LAB 3: Phân tích thuật toán (Tiếp theo)

1 Master theorem

1.1

Sử dụng Master Theorem để giải các phương trình đệ quy sau:

a. $T(n) = 9T(\frac{n}{3}) + n$

$$\begin{aligned}
&a = 9; \quad b = 3; \quad f(n) = n \\
&n^{\log_b a} = n^{\log_3 9} = n^3 \\
&f(n) = O(n^{\log_b a-2}) = n^1, \qquad \epsilon = 2 \\
\Rightarrow &f(n) = \Theta(n^{\log_b a}) = \Theta(n^3)
\end{aligned}$$

b. $T(n) = T(\frac{2n}{3}) + 1$

$$\begin{aligned}
&a = 1; \quad b = 3/2; \quad f(n) = 1 \\
&n^{\log_b a} = n^{\log_{3/2} 1} = 1 \\
&f(n) = O(1) \\
\Rightarrow &f(n) = \Theta(n \log n)
\end{aligned}$$

c. $T(n) = 3T(\frac{n}{4}) + n \log n$

$$\begin{aligned}
&a = 3; \quad b = 4; \quad f(n) = n \log n \\
&n^{\log_b a} = n^{\log_4 3} = n^{0.79} \\
&f(n) = n \log n = \Omega(n^{\log_4 3+\epsilon}) \\
&af(\frac{n}{b}) = \frac{3n}{4}\log(\frac{n}{4}) = \frac{3n}{4}\log n + \frac{3n}{4}\log 4 \leq c \log n, \quad \exists c = \frac{3}{4} < 1 \\
\Rightarrow &f(n) = \Theta(n \log n)
\end{aligned}$$

d. $T(n) = 2T(\frac{n}{3}) + n$

$$
\begin{aligned}
&a = 2; \quad b = 3; \quad f(n) = n \\
&n^{\log_b a} = n^{\log_3 2} = n^0.63 \\
&f(n) = n = \Omega(n^{0.63+0.37}), \qquad \epsilon = 0.37 > 0 \\
&af(\frac{n}{b}) = 2f(\frac{n}{3}) = 2\frac{n}{3} \leq cn, \qquad c = \frac{2}{3} < 1 \\
\Rightarrow &f(n) = \Theta(n)
\end{aligned}
$$

e. $T(n) = T(\frac{n}{2}) + n$

$$
\begin{aligned}
&a = 1; \quad b = 2; \quad f(n) = n \\
&n^{\log_2 1} = 1 \\
&f(n) = n = \Omega(n^{0+1}), \qquad \epsilon = 1 \\
&af(\frac{n}{b}) = \frac{n}{2} \leq cn, \qquad c = \frac{1}{2} < 1 \\
\Rightarrow &f(n) = \Theta(n)
\end{aligned}
$$

f. $3T(\frac{n}{2}) + n$

$$
\begin{aligned}
&a = 3; \quad b = 2; \quad f(n) = n \\
&n^{\log_2 3} = n^{1.58} \\
&f(n) = n = O(n^{1.58-0.58}), \qquad \epsilon = 0.58 \\
\Rightarrow &f(n) = \Theta(n^{\log_2 3})
\end{aligned}
$$

g. $2T(\frac{n}{2}) + n$

$$
\begin{aligned}
&a = 2; \quad b = 2; \quad f(n) = n \\
&n^{\log_2 2} = n \\
\Rightarrow &f(n) = \Theta(n \log n)
\end{aligned}
$$

1.2

Tìm độ phức tạp của từng phương trình và sắp xếp theo thứ tự tăng dần:

a. $T_1(n) = 4T(\frac{n}{2} + 1)$

$$\begin{aligned} &a = 4; \quad b = 2; \quad f(n) = 1 \\ &n^{\log_b a} = n^{\log_2 4} = n^2 \\ &f(n) = O(n^{2-2}), \qquad \epsilon = 2 \\ \Rightarrow &f(n) = \Theta(n^2) \end{aligned}$$

b. $T_2(n) = 4T(\frac{n}{2} + \sqrt{n})$

$$\begin{aligned} &a = 4; \quad b = 2; \quad f(n) = n^{\frac{1}{2}} \\ &n^{\log_b a} = n^{\log_2 4} = n^2 \\ &f(n) = O(n^{2-3/2}), \qquad \epsilon = \frac{3}{2} \\ \Rightarrow &f(n) = \Theta(n^2) \end{aligned}$$

c. $T_3(n) = 4T(\frac{n}{2} + n)$

$$\begin{aligned} &a = 4; \quad b = 2; \quad f(n) = n \\ &n^{\log_b a} = n^{\log_2 4} = n^2 \\ &f(n) = O(n^{2-1}), \qquad \epsilon = 1 \\ \Rightarrow &f(n) = \Theta(n^2) \end{aligned}$$

d. $T_4(n) = 4T(\frac{n}{2} + n^2)$

$$\begin{aligned} &a = 4; \quad b = 2; \quad f(n) = n^2 \\ &n^{\log_b a} = n^{\log_2 4} = n^2 \\ \Rightarrow &f(n) = \Theta(n^2 \log n) \end{aligned}$$

e. $T_5(n) = 4T(\frac{n}{2} + n^3)$

$$
\begin{aligned}
&a = 4; \quad b = 2; \quad f(n) = n^{\frac{1}{2}} \\
&n^{\log_b a} = n^3 \\
&f(n) = \Omega(n^{2+1}), \qquad \epsilon = 1 \\
&af(\frac{n}{b}) = \frac{4n^3}{8} = \frac{n^3}{2} \leq cn^3, \qquad c = \frac{1}{2} \\
\Rightarrow &f(n) = \Theta(n^3)
\end{aligned}
$$

$\Rightarrow T_1 = T_2 = T_3 < T_4 < T_5$