Homework1

NYU Computer Science Bridge to Tandon Course Winter 2022

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Question 1:

A. Convert the following numbers to their decimal representation. Show your work.

1. $10011011_2 = 155_{10}$

$10011011_2 = 1*2^0 + 1*2^1 + 0*2^2 + 1*2^3 + 1*2^4 + 0*2^5 + 0*2^6 + 1*2^7$

$= 1 + 2 + 0 + 8 + 16 + 0 + 0 + 128$

$= 155_{10}$

2. $456_7 = 237_{10}$

$456_7 = 4*7^2 + 5*7^1 + 6*7^0 = 237_{10}$

3. $38A_{16} = 906_{10}$

$38A_{16} = 3*16^2 + 8*16^1 + 10*16^0 = 906_{10}$

4. $2214_5 = 309_{10}$

$2214_5 = 2*5^3 + 2*5^2 + 1*5^1 + 4*5^0 = 309_{10}$

B. Convert the following numbers to their binary representation:

1. $69_{10} = 1000101_2$

Divide 69 by 2, round the result into its floor value, and record the reminders.

Result	69	34	17	8	4	2	1	0
Reminder	-	1	0	1	0	0	0	1

2. $485_{10} = 111100101_2$

Divide 69 by 2, round the result into its floor value, and record the reminders.

Result	485	242	121	60	30	15	7	3	1	0
Reminder	-	1	0	1	0	0	1	1	1	1

3. $6D1A_{16}= 0110110100011010_2$

Using hexadecimal index conversion to four binary digit.

Hex Symbol	**Binary Value**
6	0110
D	1101
1	0001
A	1010

C. Convert the following numbers to their hexadecimal representation:

1. $1101011_2 = 6B_{16}$

$1101011_2 = 1*2^0+ 1*2^1+ 0*2^2+ 1*2^3+ 0*2^4+ 1*2^5+ 1*2^6$

$= 2^0*(1*2^0+ 1*2^1+ 0*2^2+ 1*2^3) +2^4*(0*2^0+ 1*2^1+ 1*2^2)$

$= 16^0*(1*2^0+ 1*2^1+ 0*2^2+ 1*2^3) +16^1*(0*2^0+ 1*2^1+ 1*2^2)$

$= 16^0*B+16^1*6$

$= 6B_{16}$

2. $895_{10} = 37F_{16}$

Divide 895 by 16, round the result into its floor value, and record the reminders.

Result	895	55	3	0
Reminder	-	F	7	3

Question 2:

Solve the following, do all calculation in the given base. Show your work.

1. $7566_8 + 4515_8 = 14303_8$

$$
\begin{array}{rr}
 & 111 \\
 & 7566_8 \\
+ & 4515_8 \\
\hline
 & 1\,4303
\end{array}
$$

2. $10110011_2 + 1101_2 = 11000000_2$

$$
\begin{array}{rl}
 & 1\,1\,1\quad 1\quad 1 \\
 & 1\,0\,1\,1\,0\quad 0\quad 1\quad 1_2 \\
+ & 1\quad 1\quad 0\quad 1_2 \\
\hline
 & 1\,1\,0\,0\,0\quad 0\quad 0\quad 0
\end{array}
$$

3. $7A66_{16} + 45C5_{16} = C02B_{16}$

$$
\begin{array}{rr}
 & 1\,1 \\
 & 7A66_{16} \\
+ & 45C5_{16} \\
\hline
 & C0\,2B
\end{array}
$$

4. $3022_5 - 2433_5 = 34_5$

$$
\begin{array}{rr}
 & -1-1-1 \\
 & 3\,0\,2\,2_5 \\
- & 2\,4\,3\,3_5 \\
\hline
 & 0\,0\,3\,4
\end{array}
$$

Question 3:

A. Convert the following numbers to their 8-bits two's complement representation. Show your work.

1. $124_{10} = 01111100_2$

   Since 124_{10} is a positive integer, we can directly convert it into binary digits.

Result	124	62	31	15	7	3	1	0
Reminder	-	0	0	1	1	1	1	1

2. $-124_{10} = 10000100_2$

   Using the result from the previous question where 12410 = 011111002 to do a summation.

		1	1	1	1	1	1			
Positive	-	0	1	1	1	1	1	0	0	
Negative	-	1	0	0	0	0	1	0	0	+
Result	1	0	0	0	0	0	0	0	0	

$-124_{10} = 10000100_2$

3. $109_{10} = 01101101_2$

   Since 109_{10} is a positive integer, we can directly translate it into binary digits.

Result	109	54	27	13	6	3	1	0
Reminder	-	1	0	1	1	0	1	1

4. $-79_{10} = 10110001_2$

   The absolute value of -79_{10} is 79, convert 79 into binary digits.

Result	79	39	19	9	4	2	1	0
Reminder	-	1	1	1	1	0	0	1

$79_{10} = 01001111_2$

Then calculate the additive inverse of 01001111.

		1	1	1	1	1	1	1	1	
Positive	-	0	1	0	0	1	1	1	1	
Negative	-	1	0	1	1	0	0	0	1	+
Result	1	0	0	0	0	0	0	0	0	

$-79_{10}=01001111_2$

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

1. $00011110_{8\text{ bit 2's comp}} = 30$

   $00011110_{8\text{ bit 2's comp}} = 0 * 2^0 + 1 * 2^1 + 1 * 2^2 + 1 * 2^3 + 1 * 2^4 + 0 * 2^5 + 0 * 2^6 + 0 * 2^7$

   $= 0+2+4+8+16+0+0+0$

   $= 30$

2. $11100110_{8\text{ bit 2's comp}} = -26_{10}$

$11100110_{8\text{ bit 2's comp}}$ is a negative integer, calculate its additive inverse.

		1	1	1	1	1	1	1		
Positive	-	0	0	0	1	1	0	1	0	
Negative	-	1	1	1	0	0	1	1	0	+
Result	1	0	0	0	0	0	0	0	0	

The decimal representation of the additive inverse is the absolute value of $11100110_{8\text{ bit 2's comp}}$

$1\,1\,0\,1\,0_2 = 0 * 2^0 + 1 * 2^1 + 1 * 2^2 + 1 * 2^3 + 1 * 2^4$

$= 0+2+4+8+16$

26

$11100110_{8\text{ bit 2's comp}}$ is a negative integer, so it's -26 in decimal.

3. $00101101_{8\text{ bit 2's comp}} = 45_{10}$

   $00101101_{8\text{ bit 2's comp}} = 1 * 2^0 + 0 * 2^1 + 1 * 2^2 + 1 * 2^3 + 0 * 2^4 + 1 * 2^5 + 0 * 2^6 + 0 * 2^7$

   $= 1 + 0 + 4 + 8 + 0 + 32 + 0 + 0$

   $= 45$

4. $10011110_{8\text{ bit 2's comp}} = -98_{10}$

$11100110_{8\text{ bit 2's comp}}$ is a negative integer, calculate its additive inverse.

	1	1	1	1	1	1	1			
Positive	-	0	1	1	0	0	0	1	0	
Negative	-	1	0	0	1	1	1	1	0	+
Result	1	0	0	0	0	0	0	0	0	

The decimal representation of the additive inverse is the absolute value of $10011110_{8\text{ bit 2's comp}}$

$1100010_2 = 0 * 2^0 + 1 * 2^1 + 0 * 2^2 + 0 * 2^3 + 0 * 2^4 + 1 * 2^5 + 1 * 2^6$

$= 0+2+0+0+0+32+64$

$= 98$

$11100110_{8\text{ bit 2's comp}}$ is a negative integer, so it's -26 in decimal.

Question 4:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.4, sections b, c

   Write a truth table for each expression.

   (b) $\neg(p \lor q)$

p	q	$p \lor q$	$\neg(p \lor q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

   (c) $r \lor (p \land \neg q)$

r	p	q	$\neg q$	$p \land \neg q$	$r \lor (p \land \neg q)$
T	T	T	F	F	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	T	F	F

2. Exercise 1.3.4, sections b, d

   Give a truth table for each expression.

   (b) $(p \rightarrow q) \rightarrow (q \rightarrow p)$

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \rightarrow (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(d) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

Question 5:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.7, sections b, c

(b) The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.

Answer: $\neg(B \lor D \lor M \lor (\neg B \land \neg D \land \neg M))$

The negation of this compound propositions is present only one of the certificates or present none of the certificates. Present only one of the certificates can be expressed in $B \lor D \lor M$, and present none of the certificates is the negation of present all of the certificates which can be expressed in $(\neg B \land \neg D \land \neg M)$, so the condition can be expressed using logical operations like this $\neg(B \lor D \lor M \lor (\neg B \land \neg D \land \neg M))$.

(c) Applicant must present either a birth certificate or both a driver's license and a marriage license.

Answer: $B \oplus (D \land M)$

The point for this condition is that applicant can present (a birth certificate) or(a driver's license and a marriage license), but not the three. So the condition of present three certificates should be excluded. The condition can be expressed using logical operations like this $B \oplus (D \land M)$.

2. Exercise 1.3.7, sections b – e

(b) A person can park in the school parking lot if they are a senior or at least seventeen years of age.

Answer: $s \lor y \rightarrow p$

$p \rightarrow q$ can be expressed in English as q if p.

(c) Being 17 years of age is a necessary condition for being able to park in the school parking lot.

Answer: $p \rightarrow y$

$p \rightarrow q$ can be expressed in English as q is necessary for p.

(d) A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

Answer: $p \leftrightarrow s \land y$

The proposition "p if and only if q" is expressed with the biconditional operation and is denoted $p \leftrightarrow q$.

(e) Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

Answer: $p \rightarrow s \oplus y$

$p \rightarrow q$ can be expressed in English as p implies q. A person is at least 17 years of age and a person is at least 17 years of age cannot be true at the same time, so $\oplus$ should be correct logical opretion.

3. Exercise 1.3.9, sections c, d

(c) The applicant can enroll in the course only if the applicant has parental permission.

Answer: $c \rightarrow p$

$p \rightarrow q$ can be expressed in English as p only if q. It means that the only way for p to be true is if q is also true.

(d) Having parental permission is a necessary condition for enrolling in the course.

Answer: $c \rightarrow p$

$p \rightarrow q$ can be expressed in English as q is necessary for p.

Question 6:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.3.6, sections b – d

(b) Maintaining a B average is necessary for Joe to be eligible for the honors program.

Answer: If Joe want to be eligible for the honors program, then he need to maintain a B average.

"q is necessary for p" equals "if p, then q".

(c) Rajiv can go on the roller coaster only if he is at least four feet tall.

Answer: If Rajiv can to go on the roller coaster, then he must to be at least four feet tall.

"p only if q" equals "if p, then q".

(d) Rajiv can go on the roller coaster if he is at least four feet tall.

Answer: If Rajiv is at least four feet tall, then he can go on the roller coaster.

"q if p" equals "if p, then q".

2. Exercise 1.3.10, sections c – f

(c) $(p \lor r) \leftrightarrow (q \land r)$

Answer: false

p is true, so $p \lor r$ is true. q is false, so $q \land r$ is false. So $T \leftrightarrow F$ is false.

(d) $(p \land r) \leftrightarrow (q \land r)$

Answer: unknown

p is true and r is unknown, so $p \land r$ is unknown. q is false and r is unknown, so $q \land r$ is unknown. So the answer is unknown.

(e) $p \rightarrow (r \lor q)$

Answer: unknown

Hypothesis of p is true, q is false and r is unknown, so conclusion of $r \lor q$ is unknown. So the value of this conditional statement is unknown.

(f) $(p \land q) \rightarrow r$

Answer: true

p is true and q is false, so $p \land q$ is false. The hypothesis is false, and regardless of the value of conclusion, r, is unknown. The value of this conditional statement is true.

Question 7:

Solve Exercise 1.4.5, sections b – d, from the Discrete Math zyBook:

(b) **Answer:** $\neg j \rightarrow l \vee \neg r$ and $r \wedge \neg l \rightarrow j$, logically equivalent

The two sentences can be expressed in logical expressions as $\neg j \rightarrow l \vee \neg r$ and $r \wedge \neg l \rightarrow j$.

A truth table can be used to prove whether the two expressions are logically equivalent as below.

$\neg j \rightarrow l \vee \neg r$ and $r \wedge \neg l \rightarrow j$ have the same truth value for every row in the truth table, so they are logically equivalent.

j	l	r	$\neg j \rightarrow l \vee \neg r$	$r \wedge \neg l \rightarrow j$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

(c)**Answer:** $j \rightarrow \neg l$ and $\neg j \rightarrow l$, not logically equivalent

The two sentences can be expressed in logical expressions as $j \rightarrow \neg l$ and $\neg j \rightarrow l$.

A truth table can be used to prove whether the two expressions are logically equivalent as below.

Not all the truth value of $j \rightarrow \neg l$ and $\neg j \rightarrow l$ are the same for every row in the truth table, so they are not logically equivalent.

j	l	$j \rightarrow \neg l$	$\neg j \rightarrow l$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	F

(d)**Answer:** $r \vee \neg l \rightarrow j$ and $j \rightarrow r \wedge \neg l$, not logically equivalent

The two sentences can be expressed in logical expressions as $r \vee \neg l \rightarrow j$ and $j \rightarrow r \wedge \neg l$.

A truth table can be used to prove whether the two expressions are logically equivalent as below.

Not all the truth value of $r \vee \neg l \rightarrow j$ and $j \rightarrow r \wedge \neg l$ are the same for every row in the truth table, so they are not logically equivalent.

j	l	r	$r \vee \neg l \rightarrow j$	$j \rightarrow r \wedge \neg l$
T	T	T	T	F
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	T

Question 8:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.5.2, sections c, f, i

Use the laws of propositional logic to prove the following:

(c) $(p \to q) \land (p \to r) \equiv p \to (q \land r)$

Answer:

$(p \to q) \land (p \to r)$

$(\neg p \lor q) \land (\neg p \lor r)$ Conditional identity

$\neg p \lor (q \land r)$ Distributive law

$p \to (q \land r)$ Conditional identity

(f) $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

Answer:

$\neg(p \lor (\neg p \land q))$

$\neg((p \lor \neg p) \land (p \lor q))$ Distributive law

$\neg(T \land (p \lor q))$ Complement law

$\neg((p \lor q) \land T)$ Commutative law

$\neg(p \lor q)$ Identity law

$\neg p \land \neg q$ De Morgan's law

(i) $(p \land q) \to r \equiv (p \land \neg r) \to \neg q$

Answer:

$(p \land q) \to r$

$\neg(p \land q) \lor r$ Conditional identity

$(\neg p \lor \neg q) \lor r$ De Morgan's law

$(\neg p \lor r) \lor \neg q$ Associative law

$\neg(p \land \neg r) \lor \neg q$ De Morgan's law

$(p \land \neg r) \to \neg q$ Conditional identity

2. Exercise 1.5.3, sections c, d

Use the laws of propositional logic to prove that each statement is a tautology.

(c) $\neg r \lor (\neg r \rightarrow p)$

Answer:

$\neg r \lor (\neg r \rightarrow p)$	
$\neg r \lor (\neg\neg r \lor p)$	Conditional identity
$\neg r \lor (r \lor p)$	Double negation law
$(\neg r \lor r) \lor p$	Associative law
$T \lor p$	Complement law
T	Domination law

(d) $\neg(p \rightarrow q) \rightarrow \neg q$

Answer:

$\neg(p \rightarrow q) \rightarrow \neg q$	
$\neg\neg(p \rightarrow q) \lor \neg q$	Conditional identity
$(p \rightarrow q) \lor \neg q$	Double negation law
$(\neg p \lor q) \lor \neg q$	Conditional identity
$\neg p \lor (q \lor \neg q)$	Associative law
$\neg p \lor T$	Complement law
T	Domination law

Question 9:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.6.3, sections c, d

(c) There is a number that is equal to its square.

Answer: $\exists x(x = x^2)$

There exists an x, such that P(x): $x = x^2$, so it's a existentially quantified statement.

(d) Every number is less than or equal to its square.

Answer: $\forall x(x \leqq x^2)$

For all x that P(x): $x \leqq x^2$, so it's a universally quantified statement.

2. Exercise 1.7.4, sections b – d

(b) Everyone was well and went to work yesterday.

Answer: $\forall x(\neg S(x) \land W(x))$

For every variable x need to meet the condition of not sick($\neg S(x)$) and went to work ($W(x)$) at the same time.

(c) Everyone who was sick yesterday did not go to work.

Answer: $\forall x(S(x) \to \neg W(x))$

For every employee (variable x), if he was sick yesterday($S(x)$) then he did not go to work $\neg(W(x))$.

(d) Yesterday someone was sick and went to work.

Answer: $\exists x(S(x) \land W(x))$

There is someone that meet the condition of was sick ($S(x)$) and went to work ($W(x)$) at the same time.

Question 10:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.7.9, sections c – i

(c) $\exists x((x = c) \rightarrow P(x))$

Answer: false

P(c)=F, so the proposition that if x=c then P(x) is true is false.

(d) $\exists x(Q(x) \land R(x))$

Answer: true

The truth value for Q(e) and R(e) are T.

(e) $Q(a) \land P(d)$

Answer: true

The truth value for Q(a) and P(d) are T.

(f) $\forall x ((x \neq b) \rightarrow Q(x))$

Answer: true

For all the x in the domain expect b, the truth value of Q(x) are T.

(g) $\forall x (P(x) \lor R(x))$

Answer: false

When x=c, the truth value for P(c) and R(c) are F.

(h) $\forall x (R(x) \rightarrow P(x))$

Answer: true

For all the x in the set, the truth value of $R(x) \rightarrow P(x)$ is true.

(i) $\exists x(Q(x) \lor R(x))$

Answer: true

When x=a, the truth value of $Q(a) \lor R(a)$ is true.

2. Exercise 1.9.2, sections b – i

Indicate whether each of the quantified statements is true or false.

(b) $\exists x \ \ \forall y\, Q(x, y)$

Answer: true

When x=2, the truth value of Q(2,1) Q(2,2) Q(2,3) are T.

(c) $\exists y \ \ \forall x\, P(x, y)$

Answer: true

When y=1, the truth value of P(1,1) P(2,1) P(3,1) are T.

(d) $\exists x \ \ \exists y\, S(x, y)$

Answer: false

For all the x and y in the domain, the truth value of S(x, y) is F.

(e) $\forall x \ \ \exists y\, Q(x, y)$

Answer: false

When x=1, there is no y to make the truth value of Q(1, y) to be T.

(f) $\forall x \ \ \exists y\, P(x, y)$

Answer: true

For all the x in the domain, there is a y to make the truth value of P(x, y) to be T.

(g) $\forall x \ \ \forall y\, P(x, y)$

Answer: false

The truth value of P(1,2) is F, which is counterexample.

(h) $\exists x \ \ \exists y\, Q(x, y)$

Answer: true

The truth value of Q(2,1) is T.

(i) $\forall x \ \ \forall y\, \neg S(x, y)$

Answer: true

For all the x and y in the domain, the truth value of S(x, y) is F.

Question 11:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.10.4, sections c – g

(c)There are two numbers whose sum is equal to their product.

Answer:

S(x,y): The sum of x and y.

P(x,y): The product of x and y.

The statement can be expressed using logical expressions as $\exists x \exists y(S(x,y) \equiv P(x,y)$.

(d)The ratio of every two positive numbers is also positive.

Answer:

R(x,y): The ratio of x and y.

The statement can be expressed using logical expressions as $\forall x \forall y((x>0) \wedge (y>0) \rightarrow R(x,y)>0)$.

(e)The reciprocal of every positive number less than one is greater than one.

Answer:

R(x): The reciprocal of x.

The statement can be expressed using logical expressions as $\forall x((0<x<1) \rightarrow R(x)>1)$.

(f)There is no smallest number.

Answer:

The statement can be understood as there is not an x, which is smaller than all the other numbers. It is the negation of there is an x, which is smaller than all the other numbers.

M(x,y): $x<y$

$\neg \exists x \forall y(M(x,y))$

Applied with De Morgan's law, the statement can be expressed using logical expressions as

$\forall x \exists y \neg M(x,y)$.

(g)Every number other than 0 has a multiplicative inverse.

Answer:

M(x): The multiplicative inverse of x.

The statement can be expressed using logical expressions as $\forall x((x \neq 0) \rightarrow M(x))$

2. Exercise 1.10.7, sections c – f

(c)There is at least one new employee who missed the deadline.

Answer: $\exists x\ (N(x)\ \wedge D(x))$

The negation of the statement is that there is no new employee who missed the deadline, which is all the new employee didn't miss the deadline, which can be expressed using logical expression like $\forall x(N(x)\rightarrow\neg D(x))$. So the statement can be expressed in logical expression as $\neg\forall x(N(x)\rightarrow\neg D(x))$, which is $\exists x\ (N(x)\ \wedge D(x))$ simplified.

(d)Sam knows the phone number of everyone who missed the deadline.

Answer: $\exists x\forall y((x=Sam)\wedge D(y)\wedge P(x, y))$

The x, Sam, knows all the y's phone number who missed the deadline.

$\exists x\forall y(x=Sam\wedge D(y)\wedge P(x, y))$

(e)There is a new employee who knows everyone's phone number.

Answer: $\exists x\forall y(N(x)\wedge P(x, y))$

There is a new employee, x, who knows every y's phone number.

$\exists x\forall y(N(x)\wedge P(x, y))$

(f)Exactly one new employee missed the deadline.

Answer: $\exists x\forall y\ (N(x)\wedge D(x)\wedge N(y)\wedge((y\neq x)\rightarrow\neg D(y)))$

A new employee x missed the deadline, and all the other new employee did not miss the deadline, which can be expressed using logical expression like

$\exists x(N(x)\wedge D(x)\wedge\forall y(N(y)\wedge(y\neq x)\rightarrow\neg D(y)))$, and which is

$\exists x\forall y\ (N(x)\wedge D(x)\wedge N(y)\wedge((y\neq x)\rightarrow\neg D(y)))$ simplified.

3. Exercise 1.10.10, sections c – f

(c)Every student has taken at least one class other than Math 101.

Answer:

For every student x, there will be a Math y which was token by x and y is not Math 101.

$\forall x\exists y(T(x, y)\ \wedge(y\neq Math\ 101))$

(d)There is a student who has taken every math class other than Math 101.

Answer:

There is a student x, every Math class y was token by x and except Math 101.

$\exists x \forall y(T(x, y) \wedge (y \neq \text{Math } 101))$

(e)Everyone other than Sam has taken at least two different math classes.

Answer:

If a student is not Sam then the student has taken at least two math class y_1 and y_2, and y_1 and y_2 are different.

$\forall x \exists y_1 \exists y_2((x \neq \text{Sam}) \rightarrow (T(x, y_1) \wedge T(x, y_2) \wedge (y_1 \neq y_2)))$

(f)Sam has taken exactly two math classes.

Answer:

Sam has taken two math classes y_1 and y_2, and if there is a math class y_3 which is different from y_1 and y_2, then Sam did not take it.

$\exists x \exists y_1 \exists y_2 \forall y_3((x = \text{Sam}) \rightarrow T(x, y_1) \wedge T(x, y_2) \wedge ((y_3 \neq y_1 \wedge y_3 \neq y_2) \rightarrow \neg T(x, y_3)))$

Question 12:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.8.2, sections b – e

(b)Every patient was given the medication or the placebo or both.

$\forall x(D(x) \vee P(x))$

Negation: $\neg\forall x(D(x) \vee P(x))$

Applying De Morgan's law: $\exists x(\neg D(x) \wedge \neg P(x))$

English: Some patient was not given the medication and the placebo.

(c)There is a patient who took the medication and had migraines.

$\exists x(D(x) \wedge M(x))$

Negation: $\neg\exists x(D(x) \wedge M(x))$

Applying De Morgan's law: $\forall x(\neg D(x) \vee \neg M(x))$

English: Every patient did either not take the medication or have no migraines (or both).

(d)Every patient who took the placebo had migraines. (Hint: you will need to apply the conditional identity, $p \rightarrow q \equiv \neg p \vee q$.)

$\forall x(P(x) \rightarrow M(x))$

Negation: $\neg\forall x(P(x) \rightarrow M(x))$

Applying De Morgan's law: $\exists x(P(x) \wedge \neg M(x))$

English: Some patients who took the placebo had no migraines.

(e)There is a patient who had migraines and was given the placebo.

$\exists x(M(x) \wedge P(x))$

Negation: $\neg\exists x(M(x) \wedge P(x))$

Applying De Morgan's law: $\forall x(\neg M(x) \vee \neg P(x))$

English: Every patient either did not have migraines or was not given the placebo (or both).

2. Exercise 1.9.4, sections c – e

(c) $\exists x \ \forall y\ (P(x, y) \rightarrow Q(x, y))$

Answer: $\forall x \exists y(P(x, y) \land \neg Q(x, y))$

$\neg \exists x \ \forall y\ (P(x, y) \rightarrow Q(x, y))$ Negation

$\forall x \exists y \neg(P(x, y) \rightarrow Q(x, y))$ De Morgan's law

$\forall x \exists y \neg(\neg P(x, y) \lor Q(x, y))$ Conditional identity

$\forall x \exists y(\neg\neg P(x, y) \land \neg Q(x, y))$ De Morgan's law

$\forall x \exists y(P(x, y) \land \neg Q(x, y))$ Double negation law

(d) $\exists x \ \forall y\ (P(x, y) \leftrightarrow P(y, x))$

Answer: $\forall x \exists y((P(x, y) \land \neg P(y, x)) \lor (P(y, x) \land P\neg(x, y)))$

$\neg \exists x \ \forall y\ (P(x, y) \leftrightarrow P(y, x))$ Negation

$\forall x \exists y \neg(P(x, y) \leftrightarrow P(y, x))$ De Morgan's law

$\forall x \exists y \neg(P(x, y) \rightarrow P(y, x) \land P(y, x) \rightarrow P(x, y))$ Conditional identity

$\forall x \exists y \neg((\neg P(x, y) \lor P(y, x)) \land (\neg P(y, x) \lor P(x, y)))$ Conditional identity

$\forall x \exists y(\neg(\neg P(x, y) \lor P(y, x)) \lor \neg(\neg P(y, x) \lor P(x, y)))$ De Morgan's law

$\forall x \exists y((\neg\neg P(x, y) \land \neg P(y, x)) \lor (\neg\neg P(y, x) \land P\neg(x, y)))$ De Morgan's law

$\forall x \exists y((P(x, y) \land \neg P(y, x)) \lor (P(y, x) \land P\neg(x, y)))$ Double negation law

(e) $\exists x \ \exists y\ P(x, y) \land \ \forall x \ \forall y\ Q(x, y)$

Answer: $\exists x \ \exists y \neg P(x, y) \lor \forall x \ \forall y \neg Q(x, y)$

$\neg(\exists x \ \exists y\ P(x, y) \land \ \forall x \ \forall y\ Q(x, y))$ Negation

$\neg \exists x \ \exists y\ P(x, y) \lor \neg \forall x \ \forall y\ Q(x, y)$ De Morgan's law

$\forall x \ \forall y \neg P(x, y) \lor \exists x \ \exists y \neg Q(x, y)$ De Morgan's law