Dinajpur Math Club

Syllabus 2023

Final Revision

Preamble

1.1 What this is

This is a Syllabus, Topic List or even a loosely organized class plan for Dinajpur Math Club's 2023 operations.
The topic have been divided into three *ranks* (Gauss, Fermat and Euler) based on the members' general position in the academic ladder and even more specifically, their level of understanding of Olympiad Mathematics.

1.1.1 What this contains

This syllabus is sourced from multiple sources. Namely, for each rank,

- What academic knowledge is expected
- What mathematical knowledge is expected to secure a tough competitive attitude from BdMO regionals
- What knowledge fun (and important!) problems require in general

1.2 What this is *not*

This list is not *exhaustive.* This does not cover absolutely all required or important topics of competitive mathematics. **This list should not be followed strictly.** But rather, as a skeleton of what should be discussed. The topics mentioned are **not** organized by importance, dependency, or time either. Some parts of this list are left incomplete and are expected to be completed when we explore those topics in class.

1.3 Who made this?

We too, just like you- are members of this club! May the second differential of everyone's life be negative :)

Gauss

2.1 Introduction

Gauss focuses on Primary level students completely new to Olympiad mathematics.

2.2 Barebones of Problem Solving

Riddles! We put extra focus to develop a solver's attitude among the new intakes.

- We teach how to-
  1. Understand statements
  2. Approach a problem
  3. Come up with vague ideas and conjectures
  4. Find patterns
  5. Actually come up with a rigorous solution
- Brain Puzzles
  - The early classes begin with interesting problems, that don't need much mathematical knowledge, if at all. **Sourcing: BdMO Regional P1-P3**.
- Arithmetic Puzzles
  - The aforementioned puzzles should slowly turn into problems involving (some form of) mathematical intuition. Say, for example- The Census Taker problem. **Sourcing: BdMO Regional P1-P5**.

2.3 Number Theory

- Numbers
  1. Positive and negative integers
  2. Understanding of signed operations (i.e $-(-5) = 5$)
  3. Fractions, Rational and Irrational numbers
- Divisibility
  1. Integer Divisibility tricks (Divisibility criteria for 2, 3, 5 etc.)
  2. Parity
- Primes and Factors
  1. Introduction to primes
  2. Factorization of integers
  3. G.C.D and L.C.M

2.4 Counting

- Theory
  1. Addition and Multiplication principle
  2. Factorials (and why to use them)
  3. Arrangements
  4. C_r^n and P_r^n (Note: This is actually rather advanced)
- Problem Types
  1. Bruteforce solutions
  2. Counting puzzles
  3. Ways to choose n things from m things such that...
  4. Ways to arrange n people such that...
  5. Ways to construct a word with N letters such that...
  6. Ways to travel from $A_1 \to A_2 \to ... \to A_{n-1} \to A_n$
  7. Find number of n digit integers such that...
  8. Round-table versions of all arrangement problems
  9. Palindrome related problems (Basics)

2.5 Geometry

- Triangles
    1. Properties of Equilateral, Isosceles and Right triangles
    2. Theorems related to area and side lengths
    3. Congruence (SAS, SSS etc)
    4. Similarity and ratios
    5. Properties of angles of a triangle
    6. Pythagorean Theorem
- Quadrilaterals
    1. Properties of Squares, Parallelograms, Trapezoids etc
    2. Theorems related to area and diagonals
- Circles
    1. Theorems related to area and ratio of circles
    2. Properties of circumference, radius, segments, arcs etc.
- Basic Angle and Segment chasing
- Properties of different types of Polygons

2.6 Algebra

Note that class 6-7 Algebra suffices.

- Variables, Exponents, Algebraic manipulation etc.
- Solving basic equations
- Square and Cubic formulas (i.e $(a+b)^2 = a^2 + 2ab + b^2$ etc.)

Fermat

3.1 Introduction

Fermat focuses on students around advanced primary, beginner and intermediate junior level. Just like Gauss, we dedicate the first few classes to skim away from mathematics and focus on Problem Solving in general. (See *Barebones of Problem Solving*).

3.2 Number Theory

- Divisibility
    1. Integer Divisibility tricks (Divisibility criteria for 2, 3, 5 etc.)
    2. Basic Theorems (i.e $a \mid b, b \mid c \implies a \mid c$ etc.)
    3. Parity
- Primes and Factors
    1. Basic prime properties
    2. Fundamental Theorem of Arithmetic
    3. Number of divisors
- Advance usage of G.C.D and L.C.M
- Euclidean G.C.D algorithm
- Factorials (i.e Largest power of p that divides $n!$)
- Floor and ceiling functions
- Modular Arithmetic (This applies to Intermediate Juniors)
    1. Introduction to modular notation and residue classes
    2. Basic principles
    3. Last n digits problems
    4. Applying Modular tricks to equations [i.e $x^2 \equiv 0$ or 1 (mod 4)]

3.3 Counting

This section will mainly focus on listing common problem types. Note that this list is not exhaustive.

- Theory
    1. Addition and Multiplication principle
    2. Factorials (and why to use them)
    3. Subsets and Arrangements
    4. C_r^n and P_r^n
    5. Basic Bijection Principle
    6. Pigeonhole Principle
    7. Coming up with combinatorial arguments
- Problem Types
    1. Bruteforce solutions
    2. Counting puzzles
    3. Ways to choose n things from m things such that...
    4. Ways to arrange n people such that...
    5. Ways to construct a word with N letters such that...
    6. Ways to travel from $A_1 \to A_2 \to ... \to A_{n-1} \to A_n$
    7. Ways to distribute n things among m people such that... (Distribution problems)
    8. Find number of n digit integers such that...
    9. Find sum of the digits of an sequence $A_1A_2A_3...A_n$ such that...
    10. Find number of paths in a grid from A to B such that...
    11. Find number of positive solutions to the equation $ax + by = n$...
    12. Find number of diagonals of a polygon with n vertices such that...
    13. Find number of subsets of set S such that...
    14. Round-table versions of all arrangement problems
    15. Palindrome related problems
    16. Chessboard problems
    17. Game problems

3.4 Geometry

- Triangles
    1. Properties of Equilateral, Isosceles and Right triangles
    2. Theorems related to area and side lengths
    3. Congruence (SAS, SSS etc)
    4. Similarity and ratios
    5. Properties of angles of a triangle
    6. Pythagorean Theorem
    7. Cyclic triangles
    8. Circumcenter, Orthocenter, Incenter etc.
    9. Basic Trigonometry properties
- Quadrilaterals
    1. Properties of Squares, Parallelograms, Trapezoids etc
    2. Theorems related to area and diagonals
    3. Cyclic quadrilaterals
- Circles
    1. Theorems related to area and ratio of circles
    2. Properties of circumference, radius, segments, arcs etc.
    3. Sectors and angles inscribed in a circle
    4. Power of point
- Angle and Segment chasing
- Properties of Geometric figures (Polygons, 3D shapes etc.)

3.5 Algebra

Students are expected to have basic and practical knowledge about class 6, 7 and 8 algebra.
We can observe that in this level, olympiad algebra doesn't deviate much from textbook algebra, if at all.

- Pre-olympiad Algebra
    1. Variables, Exponents, Algebraic manipulation etc.
    2. Solving equations
    3. Square and Cubic formulas (i.e $(a+b)^2 = a^2 + 2ab + b^2$ etc.)
    4. Knowledge about the cartesian plane
    5. Basic set theory
    6. Correlating Algebra with Geometry
- Basics about inequalities
- Basics about functions
- Series and sequences
- Factorization

Euler

4.1 Introduction

Euler focuses on students around advanced junior, and secondary level. All topics from previous ranks apply. Please note that, these topics are expected to be studied in group study sessions. Euler doesn't have a regular mentor strategy.
Note: Some topics only apply to national level and further. Regional topics should be prioritized.

4.2 Number Theory

- Divisibility
    1. Integer Divisibility tricks (Divisibility criteria for 2, 3, 5 etc.)
    2. All well known theorems (i.e $a \mid b, b \mid c \implies a \mid c$ etc.)
    3. Parity
- Primes and Factors
    1. Well known Prime properties
    2. Fermat's Little Theorem (and prime-ness checks in general)
    3. Fermat numbers
    4. Coprimes and Euler's Totient Function
    5. Fundamental Theorem of Arithmetic
    6. Number of divisors
    7. Sum of divisors
- Advance usage of G.C.D and L.C.M
- Euclidean G.C.D algorithm
- Factorials (i.e Largest power of p that divides $n!$)

- Floor and ceiling functions
- Numerical systems (Binary, Ternary etc including generalizations)
- Modular Arithmetic
    1. Advanced usage of modular notation and residue classes
    2. Advanced principles
    3. Last n digits problems
    4. Quadratic residues and contradictions
- Diophantine Equations [Note: This section is incomplete]
    1. Introduction to Diophantine equations
    2. Application and proof techniques (i.e Modular Contradiction)
    3. Linear and non-linear Diophantine equations

4.3 Counting

For problem types, see Fermat.

- Addition and Multiplication principle
- Factorials (and why to use them)
- Subsets and Arrangements
- C_r^n and P_r^n
- Proof of various well known identities of the above
- Advanced application of Bijection Principle
- Pigeonhole Principle
- Extremal Principle and worst cases
- Inclusion-Exclusion Principle
- Counting related Graph Theory
- Fubini's Theorem
- Recurrence relations
- Two way counting
- Binomial Theorem
- Pascal's Triangle
- Probability
- Coming up with combinatorial arguments

4.4 Geometry

This section is incomplete and should be extended later on.

- Triangles
    1. Properties of Equilateral, Isosceles and Right triangles
    2. Theorems related to area and side lengths
    3. Congruence (SAS, SSS etc)
    4. Similarity and ratios
    5. Properties of angles of a triangle
    6. Pythagorean Theorem
    7. Cyclic triangles
    8. Circumcenter, Orthocenter, Incenter etc.
    9. Trigonometric properties
- Quadrilaterals
    1. Properties of Squares, Parallelograms, Trapezoids etc
    2. Theorems related to area and diagonals
    3. Cyclic quadrilaterals
- Circles
    1. Theorems related to area and ratio of circles
    2. Properties of circumference, radius, segments, arcs etc.
    3. Sectors and angles inscribed in a circle
    4. Power of point
- Angle and Segment chasing
- Properties of Geometric figures (Polygons, 3D shapes etc.)

4.5 Algebra

Students are expected to have basic and practical knowledge about class 9-10 algebra.

- Pre-olympiad Algebra
    1. Variables, Exponents, Algebraic manipulation etc.
    2. Solving equations
    3. Square and Cubic formulas (i.e $(a+b)^2 = a^2 + 2ab + b^2$ etc.)
    4. Difference of squares, and other well known formulas and proofs
    5. Knowledge about the Cartesian and Polar plane
    6. Set theory
    7. Correlating Algebra with Geometry
- Inequalities
- Functions
- Polynomials
- Series and sequences
- Quadratic equations

4.6 Proofs

This is a special section for Euler. Here we devise various proof techniques and principles important for BdMO nationals.

- Logic
- Understanding statements (If and only if, For all, There exists etc)
- Converse, Inverse and Contraposition
- Contradiction
- Induction
- Problems about existence
- Pigeonhole, Extremal and Inclusion-Exclusion principles