Angle Chasing, Congruence, Similarity and Parallelism

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Essence of Olympiad Geometry - Day 1

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Diagrams aren't provided for your "drawing" practice (and our convenience :D). Do try the problems that were not shown in class.

1 Class Problems

Problem 1.1. In $\triangle ABC$, It is given that $AB = AD$ and $\angle ABC - \angle ACB = 30°$. Find the value of $\angle CBD$.

Problem 1.2. In a trapezium $ABCD$ where $AD \parallel BC$, E is a point of AB. Show that $\angle ADE + \angle BCD = \angle CED$.

Problem 1.3. In an acute triangle ABC, M, N and L are the midpoints of AB, AC and BC respectively. Points P and Q are chosen **outside the triangle** such that, $PM = MB$ and $PM \perp AB$, and $QN = NC$ and $QN \perp BC$. Prove that $LP = LQ$.

Problem 1.4. In a right triangle $\triangle ABC$, $\angle BAC = 90°$. D is a point of BC such that $AD \perp BC$ and M is the midpoint of BC. It is given that $\angle MAD = 14°$. Find the value of $\angle ABC$.

Problem 1.5. Suppose, there is a triangle $\triangle ABC$. The angle bisector of each angle intersects at a single point I. The line through I that is parallel to AB intersects AC and BC at points M and N, respectively. Prove that $AM + BN = MN$.

Problem 1.6. Let $ABCD$ be a convex quadrilateral such that $\angle BCD = 90°$. P is a point CD such that $\angle APD = \angle BPC$ and $\angle BAP = \angle ABC$. Prove that $BC = \frac{AP+BP}{2}$.

2 Class Problems that got left out

Problem 2.1. Let $ABCD$ be a parallelogram with area 1. Let M be a the midpoint of side AD. Let $BM \cap AC = P$. Find the area of $\square MPCD$.

Problem 2.2. Let ABC be a acute triangle with circumcenter O. H is point on the altitude drawn from A onto BC. Prove that $\angle BAH = \angle CAO$.

3 Homework

Problem 3.1. [**1 point**] $\triangle ABC$ is a equilateral triangle. D and E are points on AB and BC respectively such that $AD = BE$. AE and CD intersect each other at point F. What is the value of $\angle CFE$?

Problem 3.2. [**2 points**] Suppose there is a triangle $\triangle ABC$. We draw a square $\square ABDE$ on AB and another square $\square CAFG$ on AC such that both squares are **outside the triangle**. M is the midpoint of BC. AA' passes through M such that $AA' = 2AM$. Prove that $AA' = EF$.

Problem 3.3. [**2 points**] In a $\triangle ABC$, D is a point on the side BC. BE and CF are perpendicular to **the line** AD. It is also given that $\angle BAD = \angle CAD$. Prove that $AE \cdot DF = AF \cdot DE$.

Problem 3.4. [**4 points**] Let ω be the circumcircle of a triangle ABC with $AC > AB$. Let X be a point on AC and Y be a point on the circle ω, such that $CX = CY = AB$. (The points A and Y lie on different sides of the line BC). The line XY intersects ω for the second time in point P. Show that $PB = PC$.