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# 数论

## 线性筛打质数表，质因数分解

*#define MAXIMUM 26*

int prime[1000000];

bool isprime[1000000];

void get\_prime(int listsize)

{

int primesize=1;

memset(isprime,1,**sizeof**(isprime));

isprime[1] = false;

**for**(int i=2;i<=listsize;i++)

{

**if**(isprime[i]) prime[primesize++]=i;

**for**(int j=1;i\*prime[j]<=listsize&&j<=primesize;j++)

{

isprime[i\*prime[j]] = false;

**if**(i%prime[j]==0) **break**;

}

}

}

**struct** p

{

int value;

int time;

}p[MAXIMUM];

void prime\_factorization(long long n)

{

memset(p,0,**sizeof**(p));

long long psize=0;

long long a = n;

**for**(int t = 1;1LL\*prime[t]\*prime[t]<=n;t++)

{

**if**(a%prime[t]==0) p[++psize].value = prime[t];

**while**(a%prime[t]==0)

{

p[psize].time += 1;

a = a / prime[t];

}

**if**(a<=90000)

**if**(isprime[a])

{

p[++psize].value = a;

p[psize].time += 1;

a = 1;

**break**;

}

**if**(a==1) **break**;

}

**if**(a!=1)

{

p[++psize].value = a;

p[psize].time = 1;

}

}

## 莫比乌斯函数，欧拉函数

**const** int MAX = 101000;

int mu[MAX+10];

bool isprime[MAX+10];

int prime[MAX+10];

void get\_mobius(int n)

{

mu[1] = 1;

int tot = 0;

memset(isprime,1,**sizeof**(isprime));

**for**(int i=2;i<=n;i++)

{

**if**(isprime[i])

{

prime[++tot] = i;

mu[i] = -1;

}

**for**(int j=1;prime[j]\*i<=n;j++)

{

isprime[prime[j]\*i] = 0;

**if**(i%prime[j]==0)

{

mu[prime[j]\*i] = 0;

**break**;

}

mu[prime[j]\*i]=-mu[i];

}

}

}

int euler[MAX+10];

void get\_euler(int n)

{

euler[1] = 1;

int tot = 0;

memset(isprime,1,**sizeof**(isprime));

**for**(int i=2;i<=n;i++)

{

**if**(isprime[i])

{

prime[++tot] = i;

euler[i] = i-1;

}

**for**(int j=1;prime[j]\*i<=n;j++)

{

isprime[prime[j]\*i] = 0;

**if**(i%prime[j]==0)

{

euler[prime[j]\*i] = prime[j]\*euler[i];

**break**;

}

euler[prime[j]\*i] = euler[i]\*(prime[j]-1);

}

}

}

## Miller-Rabin素性测试

**typedef** long long LL;

LL iprime[6] = {2, 3, 5, 233, 331};

LL qmul(LL x, LL y, LL mod) { *// 乘法防止溢出， 如果p \* p不爆LL的话可以直接乘； O(1)乘法或者转化成二进制加法*

**return** (x \* y - (long long)(x / (long double)mod \* y + 1e-3) \*mod + mod) % mod;

}

LL qpow(LL a, LL n, LL mod) {

LL ret = 1;

**while**(n) {

**if**(n & 1) ret = qmul(ret, a, mod);

a = qmul(a, a, mod);

n >>= 1;

}

**return** ret;

}

bool Miller\_Rabin(LL p) {

**if**(p < 2) **return** 0;

**if**(p != 2 && p % 2 == 0) **return** 0;

LL s = p - 1;

**while**(! (s & 1)) s >>= 1;

**for**(int i = 0; i < 5; ++i) {

**if**(p == iprime[i]) **return** 1;

LL t = s, m = qpow(iprime[i], s, p);

**while**(t != p - 1 && m != 1 && m != p - 1) {

m = qmul(m, m, p);

t <<= 1;

}

**if**(m != p - 1 && !(t & 1)) **return** 0;

}

**return** 1;

}

## 快速幂，矩阵快速幂

int poww(int a,int b){

int ans=1,base=a;

**while**(b!=0){

**if**(b&1!=0)

ans\*=base;

base\*=base;

b>>=1;

}

**return** ans;

}

///矩阵快速幂

**const** int N=10;

int tmp[N][N];

void multi(int a[][N],int b[][N],int n)

{

memset(tmp,0,**sizeof** tmp);

**for**(int i=0;i<n;i++)

**for**(int j=0;j<n;j++)

**for**(int k=0;k<n;k++)

tmp[i][j]+=a[i][k]\*b[k][j];

**for**(int i=0;i<n;i++)

**for**(int j=0;j<n;j++)

a[i][j]=tmp[i][j];

}

int res[N][N];

void Pow(int a[][N],int n)

{

memset(res,0,**sizeof** res);*//n是幂，N是矩阵大小*

**for**(int i=0;i<n;i++) res[i][i]=1;

**while**(n)

{

**if**(n&1)

multi(res,a,N);*//res=res\*a;复制直接在multi里面实现了；*

multi(a,a,N);*//a=a\*a*

n>>=1;

}

}

## 卢卡斯、大组合数取模

LL PowMod(LL a,LL b,LL MOD){

LL ret=1;

**while**(b){

**if**(b&1) ret=(ret\*a)%MOD;

a=(a\*a)%MOD;

b>>=1;

}

**return** ret;

}

LL fac[100005];

LL Get\_Fact(LL p){

fac[0]=1;

**for**(LL i=1;i<=p;i++)

fac[i]=(fac[i-1]\*i)%p; *//预处理阶乘*

}

LL Lucas(LL n,LL m,LL p){

LL ret=1;

**while**(n&&m){

LL a=n%p,b=m%p;

**if**(a<b) **return** 0;

ret=(ret\*fac[a]\*PowMod(fac[b]\*fac[a-b]%p,p-2,p))%p;

n/=p;

m/=p;

}

**return** ret;

}

int main(){

int t;

scanf("%d",&t);

**while**(t--){

LL n,m,p;

scanf("%I64d%I64d%I64d",&n,&m,&p);

Get\_Fact(p);

printf("%I64d**\n**",Lucas(n,m,p));

}

**return** 0;

}

*卢卡斯定理*

*O(logp(n)\*p)*

## 中国剩余定理（不互质情况）

**using** **namespace** std;

**const** int maxn=100005;

**const** int inf=0x7fffffff;

**typedef** long long ll;

void ex\_gcd(ll a,ll b,ll &d,ll &x,ll &y)*//扩展欧几里得*

{

**if**(!b) {d=a;x=1;y=0;}

**else**{

ex\_gcd(b,a%b,d,y,x);

y-=x\*(a/b);

}

}

ll ex\_crt(ll \*m,ll \*r,int n)

{

ll M=m[1],R=r[1],x,y,d;

**for**(int i=2;i<=n;i++){

ex\_gcd(M,m[i],d,x,y);

**if**((r[i]-R)%d) **return** -1;

x=(r[i]-R)/d\*x%(m[i]/d);

R+=x\*M;

M=M/d\*m[i];

R%=M;

}

**return** R>0?**R**:R+M;

}

int main()

{

int t,n;

scanf("%d",&t);

**for**(int cas=1;cas<=t;cas++){

scanf("%d",&n);

ll m[maxn],r[maxn];*//m除数，r余数*

**for**(int i=1;i<=n;i++) scanf("%lld",&m[i]);

**for**(int i=1;i<=n;i++) scanf("%lld",&r[i]);

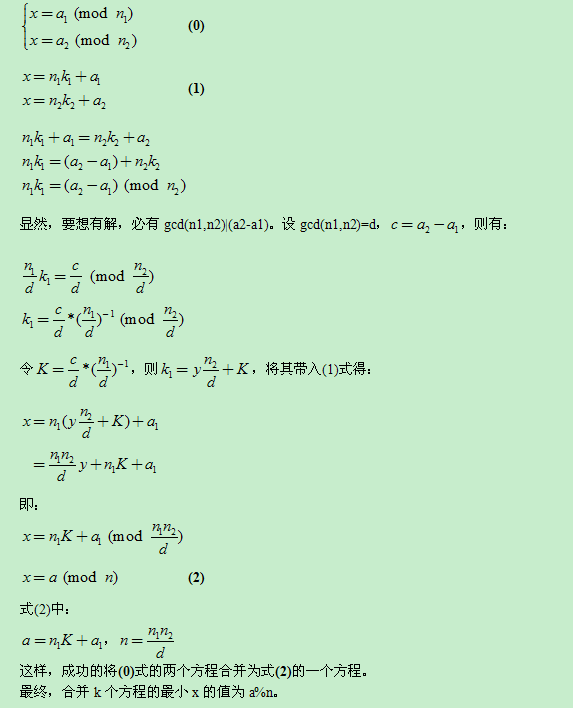
printf("Case %d: %I64d**\n**",cas,ex\_crt(m,r,n));

}

**return** 0;

}

关于原理的推导：



**typedef** **\_\_int64** int64;

int64 Mod;

int64 gcd(int64 a, int64 b)

{

**if**(b==0)

**return** a;

**return** gcd(b,a%b);

}

int64 Extend\_Euclid(int64 a, int64 b, int64&x, int64& y)

{

**if**(b==0)

{

x=1,y=0;

**return** a;

}

int64 d = Extend\_Euclid(b,a%b,x,y);

int64 t = x;

x = y;

y = t - a/b\*y;

**return** d;

}

*//a在模n乘法下的逆元，没有则返回-1*

int64 inv(int64 a, int64 n)

{

int64 x,y;

int64 t = Extend\_Euclid(a,n,x,y);

**if**(t != 1)

**return** -1;

**return** (x%n+n)%n;

}

*//将两个方程合并为一个*

bool merge(int64 a1, int64 n1, int64 a2, int64 n2, int64& a3, int64& n3)

{

int64 d = gcd(n1,n2);

int64 c = a2-a1;

**if**(c%d)

**return** false;

c = (c%n2+n2)%n2;

c /= d;

n1 /= d;

n2 /= d;

c \*= inv(n1,n2);

c %= n2;

c \*= n1\*d;

c += a1;

n3 = n1\*n2\*d;

a3 = (c%n3+n3)%n3;

**return** true;

}

*//求模线性方程组x=ai(mod ni),ni可以不互质*

int64 China\_Reminder2(int len, int64\* a, int64\* n)

{

int64 a1=a[0],n1=n[0];

int64 a2,n2;

**for**(int i = 1; i < len; i++)

{

int64 aa,nn;

a2 = a[i],n2=n[i];

**if**(!merge(a1,n1,a2,n2,aa,nn))

**return** -1;

a1 = aa;

n1 = nn;

}

Mod = n1;

**return** (a1%n1+n1)%n1;

}

int64 a[1000],b[1000];

int main()

{

int i;

int k;

**while**(scanf("%d",&k)!=EOF)

{

**for**(i = 0; i < k; i++)

scanf("%I64d %I64d",&a[i],&b[i]);

printf("%I64d**\n**",China\_Reminder2(k,b,a));

}

**return** 0;

}

## 对前n个数分解质因数

*#include<bits/stdc++.h>*

**using** **namespace** std;

*#define N 2000000*

vector <pair<long long,int>> d[2000004];

void init()

{

d[1].push\_back({1,1});

**for**(long long i=2;i<=N;i++)

{

**if**(d[i].empty())

{

**for**(long long j=i;j<=N;j+=i)

d[j].push\_back({i,1});

long long w=i\*i;

**while**(w<=N)

{

**for**(long long j=w;j<=N;j+=w)

d[j][d[j].size()-1].second++;

w\*=i;

}

}

}

}

int main()

{

init();

int t;

**while**(cin >> t)

**for**(int i=0;i<d[t].size();i++)

{

cout << "factor " << d[t][i].first << " is " << d[t][i].second << endl;

}

}

## 将根式转换为连分数形式

*#include<bits/stdc++.h>*

**using** **namespace** std;

vector<int> a;

vector<int> b;

vector<int> c;

void get\_fractions(int n)

{

a.clear();

b.clear();

c.clear();

int AA;

AA = floor(sqrt(n));

int c0 = n-AA\*AA;

int a0 = (sqrt(n)+AA)/(n-AA\*AA);

int b0 = a0\*(n-AA\*AA)-AA;

c.push\_back(c0);

a.push\_back(a0);

b.push\_back(b0);

int i=0;

**do**

{

int ccc = (n - b[i]\*b[i])/c[i];

int aaa = (sqrt(n)+b[i])/ccc;

int bbb = aaa\*ccc-b[i];

**if**(a[0]==aaa&&b[0]==bbb&&c[0]==ccc)

**break**;

c.push\_back(ccc);

a.push\_back(aaa);

b.push\_back(bbb);

i++;

}**while**(1);

printf("[%d;(",AA);

vector<int>::iterator it;

**for**(it=a.begin();it!=a.end();it++)

cout<<\*it<<(it==a.end()-1?")]":",");

*//for(auto& x : a) cout << x;*

cout << endl;

}

int main()

{

int n;

**while**(cin >> n)

get\_fractions(n);

**return** 0;

}

## Meissel-Lehmer算法（求1e11内质数个数）

*//Meisell-Lehmer*

*//G++ 218ms 43252k*

*#include<cstdio>*

*#include<cmath>*

**using** **namespace** std;

*#define LL long long*

**const** int N = 5e6 + 2;

bool np[N];

int prime[N], pi[N];

int getprime()

{

int cnt = 0;

np[0] = np[1] = true;

pi[0] = pi[1] = 0;

**for**(int i = 2; i < N; ++i)

{

**if**(!np[i]) prime[++cnt] = i;

pi[i] = cnt;

**for**(int j = 1; j <= cnt && i \* prime[j] < N; ++j)

{

np[i \* prime[j]] = true;

**if**(i % prime[j] == 0) **break**;

}

}

**return** cnt;

}

**const** int M = 7;

**const** int PM = 2 \* 3 \* 5 \* 7 \* 11 \* 13 \* 17;

int phi[PM + 1][M + 1], sz[M + 1];

void init()

{

getprime();

sz[0] = 1;

**for**(int i = 0; i <= PM; ++i) phi[i][0] = i;

**for**(int i = 1; i <= M; ++i)

{

sz[i] = prime[i] \* sz[i - 1];

**for**(int j = 1; j <= PM; ++j) phi[j][i] = phi[j][i - 1] - phi[j / prime[i]][i - 1];

}

}

int sqrt2(LL x)

{

LL r = (LL)sqrt(x - 0.1);

**while**(r \* r <= x) ++r;

**return** int(r - 1);

}

int sqrt3(LL x)

{

LL r = (LL)cbrt(x - 0.1);

**while**(r \* r \* r <= x) ++r;

**return** int(r - 1);

}

LL getphi(LL x, int s)

{

**if**(s == 0) **return** x;

**if**(s <= M) **return** phi[x % sz[s]][s] + (x / sz[s]) \* phi[sz[s]][s];

**if**(x <= prime[s]\*prime[s]) **return** pi[x] - s + 1;

**if**(x <= prime[s]\*prime[s]\*prime[s] && x < N)

{

int s2x = pi[sqrt2(x)];

LL ans = pi[x] - (s2x + s - 2) \* (s2x - s + 1) / 2;

**for**(int i = s + 1; i <= s2x; ++i) ans += pi[x / prime[i]];

**return** ans;

}

**return** getphi(x, s - 1) - getphi(x / prime[s], s - 1);

}

LL getpi(LL x)

{

**if**(x < N) **return** pi[x];

LL ans = getphi(x, pi[sqrt3(x)]) + pi[sqrt3(x)] - 1;

**for**(int i = pi[sqrt3(x)] + 1, ed = pi[sqrt2(x)]; i <= ed; ++i) ans -= getpi(x / prime[i]) - i + 1;

**return** ans;

}

LL lehmer\_pi(LL x)

{

**if**(x < N) **return** pi[x];

int a = (int)lehmer\_pi(sqrt2(sqrt2(x)));

int b = (int)lehmer\_pi(sqrt2(x));

int c = (int)lehmer\_pi(sqrt3(x));

LL sum = getphi(x, a) +(LL)(b + a - 2) \* (b - a + 1) / 2;

**for** (int i = a + 1; i <= b; i++)

{

LL w = x / prime[i];

sum -= lehmer\_pi(w);

**if** (i > c) **continue**;

LL lim = lehmer\_pi(sqrt2(w));

**for** (int j = i; j <= lim; j++) sum -= lehmer\_pi(w / prime[j]) - (j - 1);

}

**return** sum;

}

int main()

{

init();

LL n;

**while**(~scanf("%lld",&n))

{

printf("%lld**\n**",lehmer\_pi(n));

}

**return** 0;

}

# 字符串

## KMP字符串匹配

**const** int MAX = 200;

int nextt[MAX];

void getNext(string t)

{

int j, k;

memset(nextt,0,**sizeof**(nextt));

j = 0; k = -1; nextt[0] = -1;

**while**(j < t.length())

{

**if**(k == -1 || t[j] == t[k])

nextt[++j] = ++k;

**else**

k = nextt[k];

}

}

*/\**

*返回模式串t在主串s中首次出现的位置*

*返回的位置是从0开始的。*

*\*/*

int KMP\_Index(string t,string s)

{

int i = 0, j = 0;

getNext(t);

**while**(i < s.length() && j < t.length())

{

**if**(j == -1 || s[i] == t[j])

{

i++; j++;

}

**else**

j = nextt[j];

}

**if**(j == t.length())

**return** i - t.length();

**else**

**return** -1;

}

*/\**

*返回模式串t在主串s中出现的次数*

*\*/*

int KMP\_Count(string t,string s)

{

int ans = 0;

int i, j = 0;

**if**(s.length() == 1 && t.length() == 1)

{

**if**(s[0] == t[0])

**return** 1;

**else**

**return** 0;

}

getNext(t);

**for**(i = 0; i < s.length(); i++)

{

**while**(j > 0 && s[i] != t[j])

j = nextt[j];

**if**(s[i] == t[j])

j++;

**if**(j == t.length())

{

ans++;

j = nextt[j];

}

}

**return** ans;

}

# 数据结构

## 并查集

*#define MAX 10000;*

**struct** UF

{

int ranking;

int parent;

}UF[MAX];

void init(int n)

{

**for**(int i=0;i<=n;i++)

{

UF[i].parent=i;

UF[i].ranking=0;

}

}

int get\_parent(int x)

{

**if**(UF[x].parent==x) **return** x;

**return** get\_parent(UF[x].parent);

}

void Union(int a,int b)

{

a=get\_parent(a);

b=get\_parent(b);

**if**(UF[a].rank>UF[b].rank) UF[b].parent = UF[a].parent;

**else**

{

UF[a].parent = UF[b].parent;

**if**(UF[a].rank==UF[b].rank) UF[a].rank++;

}

}

## 线段树

**const** int maxn = 100007;

**struct** Tree

{

int l,r,sum;

int vis;

}t[maxn<<2];

void push\_up(int step)

{

t[step].sum = t[step\*2].sum + t[step\*2+1].sum;

}

void push\_down(int step)

{

**if**(!t[step].vis) **return**;

t[step\*2].vis += t[step].vis;

t[step\*2+1].vis += t[step].vis;

t[step\*2].sum += t[step].vis\*(t[step\*2].r-t[step\*2].l+1);

t[step\*2+1].sum += t[step].vis\*(t[step\*2+1].r-t[step\*2+1].l+1);

t[step].vis = 0;

}

void build(int l,int r,int step)

{

t[step].l = l,t[step].r = r,t[step].sum = t[step].vis = 0;

**if**(l==r) **return**;

int mid = (l+r)/2;

build(l,mid,step\*2);

build(mid+1,r,step\*2+1);

}

void update(int l,int r,int val,int step)

{

**if**(l==t[step].l&&r==t[step].r)

{

t[step].vis += val;

t[step].sum += (r-l+1)\*val;

**return**;

}

int mid = (t[step].l+t[step].r)/2;

push\_down(step);

**if**(r<=mid) update(l,r,val,step\*2);

**else** **if**(l>mid) update(l,r,val,step\*2+1);

**else** update(l,mid,val,step\*2),update(mid+1,r,val,step\*2+1);

push\_up(step);

}

int query(int l,int r,int step)

{

**if**(l==t[step].l&&r==t[step].r)

**return** t[step].sum;

int mid = (t[step].l+t[step].r)/2;

push\_down(step);

**if**(r<=mid) **return** query(l,r,step\*2);

**else** **if**(l>mid) **return** query(l,r,step\*2+1);

**else** **return** query(l,mid,step\*2)+query(mid+1,r,step\*2+1);

}

## zkw线段树

## 主席树

## 树状数组

*#include<bits/stdc++.h>*

**using** **namespace** std;

**const** int maxn = 100007;

int c[maxn];

int lowbit(int n)

{

**return** n & (-n);

}

int query(int x)

{

int res = 0;

**while**(x>0)

{

res += c[x];

x -= lowbit(x);

}

**return** res;

}

int update(int x,int val)

{

**while**(x<maxn)

{

c[x] += val;

x += lowbit(x);

}

}

int main()

{

int n;

cin >> n;

**for**(int i=1;i<=n;i++)

{

int x,y;

cin >> x >> y;

update(x,y);

}

cout << "give me your question!" << endl;

int l,r;

**while**(cin >> l >> r)

{

cout << "In [" << l << "," << r << "] ,all the number's sum is: " << query(r)-query(l-1) << endl;

}

}

# 计算几何

## 基础模板

**struct** spot *///存储点，也可指代向量*

{

double x;

double y;

double z;

};

spot cross(**const** spot &a,**const** spot &b) *///计算向量a和向量b的叉乘(有顺序)*

{

spot w;

w.x = a.y\*b.z-b.y\*a.z;

w.y = a.z\*b.x-a.x\*b.z;

w.z = a.x\*b.y-a.y\*b.x;

**return** w;

}

double dot(**const** spot &a,**const** spot &b) *///计算向量a和向量b的点乘积*

{

**return** a.x\*b.x+a.y\*b.y+a.z\*b.z;

}

double norm(**const** spot &a) *///计算向量a的模长*

{

**return** sqrt(a.x\*a.x+a.y\*a.y+a.z\*a.z);

}

# 杂项

## 罗马-数字转换

*#include<bits/stdc++.h>*

**using** **namespace** std;

char str[10]="IVXLCDM";

int num[10]={1,5,10,50,100,500,1000};

int roman\_to\_num(char s[])

{

int len=strlen(s);

int cnt=0,a[20];

**for**(int i=0;i<len;i++)

{

int f=0,t;

**if**(s[i]==s[i+1]&&i!=len-1)

{

**if**(s[i]==s[i+2]&&i!=len-2)

{

f=2;

}

**else**

f=1;

}

**for**(int k=0;k<7;k++)

**if**(s[i]==str[k])

{

t=k;

**break**;

}

a[cnt++]=num[t]\*(f+1);

i+=f;

}

int sum=0;

**for**(int i=0;i<cnt;i++)

sum+=a[i];

**for**(int i=0;i<cnt-1;i++)

{

**if**(a[i]<a[i+1])sum-=2\*a[i];

}

**return** sum;

}

string num\_to\_roman(int num)

{

char\* digit[10] = {"","I","II","III","IV","V","VI","VII","VIII","IX"};

char\* ten[10] = {"","X","XX","XXX","XL","L","LX","LXX","LXXX","XC"};

char\* hundreds[10] = {"", "C", "CC", "CCC", "CD", "D", "DC", "DCC", "DCCC", "CM"};

char\* thousand[7] = {"","M","MM","MMM","MMMM","MMMMM","MMMMMM"};

string ans;

ans = string(thousand[num/1000]) + string(hundreds[num%1000/100]) + string(ten[num%100/10]) + string(digit[num%10]);

**return** ans;

}

int main()

{

char a[101];

scanf("%s",a);

cout << roman\_to\_num(a) << endl;

cout << num\_to\_roman(roman\_to\_num(a)) << endl;

}

## Bm算法求解线性递推

*#include* *<bits/stdc++.h>*

**using** **namespace** std;

*#define rep(i,a,n) for (long long i=a;i<n;i++)*

*#define per(i,a,n) for (long long i=n-1;i>=a;i--)*

*#define pb push\_back*

*#define mp make\_pair*

*#define all(x) (x).begin(),(x).end()*

*#define fi first*

*#define se second*

*#define SZ(x) ((long long)(x).size())*

**typedef** vector<long long> VI;

**typedef** long long ll;

**typedef** pair<long long,long long> PII;

**const** ll mod=1e9+7;

ll powmod(ll a,ll b) {ll res=1;a%=mod; assert(b>=0); **for**(;b;b>>=1){**if**(b&1)res=res\*a%mod;a=a\*a%mod;}**return** res;}

*// head*

long long \_,n;

**namespace** linear\_seq

{

**const** long long N=10010;

ll res[N],base[N],\_c[N],\_md[N];

vector<long long> Md;

void mul(ll \*a,ll \*b,long long k)

{

rep(i,0,k+k) \_c[i]=0;

rep(i,0,k) **if** (a[i]) rep(j,0,k)

\_c[i+j]=(\_c[i+j]+a[i]\*b[j])%mod;

**for** (long long i=k+k-1;i>=k;i--) **if** (\_c[i])

rep(j,0,SZ(Md)) \_c[i-k+Md[j]]=(\_c[i-k+Md[j]]-\_c[i]\*\_md[Md[j]])%mod;

rep(i,0,k) a[i]=\_c[i];

}

long long solve(ll n,VI a,VI b)

{ *// a 系数 b 初值 b[n+1]=a[0]\*b[n]+...*

*// printf("%d\n",SZ(b));*

ll ans=0,pnt=0;

long long k=SZ(a);

assert(SZ(a)==SZ(b));

rep(i,0,k) \_md[k-1-i]=-a[i];\_md[k]=1;

Md.clear();

rep(i,0,k) **if** (\_md[i]!=0) Md.push\_back(i);

rep(i,0,k) res[i]=base[i]=0;

res[0]=1;

**while** ((1ll<<pnt)<=n) pnt++;

**for** (long long p=pnt;p>=0;p--)

{

mul(res,res,k);

**if** ((n>>p)&1)

{

**for** (long long i=k-1;i>=0;i--) res[i+1]=res[i];res[0]=0;

rep(j,0,SZ(Md)) res[Md[j]]=(res[Md[j]]-res[k]\*\_md[Md[j]])%mod;

}

}

rep(i,0,k) ans=(ans+res[i]\*b[i])%mod;

**if** (ans<0) ans+=mod;

**return** ans;

}

VI BM(VI s)

{

VI C(1,1),B(1,1);

long long L=0,m=1,b=1;

rep(n,0,SZ(s))

{

ll d=0;

rep(i,0,L+1) d=(d+(ll)C[i]\*s[n-i])%mod;

**if** (d==0) ++m;

**else** **if** (2\*L<=n)

{

VI T=C;

ll c=mod-d\*powmod(b,mod-2)%mod;

**while** (SZ(C)<SZ(B)+m) C.pb(0);

rep(i,0,SZ(B)) C[i+m]=(C[i+m]+c\*B[i])%mod;

L=n+1-L; B=T; b=d; m=1;

}

**else**

{

ll c=mod-d\*powmod(b,mod-2)%mod;

**while** (SZ(C)<SZ(B)+m) C.pb(0);

rep(i,0,SZ(B)) C[i+m]=(C[i+m]+c\*B[i])%mod;

++m;

}

}

**return** C;

}

long long gao(VI a,ll n)

{

VI c=BM(a);

c.erase(c.begin());

rep(i,0,SZ(c)) c[i]=(mod-c[i])%mod;

**return** solve(n,c,VI(a.begin(),a.begin()+SZ(c)));

}

};

int main()

{

int t;

cin >> t;

**while**(t--)

{

int n;

cin >> n;

printf("%I64d**\n**",linear\_seq::gao(VI{2,24,96,416,1536,5504,18944,64000,212992,702464},n-1));

}

}

## FFT求多项式乘法

*#include* *<map>*

*#include* *<set>*

*#include* *<cmath>*

*#include* *<ctime>*

*#include* *<stack>*

*#include* *<queue>*

*#include* *<cstdio>*

*#include* *<cctype>*

*#include* *<bitset>*

*#include* *<string>*

*#include* *<vector>*

*#include* *<cstring>*

*#include* *<iostream>*

*#include* *<algorithm>*

*#include* *<functional>*

*#define fuck(x) cout<<"["<<x<<"]";*

*#define FIN freopen("input.txt","r",stdin);*

*#define FOUT freopen("output.txt","w+",stdout);*

*//#pragma comment(linker, "/STACK:102400000,102400000")*

**using** **namespace** std;

**typedef** long long LL;

**typedef** pair<int, int> PII;

**const** int MX = 3e5 + 5;

**const** int INF = 0x3f3f3f3f;

**const** int mod = 1e9 + 7;

**const** double pi = acos(-1.0);

**struct** cp {

double x, y;

cp() {}

cp (double x, double y): x(x), y(y) {}

**inline** cp **operator** + (**const** cp &b) {

**return** cp(x + b.x, y + b.y);

}

**inline** cp **operator** - (**const** cp &b) {

**return** cp(x - b.x, y - b.y);

}

**inline** cp **operator** \* (**const** cp &b) {

**return** cp(x \* b.x - y \* b.y, x \* b.y + y \* b.x);

}

} a[MX], b[MX];

int r[MX];

void fft(cp a[], int opt, int n) {

**for**(int i = 0; i < n; i++) {

**if**(i < r[i]) swap(a[i], a[r[i]]);

}

**for**(int i = 1; i < n; i <<= 1) {

cp wn(cos(pi / i), opt \* sin(pi / i));

**for**(int p = i << 1, j = 0; j < n; j += p) {

cp w(1, 0);

**for**(int k = 0; k < i; k++, w = wn \* w) {

cp x = a[j + k], y = w \* a[j + k + i];

a[j + k] = x + y; a[j + k + i] = x - y;

}

}

}

}

*/\*多项式a，最高次为n，多项式b，最高次为m*

*从0到n项的系数*

*卷积结果等于后来a[].x*

*复杂度O(nlogn)，最后的最高项为n+m*

*\*/*

void solve(cp a[], cp b[], int n, int m) {

int l = 0, nn, nm = n + m;

**for**(nn = 1; nn <= nm; nn <<= 1) l++;

**for**(int i = n + 1; i <= nn; i++) a[i] = cp(0, 0);

**for**(int i = m + 1; i <= nn; i++) b[i] = cp(0, 0);

n = nn; m = nm;

**for**(int i = 0; i < n; i++) {

r[i] = (r[i >> 1] >> 1) | ((i & 1) << (l - 1));

}

fft(a, 1, n); fft(b, 1, n);

**for**(int i = 0; i <= n; i++) {

a[i] = a[i] \* b[i];

}

fft(a, -1, n);

**for**(int i = 0; i <= m; i++) {

a[i].x /= n;

}

}

int main() {

int n, m; *//FIN;*

scanf("%d%d", &n, &m);

**for**(int i = 0; i <= n; i++) scanf("%lf", &a[i].x);

**for**(int i = 0; i <= m; i++) scanf("%lf", &b[i].x);

solve(a, b, n, m);

**for**(int i = 0; i <= n + m; i++) {

printf("%d%c", (int)(a[i].x + 0.5), i == n + m ? '\n' : ' ');

}

}

## 拉格朗日插值

*#include<iostream>*

*#include<string>*

*#include<vector>*

**using** **namespace** std;

double Lagrange(int N,vector<double>&X,vector<double>&Y,double x);

int main(){

char a='n';

**do**{

cout<<"请输入差值次数n的值："<<endl;

int N;

cin>>N;

vector<double>X(N,0);

vector<double>Y(N,0);

cout<<"请输入插值点对应的值及函数值(Xi,Yi)："<<endl;

**for**(int a=0;a<N;a++){

cin>>X[a]>>Y[a];

}

cout<<"请输入要求值x的值："<<endl;

double x;

cin>>x;

double result=Lagrange(N,X,Y,x);

cout<<"由拉格朗日插值法得出结果： "<<result<<endl;

cout<<"是否要继续？(y/n)：";

cin>>a;

}**while**(a=='y');

**return** 0;

}

double Lagrange(int N,vector<double>&X,vector<double>&Y,double x){

double result=0;

**for**(int i=0;i<N;i++){

double temp=Y[i];

**for**(int j=0;j<N;j++){

**if**(i!=j){

temp = temp\*(x-X[j]);

temp = temp/(X[i]-X[j]);

}

}

result += temp;

}

**return** result;

};