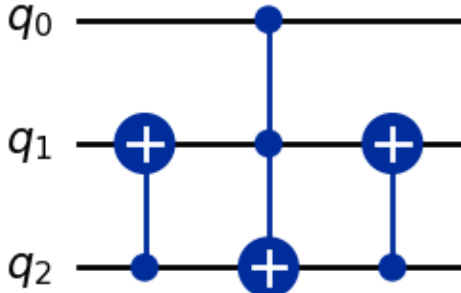
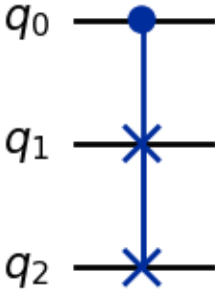
Circuit	Equivalent version (simplified)
 <p>Quantum circuit with two qubits, q_0 and q_1. q_0 has two control points connected by vertical lines to two Y gates on q_1.</p>	 <p>Simplified equivalent circuit showing two independent horizontal lines for q_0 and q_1.</p>
 <p>Quantum circuit with two qubits, q_0 and q_1. q_0 has two control points connected by vertical lines to two CNOT gates on q_1.</p>	 <p>Simplified equivalent circuit showing two independent horizontal lines for q_0 and q_1.</p>
 <p>Quantum circuit with two qubits, q_0 and q_1. q_1 has two H gates, and q_0 has a control point connected by a vertical line to a CNOT gate on q_1.</p>	 <p>Simplified equivalent circuit showing a CNOT gate with q_0 as control and q_1 as target.</p>
 <p>Quantum circuit with two qubits, q_0 and q_1. q_1 has two H gates, and q_0 has a control point connected by a vertical line to a CNOT gate on q_1.</p>	 <p>Simplified equivalent circuit showing a CNOT gate with q_1 as control and q_0 as target.</p>
 <p>Quantum circuit with two qubits, q_0 and q_1. q_1 has two H gates and a CNOT gate with q_0 as control. q_0 has H gates before and after the CNOT.</p>	 <p>Simplified equivalent circuit showing a CNOT gate with q_0 as control and q_1 as target.</p>

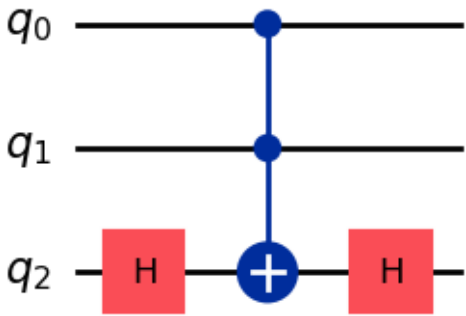
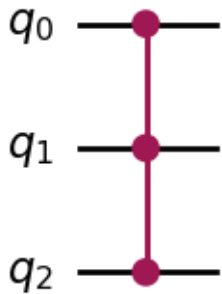
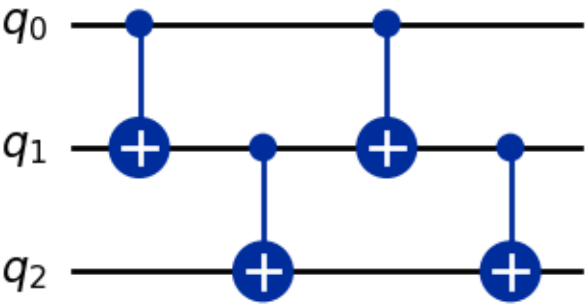
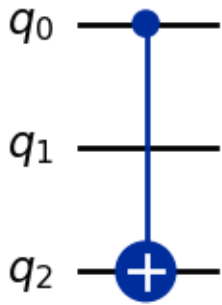
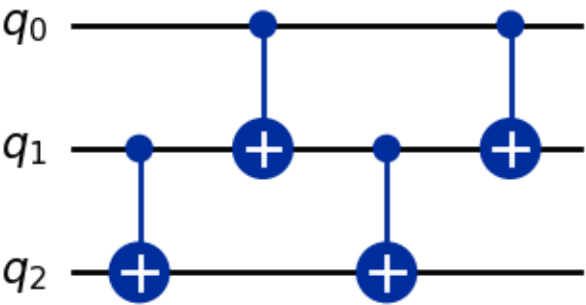
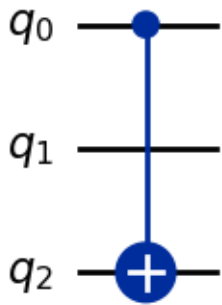
Circuit	Equivalent version (simplified)
	
	
	
	
	

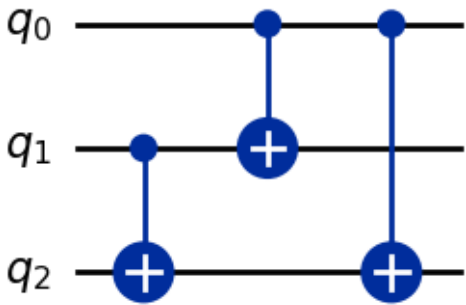
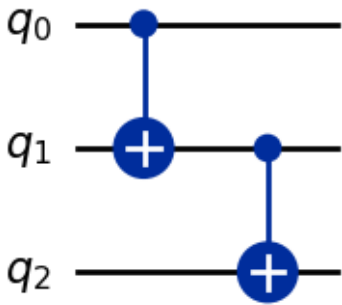
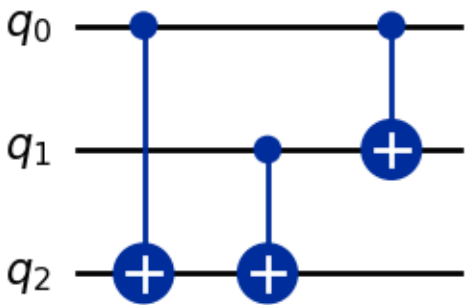
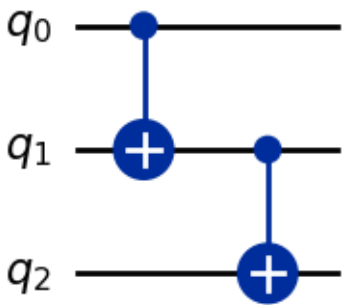
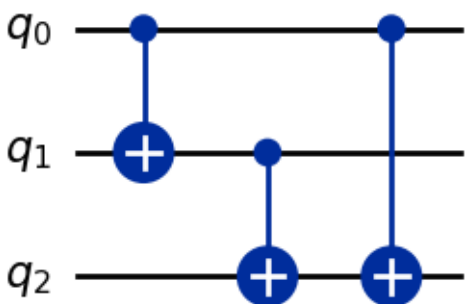
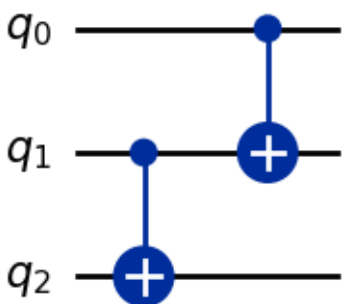
Circuit	Equivalent version (simplified)
	
	
	
	

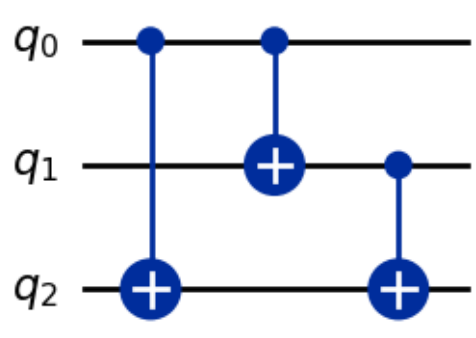
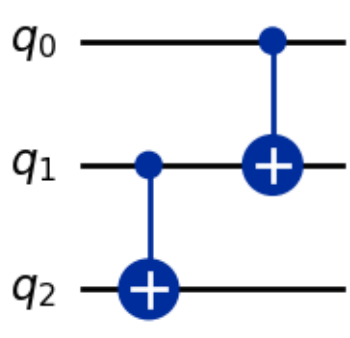
3. Three-qubit equivalences

Tabla 3: Three-qubit circuits and their simplified versions.

Circuit	Equivalent version (simplified)
 <p>Quantum circuit for three qubits q_0, q_1, and q_2. The circuit consists of two CNOT gates between q_0 and q_1. Additionally, q_2 has two CNOT gates with targets on q_0 and q_1, each preceded by a Hadamard gate.</p>	 <p>Simplified version: Three horizontal lines representing qubits q_0, q_1, and q_2 with no gates.</p>
 <p>Quantum circuit for three qubits q_0, q_1, and q_2. The circuit consists of two CNOT gates between q_0 and q_1. Additionally, q_2 has two CNOT gates with targets on q_0 and q_1.</p>	 <p>Simplified version: Three horizontal lines representing qubits q_0, q_1, and q_2 with no gates.</p>
 <p>Quantum circuit for three qubits q_0, q_1, and q_2. The circuit consists of a CNOT gate from q_0 to q_1. Additionally, q_1 has two CNOT gates with targets on q_2, each preceded by a Hadamard gate. Finally, q_2 has a CNOT gate with target on q_1.</p>	 <p>Simplified version: Three horizontal lines representing qubits q_0, q_1, and q_2. A vertical line with a dot on q_0 and crosses on q_1 and q_2 connects them.</p>

Circuit	Equivalent version (simplified)
 <p>Quantum circuit for three qubits q_0, q_1, and q_2. q_0 and q_1 have control dots connected by a vertical line to a CNOT gate on q_2. q_2 has H gates before and after the CNOT.</p>	 <p>Simplified equivalent circuit for the first circuit, showing a vertical line connecting control dots on q_0, q_1, and q_2.</p>
 <p>Quantum circuit for three qubits q_0, q_1, and q_2. q_0 has control dots connected to CNOT gates on q_1 and q_2. q_1 has a control dot connected to a CNOT gate on q_2. q_1 and q_2 have CNOT gates on q_1.</p>	 <p>Simplified equivalent circuit for the second circuit, showing a vertical line connecting control dots on q_0 and q_1 to a CNOT gate on q_2.</p>
 <p>Quantum circuit for three qubits q_0, q_1, and q_2. q_0 has control dots connected to CNOT gates on q_1 and q_2. q_1 has a control dot connected to a CNOT gate on q_2. q_1 and q_2 have CNOT gates on q_1.</p>	 <p>Simplified equivalent circuit for the third circuit, showing a vertical line connecting control dots on q_0 and q_1 to a CNOT gate on q_2.</p>

Circuit	Equivalent version (simplified)
	
	
	

Circuit	Equivalent version (simplified)
 <p>Quantum circuit diagram with three qubits q_0, q_1, and q_2. q_0 has control dots for CNOT gates on q_1 and q_2. q_1 has a target circle with a plus sign for a CNOT gate on q_2. The gates are arranged in two stages: first q_0 controls q_1 and q_2, then q_1 controls q_2.</p>	 <p>Equivalent simplified quantum circuit diagram with three qubits q_0, q_1, and q_2. q_0 has a control dot for a CNOT gate on q_1. q_1 has a target circle with a plus sign for a CNOT gate on q_2. The gates are arranged in two stages: first q_0 controls q_1, then q_1 controls q_2.</p>

1. References

- [1] C. Lomont, «Quantum Circuit Identities,» *ArXiv preprint*, 16 de jul. de 2003. arXiv: `quant-ph/0307111`. dirección: <https://arxiv.org/abs/quant-ph/0307111>.
- [2] J. C. Garcia-Escartin y P. Chamorro-Posada, «Equivalent Quantum Circuits,» *ArXiv preprint*, 13 de oct. de 2011. arXiv: `1110.2998`. dirección: <https://arxiv.org/abs/1110.2998>.
- [3] G. E. Crooks. «Gates, states, and circuits.» (2 de mar. de 2024), dirección: https://threeplusone.com/pubs/on_gates.pdf.