Circles

Sadia Islam Urme

Essence of Olympiad Geometry - Day 2
Dinajpur Math Club

September 30, 2024

Problems are not provided in order of appearance. Have fun :D - with regards from SIU.

1 Class Problems

Problem 1.1. Suppose CT is a chord in the circle U. Suppose PT is tangent to U at point T. The line PC intersects with U for the second time at point A. Prove that $\triangle PCT$ and $\triangle APT$ are simillar.

Problem 1.2. A circle (ABC) is given with center O, A line PQ is tangent to to this circle at point B. Prove that $\angle CBQ = \angle BAC$.

Problem 1.3. Suppose points A, B, C, D are concyclic. It is given that $AD = BC$. Prove that $AB \parallel CD$.

Problem 1.4. Suppose P is a point outside circle ω. PT_1 and PT_2 are two tangents. Prove that $PT_1 = PT_2$.

Problem 1.5. Let t be the tangent at point C to the circumcircle of triangle $\triangle ABC$. A line p parallel to t intersects BC and AC at points D and E, respectively. Prove that the points A, B, D and E are concyclic.

Problem 1.6. In a triangle $\triangle ABC$ let D be a point on BC such that AD is an angle bisector. Let E and F be points on the interior segments AC and AB, respectively, such that $\angle BFD = \angle BDA$ and $\angle CED = \angle CDA$. Prove that $EF \parallel BC$.

Problem 1.7. Two circles intersect at A and B. One of their common tangents touches the circles at P and Q. Prove that AB bisects the line segment PQ.

Problem 1.8. [**This got left out**] An acute-angled triangle $\triangle ABC$ is given. The circle with diameter AB intersects altitude CC' and its extensions at P and Q. Prove that the points M, N, P, Q are concyclic.

2 Homework

Problem 2.1. [**1 point**] Let P be a point of the side BC of $\triangle ABC$. The parallel line to AC through P intersects the tanget to (ABC) through A at a point Q. Prove $APBQ$ is cyclic.

Problem 2.2. [**2 points**] Let $\triangle ABC$ be a triangle with incenter I. Ray AI meets (ABC) again at L. Let I_A be the reflecction of I over L. Prove that, the points I, B, C and I_A are concyclic.

Problem 2.3. [**2 points**] Let $\triangle ABC$ be an acute angled triangle. Let BE and CF be altitudes of $\triangle ABC$, denote by M the midpoint of BC. Prove that ME and MF are both tangent to (AEF).

Problem 2.4. [**3 points**] Let $\triangle AKB$ be a triangle. D and C are points on segments AK and BK respectively such that $AB \parallel CD$. K' is a point on the segment AB. KK' intersects with CD at point L. P and P' are two points between K and K' such that $DP' \parallel AP$ and $CP' \parallel BP$. Here $\angle APB = \angle BCD$ and Q is a point on KK_1 such that $\angle DQC = \angle ABK$. Prove that $\square PCBQ$ is cyclic.