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* $[\![x]\!]^I_\rho = \rho(x)$

* $[\![f(t_1, \dots, t_2)]\!] = I(f)\left([\![t_1]\!]^I_\rho, \dots [\![t_n]\!]^I_\rho\right)$

F :

* $[\![P(t_1 \dots t_n)]\!]^I_\rho = I(P)\left([\![t_1]\!]^I_\rho, \dots, [\![t_n]\!]^I_\rho\right)$

* $[\![\top]\!]^I_\rho = T \in \mathbb{B}$ ← boolean

* $[\![\bot]\!]^I_\rho = F \in \mathbb{B}$

* $\cancel{[\![\neg]\!]^I_\rho}$

* $[\![\neg\phi]\!]^I_\rho = \neg_{\mathbb{B}} [\![\phi]\!]^I_\rho$

* $[\![\phi \wedge \phi']\!]^I_\rho = [\![\phi]\!]^I_\rho \wedge_{\mathbb{B}} [\![\phi']\!]^I_\rho$, etc. $(\vee, \Rightarrow, \Leftrightarrow)$

* $[\![\forall x . \phi]\!]^I_\rho = \bigwedge [\![\phi]\!]^I_{\rho[v/x]}$

* $[\![\exists x . \phi]\!]^I_\rho = \bigvee_{v \in D_I} [\![\phi]\!]^I_{\rho[v/x]}$

$\cancel{\exists}$ $\exists x \forall y.$

Exercice 1

A)

1) $\cancel{\exists x}$

$P(x)$ = x a réussi l'examen

$Q(x,y)$ = x a posé des questions à y

$\exists x . \neg P(x) \land \forall y \neg Q(x,y)$

2) $\forall x . P(x) \Rightarrow \exists y\, Q(x,y)$

3) $\forall x . P(x) \Rightarrow \exists y\, Q(y,x)$

4) $\forall x . P(x) \Rightarrow \forall y . \neg Q(x,y)$

5) $\forall x\ (\exists y . Q(x,y)) \Rightarrow \exists z . P(z) \land Q(x,z)$

B) I, $D_I = \{A,B,C,D\}$

1) $I(P) = \{(B,T), (C,T), (A,F), (D,F)\}$

$I(Q) = \{((A,C),T), ((A,D),T), ((B,C),T), ((B,D),T), ((D,B),T), ((C,D),T), ((x,y),F)$ pour tous les autres $x,y)$

2) $\cancel{[\![\exists x . \neg P(x) \land \forall y \neg Q(x,y)]\!]^{I}_{g}}$

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a) $[\![\exists x. (\neg P(x) \land \forall y\ \neg Q(y,x))]\!]^I_s =$

$\bigvee_{v\in D_I} [\![(\neg P(x) \land \forall y.\ \neg Q(y,x))]\!]^I_{s[v/x]}$

$\bigvee_{v\in D_I} \left([\![\neg P(x)]\!]^I_{s[v/x]} \land_B [\![\forall y. (\neg Q(y,x))]\!]^I_{s[v/x]} \right)$

$\bigvee_{v\in D_I} \left(\neg_B [\![P(x)]\!]^I_{s[v/x]} \land_B \left(\bigwedge_{p\in D_I} [\![\neg Q(y,x)]\!]^I_{s[v/x][p/y]}\right.\right.$

$\bigvee_{v\in D_I} \left(\neg_B I(P)([\![x]\!]^I_{s[v/x]}) \land_B \left(\bigwedge_{p\in D_I} \neg_B [\![Q(x,y)]\!]^I_{s[v/x][p/y]}\right.\right.$

$\bigvee_{v\in D_I} \left(\neg_B I(P)(s[v/x](x)) \land_B \left(\bigwedge_{p\in D_I} \neg_B I(Q)([\![y]\!]_{s[v/x][p/y]}, [\![x]\!]_{s[v/x][p/y]})\right)\right)$

$= \bigvee_{v\in D_I} (\neg_B I(P)(v)) \lor_B \left(\bigwedge_{p\in D_I} \neg_B I(Q)(p,t)\right)$

$= (\neg_B I(P)(A) \land_B (\bigwedge_{p\in D_I} \neg_B I(Q)(p,A))) \lor_B \ldots$

$= (\neg_B(F) \land_B (\neg_B I(Q)(A,A) \land_B \neg_B I(Q)(B,A) \land_B \neg_B I(Q)(C,A) \land_B \neg_B I(Q)(D,A))$

Exo 2

a) $\begin{cases} D_I = \{a_x\} \\ I_a = a_0 \\ I(P) = \{(a_0, T)\} \end{cases}$

$[\![P(a)]\!]^I_g = I(P)(a_0) = T$

$[\![\exists x . P(x)]\!]^I_g = \bigvee_{v \in D_I} I(P)(a_0) = T$

b)

$D_I = \{a_0, a_1\}$

$I(a) = a_0$

$I(P) = \{(a_0, F), (a_1, T)\}$

$[\![P(a)]\!]^I_g = I(P)(a_0) = F$

$[\![\exists x . P(x)]\!]^I_g = \bigvee_{v \in D_I} I(P)(v) = I(P)(a_0) \vee_B I(P)(a_1)$

$= F \vee_B T = T$

c) $a_0 \in D_1$

$I_a = a_0$

$I_P(a_0) = T$ ($P(a)$ vrai)

$[\![\exists x . P(a)]\!]^I_g =$

$\bigvee_{v \in D_I} I(P)(v) = I(P)(a_0) \vee_B \ldots \vee_B$

$= T \vee_B \ldots T \neq F$ pas possible d)

$D_I = \{a_0\}$

$I(a) = a_0$

$I(R) = \{(a_0, f)\}$

$[| P(x) |]^I_S = I(P)(a_0) = f$

$[| \exists x\ P(x) |]^I_S = \bigvee_{v \in D_I} I(P)(?) = I(P)(a_0) = f$

$= \cancel{((I(P)}$

$\cancel{[| \exists x\ P(x) |]^I_S = \bigvee_{v \in D_I}}$

$D_I = \{a_0, a_1\}$

$I_a = a_0$, $I(P) = \{(a_0, F), (a_1, T)\}$

$[| (\exists x\ P(x)) \Rightarrow P(a) |]^I_S = \left(\bigvee_{v \in D_I} I(P)(v)\right) \Rightarrow_B I(P)(a_0) =$

$\left(I(P)(a_0) \vee I(P)(a_1)\right) \Rightarrow I(P)(a_1)$

$(F \vee T) \Rightarrow F = F$

$D_I = \{a_0\}$
$I(a) = a_0$
$I(P) = \{(a_0, T)\}$

$[| (\exists x\ P(x)) \Rightarrow P(a) |]^I_S = \left(\bigvee_{v \in D_I} I(P)(v)\right) \Rightarrow_B I(P)(a_0) =$

$= I(P_0)(a_0) \Rightarrow_B I(P)(a_0) =$

$= F \Rightarrow F = T$

3) Que dire de

$P(a) \Rightarrow \exists x\, P(x)$

Elle est valide

Demonstration

Pour tout I, $[\![P(a) \Rightarrow \exists x\, P(x)]\!]^I_S = T$ valide

D_I, $I(a) = a_0 \in D_I$

$[\![P(a) \Rightarrow \exists x .\, P(x)]\!]^I_S =$

$I(P)(a_0) \Rightarrow_B \bigvee_{v \in D_I} I(P)(v) = G$

2 cas :

* $I(P)(a_0) = F$

$G = F \Rightarrow_B \bigvee_{v \in D_I} I(P)(v) = T$

* $I(P)(a_0) = T$

$G = T \Rightarrow_B \bigvee_{v \in D_I} I(P)(v) = \bigvee_{v \in D_I} I(P)(v)$

$= I(P)(a_0) \vee_B \ldots$

$= T \vee_B \ldots T = T$

$P(a) \wedge \neg \exists x .\, P(x)$ insatisfiable

D_I, $I(a) = a_0 \in D_I$

$[\![P(a) \wedge \neg \exists x\, P(x)]\!] = F$

$[\![P(a) \wedge \neg \exists x .\, P(x)]\!]^I = I(P)(a_0) \wedge_B \neg_B \left(\bigvee_{v \in D_I} I(P)(v) \right) = G$