Favourites Pt.1

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Essence of Olympiad Geometry - Day 4
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1 SIU

Problem 1.1. In a quadrilateral $ABCD$, Suppose there exist two points E and F on BC such that $\angle EAF = \angle FDE$ and $\angle EAB = \angle FDC$. Prove that $ABCD$ must also be a cyclic quadrilateral.
Or, Show that for $ABCD$ to be cyclic, it is necessary and sufficient that there exist two points E and F on side BC such that $\angle EAF = \angle FDE$ and $\angle EAB = \angle FDC$.

Problem 1.2. In a triangle $\triangle ABC$, it is given that $AC = BC$. P is a point on the circumcircle of ABC between A and B such that it lies on the opposite site of C. D is the foot of the perpendicular from C to PB. Show that $BD + PA = PD$.

2 NRS

Problem 2.1. Suppose O is the circumcenter and H is the orthocenter of $\triangle ABC$. The extension of of AO intersects the circumcircle of ABC at point K. Prove that $A, K,$ and the midpoint M of side BC are collinear .

Problem 2.2. Use the notations from the previous problem. Prove that $AH = 2OM$.

Problem 2.3. Use the notations from the previous problem again. Prove that the midpoint of OH is the center of the Nine-point circle of $\triangle ABC$.

3 Bonus problems

Problem 3.1. $ABCD$ is a convex quadrilateral such that $\angle ABD = \angle DBC, AD = CD, AB \neq BC$. Prove that $ABCD$ is cyclic.
(A convex quadrilateral is a quadrilateral having all of its interior angles measuring less than $180°$)

Problem 3.2. In $\triangle ABC, \angle BAC$ is a right angle. BP and CQ are bisectors of $\angle B$ and $\angle C$ respectively, which intersect AC and AB at P and Q respectively. Two perpendicular segments PM and QN are drawn on BC from P and Q respectively. Find the value of $\angle MAN$ with proof.

Day 3 notes can be found here-