- **Derivation of the modified Korteweg-de Vries (mKdV) equation from the full velocity difference (FVD) model.**

Let us consider the FVD model,

$$\frac{d^2\Delta x_n(t)}{dt^2}=a\left[V_F\left(\Delta x_n(t)\right)-\frac{d\Delta x_n(t)}{dt}\right]+\lambda.\left[\frac{d\Delta x_{n+1}(t)}{dt}-\frac{d\Delta x_n(t)}{dt}\right] \tag{1}$$

Equation (1) leads us to the reductive perturbation method. Around the critical point $\left(a_c,h_c\right)$, a small positive scaling parameter *ε* is introduced. Let us then define the slow scales *X* and *T* [44] [45], where the space variable n and time variable *t* are transformed as follows:

$X=\varepsilon\left(n+bt\right)$ and $T=\varepsilon^3 t$ with $0<\varepsilon\le 1,$ (2)

Where *b* is a constant to be determined. The headway distance $\Delta x_n\left(t\right)$ can be defined as follows:

$$\Delta x_n\left(t\right)=h_c+\varepsilon R\left(X,T\right) \tag{3}$$

We know, $\Delta x_{n+1}\left(t\right)=h_c+\varepsilon R+\frac{\varepsilon^2}{1}\partial_X R+\frac{\varepsilon^3}{2}\partial_X^2 R+\frac{\varepsilon^4}{6}\partial_X^3 R+\frac{\varepsilon^5}{24}\partial_X^4 R$ (4)

By expanding each term in equation (1) to the fifth order of ε we get,

$$\frac{d\Delta x_n\left(t\right)}{dt}=\frac{d}{dt}\left(h_c+\varepsilon R\left(XT\right)\right)=\varepsilon\frac{d}{dt}R\left(X,T\right)\frac{dX}{dt}+\varepsilon\frac{d}{dT}R\left(X,T\right)\frac{dX}{dt}=b\varepsilon^2\partial_X R+\varepsilon^4\partial_T R \tag{5}$$

$$\frac{d^2\Delta x_n\left(t\right)}{dt^2}=\frac{d}{dt}\left(b\varepsilon^2\partial_X R\right)+\frac{d}{dt}\left(\varepsilon^4\partial_T R\right)$$
$$=b\varepsilon^2\frac{d}{dX}\left(\partial_X R\right)\frac{dX}{dt}+b\varepsilon^2\frac{d}{dT}\left(\partial_X R\right)\frac{dT}{dt}+\varepsilon^4\frac{d}{dX}\left(\partial_T R\right)\frac{dX}{dt}+\varepsilon^4\frac{d}{dT}\left(\partial_T R\right)\frac{dT}{dt} \tag{6}$$
$$=b\varepsilon^3\partial_X^2 R+b\varepsilon^5\partial_X\partial_T R+b\varepsilon^5\partial_T\partial_X R+\varepsilon^7\partial_T^2 R=b\varepsilon^3\partial_X^2 R+2b\varepsilon^5\partial_X\partial_T R$$

$$\frac{d\Delta x_{n+1}\left(t\right)}{dt}=\frac{d}{dt}\left(h_c+\varepsilon R+\frac{\varepsilon^2}{1}\partial_X R+\frac{\varepsilon^3}{2}\partial_X^2 R+\frac{\varepsilon^4}{6}\partial_X^3 R+\frac{\varepsilon^5}{24}\partial_X^4 R\right)$$
$$=\frac{d}{dt}\left(\varepsilon R\right)+\varepsilon^2\frac{d}{dt}\left(\partial_X R\right)+\frac{\varepsilon^3}{2}\frac{d}{dt}\left(\partial_X^2 R\right)+\frac{\varepsilon^4}{6}\frac{d}{dt}\left(\partial_X^3 R\right)+\frac{\varepsilon^5}{24}\frac{d}{dt}\left(\partial_X^4 R\right)$$
$$=\varepsilon\frac{d}{dX}\left(R\right)\frac{dX}{dt}+\varepsilon\frac{d}{dT}\left(R\right)\frac{dT}{dt}+\varepsilon^2\frac{d}{dX}\left(\partial_X R\right)\frac{dX}{dt}+\varepsilon^2\frac{d}{dT}\left(\partial_X R\right)\frac{dT}{dt}+\frac{\varepsilon^3}{2}\frac{d}{dX}\left(\partial_X^2 R\right)\frac{dX}{dt}+\frac{\varepsilon^3}{2}\frac{d}{dT}\left(\partial_X^2 R\right)\frac{dT}{dt} \tag{7}$$
$$+\frac{\varepsilon^4}{6}\frac{d}{dX}\left(\partial_X^3 R\right)\frac{dX}{dt}+\frac{\varepsilon^4}{6}\frac{d}{dT}\left(\partial_X^3 R\right)\frac{dT}{dt}+\frac{\varepsilon^5}{24}\frac{d}{dX}\left(\partial_X^4 R\right)\frac{dX}{dt}+\frac{\varepsilon^5}{24}\frac{d}{dT}\left(\partial_X^4 R\right)\frac{dT}{dt}$$
$$=\varepsilon^2 b\partial_X R+\varepsilon^4\partial_T R+\varepsilon^3 b\partial_X^2 R+\varepsilon^5\partial_X\partial_T R+\frac{\varepsilon^4}{2}b\partial_X^3 R+\frac{\varepsilon^6}{2}b\partial_X^2\partial_T R+\frac{\varepsilon^5}{6}b\partial_X^4 R+\frac{\varepsilon^7}{6}\partial_X^3\partial_T R+\frac{\varepsilon^6}{24}b\partial_X^5 R+\frac{\varepsilon^8}{24}\partial_X^4\partial_T R$$
$$=\varepsilon^2 b\partial_X R+\varepsilon^3 b\partial_X^2 R+\varepsilon^4\left[\frac{b}{2}\partial_X^3 R+\partial_T R\right]+\varepsilon^5\left[\frac{b}{6}\partial_X^4 R+\partial_X\partial_T R\right] \tag{8}$$

Now

$$V_F\left(\Delta x_n\left(t\right)\right)=V_F\left(h_c+\varepsilon R\right)=V_F\left(h_c\right)+\varepsilon V_F'\left(h_c\right)R+\frac{\varepsilon^2}{2}V_F''\left(h_c\right)R^2+\frac{\varepsilon^3}{6}V_F'''\left(h_c\right)R^3+\frac{\varepsilon^4}{24}V_F^{iv}\left(h_c\right)R^4$$
$$=V_F\left(h_c\right)+\varepsilon V_F'\left(h_c\right)R+\frac{\varepsilon^3}{6}V_F'''\left(h_c\right)R^3 \tag{9}$$

$$
\begin{aligned}
&V_F\left(\Delta x_{n+1}(t)\right)=V_F\left(h_c+\varepsilon R+\frac{\varepsilon^2}{1}\partial_X R+\frac{\varepsilon^3}{2}\partial_X^2 R+\frac{\varepsilon^4}{6}\partial_X^3 R+\frac{\varepsilon^5}{24}\partial_X^4 R\right)\\
&=V_F(h_c)+\varepsilon V_F'(h_c)R+\varepsilon^2 V_F'(h_c)\partial_X R+\frac{\varepsilon^3}{2}V_F'(h_c)\partial_X^2 R+\frac{\varepsilon^3}{6}V_F'''(h_c)R^3\\
&+\frac{\varepsilon^4}{6}V_F'(h_c)\partial_X^3 R+\frac{\varepsilon^4}{6}V_F'''(h_c)\partial_X R^3+\frac{\varepsilon^5}{24}V_F'(h_c)\partial_X^4 R+\frac{\varepsilon^5}{12}V_F'''(h_c)\partial_X^2 R^3\\
&=V_F(h_c)+\varepsilon V_F'(h_c)R+\varepsilon^2 V_F'(h_c)\partial_X R+\varepsilon^3\left(\frac{V_F'(h_c)}{2}\partial_X^2 R+\frac{V_F'''(h_c)}{6}R^3\right)+\\
&\varepsilon^4\left(\frac{V_F'(h_c)}{6}\partial_X^3 R+\frac{V_F'''(h_c)}{6}\partial_X R^3\right)+\varepsilon^5\left(\frac{V_F'(h_c)}{24}\partial_X^4 R+\frac{V_F'''(h_c)}{12}\partial_X^2 R^3\right)
\end{aligned}
\tag{10}
$$

where,

$$
V_F'=V_F'(h_c)=\left.\frac{dV_F\left(\Delta x_n(t)\right)}{d\left(\Delta x_n(t)\right)}\right|_{\Delta x_n(t)=h_c},V_F'''=V_F'''(h_c)=\left.\frac{d^3V_F\left(\Delta x_n(t)\right)}{d\left(\Delta x_n(t)\right)^3}\right|_{\Delta x_n(t)=h_c}
\tag{11}
$$

Now,

$$
\begin{aligned}
&V_F\left(\Delta x_{n+1}(t)\right)-V_F\left(\Delta x_n(t)\right)=V_F(h_c)+\varepsilon V_F'(h_c)R+\varepsilon^2 V_F'(h_c)\partial_X R+\varepsilon^3\left(\frac{V_F'(h_c)}{2}\partial_X^2 R+\frac{V_F'''(h_c)}{6}R^3\right)+\\
&\varepsilon^4\left(\frac{V_F'(h_c)}{6}\partial_X^3 R+\frac{V_F'''(h_c)}{6}\partial_X R^3\right)+\varepsilon^5\left(\frac{V_F'(h_c)}{24}\partial_X^4 R+\frac{V_F'''(h_c)}{12}\partial_X^2 R^3\right)-V_F(h_c)-\varepsilon V_F'(h_c)R-\frac{\varepsilon^3}{6}V_F'''(h_c)R^3\\
&=\varepsilon^2 V_F'(h_c)\partial_X R+\varepsilon^3\frac{V_F'(h_c)}{2}\partial_X^2 R+\varepsilon^4\left(\frac{V_F'(h_c)}{6}\partial_X^3 R+\frac{V_F'''(h_c)}{6}\partial_X R^3\right)+\varepsilon^5\left(\frac{V_F'(h_c)}{24}\partial_X^4 R+\frac{V_F'''(h_c)}{12}\partial_X^2 R^3\right)
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
&\Rightarrow\frac{d\Delta x_{n+1}(t)}{dt}-\frac{d\Delta x_n(t)}{dt}=\varepsilon^2 b\partial_X R+\varepsilon^3 b\partial_X^2 R+\varepsilon^4\left[\frac{b}{2}\partial_X^3 R+\partial_T R\right]+\varepsilon^5\left[\frac{b}{6}\partial_X^4 R+\partial_X\partial_T R\right]\\
&-b\varepsilon^3\partial_X^2 R-2b\varepsilon^5\partial_X\partial_T R=\varepsilon^3 b\partial_X^2 R+\varepsilon^4\frac{b}{2}\partial_X^3 R+\varepsilon^5\frac{b}{6}\partial_X^4 R+\varepsilon^5\partial_X\partial_T R
\end{aligned}
\tag{13}
$$

We next substitute Equations (2) to (13) into Equation (1), and then, we expand the equation using a Taylor expansion of ε up to the fifth order. We thus obtain the following nonlinear partial differential equation:

$$
\begin{aligned}
&\varepsilon^3\frac{b^2}{a}\partial_X^2 R+\frac{2b}{a}\varepsilon^5\partial_X\partial_T R=\left(\varepsilon^2 V_F'\partial_X R+\frac{\varepsilon^3}{2}V_F'\partial_X^2 R+\frac{\varepsilon^4}{6}\left(V_F'\partial_X^3 R+V_F'''\partial_X R^3\right)+\frac{\varepsilon^5}{24}\left(V_F'\partial_X^4 R+2V_F'''\partial_X^2 R^3\right)\right)\\
&-b\varepsilon^2\partial_X R-\varepsilon^4\partial_T R+\frac{\lambda}{a}\left(\varepsilon^3 b\partial_X^2 R+\varepsilon^4\frac{b}{2}\partial_X^3 R+\varepsilon^5\frac{b}{6}\partial_X^4 R+\varepsilon^5\partial_X\partial_T R\right)\\
&\Rightarrow\varepsilon^2\left[b-V_F'\right]\partial_X R+\varepsilon^3\left[\frac{b^2}{a}-\frac{1}{2}V_F'-\frac{\lambda b}{a}\right]\partial_X^2 R+\varepsilon^4\left[\partial_T R-\frac{\lambda b}{2a}-\frac{1}{6}\left(V_F'\partial_X^3 R+V_F'''\partial_X R^3\right)\right]\\
&+\varepsilon^5\left[\frac{2b-\lambda}{a}\partial_X\partial_T R-\frac{\lambda b}{6a}\partial_X^4 R-\frac{1}{24}\left(V_F'\partial_X^4 R+2V_F'''\partial_X^2 R^3\right)\right]=0
\end{aligned}
$$

$$\Rightarrow \varepsilon^2\left[b-b\right]\partial_X R+\varepsilon^3\left[\frac{b^2}{a}-\frac{1}{2}V_F'-\frac{\lambda b}{a}\right]\partial_X^2 R+\varepsilon^4\left[\partial_T R-\left\{\frac{1}{6}V_F'+\frac{\lambda b}{2a}\right\}\partial_X^3 R-\frac{1}{6}V_F'''\,\partial_X R^3\right]$$
$$+\varepsilon^5\left[\frac{2b-\lambda}{a}\partial_X\partial_T R-\left\{\frac{1}{24}\left(V_F'+2V_F'''\,\partial_X^2 R^3\right)+\frac{\lambda b}{6a}\right\}\partial_X^4 R-\frac{1}{12}V_F'''\,\partial_X^2 R^3\right]=0 \tag{14}$$

where $V_F'(b)=\left.\frac{dV_F(\Delta x_n)}{d\Delta x_n}\right|_{\Delta x_n=b}$, $V_F'''(b)=\left.\frac{d^3V_F(\Delta x_n)}{d\Delta x^3{}_n}\right|_{\Delta x_n=b}$

Now, let us introduce $a_c=a\left(1+\varepsilon^2\right)$ as the neighbor to the critical point $\left(a_c,h_c\right)$ and consider $b=V_F'$. The terms in Equation (14) containing second and third orders of ε should be neglected; this allows us to simplify the equation as follows:

$$\Rightarrow \varepsilon^2\left[b-b\right]\partial_X R+\varepsilon^3\left[\frac{b^2}{a}-\frac{\lambda b}{a}-\frac{1}{2}V_F'\right]\partial_X^2 R+\varepsilon^4\left[\partial_T R-g_1\partial_X^3 R+g_2\partial_X R^3\right]+\varepsilon^5\left[g_3\partial_X^2 R+g_4\partial_X^4 R+g_5\partial_X^2 R^3\right]=0 \tag{15}$$

where the values of g_i are given in Table 1.

To derive the regularized equation, the following transformations are applied to Equation (15):

$$T=\frac{1}{g_1}T' \text{ and } R=\sqrt{\frac{g_1}{g_2}}R'(X,T'), \tag{16}$$

Here,

$$T=\frac{1}{g_1}T',\quad R=\sqrt{\frac{g_1}{g_2}}R'(X,T'),$$

$$\partial_T R=\frac{\partial R}{\partial T}=\frac{\partial R}{\partial T'}.\frac{\partial T'}{\partial R}=\sqrt{\frac{g_1}{g_2}}\frac{\partial R'}{\partial T'}g_1=\frac{g_1\sqrt{g_1}}{\sqrt{g_2}}\partial_{T'}R',\; g_1\partial_X^3 R=g_1\frac{\partial}{\partial X}\left(\sqrt{\frac{g_1}{g_2}}R'\right)=\frac{g_1\sqrt{g_1}}{\sqrt{g_2}}\partial_X^3 R'$$

$$g_2\partial_X R^3=g_2\frac{\partial}{\partial X}\left(\sqrt{\frac{g_1}{g_2}}R'\right)^3=\frac{g_1\sqrt{g_1}}{\sqrt{g_2}}\partial_X R'^3,\; g_3\partial_X^2 R=g_3\frac{\partial^2}{\partial X}\left(\sqrt{\frac{g_1}{g_2}}R'\right)=\frac{g_3\sqrt{g_1}}{\sqrt{g_2}}\partial_X^2 R'$$

$$g_4\partial_X^4 R=g_4\frac{\partial^4}{\partial X}\left(\sqrt{\frac{g_1}{g_2}}R'\right)=\frac{g_4\sqrt{g_1}}{\sqrt{g_2}}\partial_X^4 R',\; g_5\partial_X^2 R^3=g_5\frac{\partial^2}{\partial X}\left(\sqrt{\frac{g_1}{g_2}}R'\right)^3=\frac{g_1 g_5\sqrt{g_1}}{g_2\sqrt{g_2}}\partial_X^2 R'^3$$

The standard mKdV equation with the correction term $O(\varepsilon)$ is given as follows from (15):

$$\partial_{T'}R'(X,T')-\partial^3{}_X R'(X,T')+\partial_X R'^3(X,T')+\varepsilon M\left[R'(X,T')\right]=0, \tag{17}$$

Where $M\left[R'(X,T')\right]=\left[\frac{g_3}{g_1}\partial^2{}_X R'+\frac{g_4}{g_1}\partial^4{}_X R'+\frac{g_5}{g_2}\partial^2{}_X R'^3\right]$.

Table 1. The coefficients g_i of the FVD model.

$g_1=\frac{1}{6}V_F'+\frac{\lambda b}{2a}$	$g_2=-\frac{1}{6}V_F'''$	$g_3=\frac{1}{2}V_F'$
$g_4=-\frac{1}{24}\left(V_F'+2V_F'''\,\partial_X^2 R^3\right)+\frac{\lambda b}{6a}$		$g_5=-\frac{1}{12}V_F'''$

Note:

$$\frac{d\Delta x_{n+1}(t)}{dt}=\frac{d}{dt}\left(h_c+\varepsilon R+\frac{\varepsilon^2}{1}\partial_X R+\frac{\varepsilon^3}{2}\partial_X^2 R+\frac{\varepsilon^4}{6}\partial_X^3 R+\frac{\varepsilon^5}{24}\partial_X^4 R\right)$$

$$=\frac{d}{dt}(\varepsilon R)+\varepsilon^2\frac{d}{dt}(\partial_X R)+\frac{\varepsilon^3}{2}\frac{d}{dt}(\partial_X^2 R)+\frac{\varepsilon^4}{6}\frac{d}{dt}(\partial_X^3 R)+\frac{\varepsilon^5}{24}\frac{d}{dt}(\partial_X^4 R)$$

$$=\varepsilon\frac{d}{dX}(R)\frac{dX}{dt}+\varepsilon\frac{d}{dT}(R)\frac{dT}{dt}+\varepsilon^2\frac{d}{dX}(\partial_X R)\frac{dX}{dt}+\varepsilon^2\frac{d}{dT}(\partial_X R)\frac{dT}{dt}+\frac{\varepsilon^3}{2}\frac{d}{dX}(\partial_X^2 R)\frac{dX}{dt}+\frac{\varepsilon^3}{2}\frac{d}{dT}(\partial_X^2 R)\frac{dT}{dt}$$

$$+\frac{\varepsilon^4}{6}\frac{d}{dX}(\partial_X^3 R)\frac{dX}{dt}+\frac{\varepsilon^4}{6}\frac{d}{dT}(\partial_X^3 R)\frac{dT}{dt}+\frac{\varepsilon^5}{24}\frac{d}{dX}(\partial_X^4 R)\frac{dX}{dt}+\frac{\varepsilon^5}{24}\frac{d}{dT}(\partial_X^4 R)\frac{dT}{dt}$$

$$=\varepsilon^2 b\partial_X R+\varepsilon^4\partial_T R+\varepsilon^3 b\partial_X^2 R+\varepsilon^5\partial_X\partial_T R+\frac{\varepsilon^4}{2}b\partial_X^3 R+\frac{\varepsilon^6}{2}b\partial_X^2\partial_T R+\frac{\varepsilon^5}{6}b\partial_X^4 R+\frac{\varepsilon^7}{6}\partial_X^3\partial_T R+\frac{\varepsilon^6}{24}b\partial_X^5 R+\frac{\varepsilon^8}{24}\partial_X^4\partial_T R$$

$$=\varepsilon^2 b\partial_X R+\varepsilon^3 b\partial_X^2 R+\varepsilon^4\left[\frac{b}{2}\partial_X^3 R+\partial_T R\right]+\varepsilon^5\left[\frac{b}{6}\partial_X^4 R+\partial_X\partial_T R\right]$$

$$V_F\left(\Delta x_{n+1}(t)\right)=V_F\left(h_c+\varepsilon R+\frac{\varepsilon^2}{1}\partial_X R+\frac{\varepsilon^3}{2}\partial_X^2 R+\frac{\varepsilon^4}{6}\partial_X^3 R+\frac{\varepsilon^5}{24}\partial_X^4 R\right)$$

$$=V_F(h_c)+\varepsilon V_F'(h_c)R+\varepsilon^2 V_F'(h_c)\partial_X R+\frac{\varepsilon^3}{2}V_F'(h_c)\partial_X^2 R+\frac{\varepsilon^3}{6}V_F'''(h_c)R^3$$

$$+\frac{\varepsilon^4}{6}V_F'(h_c)\partial_X^3 R+\frac{\varepsilon^4}{6}V_F'''(h_c)\partial_X R^3+\frac{\varepsilon^5}{24}V_F'(h_c)\partial_X^4 R+\frac{\varepsilon^5}{12}V_F'''(h_c)\partial_X^2 R^3$$

$$=V_F(h_c)+\varepsilon V_F'(h_c)R+\varepsilon^2 V_F'(h_c)\partial_X R+\varepsilon^3\left(\frac{V_F'(h_c)}{2}\partial_X^2 R+\frac{V_F'''(h_c)}{6}R^3\right)+$$

$$\varepsilon^4\left(\frac{V_F'(h_c)}{6}\partial_X^3 R+\frac{V_F'''(h_c)}{6}\partial_X R^3\right)+\varepsilon^5\left(\frac{V_F'(h_c)}{24}\partial_X^4 R+\frac{V_F'''(h_c)}{12}\partial_X^2 R^3\right)$$

where,

$$V_F'=V_F'(h_c)=\left.\frac{dV_F\left(\Delta x_n(t)\right)}{d\left(\Delta x_n(t)\right)}\right|_{\Delta x_n(t)=h_c},V_F'''=V_F'''(h_c)=\left.\frac{d^3V_F\left(\Delta x_n(t)\right)}{d\left(\Delta x_n(t)\right)^3}\right|_{\Delta x_n(t)=h_c}$$

$$V_F\left(\Delta x_{n+1}(t)\right)-V_F\left(\Delta x_n(t)\right)=V_F(h_c)+\varepsilon V_F'(h_c)R+\varepsilon^2 V_F'(h_c)\partial_X R+\varepsilon^3\left(\frac{V_F'(h_c)}{2}\partial_X^2 R+\frac{V_F'''(h_c)}{6}R^3\right)+$$

$$\varepsilon^4\left(\frac{V_F'(h_c)}{6}\partial_X^3 R+\frac{V_F'''(h_c)}{6}\partial_X R^3\right)+\varepsilon^5\left(\frac{V_F'(h_c)}{24}\partial_X^4 R+\frac{V_F'''(h_c)}{12}\partial_X^2 R^3\right)-V_F(h_c)-\varepsilon V_F'(h_c)R-\frac{\varepsilon^3}{6}V_F'''(h_c)R^3$$

$$=\varepsilon^2 V_F'(h_c)\partial_X R+\varepsilon^3\frac{V_F'(h_c)}{2}\partial_X^2 R+\varepsilon^4\left(\frac{V_F'(h_c)}{6}\partial_X^3 R+\frac{V_F'''(h_c)}{6}\partial_X R^3\right)+\varepsilon^5\left(\frac{V_F'(h_c)}{24}\partial_X^4 R+\frac{V_F'''(h_c)}{12}\partial_X^2 R^3\right)$$

$$\frac{d\Delta x_{n+1}(t)}{dt}-\frac{d\Delta x_n(t)}{dt}=\varepsilon^2 b\partial_X R+\varepsilon^3 b\partial_X^2 R+\varepsilon^4\left[\frac{b}{2}\partial_X^3 R+\partial_T R\right]+\varepsilon^5\left[\frac{b}{6}\partial_X^4 R+\partial_X\partial_T R\right]-b\varepsilon^3\partial_X^2 R-2b\varepsilon^5\partial_X\partial_T R$$

$$=\varepsilon^3 b\partial_X^2 R+\varepsilon^4\frac{b}{2}\partial_X^3 R+\varepsilon^5\frac{b}{6}\partial_X^4 R+\varepsilon^5\partial_X\partial_T R$$

$$\varepsilon^3 \frac{b^2}{a}\partial_X^2 R + \frac{2b}{a}\varepsilon^5 \partial_X \partial_T R = p\left(\varepsilon^2 V_F'\left(h_c\right)\partial_X R + \varepsilon^3 \frac{V_F'\left(h_c\right)}{2}\partial_X^2 R + \varepsilon^4\left(\frac{V_F'\left(h_c\right)}{6}\partial_X^3 R + \frac{V_F'''\left(h_c\right)}{6}\partial_X R^3\right) + \varepsilon^5\left(\frac{V_F'\left(h_c\right)}{24}\partial_X^4 R + \frac{V_F'''\left(h_c\right)}{12}\partial_X^2 R^3\right)\right)$$

$$+\left(1-p\right)\left(\varepsilon^2 V_B'\left(h_c\right)\partial_X R - \frac{\varepsilon^3}{2}V_B'\left(h_c\right)\partial_X^2 R + \frac{\varepsilon^4}{6}V_B'\left(h_c\right)\partial_X^3 R + \frac{\varepsilon^4}{6}V_B'''\left(h_c\right)\partial_X R^3 - \frac{\varepsilon^5}{24}V_B'\left(h_c\right)\partial_X^4 R - \frac{\varepsilon^5}{12}V_B'''\left(h_c\right)\partial_X^2 R^3\right) - b\varepsilon^2 \partial_X R - \varepsilon^4 \partial_T R$$

$$+\frac{\lambda}{a}\left(\varepsilon^3 b\partial_X^2 R + \varepsilon^4 \frac{b}{2}\partial_X^3 R + \varepsilon^5 \frac{b}{6}\partial_X^4 R + \varepsilon^5 \partial_X \partial_T R\right)$$

$$\Rightarrow \varepsilon^2\left[b - V_F'\left(h_c\right)\right]\partial_X R + \varepsilon^3\left[\frac{b^2}{a} - \frac{1}{2}V_F'\left(h_c\right) - \frac{\lambda b}{a}\right]\partial_X^2 R + \varepsilon^4\left[\left(\partial_T R + \frac{1}{6}V_F'\left(h_c\right) + \frac{\lambda b}{2a}\right)\partial_X^3 R - \left(\frac{1}{6}V_F'''\left(h_c\right)\right)\partial_X R^3\right]$$

$$+\varepsilon^5\left[\frac{2b-\lambda}{a}\partial_X \partial_T R - \left\{\frac{1}{24}\left(V_F'\left(h_c\right)\right) + \frac{\lambda b}{6}\right\} - \frac{3}{4}\left(V_F'\left(h_c\right)\right)\right]\partial_X^2 R^3 = 0$$

$$\Rightarrow \varepsilon^2\left[b - V_F'\left(h_c\right)\right]\partial_X R + \varepsilon^3\left[\frac{b^2}{a} - \frac{1}{2}V_F'\left(h_c\right) - \frac{\lambda b}{a}\right]\partial_X^2 R + \varepsilon^4\left[\partial_T R - \left(\frac{1}{6}V_F'\left(h_c\right) + \frac{\lambda b}{2a}\right)\partial_X^3 R - \left(\frac{1}{6}V_F'''\left(h_c\right)\right)\partial_X R^3\right]$$

$$+\varepsilon^5\left[\frac{2b-\lambda}{a}\partial_X \partial_T R - \left\{\frac{1}{24}\left(pV_F'\left(h_c\right)\right) + \frac{\lambda b}{6}\right\} - \frac{3}{4}\left(pV_F'\left(h_c\right)\right)\right]\partial_X^2 R^3 = 0$$