Simple Mathematical Model of Successive Forgetting

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Quantitative Models of Forgetting and Some Quantitative Representations

Hermann Ebbinghaus [2, 3] proposed a number of experimental equation of the rate of evanescence of concrete memories (”forgetting curves”), of which the exponential forgetting curves [4, 1] are the simplest. According to this model, if at the initial moment of time t_0 there existed an initial amount of information I_0, after a time interval $\Delta t$the remaining amount of information will be

$$I(t_0 + \Delta t) = I_0 \cdot e^{-k \cdot \Delta t}, \tag{1}$$

where k is called the **decay rate**, by analogy with the process of radioactive decay [4]. The inverse value $S = 1/k$ is called [1] the ”**relative strength of memory**”.

Superposition Principle and Effect of Information Inferred at Discrete Time Moments

Since information is treated as an additive amount in quantitative models of forgetting, principle of superposition is valid for such a system that defines that if you have a number of influences on the system, the reaction to the sum will be equal to the sum of partial reactions. In other words, if you infer finite amounts of information I_i at moments $t_1, t_2, ..., t_n$ the resulting forgetting curve can be described as

$$I(t) = \sum_{i=1}^{n} \Delta I_i \cdot e^{-\frac{t-t_i}{S}} \cdot \theta(t - t_i), \tag{2}$$

where

$$\theta(t) = \begin{cases} 1, & t \geq 0, \\ 0, & t < 0 \end{cases} \tag{3}$$

is the so called Heaviside function [1].

[1] A comprehensive proof of formula (2) is based on some very basic results of the theory of linear systems including Laplace transform and delta functions.

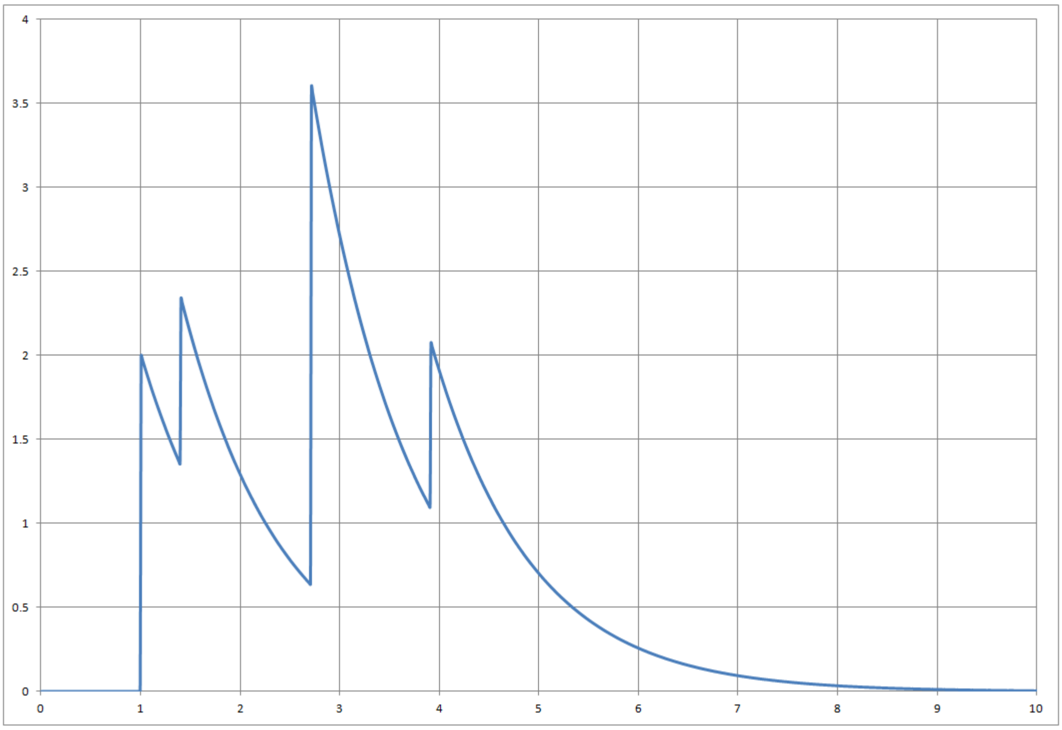

Figure 1: Resulting Imformation curve for an example with five non-equidistant information injections at $t = 1,\ 1.4,\ 2.7,\ 3.9$ with intensities $\Delta I = 2.0,\ 1.0,\ 3.0,\ 1.0$, respectively.

In Figure 1 is shown an example with four arbitrary information injections with memory strength $S = 1$. After the last injection occurs at $t = 3.9$ the amount of remaining information rapidly decays.

Special Cases

Equidistant Time Moments

If the values t_i in (2) are equidistant (which is of special interest in situations when modeling of cognition and forgetting is involved,

$$t_i = i \cdot \Delta t, \quad i = 1, ..., n, \tag{4}$$

equation (2) rewrites as

$$I(t) = \sum_{i=1}^{n} \Delta I_i \cdot e^{-\frac{t - i \cdot \Delta t}{S}} \cdot \theta(t - t_i), \tag{5}$$

Figures 2, 3 show that the average behavior of the system can be different depending on the value of memopry strength: whereas a system with a stronger memory (case 3) increases the average amount of information, a system with $S = 1$ (2) loses information on an average basis even though information is being injected at equidistant moments of time.

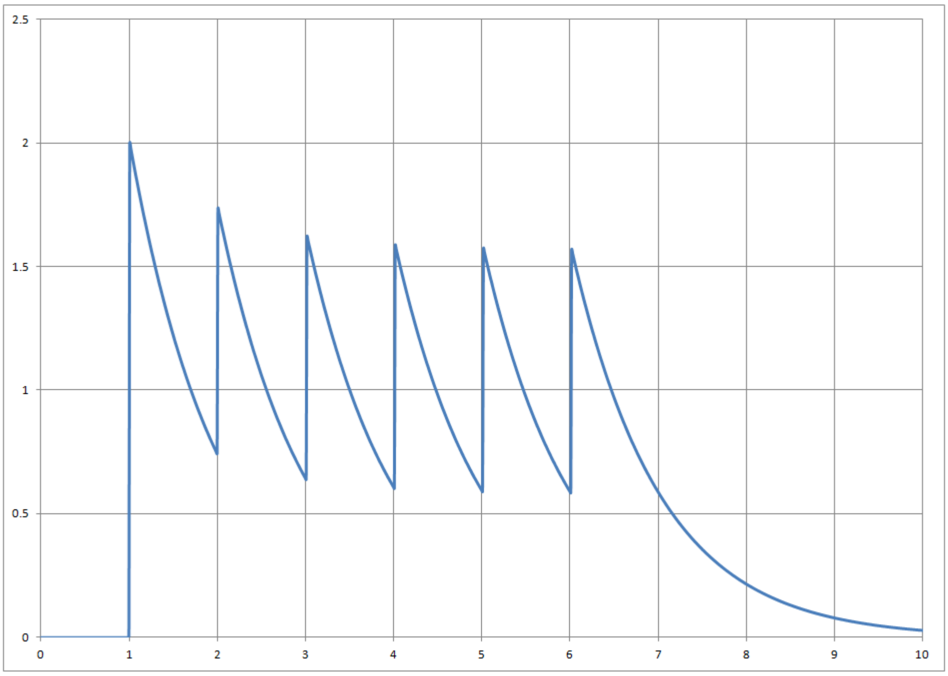

Figure 2: Equidistant information injections into a system with memory strength $S = 1$. Intensity of the first injection is $\Delta I_1 = 2$, all subsequent $\Delta I_i = 1$.

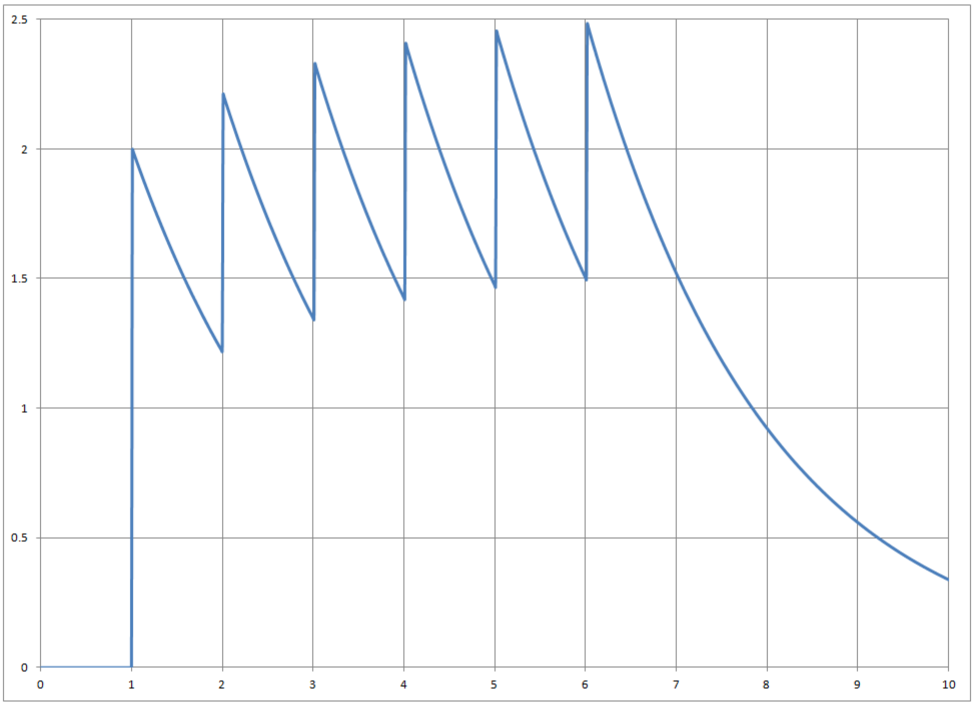

Figure 3: Equidistant information injections into a system with memory strength $S = 2$. Intensity of the first injection is $\Delta I_1 = 2$, all subsequent $\Delta I_i = 1$

Equidistant Time Moments with Equal Injection Intensities

In this case all $\Delta I_i = \Delta I = const$, and equation (5) becomes even more simple:

$$I(t) = \Delta I \cdot \sum_{i=1}^{n} e^{-\frac{t - i \cdot \Delta t}{S}} \cdot \theta(t - t_i), \tag{6}$$

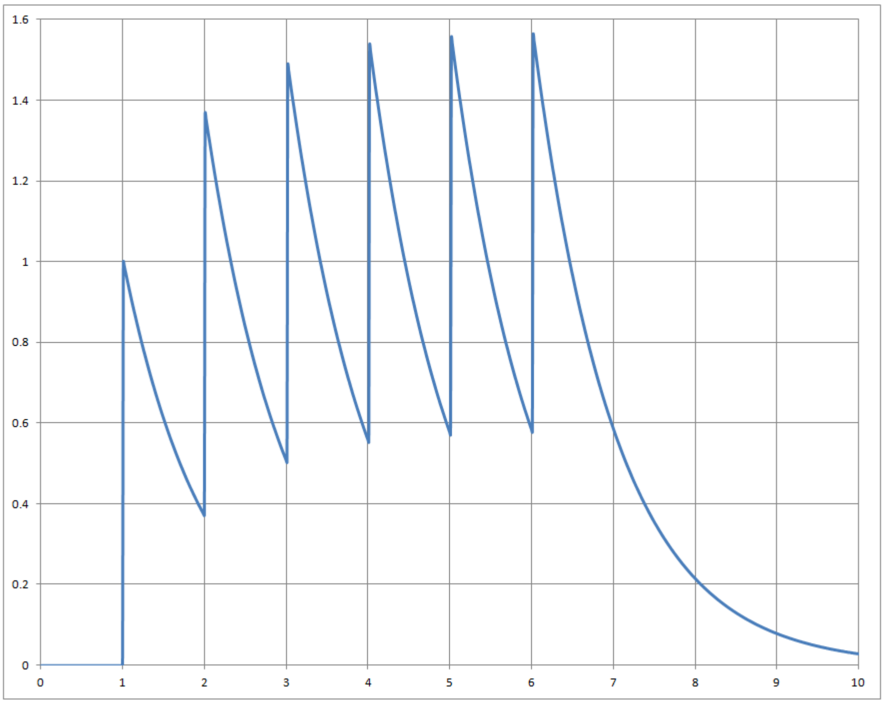

Figure 4: Equidistant information injections with equal intensities ($\Delta I_i = 1$) into a system with memory strength $S = 1$.

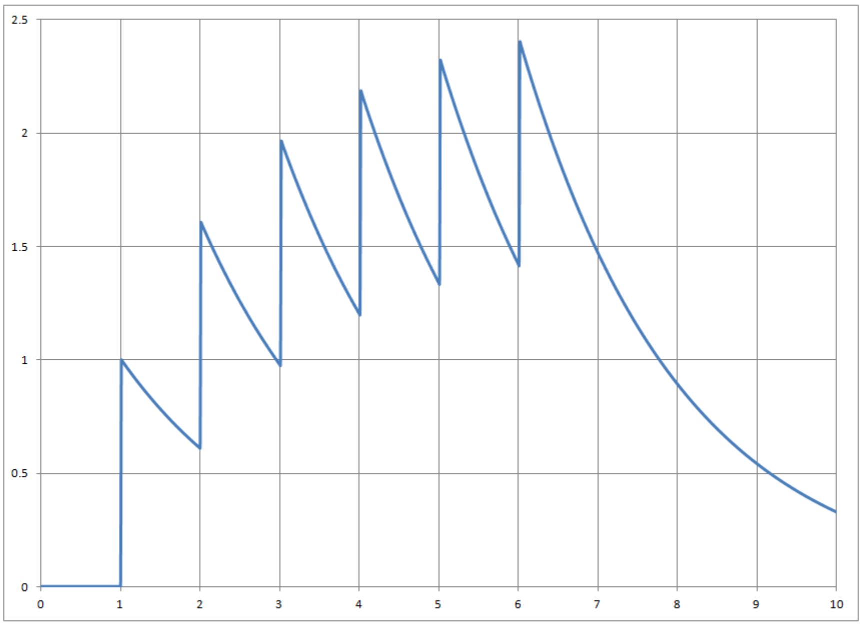

Figure 5: Equidistant information injections with equal intensities ($\Delta I_i = 1$) into a system with memory strength $S = 2$.

Figures 4, 5 show what the resulting forgetting curves look like in cases of $S = 1$ (Figure 4) and $S = 2$ (Figure 5). The figures show that, independently from the memory strength, the average amount of information grows, albeit with different rates. Measured as the average of the lower and the upper value at the injection points, it reaches some 1.8 untit with $S = 2$, but only some 1.1 with $S = 1$.

Maximum Values and Estimation of Parameters

With all injections equal, the value of an upper point can be calculated using (6): with $t = n \cdot \Delta t$ all values of the theta functions are equal to 1 and

$$I(t_n + 0) = \Delta I \cdot e^{-\frac{t}{S}} \cdot \sum_{i=1}^{n} e^{\frac{i\Delta t}{S}}.$$

The expression to sum is a sum of a geometric progression that can be calculated using known formulas so that at the end we obtain

$$I(t_n + 0) = \Delta I \cdot \frac{1 - e^{-\frac{n\Delta t}{S}}}{1 - e^{-\frac{\Delta t}{S}}}. \tag{7}$$

In formula (7) the term $e^{-\frac{n\Delta t}{S}}$ rapidly decreases with the growth of n, so that for the estimation of S the following expression can be used based on the upper values at injection points:

$$S = \frac{\Delta t}{\log \frac{I_n}{I_n - \Delta I}}, \tag{8}$$

where I_n is an upper value at an injection point distant enough from the first injection. So for instance in the example of Figure 5 the value of I_n at the last injection is ≈ 2.4. Using Equation (8 we obtain

$$S = \frac{1.0}{\log \frac{2.4}{2.4 - 1.0I}} = 1.85.$$

(In the simulation was used $S = 2$. Not so bad.)

References

[1] Forgetting curve, August 2016. URL: `https://en.wikipedia.org/w/index.php?title=Forgetting_curve&oldid=732658685`.

[2] Hermann Ebbinghaus. *Memory: A Contribution to Experimental Psychology.* University Microfilms, 1913.

[3] Lee Averell; Andrew Heathcote. The form of the forgetting curve and the fate of memories. *Journal of Mathematical Psychology*, 55, 2011.

[4] Geoffrey R Loftus. Evaluating forgetting curves. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 11(2):397, 1985.